THE TEACHING OF SECOND LEVEL CALCULUS AT SOUTH AFRICAN TECHNIKONS - A DIDACTICAL ANALYSIS OF SPECIFIC LEARNING PROBLEMS

by

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SUMMARY

This study was prompted by serious problems regarding specific teaching and learning problems in calculus at the technikon. The general aims were to identify and analyze particular teaching and learning problems relating to 2nd level engineering courses in calculus and to recommend improvements which could increase student performance in engineering calculus courses. An extensive study revealed world wide concern in calculus reform.

The empirical research instruments consisted of structured questionnaires given to staff and students from nine technikons plus interviews. Five serious problem areas were identified: student ability in mathematics, content difficulty, background difficulties, timetable pressures and lecturer's presentation.

The impact of training technology on calculus was investigated. Recommendations were that routine exercises can be done on computer with extra tutorial time for computer laboratory projects. Background recommendations suggested that schools give more time to trigonometry and coordinate geometry and that bridging courses at technikons for weaker students be developed.

KEY TERMS

Trigonometrical identities; function; tutorials; word problems; teaching problems; student problems; learning problems; time factor; student questionnaire; staff questionnaire; computer questionnaire; Didactical analysis; second level calculus.

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CHAPTER 1

INTRODUCTION TO THE PROBLEM, AIMS AND RESEARCH METHODS

1.1 IDENTIFICATION OF THE PROBLEM

There is a perception among students and staff at tertiary institutions that a significant number of students have problems in learning mathematics. In particular as most tertiary mathematics courses contain calculus as a basic component, the ability to cope with calculus is a major factor in determining the success or failure of students in tertiary mathematics courses. As a lecturer and teacher in high school, university and technikons for the past forty years the author has become aware of the particular difficulties students have with learning calculus. The problem is worldwide as Foley (1988:55) states that

> "Calculus education faces both long standing problems and new challenges."

Later Foley (1988:56) states that research results confirm that students do not learn and retain calculus in the manner we would wish. Seldon (1989:4) asks the question

> "Do average calculus students really have as strong an aversion to calculus as the response rate to our study suggests?"

Woods (1989:28) spoke of poorly prepared and lethargic students. The problem is to pinpoint from a didactic standpoint those items which affect the learning of calculus.

1.2 MOTIVATION

1.2.1 General

This study on the problems that technikon students have with calculus courses was prompted by the results of a pilot study conducted during 1989 with students and staff members at nine South African technikons. Over a period of eighteen months with the aid of a Student Questionnaire (See Annexure A) and a Staff Questionnaire (See Annexure B) the pilot study tried to find out the problem areas, for example, in Algebra and Trigonometry, likes and dislikes in notation, understanding of terms used in the calculus, study habits, methods of teaching and any areas in which students encounter problems. The results of these questionnaires revealed, for example, that students and staff have similar conceptions of difficult topics. Their top three difficulties were Trigonometric identities, word problems and reading and analysing problems. Students and staff also agreed that lectures were the best option with 55% and 60% respectively voting in favour. However, half the students had problems with limits while only one fifth of the teachers had problems with teaching limits. There was also divergence over tutorial time. Only 24% of lecturers felt tutorial time was adequate while nearly 80% of students felt tutorial time was adequate. Students who had difficulties listed the period five to six weeks after the course started as the crucial time when they failed to cope with the course. These brief comments are now amplified in more detail.

1.2.2 Student Problems

In the Student Questionnaire (See Annexure A) students were asked in Question 7 to rank ten items in order of difficulty with the most difficult as 1 through to the least difficult as 10.

The results of this ranking, when analysed, gave the following order of difficulty.

1. Word Problems
2. Trigonometric Identities
3. Reading and Analysing a problem
4. Limits of Expressions
5. Binomial Expansion of Brackets
6. Co-ordinate Geometry
7. Simplifying algebraic expressions
8. Changing symbols for other symbols
9. Algebraic Equations
10. Substituting numbers for symbols

This order reflects much of the author's experience with student problems. For example, on all the examination papers set for second level technikon students the worst results have occurred with word problems. Either students omitted the question or they misinterpreted the information with fatal results or in a few cases they did the question correctly. Sometimes only a handful of students would get full marks out of a total of two hundred students. The inability to formulate mathematically from words is all too often a common difficulty with students. Hall, Kibler, Wenger and Truxaw (1989:223) quote how eighty five junior and senior computer science undergraduates had difficulty or failed to solve a simple word problem such as "Mary can do a job in 5 hours and Jane can do it in 4 hours. If they work together, how long will it take to do the job?"

Some of the other difficulties need investigation. Trigonometry which appears second on the above list still suffers from being the numerical cousin of geometry. High school teachers spend more time on geometry than trigonometry which lowers the value of

trigonometry. Certainly from the technikon students' point of view trigonometry should hold a greater priority than geometry. Allendoerfer, (1963:56) pointed out that in the first half of this century calculus was a course for college juniors, or sophomores who were able to complete college algebra, trigonometry and analytical geometry in their freshman year. This prerequisite seems to have been watered down with topics being eliminated. Again Chang and Razicka (1985:482) point out there is little knowledge or growth among United States precalculus 2nd graders on the topics of co-ordinate geometry functions and graphing prerequisites for success in calculus. Whilst these may be examples from the United States they confirm the sort of difficulties this study has sought to investigate. The limits of an expression is an understandable difficulty. Tall (1985:49) works at this in depth and postulates that obvious results for limits are not so obvious to students. It is said for example 0,9 0,99 0,999 ---> 1, that is, the limit is 1.
The student however will say the limit is 1 but 0,9 0,99 0,999 ---> $0,\dot{9}$.

The usual limit of a chord being a tangent was challenged by one student who noted the chord lay on both sides of the tangent: "The tangent can't be a limit because you can't get past a limit and part of the chord is already past it."

The explanation of a limit requires words used by the lecturers but these are not necessarily the words the student would use.

1.2.3 Staff Problems

In the previous section student problems were discussed with regard to word problems, trigonometric identities, reading and analysing and limits which logically leads on to staff problems with regard to reading and analysing problems, proof, tutorials, shortage of time and tests. There is no clear division between these problems since there are many joint issues which affect both staff and students. However the perceptions of staff and students can be vastly different to the same problem. For example in the pilot study staff were also asked in Question 7 of their questionnaire (Annexure B) to order the difficulty of topics.

Staff	Students
1. Reading and Analysing	1. Word problems
2. Word problems	2. Trigonometric Identities
3. Trigonometric Identities	3. Reading and Analysing
4. Simplifying algebraic expressions	4. Limits
5. Limits	5. Binomial Expansion
6. Binomial Expansion	6. Co-ordinate Geometry
7. Changing Symbols	7. Simplifying Algebraic Expression
8. Algebraic Equations	8. Changing symbols
9. Co-ordinate Geometry	9. Algebraic Equations
10. Substituting numbers	10. Substituting numbers

It is obvious that students and staff may have a different order for the first three topics but list the same three difficulties. As a staff member "reading and analysing" can be interpreted as how you would start a problem in calculus or what method you would adopt. Students generally need some assistance to start solving a problem. The lecturers job is to provide enough information for the student to approach a problem in several different ways. One of the common staff problems is in what detail should a lecturer deal with proof, theory and background to a particular topic when lecturing to engineering students at technikons. Sometimes the lecturer can slot in a very good explanation of a formula to be used but on other occasions the sheer difficulty of the algebra involved is very daunting for the students. A lecturer of mathematics is a mixture of both a pure mathematician and an applied mathematician. In the training of mathematics lecturers, equal emphasis is usually given to both aspects, which may account for a dislike of the term pure or applied mathematician. There should only be the one term mathematician. It is very desirable that the mathematician should be flexible in changing to either pure or applied methods when circumstances warrant it. Christopherson (1967:74) in talking about what sort of mathematics do engineers need, states that

> "You may think that the conclusion towards which I am tending is that, while applied mathematics is our friend, pure mathematics is, if not directly a foe, at least our rival in that it leads away from the path of rectitude many who would be very useful on work with practical applications. In fact, I am going to argue just the opposite; that part at least of our difficulties arises from the fact that, throughout our educational system, too much emphasis has been placed on mathematics as a means to an end - on the particular bits of mathematics that are assumed to be relevant to certain parts of physics or engineering, or indeed accountancy or life insurance, and not enough attention has been paid to the main object of the exercise, to give people an understanding of what mathematics is and does - not for utilitarian ends, but simply as part of the equipment of the trained mind."

Another staff problem is time. The sixteen week semester available for engineering students at technikons means that time is precious. Each lecture has to consist of a certain amount of content or application so a strict schedule is necessary. Students who miss lectures can very quickly lose the thread of continuity in calculus. Tutorials are vital in filling the gaps but here again they are not compulsory so the lecturer finds the ones who really need the help may not attend. Tests and examinations can lead to major problems for lecturers due to the frequent changes in syllabus which will ultimately lead to a bachelors degree in technology in the present transition stage at technikons. The frequency of tests and the amount of time for each test varies from technikon to technikon, for example, Technikon Port Elizabeth has four two-hour common tests per semester while Cape Town Technikon has three tests minimum with several lecturers giving four tests. Mangosuthu Technikon has three one-hour tests and Technikon Natal has two major tests each of one and a half hours, plus an assignment or project. Multiple choice tests are used as a part of the testing procedure so time must be allocated for this. All of this leads to heavy responsibility on staff to compose tests, mark tests and to give time to going over tests.

1.2.4 Content problems

Over the last five years the content of the 2nd level mathematics course for engineers at the technikon has been changed or added to at least twice. Basic differentiation and integration has remained fairly constant except that hyperbolic functions were moved out for a while and then moved back again. Complex functions over the last year have been slotted into first level along with some second level integration methods. Differential equations have gradually been introduced over three semesters and will form a separate module in the new syllabus. Matrices were removed but reappeared as a new module in the modular system from July 1992. Numerical methods have been in the syllabus for three semesters but will be excluded in the new modular system. Statistics which was given to all sections for three semesters but as from July 1992 it will be given as a module for civil engineers only.

This brief summary of changes shows that lecturers have had to restructure their courses every year and change their priorities on various topics. The new modular syllabus envisages four, four-week modules with two common modules on differentiation and integration and then a choice of two further modules from a selection of four including differential equations, matrices, statistics and linear programming. There are three engineering courses at the second level at technikons labelled IIA, IIB and IIC. They are

IIA Differentiation, Integration, Differential Equations, Matrices
IIB Differentiation, Integration, Statistics, Linear Programming
IIC Differentiation, Integration, Differential Equations, Linear Programming

These changes in syllabi are unsettling for staff and students and usually lead to a drop in pass rates until both staff and students are accustomed to the new material. Also it is rare that books are available to cover all the topics in the manner required so lecturers have to improvise with their own notes or transparencies.

Christopherson (1967:75) discusses how far a mathematics syllabus for engineers has to be practical or theoretical. He is of the opinion

> "that practical devices may come and go but differential equations and calculus of finite differences are not subject to the tides of fashion."

Christopherson (1967:78) then goes on to say:

> "In my view our approach to our mathematical colleagues should not be to say, "We want our men taught this, that and the other," but to say "We want you to give us a list of items which in your view will be useful in giving our men an understanding of what mathematics is, and how mathematicians think which will develop to the full whatever mathematical ability they have."

Landbeck (1991:673) has an opposite point of view:

> "A problem based service course developed around the issues in the students major subject, containing only the mathematics essential to this subject might be much more motivating for students. The service course concept could disappear and the extra time gained or used to expand the major subject to deal with mathematical applications as they arose naturally in the development of the subject."

In the technikons of South Africa the way mathematics for engineers is organised reflects the debate on the issue of theoretical and practical syllabi. Some technikons still have mathematics taught by one of their own engineering sections as they feel such people know the relevant practical applications. On the other hand several technikons have specific mathematics departments who service the engineering departments although lecturers still show a strong affinity for one engineering discipline with which they are more familiar or previously served as a staff member. In at least one technikon the mathematics department is linked to the science department for example Port Elizabeth Technikon has a "Mathematics and Physics Department."

1.3 STATEMENT OF PROBLEM

A pilot study amongst students and staff at nine South African technikons revealed serious problems regarding specific teaching and learning areas in calculus at 2nd level courses in engineering at technikons. The main areas that were identified are

1. Student ability in mathematics
2. Content difficulty
3. Background difficulties
4. Timetable pressures
5. Lecturer's presentation.

These can be further specified and analysed under the following:

* students specific ability in understanding, manipulating and applying algebra or trigonometry, which has an important functional role in the structure of the calculus
* utilising tutorials in teaching and learning
* the time students spent in learning calculus
* sequencing and difficulty level of learning content
* previous learning success or failure when a particular topic occurred.

This study will focus on a closer scrutiny of these problems and will recommend relevant improvements in this respect.

1.4 ANALYSIS OF THE PROBLEM

It is very relevant for lecturers and teachers to realise that their courses in calculus will not succeed if they ignore the students' difficulties with the calculus. The great need for technical staff at all levels of industry is hindered by the inability of some students to complete their mathematics courses. It is also possible that more confident mathematical technologists could save South African expenditure on overseas personnel who presently are recruited to deal with research and industrial problems. A solution of the learning problems of the calculus could materially assist students, lecturers and ultimately the economy as a whole if more students were able to master the calculus and continue with technical and engineering courses.

The main problem is to identify the causes of the difficulties experienced by students taking calculus courses at 2nd year level at technikon and correspondingly also the difficulties experienced by lecturers giving these courses to students.

1.4.1 Student ability in Mathematics

Lecturers will often generalise and say that a low symbol in school mathematics is the cause of a student's difficulties with calculus. A certain minimum attainment such as a D symbol in higher grade mathematics would help, particularly with regard to trigonometry but students with lower grades have succeeded whereas students with higher grades have failed. The situation is more complex involving intangibles such as motivation by the student, encouragement by the lecturer, size of classes, specific abilities, etc.

Steen (1992:3,4) states that:

* Mathematics can be learned by most students
* The cost of failure is often higher than the cost of success

and he mentions that a first step toward improved success is a good up to date understanding of the preparation and motivation of the students. He also poses the question:

> "Are you committed to teaching the students you have?"
> (Steen, 1992:4)

His answer is below.

> "It is all too easy for faculty to covet students who fit an imagined mold of young scholars created in the faculty's image, or to treat every first year student as a potential mathematics major. Instead, the department's priorities should match the actual student population. Instructional practice based on false assumptions yields disillusionment for both students and faculty. Effective instruction harmonizes the goals of the institution with the expectations of its students." (Steen, 1992:4).

1.4.2 Content Difficulty

Students will say that calculus courses are too difficult without specifying what makes them difficult. They will usually be reluctant to admit that a certain section gave them difficulty, rather hiding behind the statement that the whole course is too difficult. This is why the aim of the pilot study was to rank content difficulties such as limits, functions, derivatives, graphical concepts, etc. and bring to their attention the sections likely to cause them problems. (See 1.2.2; 1.2.3).

On the other hand teachers may feel certain sections are difficult to teach or that some sections should either be done earlier or later. The demands of service departments very often handicaps the mathematician in setting up the correct order of events.

Straesser and Thiering (1986:131) states that

> "Sometimes teachers are forced to teach content they would not choose such as operations with fractions, because students seek reassurance about their ability to cope with school topics and because of entrance tests for employment or study."

Whilst this was stated with regard to adult education it equally applies to technical education where the drag of school topics still impinges on 1st level courses and filters into 2nd level courses. This leads on to the next section dealing with background difficulties.

1.4.3 Background difficulties

In the student questionnaire students were asked whether they came straight from school, whether they came directly from military service or from business or from some other institution, say university. The entry point usually has a significant effect on background difficulties. The student who comes straight from school is likely to remember more about mathematics than a rather older student with a gap of several years in business or military service. A student from university will usually have an advantage with a deeper background of first year calculus. It is therefore difficult for the lecturer to always hit the right level for his lecture. Some students will still have problems with gradient and never seem to grasp the essentials of turning points because they lack graphical skills. Other students cannot extract the essentials of differentiation and integration so that whatever symbols are used the situation remains the same. If students have always been rigidly controlled and directed they lose the individual understanding relying on the lecturer to always point to the correct method or approach. Sometimes while students may vary in academic background the individual response may depend on training and experiences outside of academic institutions such as business or military service. The motivation of these business or military students may be quite strong which may lead to better results than school students or even ex university students.

There are also academic background difficulties such as the ability to solve equations, the ability to handle trigonometrical identities, sketching of graphs which will be discussed as sub problems in this study.

1.4.4 Timetable Pressures

When students do a semester course at second level they are usually doing six courses in engineering including the mathematics course. Each course will have at least six periods while some may have eight periods. Each day there is a possibility of 10 periods. So from Monday to Thursday they have 40 periods and Friday, being usually the morning only, another six periods, which gives 46 periods in total. Students could conceivably be engaged for 40 out of these 46 periods. This leaves only six periods for any student for preparation during normal hours. Many students stay away from lectures when it comes to tests or projects so that a lecturer is left with a reduced class while students revise for the other lecturer's test. The testing procedure is timetabled by the mathematics department and some technikons have their tests on Friday afternoon or on an evening after normal lectures. This scheduling helps the problem but as other departments do not do this, the mathematics departments suffer when their tests interfere with lecturing time. The allocation of periods does sometimes leave difficult times for mathematics, for example, 15:00 to 17:00 or 19:00 to 21:00 when the concentration of the students is rather low.

1.4.5 Sub Problems

1.4.5.1 Students' ability in Algebra and Trigonometry

The students' ability in algebra and trigonometry plays a key role in the understanding of calculus as with the successful completion of problems connected with calculus. Calculus deals with indices or powers when either differentiating or integrating. The understanding of roots of expressions can be crucial in these operations. The inverse of the square root of x, for example, when either differentiated or integrated gives many student's problems purely because their handling of indices is weak. This is an algebraic problem. Similarly manipulating trigonometrical expressions, for example, simplifying products of sines and cosines, powers of sines, cosines and tangents can lead to a major breakdown in any given problem. Explanations of various trigonometrical limits depend on radian measure, formulae and handling of fractions. The lack of knowledge in these areas is always going to make the lecturer's task more difficult.

1.4.5.2 Tutorials

Students and lecturers have different views on the use of tutorials. Students use them to ask questions about tests and examinations or to revise topics they find difficult. Lecturers use tutorials to set questions which amplify their lectures, give multiple choice tests and answer questions from students. Unfortunately the very students who need all of these items do not attend tutorials regularly. The tutorial is an essential part of the course so students who miss them lose out on certain essential practice with problems.

1.4.5.3 Time spent by students on their own.

The time spent by students on their own was one of the questions in the Student Questionnaire (Annexure A). It was noted that 36% of the students spent less than four hours a week on their mathematics course and a further 34% spent from four to six hours. In daily terms this means an hour or less per day even on a five day week. This is hardly adequate to cover the examples which need to be done by the students to consolidate the lectures. The problem is that five other subjects also need time. If they were all as difficult then six times four means twenty four hours a week which is about the limit a student can give outside lecture time. However some topics do not need the extra time so students could give more to mathematics. Of students 12% spent over eight hours a week on their own and 5% spent over ten hours a week.

Students have said in interviews that Mr X achieves in two hours what Mr Y does in four hours. Another cynical student said he looks at something for two hours and gets nowhere whereas someone else looks at the same item for two minutes and sees the solution. A new phrase is "quality time" and students must not think sitting and staring at formulae for hours is "quality time." A group effort may be a possible solution but tests and examinations are individual so sooner or later the student must work things out himself.

1.5 AIMS OF THIS STUDY

Statement and analysis of the problem of this study necessitates explication of the general aims and specific objectives of the study.

1.5.1 General aims

The general aim of this study is to identify and analyse learning problems in calculus for South African technikons. Following from this the aim is to identify and analyse particular teaching and learning problems which affect different engineering groups at 2nd level. Having established these problems the subsequent aim is to investigate and recommend relevant improvements which would increase the prospect of improving students performance at 2nd level calculus courses in engineering at technikons.

1.5.2 Objectives

In realising the general aims and finding solutions to the problems as stated specific objectives are relevant. This study will, therefore, try to answer the following research questions:

1. What are the important background items which affect the learning of calculus?
2. Which topics within the first year calculus course are the essential items to understanding the course?
3. What crucial effect and implication does the identified teaching-learning problems have on the performance in calculus of 2nd level students?
4. What recommendations can be made regarding the relevance of content, the emphasis which should be placed on various items in the syllabus and the deficiencies and weak points in the present courses?
5. What impact do particular computer software programs, have on the identified teaching and learning problems of the calculus?

1.6 CLARIFICATION OF TERMS

In the title of this study, namely "The teaching of 2nd level calculus at South African Technikons with particular reference to a didactic analysis of specific teaching and learning problems," there are certain key aspects which need clarification.

1.6.1 Second level calculus

This refers to the calculus done in the 2nd semester course in mathematics for engineers, now known as Mathematics II.

1.6.2 Calculus

Calculus is defined as a section of mathematics dealing with differentiation and integration with applications in various engineering fields.

1.6.3 Technikons

These are tertiary teaching institutions offering vocational education through the higher technical qualifications such as diplomas, higher diplomas and masters diplomas.

1.6.4 Didactic analysis

This refers to a scientific study of, and reflection on teaching and learning against the background of the three constituent components of the didactic situation, that is the student, the lecturer or teacher and the learning content.

1.6.5 Teaching and learning problems

These are those problems experienced in terms of facilitating learning by staff when they teach calculus and learning problems experienced in terms of comprehension, manipulation and application by students in learning calculus.

1.6.6 Students

These are students who study civil, mechanical, electrical, marine and chemical engineering courses at South African Technikons.

1.7 RESEARCH METHODS

This study began with investigations on teaching and learning problems in Calculus in 1989 at Technikon Natal. These investigations served as a pilot study for the research undertaken later. They were followed up by joint investigations at Technikon Natal, M.L. Sultan Technikon and Mangosuthu Technikon all situated in the greater Durban area. Then the investigations were extended to include the following technikons: Peninsula Technikon, Port Elizabeth Technikon, Pretoria Technikon, Cape Technikon, Technikon Witwatersrand and Vaal Triangle Technikon.

The following procedures and research strategy were followed in this research:

1.7.1 Empirical Investigation

The empirical investigation was conducted by means of an analysis of the responses to various structured questionnaires given to staff and students at nine technikons during 1988/89 on the calculus and on a computer package in use at Technikon Natal during 1990/91/92.

As an addendum to the questionnaire structured interviews were conducted on a group and individual basis with some of the students who answered the questionnaire. This was aided by tape and video recordings. Comments were subsequently extracted and analysed.

1.7.2 Literature Survey

Whilst there is little definitive literature on problems in learning and teaching the calculus at South African technikons the author has drawn on international studies on the difficulties students have in learning calculus.

In addition as many relevant publications from South Africa, including engineering reports and government studies are included. Where applicable confirmation from tertiary institutions other than technikons of similar problems is quoted.

1.7.3 Subsequent chapters

Having identified the problems in calculus with respect to teaching, learning and content in Chapter I, it was necessary in Chapter 2 to thoroughly investigate the technikon as a didactic environment dealing particularly with its mission, the structure of engineering courses, mathematics as a supporting science in training technical engineers and the role of calculus in the mathematics curriculum for engineers at the technikon. In Chapter 3 there is particular emphasis on the teaching and learning problems in Calculus. In Chapter 4 a case study of teaching-learning problems in calculus at South African Technikons is presented. It also investigates possible strategies to overcome the didactical problems identified in the calculus which could lead to more successful studies in engineering mathematics at technikons.

Chapter 5 investigates the empirical evidence collected by the author by means of a structured questionnaire given to eight hundred second-level technikon students who had taken a first level course in calculus. It also involves a study of group and personal interviews with students on their difficulties in calculus. A further questionnaire investigates the effects of a computer package on teaching calculus. A staff questionnaire is also analysed to provide comparisons between staff and student perceptions of calculus. Chapter 6 will summarise the conclusions and recommendations to be drawn from this research into specific learning problems of the calculus.

CHAPTER 2

TEACHING CALCULUS AT THE TECHNIKON

2.1 THE TECHNIKON AS A DIDACTIC ENVIRONMENT

In Chapter 1 the problem of learning in mathematics was identified. This led to a statement of the specific teaching and learning problems in calculus at 2nd level courses in engineering at technikons. In the analysis of the students' difficulties various items such as student ability in mathematics, content difficulties, background difficulties and timetable pressures were briefly investigated. In Chapter 2 there will be a detailed study of what is a technikon, its role in higher education, the type of training it caters for, and the type of student, lecturer and learning content involved. There will be a discussion on students' and lecturers' perspectives on the training of technician engineers and their status in tertiary qualifications. A review of mathematics as a supporting science in training technical engineers is followed by the role of calculus in the mathematics curriculum for engineers at the technikon. A summary will review and synthesize the very important issues studied in Chapter 2.

2.1.1 Historical overview of the Technikon

The information given in this historical overview is to a large extent gleaned from Pittendrigh (1988: 108-120).

South African technikons owe their existence to the development of railways and mining activity at the end of the 19th century. At first there were only technical schools, for example, the South African School of Mines founded in Kimberley in 1896. This eventually became the Transvaal Technical Institute which in turn was the predecessor of the Witwatersrand College for Advanced

Technical Education. The history of technical institutes and university colleges are intertwined. In both the Transvaal and Natal the technical institute was established first, in the case of the Transvaal in 1903 and in Natal in 1907. In the Cape Province technical training was part of the university college programme from 1907 and did not break away till 1917 when the Cape Technical College was established. In Pretoria the Pretoria Polytechnic was established in 1907 and in 1908 it was transferred to the Transvaal Education Department. In the early stages, therefore, technical education was a provincial responsibility. This was, however, changed by the Higher Education Act No. 30 of 1923 which constituted technical colleges as places of higher education. This meant that technical colleges were controlled by autonomous councils and subsidized by the Union Education Department. In 1926 all the technical colleges formed an association which met regularly at one or other of the colleges attended by the principals and chairmen of the college councils.

The progression from a technical institute to a technical college to a college for advanced technical education and finally to a technikon has taken nearly a century of hard work and endeavour by many concerned individuals in education, industry and commerce. A typical example of this progression is Technikon Natal.

2.1.1.1 The Hunter Commission (Rees, 1957 : 1-75).

In December 1904 the colonial government set up a commission chaired by Sir David Hunter to enquire into technical education in Natal. The commission's report was published in 1905 amid a period of financial stringency, indifference and downright opposition to technical education by some officials of the Education Department as well as visions to mount a private campaign to establish a technical institute in Durban. The report recommended that a technical institute should be established as part of a university college situated at Pietermaritzburg.

However, local business, railways and Durban corporation managed to persuade government that a Durban Technical Institute was the better proposition. This was established in 1907 with Dr Sam Campbell as the first chairman of the council of the institute.

2.1.1.2 Higher and Technical Education

During the period 1907 - 1922 the institute grew despite financial difficulties due to the tardiness of government grants and the lack of knowledge of technical institutions. There were strong moves to downgrade the institute to a trades school but these were resisted. As Rees (1957:76) points out

> "perhaps in twenty years time this Institute would become a university college."

Certain people were worried that the institute was aiming at too high standards. Again Rees (1957:76) quotes the following paragraph taken from the report of the Under-Secretary for Education (1912):

> "That for the purposes of the South African Act, Higher Education should include education beyond the standard of Matriculation, or a standard considered by the Minister to be equivalent thereto which is carried on in an institution established under a special statute, and any extension or continuation courses carried on in connection with such an institution which the Minister may approve and courses for the training of teachers followed in institutions to be afterwards named."

2.1.1.3 The Fisher Act of 1918 (Rees, 1957: 102-103)

Technical education at that time may have failed to be termed higher education on the basis of a matric but could qualify on the basis of equivalent standard as determined by the minister. This left the door open for certain institutions and courses. During this period, that is from 1907 - 1922, the Fisher Act of 1918 made it compulsory for young people in the 14 - 18 year age group to attend continuation schools for a minimum of eight hours a week in 40 weeks of the year. This plus the fact that enrolments at the Durban Technical Institute had risen by 50% put great pressure on facilities at the institute.

2.1.1.4 The Financial Adjustment Bill of 1922 (Pittendrigh, 1988 : 115)

Extensions to the Institute were definitely required. A memorial technical university college combining both university and technical work was envisaged. A lot of discussion took place over what subjects should be handled by the university section and the technical section. Finally in 1922 the Financial adjustment bill was passed. This bill defined higher education and empowered the minister to declare certain technical colleges as places of higher education. In the Government Gazette of 29 September 1922 the Technical College Durban was declared a place of higher education as from 1st April 1922. This enabled finance to be given from the Union Government and an agreement to be reached with Natal University College for courses in commerce and engineering to be conducted at Durban Technical College at university level for a transitional period of five years. A short time later the name of the technical college was changed to 'The Natal Technical College Durban' as the college served many areas outside of Durban. The dual status of Natal Technical College to conduct both university and technical courses was challenged by the Van Der Horst commission of 1928 which sought to downgrade technical colleges to the level of schools offering specialised secondary education.

The two colleges were separated as from 1st August 1931. The new buildings constructed in 1930/31 for the technical college were handed over to the council of the University College of Natal (Durban). Also transferred were two hundred students, ten professors and lecturers plus furniture and equipment and a college campus of fifty acres. As Rees (1957:206) indicated

> "It would be difficult in the annals of educational progress in South Africa, or indeed in any country to find a parallel for the free gift which the Natal Technical College handed over to the University College in 1931."

2.1.1.5 The Vocational Education Bill of 1955 (Pittendrigh, 1988 : 150-157).

The next significant date was 1955 when the Vocational Education Bill was passed. This bill made provision for taking over various technical colleges by the Department of Education, Arts and Science as full State institutions. These included Free State, East London, Pietermaritzburg and Port Elizabeth Technical Colleges but excluded the Cape, M L Sultan Natal, Pretoria and Witwatersrand Technical Colleges which remained autonomous. The bill sought to upgrade technical colleges and separate the schools offering technical and commercial courses from the colleges. In 1962 the National Advisory Council on Education was founded. In 1963 the council reported that the most acute problem was the divided control of secondary education.

2.1.1.6 The Advanced Technical Education Act of 1967 (Pittendrigh, 1988: 170-173)

A further step in the process of change was the Advanced Technical Education Act of 1967 which established Colleges for Advanced Technical Education (CATE's). This also brought the remaining technical colleges under the State system prescribing various conditions for their existence which severely limited their autonomy. Six CATE's were established: Cape, Pretoria, Natal, Witwatersrand, Vaal Triangle and a new campus at Port Elizabeth. Efforts to upgrade courses led to conflict over semester and trimester courses, course marks of 40% or 25%, national certificates as opposed to diploma courses. Amendments to the 1967 Act were introduced in 1973 which helped to define courses given at CATE's or University. It was felt that CATE's should mainly confine themselves to advanced technical and teacher training extending from more or less than Standard 10 level to a level somewhat lower than the university level in that particular field, providing that such training is of a more practical nature than the corresponding university training. There was also a clear statement that technical colleges would not develop into technological universities.

2.1.1.7 The Goode Report of 1978 (Pittendrigh, 1988: 192-195).

The final step in the establishment of the Technikons came about via the Goode report. The Goode committee was appointed in 1973 and finally reported to the Minister in 1978. One of its tasks was to report on the award of certificates and diplomas and the recognition thereof by industry and others. It was a direct result of the Goode report that technikons became responsible exclusively for the training and education of technicians (skilled in the techniques and application of a scientific discipline) and technologists (involved in the development and application of a discipline). Technical courses for a diploma, higher diploma and fifth year diploma courses were defined being known as T- courses. It was also accepted that higher technical qualifications would be obtained by research. The name of the CATE was changed to technikon to distinguish it from a technical college offering artisan courses known as N-courses. The T-courses are usually offered on a semester or bi- annual basis while the N-courses are offered on a tri-semester basis or three term basis. These changes were embodied in the Advanced Technical Education Amendment Act of 1979.

2.1.1.8 The National Education Report of 1983

The annual report of the Department of National Education for 1983 stated that greater autonomy should be given to technikon councils with regard to staffing and examination authority. The National Education Bill of 1983 in many ways reversed the provisions of the 1967 bill restoring the rights of technikons to, for example, borrow and lend money, choose their own principal, elect their own councils, be responsible for their own examinations subject to final certificates and diplomas to be recognised by the minister and director general.

Two important bodies were established to liaise with the minister on tertiary education. These were the committee of university principals (CUP) and the Committee of Technikon Principals (CTP). Both of these were formed after the Universities and Technikons Advisory Council Act 1983 (Act No 99 of 1983). Not only did these two committees advise the Minister of their own problems but they decided to meet jointly on an annual basis to discuss matters of mutual interest. The committees represented whites, indians and coloureds. Blacks were covered by a council for University and Technikon affairs (Education and Training (CUTA)). A new dispensation currently underway will remove these distinctions so that one body will be responsible for all technical education.

2.1.1.9 The current situation

Additional technikons were added to the original six namely Technikon Orange Free State, Peninsula Technikon, Mangosuthu Technikon and Vaal Triangle. The Peninsula Technikon was a special case being governed by the Peninsula Technikon Bill which changed the conditions under which higher coloured education was administered. Previously the Peninsula Technical College was administered under the Coloured Persons Education Act (Act No 47 of 1963). The new bill was to enable the Peninsula Technikon to acquire the necessary expertise as soon as possible and to obtain its own funds so that it would eventually be fully autonomous, like other technikons.

Current changes in the technikons include a strong emphasis on research and upgrading of staff qualifications to enable the present courses to lead to a Bachelor of Science degree in Technology, the certification of courses which has led to pruning of courses unsuitable for a technikon or transfers to technical colleges. In some cases a large building programme to improve facilities for the larger numbers expected in 1993 - 2000 has begun. Greater co- operation between black and white technikons for example M L Sultan Technikon, Natal Technikon and Mangosuthu Technikon all situated in Durban are currently involved in discussions to establish one technikon for Durban.

2.1.2 The Mission of the Technikon

The technikons of South Africa are a relatively new concept so defining their mission depends upon their present achievements and future expectations. Departments within the technikon differ greatly with some departments contemplating starting Bachelor of Science degree courses in Technology in 1995 while others doubt whether their industrial discipline requires or needs such a high qualification, for example, art and textiles. In Durban, for instance, Technikon Natal has spent much of 1993 redefining their mission with each department submitting their own mission statement. It is therefore fruitful to examine earlier statements related to the mission of a technikon according to Pittendrigh (1988 : 315).

> "The function of a technikon can be stated as being to train people to function as technicians and technologists."

Pittendrigh then goes on to define a technician and a technologist. The former he believes will have acquired detailed knowledge and skills in one specialist field plus knowledge and skills to a lesser degree in other specialist fields. The technician will be involved in supervising the work of others and will normally be engaged for more than 40% of his time in intellectual work rather than in tasks requiring normal skills. The technologist on the other hand will be capable of applying scientific and analytical methods to the solutions of technological problems. He should be able to make contributions on his own account to the advancement of the technology of his discipline. His work will be predominantly intellectual requiring original thought and judgement both in design, research, development and related matters. It will also include the supervision of technical and administrative work of other staff (Pittendrigh, 1988 : 315).

Bearing in mind that Pittendrigh published his book in 1988 it is not surprising to find in his book a statement that the function of a technikon is clearly different to the function of a university. His view at that time was that programmes offered at the technikon should prepare a student for a specific vocation or band of vocations. The author's view is that in the highly volatile economic and political climate of South Africa one cannot guarantee that vocational training will be enough to satisfy employers. For example students have been trained at Technikon Natal for textile firms who find that purely vocational skills without managerial expertise render the diplomates a liability rather than an asset. Is it then necessary for technikons to incorporate managerial training as part of their courses?

Pittendrigh (1988 : 313) advances the original function of the technikon by the following statement with regard to the technikon graduate:

"During his career there will be a need for the personal development of the technikon graduate, apart from his need to advance his technical knowledge, and this will in all probability take him into the management field."

Pittendrigh mentions further the ladder of qualifications and the availability of advanced courses on a part-time basis. Recent developments such as the Masters Diploma in Technical Education do provide that incentive to rise on the ladder of qualifications. It is interesting to examine the present mission statement of Technikon Natal.

In response to the needs of its community and country,
Technikon Natal,
as a unique and dynamic institution,
shall
....
educate and train
in the true spirit of tertiary education
high-level career manpower
for leadership roles in the South African society,
....
advance technology through research and development
....
and offer its specialised knowledge, skills and resources
to commerce, industry and the people
of the community it serves.

The emphasis here is on four things: the needs of the community, high level career manpower, leadership roles and research and development. This seems to the author to be almost identical to a university mission statement. The progression from a technical institute to a technikon offering degrees is nearly complete.

Engineering Shortages

Van Zyl (1971:284) highlights the different categories of support personnel who are needed in the engineering industry.

Percentage of population	Intelligence Quotient	Level of training in terms of IQ
	.	
	.	
	.	
	.	
	.	
	130	professional or academic engineers
25	125	
	120	
	115	high-grade technicians or technician engineers
	110	
25	105	technicians or advanced artisans
	100	
25	95	artisans (mechanical, electrical, etc.)
	90	
	85	artisans (painting, bricklaying, etc.)
25	80	
	.	
	.	
	.	

NOTE: IQ is not the only determinant of ability. Categories overlap more than is shown, depending on other personality factors

The table on the previous page shows the different levels of training required in terms of intelligence quotient although as is pointed out I.Q. is not the only determinant of ability. It is quite clear that universities and technikons compete for the same students in the intelligence quotient range of 115 - 125 which was borne out by the author's research on qualifications on entry to technikon when 16% had matric grades of C and above on higher grade in mathematics. (See Annexure A).

Van Zyl (1971 : 283) also mentions that for administrative and organizational purposes it would be preferable to train staff at levels where fewer people are required, that is the highest levels, but the demand is for many levels and categories. Many more technicians are required. In fact Van Zyl (1971 : 260) advocates that a ratio of three technicians to one engineer trained at university. Van Zyl (1971 : 258) states

> "In 1968 4317 students were being trained at university for engineering while 2089 technicians were being trained at CATE's for engineering."

This imbalance has swung more in favour of technicians. Pittendrigh (1988 : 280) gives 4296 students enrolling for engineering in 1974 at CATE's but it has still not reached the 3 : 1 ratio with universities advocated by Van Zyl.

Lloyd and Plewman (1984 : 1) comments on shortages of engineers at a comparable date:

> "In 1983 over 1200 engineers entered the job market, but there remained nearly 1600 vacancies giving a total demand of some 2800."

This is confirmed by different figures from the Human Sciences Research Council (Human Sciences Research Council, 1981:33).

> "The average demand for engineers is approximately 1 000 per annum while the average supply estimated on the basis of graduation trends comes to approximately 875, a shortfall of 125 engineering degrees a year."

The shortages of engineers at the top compounds the training of technicians and technologists who find they may have to do the work of a university trained engineer.

2.1.3 Lecturer perspective

The diverse routes which lead lecturers to the technikon accounts for the many perspectives lecturers have concerning the technikon. There are for example those lecturers who are inborn products of the technikon qualifying at the technikon then returning to lecture with little or no teaching experience. Then there are lecturers who come via the high schools with little or no industrial experience. A further group are migratory lecturers from the universities possibly with no teaching or industrial experience. Then some may come directly from business or industry when times are hard having been executives but with no classroom experience. For example, in the Mathematics department at Natal Technikon four lecturers came directly from school, two from university, two from commerce and one from industry. Similarly, in the mechanical engineering department two lecturers came directly from high school, with degrees and educational qualifications, three lecturers were directly trained at the technikon, while three came

from university with industrial experience, another three came from industry, technically trained, and a final one university trained from business.

In view of the differences in training and tertiary institutions lecturers attended as students, strong debates are being generated about what a technikon should offer to its students. The author has listened to lecturers who strongly believe that the technikon is merely a place to obtain a diploma whilst others believe that the technikon should produce high quality students possibly with degrees. The acceptable qualification for the mass of students who are entering technikons is still debatable with input from firms, professional organisations and lecturers being canvassed in preparation for a new bill to be introduced to parliament.

2.1.4 Student Perspective

There are similarities between lecturer's and student's perspective. In the author's research with nine technikons 48% of the technikon students came directly from school. This group may feel that technikon courses are easier than university courses. This perspective changes rapidly after their first tests. The author's research indicated that approximately 25% felt unable to cope with the calculus course after three or four weeks and a further 31% after five or six weeks when the first test was given. On the other hand over 30% of students were prepared to spend six or more hours on their own doing mathematics. This latter group usually comprises those who excel at their studies or pass comfortably.

Another group of students whose perspective is rather different comprises ex business students. They are more mature and realistic about their goals. In the author's experience they don't expect to get A grades but work hard to pass their subjects as they realise their firms will not upgrade them if they fail.

A further group of ex-university students tend to float along for a while feeling they needn't bother until things get difficult or new items occur. This perspective is false because the technikon approach is rather different to the university approach. It may not have the rigour of proof but the courses cover various items rather quickly.

Many students believe that technikon lecturers should be like their teachers at high school continuing at a relatively slow level giving individual assistance when required. This perception is more acute at T1 level but seems to persist with some students until T3 level. Parents also seem to think that students will not be overtaxed with technikon courses. The students perspective on examinations is also interesting. Many feel they shouldn't fail irrespective of the effort they put into their work. Somehow the ogive should ensure the majority will pass. It is therefore a shock when a 60% pass rate in mathematics is considered normal.

There is some inconsistency in the student perspective with regard to lectures. Most lecturers plan their work so that every lecture contributes to the whole course. In the author's experience students do not realise this fact. In small classes it is possible to achieve very good attendance but in large classes of 80 plus students can easily miss lectures.

2.1.5 Teaching - learning activities, opportunities and experiences.

Over the past decade the technikon has incorporated modern approaches to teaching. In the International Journal of Mathematics Education in Science and Technology (1987:651) Professor Geymonat of the Politechnic di Torino pointed out that changes induced in industry and society by the development of computer science were very evident. He goes on to show that fitting these developments into the curriculum of an engineering student is seen as an outstanding problem of teaching. It is found that some chalk and talk lectures are being replaced by computer simulated situations. For example electrical circuits can be planned and altered on computers. In mechanical engineering computer assisted design packages are used instead of the usual pencil on paper. In mathematics graphical packages and logical packages are used to solve equations, differentiate, integrate and to do extensive work on differential equations.

Slides and films also play a useful role in providing background to learning. Some departments use visiting lecturers or business executives to provide the industrial point of view. Alternatively other departments have visits to factories where the latest equipment is being used and can be viewed "on site". Students learn by talking to the operatives of such equipment and by the questions asked by the more perceptive students. For example, mechanical engineers at Durban Technikon visit Toyota's factory where the latest assembly line is operating and the most sophisticated paint shop in South Africa is in use. Textile students always visit Frame's factories where modern spinning and weaving techniques are illustrated. Another avenue which assists learning is through projects which students have to complete during their course. They usually have a choice of topic so this is an incentive to choose one they are really interested in. The results obtained with projects generally exceed the standard tests so this helps students motivation.

Waks (1987 : 381) states that

> "Research literature shows that inclusion of applied examples in the curriculum improves memorization of the fundamental principles."

Also Waks (1987 : 381) states that

> "when a piece of knowledge is classified as more vital and usable, it is stored in an easier to retrieve memory location."

Many students experience for the first time the responsibility of assembling a piece of equipment given its basic components. This is something very different to talking about how equipment works. The change in students from the new T1 students to the confident T3 students is largely due to the experience gained in the workshops, laboratories and practical projects completed during their courses. Prizes for excellence in studies, competitions to design items for the technikon, grants put up by firms plus offers of employment to students qualifying in certain areas provide unlimited opportunities for the student prepared to work hard with reasonable ability.

2.2 PERSPECTIVE ON THE TRAINING OF TECHNICIAN ENGINEERS

2.2.1 Student perspective

Engineering students have to satisfy their teaching departments that they possess minimum skills in mathematics and science. This is usually at least a D symbol in standard grade with an equivalent E symbol in higher grade. However with the vast increase in students applying for technikon there is no guarantee that students will be admitted with minimum qualifications. It is therefore wiser to aim for a C symbol in standard grade or a D symbol in higher grade.

The author's research (See the Student Questionnaire in Annexure A) indicates that in 1989 32% of the students had qualifications in mathematics at either higher or standard grade with at least a C, B or A symbol. On the other hand 25% had minimium qualifications but most of these failed to pass T1. The situation has changed over the last four years with possibly 2/3 of candidates possessing higher grade qualifications as opposed to a 50:50 split between higher and standard grade previously. Student ability has therefore improved but students tend to underestimate the dedication required to pass mathematics or courses related to mathematics.

Shell, Toyota, Anglo-American, Barlows and other large firms sponsor their employees to attend technikon courses. If they pass their courses then a bright future avails them. Others not so fortunate may have to pass at least T1 courses before a firm will sponsor them.

All students have to have eighteen months experience with a firm as well as eighteen months in the technikon. The original intention was six months at technikon followed by six months with a firm. This is not always possible so students may take two semesters, that is one year, to take T1 and T2, then work with a firm for a year, then return to take T3 and then do a further six months with a firm. Again because of difficulties in getting a permanent job some students are continuing to do a T4 course which occupies another year. If a student wishes to work as a consulting engineer at a factory or mine then he will have to get a government ticket or certificate of competency. This will involve his diploma, three years of experience plus two further examinations in law and engineering machines. For those with a higher diploma then the aim may be to become a professional engineer (Pr.Eng.) who is allowed to certify standards have been adhered to in engineering operations, for example, bridges, roads, etc.

2.2.2 Lecturer perspective

In the past it was not essential for a lecturer, senior lecturer or even a head of department to have a degree in engineering so even today there are departments having more than 50% of their staff with diplomas or higher diplomas in their subject. Usually the director, head of department and associate directors will have a

BSc or MSc in engineering. Much will depend on where lecturers have worked previously. Those transferring from university will obviously have higher qualifications than those only circulating in the technikon environment. A considerable change is taking place in the technikon with regard to qualifications with many staff upgrading their qualifications to master level. Teaching qualifications are also on the increase but are still a minority among the technical qualifications.

The type of qualification a lecture possesses affects his perspective on the training of technician engineers. For example if a lecturer has university experience then he may try to convert technician engineers into university engineers emphasising higher academic standards. This initial perspective usually changes after a couple of semesters when students reject a purely university approach to their work. Then there are lecturers who feel that the practical approach is the only realistic way to train technician engineers. They seem to think that theory can be slotted in when needed which is not logical in mathematical courses serving engineers. Some lecturers do not see the need to train engineering technicians to a high level and would be quite happy to stop at the diploma level or even lower. They point out that the intake of black students will concentrate attention on the diploma level.

Pittendrigh (1988:318) has an interesting discussion about the domain of university and technikon courses. He states that study in a technikon should be of an applied nature and not have the basic subject discipline characteristic of university study.

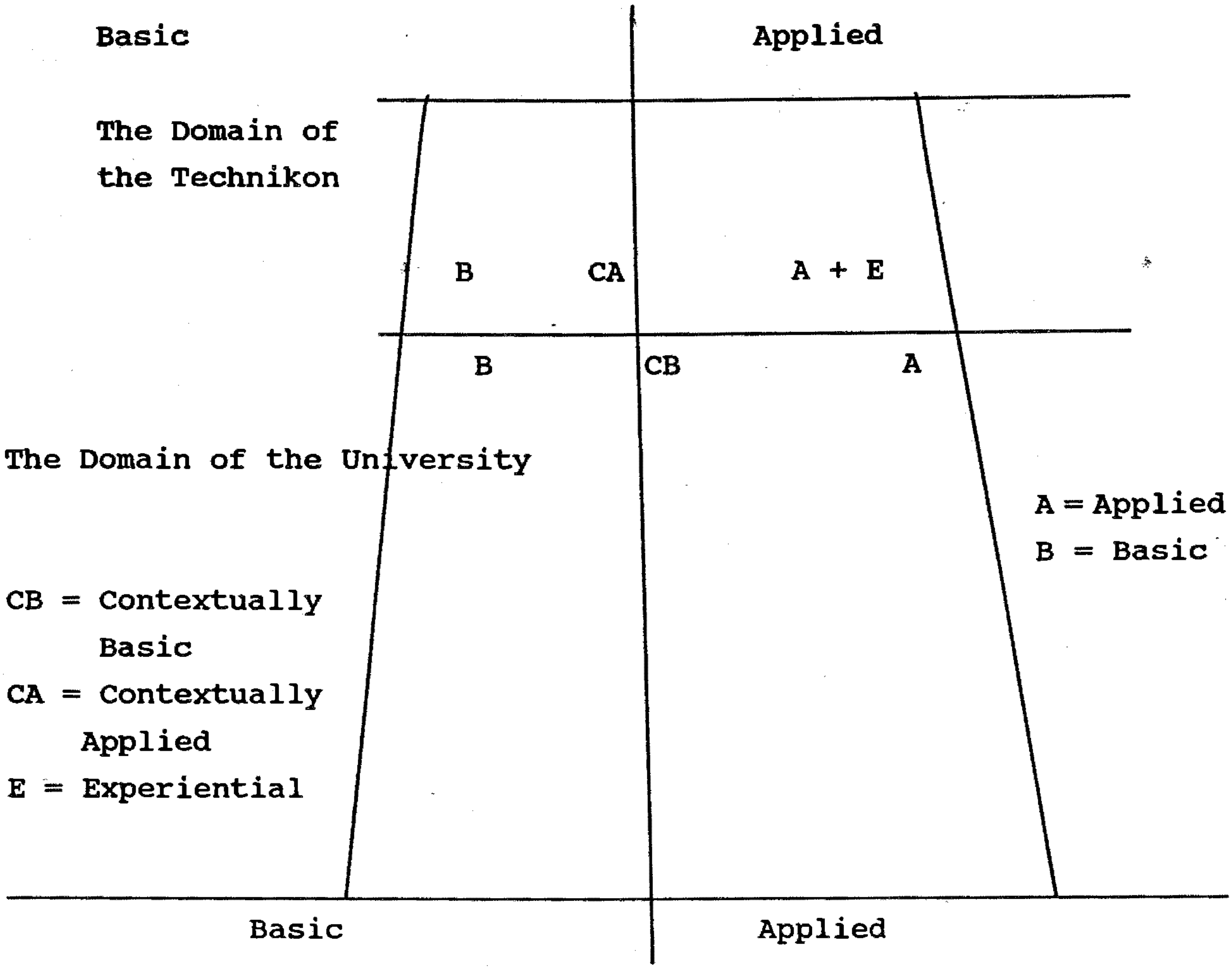
Then he defines topics of study such as applied, basic, contextually basic, contextually applied and experiential.

Pittendrigh (1988:318) discusses the terms contextually basic and contextually applied. A contextually basic subject is one which is not included in a list of basic subjects and which becomes a basic subject within the context of the discipline being studied. In this way it is seen that economics is a basic subject while agricultural economics is an applied subject. When a student of agriculture studies agricultural economics it becomes a basic subject within the context of the study of his discipline of agriculture. The characteristics of a contextually applied subject are somewhat different. Such a subject is selected from a list of basic subjects. Suitable material, which is applicable to the vocation for which the student is being prepared, is selected to form the subject syllabus and the manner of presentation of the subject, with all examples dealt with, is totally related to the vocation.

A contextually basic subject can be taught to a non-homogeneous class while a contextually applied subject must be taught to a homogeneous group.

Diagram 6.1 is taken from Pittendrigh (1988: 319).

Diagram 6.1 The Domain of University and Technikon courses



Applied lecturers are divided between basic courses and applied courses. The content of basic courses is generally straightforward but the content of applied courses can be subjective where personal preferences may influence the course. Unfortunately rapid developments in industrial technology may affect applied courses' validity.

Pittendrigh (1988 : 320) states the following:

> "There is as much need today for virile institutions reacting rapidly to industries needs and delays of two years in innovation in courses hardly meet this requirement."

Experience in industry is another area where lecturer's have different qualifications. In some cases lecturers have ten years experience in a particular specialised field while in others only two years has been spent with a general engineering firm. Others have run small companies and sometimes even continue to do so while lecturing at technikon. It is obviously desirable that a lecturer should have worked in industry but for how long or in what field is debatable. The main benefit for students is that lecturers have contacts in industry so they can keep abreast of new developments.

2.2.3 The structure of technician engineers courses

The technical engineer has 3 semester courses at the technikon as a full time student and eighteen months in industry gaining practical experience. However many students do the first two semesters at technikon followed by a year in industry and then return for the third semester at technikon and a final six months in industry. Each semester course consists of a minimum of six topics including one mathematics section, for example:

T1 Mechanical Engineering -	COMMUNICATION)	COMPULSORY
	ENGINEERING MATHEMATICS)	COMPULSORY
	ENGINEERING MECHANICS)	COMPULSORY
	MACHINE DRAWING)	COMPULSORY
	ENGINEERING SCIENCE }	CHOOSE 2
	ELECTRICAL ENGINEERING }	CHOOSE 2
	MOTOR VEHICLE ENGINEERING }	CHOOSE 2

Each topic involves practical work or laboratory experiments. The timetable is usually very full with only a handful of free periods. On the semester basis tests occur every month so that the pressure builds up on the student. Some topics may cause little diffuculty but others particularly those with mathematical content tend to cause difficulty. Failures in more than two subjects means repeating the course and repeats on a subject are now only allowed twice.

First semester courses usually have six periods a week but second and third semester courses can have seven or eight periods a week involving say four theory, two tutorial and then either one or two periods for practical or laboratory work, for example:

Fluid Mechanics T2 - Theory 4
Laboratory Practical 1
Tutorial 2

The full diploma course in engineering requires eighteen courses to be completed as opposed to commerce where only twelve need to be completed. This means there is a greater pressure on engineering students both in lecture load and examination requirements. For example engineering students will be fully occupied with four or even five afternoons of practicals and tutorials whereas commerce students will only have two afternoons.

2.2.4 The status of engineering diplomates

The status of engineering diplomates is another cause for concern. Some firms do not integrate the diplomate into their higher sections so frustrating the diplomate and losing the value of his training. In Germany the status of a technically trained person is on a par with a similar university trained person.

Marcus (1989 : 2) states

> "We have failed to ensure the correct employment of graduate engineers, and have done very little to enhance the standing of diplomates from technikons. In Germany and other European countries an equivalent to the technikon diplomate is not perceived by society as a person who has failed to obtain an engineering degree. He is a highly valued member of the engineering team and his contribution to the overall engineering effort is recognized."

Saunders (1992 : 2) suggests that universities, technikons, technical colleges and teacher training colleges form a "federation" of co- operating educational institutions. In terms of the education federation, universities need to be preserved and technikons need to become technical universities giving appropriate degrees.

2.2.5 Mathematical qualification for technician engineers

Entrance standards to both university and technikon engineering departments inevitably raise the problem of qualifications in mathematics. In many instances they (the university and technikon) compete for the same students.

Hugo, Kemp and Rohde (1989 : 7) state in the conclusions and recommendations that

> "It is recommended that the quality of civil engineering education in South Africa be improved by ensuring minimum entry standards of a "C" in mathematics and science and that as an absolute minimum D levels be required by the SACPE accreditation terms."

Also Hugo, Kemp and Rohde (1989 : 6) states that

> "It is recommended that further attention be given in South Africa to the quality and availability of higher grade mathematics and science teaching with the objective of accommodating over 10% of the peer group population."

A similar pattern in Technikon engineering departments emerges as they also ask for "C" levels in mathematics and science; but not necessarily in higher grades. However the author's investigations over a three year period in nine technikons (See the Student Questionnaire in Annexure A) show that at the second level in engineering mathematics courses the ratio of higher grade mathematics students to standard grade mathematics students has risen from a ratio of 1 : 1 to a ratio of 3 : 1 in favour of higher grade with 16% having a C on higher grade. This bears out previous observations that universities and technikons are competing for the same student. In future with degrees being granted at technikons co-operation will be a necessity so that scarce resources (well trained staff and students) are not squandered.

This perspective on the training of technician engineers highlights the difficulties faced by students in completing a diploma in engineering problems generated by technikon degrees, the prior image of technician engineers with employers and public in South Africa as opposed to Germany and other countries who treat their technician engineers and university engineers as equals.

2.3 MATHEMATICS AS A SUPPORTING SCIENCE IN TRAINING TECHNICAL ENGINEERS.

2.3.1 The Role of Mathematics

A certain amount of mathematics is deemed necessary for all engineering courses

Barry and Harley (1990 : 344) agree that a core curriculum for professional engineers should include

Analysis and Calculus
Linear Algebra
Statistics and Probability
Discrete Mathematics

and that numerical methods should be integrated into the curriculum.

In a similar vein Boieri and Steele (1987 : 655) have the following topics for the modern electronics engineer:

Probability and Statistics
Random Signals
Classical Analysis
La Place and Fourier Transforms
Z-and discrete Fourier Transforms
Matrix Analysis
Approximate methods and algorithms

Each department of engineers has its own priorities, for example, electrical engineers require detailed analysis of wave motion, chemical engineers may require differential equations earlier than other groups in order to explain rates of decay or expansion whilst civil engineers require statistics and linear programming at an early stage. Mathematicians have to link together these priorities into a comprehensive course. Whenever possible examples are given which are relevant to the group being lectured. The role of mathematics is to provide the basic background so that students can solve engineering problems which involve mathematics. It is also helpful that other lecturers of engineering refer to items taught in the mathematics section even though the symbols may not necessarily be the same.

2.3.2 The type of mathematics given to engineers at technikons

At a group conference held at Witwatersrand Technikon in May 1991 new syllabi for engineering mathematics were formulated. In the first level course determinants, logarithms, exponents and equations associated with these processes form the basic algebra, while trigonometrical identities, wave motion and solution of trigonometrical equations form the basic trigonometry, complex numbers and statistics are options for electrical, mechanical and civil engineers. Differential calculus from basic principles and the various rules plus applications to rates of change and maximum and minimum give students an inkling of what the calculus is all about. Integral calculus is introduced but rarely gets more than two weeks of lectures so is rather brief. In the 2nd level course the emphasis is on modules so the first module is on differentiation including partial differentiation. This covers a wide range of functions including combination of logarithms, exponents and trigonometrical functions. Errors, rates of change, maximum, minimum and points of inflection are also extended to harder functions and functions of two variables. The second module is on integration which is a very comprehensive investigation of integration both with respect to special methods and general methods of integrating functions. It also includes applications to areas, volumes, rootmean square, centroids and centre of gravity.

The third and fourth modules are now chosen from a group of four modules: Differential Equations, Matrices, Linear Programming and Statistics. For example mechanical engineers and electrical engineers take differential equations and matrices while civil engineers do Linear Programming and Statistics. Over the last six years many changes have taken place in courses at technikons. A conference of all technikons took place in Scottburgh in 1987, a further conference took place at Witwatersrand Technikon in 1991 and a further one day conference in 1993.

After the first conference in 1987, which reviewed the syllabi of technikon engineering mathematics, Strauss and Diab (1988 : 135) have the following categories in Table 6:

School Mathematics	Further Mathematics
Programming	Introductory Statistics
Introductory Mathematics	Calculus
Introductory Calculus	Further Statistics

The author's investigations do not cover the third and fourth level of mathematics at Technikon for engineers but the topics of La Place Transforms, Partial Differential equations, Z-transforms, Fourier analysis, Numerical Methods, Probability and Statistics comprise the courses available. An examination of the earlier core curriculums in 2.3.1 show that university engineering courses follow the same topics (of mathematics). Van Rooy (1988 : 5), summarising those aims at technikon that are regarded as most important, has the following items:

The ability of mathematics to train the mind
The specific utility value of mathematics
Basic mathematical activities
Mathematical self-realisation

This concludes that the general aims of teaching mathematics are to a large extent relevant for the teaching of mathematics at the technikon.

The author's investigations show that over a period of five years with two major changes of syllabi, technikon mathematics and university mathematics for engineers have drawn much closer especially in the second and third years of study. The aims in both institutions, that is university and technikon, for engineering mathematics would appear to be very similar and may become indistinguishable when a bachelors degree in technology is in place.

A certain portion of the course mark, usually twenty per cent, is allocated to a project which will require students to study a part of the syllabus in more depth or to do something on his own which is not lectured. Examples of this are graph work, hyperbolic functions and statistics. The author has used a computer package on the graphical approach to the calculus which students use to tackle various tutorials on differentiation, integration, differential equations, etc.

2.3.3 Teaching - learning problems

The previous two sections show that considerable variety in courses for different sections of engineering can give rise to teaching-learning problems. Students tend to lose interest when topics are not directly relevant to their particular engineering section. For example, civil engineers are not interested in complex numbers whereas electrical engineers find them very relevant. Mathematics lecturers cannot always give students exactly what is relevant to their other courses. Building a basic course requires certain essential parts so deleting too drastically is very short-sighted. There is also another problem which affects lecturers: the distinction between engineering mathematics and mathematicians mathematics. There seems to be an illusion that much easier mathematics exists for engineers as opposed to mathematicians therefore correspondingly engineers will do better than their colleagues on purely academic mathematics courses. The author's experience in university and technikon indicates that difficulties, syllabi and results are very similar in both institutions. For example, many students in both institutions find it difficult to solve quadratic equations especially if they are related to exponents. The ability to simplify algebraic expressions seems to be sadly lacking in too many students. This also overflows into trigonometry where simplification involves intelligent use of identities. Student problems arise from the lack of these basic requirements.

Lecturers have problems during their lectures when they automically simplify expressions in order to move on to the next step, which leaves some students in difficulties. At technikon a lecturer may have to explain more fully his steps in mathematical sequence. On the other hand some lecturers may feel this is unnecessary due to time constraints as will be discussed in the next section of this study. The syllabi organized for engineers may look similar to academic courses but will differ perhaps in depth and applications. Lecturers used to one academic course may find it difficult to switch to another non-academic course.

2.3.4 Constraints on mathematics education for technical engineers.

2.3.4.1 Constraints from engineering departments

Mathematics education has constraints imposed by each department it services. They require a certain proficiency or knowledge of processes in order to proceed with their engineering, commerce or science module. The topics they require may not follow a logical mathematical order so that it is necessary to fill up the gaps in order to lecture on these topics. For example, complex numbers are required by electrical engineers but to do these a good knowledge of trigonometrical ratios, radians and exponential expressions is required. Several departments want a bit of statistics included in their courses but this requires quite a lot of practice, particularly if hypothesis testing is needed. Time is always a factor with a maximum of sixteen weeks for half year semester courses so the pressure is always there. There is a constant battle between rigour with regard to mathematical explanation and the use of a mathematical result. If too little explanation is given then all mathematics becomes rote substitution which sooner or later torpedoes progress into higher courses. Terminal courses, namely those only requiring one semester, may possibly survive with minimum explanation but most students require two or more semester courses and failures on the second semester course are usually due to the sketchy knowledge or explanation given in the first semester course. If the student takes a very limited viewpoint of barely passing mathematics in his first course then there is little hope in the second course.

2.3.4.2 Constraints from industry

The technikon has to abide by the constraints imposed by industry. Industry requires students to study six months at technikon and work six months on the job. Sometimes this is to the disadvantage of mathematics continuity as students forget mathematical formulae and concepts. The second semester courses usually involve vital issues such as differentiation, integration, differential equations, numerical methods and statistics. Students find a dramatic increase in learning content after a session at work severly strains their capacity to absorb the essentials of each stage. Lecturers have to press on leaving weaker students on the way otherwise they would not complete their syllabi.

2.3.4.3 Different lecturing styles

There are different approaches due to the background of the lecturer. Some lecturers have had a special link with a specific engineering department, say electrical or mechanical, in the past so they can readily supply examples relevant to these fields. Others come from a university background so do not have this familiarity with technical topics while yet others have graduated from school teaching so require time to acclimatise to technical jargon. Different styles are evident as a result of the above, some lecturers feel fifty minutes should be spent talking or explaining the transparencies they display on the overhead projector. Other lecturers feel student involvement is still necessary so allow time for individual responses rather than have a group writing furiously during the lecture. Other lecturers hand out notes and lecture to those notes but this is more the exception than the rule owing to the expense of printing. The approach to tutorials also differs, some lecturers feel quantity of examples is what counts while others feel quality of examples is what matters. Some lecturers give hints to questions while others do not. Some lecturers sit at their desk and answer queries while others circulate and see if problems are arising.

2.3.4.4 Constraints from the timetable

As a service department mathematics has to fit in with a variety of timetables which can lead to staffing problems such as splitting courses between two lecturers. The service department also has to contend with the test schedules of the departments it services which usually results in a loss of lecturing time. Individual departments also vary in their priorities so the mathematics department has difficulties in planning a general course for everyone so has to include options, for example, civil, electrical and mechanical engineering in their mathematics I or mathematics II courses. Technikons have quite a number of external students who attend classes after work so their courses begin in the later afternoon or evening. These students usually have to travel a fair distance from their work so tend to suffer from fatigue and loss of concentration. This does adversely affect their performance in mathematics. They have less access to lecturers outside the lecture time so are at a disadvantage with respect to the day students.

2.3.4.5 Constraints due to numbers of students taught.

Numbers of students is also a problem with service departments. For example, the Department of Mathematics at Technikon Natal may have 360 first semester mathematics students which would require ten lecturers to have small classes of 36 students each. This staffing ratio is impossible so five lecturers lecture 72 students each or four lecturers lecture 90 students each. Large groups in mathematics leads to supervision problems and dropouts with respect to attendance. Tutorials are usually kept to 30 - 40 students so supervision is undoubtedly much better. Smaller groups usually do better in tests and examinations as is evident with some of the departments serviced by the Department of Mathematics at Technikon Natal. Their individual total of students is 30 or less so in the rest of their papers they do better than mathematics. A pass rate of 75 - 80% is not unusual in some of their other papers whilst in mathematics lecturers battle to get 50 - 60% through their courses.

2.4 THE ROLE OF CALCULUS IN THE MATHEMATICS CURRICULUM FOR ENGINEERS AT THE TECHNIKON

2.4.1 Calculus content of engineering courses

At the technikon all engineering courses in mathematics include a substantial percentage of calculus work in their courses. At the Mathematics I level it forms at least 50% of the course with introduction to differentiation and integration. In Mathematics II the percentage rises to 70% with direct work in differentiation, partial differentiations, integration and differential equations plus a further 10% on applied work such as series, maxima and minima, centres of gravity, centroids and rootmean square. At the Mathematics III level differential equations, La place transforms & Z-transforms form the majority of the course. Mathematics IV and V include complete courses in further aspects of the calculus. It is also at the basis of much that is done in numerical methods and even statistics has links with the calculus. Depending on the engineering course students are required to take at least Mathematics I and usually take Mathematics II. Electrical engineering and chemical engineering take Mathematics III plus a smaller number of Mechanical Engineering students. The figures in

Technikon Natal per semester are 400, 250 and 100 taking Mathematics I, II, III. For those who wish to pursue a higher diploma the lack of performance in calculus at an earlier stage is a definite handicap.

2.4.2 Uses of calculus in various engineering situations

The language of the calculus and its methods are used to problems involving rates of change and movements of bodies requiring moments of inertia, for example, rotation of bodies about fixed axes. Dynamic change and stresses within systems can usually be reduced to differential equations. Work done on a body is calculated by area under the force/distance curve which involves integration. Reactions by chemicals, rates of decay of nuclear substances, growth of bacteria all require exponential equations solved by the calculus. Electrical systems can be explained using sine and cosine waves and changes in these require Fourier series based on the calculus. There are a multitude of situations which can be explained more concisely if calculus is used. Even if only one item is required each semester such as rate of change or moment of inertia or differential equations by particular engineering departments, the mathematics department must present a logical course in the calculus building up to this item. A reasonable proficiency must be attained at each stage. Basic items are being increased at each stage so that applications in complex numbers integration, series and harder differentiation are now included in Mathematics I while more work in differential equations has been added to Mathematics II and Mathematics III courses now include Z - transforms.

2.4.3 Applications of calculus to various engineering departments

One of the common problems in engineering is to use materials economically. Various shapes have a maximum volume when made from a given area of material. Examples of this are closed cylinders (cans) cuboids (blocks) and cones. The calculus enables engineers to set the dimensions accordingly. For example a closed cylinder has maximum volume when the height is equal to the diameter of the circular base. The inertia of moving parts such as flywheels in engines involves moment of inertia which inevitably leads us to integration, which forms a major part of the 2nd level course in engineering mathematics.

Davis and Hersh (1981:83) give an interesting application of the theory of differential equations:

> "Let us suppose we have an application, say, of the theory of partial differential equations to the mathematical theory of elasticity. We may now inquire whether elasticity theory has an application outside itself. Suppose it has in theoretical engineering we may inquire now whether that theory is of interest to the practical engineer. Suppose it is, it enables him to make a stress analysis of an automobile door. Again we raise the question, asking how this might affect the man in the street. Suppose the stress analysis shows that a newly designed door satisfies minimal strength requirements set by law. In this way we can trace the application of mathematics from the most abstract level down to the consumer level."

As the technikon does not pursue mathematics for its own sake but looks for those parts which can be applied to a particular engineering department, the calculus predominates in all the courses given in mathematics as it can be applied to many situations.

In the foreseeable future no one has a viable alternative to the calculus so it is assumed that calculus will continue to occupy students attention in a major role for their mathematics courses in the technikon.

2.5 SYNTHESIS

In this chapter the development of the technikon from its early days as a Technical Institute founded in 1907 through the changes to finally Technikon Natal in 1979 was reviewed. The history of Natal Technikon is a typical progression for most of the present technikons. An examination of the technikon from the point of view of the lecturer revealed that their perspectives varied according to the background and training they had received, for example university, business, commerce, technikon or school. The student perspective similarly was affected by the background and training they had received, for example, if they came straight from school or if they came from commerce after working in a firm or if they had taken higher grade matric or standard grade. The type of teaching-learning activities available at technikon were examined with reference to new approaches via computers, films, transparencies, outside speakers and visits to factories.

The training of technician engineers at technikon was discussed with regard to the various courses available and with regard to the difference between a technician and a technologist. Lecturer's expectations for their courses and students varied from those who wanted very high standards with a lower pass rate to those who felt the majority of those who entered should pass even if the standards had to drop a little. Experience in industry is not always possible although it is desirable.

With regard to mathematics as a service course for engineers, the role played by mathematics in engineering courses was investigated. This included the type of mathematics required in engineering courses. It was observed that different faculties of engineering had different priorities, for example, chemical engineering used differential equations sooner than anyone else whilst electrical engineering need complex numbers and wave motion at a very early stage of their courses. The teaching - learning problems of mathematics service courses included the number of topics required by engineers which don't always form a coherent mathematical entity as well as the compressed nature of semester courses and the size of classes.

Finally the role of calculus in the mathematics curriculum was discussed. It was observed that calculus comprises more than half of any mathematics course given to engineers. Calculus enables engineers to explain and quantify various engineering problems for example rotation of bodies, growth and decay of substances and electrical systems.

CHAPTER 3

TEACHING AND LEARNING PROBLEMS IN MATHEMATICS AND CALCULUS

3.1 GENERAL TEACHING AND LEARNING PROBLEMS IN MATHEMATICS

In Chapter I the didactical problems associated with calculus were identified. They included the problems students have in learning calculus, the difficulties staff have in lecturing calculus and the particular problems associated with the content of semester mathematics courses for engineers at technikons. In Chapter 2 the technikon was viewed as a didactic environment with references to its special background and growth which sets it apart from other tertiary institutions. A detailed perspective on the training of technician engineers followed which was closely linked to mathematics as a supporting science in training technical engineers. Then more specifically the role of calculus in the mathematics curriculum for engineers at the technikon was examined. In Chapter 3 a detailed analysis of lecturing and learning problems in mathematics will be linked to the additional lecturing and learning problems in calculus. Aims and methods in calculus will be detailed. The learning process is examined to assess how students learn calculus. The changes in the calculus curriculum leading to lecturing and learning opportunities are discussed.

Mathematics teaching creates frustration in both the lecturer and the student. How often do lecturers of mathematics say: I can't get them to understand x y z, or how often do students say: I don't know what you are talking about. Both these groups have genuine problems and it is important to find out what the problems are before offering any solutions.

3.1.1 Problems of communication in mathematics

Briginshaw (1987a.: 328) claims that lecturers require two qualities: knowledge and the ability to communicate. The former, that is knowledge, can be measured to some extent by the possession of degree certificates to whatever standard is necessary.

Briginshaw(1987a. : 328) states further that:

> "The second of the qualities however, ability to communicate, more ephemeral, less easily focussed, attracting less attention, is susceptible only to objective assessment. Who judges good communication? The consumer? Perhaps, but how? With what formality? With what safeguards? With what weight?"

A major problem for the lecturer is the method used to disseminate knowledge and the language used to explain concepts. Mathematical language is different to everyday language, for example vulgar fractions refers to common or simple fractions and not to obscene parts, a function of x is an expression which has a unique value for each value of x which is very different to a social function or special activity. Very often students do not have a good mathematical vocabulary so that many of the words used by the lecturer are not understood.

Countryman (1992 : 11) amplifies this theme.

> "When students learn to use language to find out what they think they become better writers and thinkers. Our students need more classroom opportunities to do informal writing, to make sense by making meaning, to create for themselves the underlying concepts of mathematics."

On many occasions when students are asked to express algebraic formulae in words, their explanations indicate the reasons for errors, for example, x plus y squared is different to x plus y all squared. In symbols the former is $x + y^2$ and the latter is $(x + y)^2$.

Countryman (1992 : 2) showed that even the most successful school students of mathematics claim they can do problems but can't explain them. Teachers really understand problems by writing out their own explanations. It is no wonder that students do not understand problems if they don't try to convert the steps taken into words.

Azzolini (1990 : 92) states that

> "Because writing is a way of clarifying and refining one's own thoughts as well as communicating with others, mathematics has a rich history of using writing to learn."

Azzolini (1990 : 96) advocates that students write their own word problems.

She gives three possibilities:

1. Write your own word problem similar to the ones you have been doing in class.
2. Change the problem you write in question 1 so that it is an easier problem.
3. Change the problem you wrote in question 1 so that it is a harder problem.

Other ideas propogated by Azzolini include word banks or a list of mathematical words used for explanation of a technique and debriefing which means going back over the steps of various procedures with students listing for themselves the steps and summarizing the format or topic.

The method used by the lecturer to communicate may also be a stumbling block. For example, a weak voice may be useless in a large room, similarly a poorly prepared transparency may be worse than a good exposition on the blackboard. The questions asked by the lecturer may be mainly rhetorical rather than assisting students to formulate answers. The fluency of the lecturer with breaks or pauses may greatly influence the students understanding of the material.

Briginshaw (1987a. : 329) identifies three categories of lecturers who lack communication skills:

> "(i) Mumblers and scrawlers
>
> Persons who are pathologically incapable of speaking and/or writing are clearly a *priori* poor communicators.
>
> (ii) Lack of training
>
> Who trains teachers of undergraduates and by what means and for how long? Most practitioners are self-taught teachers and, in my opinion,those that are not are poorly trained, by the wrong means and the wrong instructors.
>
> (iii) Lack of innate skill
>
> It is the existence of innate skills, of timing, of design, of emphasis, etcetera that distinguish the great communicator from one who is just competent. I do not know if such qualities can be acquired 'post facto' of appointment, I suspect not."

The problems of communication in mathematics have raised two problems:

1. Is the lecturer really conveying a clear picture to the student of what should be learnt?

2. Has the student a sufficient repetoire of mathematical language, formulae, identities and set routines to understand a new concept?

The first point leads to a discussion of the methods used by lecturers involving transparencies, questioning and timing of the lecture. The second point was examined with suitable comments by Countryman (1992 : 2-12) and Azzolini (1990 : 92-100) on the way mathematical language can assist the learning process. This continues in the next section on problems created by deficiencies in Mathematical knowledge.

3.1.2 Problems created by deficiencies in Mathematical knowledge

A major problem for the student is that each new piece of mathematics learning content depends on previous content he has learned. If there are gaps in his knowledge or weak points then the new learning content will not bond to previous content. Even one poorly prepared area such as solving a quadratic equation or the ability to draw a graph or sketch will seriously handicap the process of learning. The learning process in mathematics depends on logical steps and assumptions that each step will be understood. Teachers usually repeat learning steps but in tertiary education the pace of work may limit the repeating of information. There is a fundamental change from school learning where the student has small packages of mathematics learning content which are repeated until they are familiar. This is replaced in tertiary learning by a fast moving syllabus in which a study package is lectured once and then reinforced by a tutorial. This leads to continuous pressure on the student to learn thoroughly what is being presented. The learning curve is a very contemporary issue when teaching a new subject or a computer package. This assumes that the normal person requires "x" hours to master a subject or package. The lecturer can provide four or five hours a week for the subject but only the student can learn those items. His learning is inhibited by past success or failure. Thus, if he has consistently failed to master trigonometrical identities then whenever these are needed for something else he will be hesitant over their use. Many students falter when these gaps or deficiencies in their learning process are exposed when doing problems or learning new work.

Tall and Razali (1993 : 218) in an investigation into difficulties experienced by students in Malaysia during pre-requisite courses for mathematics for engineering studies, distinguish between the more able and the less able learners:

> "The more able need to remember less because they can reconstruct more. The less able see more information as a burden as even more disjoint pieces of information to remember - an increased burden on a weaker back leading to greater probability of inevitable collapse."

Tall and Razali (1993 : 218) also postulate that the less able have a fundamentally different viewpoint mentioning those points which cause them difficulties. These are firstly being less able to crystallize or condense processes into concepts they can handle. Secondly they have fewer concepts so they rely on familiar routines. Thirdly, although they handle these routines, they don't relate them or give them a meaning.

3.1.3 Problems associated with recall of knowledge

Briginshaw(1987a. : 331) discusses assimilation and recall. By using a paper with ten questions with 102 marks possible from six questions and an average class mark of 50 he concludes the average performer only appreciates well 28% of the syllabus at the time of the examinations. But he also concludes a memory slip of 90% from year one to year two so in fact only 2,8% can be recalled out of the blue. If the memory is jogged first then the memory slip is only 30%, but this still means only 19,6% of first year work is available in memory for year two.

This complicated formula might lead one to despair for your second year students but Briginshaw(1987a.: 332) also concludes that much of the first year work is 'throw away' that is not required after first year. Therefore provided the student can recall the right material he will survive.

A good example of the right material to remember from first year calculus would be the rules of differentiation such as

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

which could be applied to second year work. This section shows that constant revision or selective jogs to the memory are necessary to achieve a successful outcome to mathematical learning. The need for advance organisers which introduce the new material at a higher level of generality may work with better students but rarely do weaker students grasp the general first without specific examples leading to general principles.

3.1.4 Problems associated with the choice of method

Another problem for the lecturer of mathematics is what method should he use to introduce a mathematical concept? There may be a general approach using a formula or there may be a particular method which can later be changed to a general method.

For example, a quadratic equation can be solved by a general formula, that is

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

where a, b and c are the coefficients of the equation $ax^2 + bx + c = 0$ or you can factorise or complete the square. Eventually the general method will prevail but the factorisation and completion of the square will be needed in other situations with quadratic expressions so learners miss them out at their peril. The selection of suitable problems is something the mathematics lecturer has to wrestle with every day. Should he give a couple of simple examples which the student can follow or should he give one difficult example which the student will have problems following? The student body will vary greatly in their response to examples so this must be borne in mind.

3.1.5 Problems with problems and tutorials

3.1.5.1 The problems students have with problem solving as part of the calculus course.

Lecturers in mathematics distinguish between routine exercises which repeat the same kind of example with different numerical answers and problem solving where each problem requires a formulation from the facts given so that a mathematical method can be used The routine exercise for example would state: "solve the following equation" whilst the problem type question would require the student to formulate an equation from given information and then solve it giving meaning to the solution in the context of the given facts.

Bell (1978 : 311) states

> "Problem solving is a fundamental process in mathematics and constitutes a considerable portion of the work of mathematicians. Consequently, students can better learn about the nature of mathematics and the activities of mathematicians if they solve mathematics problems."

Students find problem solving a difficult art. In Question 7 of the Student Questionnaire "logic," that is a combination of word problems and reading and analysing a problem (See Annexure A), was placed first in a rank order of difficulty by 64% of the students. The reasons for this were that students find difficulty in converting the words of a problem into the symbols of a mathematical method or equation. They also have difficulty in understanding what the problem really means.

Bell (1978 : 311) indicates the following difficulties:

> "However problem solving can also decrease motivation if speed, precision, format, neatness and finding the correct answer become the objectives of problem solving in school. Problem solving is difficult and it can be very frustrating for students if their teachers do not exhibit patience and understanding and offer unobtrusive assistance".

Calculus as part of mathematics does lend itself to problem solving. Early items in the calculus such as rates of change, acceleration, velocities, maximum and minimum situations all lead to word problems. The classic example is to find the relationship between the height and the diameter of a closed tin when you get maximum volume from a certain given area of metal. The above wording is already formulated mathematically and the question would normally be phrased in a less structured manner. Variables have to be introduced and a suitable method evolved. Students have difficulty deciding which items are variable and which are constant. Differential equations leads to a rich harvest of problems for example the decay of radioactivity, the growth of bacteria, the behaviour of electrical circuits and the vibrations in bridges to mention but a few. Students would rather be given a differential equation than have to sort out the problem dealing with doubling the growth of bacteria in a given time when the rate of change of bacteria is proportional to the number of bacteria present at a given time.

It is important for students to use this method in the calculus as ultimately many of them will be problem solvers for their firms so the methods used to solve mathematical problems will inevitably be used in their engineering background when they leave technikon.

Bell (1978 : 311) states

> "Principles that are learned and applied in classroom problem-solving situations are more likely to be transferred to other problem-solving situations than principles that have not been applied in solving problems"

Furinghetti and Paola (1991 : 721 - 724) investigate the construction of a didactic itinerary for the calculus and one of their stages is to give students a good command in solving problems through the application of their new knowledge. These two conflicts of interest have to be reconciled.

Since tutorials are used for problem solving by students it may be opportune to consider tutorials as a didactic mode and to consider the problems which arise over their use by lecturers and students. Furinghetti and Paola (1991 : 723) make the point

> "The activity of problem solving allows construction of the cognitive goals and abilities and at the same time prepares the ground for the following rigorous settling. This settling must become considered necessary by students."

They further indicate this is a way of fixing an image in the mind of the students which is easier to recall. Another similar problem is that one group of students want the maximum of theory and a minimum of examples done by the lecturer while a second group want the minimum of theory and a maximum of examples done by the lecturer. The first group of students like to feel they have solved the examples themselves while the second group really want as much explanation as possible to help them solve examples. These two conflicts of interest have to be reconciled.

3.1.5.2 Tutorials as a didactic mode and the problems arising over their use by lecturers and students.

Mathematics courses at tertiary level usually include one or two tutorial periods. The use of these periods seems to play an important role in the better understanding of the course as a whole. Van Rooy (1984 : 63) discussing the main aims of the tutorial method emphasises the personal contact with students so that individual problems with study material can be ironed out. This is the ideal situation if tutorial classes are kept small, say 20 - 25 students, and students needing help do attend. In practice classes may be large, up to 50 students whose attendance will be erratic so the personal contact may be lost.

Van Rooy (1984 : 64) states

> "Tutoriaalklasse aan groot groepe ontaard dikwels in niks meer as 'n reeks minilesings nie."

A very important aspect of tutorials is the choice of problems for students to do in the tutorial. Some will be routine relating directly to the preceding lectures but one or two should be challenging so the brighter students do not get bored. A problem for the lecturer is what sort of guidelines or assistance should be given at the beginning of the tutorial. A list of formulae, hints on beginning questions or a method which should be used for a particular question for example integration are possibilities.

Searle (1979 : 553) says one of the aims of tutorials is

> "To acquire experience and confidence in the handling of mathematics by the knowledge gained from problem solving i.e. the mastery of study material content."

Searle (1979 : 554) also mentions that tutorials give students the chance to be actively involved in the learning process.

The weaker students may have problems in completing tutorial questions so there is a need for assistance from brighter students or the lecturer. On some occasions an open forum with question and answer (not necessarily from the lecturer) would solve the weaker student's difficulties.

In the Staff Questionnaire (See Annexure B) dealing with time for tutorials only 25% of staff felt tutorial time was adequate whereas in the Student Questionnaire (See Annexure A) nearly 80% of students felt tutorial time was adequate. Staff may feel that more tutorial time would ensure a greater number of examples are tackled under supervision while students may feel that they would prefer to tackle the examples outside of the lecture time. However, in the Student Questionnaire (See Annexure A) in Question 18 dealing with time spent on study, 70% spent less than 6 hours a week or one hour per night studying on their own. It seems unlikely that students will do enough in their spare time so tutorials are an essential part of their course. A final point of view expressed by several colleagues is "do we really need tutorials, couldn't we turn part of each lecture into a tutorial?" This may be possible at the first level but the pressure of the syllabus precludes this at the second and subsequent levels.

3.1.6 Problems associated with the time allocation for various topics in the syllabus

In a semester syllabus of sixteen weeks each topic is allocated a certain amount of time. For example, in the Mathematics II syllabus for Mechanical Engineers four weeks is allocated to differentiation both direct and partial plus applications. Similarly integration is allocated four weeks, differential equations is allocated four weeks and matrices four weeks. The testing procedure makes it difficult to alter these allocations as a test is usually set at the end of four weeks. The topics do not necessarily fit within the time frames and depending on the background of the students they may need more or less time for a topic. In addition several groups follow the same syllabus for example mechanical and electrical engineers follow the same Mathematics II course so there has to be synchronisation of topics that is starting and finishing at the same time.

Also within each topic there are several sub-divisions which require two or three lectures to cover. A good group may need two lectures, a poor group may well require three lectures. If this ratio is maintained throughout a four week section then the good group will cover the work in three weeks while the poor group could require four and a half weeks. The weaker students fall behind on the above time schedule and inevitably cannot cope with the full syllabus.

In Question 15 of the student questionnaire (See Annexure A) it showed that over 70% had difficulty with time after six weeks of a sixteen week course.

3.1.7 Impact of success or failure on students' motivation

At some stage during a mathematics course a tertiary student will have to experience success otherwise the motivation to continue will disappear. This is a major difficulty to learning if the students past record indicates only failure. From experience one small area of success will keep a student going through several difficult sections. In planning a course this encouragement should be built in with possibly an easy short test. Students who are motivated will pay more attention to lectures, will attempt all the examples given and query those points they don't understand. On rare occasions a student may also give the lecturer a new method or approach to a problem which the lecturer has not done previously. When a student does this his self-esteem rises dramatically.

Protter (1991 : 4) states

> "The second class of students for whom self pacing is useful are those who have the ability to learn mathematics but are slow at it. In a regular course such students will get about a C and thus learn a random 70% of the material. However in a self paced course, the same students will learn thoroughly the first 70% of the course set a good grade but earn three units instead of four units. As a bonus the student learns to read mathematics gains confidence and comes to appreciate the positive reinforcement that goes with passing the segment tests."

Conley et al (1992 : 176) emphasise that

> "The degree to which the student participates in learning activities has a major impact on what is learned. It is not only overt participation in learning activities that determines what is learned, however. The mental activity that takes place during the activity by each individual is as important as what the individual appears to be doing"

What both of those authors are saying is that the more students become involved in the activity of learning the more likely they are to succeed and complete their courses.

Smith (1991 : 153) mentions that

> "Success is fun. Your enjoyment in seeing the positive results of your efforts, reflected in both increased learning and higher grades should feed upon itself and further motivate you to try your hardest in the course."

3.2 TEACHING AND LEARNING PROBLEMS IN CALCULUS

In the previous Section 3.1 some general teaching and learning problems in mathematics were discussed. These affect the teaching and learning problems in calculus. Several special problems in calculus will now be discussed relating to language, extension of basic processes, problems with calculators and computers, mathematical modelling in calculus and use of calculus in real situations.

3.2.1 Language problems in calculus

When the student begins the calculus he is presented with a new language with words like infinitesimals, differential, coefficients, primitive, limiting values, etc. Sometimes there is a learning problem because the student never really understands these words. In this regard the problem of the calculus is no different to the general mathematic problem where new terms continually occur. The major difference occurs with symbols which all have a special meaning in calculus, for example,

$$dx, dy. \frac{dy}{dx}, \frac{\delta y}{\delta x}, \int, f'(x), f''(x)$$

A basic premise in calculus is that of limits, a result achieved when a certain value of x tend to zero in an expression, for example,

$$\frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

$$\textit{limit as } \Delta x \to 0$$

This requires a lot of practice in symbols and in diagrams which illustrate what practically happens. The lecturer or teacher tries to explain this but finds the student is only interested in the result, in this case finding

$$\frac{dy}{dx}$$

or the gradient using a set rule. The problem of notation is always present in mathematics but is more acute in the calculus. Students may well be lost after the first three weeks of a calculus course because they find it impossible to absorb the new notation.

Teaching the calculus can be a hard exercise when students cannot link the various strands together. They fail to identify, for example, the graphical interpretation of a gradient with the first derivative of y with respect to x or with the expression

$$\frac{dy}{dx}$$

or later with the rule of change of y with respect to x. Merely being aware that

α

$$y=f(x)$$

is governed by a rule, is very inadequate for future work in the calculus. Examples must be selected to correlate the different approaches. The inverse function denoted by

$$f^{-1}(x)$$

poses great problems for students and lecturers. Even (1992 : 557 - 562) in a study at the Weizman Institute of Science in Israel found that less than half (53 out of 123 students) could answer the following question on inverses:

Given

$$f(x)=2x-10$$

and

$$f^{-1}(x)=\frac{x+10}{2}$$

find

$$f^{-1}.f(512,5)$$

This question is logical, not numerical, since the combination of an inverse of a function and the function itself gives the identity which leaves the x value unchanged

$$(f^{-1}.f)(512,5)=I.(512,5)=512,5$$

Over half the correct answers contained a calculation which indicates a lack of confidence in the notation. Even (1992 : 557-562) concludes that both procedural and conceptual knowledge and the relationship between them is needed.

Ervynck (1992 : 217) states the following:

> "Mathematics, although an abstract discipline, relies heavily on the use of an appropriate language. A closer examination of the way mathematicians "talk" about their theories reveals that this language is above all a written language, introducing specific symbols, which represent concepts rather than words, intermingled with bribes of natural language. We call this language Math."

Ervynck (1992 : 220) later likens 'Math' to an extinct language such as ancient Egyptian where a page of text is not read at an eyeglance but has to be decoded. Unless students comprehend the special terms and shortened forms used in mathematics and calculus in particular they will not be able to read mathematics. There have to be various additions to the mathematical vocabulary at each stage so that students can understand what is written on the blackboard or what appears in a textbook.

3.2.2 Problems with extending basic processes in calculus

Another aspect the student has problems with is when the differential process has to be extended to second and further derivatives. This raises the question of simplifying the first derivative before proceeding to the second and further derivatives. This simplification will be essential when expressions are complicated, for example, products involving square roots and trigonometric functions. The teacher is continually having to remind students of simple identities in trigonometry or showing students how to simplify fractions involving square roots. This lack of fluency slows down the progress in the calculus. This happens despite the time given in the first mathematics courses on algebra and trigonometry. It is a minefield for many students and only a few reach the other side without mishap. The teacher cannot remove all the obstacles, the student has to eventually remedy the defects either by experience with several examples or by devoting some time to his weak points, for example, logarithms or trigonometrical identities.

Tall and Razali (1993 : 209 - 222) highlight various difficulties students have throughout their mathematical processes. For example, order of operations, meanings of logarithmic expressions, solutions of equations with symbols or both sides and simplification of expressions of the form

$$\frac{x-1}{2-x}-\frac{1}{x-2}$$

In simplifying this expression only 6% of the lower group of students, that is the lowest quartile, were able to give the correct answer.

3.2.3 Problems with calculators and computers

Another modern difficulty which affects students is their reliance on a calculator. There are parts of the calculus which are numerical so a calculator is a help but again there must be understanding of the calculus to avoid errors when you press the buttons. For example, using radians when substituting for trigonometric expressions in integration.

Lecturers must go some way to solving this problem and be alert to student difficulties. A similar problem exists with computers.

Furinghetti and Paola (1991 : 727), commenting on this problem, state

> "We point out that, when dealing with information technology, the didactic materials have to be chosen in such a way that the attention of the students is not addressed to problems concerning programming, but to the contents of calculus that we intend to discuss ... When using information technology as a help in learning mathematics, be conscious that they are doing mathematics and not computer science."

3.2.4 Real situations and mathematical modelling

The model used in mathematics is crucial to understanding a new concept. For example equations are usually associated with the algebraic model which handles symbols such as x, y and z according to set rules of behaviour. These rules include such items as the commutative law, distributive law, the four arithmetic operations of addition, subtraction, multiplication and division plus factorisation and simplification needed to solve an equation. Geometrical models use space and shape to examine invariant properties when bodies are rotated or stretched. Designers and artists favour this model to make sense out of their structures. The trigonometrical model originally used for calculation of heights and distances has increasingly been used to solve wave equations, expand our knowledge of Fourier Series, to provide parametric alternatives to various x, y equations and to contribute to various particular solutions of differential equations. Another model used in the calculus is the graphical model which was used to illustrate results but with the advent of the computer it can be used quickly to give numerical answers instead of the logical route via equations.

When the student is presented with a written set of information then decisions have to be made about the model to be used. The words are translated into symbols with given values and then a decision whether to use one of the models has to be taken. The problem for some students is that they are not versatile so that they will always use the algebraic model when a graphical model or trigonometrical model would be much quicker.

When calculus was invented in the seventeenth century astronomy was crying out for accurate methods of determining lengths of curves, areas under curves and volumes when curves were rotated. These methods could be said to be 'real situations' when the answer is immediately applicable to a scientific problem. The area of a circle was one of these, the length of an elliptical orbit was another. It is therefore logical that students should be able to solve situations such as finding the centre of gravity of a hemisphere or discussing the rates of change of sliding bodies from fixed axes (for example, a sliding ladder against a fixed wall and ground) or finding the maximum area of a given curve with certain initial conditions. Word examples as opposed to rote examples add another dimension to the difficulties experienced by students. The translation of words into symbols and then into equations which can be operated on by the rules of the calculus generally proves a tough exercise. Many teachers try to avoid such examples or at best give a few simple ones. It is these examples which give the real meaning to the calculus so it should be essential to face some as early as possible.

3.2.5 Review of teaching and learning problems in calculus

The problems discussed in Section 3.2 relate to the structures developed by the student. Language forms one part of that structure. Imperfect use of language can seriously handicap a student. The extension of basic processes depends on a solid base of knowledge in algebra, trigonometry and graphs. The problem is how to retain this knowledge for later use. While calculators and computers can ease the load of solving equations they also generate language problems. The use of the correct model in calculus is also a problem for some students who are not capable of switching from say the algebraic model to the trigonometrical model when this is simpler.

In the two previous sections (3.1 and 3.2) detailed analysis of teaching and learning problems in mathematics and then more specifically in calculus have been examined. Section 3.3 will concentrate on the problems the lecturer has in achieving some of the aims of the calculus course. Then in Section 3.4 various viewpoints on the learner's problems are reviewed. The final section completes the didactic triangle by looking at the problems generated by the structure of the curriculum.

3.3 AIMS AND METHODS AS VARIABLES IN TEACHING CALCULUS

One of the aims of teaching calculus to engineering technikon students is to link together the various engineering disciplines with a common background in calculus. So whether a student studies mechanical, civil or electrical engineering the objectives should be the same. The problem for the lecturer is that each engineering department has different priorities so they fragment rather than unify the syllabus. Therefore the aim of linking together disciplines is lost when alternatives have to be pursued for different disciplines and only half the second level course is common. Bressoud (1992 : 615) articulates students' expectations as follows:

> "They know it is going to be hard, but they also expect that this will be the course that draws together the mathematics that they have learned and transform it into an instrument for comprehending the world around us."

Later Bressoud (1992 : 616) states

> "We teach Calculus because it is important for an understanding of who we are as a society."

Another aim of teaching calculus is that all students become successful in solving problems involving calculus.

This aim seems to be unattainable for some students. As Tall and Razali (1993 : 209) so aptly state

> "The aim of mathematical education is surely success for all pupils, yet it seems to be a fact of life that whilst a few prosper in mathematics a much greater number find mathematics difficult."

At a lower level the lecturer may be satisfied that the majority of his students can differentiate and integrate polynomials whereas at a higher level the lecturer would be happy if more than half his students could solve problems involving 2nd order differential equations. In mathematics the 100% aim is usually very unrealistic especially with applied problems.

One aim which eludes many lecturers of mathematics is how to persuade students to read books on mathematics as intelligently as they would read a book on a great statesman.

Toumasis (1993 : 553) enlarges on this difficulty:

> "as mathematics teachers, we often voice complaints about our students reluctance to read their mathematics textbooks or study mathematics by themselves. The typical mathematics student expects a teacher to explain what the book says. Students may read a chapter in a history text and answer questions without asking for any explanation, but in mathematics, after the student reads the material the typical response is "Fine, I read it. Now what does it mean?" Material and students cry out for additional explanation."

Cowen (1991 : 50) argues

> "We should teach our students to read and understand mathematics. And to reinforce our effort, we must test their ability to do so."

Cowen feels that learning to read should not be left to chance. There should be opportunities for students to practice by restating theorems in another way and drawing out corollaries.

During the author's teaching experience there has been a considerable diminution in the number of theorems required for matriculation and formal writing in mathematics has been ignored in favour of doing examples.

Cowen (1991 : 53) ends his article thus

> "We will all be better off if the students view the textbook as a source of information, not just as a list of exercises interspersed with messages for the instructor."

The modelling of situations in science and technology is a powerful weapon in the hands of the scientist and engineer. An important aim in calculus teaching is to help students build up and model for further reference. An example of this is the model for maximising and minimising a function. Raw data have to be transformed into suitable equations, variables eliminated until there is a function of one variable and then the rules for the model can be applied. At a higher level factory processes can be similarly modelled to give differential equations solved by the appropriate method.

Laridon (1981 : 158) states

> "By using mathematical modelling techniques the irrelevant aspects of a situation may be eliminated and the essential couched in the language of mathematics... The calculus is frought with possibilities here ... The classical optimisation problems are only one class of example."

Mathematics lecturers find that aims reflect their expectations so there is a wide range of possibilities depending on the institution, the discipline, the quality of the students and the time allocated to the course. Within those constraints lecturers accept certain standards or limit their aims. The examples quoted were a unified syllabus, success in solving problems, persuading students to read mathematical texts intelligently and helping students to model mathematical situations. None of the above could be achieved for 100% of the student population. In fact a 50% rate of achievement in the first two is more than likely.

3.4 THE LEARNER AND LEARNING PROBLEMS IN CALCULUS

Most students carry learning problems from one stage of mathematics to the next. There is no 'clean slate' to work with so teachers have to attempt to erase these learning problems and add further processes to learn.

Tall (1993 : 209) adds the following:

> "In this paper we will discuss a theory which indicates that there is a continual divergence in performance between those who succeed and those who fail which is exacerbated by qualitative differences in their thinking processes."

It was previously mentioned in Section 3.1.2 that deficiencies occur in student's knowledge. Tall takes the view that the less able find it difficult to retrieve previous information.

Tall (1993 : 219) states

> "The mind of the less able is like a flawed computer diskette. Sometimes, it will respond well to some mathematical problems (usually the easier ones) and will 'blank out' to more difficult ones. Knowledge makes an imprint on the mind of the learner but in the case of the less able this imprint is not clear enough so only a garbled version can be recalled. In many ways this is worse than no knowledge of the process."

Repeatedly errors indicate this garbled recall, for example

$$\frac{1}{a-b}=\frac{1}{a}-\frac{1}{b}$$

which creeps into calculus.

$$\int\frac{dx}{sinx-cosx}=\int\frac{dx}{\sin}-\int\frac{dx}{cosx}$$

Tall (1993 : 218 - 219) postulates

> "The more able need to remember less because they can reconstruct more ... Learning occurs not in the act of remembering but in the gradual development of mental frameworks unique to each individual. In other words students learn by modifying their mind's program, not by storing new data in their mind's memory."

Mathematics and calculus in particular do not merely consist of a bunch of facts to be learned, there are structures which fit together or a jigsaw with so many pieces which lock into place. The learner has to construct his own answers from these structures.

Toumasis (1993 : 556) observes

> "The number of students who do not participate actively and who passively wait to be supplied with the answers, without thinking for themselves or without going through the process of reasoning is increasing. Few students have the willingness to learn mathematics and study it voluntarily."

To help the learner Toumasis proposes a new approach to the traditional lecture and drill format which can be summarized as presentation, reading, discussion and exercise. The lecturers' part is the presentation or overview of the topic, the reading and discussion take place in the classroom or as Toumasis (1993 : 565) says to foster auto-learning habits in the students. The exercises are solved from the text and checked. Guides are given to the reading but an active approach to learning does seem to result.

3.5 THE CURRICULUM AS A VARIABLE IN TEACHING CALCULUS

There is considerable debate on how the curriculum on calculus should be structured. The standard approach is to teach the basic elements of calculus starting with real numbers, limit, continuity derivative and integrals before moving on to differential equations.

Barnes (1992 : 73) has the following viewpoint for non-specialists:

> "I believe the most important ideas are those of differential equations and the ways they can be used to model processes involving gradual change, in either the natural or constructed world. I do not believe there is a need in a first course for work on limits or differentiation from first principles. I think it is also irrelevant for non-specialists to spend time learning techniques for curve sketching and finding maxima and minima both of which can be done far better using a graphic calculator or computer graphing utility."

Norman (1992 : 71 - 72) feels that not only non-specialists but many of specialist calculus students would benefit from a so called non- specialist course.

Smith (1992 : 67) states

> "Then in the early part of the course the concept of a limit can cause confusion. Students endure our explanation but long for the 'real' calculus to begin".

In the U.S.A. engineering faculties are revising their calculus programmes to make them more practical and responsive to industry.

At the University of California in Berkeley, for instance, a four year "mechatronics" programme has begun which merges mechanical and electrical engineering courses. Cornell University has developed a combined civil and geotechnical engineering programme.

McWilliams (1993 : 47) states that

> "In another key change some schools have begun teaching maths and science within engineering courses rather than in separate first and second year classes. Carnegie Mellor University now teaches differential calculus as part of a sophomore electrical engineering course. This "just in time" approach means students learn calculus or chemistry in the context of engineering problems."

There is an opposing point of view put forward by mathematicians who admit there is a small lag in the transmission of new techniques to their methods lecturers and a longer lag to the service lecturers of engineers but they still feel it would be unwise for engineers to inflict on students their brand of mathematics from their student days.

Briginshaw (1987b. : 217), referring to dissatisfied engineering faculty members, says

> "sometimes this dissatisfaction attains the level of a demand to "teach our own mathematics by which is meant to teach the techniques and notation that I learned twenty years ago". But if that is surely a recipe for disaster with the partially sighted, hearing and speaking leading the same then teachers of mathematical methods at this level must equally well make sure that they are not being irrelevant, over-formal and excessively abstract in the presentation of their course."

The debate is also intense and continuous about who should teach engineers mathematics. At a symposium held at Pretoria Technikon in 1993 there was a clear cut division between Pretoria Technikon who are going back to mathematics taught by engineers and the other technikons who felt there is a place for a separate mathematics department. The symposium placed considerable emphasis on the need for more communication between mathematics lecturers and clients (users) and the use of up to date techniques such as computer packages. It is clear from these quotations that the calculus curriculum will change due to outside pressures but it is doubtful if outsiders will teach mathematics better. In the U.S.A. the engineering schools hope to increase their numbers if the mathematical pre-requisites are incorporated in their engineering courses on a 'need to know' basis. Mathematicians are aware of change and it would be better if they revised their curricula and updated their techniques while still operating their own service departments.

3.6 SYNTHESIS

In this chapter it has been shown that the language of mathematics can be a major problem for students as each new section produces new words or phrases. For example, the concept of differentiation cannot be adequately explained if students do not understand the idea of a limit or have no notion of what is a function. The translation of everyday language to a mathematical form requires practice by students. Lecturers can help by giving a varied teaching model which includes advance organisers and clear patterns to follow. The tutorial as a didactic mode was investigated with various possibilities for making the best use of the time, for example, the introduction should be brief but relevant with formulae and methods for the questions being given in the tutorial. Explanations can be 'one to one' or use can be made of brighter students for explanations. Good use of time over the whole course was stressed with the difficulties weaker students encounter in this regard being investigated. A possible solution by Protter allows students to take 70% of the course and earn three units instead of four units. The model to be used in calculus was a vital question which was investigated. They included the algebraic, trigonometrical graphical and computer models. All have a part to play in understanding calculus. Using the techniques of able students to help weaker students was discussed with reference to research by Tall and Razali. Finally changes in curriculum such as merging mathematics and engineering into one course were investigated. Mathematicians are against this change and feel it is better to revise and update the techniques of present mathematics service courses.

CHAPTER 4

A CASE STUDY OF TEACHING AND LEARNING PROBLEMS IN CALCULUS AT SOUTH AFRICAN TECHNIKONS

4.1 MAIN AREAS OF DIFFICULTY

In this case study of teaching and learning problems in Calculus at South African Technikons there are certain main areas of difficulty. These include the ones presented in Chapter 3 such as language, symbols, time and methods of lecturing with additional items such as background knowledge of students and lecturers. A detailed study will be conducted of what variables affect the student at technikon such as academic ability, age of the student, previous institution of learning, background knowledge of algebra and trigonometry and time spent outside lectures on mathematics. A similar study of the lecturer dealing with academic qualifications, previous institution of learning background knowledge and time spent on lecture preparation is investigated. This is followed by the variables in the content of semester courses in engineering mathematics referring to changes in the syllabi and the increasing use of training technology. The next section brings together the common problems of lecturer and student and how they interface with content. The last section deals with possible strategies to overcome the didactical problems identified in the calculus covering such items as cooperative learning, use of appropriate language, variable credit course and use of mathematical technology.

4.1.1 Language and Symbols

It is apparent from the results of the Student Questionnaire, (See Annexure A), that students have considerable difficulty with mathematical words and symbols. For example in Question 8 the word calculus meant differentiating and integrating functions to 75% of the students questioned and only 8,8% felt it was a branch of mathematics that enabled areas and volumes to be calculated. In question 9 the symbol $\frac{dy}{dx}$ only represented one meaning to 73% of students such as gradient, first derivative and rate of change and only 27% recognised it could have all of these meanings. In question 12, 55% felt the integration sign meant the opposite to differentiation with 19% indicating it gave area under a curve.

It is small wonder that difficulties with language can leave the student confused about what he is doing when he deals with differentiation and integration. Linked with the language are the symbols which translate these new words into a mathematical code. Thus for example the first derivative of y with respect to x is $\frac{dy}{dx}$ or the the derivative of f(x) with respect to x is f'(x). The code must be known for without it the student will have great difficulty in extending the calculus. When students are presented with the rate of change of volume with respect to distance they don't automatically recognise this as $\frac{dv}{ds}$ or similarly when students work at a velocity time graph they don't automatically see the gradient as $\frac{dv}{dt}$ with the acceleration as an alternative. They appear to be stuck with $\frac{dy}{dx}$ and do not appreciate there is a family of derivatives of which $\frac{dy}{dx}$ is one possibility.

Sullivan (1991 : 165 - 166) has two lists of four notations for the first derivative

List 1 $f'(x)$, $\frac{d}{dx}(f(x))$, $f^1(x)$, $D_x f(x)$

List 2 y', $\frac{dy}{dx}$, $y^1(x)$, $D_x(y)$

He states:

> "Although it may seem desirable to have only one notation, each of the above notations is convenient at particular stages in the development of the ideas of calculus. In each list the first is brief, the second fits with differentials, the third is suitable in Taylor's theorem, and the fourth is relevant to the theory and application of differential operators. The only way to ease the problem of proliferation is to defer each notation until it is needed."

It is difficult to convince students they need more than one definition of the derivative. The better students do eventually realise the advantages of several definitions but the weaker students use only the first notation given to them whether this is the functional one, f (x), or the differential one.

There are many symbols which are continually used in calculus. For example,

$\sum$	- summation
$x \to 0$	- x tends to zero (rather different than x = 0)
$x \to \infty$	- x tends to as large a number as you wish
$\frac{\partial y}{\partial x}$	- the partial derivative of y with respect to x keeping all other variables constant.

Some students need constant reminders when lecturers use such symbols in their explanations.

For example

$$\sum_{\delta x \to 0} y\delta x = \int y\,dx$$

is a concentrated way of dealing with area under the (x;y) curve.

4.1.2 Time

The amount of time devoted to sections of the calculus is a main difficulty in the Technikons. Courses are allocated sixteen weeks of lectures to cover at least a year's work at school. This change of pace may stimulate the better students but can prove too much for the average to weaker students. Syllabuses have been increased and changed which puts pressure on lecturers to cut down the time given to earlier sections in order to complete the syllabus. Time is constantly on the minds of both lecturer and student with deadlines for tests, tutorials and examinations always pretty close. Any time lost through holidays, special occasions or the illness of student or lecturer can be crucial in the final result.

4.1.3 Methods of teaching

The methods used for teaching or lecturing can be classified as a main area of difficulty. Some students have difficulty following Mr X while others will consider Mr Y as the best teacher of mathematics they have ever had. What makes the difference? Each lecturer will adopt a certain style. It could be one of the following:-

a) A continuous lecturing style which does not allow any interruptions
b) A prepared transparency style which gives solutions to problems on transparencies but allows questions from students
c) An exposition of theory on the blackboard followed by a problem on transparency
d) A combination of blackboard and overhead projector to explain theory and diagrams.

Meyer (1993 : 15) has a parable which describes what is heaven like and what is hell like? It describes three professors of mathematics. Professor A is a good lecturer, clear well organised and enjoys a large class audience. Professor B does not lecture much but works on a problem basis giving out individual or small group projects. Professor C uses computers as a research and teaching tool, rarely lecturing but spends her time circulating among her students answering questions, posing problems and directing student efforts. Meyer suggests that heaven is when these three professors operate in their best environment. Meyer takes the opposite situation when the Professors try to change their successful routine for another routine which doesn't work for them. This he regards as hell for the students.

The lesson to be learned in this parable is that variety should be allowed in lecturing styles provided the lecturer is successful with the particular style he uses. Variety and breaks may make it easier for the student to follow so the style is not necessarily the only problem. The notation used by the lecturer may clash with previous work so adapting to new symbols may be a hindrance. Obviously the delivery of the lecturer, the confidence he displays, the enthusiasm for the subject and the general rapport with the audience will make a great difference to students.

4.1.4 Student Response

The way students respond to both the lecturer and the material presented depends on several didactic issues. The first one is the introductory phase of the lecture. The advance organiser consists of an introductory structure in general terms which can be used in the lecture to focus student's attention.

Bell (1978 : 232) states

> "The advance organiser when properly structured and received in a meaningful way by students is designed to assist students to develop mental structures which will help them comprehend new learning material and integrate it with other material."

The advance organiser can be a verbal statement, a demonstration, a group discussion, a game, a laboratory exercise, a model or a film. The subsequent material will be related to the advance organiser in a more specific form.

Mathematics teaching and lecturing require intelligent advance organisers which set the scene for the student. It is probable that lack of response by students may indicate that this early structure was poorly presented. In calculus the advance organiser may involve for instance the graphical knowledge of gradient in preparation for a discussion of the first derivative or it might be a general set of identities in trigonometry in preparation for the limiting problems encountered in differentiation of trigonometric functions.

The second didactical issue is that of problem posing. Students have difficulties if they do not understand the problem posed by the lecturer. For example, in mathematics the solution of a quadratic form of equation is not obvious to the student. The lecturer will have to pose one or two suitable questions to identify the equation as basically a quadratic and then reconcile it to the usual quadratic equation and solve it. Closely linked to problem posing is problem solving.

Bell (1978 : 311) says the following:

> "Problem solving is a fundamental process in mathematics and constitutes a considerable portion of the work of mathematicians. Consequently students can better learn about the nature of mathematics and the activities of mathematics if they solve mathematical problems."

The problem solving model consists of five steps according to Bell (1978 : 312). These are

Step 1 Present the problem in a general form.
Step 2 Restate the problem in an operational (solvable) form.
Step 3 Formulate alternative hypotheses and procedures for attaching the problem.
Step 4 Test hypotheses and carry out procedures to obtain a solution or sets of potential solutions.
Step 5 Analyse and evaluate the solutions, the solution strategies, and the methods which led to discovering strategies for solving the problem.

The lecturer will have to demonstrate these ideas to students criticising the various hypotheses in Step 3 so that the best one is found in Step 4. Step 5 is very important to recapitulate the whole process and relate the solutions to the original problem so there is no invalid solution accepted. Typical examples of invalid solutions are negative values if the original problem involved a logarithm of that value. Students usually respond to problem solving if the lecturer asks the right questions. In technikons the choice of problem may determine the response of the students. Mechanical problems will invoke a better response with mechanical engineers rather than electrical problems. Unfortunately some topics have more applications in one branch than another. For example wave motion problems are basically electrical problems while centres of gravity belong to mechanical and civil engineering problems.

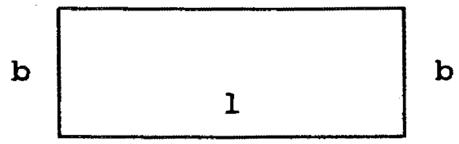
Many students fail to ask questions but this does not mean they have no problems. It could mean they don't know how to ask their question. In the enquiry teaching model the first step is formulating a question. Lecturers should engage their students in verbal discussion so that questions will arise naturally. Good questions by students themselves lead to weaker students responding much more positively to a lecture. A test taken at the end of the lecture may be a good guide to the student response but will need careful analysis to extract the main difficulties. Recognition of students existence and their response to what is said should be recognised as a major factor in helping students over difficulties in any subject but particularly with a logical topic like the calculus.

4.1.5 Applications

The applications of the calculus can occur in first level, second level or third level. As previously mentioned in the problem solving model a written general form has to be restated in an operational (solvable) representation. This means in calculus that students have to identify the variables, form equations using these variables and then solve these equations by calculus. All of these steps present difficulties to students. A written statement such as

> "A farmer wants to enclose three sides of a rectangular silo with 100 m of electric fencing, the fourth side being along a hedge. Find the dimensions of the largest rectangular silo he can enclose with the 100 m of electric fencing"

requires the student to make a representation of the facts. Basically this involves a rectangle



in which one side (that is the hedge) is fixed. The 100 m of wire will surround the three sides. The variables are the length, breadth, and area although because of the fixed length of wire it is possible to express the length in terms of the breadth

$100 = l + 2b \qquad l = 100 - 2b$

$A = lb$

$A = (100 - 2b).b.$

The equations have reduced to one equation to which the rules of the calculus can apply. These give that $l = 50$ and $b = 25$ are the dimensions to give maximum area or the largest rectangle.

In this application the student has to set up a diagram, see what is constant and variable, think of two possible equations, reduce them to to one then use the calculus. A "real problem" in industry usually involves considerable details with many variables requiring lots of good mathematics to solve it. The only way students can deal with "real mathematics" is to simplify situations, reduce the variables, tidy up or approximate the numbers and so limit the amount of mathematics needed to solve the equations.

All applications require a three way translation process namely

English words -----> Mathematical words -----> Mathematical equations

For example, a problem stated in English words would be: The rate at which a body cools is dependent on the mass of the body being cooled. The mathematical words would be: The rate of change of mass with respect to temperature is proportional to the mass. The symbolic equation to express the two statements would be:

$$\frac{dm}{dt} \propto M$$

Bookman (1992 : 1), in reviewing the National Science Foundation Workshop in Washington DC, states that students should be able to, among other things,

> "1. transfer techniques of calculus to other disciplines and novel situations;
>
> 2. Reason analytically and communicate mathematical ideas symbolically, in writing and orally."

This endorses the translation process required in applications.

4.1.6 Background Knowledge of students

The background knowledge of students particularly in algebra, trigonometry and graph work may be weak which leads to great difficulties in assimilating calculus ideas. For example the inability to simplify fractions in algebra is a major stumbling block in many calculus processes. A favourite one quoted by lecturers at university, technikon and other tertiary institutions is

$$1 + \frac{x}{y} = \frac{1+x}{y}$$

This clearly indicates that the student does not know that

$$1 = \frac{y}{y}$$

which means that

$$1+\frac{x}{y} = \frac{y}{y}+\frac{x}{y} = \frac{y+x}{y}$$

Confusion exists with regard to division in algebra largely due to lack of knowledge of division in arithmetic. Division in arithmetic consists of repeated subtraction of the divisor until there is a remainder of zero or a remainder smaller than the divisor.

For example:

$$7 \div 2 \textit{ equals } 3 \textit{ rem} 1 \textit{ Statement}$$

$$7-2-2-2=1$$

$$\frac{7}{2} \textit{ equals } 3+\frac{1}{2} \textit{ Equation}$$

In algebraic division the same procedure is applied until the remainder is of a lower degree than the divisor

$$(x^2+3x+3)\div(x+1) \textit{ equals} (x+2) \textit{ rem} 1 \textit{ Statement}$$

$$\frac{x^2+3x+3}{x+1} \textit{ equals } x+2+\frac{1}{x+1} \textit{ Equation}$$

The ability to use formulae especially trigonometrical formulae is a major problem with many students. A simple formula such as

$$\cos 2A = 2\cos^2 A - 1$$

is not recognised by the student in the form

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

which is required for the integration of

$$\cos^2 A$$

Tall and Razali (1993 : 211) speculate

> "An initial scan of the errors reveals a wide array of difficulties with specific knowledge. However the nature of the process failures followed a much smaller number of different patterns. We hypothesise that these patterns are indicative of a qualitative difference in the thinking between more and less able students."

It seems therefore that while background knowledge may be similar the errors in thinking will separate the less able from the more able. Tall and Razali list order of operations, logarithms, simplifying algebraic expressions, the inverse function and the modulus sign as basic difficulties.

Tall and Razali (1993 : 212) state

> "Thus one may hypothesise that the less able see the symbols as a process to be carried out in a written order while the more able can chunk the symbols together as sub expressions to be carried out to given conventions."

Other researchers have similar views on student difficulties.

Ferrini-Munday and Graham (1991 : 629 - 631) list several problem areas such as function, the derivative and the integral.

On function, Ferrini-Mundy and Graham (1991 : 629) state

> "This is one of the most fundamental concepts underlying the calculus. Despite its importance, students come to calculus with a primitive understanding of the concept and hold deeply rooted and firm misconceptions."

On derivative, Ferrini-Mundy and Graham (1991 : 630) quote research by Orton:

> "Orton found that the routine aspects of differentials were well understood. Answers on more conceptual tasks were far from satisfactory and the results suggest little "intuitive understanding of the derivative, as well as some fundamental misconceptions."

Similarly on the integral Ferrini-Mundy and Graham (1991 : 631) quote research by Orton:

> "Orton found that the procedure of dissecting an area or volume making use of the limit process, and the reasons why such a method works, were not part of most students' understanding of the integral."

4.1.7 Background knowledge of lecturer

In teaching calculus at the technikon the lecturer needs a good background of how the calculus fits into the engineering topics his students must study. The lack of this background can be a difficulty for his students. They want to relate to the calculus which is easier if the lecturer can point out the correlation with an engineering situation. For example, area under force/distance curve gives the work done so an integral of the force/distance relationship will give us the work done.

Andrie (1985 : 161) stresses

> "The selection of mathematical topics orientated towards practical needs motivates the students and suggest an adequate didactic method ... necessary to understand the rapid changes of technical development."

Consulting between mathematics and its service users will help the spread of the latest developments in both directions. Usually one only thinks of new ideas coming from say engineering to mathematics but the converse may also be very relevant.

4.2 STUDENT VARIABLES

In the previous paragraphs of Section 4.1 the discussion has centred on general problems such as language and symbols, methods of teaching and background knowledge of teacher and students. This section discusses the specific way students vary due to several variables such as the range of academic ability,the wider range of ages in technikon classes, the way previous institutions of learning may have an impact on the difficulties students have in their engineering mathematics courses at technikon, the specific background knowledge in algebra and trigonometry (which is vital to the processes of calculus) and finally the time spent outside of lectures in mathematics. Each of these variables can affect the difficulties students have in their engineering mathematics courses and in particular the calculus part of the mathematics course.

It has been said on many occasions that every student is different. This may well be true but the author has frequently seen repeats of certain student patterns either good or bad. It is these patterns that should be investigated.

4.2.1 Range of academic ability

Engineering students at technikons are admitted with a range of mathematical grades in matric. In 1990 it was possible to enter with an E standard grade. However, this was changed to a D standard grade in 1991. Later some technikons insisted on a C standard grade or E higher grade as the minimum standard. Owing to the time lag of students entering technikon it is possible to have a range of academic ability varying from an E standard grade to A higher grade.

The percentage of students with higher grade A, B and C symbol is much smaller than the remainder but in the author's research approximately 16% had these qualifications. The split between higher grade and standard grade is also relevant on the basis of two to one in favour of higher grade. These differences affect student performance. A student who has done a higher grade mathematics course is likely to have a wider grounding in algebra and trigonometry and to appreciate the idea of proof and identity. The standard grade student has been restricted to narrower domains of solution and has rarely examined alternatives to a given solution. Higher grade students who obtained an A, B or C will have most probably done the additional mathematics course where calculus, algebra and further trigonometry will feature as some of the options in the course. This will help the earlier stages of the calculus course. This first variable will have a major role in a students success with the calculus. Campbell (1991 : 93) speaking about students taking College algebra courses states

> "The students: many students have not completed the necessary prerequisite course(s) for taking College algebra. Some never took much mathematics before entering college, and now they have made changes in career plans which require a college algebra course. Many students who have "successfully" completed the prerequisite course(s) have skill levels woefully below what is needed for success in college algebra."

While the above refers to college algebra in the USA many lecturers in other countries also find that students are inadequately prepared for tertiary courses in mathematics.

In a similar vein Piccarelli et al (1990 : 369), writing about university entrants in the Science and Engineering Faculties of the University of Beri (Southern Italy), confirm

> "At the end of the secondary high school as many as 30 - 40 per cent of students had not mastered the skills of hypothesizing indentifying variables operationally defining variables designing investigations graphing and interpreting data which are commonly known as integrated process skills."

4.2.2 Age of students

Approximately one third of the second level students were aged 18 - 19 years and approximately one third were from 20 - 21 years of age. The balance were over twenty two. The first group came directly from school so that there was no break in their mathematical learning, the second group either due to military service, other tertiary experience, business experience or other reasons had a break with mathematics. The third group may have worked for many years and then decided to come to the technikon. In the third group you may find students who have not done mathematics for ten years.

The changes in school syllabi make assumptions about what students know rather complicated when the age of a student is a further variable. The first group will have had exposure to calculus at school whilst the older members of the second group and the whole of the third group will not have taken any course in calculus. Trigonometrical knowledge will also differ according to age.

4.2.3 Previous institution of learning

In the last section the age of the student was discussed. A link does exist with this section. The group of students aged 19 - 20 years will have left school the previous year. The interesting group will be from 21 - 24 years. Some of these may have been at another tertiary institution such as university, training college or technical college which could give them a temporary advantage over their colleagues. Others may have been in business or industry which could benefit their practical experience but rarely improves their theory or mathematical background.

Within these general categories it is sometimes possible to say that students from a particular school are better prepared in mathematics then students from another school. Some schools will insist all boys do higher grade mathematics while others will only put their best students in for higher grade. Even an E grade at higher mathematics will indicate the student has studied a wider course and were exposed to more difficult concepts.

4.2.4 Background knowledge of algebra and trigonometry

In mathematics and calculus in particular the lack of understanding of algebraic and trigonometrical concepts can be the greatest stumbling block to learning. At each stage in calculus it is necessary to simplify expressions which requires knowledge of factors both algebraic and trigonometrical. Many equations require algebraic formulae to solve and manipulation of expressions. Differentiation and integration require many trigonometrical ratios which in turn lead to various identities needing alternative forms for calculation. For example, to differentiate the sine ratio of x, we need to have the formula for simplifying $\sin(x+h) - \sin x$.

In integration the completion of the square comes into many examples involving a square root. Substitutions of

$$y = a \sin\Theta; \; x = a \cos\Theta$$

will lead to trigonometrical identities. Parametric forms are usually trigonometric.

Simple items such as signs, factors, expanding brackets and calculating trigonometrical expressions in integration can all prove to be difficulties. Mathematics lecturers talk of marking a question until the student breaks down. These break downs happen when students either make a simple error or cannot simplify due to insufficient background knowledge.

The simplification of formulae or expressions does require the manipulation of symbols. In the calculus continued processes like differentiation to the 2nd or 3rd derivative or continued integration by parts are very much simpler if at each stage the expressions are 'tidied up'.

Standler (1990 : 3) states

> "Students need practice in algebraic manipulation to put the solution in a form that is easy to appreciate. It is difficult to state specific criteria for 'easy to appreciate' but people who are fluent in mathematics seem to be able to agree that some expressions are, "simpler" or more easy to appreciate than others."

Sheets of formulae are not the answer to lack of background knowledge. They are helpful when the student understands what the formulae mean in various engineering situations.

4.2.5 Time spent outside lectures on mathematics

The official time spent on lectures and tutorials in engineering mathematics courses may occupy approximately five hours. A rough guide to the time needed to do the examples and understand the lecture is an equal amount of time so students should be setting aside an extra hour a day for their studies in mathematics. The results of question 1 in Annexure A showed that 36% were spending less than four hours a week or put another way, 64% were spending more than four hours a week outside the classroom. It would be presumptious to assume that the failures all belong to the 36% who spent less than four hours and that the successes belong to the 64% who spend more than four hours but there would in all probability be a strong correlation between success and time spent outside lecturers.

In interviews conducted by the author one student said he could spend hours trying to solve a problem whereas his friend could solve the problem in a few minutes. It is important therefore that the time variable is used wisely.

4.3 LECTURER VARIABLES

In Section 4.2 student variables such as range of academic ability, previous institution of learning, background knowledge of algebra and trigonometry and time spent outside lectures on mathematics were discussed. In Section 4.3 a similar discussion of lecturer variables will be made with changes in emphasis from the problems of the student in learning to the problems of the lecturer in imparting calculus to his students due to the effect of the lecturer variables.

4.3.1 Range of academic ability

Lecturers in mathematics departments at technikon are generally well qualified with at least a degree and a higher educational diploma. Within this proviso the author's research found that 64% had three years of mathematics at university and 55% had an Higher Education Diploma. A further 20% had four years of mathematics at university or in other words an honours degree while 13% had a five year or master's qualification.

Owing to the diversity of academic ability the methods used by lecturers may also differ considerably. For example, a lecturer with high qualifications may be used to general methods and use these in preference to specific approaches. A lesser qualified lecturer may prefer the specific approach first and then the general. A lecturer who is keen on teaching may still use the questioning approach to topics expecting the student to answer some of the lecturer's questions. Students do adjust to lecturer's methods but lecturers also may have to modify their approach so that most of the students can understand the various topics which make up calculus. Simpler examples may be necessary on occasions so the main principles can be explained fully. The brighter students will need one or two challenging examples to keep their interest. The changes in engineering mathematics syllabi have raised the standard at each level so lecturers have to adjust to these changes.

This means that topics which were previously in 3rd level only now are in 2nd level, for example, differential equations. Similarly topics which were in 2nd level are now in 1st level, for example, complex numbers. This will be no problem to a lecturer used to the higher levels of teaching but will be a problem to a lecturer who only lectures at the lower level. The academic standard expected of lecturers is rising so it is likely that a master's degree will be required in future posts.

4.3.2 Previous institution of learning

Lecturers at a technikon particularly in mathematics are likely to come from a variety of institutions. This will colour the approach to mathematics which may be quicker than is required at a technikon or may be slower than is required. If the previous institution was a university then the pace may be too quick, if the previous institution was a school the pace may be too slow. Each lecturer will need some time to acclimatise to the pace required. The level of comprehension will again be different in previous institutions. In a university proof and analysis will be very important whereas in the technikon the use of formulae rather than proof will prevail. The lecturer brings a different attitude if his previous institution insisted on proof of statements.

Previous institutions may have had better qualified students in matric mathematics or less qualified students in matric mathematics. This preconditions a lecturer's expectations which can be his weakness when he changes institutions. Even if the previous institution was a technikon there can be subtle differences in both the syllabi and the student approach. There is a minimum syllabus practised by some institution or a maximum by others. For example, one technikon will only do the minimum of first order differential equations whilst another does a maximum of both first and second order differential equations in the second level.

4.3.3 Background knowledge of lecturer

In the technikon as didactic environment the background knowledge of the lecturer in mathematics is very important. If he is able to point out the direct way that mathematics links to the engineering discipline then this will provide good motivation for the student to try and understand that bit of mathematics. Some lecturers were originally linked to and lectured in various engineering sections. This is a great help when they form part of a mathematics department. It is possible to acquire the necessary background by delving into engineering textbooks. The variable of background knowledge will apply when lecturers only see their task to lecture on mathematics whereas others see their real function to provide the necessary mathematics to solve engineering problems.

Criticism is levelled by engineers that students are not receiving the relevant mathematics for their courses in engineering. Standler (1990 : 1) seeks to influence mathematics instructors to consider the practical and logical needs of students who will take engineering courses. The idea of problem solving is a theme or thread going through Standler's article.

Few problems can be solved immediately. It is critical that engineering students develop persistance at solving problems. Often the 'best' way does not come instantly or even easily; one has to try various methods and see what happens.

> "Doing derivations help the student develop a logical thought process, a discipline of problem solving which is essential for solving engineering problems of many kinds." Standler (1990 : 1).

A collection of engineering problems which relate to the mathematics being taught or lectured will be a great asset to engineering students at technikon but the time to use them is always limited. However the impact on students of a few relevant problems can be of inestimable value in the totality of their mathematics course.

4.3.4 Time spent on lecture preparation in mathematics

This is a topic similar to the time students spend outside lectures.

Trying to give a formula for the amount of time is difficult because it depends whether the course is being lectured for the first time or the second or subsequent times. The more times the course is lectured the more duplicated material is available or the number of good transparencies increases. However, a good guide for lecturers might be two hours per day, that is ten hours per week. In Question 14 of the Staff Questionnaire (See Annexure B), only 20% did more than eight hours a week preparation and 46% did less than four hours a week. The changes in mathematics syllabi over the last five years have meant reorganising courses, so much more time has been spent in preparation. This leads naturally to the next section dealing with content variables.

4.4 CONTENT VARIABLES

The content of mathematics courses to engineering students at technikon has been constantly changing over the last ten years and particularly during the last five years due to the upgrading of qualifications from diploma to higher diploma to master's diploma and now ultimately to a bachelors degree in technology. In this section there will be a discussion on changes in syllabi, the use of training technology and the incorporation of specialised approaches such as computer programmes, student learning texts and other similar items into the content of mathematics courses. These all affect lecturers either in extra training or readjustment of their courses.

4.4.1 Changes in Syllabi

The changes in syllabi of mathematics courses for engineers at technikons are partly due to the change in status of technical education. The original technical college only issued first diplomas over three years. Its successor the colleges of advanced technical education went on to higher diplomas over four years whilst the present technikon added master's diplomas (five years) and laureatus (six years) qualifications and the expectation in 1995 is to begin B Sc (Tech) qualifications.

To achieve higher qualifications the base qualification, that is the diploma, usually has to be strengthened which means the introduction of topics in 3rd, 2nd and 1st level in order to do the 4th and 5th levels. An example of this is differential equations which were introduced to 2nd level since many engineering students do not do 3rd level mathematics and yet many wish to do a higher diploma involving differential equations.

It is also partly due to a complete revision of school syllabi which introduced elements of the first course in mathematics at the technikon into the final matric examination, especially if students did higher grade and additional mathematics, for example, elementary differentiation in the calculus. Engineers at technikons held three conferences, the first at Scottburgh in 1987, the second at the University of the Witwatersrand in 1990 and the third at Pretoria Technikon in 1993. Mathematics departments were represented at these conferences. The needs of engineers for items such as complex members, linear programming, matrices, statistics and differential equations at lower levels led to successive revisions of first and second level mathematics courses for engineers at technikons. Complex numbers were introduced to first level mathematics courses after the second conference. The second level course was extended to include extra modules such as differential equations, linear programming, matrices and statistics. The differentiation and integration modules still remained although one or two items were passed down to the 1st level, for example, integration by parts. The net effect was an increase in standard in the 1st level course and a much heavier load for students and staff in the 2nd level course.

Flashman (1992 : 93) endorses the idea of change when he states

> "The student taking calculus courses today have backgrounds, needs and goals different from those of students of 20 years ago. We need to adjust in a meaningful way to the legitimate demands for change."

4.4.2 The use of technology in training

Lecturers have various viewpoints on using technology to assist their lecturing techniques. Many lecturers still use the blackboard only relying on a clear style with chalk. However there is increasing use of the overhead transparency as more material can be covered. In mathematics examples on transparency can be checked for errors in sign or numerical errors before exhibiting them to students. The use of coloured pens helps to emphasise important points. Another advance in technology is the laser printer which can produce extremely clear and bold transparencies. The only problem which can arise is that lecturers tend to rely completely on transparencies which excludes the advantage of variety and change of direction in teaching. Students can be very passive attempting to take down everything on transparency. In question 16 of the Staff Questionnaire (See Annexure B) staff ranked the overhead projector as the most useful aid with computers second most useful aid. It is necessary to discuss specialised approaches such as computer packages, video tapes, cassette tapes and student learning texts in a separate section.

4.4.3 The use of specialised approaches

The number of black students matriculating has steadily risen over the last five years which is reflected in corresponding increases in tertiary institutions. To assist students in mathematics and science computer packages have been developed. They range from school packages suitable for matric students which assist students improve their grades or help those whose facilities are poor to tertiary packages which take out the routine difficulties of differentiation and integration plus excellent graphing facilities. One example is graphical approach to calculus by Dr Tall. Other examples are two logical approaches called Derive and Macysma. At a post graduate level Mathematica provides for those who wish to do research. Computer laboratories are exceedingly popular with students especially if printing facilities are available. Lecturers are incorporating these packages onto their programmes with the help of liquid crystal displays to demonstrate items which take a long time on the blackboard for example showing the limiting process with gradients or illustrating the closeness of areas under a curve to the definite integral. Along with computer packages there are student learning texts used by distance learning centres. Notes, tutorials, tapes and videos can all be a part of this. Work is despatched at regular intervals with tests and finally an examination is set at the end. The University of South Africa and Technikon South Africa are two large centres who specialise in this work but many private ones exist.

There is also increasing use of television to assist students. The South African Broadcasting Corporation has put out many educational programmes for school students, for example, mathematics and science by William Smith while at a higher level some very useful scientific programmes have been given for tertiary students. Closed circuit television is also used in universities, training colleges and in technikon with one lecturer taping a good lecture or series of lectures for use in several venues simultaneously.

Morgan (1990 : 981) states

> "Traditional teaching methods should be re-examined and consideration given to individual, self paced, self-study, including general reading, programmed learning, computer aided learning, interactive video, slides tapes etc."

4.4.4 Synthesis

This section has illustrated that traditional approaches to lecturing are being replaced with more sophisticated methods involving transparencies, computer packages, distance learning, video courses and slide presentations. Lecturers who were previously quite happy with traditional approaches are now forced to change over to some of these new approaches. This can cause problems of readjustment and possible uncertainty with courses.

4.5 INTERRELATEDNESS OF THE STAFF AND STUDENT PROBLEMS

4.5.1 Content problems

Lecturers and students do have some common problems with content of courses. For example, in the results of Question 7 of the Student Questionnaire given to technikon students (See Annexure A) students listed limits in calculus as the 4th most difficult topic which lecturers. In the results of Question 6 of the Staff Questionnaire given to technikon staff (See Annexure B), respondents listed limits as the fifth most difficult topic. Similarly students listed word problems as the most difficult topic while lecturers placed word problems second on their list. It appears that what staff find difficult topics to lecture, students find difficult topics to absorb and use.

4.5.2 Concept problems

Several recent articles have stressed that both teachers and students have difficulties with basic concepts of mathematics.

Morgan (1990 : 981) states

> "Teaching methods must be changed in order to produce conceptual understanding, not just mechanical skill with standard problems. Students must be encouraged to think in a flexible and inferential mode, rather than in a mechanical fashion."

Olayi (1990 : 698), referring to third world curriculum for mathematics teachers, states

> "It has been generally accepted that much of the preparation of a mathematics teacher lies in the task of studying the development of mathematical concepts and skills at the level he is going to teach."

Orton puts it in a different way when he discusses"instrumental understanding" which means knowing how to use knowledge as an instrument or a vehicle to achieve a certain result as opposed to relational understanding which means knowing how various results link together.

> "In terms of differentiation and integration, instrumental understanding is relatively easy to achieve but relational understanding appears to be sadly lacking. Meeting up with mathematics graduates in teacher training one is forced to conclude either that the teaching of calculus which they experienced was geared towards instrumental understanding or that relational understanding was striven for but could not be achieved."

Orton (1986 : 660).

4.5.3 Student and Lecturer Thinking Problems

The students' thinking problems are very often due to the problems that lecturers encounter when explaining their lecture to students. If lecturers assume that students think as they do or as quickly as they do then students are bound to have problems. Students' thought processes are restricted due to lack of experience, so cognisance of this will require the lecturer to think ahead and organise a suitable structure within which the student can operate. For example, emphasising the process, say maximising, at least four times during a lecture independently of the examples given during the lecture. The continual focusing of students thoughts will clarify the examples given. Olayi (1990 : 69) confirms that his thinking processes are very different to his students'.

> "My assignment was to teach form 3 mathematics. Despite all my training in MC2 level (that is core mathematics areas as well as applicable mathematics, statistics and computer science) and the fact that I had a very good pass in school level mathematics I had problems carrying out the assignment. I was looking at the problems from my own level and this adversely affected my communication. What seemed perfectly obvious to me was Greek or impossible magic to them."

In this example Olayi is speaking of school mathematics but the same experiences are encountered at tertiary level.

4.5.4 Problem solving

The activity of problem solving has always been present in mathematics but during the 1980's and 1990's there have been strong moves in the USA and Europe to use problem solving as a teaching technique.

Lerman (1983 : 60) proposes the following:

> "Clear distinctions can be seen between only two perspectives of teaching mathematics, as a body of knowledge or as a way of thinking."

Lerman (1983 : 62 - 63) discusses these two perspectives showing that the first perspective means building up or accumulating a body of knowledge, learning methods first and then understanding uses, applications and relevances afterwards. The other perspective is really problem solving first without being given a method encouraging students to propose ideas and suggest methods and then later to generalise whatever methods they find to solve the problem.

Lerman (1983 : 59) states

> "As a consequence, two teaching perspectives can be identified, knowledge-centred and problem-solving respectively. In describing these styles, I criticise the former and propose that the latter can initiate substantial changes and advances for school mathematics programmes."

Using problem solving techniques is certainly very challenging to lecturers as it involves dialogue with students, the loosening of formal approaches and some uncertainty of the outcome. Some lecturers find this technique a problem, others welcome the challenge. Students also are divided with many preferring the methods approach although some students are stimulated by the extra effort involved with the problem solving approach.

Wilson (1978 : 423) states

> "A characteristic displayed by many university students in mathematical fields is an inability to use their mathematics to solve problems."

4.5.5 Time factor

The time factor affects both lecturers and students. A very fine balance must be kept on the amount of content required during the course, the time taken for small tests, longer tests and semester exams plus the various exercises given to students. Add to the above the use of computer packages then the task becomes very difficult to cover the syllabus in the fifteen to sixteen weeks allocated for second level engineering mathematics. The items mentioned are organized by the lecturer in conjunction with his colleagues which involves a fairly regular series of meetings which also requires extra time. Students find the time factor difficult because they have another five or six subjects to cover apart from mathematics each with all the components mentioned. Some depend heavily on practical work so the workshop sections in the afternoons are vital which makes engineering timetables very full.

4.5.6 Synthesis

In Section 4.5 it was shown that lecturers and students have joint problems which arise from the same topic in the syllabus for example "limits". It has illustrated that students and lecturers' can have very different thought processes which leads to misunderstanding and frustration either with the lecturer or with the students. An important discussion on teaching techniques highlighted two main possibilities, firstly knowledge centred techniques and secondly problem solving techniques. Also mentioned were the problems caused by shortage of time in a semester course involving the wide syllabus, tests, practical work, computer packages and examples.

4.6 POSSIBLE STRATEGIES TO OVERCOME THE DIDACTICAL PROBLEMS IDENTIFIED IN THE CALCULUS LEADING TO SUCCESS IN CALCULUS.

Since most tertiary mathematics courses contain calculus as a basic component, the ability to cope with calculus is a major factor in determining the success or failure of students in tertiary mathematics courses. Statistics in South Africa and other parts of the world indicate pass rates of approximately 50%.

Beckmann (1992:93) states:

> "Roughly 100 000 students in the USA study calculus each year and 50% of them withdrew or failed the course."

Here in South Africa at Technikons the pass rate is very similar. The author has lectured second level mathematic courses for eight years at Technikons in Natal and finds that on average 20% do not achieve a course mark enabling them to write the final examination and of the remaining 80% there is a pass rate of approximately 65%. Thus the overall pass rate is 52%. This chapter will investigate possible strategies to increase the success rate in calculus including cooperative learning, the better use of mathematical language, variable credit courses and the intelligent use of mathematical technology.

4.6.1 Cooperative learning

The student approach to learning is the basic lecture where one person delivers the content and students take notes followed later by a tutorial where problems are handed out and questions are answered. This leads to a wide spread of results indicating that learning has not really taken place as lecturers expected.
Tall and Razali (1993:209) conclude:

> "there is a continual divergence in performance between those who succeed and those who fail which is exaggerated by qualitative differences in their thinking process."

Later Tall and Razali (1993:218) state:

> "The more able need to remember less because they can reconstruct more. The less able see more information as burden, as even more disjoint pieces of information to remember - an increased burden on a weaker back leading to the greater probability of an evitable collapse."

The weaker student is always asking the lecturer to go back several steps as he has missed one vital formula conversion. It is important to plan beforehand to put on the blackboard or as a handout the formulae which will be used in the lecture. Then at the appropriate stage the lecturer can refer to the one being used.

For example:

$$\cos^2\Theta = \frac{1+\cos 2\Theta}{2}$$

is frequently needed but students look blank if they are not shown how this arises from $\cos 2\Theta = 2\cos^2\Theta - 1$

Similar examples are: $\sin\Theta = \sqrt{1-\cos^2\Theta}$

and $\cos\Theta = \sqrt{1-\sin^2\Theta}$

which arise from $\sin^2\Theta + \cos^2\Theta = 1$

Tall and Razali suggest that students should not only look at the errors they make but at the strategies that more able students use to succeed. In fact it is advocated that problem solving strategies should be taught separately to mathematical content courses. The author has tried this procedure with structures and steps to solve problems in integration.

For example:

$$\int_0^{\frac{\pi}{2}} \sin^5 x \cos^3 x\,dx$$

PROBLEM: What method shall we use?

CLASS REPLIES: (1) Integration by parts
(2) Trigonometrical Substitution
(3) Derivative

EXAMINATION OF CHOICES (1) - (3)

(1) Too complicated
involves odd powers of sinx and cosx
(2) Not directly
(3) Seems relevant

$$\int_0^{\frac{\pi}{2}} \sin^5 x\cos^3 x\,dx = \int_0^{\frac{\pi}{2}} \sin^5 x\cos^2 x\cos x\,dx$$

$$= \int_0^{\frac{\pi}{2}} \sin^5 x(1-\sin^2 x)\cos x\,dx$$

$$= \int_0^{\frac{\pi}{2}} (\sin^5 x-\sin^7 x)\cos x\,dx$$

u = sinx
du = cosxdx

$$= \int_0^{1} (u^5-u^7)\,du$$

$$= \left[\frac{u^6}{6}-\frac{u^8}{8}\right]_0^1$$

$$= \frac{1}{6}-\frac{1}{8}$$

$$= \frac{1}{24}$$

The brighter student investigates several possiblities before deciding on the best approach. He does not accept one method for all problems but seeks the neatest approach. This very often leads to a shorter answer. Another strategy to achieve greater success is to involve students in formulating a problem and then solving it.

4.6.2 Use of appropriate language

Another strategy for assisting success in calculus is to get students who do not understand problems to put their steps into words. When students help each other it is usually by words that the weaker student understands.

Joan Countryman (1992:11) amplifies this theme:"

> "When students learn to use language to find out what they think they become better writers and thinkers. Our students need more classroom opportunities to do informal writing, to make sense by making meaning, to create for themselves the underlying concepts of mathematics."

The author has on many occasions asked students to express algebraic formulae in words. Their explanations indicate the reasons for errors, for example, x plus y squared is different to x plus y all squared. In symbols the former is $x + y^2$ and the latter is $(x + y)^2$. Joan Countryman's investigations showed that even the most successful school students of mathematics claim they can do problems but can't explain them. Teachers really understand problems by writing out their own explanations. It is no wonder that students do not understand problems if they don't try to put their steps into words.

Azzolini (1991:92) states that:

> "writing is a way of clarifying and refining one's own thoughts as well as communicating with others mathematics has a rich history of using writing to learn. Lecturers are challenged when they invent word problems so why not ask students to do likewise?"

Azzolini (1990:96) gives three possibilities:

1. Write your own word problem similar to the ones you have been doing in class.
2. Change the problem you wrote in question 1 so that it is an easier problem.
3. Change the problem you wrote in question 1 so that it is a harder problem.

4.6.3 Variable credit courses

A major problem with students learning calculus is their ability to understand and retain information. At Technikon Natal the second level course has 4 modules. The first two are differentiation and integration each of 4 weeks duration followed by a choice of two more modules from differential equations, matrices, linear programming and statistics. The mechanical and electrical engineers take as their third module differential equations so calculus occupies 3/4 of their course. Our experience is that initially students do quite well on their 1st test on differentiation but decline on their 2nd test on integration and their 3rd test on differential equations. The final semester examination includes all the modules so they can pick up a better mark with the help of the 4th non-calculus module. What can be done about students who fade about half-way through the course?

To assist in answering this question consider the following:

An interesting programme is being used in the department of Mathematics at the University of California at Berkeley, called the "Self-Paced calculus program". It was started in 1972 by Murray Protter. The idea behind the programme was to split the first year calculus and analytic geometry syllabus in 30 equal parts called segments. Each segment normally would be completed in one week. A study guide is provided for each segment. Students work at their own pace and take the test for a segment when they are ready. Each test has three questions and a student must pass all three questions except for minor errors. If he fails then he goes back to the guide and discusses his failure with a tutor and then takes another similar test.

Protter (1991:246) enlarges on the grading system:

> "At Berkeley the regular first year calculus course provides four units credit each semester. Thus if a student completes 15 segments of the self paced course each semester he is credited with four units. However the course actually is denoted a "variable credit course."

Students can earn two three or four units depending on the number of segments they complete. The final examination also provides a choice whereby students only answer those problems which refer to the segments they have covered.

Protter mentions that after 18 years one third of 1st year students opt for the self-paced course and two thirds take the usual lecture discussion course.

The above scheme does seem to obviate the time problems experienced by lecturers but has certain administrative difficulties for the technikons of South Africa who don't work on a units credit system for mathematics as outlined by Protter.

At some stage during a mathematics course a tertiary student will have to experience success otherwise the motivation to continue will disappear. This is a major difficulty to learning if the student's past record indicates only failure. From experience one small area of success will keep a student going through several difficult sections. In planning a course this encouragement should be built in with possibly an easy short test. Students who are motivated will pay more attention to lectures, will attempt all the examples given and query those points they don't understand. On rare occasions they may also give you a new method or approach to a problem which you have not done previously. When a student does this his self-esteem rises dramatically. This type of student may be your brightest or he could be just an average student who tries to solve your problem.

Protter (1991:248) states:

> "The second class of students for whom self pacing is useful are those who have the ability to learn mathematics but are slow at it. In a regular course such students will get about a C and thus learn a random 70% of the material. However in a self paced course, the same students will learn thoroughly the first 70% of the course set a good grade but earn three units instead of four units. As a bonus the student learns to read mathematics gains confidence and comes to appreciate the positive reinforcement that goes with passing the segment tests."

M R Conley et al (1992:176) emphasise:

> "The degree to which the student participates in learning activities has a major impact on what is learned. It is not only overt participation in learning activities that determines what is learned, however. The mental activity that takes place during the activity by each individual is as important as what the individual appears to be doing.The confidence that one feels about successful completion and the interest in the activity, also affect the quality of a learners participation and what is actually learned from doing the activity."

What both of those authors are saying is that the more students become involved in the activity of learning the' more likely they are to succeed and complete their courses.

Smith (1991:153) mentions in his conclusion:

> "Success is fun. Your enjoyment in seeing the positive results of your efforts, reflected in both increased learning and higher grades should feed upon itself and further motivate you to try your hardest in the course."

4.6.4 Use of mathematical technology

The approach to learning and teaching calculus is changing rapidly with the advent of graphic calculators, computer packages and laboratory calculus courses. It is also changing the curriculum by querying whether drill work is necessary or whether proofs should be included. A review of a couple of projects will illustrate the new trends.

At Duke University (USA) "Calculus as a laboratory course" is being run by Lawrence Moore and David Smith. They have developed a three semester calculus programme based on a computer laboratory. The students working in pairs, explore real world problems with real data conjecture and test their conjectures, discuss their work with each other and write up their results on a technical word processor.

Moore and Smith (1992:100) give details of their course.

The group meets for three 50-minute periods in a classroom equipped with one computer for instructor demonstrations. Each section (maximum of 32 students) splits into two laboratory groups; each group has a scheduled two-hour laboratory each week. Each laboratory team (two students) submits a written report almost every week; three or four of these a semester are formal reports that involve submission, review by the instructor, and resubmission for a grade. The remaining reports are "fill in the paragraph", often finished during the laboratory period itself. In the classroom, teams of four work on activities that lead to a variety of reports. In weeks that do not include a test, each student has a week-long assignment of routine computations and exercises embedded in the reading. One class period each week is "group office hours"; the instructor responds to student problems but does not initiate new material.

During the first semester, students use Mathcad (Mathematical Computer Assisted Design) for the mathematical portion of the labs and write their reports on EXP (Executive Programme). In the second semester Derive is added right after the students have begun to wrestle with the problem of finding antiderivatives. In the third semester MPP (Mathematics Parametric Plotter) is added for parametric curves and numerical evaluation of double and triple integrals, and Surface Plotter for investigation of surfaces.

The main difference to traditional courses is that formal lecturing is very much reduced. Team reports, classroom discussions and illustrations on the computer give much greater involvement to students.

This course needs a confident lecturer well versed in mathematical software and word processing. A similar situation exists with the next project originating at Purdue University USA.

Purdue University have a project called 'Calculus, Concepts and Computers' which uses two software systems, first, a symbolic computer system (SCS), for example, Maple, Derive plus a mathematical programming language (MPL), for example, ISETL (Interactive Set Language).

Schwingendorf (1992:97) states

> "Two common questions raised by others about our project are: Can students learn Calculus and learn to program at the same time? How can programming help students learn? The answers to these questions lies in the very powerful and relatively easy manner in which students can construct computer implementations of mathematical processes and objects in ISETL. Using a SCS that performs operations on mathematical objects is just another way of showing students mathematics, and this type of activity is just not enough if we want our students to learn ideas and concepts.
> On the other hand, our research suggest that having students write computer programs in a programming language, such as ISETL, with a syntax very close to standard mathematical notation, which is easy for students to learn and convenient to use, and that treats functions as first-class objects (i.e. can accept a function as an input and return a function as an output), appears to help students gain a deeper understanding of the concept of function (which is fundamental to all of mathematics, especially calculus) and the basic concepts of calculus."

Both the projects at Duke University and Purdue University have a common theme which is the greater involvement of students. Students are reluctant to abandon their passive role, especially in mathematics, but tend to be more receptive to work on the computer. The success rate on these projects appears to be greater than traditional courses.

Nichols (1992:4) states:

> "During the spring of 1991, 38 mathematics instructors from numerous institutions throughout the USA attended a 17 day workshop at Purdue University. The goal of the workshop was to prepare them to teach Calculus as they had never taught it before."

Of the 38 instructors (or piloters) 25 went back and implemented the course immediately while 9 planned to implement it in the future while only 4 felt they did not wish to implement the course. The feedback from those who implemented was very positive with pass rates considerably higher than traditional. For example, at Mt Hood Community College, 19 out of 24 students finished the course compared to 50% in traditional courses.

The author has used a computer package in graphic approach to the calculus by Dr David Tall for five years at Technikon Natal.

This is not computer programmed learning but rather provides various frameworks that can be used for drawing graphs, finding the gradient of any function, integrating practically any function, using parameters, solving equations etc. The student has to input his own information and then use the package to give various results from a graph. It is not merely pushing buttons as the student will, for instance, have to suggest a derivative which will involve differentiating the function displayed and similarly the student will have to give a primitive when doing integration. Domain and range play a very important role in the graphs which are drawn. The initial programme on graphs is quite testing when you have to identify various graphs of linear, quadratic, trigonometric and other functions.

It has produced a better understanding of differentiation and integration but the author has no conclusive evidence it has increased the pass rate but is has helped more students to gain a course mark.

Graphic calculators are also gaining popularity. Various studies are being conducted in the USA. One of these is at Grand Valley University.

Beckmann (1992:110) reports as follows:

> "Preliminary results of research currently being conducted through Grand Valley State University supports the use of graphing calculators in developing student understanding of Calculus at both the high school and college level (Beckmann, in progress). Students enrolled in a discovery-based conceptual Calculus course in which concepts were investigated through the use of graphing calculators and supplemental discovery laboratory activities are being interviewed concerning their understanding of the concepts of function, limit, continuity, derivative, and integral upon completion of the course. Two college students (TV and JP) and three high school students (JA1, JA2 and MV) have been interviewed. The high school students were interviewed 2.5 months after the course ended. TV was interviewed 8 months after he completed Calculus II and 15 months after Calculus I. JP was interviewed 15 months after she completed Calculus II and 20 months after Calculus I. Preliminary results with the college students suggest that students' conceptual understanding of Calculus might continue to be strong long after their completion of the course."

4.6.5 Synthesis

In conclusion success is not only increased by passing examinations but should include the ability to reconstruct concepts some time after the course is finished. Since mathematics courses are dependent on previous knowledge or concepts the second point could be more important to future success in mathematics. This Section has examined new approaches which involve enthusiastic teachers or lecturers. Successful students generally look forward to their lectures with enthusiasm. Now even old approaches can generate enthusiasm with some students but perhaps weaker students need a new look at calculus through cooperative learning, mathematical technology and more suitable targets for their ability so they can become enthusiastic and successful.

4.7 SUMMARY

The understanding of calculus discussed in this case study was dependent on the ability of students to understand a particular kind of language and a series of mathematical symbols. The problem for lecturers was to translate this mathematical language into a recognisable form which students could handle. The study showed that students have great difficulty in dealing with more than one form of a calculus operation, for example, 'differentiation'. Another factor which hindered students and staff was the concentrated time schedule of the semester courses. Styles of teaching were discussed ranging from pure lecturing to part lecturing with problem sessions to laboratory or small group projects. The way students respond gave rise to a discussion on advance organisers applications and problems were also related to the earlier discussion on language and symbols.

Problems relating to students' background highlighted many areas such as fractions in algebra, logarithms simplification, inverse functions and modules of an expression. These impinged on the calculus particularly in differentiation and integration. The range of students ability plays an important role in the success of a particular engineering group. The time spent by students outside of lectures can be a strong factor in overcoming difficulties or creating confidence. Students interact so lecturers should encourage this interaction with sensible problems and use bright students to help others.

The problems of lecturers were related to the previous background of the lecturer such as school, university business or industry. The teaching qualifications were investigated noting that nearly half had no teaching qualification. It is therefore not surprising that some lecturers have difficulties in communicating with their students. The solution to these problems involve staff orientation and training in methodology to the technikon environment.

The changes in syllabi and content have presented difficulties due to the sudden switches or deletions of topics. New technology such as computer packages, video tapes, student learning tests and television were shown to be on the increase. These could assist many weaker students and change the calculus format so that differentiation and integration could well be done using a logical package like 'derive' so allowing more time for differential equations at an earlier stage.

Strategies to help weaker students included, firstly, studying the methods used by brighter students to solve problems. A "Holistic" approach may bring better results than merely pin-pointing errors. Secondly, the language used by students to explain mathematical problems is a vital literacy component which must be improved if students are to succeed in solving word problems in mathematics. Thirdly, variable credit courses which allow students to cover part of the full course and gain credits for this part as, for example, at Berkeley, were discussed. Fourthly, a more detailed examination of mathematical technology with respect to graphic calculators, computer packages and laboratory courses was investigated with particular reference to projects at Duke University, Purdue University and Technikon Natal.

CHAPTER 5

DESCRIPTION OF THE EMPIRICAL RESEARCH

5.1 INTRODUCTION

This chapter will examine the research data taken from the empirical survey conducted at South African Technikons during 1989-1990 dealing with teaching and learning problems in calculus. This research data included the results of a Student Questionnaire (Annexure A) given to 800 Technikon second level students in engineering drawn from nine technikons in South Africa who had all taken a first level course in calculus and at the time of the questionnaire being given were taking a second level course in calculus. To gain greater insight into the results of this questionnaire a structured interview was conducted with five students of varying background and ability who had answered the questionnaire. This gave the students the opportunity to comment on difficulties they encountered in calculus which were not necessarily in the questionnaire. A further part of the research data were the results of a Staff Questionnaire (Annexure B) answered by over 70 staff (drawn from the same nine technikons as the students) which were analyzed for staff responses and also to provide comparisons between students and staff responses where this was appropriate. An additional component to the research data was a further questionnaire (Annexure C) based on the effect a computer package had on the learning of calculus. This was further strengthened by structured interviews with four pairs of students who had used the package.

5.2 ANALYSIS OF THE DATA OBTAINED THROUGH THE STUDENT QUESTIONNAIRE

5.2.1 Aims of the Questionnaire

The first aim of the questionnaire was to gain insight into the learning difficulties experienced by students when faced with the calculus for the first time. The second aim was to assess students' views on lectures, tutorials and technology used in the classroom such as, for example, overhead transparencies and computers. The third aim was to provide some information on student ability and background in engineering courses at the Technikon.

5.2.2 Modus Operandi of Questionnaire

A structured questionnaire (See Annexure A) was prepared in both English and Afrikaans so that students could answer in their home language but home language differences were not pursued in this study. Preliminary questions included biographical information such as age group and matric results in mathematics. The main group of twenty questions dealt with the following categories (question numbers are given in brackets each time):

1. Previous activities prior to enrolling at the Technikon (1,2,3)
2. The desirability and form of a two week pre course in calculus (4,5)
3. Whether the respondent had passed a course in mathematics which included calculus (6)
4. Ranking ten mathematical items in order of difficulty (7)
5. The meaning of terms used in calculus (8,9,12,14)
6. Simple questions on differentiation (10,11)
7. Whether the respondent has problems with limits (13)

8. The way time difficulties affected the calculus course (15, 16, 17, 18)
9. Views on the methodology used in learning (19)
10. Finally ranking various differentiation problems for difficulty (20)

A pilot group of thirteen engineering students having a wide spread of matric qualifications, age and previous business experience initially filled in the questionnaire. Their results were analyzed and then discussed with the students. These discussions were very valuable and led to the revision of several questions. The revised questionnaire (Annexure A), was then given to 231 second year engineering students in M L Sultan Technikon, Mangosuthu Technikon and Technikon Natal. This was followed by a tour in April of six other Technikons in Pretoria, Johannesburg, Cape Town and Port Elizabeth. The personal supervision of the questionnaires during this tour by the author plus the excellent cooperation by the heads of the various Technikon mathematics departments ensured a representative sample of nearly 800 responses. Times for answering the questionnaires were synchronized as far as possible to be the same usually from 10:00-12:00.

The results were analyzed checking how many responses occurred to each question. For example if one group consisted of 51 students and for question A only 49 responded the valid response was 96.1%. Valid responses and percentages were tabulated for individual Technikons and for the total group of Technikons. A group of five students volunteered to be videoed on their comments on the questionnaire. The interplay was quite spontaneous and gave rise to discussion of further problems the students experienced relating to lectures and content.

5.2.3 Analysis of the Questionnaire

5.2.3.1 Analysis of biographical data.

(A) Mathematics symbols for higher and standard grades (Figure 1).

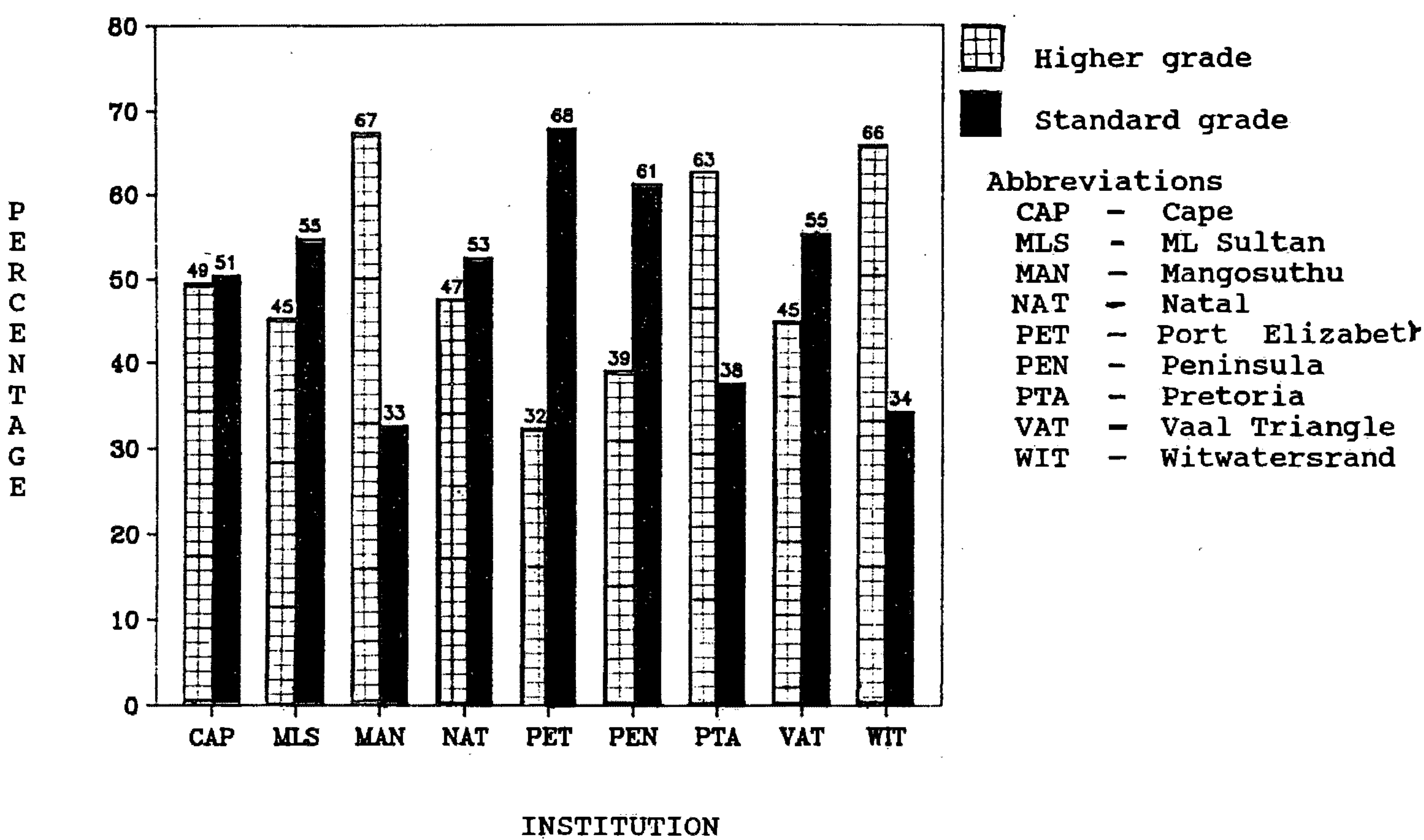


FIGURE 1 MATRIC GRADES BY INSTITUTION

There was a wide divergence between the nine Technikons, some have over 2/3rd of their engineering mathematics students with higher grade qualifications while others have just the reverse with only 1/3rd higher grade. Technikon Natal has approximately 1/2 of its students with higher grade. The overall totals show 52% higher and 48% standard grade.

The graph of matric symbols (Figure 2) also indicates that 16% of students would have qualified for university engineering courses and 25% have the minimum qualifications for Technikon engineering courses.

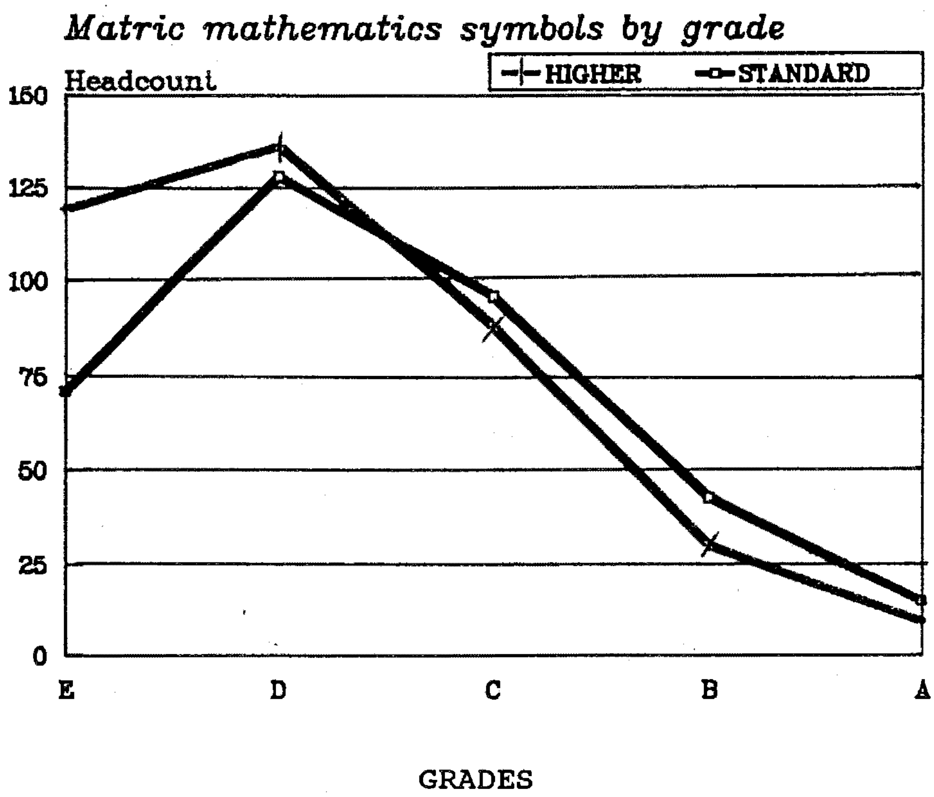


FIGURE 2 MATRIC MATHEMATICS SYMBOLS BY GRADE

(B) Years since matric (Figure 3).

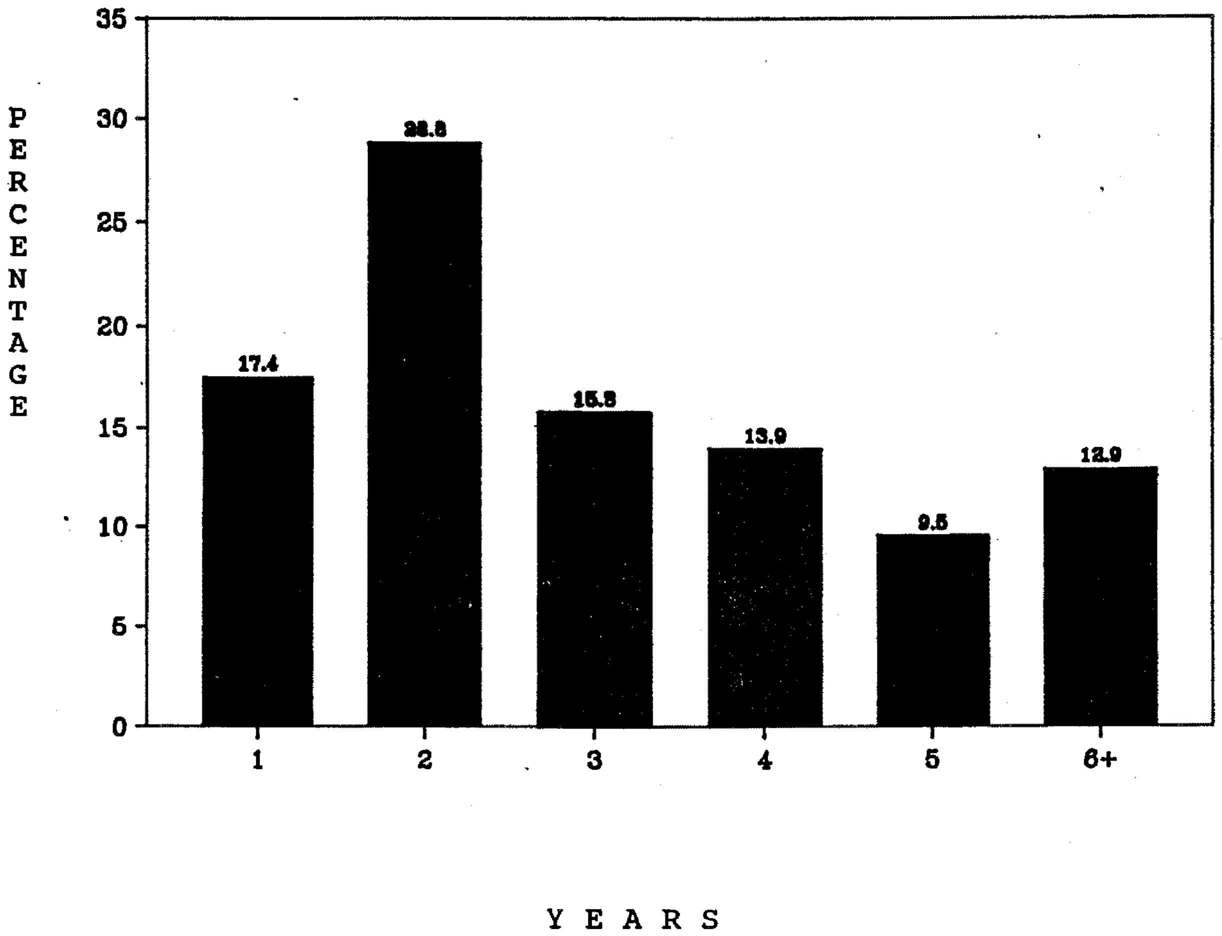


FIGURE 3 YEARS SINCE MATRIC AS PERCENTAGE OF TOTAL STUDENTS

A significant number of students, namely 36% took matric more than four years before this study was conducted. This does affect their performance in mathematics, as school syllabi changed since then to include calculus.

(C) Age of students (Figure 4).

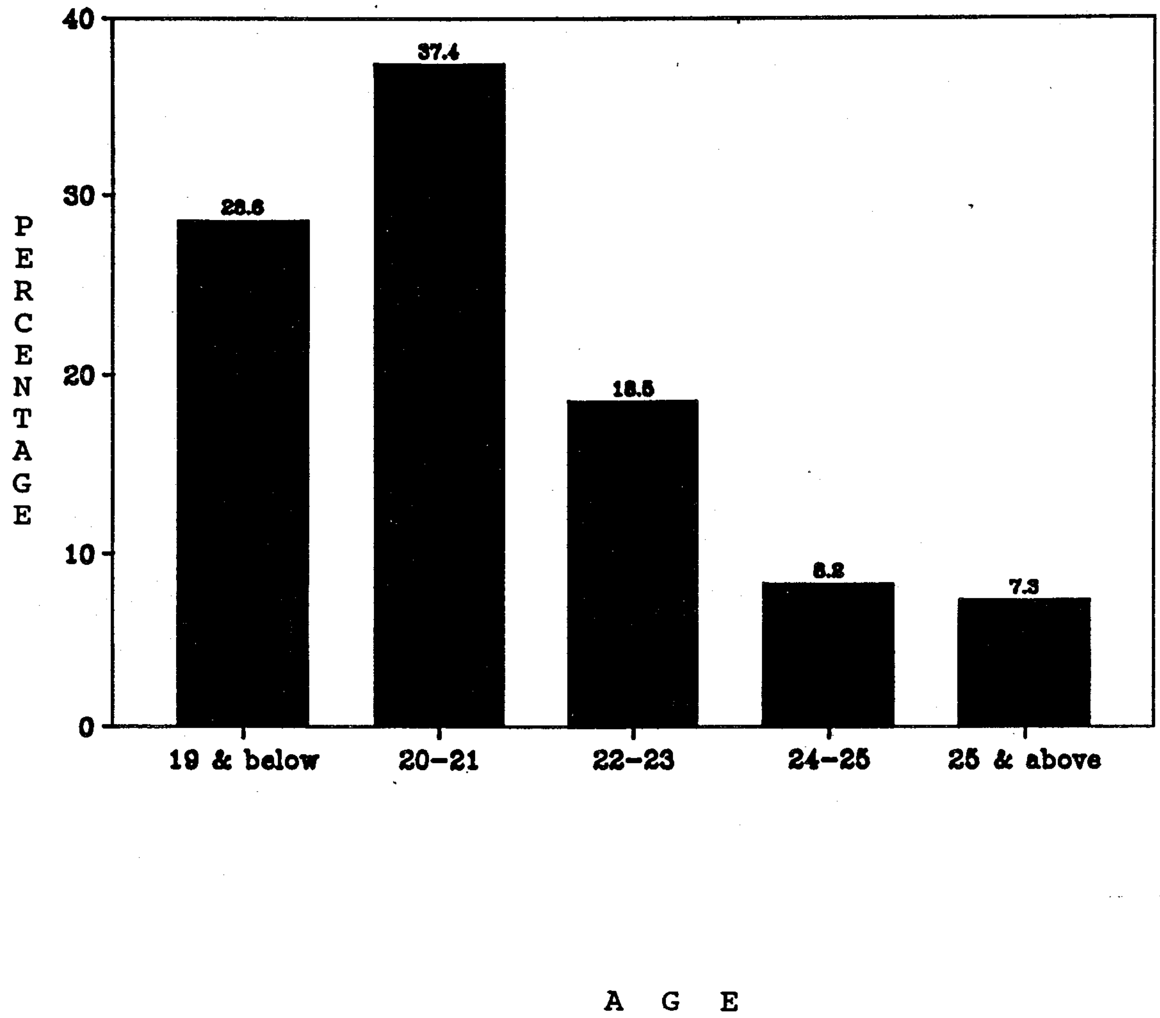


FIGURE 4 DISTRIBUTION OF STUDENT'S AGES

The age distribution shows that 34% of students are over 21 years of age which correlates with the previous table (Figure 3) on years since matric where 36% of students had taken matric four years before the study was conducted.

5.2.3.2 Analysis of responses to specific questions

QUESTION 1-3 Previous activities of engineering mathematics students (Figure 5)

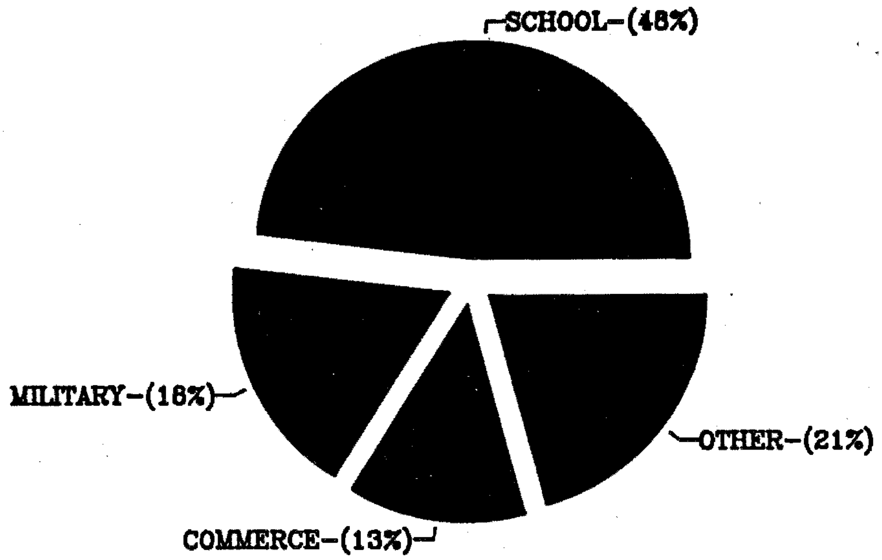


FIGURE 5 PRE-TECHNIKON OCCUPATION

Approximately half of the students came directly from school, whilst the balance was split between military, commerce and other institutions. The break after leaving school can affect the retention and transfer of mathematical knowledge and skills obtained up to that stage. For example, those doing military service, as they tend to forget most of the formulae in algebra and the identities in trigonometry. It may, however, be an advantage for those who worked in commerce or industry owing to their wider experience of applications of mathematics in the workplace.

QUESTION 4 Interest in Pre-course in Mathematics.

Approximately 65% of the respondents indicated that they would be interested in a pre-course if this was available in January.

QUESTION 5 Form of Pre-Course (Figure 6)

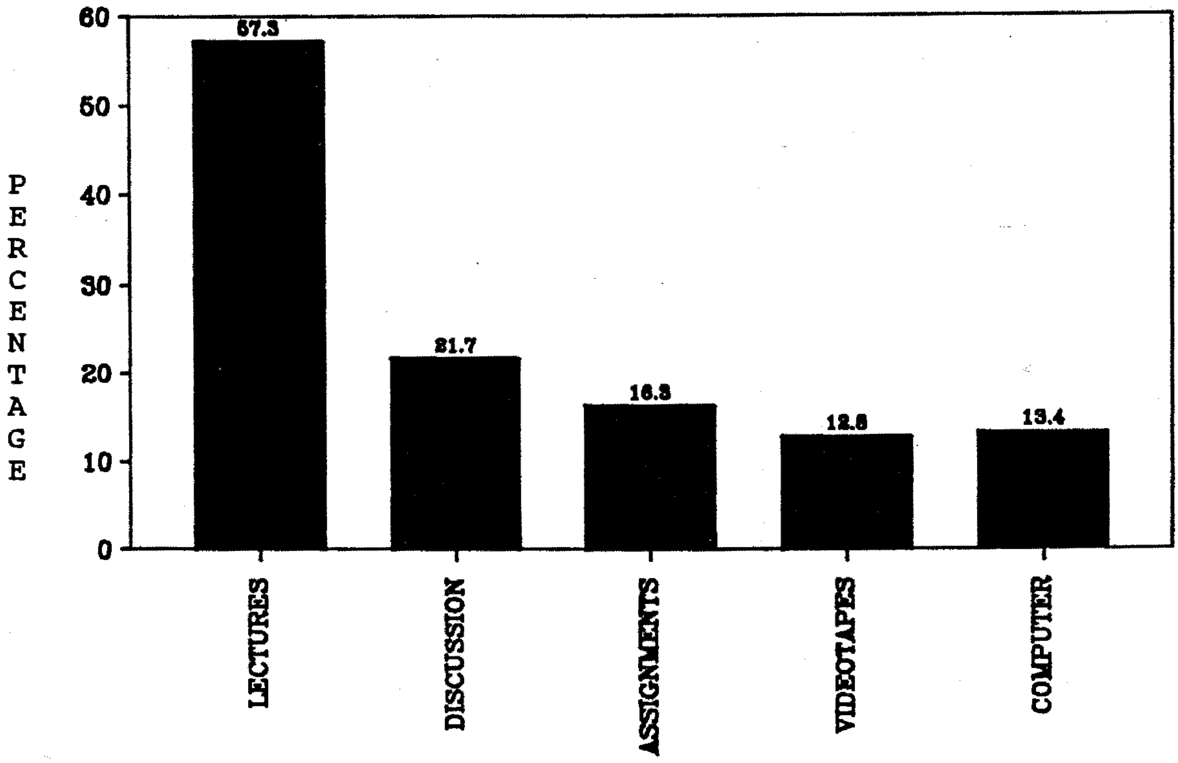


FIGURE 6 FORM OF PRE-COURSE

Analysis of the responses on the form of a pre-course in calculus shows that over half of the students still preferred lectures in preference to video tapes and computers. However with the rapid advances with computers and video tapes the figures could well alter significantly with time.

QUESTION 6 This question asked if students had passed a course in technical mathematics which included calculus. Of the respondents 88% had passed such a course. The other 12% had taken a course which included calculus but had not passed the course at the time of the questionnaire. Theoretically all students at second level mathematics should have passed the first level mathematics which includes calculus so the figures indicate some students who were allowed to go on to second level mathematics without passing the first level mathematics course.

QUESTION 7 Degree of difficulty of topics in first year mathematics.

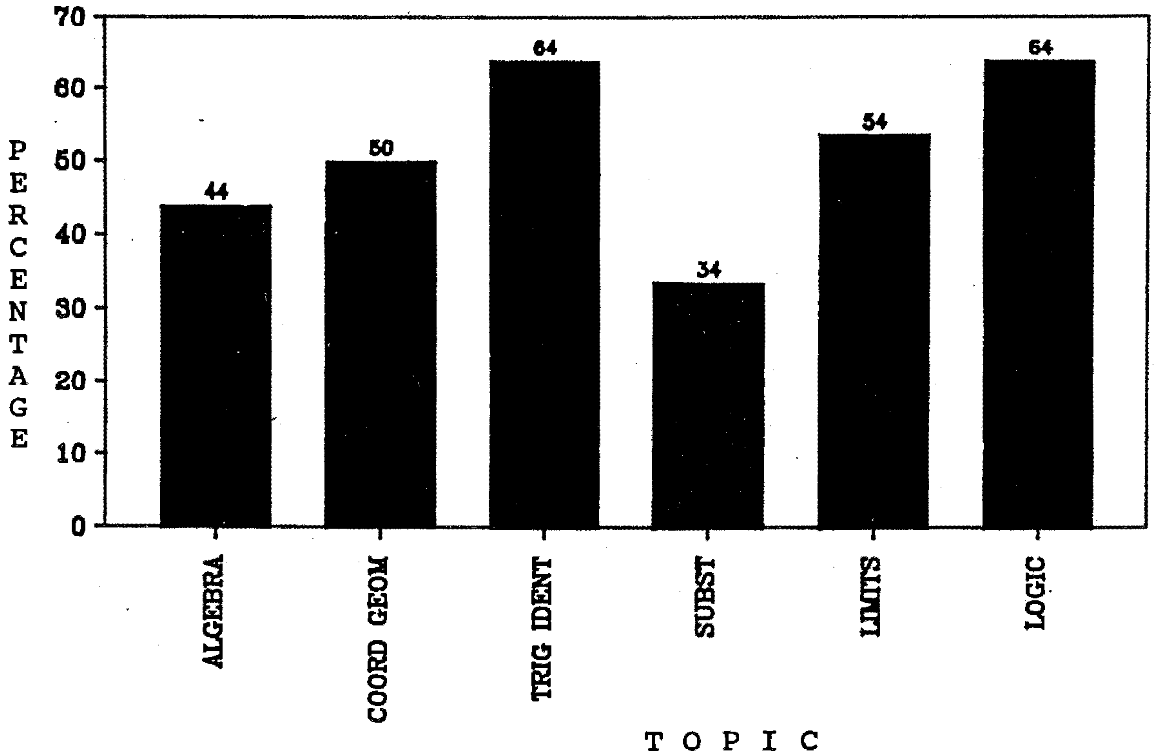


FIGURE 7 DEGREE OF DIFFICULTY OF TOPICS IN FIRST YEAR MATHEMATICS

The ten options were condensed into six items for the purpose of illustration, thus Algebra was an average combination of Algebraic equations, binomial expansion of brackets and simplifying algebraic expressions. Logic included an average of word problems, mathematical modelling as well as reading and analyzing a problem. Substitution was an average of substituting numbers for symbols and changing symbols for other symbols. Allowing for this it shows that trigonometric identities and logic caused the greatest difficulties followed by limits.

QUESTION 8 In this question students were asked to choose a meaning for the name calculus. Of the respondents, 75% chose a branch of mathematics involved with differentiating and integrating functions, 13% chose a branch of mathematics that deals with variable quantities and 9% chose a branch of mathematics that enables you to calculate areas and columns. 1.7% chose a type of addition. It was useful to test student perceptions of the name calculus.

QUESTION 9 Students interpretation of " $\frac{dy}{dx}$ " (Figure 8)

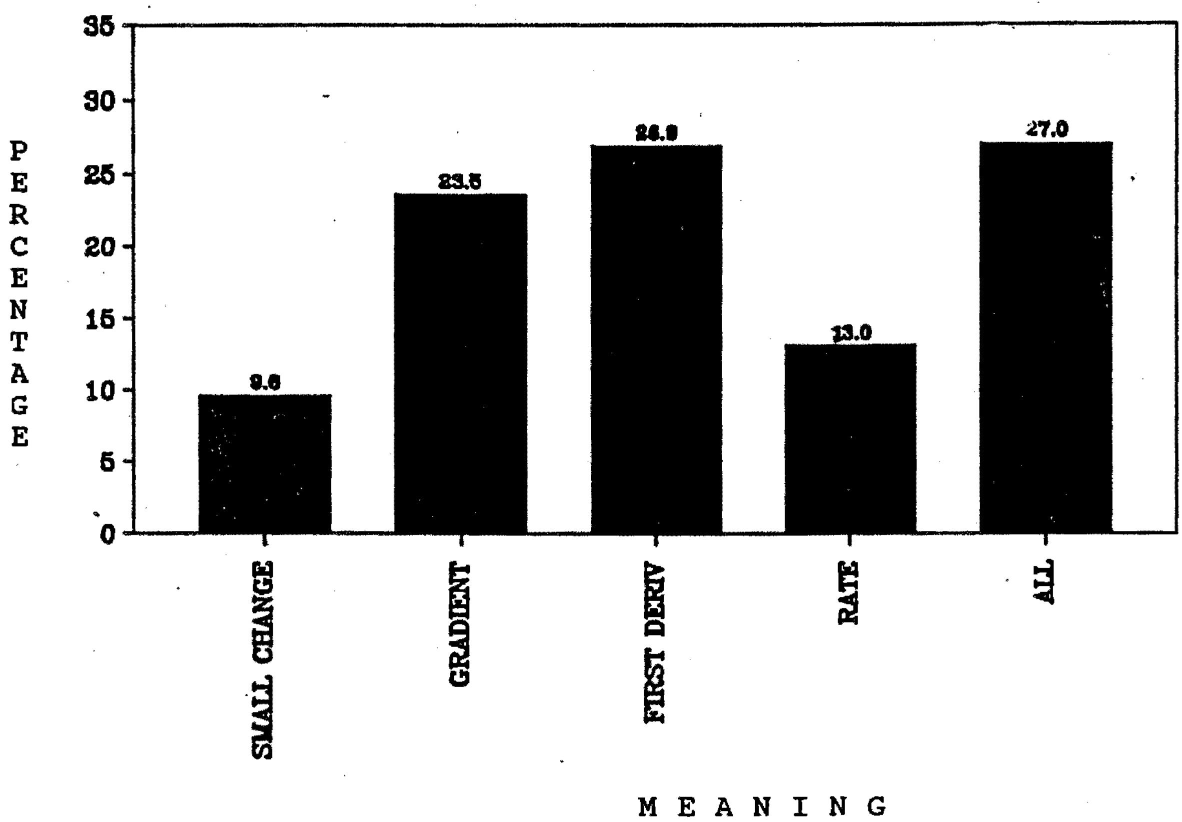


FIGURE 8 **STUDENTS' INTERPRETATION OF " $\frac{dy}{dx}$ "**

Student were divided in their responses with approximately 25% realising that all the first four related to the interpretation of $\frac{dy}{dx}$ whilst another 25% felt it related to the first gradient. The object of this question was to illustrate that students only think of one definition of $\frac{dy}{dx}$ which limits them when applications need other definitions.

QUESTION 10 Success in Differentiating $\frac{1}{x^2}$ (Figure 9)

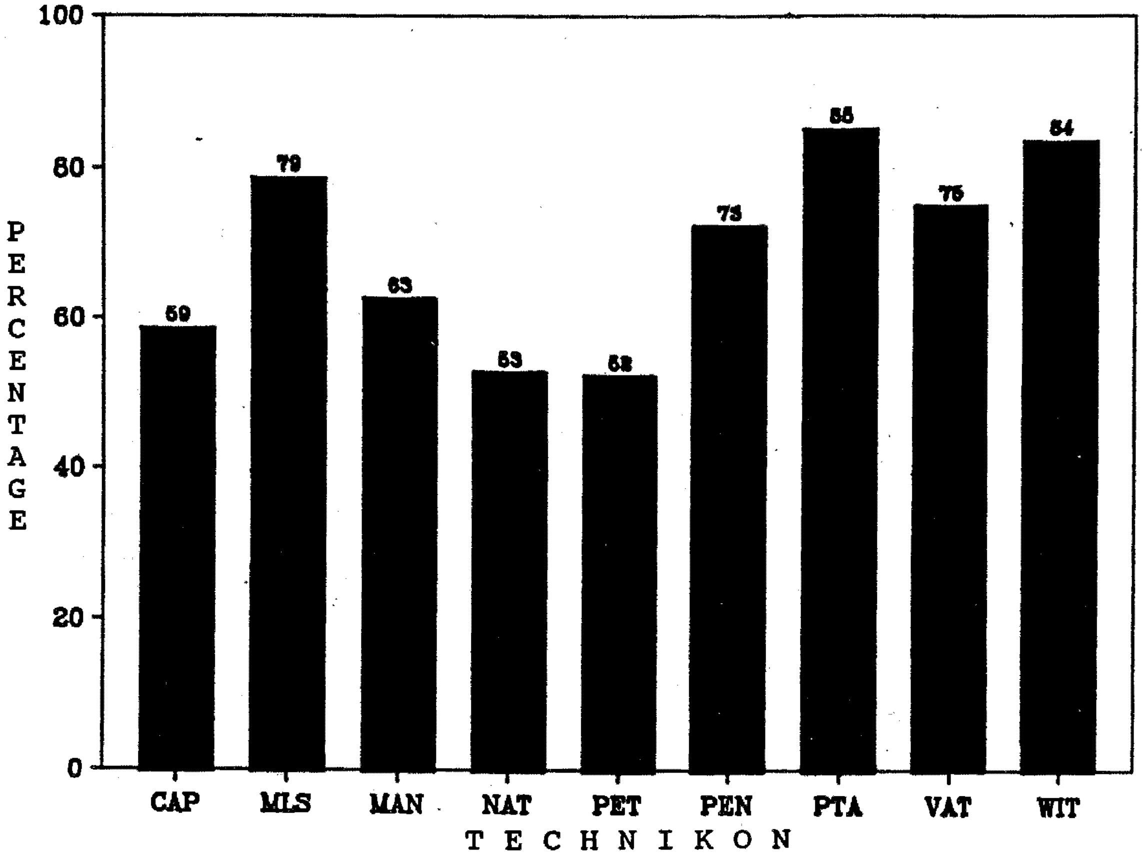


FIGURE 9 **SUCCESS IN DIFFERENTIATING** $\frac{1}{x^2}$

The function $\frac{1}{x^2}$ is a relatively simple expression to differentiate yet the overall success rate was approximately 70% with a range of success from a low of 52% to a high of 85%. The reasons for this range of success may well correlate with matric grades. For example on Figure 1 Pretoria Technikon has the highest percentage of higher grade passes (63%) and on Figure 9 has the highest percentage of success in differentiating $\frac{1}{x^2}$ (85%).

QUESTION 11 Differentiation of cos 2x added to sin 2x (Figure 10)

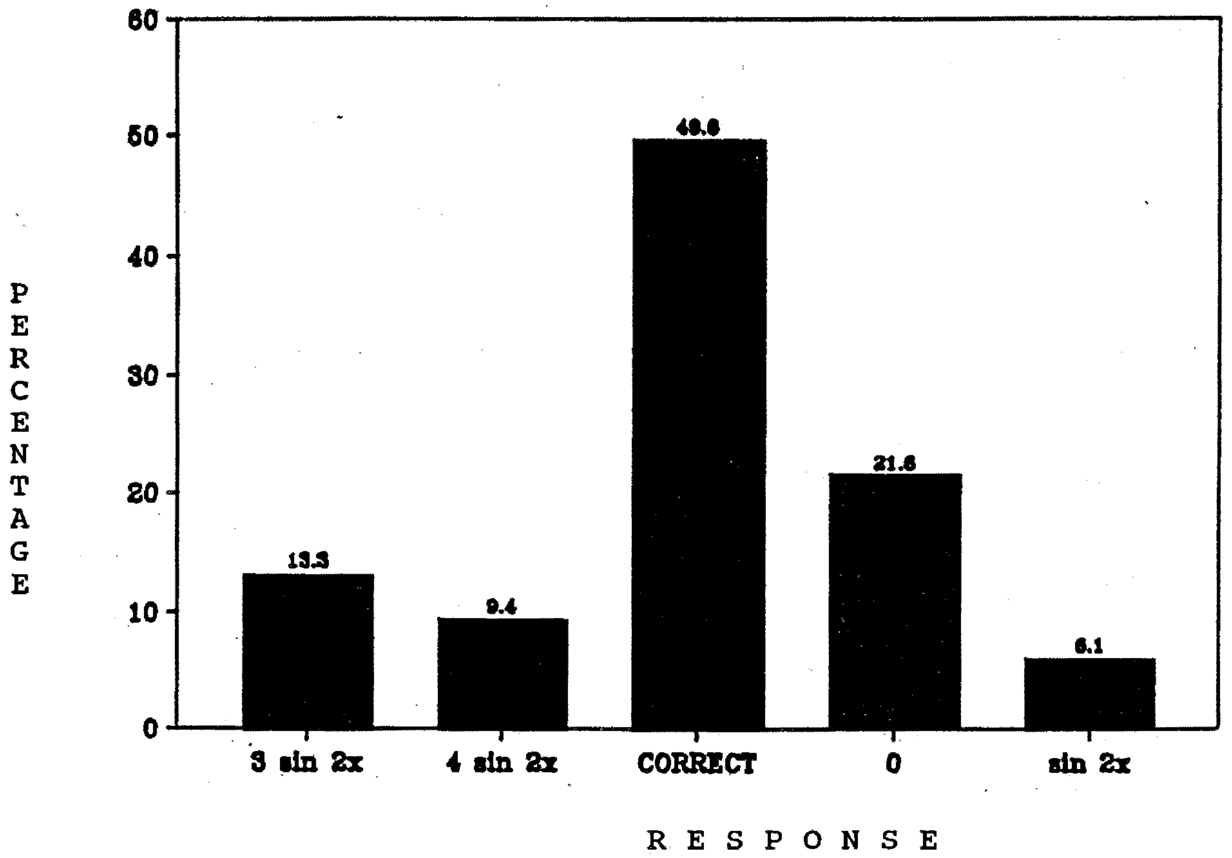


FIGURE 10 SUCCESS IN DIFFERENTIATING (COS 2X) AND ADDING RESULT TO (SIN 2X)

The success rate on this question was approximately 50% with a range from 16% - 74% at various Technikons. In Question 7 students listed trigonometrical identities as most difficult so this may tie in with the results of this question. Another point was the use of $\frac{d}{dx}$ which some students found unfamiliar.

QUESTION 12 Assigning a meaning to $\int$ (Figure 11)

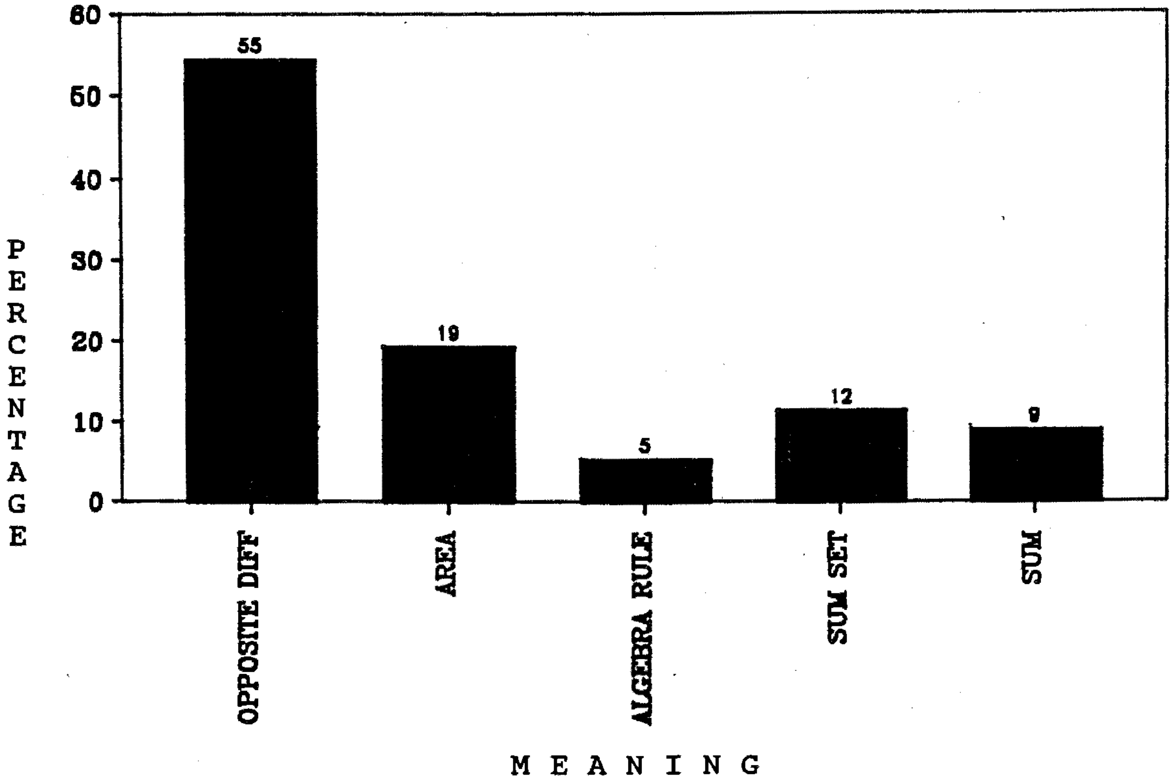


FIGURE 11 INTERPRETATION OF INTEGRAL SIGN

Approximately 55% of the respondents interpreted the integral sign to mean "do the opposite to differentiation" while a significant number of students (19%) interpreted it to mean area under a curve. There was no wrong answer, it merely indicated that students tend to fix one meaning to the integral sign.

QUESTION 13 Problems with Limits (Figure 12)

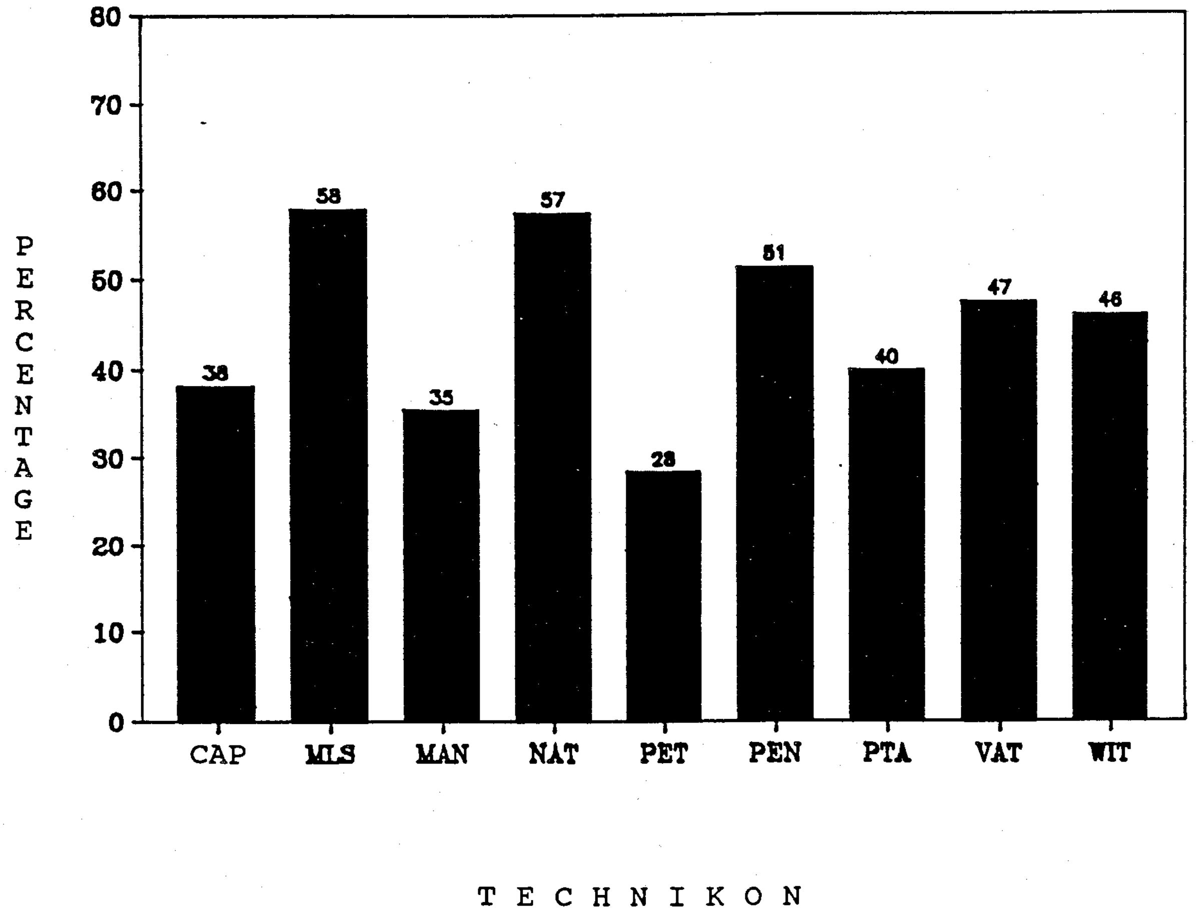


FIGURE 12 PROBLEM WITH LIMITS COMPARISON OF RESPONSES FROM THE DIFFERENT TECHNIKONS

On average 45% of students said they had problems with limits. The comparison between various Technikons shows a range from 28% - 57%. Limits obviously causes considerable difficulty to many Technikon engineering students.

QUESTION 14 Preferred Symbol for Derivative (Figure 13)

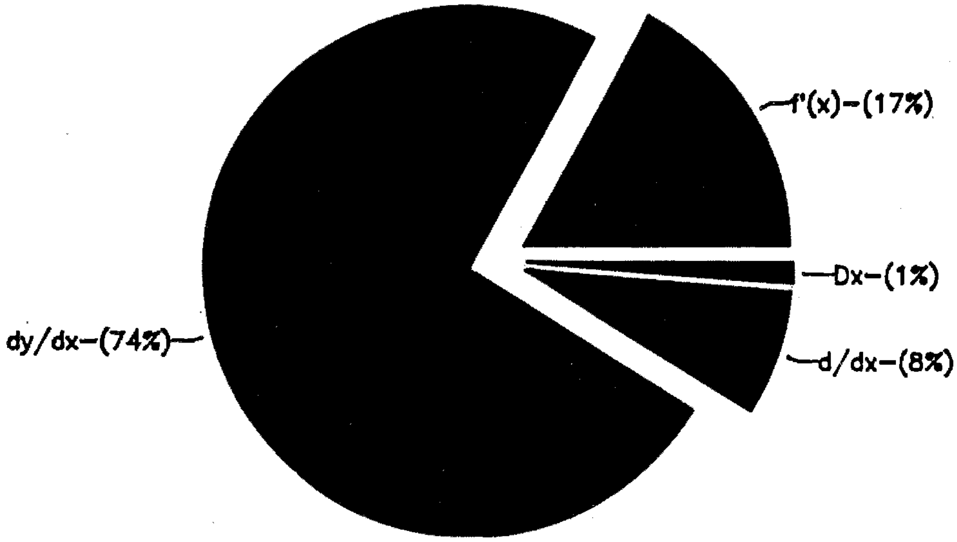


FIGURE 13 PREFERRED SYMBOL FOR DERIVATIVE

Approximately three out of four students preferred $\frac{dy}{dx}$ while approximately one in six students preferred $f'(x)$. This shows students are rather limited with their notation and find it difficult to use alternatives.

QUESTION 15 Inability to cope in terms of time (Figure 14)

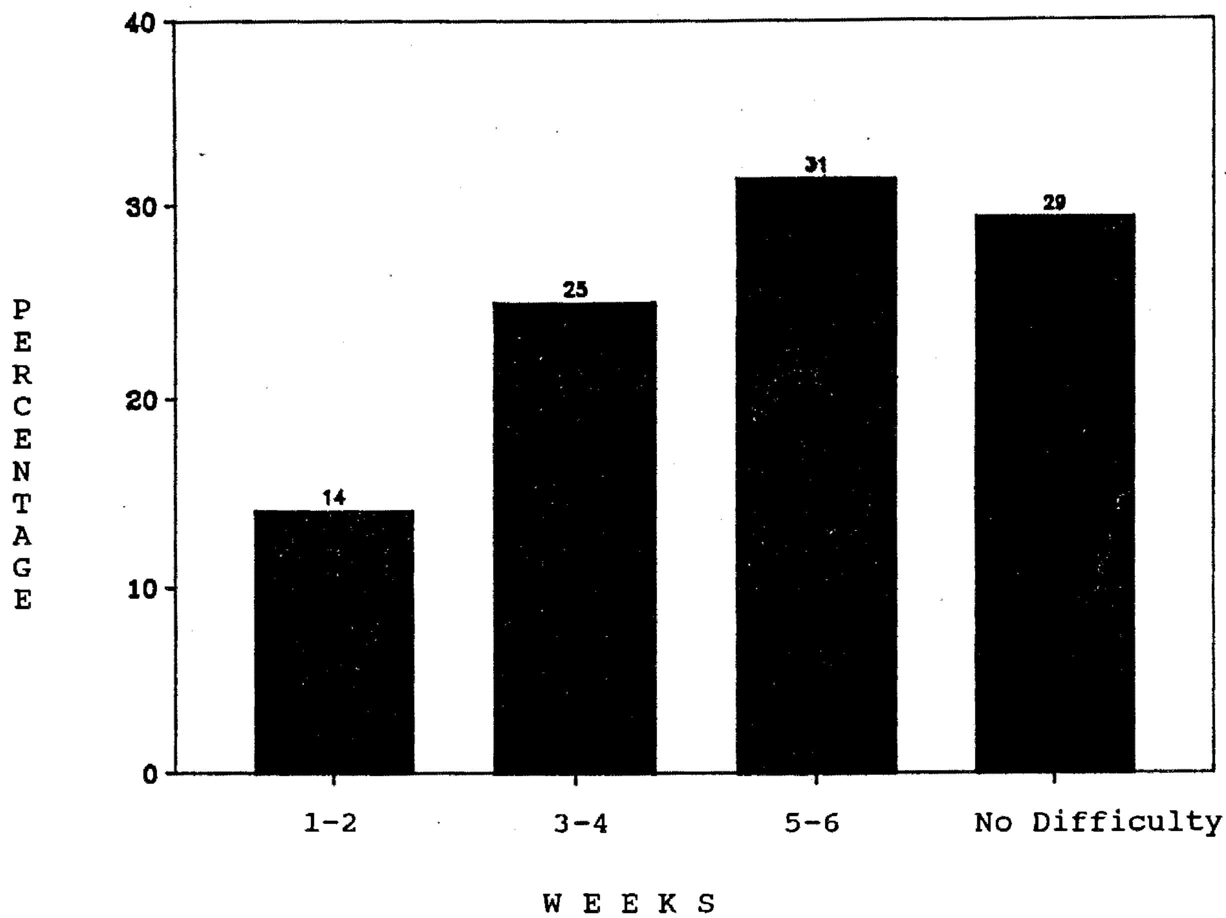


FIGURE 14 INABILITY TO COPE IN TERMS OF TIME

Adding up the various sub totals shows that 70% of students were having difficulty with time 5-6 weeks into the course. This causes a backlog in learning which is reflected in the second and third test results. For example at Technikon Natal one engineering group had results of 62%, 47%, 35% of students passing for the first three tests.

QUESTION 16 Adequacy of Lectures and Tutorials (Figure 15)

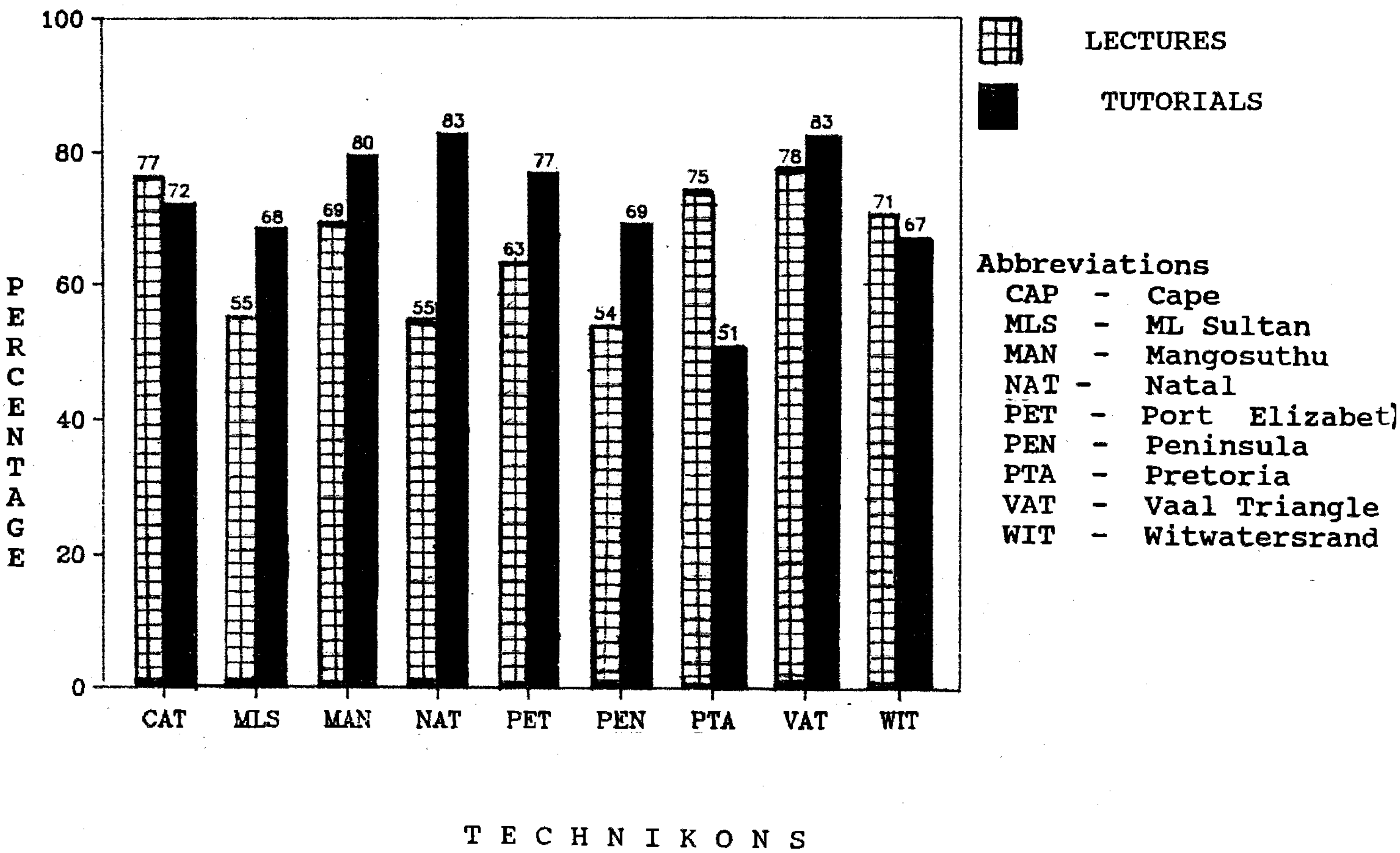


FIGURE 15 PERCENTAGE OF STUDENTS RATING LECTURES/TUTORIALS AS ADEQUATE

This question referred to the adequacy of time devoted to lectures and tutorials in calculus during a semester. Overall 68% of students rated the time devoted to lectures as adequate and 72% of students rated the time devoted to tutorials as adequate. A comparison of the nine Technikons shows a wide range of differences from 55% to 78% for lectures and 57% to 83% for tutorials. Some of these differences may be due to the total number of periods available in each Technikon for lectures and tutorials. This is investigated in the next question.

QUESTION 17 Preferred number of Lectures and Tutorials (Figure 16)

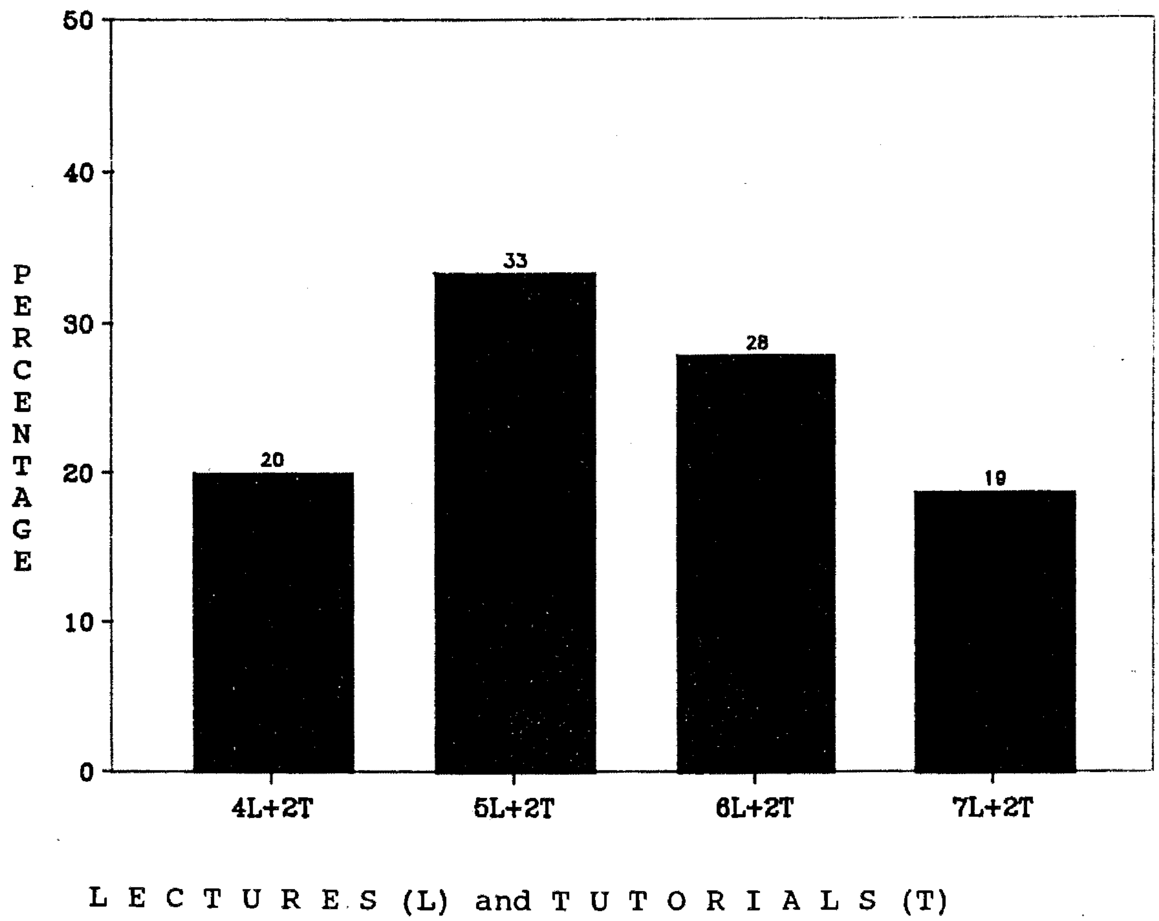


FIGURE 16 PREFERRED NUMBER OF SESSIONS PER WEEK

This Question analyzed the preferred number of periods for lectures and tutorials per week. Seven periods a week gained the highest percentage of responses (33%) while eight periods was nearly as popular (28%). This confirms students generally vote for one more period than they were presently receiving.

QUESTION 18 Hours per week spent on study (Figure 17)

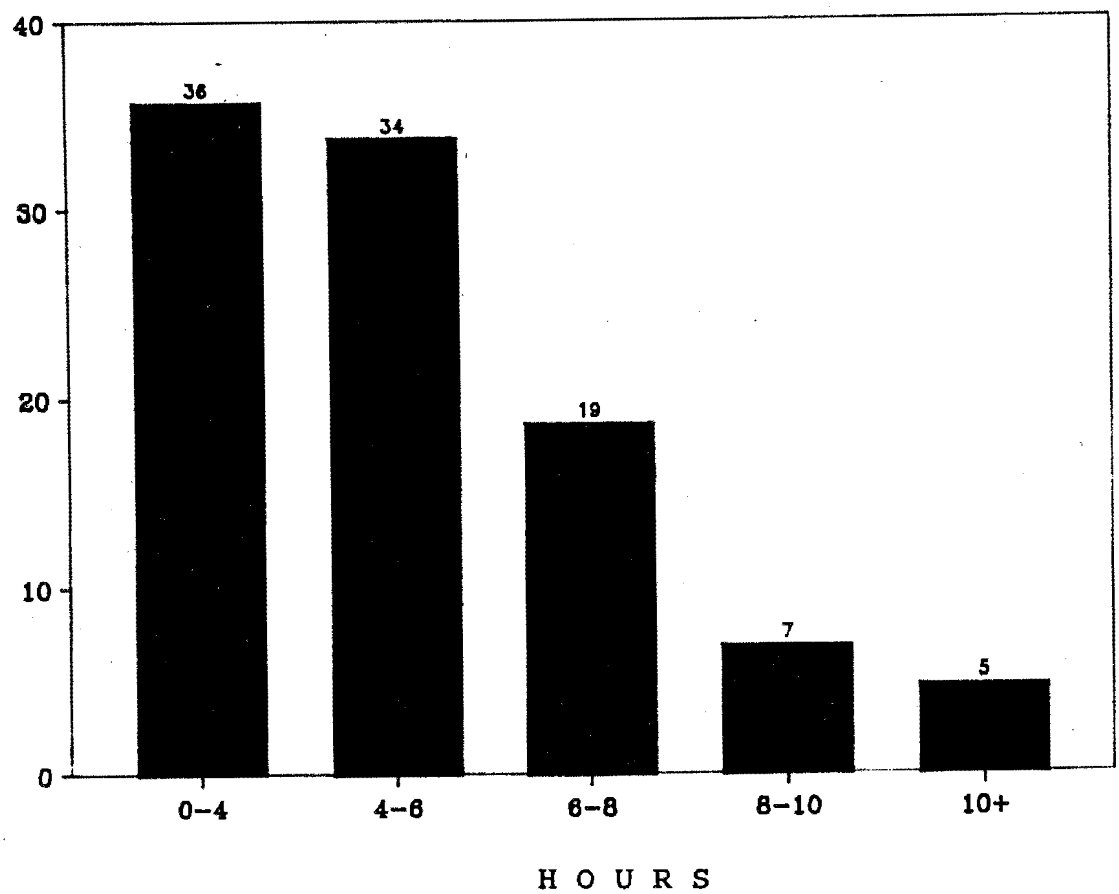


FIGURE 17 HOURS PER WEEK SPENT ON STUDY

Approximately one third of the respondents spent four hours or less and approximately two thirds spent six hours or less on study outside lectures. A comparison of results for Question 18 with matric grades (See Figure 1) showed that 22% of Higher Grade A candidates were spending over ten hours per week. Of Higher Grade B candidates and Standard Grade A, 13% were also prepared to spend more than ten hours per week. At the other end of the scale only 2% Higher Grade E candidates were prepared to spend ten hours of more with 7% Standard Grade E candidates also prepared to spend ten hours or more. This question was raised in the interviews and student comments will be discussed at that time. (See 5.2.3).

QUESTION 19 Various views on study guides, lectures and tutorials.

Students were asked to put a cross if they agreed with the statement or leave it blank if they disagreed. The results in percentages are given below:

			%
19.1	Textbooks are only useful for examples	19.1	20
19.2	Lectures give you all the explanation you need	19.2	26
19.3	Study guides are essential for any course.	19.3	43
19.4	Textbooks are useful for explanations and for examples.	19.4	64
19.5	Tutorials are essential	19.5	59

Several conclusions can be drawn from the results. The first is that only 26% of students think that lectures are enough or, put another way, approximately 75% of students require more than lectures. Secondly a majority of students feel textbooks are helpful with explanations and examples and nearly 60% of students regard tutorials as essential.

QUESTION 20 Difficulties with Differentiation (Figure 18)

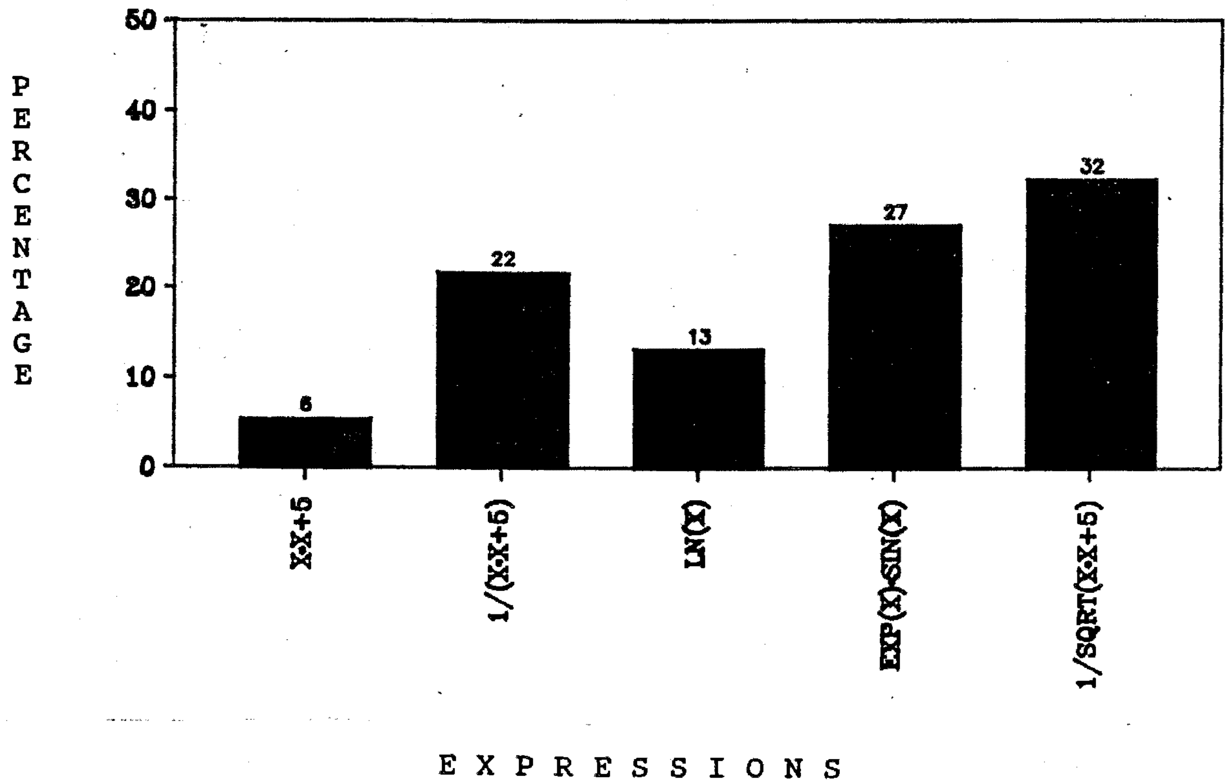


FIGURE 18 DEGREE OF DIFFICULTY OF SOME DIFFERENTIATION PROBLEMS

Five expressions to be differentiated were to be ranked according to difficulty. The greatest difficulty occurred with inverse square roots whilst a product of an exponential multiplied by a trigonometrical expression was next in order of difficulty. The object of this question was to confirm whether certain standard items consistently cause difficulty with differentiation.

5.2.4 A review of students' comments on the Student Questionnaire.

There are various ways of obtaining research data. The structured questionnaire is a very useful instrument as it is objective and the facts can be analyzed mathematically. However, personal interviews can reveal rather deeper observations which can confirm the results of the questionnaire or can be used to simplify those results or suggest amendments to the questionnaire. It was these considerations which prompted the group interview. Five students whose age, academic background and experience varied widely volunteered for the interview. The proceedings were videoed with the aid of two video cameras using the staff from the video department at Technikon Natal. At the time of the interview the results of the student questionnaire for the three Durban Technikons: Natal, M L Sultan and Magosuthu were known but the students did not know them.

The author first asked the students what question was the most difficult to answer. The students replied that Question 7 which asked students to rank in order of difficulty ten mathematical items, was the most difficult as ten items are rather a lot to place in order. They had no problem placing the top three or four but they found the last two or three a problem. This group placed trigonometrical identities and reading and analysing a problem as the top two topics for difficulty. They were quite surprised when the Durban results gave limits as the most difficult topic since limits did not cause them much difficulty. When asked which question they liked, the reply was Question 11 as it involved trigonometrical differentiation. Question 11 asked students to choose the correct result for differentiating cos 2x and then adding sin 2x.

Their main comment was that they did not like $\frac{d}{dx}\cos 2x$. They would have preferred y = cos 2x so that $\frac{d}{dx}\cos 2x=\frac{dy}{dx}$. The question would then have been written: y= cos 2x find $\frac{dy}{dx}+\sin 2x$. Their mistake was to interpret $\frac{d}{dx}(\cos 2x)+\sin 2x$ as $\frac{d}{dx}(\cos 2x+\sin 2x)$

When asked which question should be changed they indicated Question 19 which dealt with views on study guides, lectures and tutorials. They felt a straight forward question which asked whether you prefer a good lecture or a good textbook would have been better. This led to a discussion on how do you judge a good lecturer? This can be subjective so it is a difficult thing to quantify. The group indicated that a good textbook is very useful at home when you try to work out problems because you can compare simpler examples in the book with more difficult ones done in the lectures.

The group were then asked to suggest questions they felt should have been included in the questionnaire. One question suggested was to ask students whether they saw mathematics as a learning subject. Mathematics seemed to consist of learning routines, in other words, do it this way and it will give the correct answer. One student really wanted to know what was behind the routines but he said he didn't have the time to spend reasoning out why the routines worked. Another question they felt should be asked concerned standard of entry to mathematics courses. Technikon mathematics departments vary with some accepting E Standard Grade while others insist on at least a D Standard Grade. This group did not like restrictions and felt if a student passed matric mathematics he should be admitted. Their reasons were that Technikon mathematics was different and students varied in the early stages with slow starters coming out better in the end. This slow start could be due to the gap between matric and starting Technikon with factors such as military service or working to obtain funds.

One of the weaker students who had not done mathematics for some years felt that better selection of students for each class could have helped his progress. If two classes were possible then one could be for the brighter students and one for the weaker students. He said during the first course in calculus the pace was too quick for him. The counter argument is that brighter students can help the weaker student with explanations and examples if they are in the same class.

Another question they commented on was Question 18: "How much time do you spend in hours per week on your own doing mathematics?" In this small group the lowest amount spent by one student was four hours a week while the highest amount was ten hours a week. Their comments included one student mentioning he could try to solve a problem for three hours whereas his friend could solve it in fifteen minutes so the amount of time spent does not necessarily guarantee that progress will be satisfactory and success is assured.

5.2.5 Synthesis

The Student Questionnaire confirmed that students have certain basic difficulties with learning calculus. These difficulties can be split into difficulties inherited from previous mathematical experience in, for example, Algebra and Trigonometry and difficulties which belong to Calculus such as notation and various early concepts. The algebraic difficulties included translating words into symbols, simplifications, solving algebraic equations and using the binomial theorem. In trigonometry the use of identities was a major stumbling block to many students as illustrated by the rating of top difficulty by 64% of students (See Figure 7). The difficulties which belong to the calculus were understanding of notation such as derivative (See Figure 8), for example, and concepts such as limits (See Figure 12) and integration (See Figure 11). Other items on the questionnaire referred to time difficulties of a semester course. Students had two problems. One was to cope with the content of the course and the second was the time devoted to lectures and tutorials. As previously mentioned (See Figure 10) 70% of students were having difficulty with time after 5-6 weeks of the course and students generally wanted one more period a week than what they were receiving at the time of the questionnaire (See Figure 16). Modern technology was examined but results still placed computers and videos below lectures and assignments (See Figure 5). Biographical information showed the diversity of entrants with regard to matric results ranging from Higher Grade A to Standard Grade E, ages ranging from 18 to 26 plus and previous occupation ranging from school, university, business, commerce and industry.

Interviews with students raised other matters such as: Is mathematics a learning subject and should students be excluded with low mathematical grades? On the first point students felt mathematics involved routines rather than learning what it all means, on the second point they did not want students excluded, as some weak students flourished in the technikon environment. The quality of time spent outside lectures was a good point discussed by students.

5.3 ANALYSIS OF THE DATA OBTAINED THROUGH THE STAFF QUESTIONNAIRE

5.3.1 Aims of the questionnaire

The first aim of this questionnaire was to gain insight into the difficulties experienced by staff when lecturing on the calculus. (See Annexure B for the questionnaire detail).

The second aim was to compare results of several questions which were common to both the Staff and Student Questionnaire. For example, Question 6 on the Staff Questionnaire (order of difficulty of ten mathematical topics) was the same as Question 7 on the

Student Questionnaire which also dealt with ranking mathematical topics in order of difficulty. Questions 7-9 on the Staff Questionnaire which dealt with symbols and names were the same as questions 10-12 on the student questionnaire. In Question 10 of the Staff Questionnaire staff were asked if they had problems teaching limits while in Question 13 of the Student Questionnaire students were asked if they had problems with limits. Question 17 on the Student Questionnaire and Question 13 on the Staff Questionnaire both asked how many periods or lectures were necessary per week.

The third aim was to obtain biographical information about the qualifications, age, etc. of staff.

5.3.2 Modus operandi of the questionnaire

A structured questionnaire was prepared (See Annexure B) in both English and Afrikaans so that staff could answer in their home language but home language differences were not pursued in this study. The group of sixteen questions dealt with the following categories (question numbers are given in brackets each time):

1. Biographical information (1,2,3)
2. The desirability and form of a pre-course in calculus (4,5)
3. Ranking ten mathematical items in order of difficulty (6)
4. The meaning of terms used in calculus (7,8,9,11)
5. Whether the respondent has problems with teaching limits (10)
6. The way time difficulties affected the calculus course (12,13,14)
7. Views on the methodology used in teaching and learning (15,16)

Staff were invited to fill in the questionnaire on a voluntary basis. Seventy responses from nine technikons indicated a high percentage of possible staff members responded. Each technikon was provided with the average total results and its own particular results without using names of individuals.

5.3.3 Analysis of Staff Questionnaire

5.3.3.1. Analysis of biographical data

QUESTION 1 Highest Mathematical Qualification (Figure 19)

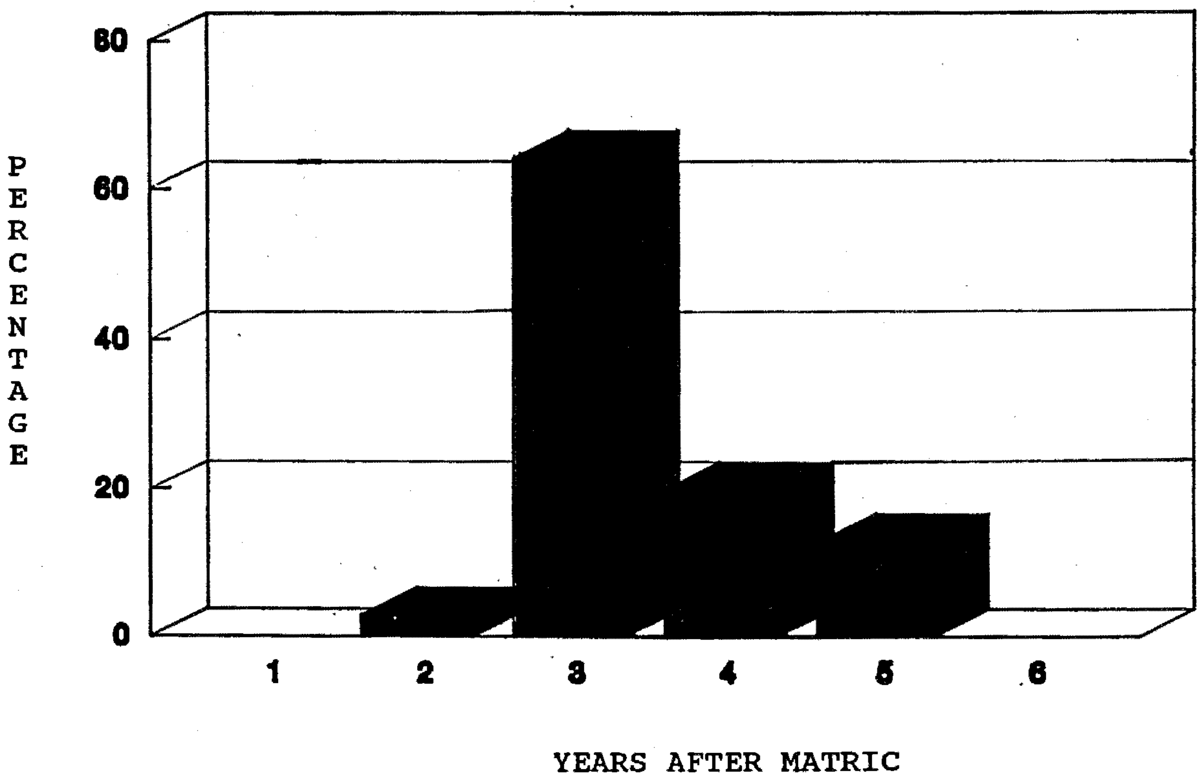


FIGURE 19 LEVEL OF MATHEMATICS TRAINING

The responses to this question showed that over 60% of staff had three years of mathematics training, 20% had four years and 13% had five years of mathematics training. In view of increasing standards and the future prospect of a bachelor degree in technology for engineering, mathematics departments may be asking for a minimum of four years of training and for some posts five years of training in mathematics.

QUESTION 2 Highest Teaching Qualification of Staff (Figure 20)

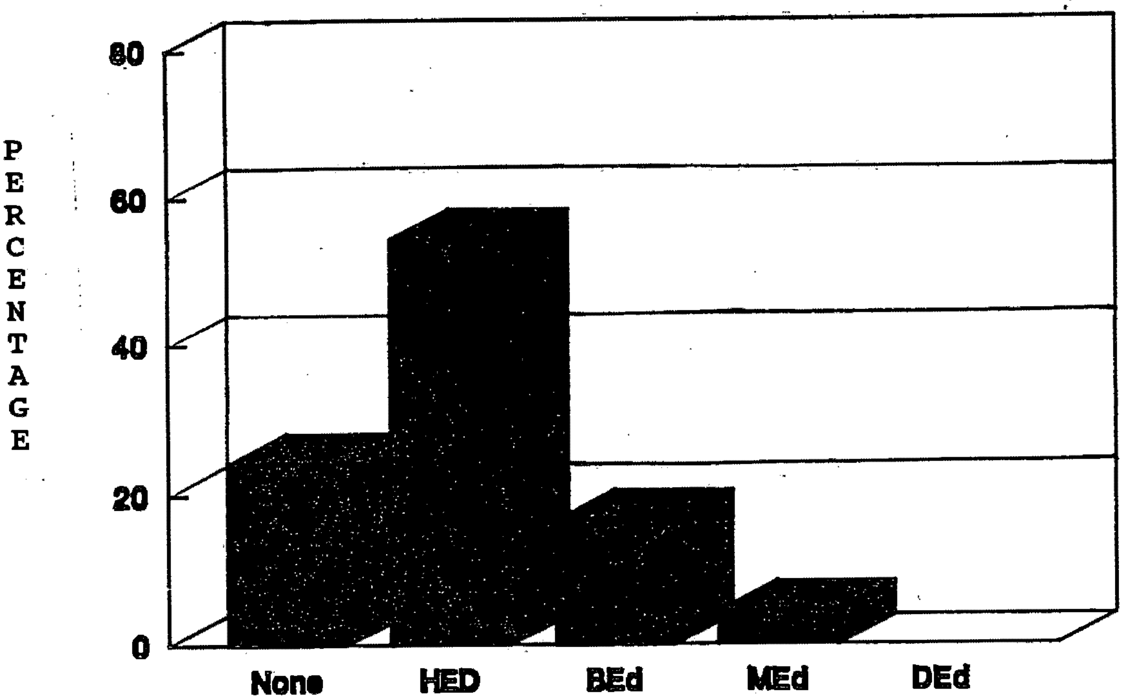


FIGURE 20 HIGHEST TEACHING QUALIFICATION

An examination of the responses to Question 2 showed that over 20% had no teaching qualifications while over 50% had one year higher education diploma, 17% had a Bachelor of Education and only three had a Master of Education. This reflected the diversity of training or lack of training in education. During the last five years with the increase of higher diplomates, laureatus and the possibility of degree courses many staff at various technikons have studied for masters degrees.

QUESTION 3 Age of Mathematics Lecturers (Figure 21)

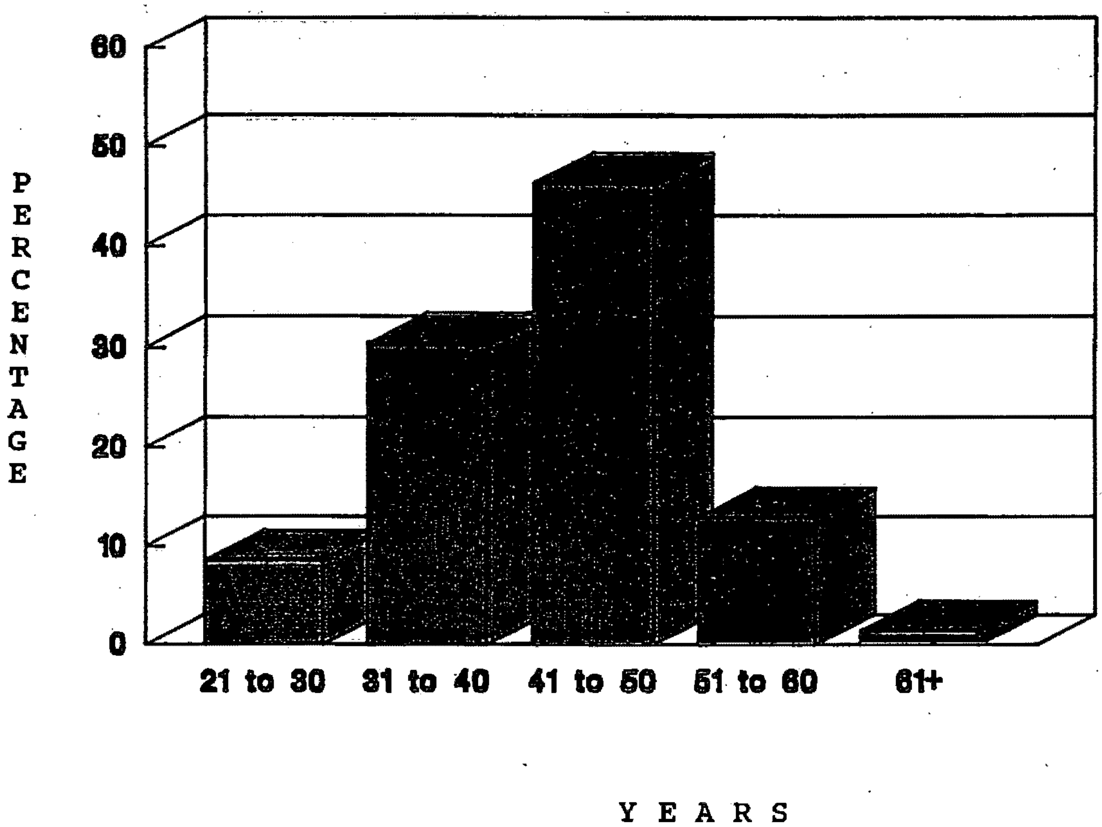


FIGURE 21 AGE OF MATHEMATICS LECTURERS

The responses to Question 3 showed that 46% were aged from 41-50 years and approximately 30% were aged from 31-40 years. Only 13% were aged from 51-60 years and only one person was over 60 years of age. Apparently lecturers in the over 50 years age group tend to move to other tertiary institutions such as universities or technical colleges.

5.3.3.2 Analysis of responses to specific question

QUESTION 4 Need for a mathematics pre-course for weaker students (Figure 22)

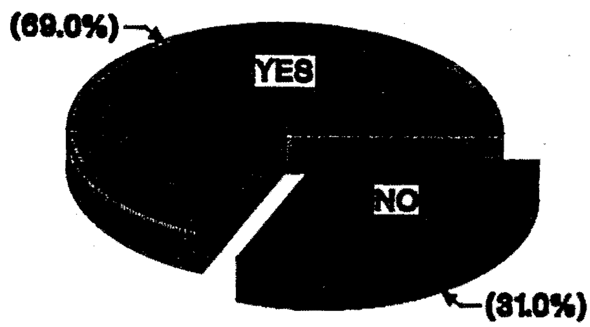


FIGURE 22 NEED FOR A MATHEMATICS PRE-COURSE FOR WEAKER STUDENT

Over 2/3 of staff answered that a pre-course in mathematics is necessary. Many bridging courses in mathematics have been evolved for university students but technikons have been slower to implement this idea. Mangosuthu Technikon run a full semester course for pre-technical students in engineering which includes mathematics. Technikon Natal recommends that all students with D or E Standard Grade symbols for mathematics should attend a short course at the beginning of their first semester. Port Elizabeth Technikon who had quite a low entry standard in mathematics at the time of the questionnaire have helped their students with extra tutorials.

QUESTION 5 Form of the Pre-Course in Order of Preference (Figure 23)

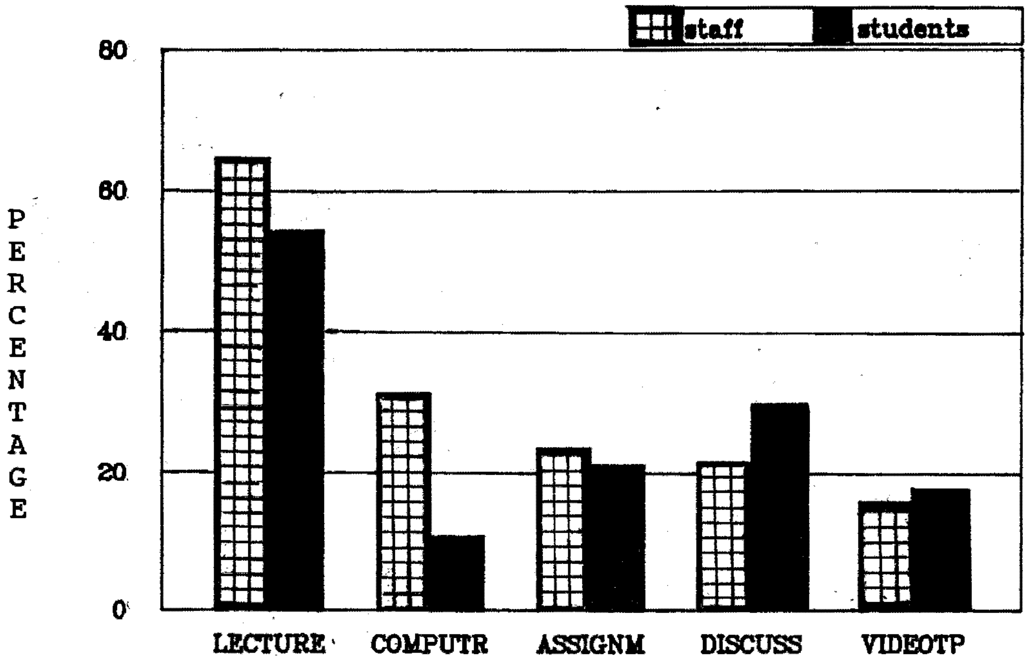


FIGURE 23 FORM OF THE PRE-COURSE

Respondents who felt a need for a pre-course in mathematics in Question 4 were asked to select the method they would prefer to be used on a pre-course. Approximately 2/3 of staff put official lectures as their first choice while approximately one third put computer programmes on an individual basis as their second choice. Assignments, small discussion groups and video tapes covering specific items attracted only ± 20% of respondents' third, fourth and fifth choices.

If a comparison is made of student and staff responses the first choice is still official lectures for both but the students' second highest choice was discussion groups as compared with staff's second choice of computer programmes. The students would probably place computer programmes much higher over time since many computer packages have lately been incorporated into their courses.

QUESTION 6 ORDER OF DIFFICULTY OF TEN MATHEMATICS TOPICS

STAFF	STUDENTS
1. Reading and analysing	1. Word problems
2. Word Problems	2. Trigonometry identities
3. Trigonometry identities	3. Reading and analysing
4. Simplifying alg expressions	4. Limits
5. Limits	5. Binomial expansion
6. Binomial expansion	6. Coordinate geometry
7. Changing symbols	7. Simplifying alg expressions
8. Algebraic equations	8. Changing symbols
9. Coordinate geometry	9. Algebraic equations
10. Substituting numbers	10. Substituting numbers

Analysing the staff responses to Question 6 where ten mathematical topics had to be ranked for difficulty from 1 (most difficult) to 10 (least difficult) shows that the first three in order of difficulty were reading and analysing a problem, word problems and trigonometrical identities whilst the three least difficult were algebraic equations, coordinate geometry and substituting numbers. The most difficult topics involve mathematical modelling which lecturers try to impart to their students. Students on the other hand prefer set routines.

A comparison of staff and student responses shows that the three most difficult topics reading and analysing a problem, word problems and trigonometrical identities were the same except the order changed. Students put words problems as the most difficult which agrees with the low marks achieved on such questions in examinations conducted by the author over the last eight years at Technikon Natal. At the other end of the scale both students and staff felt substituting numbers as the least difficult. Limits were placed fourth in order of difficulty by students and fifth in order of difficulty by staff so this topic seems to cause equal difficulty to staff and students.

QUESTION 7 Definition of Calculus

STAFF		STUDENTS	
65%	Differentiation and Integration of functions	75%	Differentiation and Integration of functions
23%	Deals with variables	13%	Deals with variables
10%	Calculates rates of change	8,8%	Calculates areas and volumes
1,4%	Calculates areas and volumes	1,7%	A type of addition
0%	A type of addition	0,9%	Calculates rates of change

Staff were asked to put a mark to the statement which most fits the meaning of calculus. About 2/3 responded to a branch of mathematics dealing with differentiating and integrating functions. It is somewhat surprising that nearly 25% of staff responded to a branch of mathematics that deals with variable quantities. A comparison of the same question for students shows that the majority (75%) responded to differentiating and integrating functions somewhat higher than the staff response of 65% with a corresponding reduction in the response to deal with variables for students (13%) and for staff (23%).

QUESTION 8 Definition of $\frac{dy}{dx}$

D E F I N I T I O N O F $\frac{dy}{dx}$

STAFF		STUDENTS	
52%	Combination of those given	27%	Combination of those given
19%	Rate of change of y wrt x	27%	First derivative of y wrt x
16%	First derivative of y wrt x	23%	Gradient of y=f(x)
6,3%	Small change of y wrt x	13%	Rate of change of y wrt x
6,3%	Gradient of y-f(x)	9,6%	Small change of y wrt (x)

Approximately half of the staff responded to a combination of several definitions of $\frac{dy}{dx}$. The surprising conclusion to this question is that many staff see $\frac{dy}{dx}$ as having only one meaning. If a comparison is made with the student responses then only approximately one quarter of students as opposed to approximately one half of staff agreed that $\frac{dy}{dx}$ could have several definitions.

This is a major difficulty when word problems are given to students. This links with the high ranking of difficulty with word problems given by students in response to Question 7 on the Student Questionnaire.

QUESTION 9 Definition of integral sign (Figure 24)

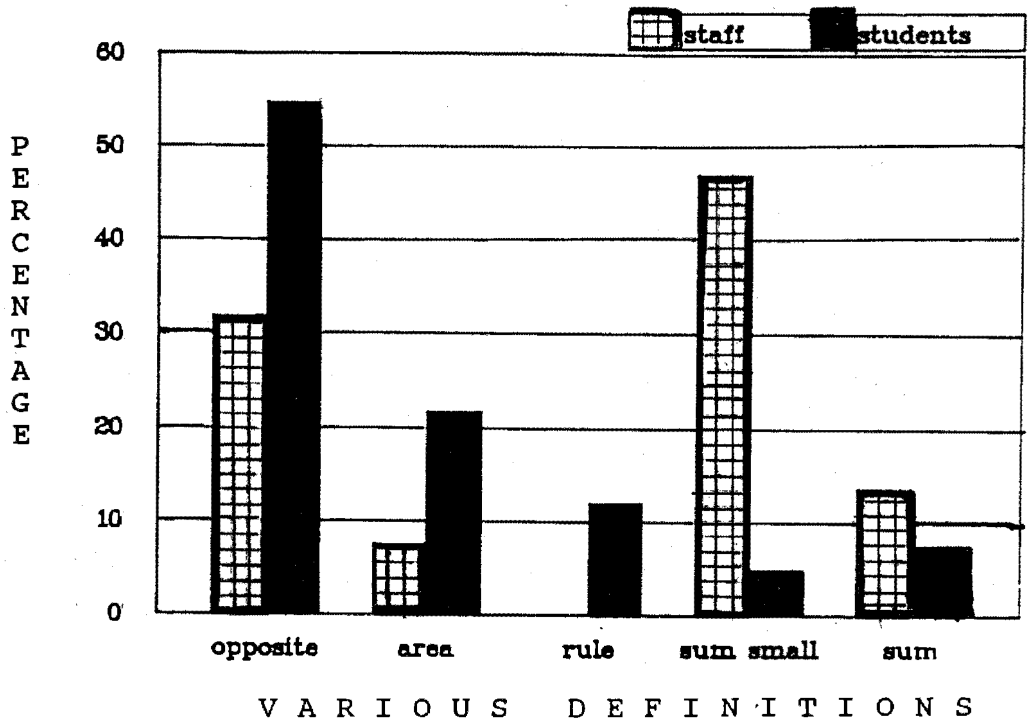


FIGURE 24 DEFINITION OF INTEGRAL SIGN

Nearly half the staff responses marked the definition 'sum of a set of small quantities' and approximately a third of the staff responses marked the definition 'the opposite of differentiation'.

A comparison with staff and student responses showed a much higher percentage of students (55%) marked the definition 'do the opposite to differentiation' as opposed to 32% for staff while a very small percentage of students (5%) marked the definition 'the sum of a set of small quantities' as opposed to 47% for staff. This question again shows how limited is the perception of integral as exemplified by the symbol of integration.

QUESTION 10 Problems with limits (Figure 25)

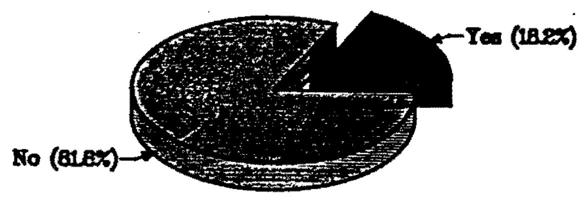


FIGURE 25 STAFF PROBLEMS WITH LIMITS

The results of Question 10 show that approximately one in five staff have difficulty teaching the ideas on limits which are fundamental in the early concepts of differentiation and later in integration.

QUESTION 10 Problems with limits (Figure 26)

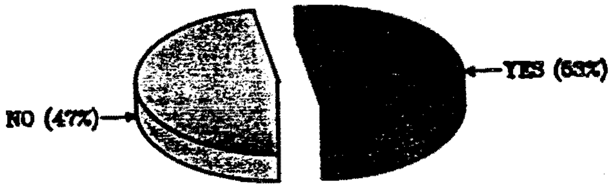


FIGURE 26 STUDENTS' PROBLEMS WITH LIMITS

When student responses are compared with staff there is a wide divergence as 53% of students said they failed to understand limits as opposed to 18% of staff who had difficulty teaching the ideas on limits. The modern viewpoint is tending towards eliminating time spent on limits and differentiation from first principles with greater use of computer packages. For example, Barnes (1992:73) asks for non-specialists

> "which parts of the traditional curriculum could be left out? I do not believe there is any need in a first course for work on limits or differentiation from first principles."

QUESTION 11 Symbol for Differentiation (Figure 27)

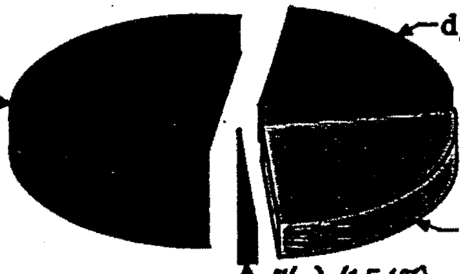


FIGURE 27 SYMBOL FOR DIFFERENTIATION

The staff replies to Question 11, which dealt with various symbols, used to indicate differentiation, showed just over half had no preference but approximately a quarter preferred $\frac{dy}{dx}$ with a very small percentage (1,54) preferring $f'(x)$. It is necessary to be versatile with notation in calculus but as shown in Figure 26 lecturers can be selective which will affect student choices also. If reference is made to Figure 13 then it will be seen that students have a very strong preference for $\frac{dy}{dx}$ (74%) as opposed to staff (26%). This divergence can possibly be explained by the examples given in the first level where $\frac{dy}{dx}$ is used more often than $f'(x)$.

QUESTION 12 Lecture and tutorial time (Figure 28)

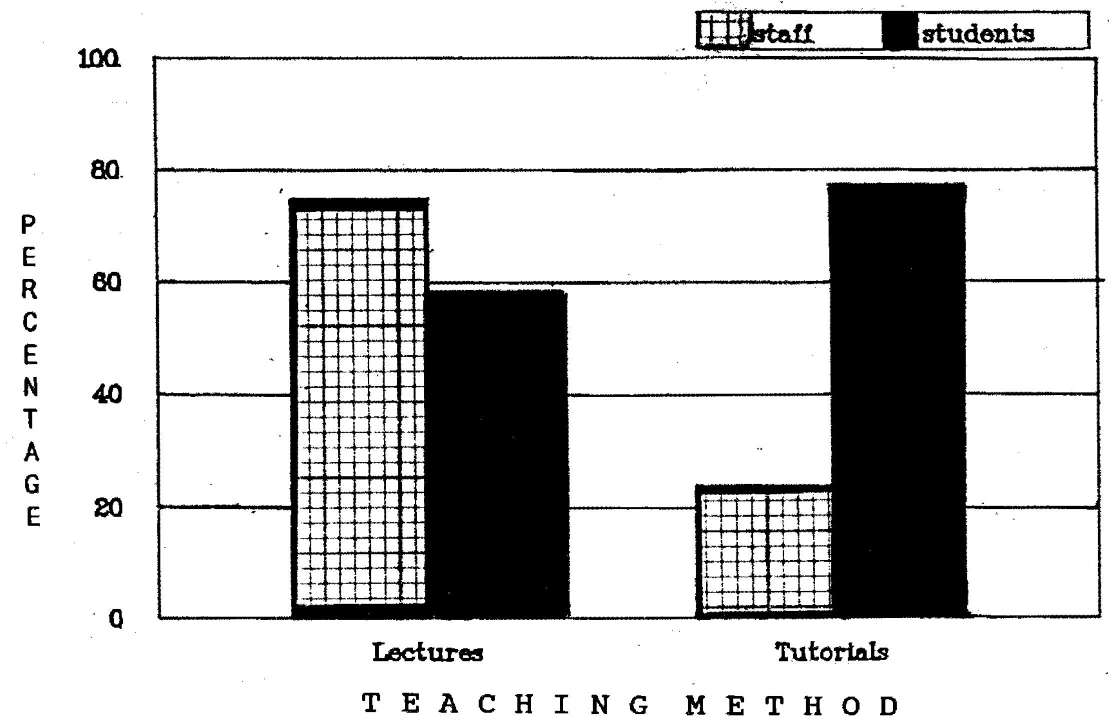


FIGURE 28 LECTURE AND TUTORIAL TIME

Approximately 80% of staff agreed that lecture time was adequate while only 22% of staff felt tutorial time was adequate. Students were less emphatic with whether time was adequate for lectures (60%) as opposed to approximately 80% for staff. The largest difference occurred with time for tutorials where nearly 80% of students felt tutorial time was adequate as opposed to 22% of staff. A revision of the lecturing time is being done overseas as more computer time is required. In the USA some courses are built around computer packages so that formal lectures are reduced and laboratory work is increased.

Moore and Smith (1992:100) states

> "Now we are concentrating on the student: what activities should students be doing to help them construct the mathematical knowledge they need? As we thought about this we have lectured less and less, but instead, developed a three semester calculus program based in a laboratory science model which is in our case a computer lab."

They advocate students working in pairs with information outside normal textbook examples which they examine and reach conclusions after discussions and then write up their results and conclusions using a technical word processor. This puts a greater onus on students to really understand what they are doing as opposed to merely observing facts and views from lecturers.

QUESTION 13 Number of periods per week needed for mathematics (Figure 29)

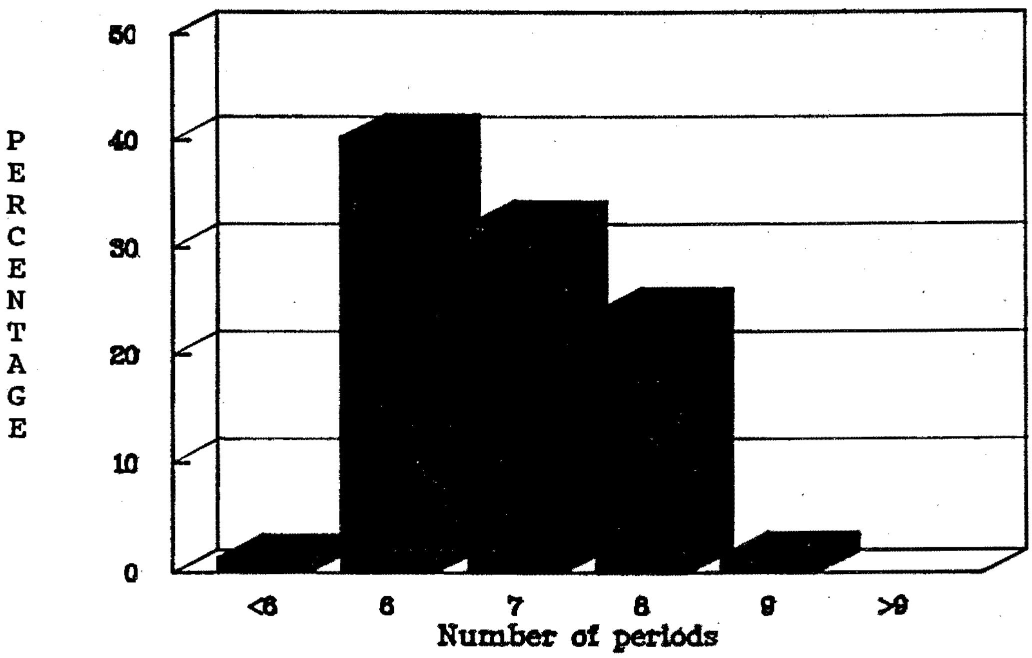


FIGURE 29 NUMBER OF PERIODS PER WEEK NEEDED FOR MATHEMATICS

Of staff 40% felt that six periods were necessary for mathematics courses and 30% of staff felt seven periods were necessary and over 20% felt eight periods were necessary. This may reflect the variance of periods in different technikons for example Natal has six periods while Magosuthu has eight periods. A comparison with student responses (Figure 17) showed that students in general opted for one more period than staff, only 20% went for six periods as opposed to 40% of staff while 33% wanted seven periods as opposed to 30% of staff and 28% of students wanted eight periods as opposed to 20% of staff.

QUESTION 14 Time spent on preparation for lecturing MATHEMATICS (Figure 30)

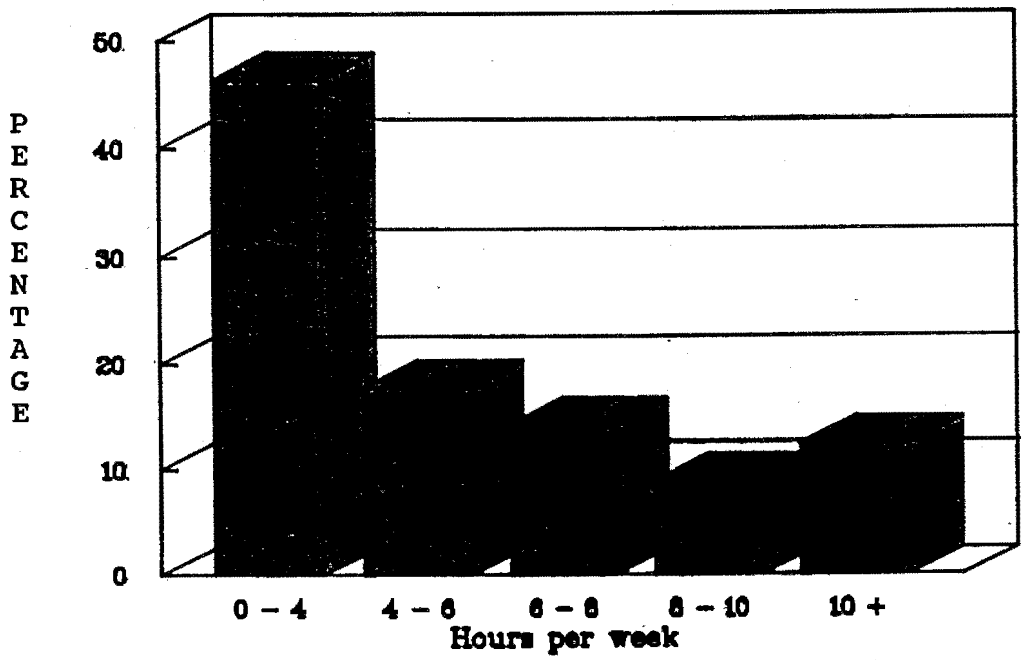


FIGURE 30 TIME SPENT OF PREPARATION FOR LECTURING MATHEMATICS

An examination of the responses to Question 14 reveals the 46% spend less than four hours a week preparing their mathematics lectures. This may be due to staff keeping the same courses for several years or it could be that courses have been covered with a batch of transparencies which are not changed. This is balanced by about 12% who are prepared to spend over ten hours a week on preparation which can only benefit students.

QUESTION 15 VIEWS ON LECTURES, TUTORIALS AND TEXTBOOKS

VIEWS ON LECTURES, TUTORIALS AND TEXTBOOKS

65%	A good lecturer is essential
58%	Poor text and good lecturer better than vice versa
55%	Good textbook better than poor tutorial
45%	Tutorial attendance should be compulsory
24%	A good textbook can replace a poor lecturer

65% of staff felt a good lecturer was essential. Even if a lecturer was not up to standard only 24% felt he could be replaced with a good textbook. A good lecturer and a poor textbook were felt to be better than a poor lecturer and a good textbook by 58% of staff.

The general conclusion from staff responses is that lecturers are still very necessary for courses despite textbooks.

QUESTION 16 Visual aids in order of usefulness (Figure 31)

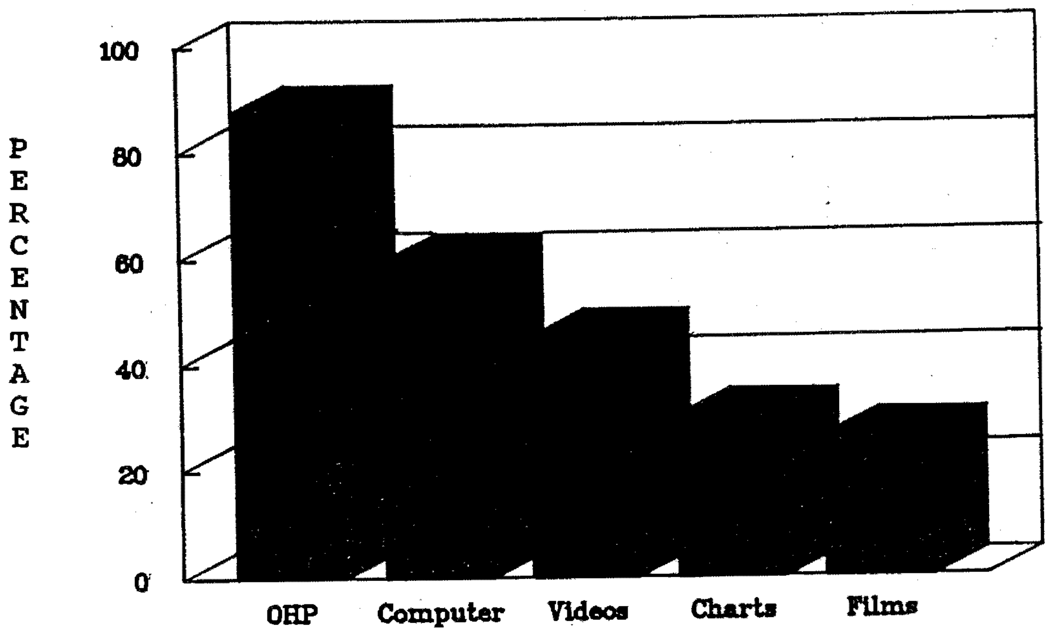


FIGURE 31 VISUAL AIDS IN ORDER OF USEFULNESS

It is interesting to analyse the results of responses to Question 16 of the staff questionnaire. Over 80% of staff rank the overhead projector as the most useful aid to teaching which is not surprising considering the number of overhead projectors available in Technikons. The real surprise was that 63% rank the computer as the next most useful aid to teaching considering the fact that computers are only recently becoming readily available for teaching purposes at the technikon. Videos also play a significant role in some courses so they occupied third place with charts and films fourth and fifth respectively.

5.3.3.3 Synthesis

One of the main difficulties experienced by staff was what to do with weaker students. In the questionnaire the majority of respondents felt the need for a pre-course for weaker student (See Figure 22). The order of difficulty for ten mathematical topics (Question 7 of Annexure B) underlines the problems staff have in lecturing to students. The weaker students do not cope adequately with reading and analysing a problem , converting word problems into mathematical problems and using trigonometrical identities. These areas can be improved by a pre-course. Staff have got problems with definitions of $\frac{dy}{dx}$ the integral sign (See Figure 24) and limits (See Figure 25). It was also found that symbols in common use for the same item can lead to problems, for example, $\frac{dy}{dx}, f'(x)$ and $\frac{d}{dx}$ used for differentiation (See Figure 25). Staff have genuine difficulties with time for the compressed courses they administer, listing inadequate tutorial time as a major difficulty (See Figure 28). Staff responses indicated that a majority felt they are essential for student success (See Question 15). The biographical data indicated the qualifications at the time of the questionnaire which showed a large percentage (over 60%) had at least 3 years of mathematics training and a similar percentage had at least one year of education training. Changes in these percentages will be desirable in the future due to the establishment of Bachelor of Science in Technology courses.

5.4 ANALYSIS OF THE DATA OBTAINED THROUGH THE COMPUTER QUESTIONNAIRE

5.4.1 Background information

In 1989 the mathematics department of Technikon Natal began using a computer package called "A graphical approach to Calculus" by David Tall. Initially two groups of twenty second year engineering students under the author's supervision attended a two hour session in the computer room above the library. After some initial work on how to handle the computer and the language required students were required to work through four tutorials on graphs, differentiation, integration and parametric equations (See Annexure D). Tutorials had to be completed either at the time or within three or four days. These were marked and constituted 10% of the course mark. As a feedback interviews were conducted with pairs of students using a tape recorder. A report on these interviews is given in 4.4.3.

In 1990 the number of second year engineering students was increased to 150 and the method of dealing with tutorials was changed. Students were issued with a disk and documentation After an initial demonstration using a liquid crystal display they were then given the first tutorial which had to be handed in two weeks later followed by four others at regular intervals of two weeks. Computer facilities had improved so several departments serviced by the Mathematics Department had small computer rooms which could be used by students plus an increasing number of personal computers owned by students. To assist students the author made himself available on Period 6 on Fridays and the lunchtime in a smaller computer room, for queries and assistance. Other times could be arranged if necessary. The emphasis was on the student doing the necessary work on the programme to complete his tutorial.

In each semester during 1990 approximately 50% of all students who took the computer component volunteered to answer the questionnaire. In the first semester 44 students answered the questionnaire and in the second semester there were 77 students. Staff were present when the questionnaire was answered which prevented group answering of questions.

The questionnaire (See Annexure C) was split into three parts, the first part comprising Question 1 to Question 3 dealt with biographical data of the students such as matric mathematics symbol, age group and previous activity before joining the Technikon The second part comprising Question 5 to Question 12 dealt with previous computer experience, the programme itself and the five tutorials which were given to students (See Annexure D). The third part comprising Questions 13 to Question 20 consisted of a series of graphs of linear, quadratic, trigonometrical and exponential functions. Students had to choose the correct equation of the graph from 5 choices or find the gradient or the area under the curve. The results for each semester are compared by means of block charts in percentage of valid responses.

5.4.2 Analysis of biographical data (Questions 1-3)

QUESTION 1 Matric mathematics symbol (Figure 32)

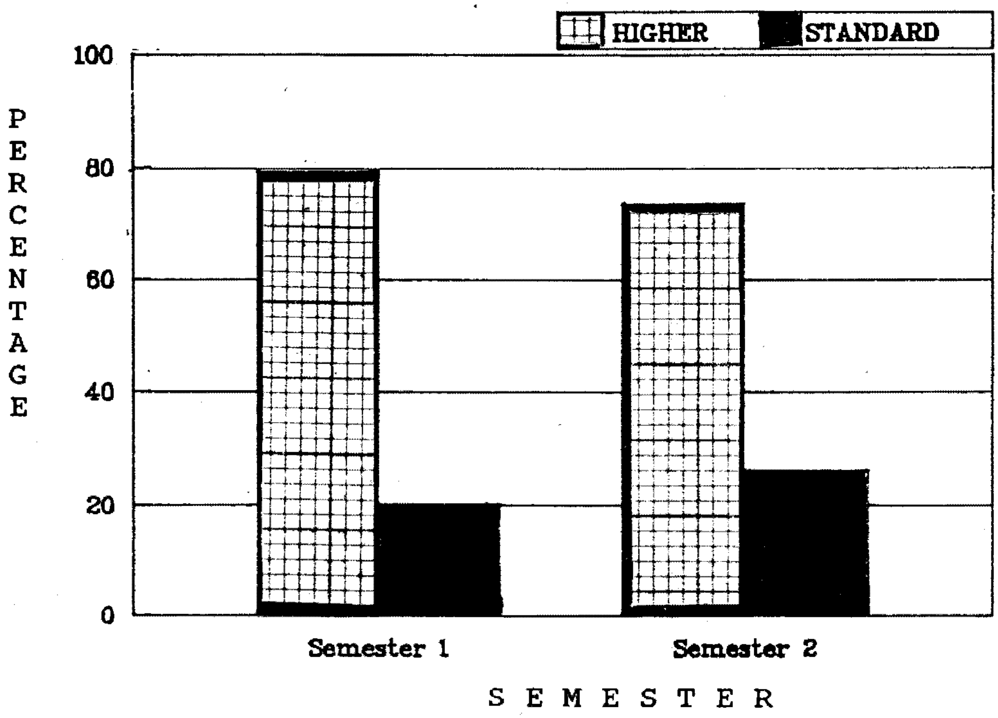


FIGURE 32 MATRIC MATHEMATICS SYMBOL

There was an overall drop of 6% in higher grade candidates in the 2nd semester and a corresponding increase in standard grade candidates. Another way of putting this was that the ratio of higher to standard grade dropped from 4:1 in the 1st semester to 3:1 in the second semester. Two years previously this ratio was 2:1. There has been an increase in standard of entry with higher grade mathematics students being preferred to standard grade so in the 1st semester higher grade will be in the majority. In the second semester there are a good number of repeat students who being weak students will probably have standard grade mathematics qualification so the ratio of higher to standard grade tends to drop.

QUESTION 2 Age group (Figure 33)

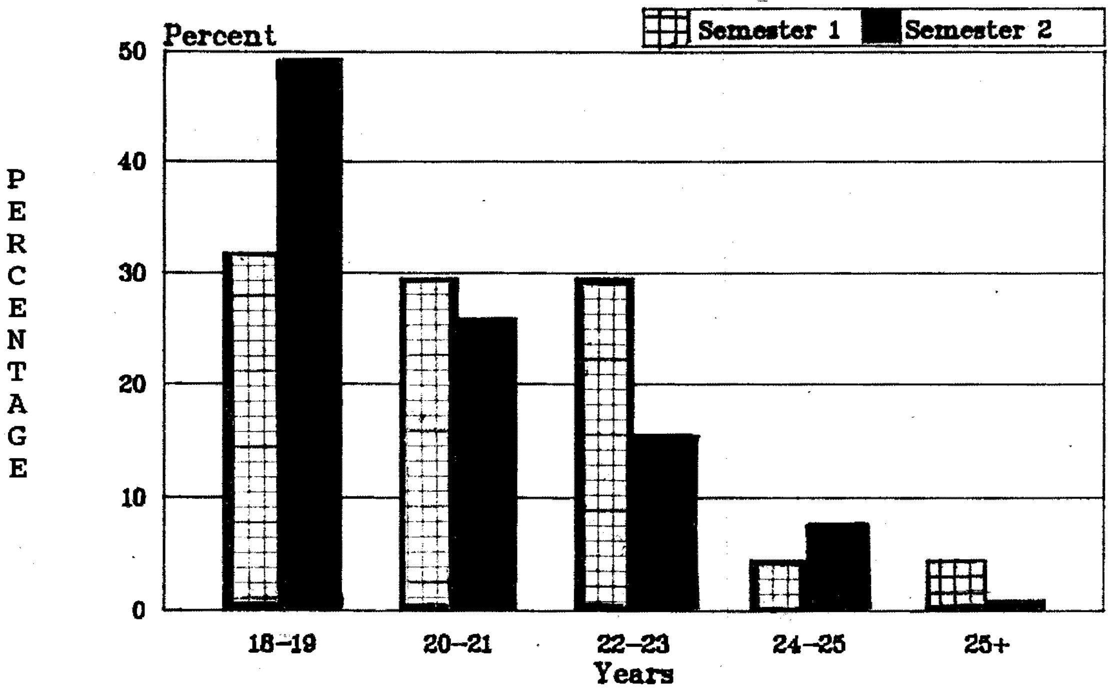


FIGURE 33 AGE GROUP

There was a big change between the two semesters with the age group 18-19 years rising from 32% to 49% and the age group 22-23 years falling from 30% to 16%. A possible reason for this is that repeat students from the previous year often return in July after six months in industry.

QUESTION 3 PREVIOUS ACTIVITY PRIOR TO TECHNIKON (Figure 34)

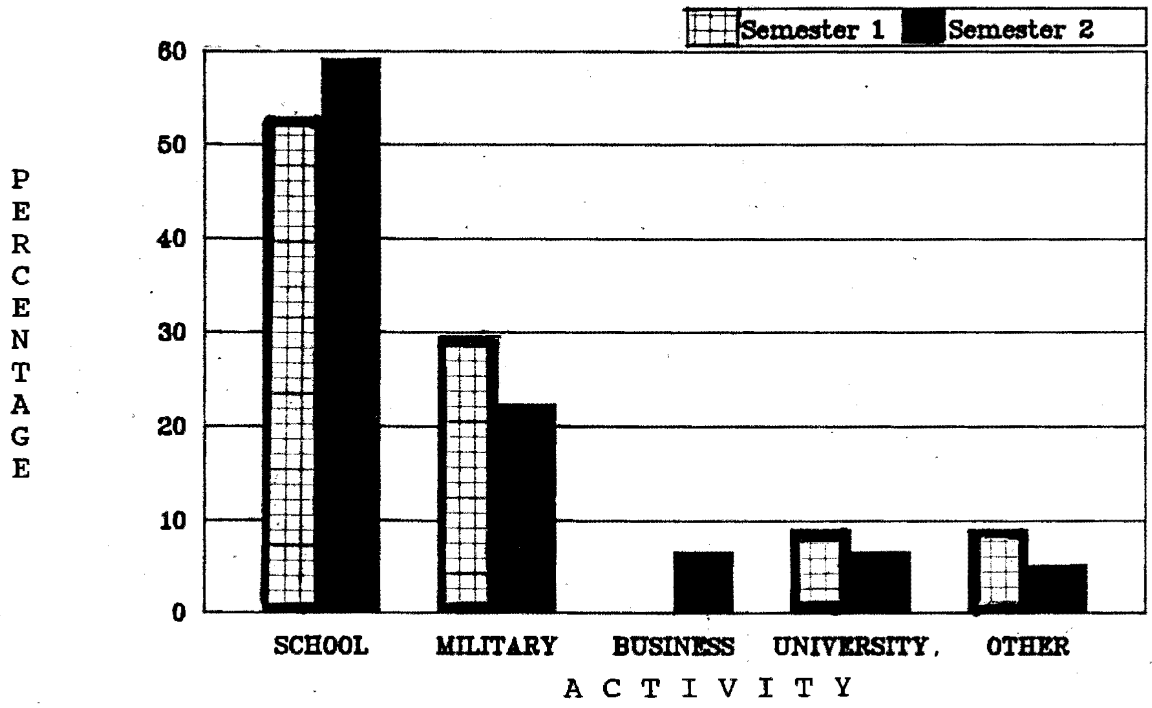


FIGURE 34 PREVIOUS ACTIVITY PRIOR TO TECHNIKON

There was an increase of 7% for ex-school candidates and a drop of 8% for ex-military candidates. A comparison with Question 2 shows that an increase of the 18-19 age group in the second semester fits in with an increase of ex-school candidates and similarly a drop in the 22-23 age group fits in with a drop in ex-military candidates who would expected to be in the older group.

5.4.3 Analysis of responses pertaining to the computer programme and tutorials.

QUESTION 4 Computer experience (Figure 35)

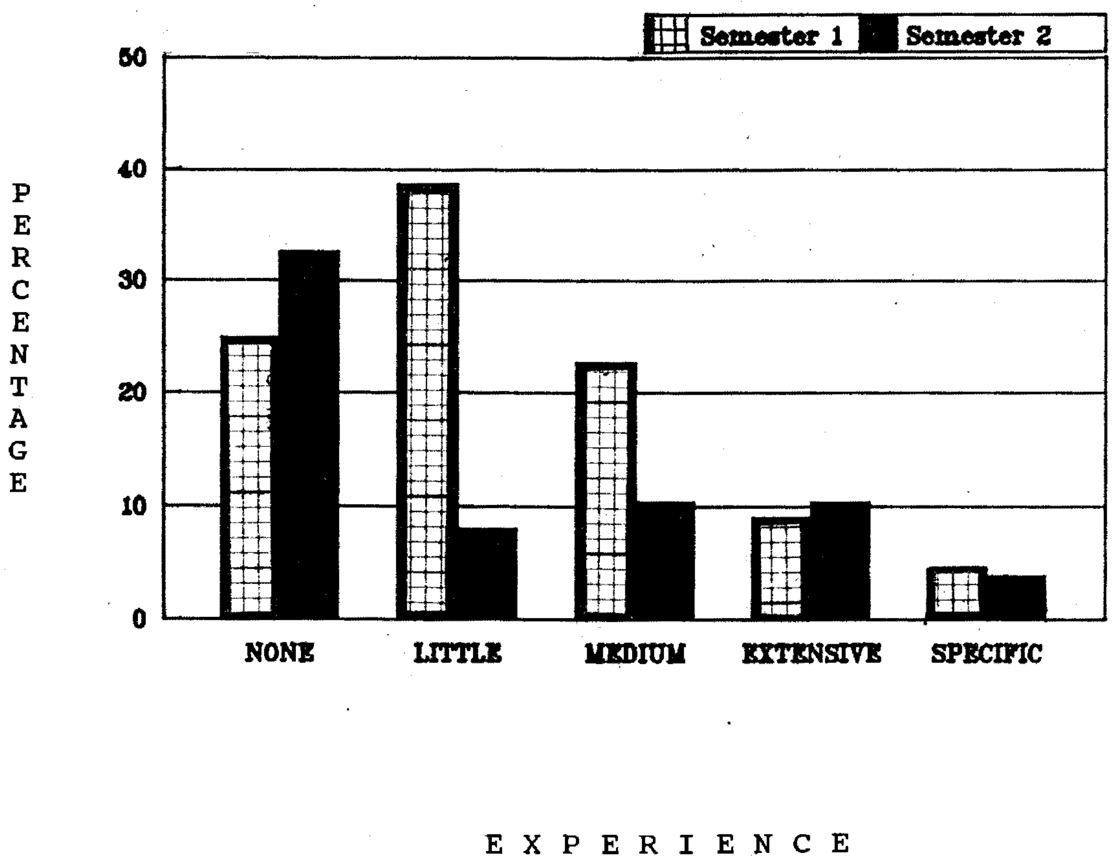


FIGURE 35 COMPUTER EXPERIENCE

Approximately 40% had reasonable computer experience which is a fourfold advance from 1989 where the figure was 10%.

QUESTION 5 Helpfulness of program (Figure 36)

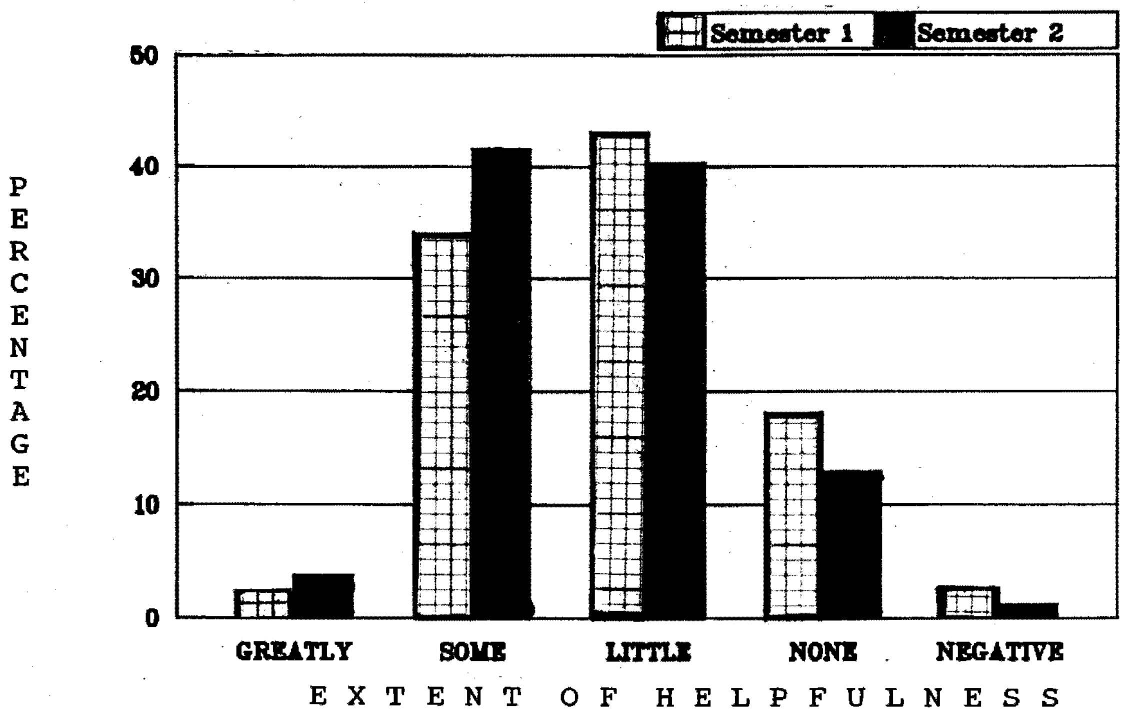


FIGURE 36 HELPFULNESS OF PROGRAM

Question 5 dealt with whether the programme promoted understanding of calculus. In the overall situation 80% in the first semester and 86% in the second semester said it did help to varying degrees with the understanding of the calculus.

QUESTION 6 Difficulty encountered with straight line graphs (Figure 37)

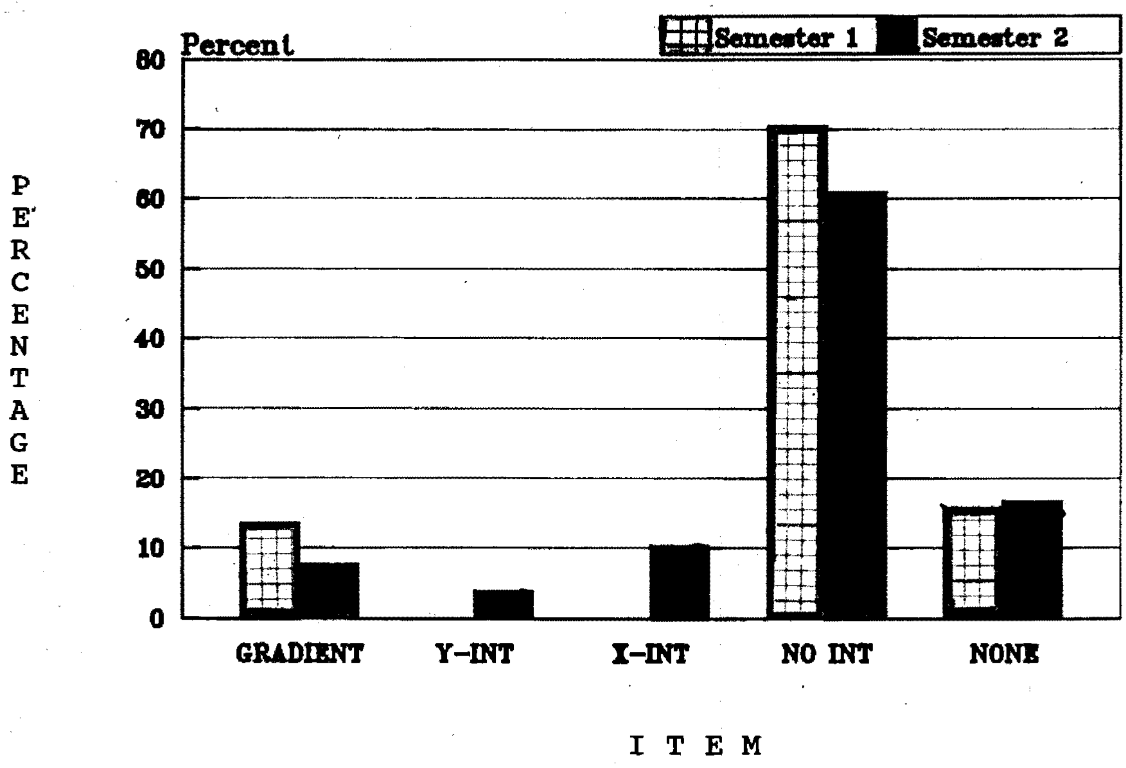


FIGURE 37 DIFFICULTY ENCOUNTERED WITH STRAIGHT LINE GRAPHS

Question 6 dealt with students' difficulty in identifying equations of three straight lines when these appeared on the computer screen (See Annexure D,Tutorial 1) The significant difference between the two semesters was that whereas 70% in the first semester had difficulty identifying the straight line when no intercept was given 60% in the second semester had similar difficulties but 10% in the second semester had difficulties with x - intercept while no student recorded difficulties with this point in the first semester.

QUESTION 7 Difficulty encountered with determination of a Quadratic Equation (Figure 38)

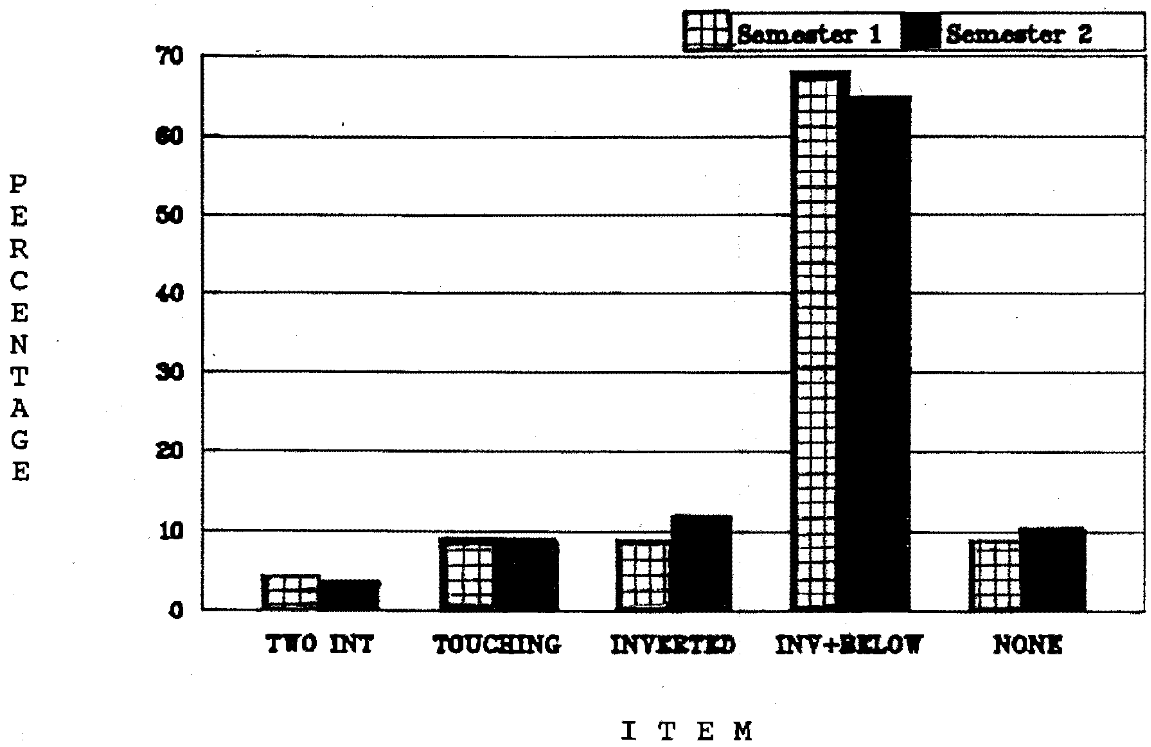


FIGURE 38 DIFFICULTY ENCOUNTERED WITH DETERMINATION OF A QUADRATIC EQUATION

Question 7 dealt with the students' difficulty in the identification of three quadratic equations when these appeared on the computer screen (See Annexure D, Tutorial 1). There were no significant differences in the two semesters with the main difficulty being the identification of a quadratic inverted and below the axis.

QUESTION 8 Difficulty encountered with determination of a Trigonometric function (Figure 39)

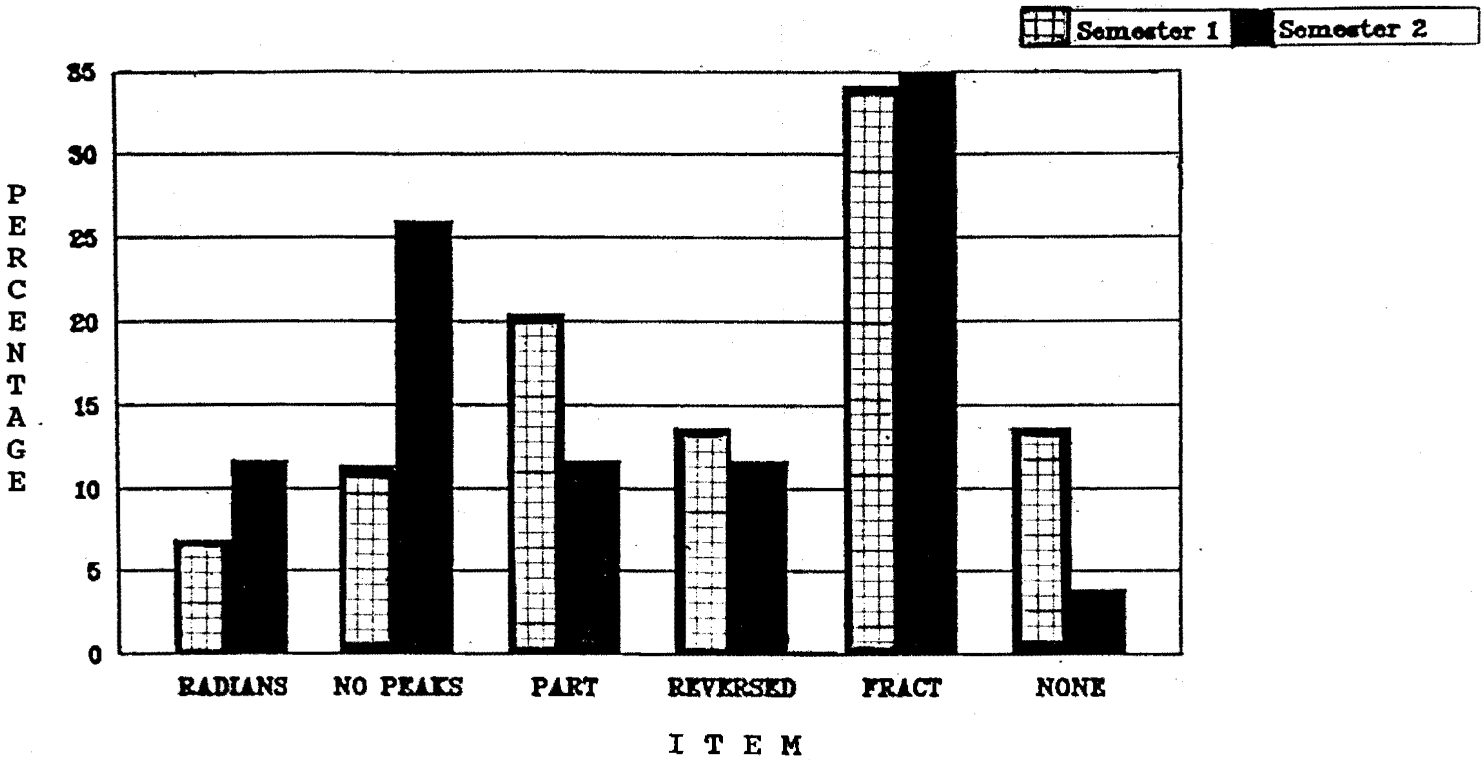


FIGURE 39 DIFFICULTY ENCOUNTERED WITH DETERMINATION OF A TRIGONOMETRIC FUNCTION

Question 8 dealt with students' difficulties in identifying three trigonometric equations when these appeared on the computer screen (See Annexure D, Tutorial 1). Over 30% in both semesters had difficulty identifying trigonometrical functions when fractions occurred in the functions. A possible explanation is that students spend very little time on graphs being content with the shape of $\sin\Theta$, $\cos\Theta$ and $\tan\Theta$ without being aware of the effect on the period of the wave when for example $\sin\frac{1}{4}\theta$ is given. There was a rise from 11%-26% in the second semester of students having difficulty in recognizing the correct trigonometrical function when the peaks were cut off and a drop from 20%-12% of those students having difficulty in recognizing the correct trigonometrical function when only part of the period was given.

The removal of the peaks takes away the maximum value of the function so students have to apply other methods to find the functions. A similar procedure applies when only part of the period is given when students have to extrapolate the curve to complete the period and hence find the equation of the curve. This is a problem for the weaker students.

QUESTION 9 Difficulty encountered with gradient of a function (Figure 40)

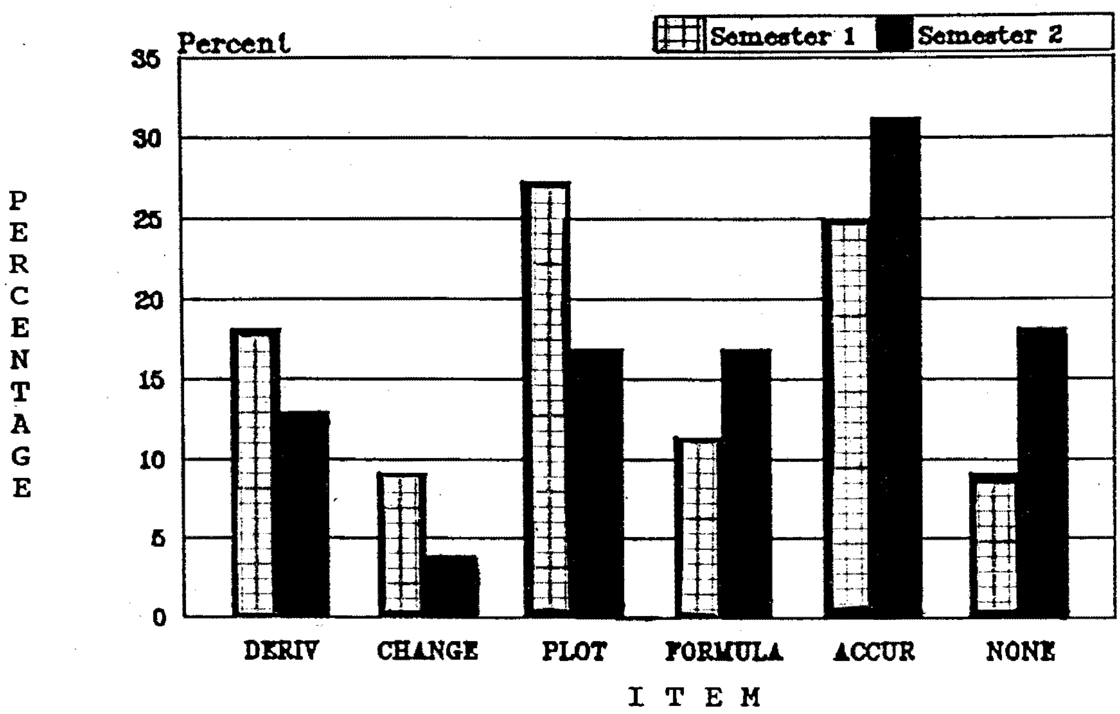


FIGURE 40 DIFFICULTY ENCOUNTERED WITH GRADIENT OF A FUNCTION

Question 9 dealt with the second tutorial (See Annexure D) on the gradient of a function. The main features of the results were that students had less difficulty in the 2nd semester with understanding what happened when they plotted the gradient but in both semesters students found problems with knowing when to stop the programme to get the right accuracy.

QUESTION 10 Difficulty encountered with graphical solution of Equations (Figure 41)

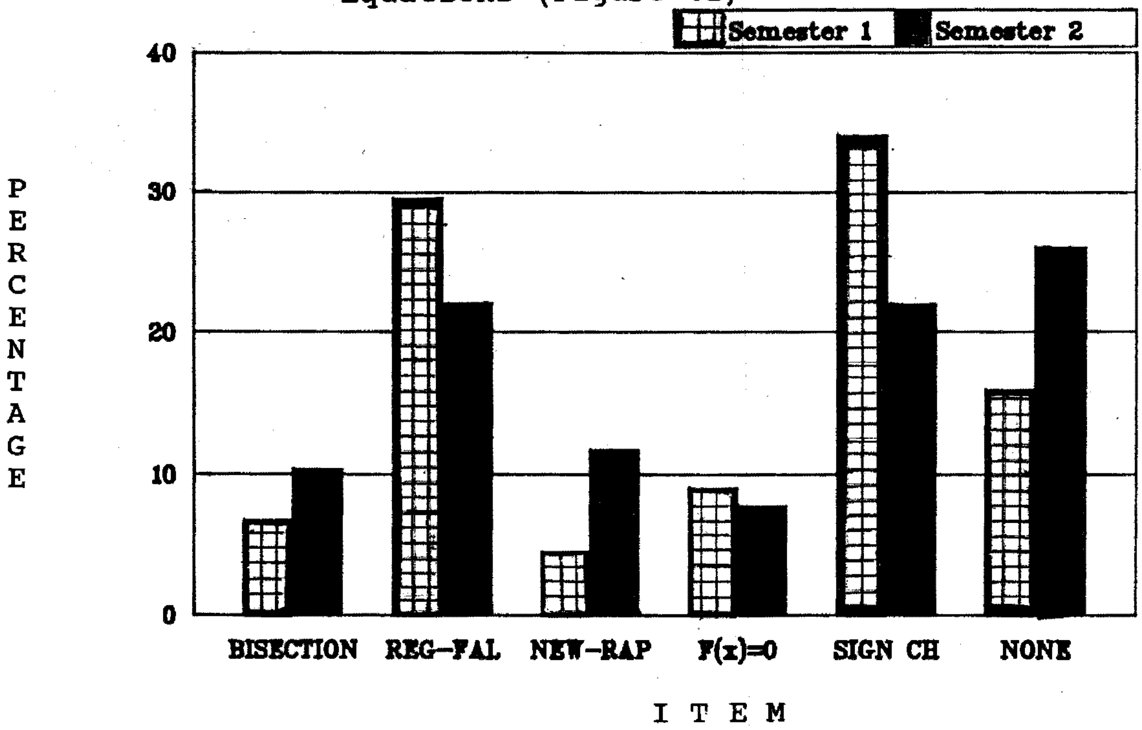


FIGURE 41 DIFFICULTY ENCOUNTERED WITH GRAPHICAL SOLUTION OF EQUATIONS

Question 10 referred to the fifth tutorial on solving equations (See Annexure D). Of the three methods the regula-falsi was the most difficult to understand and approximately 30% in the first semester and over 20% in the second semester seemed to have problems with the idea that when a curve passes through a root the sign of the function changes.

QUESTION 11 Difficulty encountered with evaluating definite Integrals (Figure 42)

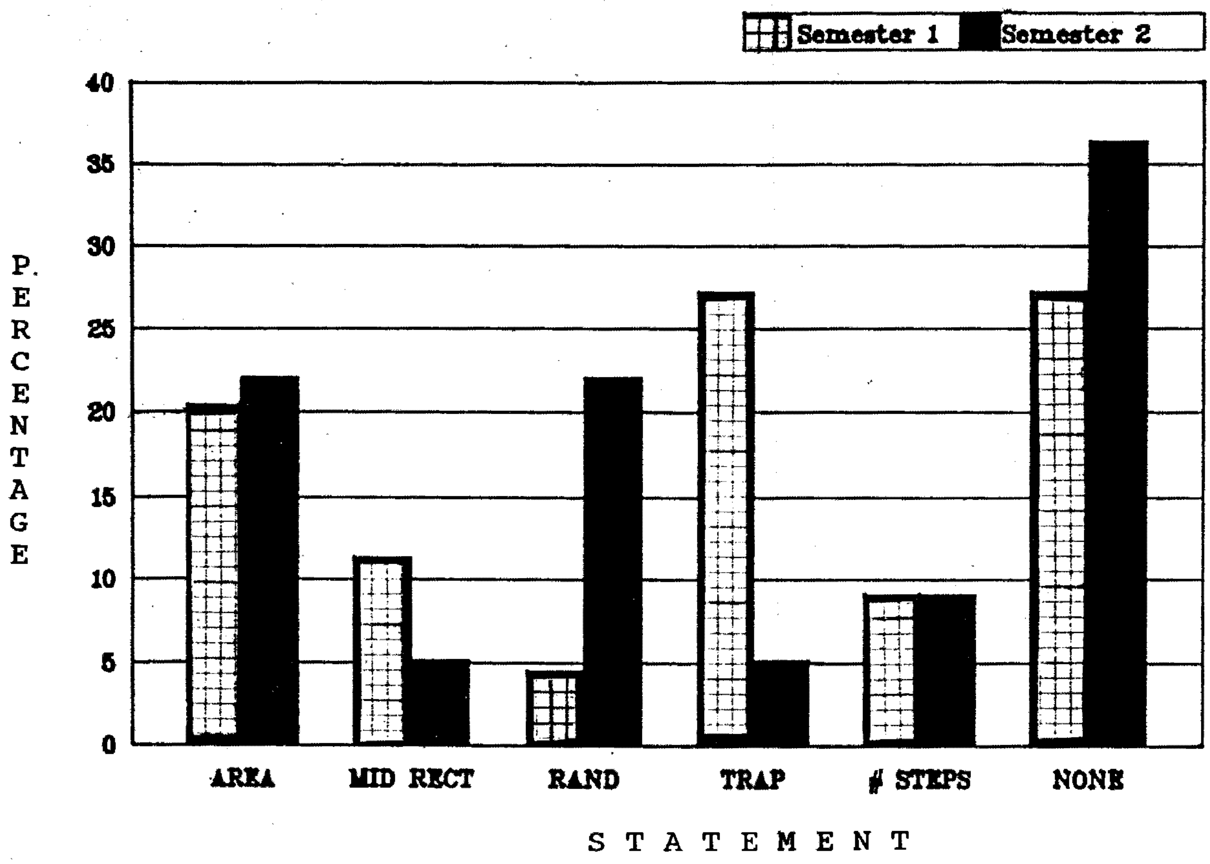


FIGURE 42 DIFFICULTY ENCOUNTERED WITH EVALUATING DEFINITE INTEGRALS

Question 11 referred to the third tutorial (See Annexure D) where six numerical methods were used to find the area, that is, the definite integral. The surprising result was that in both semesters over 20% of the students had difficulty identifying the area under the curve with the definite integral. The change between semester 1 and semester 2 on the trapezium rule can be attributed to a change in syllabus which included the trapezium rule during lectures.

QUESTION 12 Difficulty encountered with parametric Equations (Figure 43)

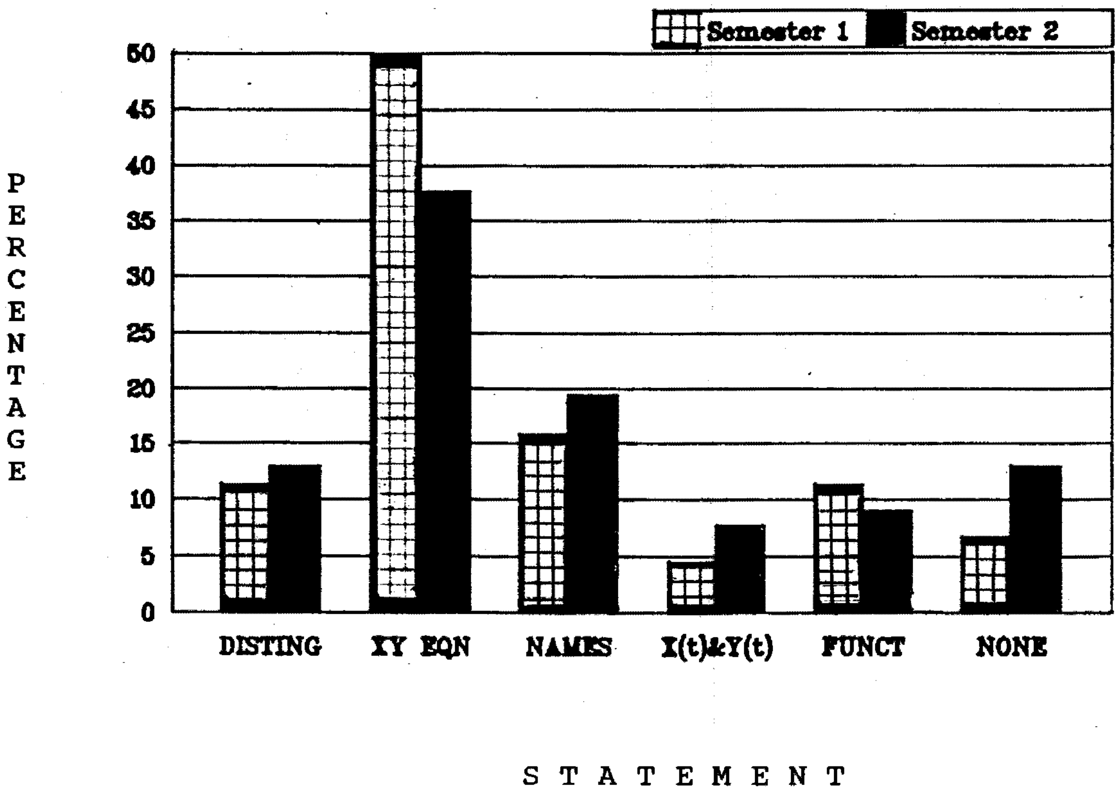


FIGURE 43 DIFFICULTY ENCOUNTERED WITH PARAMETRIC EQUATIONS

Question 12 referred to the fourth tutorial (See Annexure D) on parametric equations. In each semester finding the x,y equation was the greatest difficulty but there was a drop from 50% - 38% in the two semesters.

5.4.4 Analysis of responses to graphs of specific functions

QUESTION 13 - 20 Selecting correct equations for specific Graphs (Figure 44)

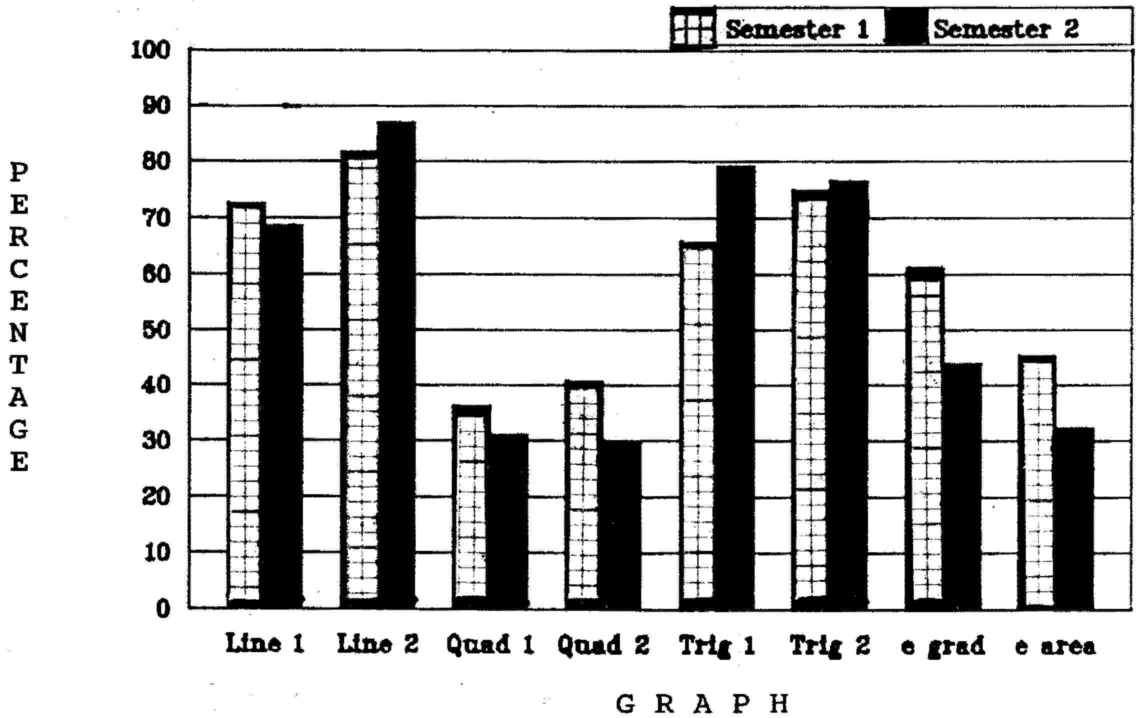


FIGURE 44 SELECTING CORRECT EQUATIONS FOR SPECIFIC GRAPHS

In Questions 13 and 14 students had to identify one straight line graph given five choices. Approximately 70% and 80% respectively identified the lines correctly. In Question 15 and 16 students were asked to identify two quadratic graphs (Quad 1; Quad 2) given five choices. Only 30-40% were able to give the correct choice. Questions 17 and 18 dealt with trigonometric functions (Trig 1; Trig 2) which resulted in 70-80% correct choices which seemed to indicate the computer programme may have helped the weaker students. In Question 19 students were asked to choose the correct gradient for an exponential function (e grad). About half of the students were able to do this correctly. The final question number 20 dealt with the area under an exponential curve between two limits (e area). This was less successful with 30-40% being successful. These questions did provide useful clues to the areas students have most difficulty with namely the quadratic and exponential graphs.

5.5 A REPORT ON INTERVIEWS WITH STUDENTS ON THE COMPUTER PACKAGE

5.5.1 Background Information

During 1989 Mechanical Engineering students at the T2 level have been using a computer package on a graphical approach to the calculus by David Tall. In two groups of twenty students they have used the library complex where each student had a computer to work with. Five tutorials were given (See Annexure D) the first one dealt with graphs, the second one was based on differentiation, the third one was dealing with integration while the fourth one was based on parametric equations. Later a fifth tutorial on the solution of equations was added but comments on this tutorial were not included in the interviews. The time allocated to each tutorial was approximately one hour and forty five minutes but often students took work away to complete. In some cases extra time was available as students from Group A could come into Group B the following week if they wished.

In order to assess the value of the computer course two approaches were used. The first involved a questionnaire (See Annexure C) whose results have been discussed in 4.4.1 and the second structured interviews with pairs of students. The structured interview dealt in detail with each tutorial and asked the students what they had learned about various items which they were not aware of previously. The personal opinions expressed in the interviews reinforced what was in the questionnaire but very often gave much wider information on the difficulties experienced by students using the programme.

5.5.2 Modus Operandi for Taped Interviews

Students were interviewed in pairs which helped any embarrassment when facing a tape recorder. Students were chosen to give a range of ages from 18-25; a range of previous activities such as business, school and the army and a range of results in matric ranging from an E Standard grade in mathematics to a C Higher grade in mathematics.

In the first two pairs academic qualifications were very different: an E Standard grade and a D Higher grade, where as in the two other pairs they were similar. The sample of eight students was representative of the total population of 121 students who answered the computer questionnaire.

Prior to the interviews the eight students had answered the Student Questionnaire (See Annexure A) which was discussed in 5.2. A summary of their biographical data and whether they answered Question 10 and Question 11 correctly is indicated below:

	Matric Maths Symbol Higher	Standard	Years since Matric	Age Group	From Bus	From School	From Army	Ques 10	Ques 11
Martinus	D			22-23	Yes			1	1
Theo		E	4	24-25	Yes			1	0
Bruce		E	2	18-19		Yes		1	-
Alan	D		2	18-19		Yes		1	0
Oliver		A	1	18-19		Yes		1	0
Broderick	D		2	20-21			Yes	0	0
Chris	D		2	20-21			Yes	1	0
Rolf	C		1	18-19		Yes		0	0

During the interviews questions were asked about the tutorials (See Annexure D) which had been done by the students during their semester course. The interviewer had the results of the tests which the students had answered. Each tutorial was discussed separately to find out what students had learned and to elicit additional items of difficulty or suggestions for improvements.

5.5.3 Comments on Tutorials

Comments on Tutorial 1

This tutorial dealt with graphs of linear quadratic and trigonometric functions. The following questions were asked:

Question 1.1 What did you learn about a straight line graph which you didn't know before?

Question 1.2 What did you find was new about quadratic graphs which you didn't know previously?

Question 1.3 What struck you as different with the trigonometric graphs compared with you previous knowledge of trigonometric graphs.

Question 1.4 Did seeing graphs on a computer help your general understanding of the second level course?

The students responses which follow involve comments on areas not in the Student Questionnaire. Also since each interview is independent of the other three interviews comments may be repeated.

The replies from Martinus and Theo were that on the computer you could automatically see your mistake if you identified the wrong graph. They felt revision was necessary before you tackled the programme. The scales were marked at wide intervals but if you used the zoom effect this could be corrected. Bruce said it brought back your knowledge of Matric graphs. Alan felt it helped you in clarifying domain and range. Bruce and Alan noticed that the domain for trigonometric graphs was in radians which changes the values one normally sees on a trigonometric graph. Broderick and Oliver mentioned the standard scales used, that is $-3 \leq x \leq 3$. They also said they were rusty on routines and didn't know where to begin.

Eventually they managed to cope with the tutorial which they felt helped their understanding of graphical procedures. Rolf said that you understand things better when you saw a lot of them on the screen. Chris appreciated the adjustment of the domain and range to get the whole picture. He pleaded for more time for this tutorial as he felt it was all done rather quickly. Obviously it would be better to do it in your own time.

The gist of the above comments was that the visual effects on the computer forced students to look carefully at the variables involved in obtaining a good picture that is domain, range and scales. They also found weaknesses in their own knowledge of graphs which hindered them in identifying graphs quickly hence the pleas for more time.

Comments on Tutorial 2

In this tutorial students were asked to find the gradients of several functions at given points, to suggest the gradient function of several more complicated functions and to plot derivatives of other functions. The value of the gradient and the plot of the derivative could be done on the programme, the other questions required the students to differentiate the more complicated functions using the theory from their normal lectures.

The questions asked were:

2.1 Did you understand the two methods of differentiation used?

2.2 Did the graph of the derivative or gradient function help your understanding of the change of a function?

2.3 Did you find the function notation helpful?

The students responses were as follows:

Martinus said with the two methods you were not merely getting an answer. Alan said the suggest derivative was a good check on your own work. He also said he didn't realise the tangent was the derivative. Bruce thought it would have been very helpful in T1 where you deal with the derivative for the first time. Oliver said he hadn't associated differentiation with a graph. Rolf and Chris had no difficulty with this tutorial and said they were taught at school to use the functional notation.

This tutorial went smoother than the first tutorial possibly due to some questions requiring numerical answers but greater clarity on the derivative and its relation to gradient and rate of change was obvious from the students' remarks.

Comments on Tutorial 3

In this tutorial the emphasis was on students calculating areas under a curve by seven methods. Four involved rectangles with the left top corner on the curve, right top corner on the curve, mid point of top of the rectangle on the curve and random combination of these three. The fifth involved trapeziums the sixth was Simpson's rule and finally using theory the primitive or integral without constant had to be typed in and a further calculation made.

The questions asked about this tutorial were:

3.1 Did you have any difficulties with operating the integration programme?

3.2 Do the various methods explain how integration is linked to area?

3.3 Would you prefer to use a computer for integration?

The students responses were as follows:

Bruce and Alan felt this tutorial was very suitable for T2 and they liked the link with theory through finding the primitive. Undefined values also occurred with square roots which required careful handling. Oliver was confused with the six methods and felt an explanation session could have helped. Chris said he had an advantage since he had done 1st year Physics at University so knew about applications of integration. Rolph felt quite happy with integration.

It was interesting to help students when there were differences in the numerical values and when "undefined" came up on the screen. They realised for the first time very often that certain graphs are restricted to special domains especially where square roots are concerned. This came out in the students comments above.

Comments on Tutorial 4

In this tutorial students basically were asked to use parametric equations of x as a function of t and y as a function of t, to give a plot of the x,y-curve and then to determine its x,y-equation. Then they had to use the integration methods to find the area under the curve.

The following questions were asked in connection with this tutorial.

4.1 What did you learn about parameters?
4.2 Did the graphs generated parametrically help you to identify the various conic sections?
4.3 Do you think that parametric equations are better than the x,y-equations?

The responses by students were as follows:

Theo and Martinus thought that computers generated a picture which was different, for example, a circle looked like an ellipse which led to studies of the differences between a circle and an ellipse. The domain and range played a crucial role in the presentation on the screen. Oliver found this tutorial difficult, particularly the connections between (x,y) and the parameters. Chris said the programme can be done in 45 minutes but added he worked with several others so many hands make light work. Bruce and Alan felt two sessions on this tutorial would have been beneficial.

As with tutorial one the visual interpretation was important and again domain and range was mentioned.

5.5.4 General Comments on the Computer Package

Several of the students suggested that series might be usefully illustrated by the programme.

For example:

Plot sinx and *plot* $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}$ to see how close one gets to sinx for 4 terms. Rolf suggested that he would like moment of inertia on this programme which, since it involves integrals, should be possible.

Some general comments on the programme were a plea for some simpler exercises to get used to the section. Oliver wanted a text book related to the programme. Broderick wanted a guide not a text book. A simplified set of instructions were given on the tutorials but very often students ignored these and went cold into the programme.

These comments were noted and in subsequent work with the computer package some simpler exercises were included. A handout was provided which also allowed them more time to complete the tutorials as work could be done on any computer either at home or at the Technikon.

5.5.5 Calculus Package Interviews on Video

5.5.5.1 Modus operandi for Video Interviews

The interviews on tape were taken separately and the same questions were asked of each pair of students. The interviews recorded on video followed on from one another so the questions were not the same. The main aims of the questions was to find whether the computer programme helped with the main lecturing programme and whether the programme would help in their future work situation. The six students represented the three culture groups: two Indian, two White and two black students. The interviews were relaxed and brought out some different viewpoints to the more formal structured interviews on tape. The reason for the video interviews was in order to show students and staff the feedback on the programme. It was also shown at several conferences both locally and overseas.

5.5.2 Comments on the Tutorials

Comments of Tutorial 1

The two Indian students found they had to dig for information regarding quadratic and trigonometric graphs in order to find the answers to the equations of the graphs shown on the screen. One of the white students said he didn't have the basic knowledge of graphs which correlated with his low academic results and E Standard grade in matric mathematics. The two black students both mentioned the problems they had identifying trigonometric graphs when fractions occurred in the equations. A comparison with the taped interviews 5.5.3 shows that each group found weaknesses in their own knowledge of graphs.

Comments on Tutorial 2

Students found that computing gradients was easier using the package which relates well to the comments in 5.5.4 where students gave favourable comments on the visual explanations of tangent and gradient. They also had no difficulties with this tutorial.

Comments on Tutorial 3

The Indian students felt that the pictorial view of integration via areas was very helpful. The two white students found the various methods for areas explained this concept from several different standpoints. The black students were generally unaware of the area methods. The previous group of students also found the tutorial helpful especially the link with the theory given to them in the lectures. Only one said he was confused with the six methods of finding the area.

Comments on Tutorial 4

This was not commented on in the video interviews.

5.6 INTERPRETATION OF THE RESULTS OF THE EMPIRICAL RESEARCH

5.6.1 Introduction

In Chapter 5 there has been a critical analysis of responses to three questionnaires (Annexures A,B and C) and four tutorials (Annexure D) plus a study of taped interviews with four pairs of students and video interviews with three pairs of student. It is now important to interpret the results of all these instruments of research. The interpretation falls logically into three categories. Firstly the student problems, secondly the staff problems and thirdly the computer problems.

5.6.2 Student Problems

The first question to ask is what general problems in mathematics and particularly calculus were identified for students? The responses to Question 7 of the Student Questionnaire (Annexure A) identified translating word problems into mathematical equations, dealing with trigonometric identities, graphs, limits and notation in calculus as major problems. Also students had difficulty with differentiating simple inverses and trigonometric functions. As the results of responses to Question 12 and Question 13 of the Student Questionnaire (See Figure 9 and Figure 10) show, this was further amplified in Question 8 of the Computer Questionnaire (Annexure C), where students had difficulty with identifying trigonometrical functions. Also in the video interviews the comments from students again identified trigonometrical functions as a problem. The consistency of responses in the three instruments of research confirms that trigonometry is a major problem for students.

Whilst the content of courses may be a major issue the time to cover a semester course was also an important issue with students. This was also relevant to the other courses the students take in engineering so the cumulative effect had serious repercussions after only a few weeks into the course. In Question 15 of the student questionnaire 70% of the students had difficulty with time after 5-6 weeks. On the other hand the time spent outside lectures by students plays an important role in their ability to cope with the course in calculus. The results of responses to Question 18 (Figure 17) of the student questionnaire indicated that one third of the students were spending four hours or less per week outside of lectures which is below the minimum of one hour per day needed to do the necessary exercises and applications of calculus. Another problem which affects students in varying degrees is the language of mathematics, that is the symbols, phrases and alternative meanings to familiar english words.

Question 8/9/12/13/14 of the student questionnaire all dealt with meaning and symbols such as the integral sign, the derivative, limits and $\frac{dy}{dx}$. The research indicated that students lacked clarity identifying these items. In the interviews mathematical words were not understood, for example "primitive" with regard to integration. In general students mathematical vocabulary is rather small and lacks alternative definitions of basic terms.

5.6.3 Staff Problems

The problems encountered by mathematics staff at technikons have several facets which mirror or complement student problems. For example Question 6 of the Staff Questionnaire revealed that topics staff have difficulties teaching are the same topics students claim to be most difficult. These are word problems or applications which involve translating words into symbols and then setting up follow on in order of difficulty. The time factor was another problem with staff being dissatisfied with tutorial time where in Question 12 of the Staff Questionnaire (Annexure B) only 22% rated tutorial time as adequate and in Question 13 50% of staff responded to seven or eight periods a week for mathematics. The main interpretation to the time question is one of anxiety for staff that the syllabus will not be covered in time or it will be inadequately completed as far as exercises are concerned. Another area of concern for staff is the impact of technology which is moving quicker than the capacity of lecturers to absorb and use it. Students are equipping themselves with sophisticated calculators and even computers whereas lecturers battle to catch up with software on existing models. Staff did rank the computer as the 2nd most useful visual aid (See Figure 31) after the overhead transparency.

5.6.4 Computer Problems

As mentioned at the end of the staff problems the computer also generates problems of its own. The video interviews show that without knowledge of graphs students will have problems using a package which illustrates graphically the derivative and the primitive. As Rick in Interview 2 said "The basis of graphs I don't know" and later "Yes I can handle the computer but I don't know graphs." It is assumed that the computer will solve everything but without the necessary background knowledge the results of the package will be meaningless. If certain conditions are not adhered to then the computer will refuse to function correctly. For example, the syntax of expressions must be mathematically correct, real values should exist as for example, in square roots and the domain may only be valid for positive values as for example logarithms. Students do have problems with handling computers and interpreting their results or failure to obtain results.

It will be left to Chapter 6 where conclusions and recommendations are made to develop and widen the interpretation of the empirical research.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 INTRODUCTION

Before conclusions are drawn it is necessary to review the aims and objectives of this study (See 1.5.1 and 1.5.2).The aims included identifying and analysing learning problems in calculus in general and then particular teaching and learning problems for second level engineering groups at South African Technikons. The objectives split the aims down to background items which affect the learning of calculus, concepts in the first year calculus course which are essential to the understanding of the course and how the teaching and learning problems affect the performance of students in calculus. Another important objective was to investigate the impact of computer software programmes on the identified teaching and learning problems of calculus. There were several sub-issues including timetable pressures, tutorials, time for lectures and time spent outside lectures. All these threads will be drawn together in the following paragraphs.

6.2 CONCLUSIONS

6.2.1 Problems identified in the background knowledge of student which affect the learning of calculus

The major problems students have can be traced backwards to earlier work in mathematics or the omission of items which are essential to understanding calculus. The survey identified background knowledge of trigonometry as a major problem for 64% of the students questioned. The differentiation and integration of trigonometric expressions involves many trigonometrical identities so the more experience students have in these areas the less problems students will have in completing their work. It must be concluded that trigonometry does not receive enough attention at school level particularly with standard grade students. Weaknesses in graphical work came out strongly in interviews with students.

Since the early understanding of calculus depends on graphs of various elementary functions a weak graphical background tends to handicap students in their initial work with calculus. Linked to graph work is coordinate geometry which caused 50% of students problem. This is a topic not studied by most school students and it only briefly mentioned in first level courses at technikons. The concept of gradient, the study of conics and various equations of the straight line are some of the useful items which belong to coordinate geometry which are needed for calculus. Another major background problem for students came under the heading of algebraic processes where, for example, simplification of fractions can adversely affect the success of differentiating a complicated expression. Division of algebraic expressions can similarly affect the integration of a fraction where the numerator is of higher degree than the denominator.

6.2.2 Concepts in the first year calculus course which are essential to understanding calculus

It can be concluded that the concepts of a limit, gradient, derivative, rate of change, maximum and minimum value and area under a curve linked to integration are fundamental to understanding calculus. It is not a whole list of formulae rules and routines which give understanding but rather a deeper knowledge of concepts. Students show every evidence of having suffered from their pursuit of meaningless realistic manipulation of symbols. Mathematics and calculus in particular do not merely consist of a bunch of facts to be learned, there are structures which fit together or a jigsaw with so many pieces which lock into place. The learner has to construct his own answers from these structures.

6.2.3 Teaching - learning problems which affect the performance of students in calculus

Several general teaching-learning problems were identified such as mathematical language, mathematical symbols, the mathematical model used by the lecturer and the effort applied by both students and lecturer's outside formal lecture time.

6.2.3.1 Mathematical Language

It is possible to conclude that mathematical language is not easily assimilated which presents a problem to students who listen but do not understand and to the lecturer who knowing his audience is illiterate in correct mathematical language has to constantly divert into popular language to express his point. The research instruments indicated that students were confused about words such as derivative and primitive.

6.2.3.2 Symbols

A very close link exists with regard to mathematical language and mathematical symbols. A problem arises when the same symbol is used for different purposes by mathematicians and engineers. Lecturers and students have to forge an understanding with regard to symbols so the appropriate ones are used in the right context. The main conclusion to be drawn is that students have difficulty in using mathematical symbols correctly because their mathematical language is incomplete or at a lower level than is desirable.

6.2.3.3 The Mathematical model used by the lecturer

This study has analyzed different models used by lecturers. There are various models such as the algebraic, geometric, trigonometric, graphical, problem solving and computer laboratory model. Students may function well using one of these models say algebraic, for example, but fail badly if the graphical, model has to be used. Engineers lecturing their students may expect them to be fluent in different models where the visual or geometrical model may be more predominant over the abstract or algebraic model. For example, civil engineers require many diagrams for design of structures. The computer model was examined which puts more onus on the student to complete work using computer packages. It also changes the role of lecturer to one of adviser who demonstrates mathematical topics on computer but leaves all the detailed work to pairs of students who write up reports every week. Regular tests are also scheduled on the computer. The conclusion to be drawn is that one mathematical model is insufficient to explain calculus and that students relying on one model may be handicapped in their engineering studies.

6.2.3.4 Effort applied by students and lecturers outside formal lecture time

This study investigated the time spent by students outside lecture time and time spent by lecturers on preparation for lecturing mathematics. It was found that approximately one third of the students were spending less than four fours a week on study outside lectures and correspondingly 46% of staff were spending less than four hours a week preparing their lectures. In both cases neither staff nor students can expect a satisfactory result in their work if they put in the minimum of effort outside the formal lecture time. Comments were made that the number of hours spent outside lecture time in studying were not necessarily guaranteed to give success. In conclusion it can be stated that the performance of students and lecturers can definitely be enhanced with regular work outside the formal lecture time.

6.2.4 The impact of computer software on teaching - learning problems in calculus

Reference was made of the use of specialised approaches including computer software. The conclusions to be drawn are that computer software can provide a basis for developing the calculus in different ways, for example, graphically as demonstrated by the graphical approach to calculus (Tall) or logically using Derive or Maple. Computer software can also be used for bridging courses with pre-calculus courses such as Plato. The spare time time due to results being available on the computer for differentiation and integration could be utilised in developing real life examples in engineering.
It must be concluded that use of computer software will grow and that full courses on calculus in computer laboratories will become the norm instead of the exception.

6.2.5 Subsidiary issues affecting students and lecturers in semester courses.

There were three subsidiary issues which were analyzed in the questionnaires which relate to time. Firstly there was the time devoted to tutorials. Staff definitely felt more time was needed for tutorials (only 22% felt the time was adequate) while students were relatively happy with the tutorial time (80% felt it was adequate). Secondly the time for lectures was linked with this so that students (60% felt lectures were adequate) seem to feel they need more lectures whereas lecturers (80% felt lectures were adequate) do not need more lecturing time. Thirdly the pressure of time to complete the semester course was a problem for both lecturer and student. Students responses showed that 70% were having difficulty with time after 5-6 weeks of the course. The conclusion to be drawn is that there will always be a problem with time as long as 16 weeks with 6 periods a week is considered adequate for 4 modules of intensive work in calculus.

6.2.6 Summary of conclusions

In terms of background knowledge four problem areas were identified, consisting of trigonometry, graphical work, coordinate geometry and algebraic processes.

Six concepts essential to understanding calculus were identified: limit, gradient, derivative, rate of change, maximum and minimum value and area under a curve leading to integration.

Three areas of teaching-learning problems which affect the performance of students in calculus were investigated. They included mathematical language, mathematical symbols and the mathematical model used by the lecturer.

The conclusions drawn were that mathematical language is not easily assimilated and that students have difficulty using mathematical symbols correctly because their mathematical language is incomplete. Mathematical models it was concluded must be varied to give sufficient explanation of the processes of calculus.

The conclusions drawn on the impact of computer software on teaching-learning problems in calculus were that computer software can play a significant role in the pre-calculus courses. It will increasingly assist students to remove the drudgery from the traditional differential and integration exercises, allowing time for more realistic examples in engineering.

6.3 RECOMMENDATIONS

6.3.1 Background knowledge

It is recommended that school syllabuses should take cognisance of the desirability of increasing the time spent on trigonometry especially with regard to identities and to include some basic coordinate geometry for all students. Some practical work on graphs would also assist many students entering technikon engineering courses. Over and above the school programme there is a need for some bridging course for those students requiring assistance which could fill in the gaps listed above.

6.3.2 Concepts essential to understanding calculus

It is recommended that understanding of concepts have a higher priority then the number of routine examples on differentiation and integration. To achieve this examples relating to engineering topics in the language used by engineers should be used for students in mathematics.

6.3.3 Teaching-learning problems which effect the performance of students in calculus

It is recommended that at each stage in mathematics a list of words and processes should be given to students with the appropriate mathematical meaning or meanings. In this way a mathematical vocabulary will be encouraged. It is similarly recommended that symbols be thoroughly explained before using them in lectures and a similar handout of explanations with regard to symbols should be available for students. With regard to mathematical models there should be the opportunity for students to pursue alternative mathematical models via the computer to offset the bias which might occur with one model only being pursued by the lecturer, for example, the algebraic model.

6.3.4 Impact of computer software on teaching-learning problems in calculus

It is recommended that students be given the opportunity to use computer software from the earliest courses in calculus so that concepts can be shown as clearly as possible and that courses change to incorporate a computer laboratory tutorial for all levels.

6.3.5 Subsidiary issues affecting students and lecturers in semester courses

It is recommended that students who have completed semester courses be canvassed on their views with respect to time allocations for semester courses. It is further recommended that lecturers look at the content of semester courses with a view to eliminating some items instead of the usual practice of adding items rather haphazardly. Serious consideration should be given to either incorporating an extra period for tutorials or for a computer laboratory period.

6.4 FINAL COMMENTS

This study has sought to contribute some partial solutions to serious teaching-learning problems in calculus at the Technikon. It is, however, hoped that this study will stimulate lecturers and students in calculus, as well as all other stakeholders, to seriously reflect on their teaching and learning in calculus in order to strive for an improved, effective and didactically sound performance in calculus. Calculus reform is a world wide movement so now is the time to take advantage of the solutions being offered which can help a greater range of students now entering technikons and other tertiary institutions. A new approach to calculus may juxtapose conveniently with a new approach to education being negotiated for all South Africans.

ANNEXURE A – STUDENT QUESTIONNAIRE

The purpose of this questionnaire is to gain insight into the difficulties experienced by students when faced with the calculus for the first time.

All information will be treated in the strictest confidence and under no circumstances will any individual be identified. Names are required solely for clerical checking and for follow-up and clarification where needed.

BACKGROUND INFORMATION

SURNAME ______________________ INITIALS

COURSE ______________________ DEPARTMENT

INSTITUTION ______________________

MATHEMATICS SYMBOL (RING YOUR SYMBOL PASSED IN MATRIC)

HIGHER GRADE A B C D E

STANDARD GRADE A B C D E

HOW MANY YEARS SINCE YOU PASSED MATRIC MATHS? (PUT AN X IN THE APPROPRIATE BOX)

0	1	2	3	4	5	MORE THAN 5 YEARS

AGE GROUP (PUT AN X IN THE APPROPRIATE BOX

18-19	20-21	22-23	24-25	OVER 25

ANSWER YES OR NO TO QUESTIONS 1, 2, 3 AND 4

1.	Did you start Tech straight from School?	1.1	YES	
		1.2	NO	
2.	Did you start Tech directly after military service?	2.1	YES	
		2.2	NO	
3.	Did you start Tech directly from business or commerce?	3.1	YES	
		3.2	NO	
4.	If a two week pre-course in Mathematics was available in January, would you be interested?	4.1	YES	
		4.2	NO	

5. What form should the pre-course take? (Put an X in the appropriate boxes).

5.1 Official lectures	5.1	
5.2 Small Discussion groups	5.2	
5.3 Assignments	5.3	
5.4 Video tapes covering specific items	5.4	
5.5 Computer programmes on an individual basis	5.5	

6. Have you passed a course in technical Mathematics which included calculus?

6.1	YES	
6.2	NO	

7. Which of the items below causes you difficulty. Rank the items in order of difficulty starting with the most difficult as 1 and the least difficult as 10.

7.1 Algebraic Equations	7.1	
7.2 Co-ordinate Geometry (Simple Processes)	7.2	
7.3 Binomial Expansion of Brackets	7.3	
7.4 Trigonometric Identities	7.4	
7.5 Word problems	7.5	
7.6 Substituting Numbers for Symbols	7.6	
7.7 Simplifying Algebraic Expressions	7.7	
7.8 Limits of Expression	7.8	
7.9 Changing of symbols for other symbols	7.9	
7.10 Reading and analysing a problem	7.10	

The following questions relate to the calculus. Each question has several alternative answers. Put an X against the one you think most fits the mathematical meaning of the word or phrase given.

8. The name "Calculus" means:

8.1 A stone formed in the body 8.1 ☐

8.2 A type of addition 8.2 ☐

8.3 A branch of mathematics involved with differentiating and integrating functions 8.3 ☐

8.4 A branch of mathematics that deals with variable quantities 8.4 ☐

8.5 A branch of mathematics that enables you to calculate areas and volumes 8.5 ☐

9. The symbol "$\frac{dy}{dx}$" means:

9.1 A small change of y w.r.t. x 9.1 ☐

9.2 The gradient of the function y=f(x) 9.2 ☐

9.3 The first derivative of y w.r.t. x 9.3 ☐

9.4 The rate of change of y w.r.t x 9.4 ☐

9.5 A combination of some of the above possibilities 9.5 ☐

10. $\frac{d}{dx}\left(\frac{1}{x^2}\right)$ is equal to:

10.1 $\frac{1}{x^2}$ 10.1 ☐

10.2 $\frac{-1}{x}$ 10.2 ☐

10.3 $\frac{2}{x^2}$ 10.3 ☐

10.4 $\frac{-2}{x}$ 10.4 ☐

10.5 $\frac{-2}{x^3}$ 10.5 ☐

11. $\frac{d}{dx}$ (cos2x) + Sin2x is equal to:

11.1	3 sin2x	11.1
11.2	4 sin2x	11.2
11.3	- sin2x	11.3
11.4	0	11.4
11.5	sin2x	11.5

12. The sign $\int$ is used in the calculus. Put an x against which of the following statements explains its meaning fully.

12.1	The sign tells you to do the opposite to differentiation.	12.1
12.2	This sign gives you the area under a curve.	12.2
12.3	This sign means add one to the power of x and divide by the new power.	12.3
12.4	This sign means sum a set of small quantities	12.4
12.5	This sign means "sum"	12.5

13. Do you have problems with limits in the calculus?

13.1	YES	
13.2	NO	

14. Which symbol did you like best for differentiating?
(Put an x in the appropriate box)

14.1	$f^1(x)$	
14.2	$\frac{dy}{dx}$	
14.3	$\frac{d}{dx}$	
14.4	Dx	

15. At what stage did you feel unable to cope with the calculus course?

15.1	After ONE TO TWO WEEKS	15.1
15.2	After THREE TO FOUR WEEKS	15.2
15.3	After FIVE TO SIX WEEKS	15.3
15.4	No difficulties at any stage	15.4

16. Put an x against the statement which reflects your views on the length of time devoted to the calculus during a semester.

16.1 The lecture time was inadequate	16.1	
16.2 The tutorial time was inadequate	16.2	
16.3 The time was adequate for both lectures and tutorials	16.3	

17. How many lectures per week do you feel are necessary in mathematics?

17.1 Four lectures plus 2 tutorial periods	17.1	
17.2 Five lectures plus 2 tutorial periods	17.2	
17.3 Six lectures plus 2 tutorial periods	17.3	
17.4 Seven lectures plus 2 tutorial periods	17.4	

18. How much time do you spend in hours per week on your own doing mathematics?

18.1 up to 4 hours	18.1	
18.2 4 hours to 6 hours	18.2	
18.3 6 hours to 8 hours	18.3	
18.4 8 hours to 10 hours	18.4	
18.5 Over 10 hours	18.5	

19. Various views on textbooks, study guides, lectures and tutorials are listed below. Put an x against a view you agree with. Leave blank those you disagree with.

19.1	Textbooks are only useful for examples	19.1	
19.2	Lectures give you all the explanation you need.	19.2	
19.3	Study guides are essential for any course.	19.3	
19.4	Textbooks are useful for explanations and for examples.	19.4	
19.5	Tutorials are essential	19.5	

20. Rank the following differentiation problems according to the difficulties they cause you, starting with the most difficult as 1 and the least difficult as 5.

20.1	$\frac{d}{dx}(x^2 + 5)$	20.1	
20.2	$\frac{d}{dx}\left(\frac{1}{x^2 + 5}\right)$	20.2	
20.3	$\frac{d}{dx}(\ln x)$	20.3	
20.4	$\frac{d}{dx}(e^x \sin x)$	20.4	
20.5	$\frac{d}{dx}\left(\frac{1}{\sqrt{x^2 + 5}}\right)$	20.5	

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MATHS DEPT

ANNEXURE B – STAFF QUESTIONNAIRE

The purpose of this questionnaire is to gain insight into the difficulties experienced when lecturing on the calculus.

All information will be treated in the strictest confidence and under no circumstances will any individual be identified. Names are required solely for clerical checking and for follow-up and clarification where needed.

BACKGROUND INFORMATION

SURNAME ______________________ INITIALS

INSTITUTION ______________________ DEPARTMENT

1. Highest Mathematical Qualification in years of training at a tertiary institution [Put X in appropriate box]

1	2	3	4	5	6

2. Highest Teaching Qualification [Put X in appropriate box]

1 HEd	2 BEd	3 MEd	4 DEd	5 NONE

3. Age group [Put X in the appropriate box]

1 21–30	2 31–40	3 41–50	4 51–60	5 OVER 60

4. For this question only answer YES or NO.
Do you feel a pre-course in Mathematics is necessary for weaker candidates?

YES	
NO	

5. If you answered YES to Question 4, then fill in Question 5. If you answered NO proceed to Question 6. What form should the pre-course take?
[Put X in appropriate box]

1. Official lectures	1	
2. Small discussion groups	2	
3. Assignments	3	
4. Video tapes covering specific items	4	
5. Computer programmes on an individual basis	5	

6. In your capacity as a lecturer or teacher rank the following items in order of difficulty wrt teaching or lecturing. Rank the least difficult as 0 rising to the most difficult as 9.

1	Algebraic Equations	1	
2	Co-ordinate Geometry (Simple Processes)	2	
3	Binomial Expansion of Brackets	3	
4	Trigonometric Identities	4	
5	Word problems	5	
6	Substituting Numbers for Symbols	6	
7	Simplifying Algebraic Expressions	7	
8	Limits of Expression	8	
9	Changing of symbols for other symbols	9	
10	Reading and analysing a problem	10	

7. The following questions relate to the calculus. Each question has several alternative answers. Put an X against the one you think most fits the mathematical meaning of the word or phrase given.

The name "Calculus" means:

1	To calculate instant rates of change	1	
2	A type of addition	2	
3	A branch of mathematics involved with differentiating and integrating functions	3	
4	A branch of mathematics that deals with variable quantities	4	
5	A branch of mathematics that enables you to calculate areas and volumes	5	

8. The symbol "$\frac{dy}{dx}$" means:

1 A small change of y w.r.t. x	1
2 The gradient of the function y=f(x)	2
3 The first derivative of y w.r.t. x	3
4 The rate of change of y w.r.t x	4
5 A combination of some of the above possibilities	5

9. The sign $\int$ is used in the calculus. Put an x against which of the following statements explains its meaning fully.

1 The sign tells you to do the opposite to differentiation.	1
2 This sign gives you the area under a curve.	2
3 This sign means add one to the power of x and divide by the new power.	3
4 This sign means sum a set of small quantities	4
5 This sign means "sum"	5

10. Do you have teaching problems with limits in the calculus?

1	YES	
2	NO	

11. Which symbol do you like best for differentiating?
(Put an x in the appropriate box)

1	$f'(x)$	
2	$\frac{dy}{dx}$	
3	$\frac{d}{dx}$	
4	Dx	
5	NO PREF	

12. Put an x against the statements which reflect your views on the length of time devoted to the calculus during a semester.

1 The lecture time was adequate	1
2 The tutorial time was adequate	2

13. ~~How many~~ periods per week do you feel are necessary in mathematics? [Take 45 mins as usual period time]

1 Six periods	1
2 Seven periods	2
3 Eight periods	3
4 Nine periods	4
5 More than nine periods	5
6 Less than six periods	6

14. How much time do you spend in hours per week on your own preparations for lecturing mathematics?

1 up to 4 hours	1
2 4 hours to 6 hours	2
3 6 hours to 8 hours	3
4 8 hours to 10 hours	4
5 Over 10 hours	5

15. Various views on lectures and tutorials are given below. Put X against views you agree with. Leave blank those you disagree with.

1 A good lecturer is essential to the success of his students
2 A good textbook can replace a poor lecturer.
3 Attendance at tutorials should be compulsory.
4 A poor textbook and a good lecturer are preferable to a good textbook and a poor lecturer.
5 A good textbook is preferable to a badly organised tutorial

1	
2	
3	
4	
5	

16. In your lecturing you can use various visual aids. Rank the usefulness of the following. Put an 0 for the least useful through to 4 for the most useful.

1 OHP
2 Charts
3 Computer programme
4 Films
5 Videos

1	
2	
3	
4	
5	

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ANNEXURE C – COMPUTER QUESTIONNAIRE

The purpose of this questionnaire is to assess the reactions of students to the use of the computer programme during the T2 Engineering Mathematics course.

All information will be treated in the strictest confidence and under no circumstances will any individual be identified. Name are required solely for clerical checking and for follow-up and clarification where needed.

SURNAME________________________ INITIALS_______________

COURSE ____

DEPARTMENT____

NAME OF TERTIARY INSTITUTION_________________________

BACKGROUND INFORMATION

1. MATHEMATICS SYMBOL

HIGHER GRADE	A	B	C	D	E
STANDARD GRADE	A	B	C	D	E
ALTERNATIVE EXAM	A	B	C	D	E

HIGHER	
STANDARD	
ALTERNATIVE	

2. AGE GROUP

18 – 19	A
20 – 21	B
22 – 23	C
24 – 25	D
OVER 25	E

AGE GROUP	

3. STUDENT'S ORIGIN PRIOR TO TECHNIKON

SCHOOL	A
MILITARY	B
BUSINESS	C
UNIVERSITY	D
OTHER	E

ORIGIN	

The remaining questions refer to the computer package and the five tutorials you handed in during this semester. Indicate your choice in the box provided.

4. WHEN YOU STARTED THIS PROGRAMME HAD YOU PREVIOUSLY DONE WORK ON COMPUTER PACKAGES?

NO WORK AT ALL	A
A LITTLE WORK	B
A FAIR AMOUNT OF WORK	C
A LOT OF WORK	D
HAD DONE THE PROGRAMME BEFORE	E

COMPUTER WORK	

5. DO YOU CONSIDER THE PROGRAMME HELPED YOUR UNDERSTANDING OF THE CALCULUS?

HELPED A LOT	A
DID NOT HELP AT ALL	B
HELPED A LITTLE	C
HELPED WITH SOME ASPECTS	D
MADE THINGS MORE DIFFICULT	E

UNDERSTANDING PROGRAMME	

6. THIS QUESTION REFERS TO THE STRAIGHT LINE GRAPHS. INDICATE YOUR GREATEST DIFFICULTY IN THE BOX PROVIDED.

FINDING THE EQUATION USING THE GRADIENT	A
FINDING THE EQUATION WHEN THE Y- INTERCEPT WAS SHOWN	B
FINDING THE EQUATION WHEN THE X-INTERCEPT WAS SHOWN	C
FINDING THE EQUATION WHEN NO INTERCEPT WAS SHOWN	D

LINE DIFFICULTY	

7. THIS QUESTION REFERS TO GRAPHS OF QUADRATIC FUNCTIONS. INDICATE WHICH ITEM GAVE YOU THE GREATEST DIFFICULTY.

FINDING THE EQUATION WHEN THE CURVE HAD TWO X INTERCEPTS	A
FINDING THE EQUATION WHEN THE CURVE TOUCHED THE X AXIS	B
FINDING THE EQUATION WHEN THE CURVE WAS INVERTED	C
FINDING THE EQUATION WHEN THE CURVE WAS INVERTED & BELOW THE X AXIS	D

QUADRATIC DIFFICULTY	

8. THIS QUESTION REFERS TO GRAPHS OF TRIGONOMETRIC FUNCTIONS. INDICATE WHICH ITEM GAVE YOU THE GREATEST DIFFICULTY.

FINDING THE EQUATION WHEN THE X AXIS WAS MEASURED IN RADIANS	A
FINDING THE EQUATION WHEN THE PEAKS OF THE CURVE WERE REMOVED	B
FINDING THE EQUATION WHEN ONLY PART OF THE PERIOD WAS SHOWN	C
FINDING THE EQUATION WHEN THE WAVES WERE REVERSED	D
FINDING THE EQUATION WHEN FRACTIONS OCCURED IN YOUR ANSWER	E

TRIG DIFFICULTY	

9. THIS QUESTION REFERS TO THE 2ND TUTORIAL ON GRADIENT OF A FUNCTION. INDICATE WHICH ITEM GAVE YOU THE GREATEST DIFFICULTY.

FINDING THE SUGGESTED DERIVATIVE	A
CHANGING THE DOMAIN AND RANGE WHEN REQUIRED	B
UNDERSTANDING WHAT HAPPENED IN THE PLOT OF GRADIENT	C
APPRECIATING THE FORMULA USED IN THE GRADIENT FUNCTION	D
KNOWING WHEN TO STOP THE PROGRAMME TO GET THE RIGHT ACCURACY	E

GRADIENT DIFFICULTY	

10. THIS QUESTION REFERS TO THE 5TH TUTORIAL ON SOLVING EQUATIONS USING THREE NUMERICAL METHODS. INDICATE WHICH ITEM WAS MOST DIFFICULT TO UNDERSTAND.

THE BISECTION METHOD INVOLVING A CONTINUOUS AVERAGE TECHNIQUE	A
THE REGULA-FALSI METHOD WHICH JOINS POINTS ON EITHER SIDE OF X AXIS	B
THE NEWTON RAPSON METHOD WHICH USES TANGENTS TO GET CLOSER TO THE ROOT	C
THE IDEA THAT A ROOT LIES ON THE X AXIS ie WHERE F(X) = 0	D
THE IDEA THAT WHEN A CURVE PASSES THROUGH A ROOT THE SIGN OF THE FUNCTION CHANGES	E

EQUATION DIFFICULTY	

11. THIS QUESTION REFERS TO THE 3RD TUTORIAL WHERE YOU LOOKED AT SIX NUMERICAL METHODS FOR FINDING A DEFINITE INTEGRAL. INDICATE WHICH STATEMENT GAVE YOU THE MOST DIFFICULTY.

THE AREA UNDER A CURVE IS A NUMERICAL VALUE OF THE INTEGRAL OF FUNCTION REPRESENTING THE CURVE	A
THE MIDDLE RECTANGLE AREA CALCULATION	B
THE RANDOM RECTANGLE AREA CALCULATED	C
WHY THE TRAPEZIUM RULE GAVE A BETTER APPROXIMATION THAN RECTANGLES	D
THE INCREASE IN THE NUMBER OF STEPS BROUGHT ALL RESULTS CLOSER TO THE CORRECT ANSWER	E

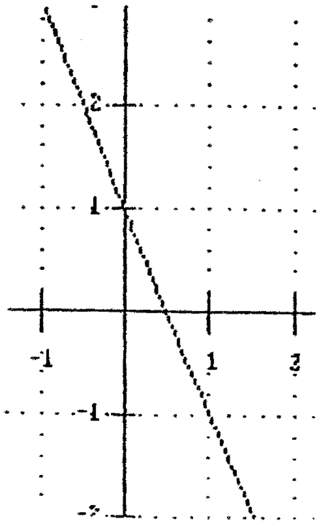
INTEGRATION DIFFICULTY	

12. IN THIS QUESTION THE STATEMENTS REFER TO THE 5TH TUTORIALS ON DIFFERENTIAL EQUATIONS. INDICATE THE STATEMENT WHICH GAVE YOU THE GREATEST DIFFICULTY.

SETTING THE ARROW CORRECTLY	A
ACTUALLY PINPOINTING THE GIVEN X-VALUE	B
DECIDING WHETHER TO USE NORMAL OR QUICK	C
REVERSING THE ARROW	D
UNDERSTANDING WHAT THE QUESTION REALLY REQUIRED	E

DIFFERENTIAL DIFFICULTY	

13. IN QUESTIONS 13–18 SELECT THE CORRECT EQUATION FOR THE GRAPH SHOWN FROM THE GIVEN CHOICES:



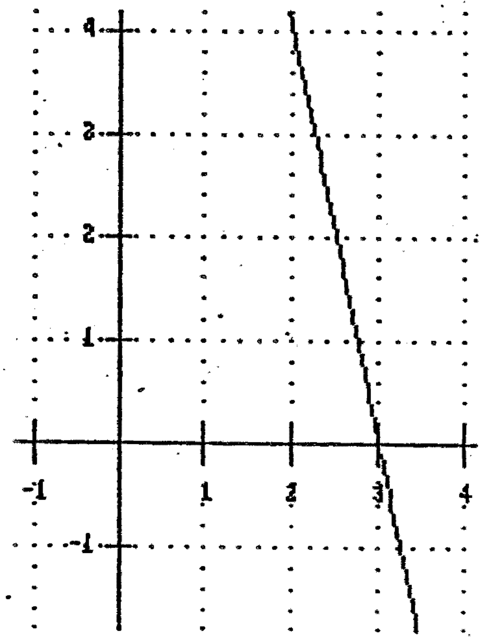
f(x) = −x + 1	A
f(x) = −1/2x+ 1	B
f(x) = 3/4 x + 1	C
f(x) = −3/2x + 1	D
f(x) = −2x + 1	E

LINEAR Y INTERCEPT	

14.

f(x) = 1/4x + 3	A
f(x) = 4x + 3	B
f(x) = −1/4x + 3	C
f(x) = −4x + 3	D
f(x) = −4x + 12	E

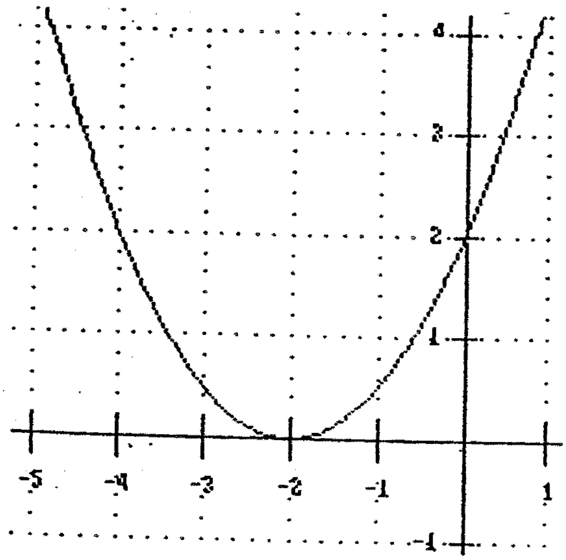
LINEAR X INTERCEPT	



15.

$f(x) = 1/2(x + 2)^2$	A
$f(x) = (x + 2)^2$	B
$f(x) = (x - 2)^2$	C
$f(x) = 1/2(x - 2)^2$	D
$f(x) = x^2 + 2$	E

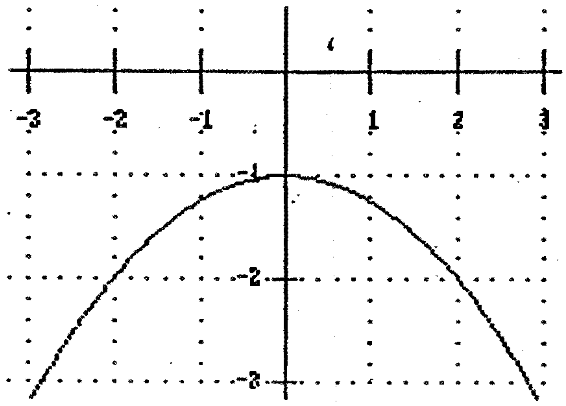
QUADRATIC TOUCHING AXIS	



16.

$f(x) = -x^2 - 1$	A
$f(x) = -1/2x^2 - 1$	B
$f(x) = -2x^2 - 1$	C
$f(x) = -1/4x^2 - 1$	D
$f(x) = -3/2x^2 - 1$	E

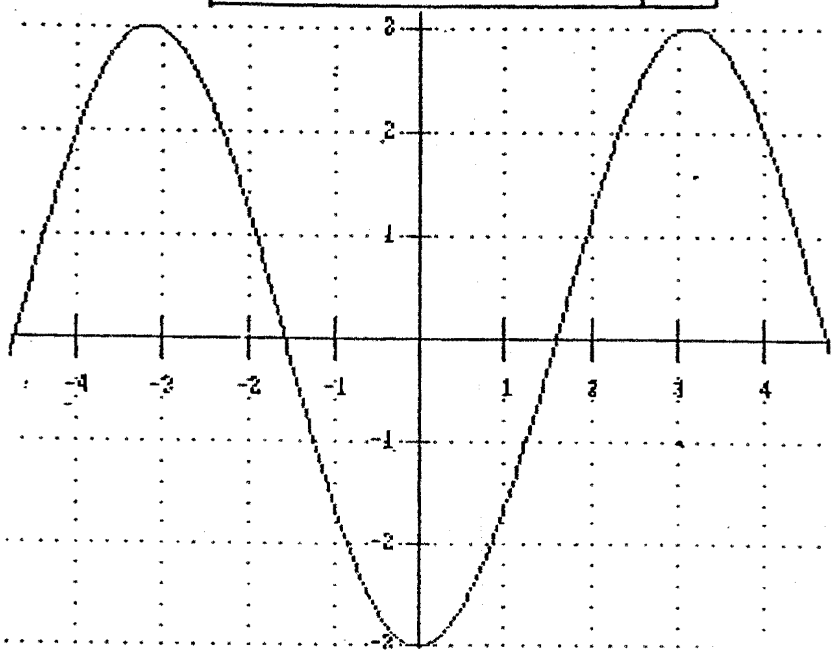
QUADRATIC NOT TOUCHING AXIS	



17.

$f(x) = 3 \sin x$	A
$f(x) = 3 \cos x$	B
$f(x) = -3\cos x$	C
$f(x) = -3\sin x$	D
$f(x) = 3(\sin x - \cos x)$	E

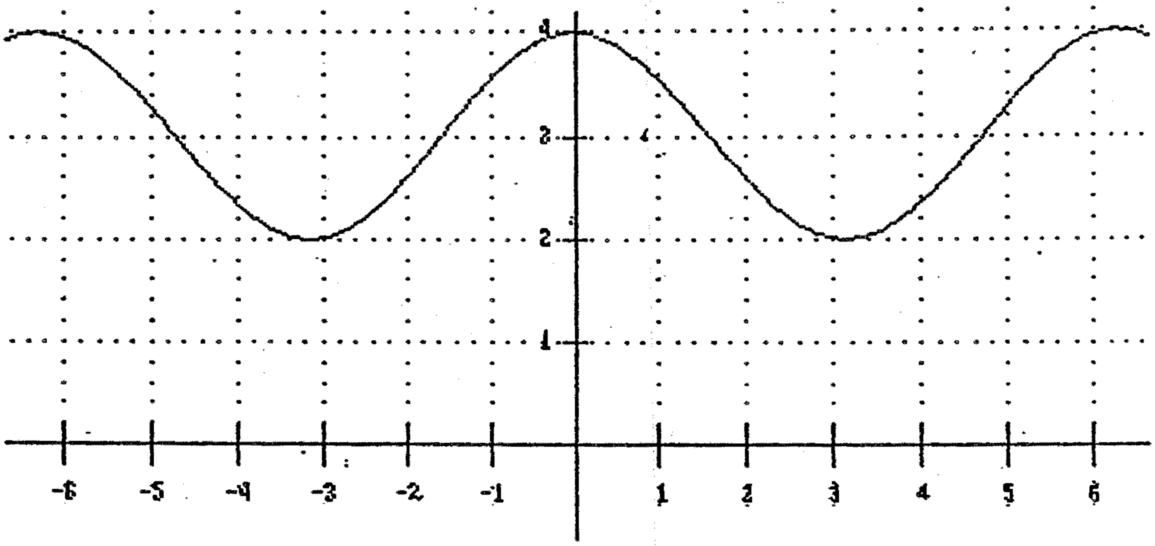
TRIG CUTTING X AXIS	



18.

f(x) = 4 cos x	A
f(x) = cosx + 3	B
f(x) = cos2x + 2	C
f(x) = 2 cosx + 4	D
f(x) = −2sinx + 4	E

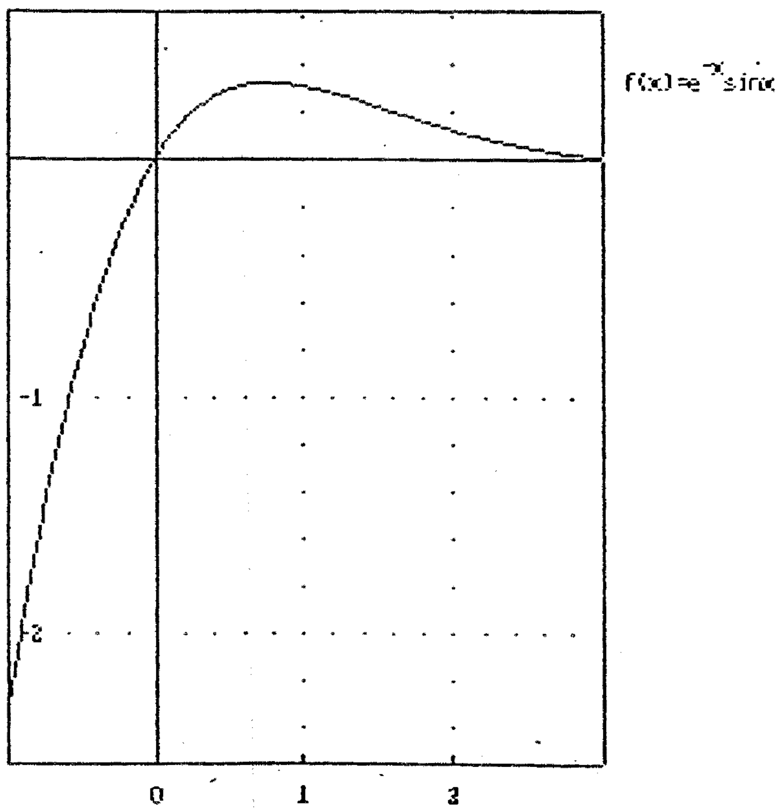
TRIG ABOVE AXIS	



19. THE GRADIENT OF $f(x) = e^{-x}\sin x$ at $x = 0$ IS ONE OF THE FOLLOWING CHOICES:

e	A
$e - 1$	B
0	C
1	D
−1	E

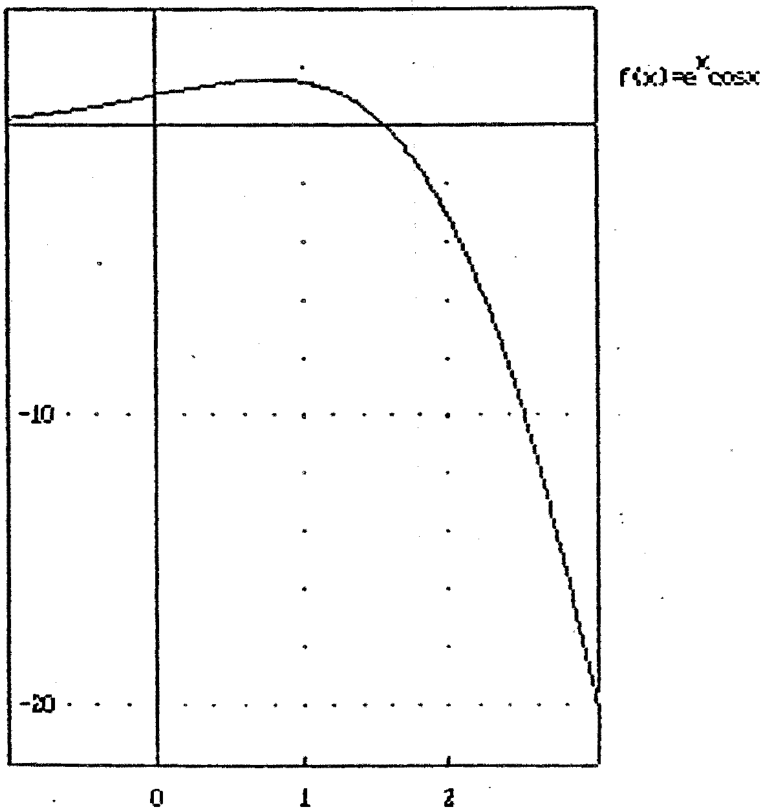
GRADIENT EXPONENTIAL	



20. THE AREA UNDER THE CURVE FROM X = 0 TO X = π/2 IS ONE OF THE FOLLOWING CHOICES APPROXIMATELY:

1,9	A
2,9	B
3,8	C
5,8	D
23,1	E

INTEGRATION EXPONENTIAL	



ANNEXURE D-TUTORIALS

TUTORIAL 1

AIMS:

1 To investigate various linear, quadratic, trigonometric, inverse trigonometric and exponential functions.

2 To illustrate some results in the form of graphs with the equations and domain and range.

PRACTICAL DETAILS:

1 Read the introductory information pages 1-7

2 Activate the computer with a suitable DOS

3 Put in program disk and type GC and press enter

4 When the menu is displayed press 1

Press 1. "looking for the formula"
The first program in the pack allows users to test their knowledge of the shapes of graphs by "looking for the formula". The computer draws a graph and the student is invited to suggest a formula for the function. It is possible to draw the graph of the suggested formula as a visual check on the aptness of the suggestion.

This programme has three major options:

1 Beginning level

2 Medium level

3 Give a function oneself

When you press 1 or 2 you get five choices:

Linear
Quadratic
Complicated Quadratic
Trigonometric
Mixture

Each one of the above has 16 possibilities. Choose a number from 1-16. A graph is then drawn (original). Your guess is then made and displayed. If you are correct then your guess overprints the original and FOUND appears on the screen. If you are wrong, the computer asks you if you wish another try (Y/N). It also asks you if you want a clear picture (Y/N).
If you answer Y to both then it clears the guess away from the screen and starts again. The old try is written above for reference.

NB: For Linear press L
Quadratic press Q
Trigonometric T

1. Using programme 1 looking for a formula work through the following choices and print your own results. Photostats will not be accepted.

Using program 1 "looking for a formula" try 3 choices of linear, quadratic and trigonometric functions at the beginning level and 3 choices at the medium level of linear, quadratic and trigonometric functions. Record the correct results of your choices.

2. Using the graph programme 2 make a printout of the following graphs:

$f(x) = \arcsin x$
$f(x) = \arccos x$
$f(x) = e^{x} + e^{-x}$
$f(x) = \ln(\tan x + \sec x)$

For each graph give the domain.

DEPARTMENT OF MATHEMATICS

TUTORIAL 2

AIMS:

1. To differentiate certain functions. i.e. find gradient function.

2. To plot certain graphs.

PRACTICAL DETAILS:

1. Activate programme.
2. Press 4
3. Asks you for a function
   e.g. $f(x) = x^2 - 2x$
   Type in your function
   Use ↑ for powers of x
4. Press return key
5. If domain and range are satisfactory - Press F1
6. Two choices are given
7. Press 1 for drawing chords from one point
8. Choose a point, say, 1,5
9. Press return
10. Press plot or P
    This gives gradient at point to one place of decimals
11. Press P again for second place of decimals etc
12. use zoom to get finer detail
13. Answer in this case approaches 1
14. Press escape
15. Press 2 for gradient functions
16. Your range may need altering to show all gradient functions for negative values: e.g. for $f(x) = x^2 - 2x$
17. Suggest the gradient function i.e. $f'(x) = 2x - 2$
    Press D before you type in your answer.

TUTORIAL

Using programme 4 find the gradient of the following functions at the given points.

1. $f(x) = 2^x$ at $x = 2$
2. $f(x) = \ln x$ at $x = 10$
3. $f(x) = \arcsin x$ at $x = 0,5$
4. $f(x) = e^x \sin x$ at $x = 0,75$
5. Suggest the gradient function of $e^x \sin x$
6. Find the gradient function of $x \ln x$ at $x = 3$
7. Plot the gradient function of $x \ln x$
8. Find the derivative of $\sqrt{1 - x^2} \arcsin x$ at $x = 0,4$
9. Suggest the derivative of $\sqrt{1 - x^2} \arcsin x$
10. Find the gradient of $x^2 \cos 2x$ at $x = \pi$
11. Plot the derivative of $x^2 \cos 2x$
12. Find the gradient of $x^2 e^x$ at $x = 4$
13. Find the derivative of $\sinh x \, . \, \sin x$ at $x = 3$
14. Plot the derivative of $\sinh x \, . \, \sin x$

DEPARTMENT OF MATHEMATICS

TUTORIAL 3

INTEGRATION USING THE AREA UNDER THE CURVE

1. Activate programme
2. Press 5
3. Enter function you wish to integrate, e.g. x^2
4. There are six approaches to plotting and calculating the area. 1.2.3.4 concern splitting the area into rectangles, beginning from the left, middle, right or at random. 5 by means of trapeziums and 6 by Simpson's rule.
5. Press plot i.e. (P)
6. Choose your option. This shows you clearly how each option works by graphing the rectangles or trapeziums.
   1. 18,36
   2. 17,82
   3. 18,36
   4. 18,94
   5. 18,36
   6. 18
7. Press ESC.
8. Press calculate i.e. (C). This also does all the options but does not show how it happens on the graph. To operate press option and Go (G).
9. For primitive press 7 and G
10. Then suggest primitive e.g.

$$\frac{x^3}{3}$$

11. It will also plot the primitive
12. Calculate the area from the primitive

Find the definite integrals by means of areas under the following curves:

1. $\int_0^4 x^4 \, dx$

2. $\int_1^2 \sqrt{x - 1} \, dx$

3. $\int_0^{\pi} \sin x \, dx$

4. $\int_0^{\frac{\pi}{4}} \cos^2 x \, dx$

5. $\int_1^5 e^{2x} \, dx$

6. $\int_1^5 e^x \cos x \, dx$

7. $\int_1^3 x^2 \ln x \, dx$

8. $\int_0^{\frac{\pi}{4}} \tan^2 x \, dx$

9. $\int_0^{\frac{\pi}{2}} \sin 7x, \cos 5x \, dx$

10. $\int_0^1 \frac{(x^2 - 2x) \, dx}{(2x + 1)(x^2 + 1)}$

In each case give results for six methods. Also suggest the primitive for each problem. Calculate the area using the primitive.

TECHNIKON NATAL

TUTORIAL 4

PARAMETRIC REPRESENTATION OF CURVES

ROUTINES:

1. Activate programme
2. Press A
3. Note $x(t) = \sin t$
   $y(t) = \cos t$
4. Press F1 ready
5. Note the curve is a computer circle i.e. diameter are equal
6. Press L for enlargement

QUESTIONS:

Question 1:

1.1 Plot the curve with parametric equation $x = 2\sin t$; $y = 3 \cos t$.
1.2 What is the name of this curve?
1.3 Find the (x;y) equation of this curve.
1.4 Make a print of this curve with its equation in terms of x and y.

Question 2:

2.1 Plot the curve with parametric equations $x = 2t^2$; $y = 4t$ with t values from $t = 0$ to $t = 4$.
2.2 Find the (x;y) equation of this curve.
2.3 Make a print of this curve with its equation in terms of x and y.

Question 3:

Repeat question 2 with $x = 2\cosh t$; $y = 3\sinh t$.

Question 4:

Find the area of the curve in Question 1 using integration programme.

Question 5:

If $x = 5(2t - \sin 2t)$ $y = 10 \sin^2 t$ find the area enclosed by the curve and the x axis between $t = 0$ and $t = \pi$.

TECHNIKON NATAL

DEPARTMENT OF MATHEMATICS

TUTORIAL 5

SOLUTION OF EQUATIONS

Read pages 24-26 of handout. There are 3 methods.

1. Bisection method
2. The method of false position
3. Newton-Raphson

Example: Solve $x^2 - x - 5 = 0$

Routine 1. Bisection Method

Press 6
Press 1
Type in $f(x) = x^2 - x - 5$
The graph will appear
From the graph decide on a point before graph cuts x axis and a point after,

e.g. $x = 2$	LEFT	END	POINT	TYPE	ENTER
$X = 3$	RIGHT	END	POINT	TYPE	ENTER

Press PLOT - a value of x will appear and a value of f(x) (or y). Repeat ad nauseum till the y value gets very close to zero. You can use the zoom to enlarge. Record value of x.

x = 2,791503g y = 0,0009902

Press 2
Regular, Falsi or method of false position. Again start with interval in which the function changes sign.

Press PLOT repeat as for method 1.

Press 3
NEWTON RAPHSON
Required to give a starting point, say, $x = 2$
Also required to give $f'(x)$ i.e. $2x - 1$
When this is done the tangent is drawn at $x = 2$ and the next x value is where this line cuts the x axis. After 3 steps
x = 2,79130435
y = 0,0000756144

Using the bisection method solve the following equations for one positive root.

1. $2x^2 - x - 5 = 0$
2. $x^4 - x^2 - 3 = 0$
3. $e^{2x} + e^x - 3 = 0$

Using the method of false position solve the following equations for one root.

4. $3 \sin x + 4 \cos x - 3 = 0$
5. $2 \sin^2 x + \sin x - 2 = 0$
6. $\sin 3x + \sin x + 1 = 0$

Using the Newton Raphson method solve the following equation for one positive root.

7. $7 \sinh x + 20 \cosh x = 24$
8. $2x^2 - x - 5 = 0$
9. $3 \cosh 2x - \sinh 2x - 3 = 0$
10. $e^{\sin x} - 2 = 0$

In addition a sketch or print out is required for 1/3/5/7/10

BIBLIOGRAPHY

1. Allendoerfer, C.B. 1993. The case against calculus. Mathematics Teacher. No. 56, p.482-485.

2. Andrie, M. 1985. Application-orientated mathematics in the education of engineers. International Journal of Mathematics Education in Science and Technology. Vol. 16, No. 2, p.157-162.

3. Azzolini, A. 1990. Writing as a tool for teaching mathematics: The silent revolution National Council of Teachers of Mathematics. 1990 Yearbook. Western Michigan University. p.92-100.

4. Barnes, M. 1992. Calculus for non specialists. Proceeding of Working Group 3 on students difficulties in calculus. Seventh International Congress on Mathematical Education. Quebec 1992. College de Sherbrooke Canada. p.64-65.

5. Barry, M.D.J. & Harley, G.F. 1990. The Fifth European Seminar on mathematics in engineering education. International Journal of Mathematics Education in Science and Technology. Vol. 21, No. 3, p.343-61.

6. Beckman, C. 1992. Renovation in the Calculus Curriculum. A United States perspective. Proceeding of Working Group 3 on students difficulties in calculus. Seventh International Congress on Mathematical Education. Quebec 1992 College de Sherbrooke Canada. p. 109-113.

7. Bell, F.H. 1978. Teaching and learning Mathematics (In Secondary Schools) Dubuque, Iowa: C. Brown Company Publishers.

8. Bloom, B.S. Hastings, J.H. & Madaus, G.F. 1971. Handbook on Formative and Summative Evaluation of Student Learning. New York: McGraw-Hill.

9. Boitri, P. & Steele, N.C. 1987. Mathematics in the professional life of an engineer. International Journal of Mathematics Education in Science and Technology. Vol. 18, No. 5, p.651-656.

10. Bookman, J. 1992. Natural Science Foundation Workshop on assessment in Calculus Reform efforts. Undergraduate Mathematics Education Trends. Vol. 4, No. 4, p.1.

11. Bressoud, D.M. 1992. Why do we teach calculus? The American Mathematical Monthly. Vol. 99, No. 7, p.615-618.

12. Briginshaw, A. 1987a. On teaching mathematics to undergraduate engineers and others. International Journal of Mathematics Education in Science and Technology. Vol. 18, No. 3, p.327-334.

13. Briginshaw, A. 1987b. Myth and reality in teaching undergraduate mathematics. International Journal of Mathematics Education in Science and Technology. Vol. 18, No.2, p.215-220.

14. Campbell, P.J. 1991. What's wrong with College Algebra? Undergraduate Mathematics Education Trends. Vol. 12, No. 1, p.93-97.

15. Chang, L.C. & Razicka, J. 1985. Second International mathematics study United States technical report 1. Champaign: Stikes I.L. Publishing.

16. Christopherson, D.G. 1967. The Engineer in the University. London: English Universities Press.

17. Conley, M.R. et al. 1992. Students perception of projects in learning calculus. International Journal of Mathematics Education in Science and Technology. Vol. 23, No. 2, p.175-192.

18. Countryman, J. 1992. Writing to learn mathematics ; Strategies that work. London: Heinemann.

19. Cowen, C.C. 1991. Teaching and testing Mathematics reading. The American Mathematical Monthly. Vol. 98, No. 1, p.50-53.

20. Davis, G.J. & Hersh, R. 1981. The Mathematical Experience. Boston: Houghton Miflin.

21. Dubinsky, E. 1988. An epistemological approach using computers in mathematical education. Unpublished lecture given to the 6th International Congress on Mathematical Education. Budapest 1988.

22. Eisenberg, T. 1992. Renovation in the Curriculum of Calculus Courses. Proceeding of Working Group 3 on students difficulties in calculus. Seventh International Congress on Mathematical Education.Quebec 1992. College de Sherbrooke Canada, p.91-95.

23. Ervynck, G. 1992. Mathematics as a Foreign Language. Proceedings of the 16th Psychology of Mathematics Education Conference, University of New Hampshire, Durham NH, U.S.A. 1992. p.217-233.

24. Even, R. 1992. The inverse function. Prospective teachers' use of 'undoing'. International Journal of Mathematics Education in Science and Technology. Vol. 23, No. 4, p.557-562.

25. Ferrini-Munday, J. & Graham, K.G. 1991. An overview of the Calculus Reform Effort Issues for Learning Teaching and Curriculum Development. American Mathematical Monthly. Vol. 98, No. 7, p.627-635.

26. Flashman, M. 1990. A sensible calculus. Undergraduate Mathematics Applications. Vol. 11, No. 1, p.93-96.

27. Foley, G.D. 1988. Timeless and timely issues in the teaching of calculus. Amatyc Review. Vol. 9, No. 2, p.55-60.

28. Furinghetti, F. & Paola, D. 1991. The construction of a didactic itinerary of calculus starting from students' concept images (ages 16-19). International Journal of Mathematics Education in Science and Technology. Vol. 22, No. 5, p.719-729.

29. Hugo, F., Kemp, A. & Rohale, A. 1989. Report of an international investigation of University Education in Civil Engineering. Executive Summary: (Vii-Xii) [S.1:Sn]

30. Human Sciences Research Council. 1981. The supply of and demand for engineers in 1987. Pretoria. Human Sciences Research Council, Institute for Manpower Research, 1981.

31. Hundhausen, J.R. 1992. What is the role of calculus in a Scientific/Technical Education? Proceeding of Working Group 3 on students difficulties in calculus. Seventh International Congress on Mathematical Education. Quebec 1992. College de Sherbrooke Canada. p.69-70.

32. Landbeck, R. 1991. What environmental studies students think about mathematics and what could be done about it. International Journal of Mathematics Education in Science and Technology, Vol. 22, No. 4, p.669-674.

33. Laridon, P.E. 1981. Curriculum Development in Secondary School Mathematics. The creative teaching of Calculus. Unpublished M. Ed Thesis. Johannesburg: Randse Afrikaanse Universiteit.

34. Lerman, S. 1983. Problem solving or knowledge centred: the influence of philosophy on mathematics teaching. International Journal of Mathematics Education in Science and Technology. Vol. 14, No. 1, p.59-60.

35. Lloyd, P.J.D. & Plowman, R.P. 1985. The supply and demand for engineers. The Federation of Societies of Professional Engineers Journal. Vol. 347, p.1-13.

36. Mackie, D.M. 1992. An evaluation of computer assisted learning mathematics. International Journal of Mathematics Education in Science and Technology, Vol. 23, No. 5, p.731-737.

37. Marcus, R. 1989. Don't kill the world: The mechanical engineer's responsibility to society and himself. Unpublished lecture given to Society of Mechanical Engineers, Johannesburg. July 1989.

38. McNab, D. Mickaseh, H.G. & Georgi, W. 1977. Description and Assessment of Difficult Methods of Teaching. International Journal of Mathematics Education in Science and Technology. Vol. 8, No. 2, p.219-228.

39. McWilliams, M. 1993. Coming off the drawing board. Better Engineers. Business Week, 2 August 1993,p.46-47.

40. Meyer. N.C. 1993. A Parable. Undergraduate Mathematics Education Trends. Vol. 5, No. 4, p.16.

41. Moore, L. & Smith, D. 1992. The Calculus Project. Calculus as a laboratory course. Proceeding of Working Group 3 on students difficulties in calculus. Seventh International Congress on Mathematical Education. Quebec 1992. College de Sherbrooke Canada. p.100.

42. Morgan, A.T. 1990. A study of difficulties experienced with mathematics by engineering students in higher education. International Journal of Mathematics Education in Science and Technology. Vol. 21, No. 6, p.975-988.

43. Nichols, D. 1992. Calculus, Concepts, Computers and Co-operative learning. Undergraduate Mathematics Education Trends. Vol, 4, No. 4, p.4-7.

44. Norman, F.A. 1992. Issues regarding Calculus for the non-specialist. Proceeding of Working Group 3 on students difficulties in calculus. Seventh International Congress on Mathematical Education. Quebec 1992. College de Sherbrooke Canada. p.71-72.

45. Nyondo, A.C. 1993. Mathematics courses with a P.C. International Journal of Mathematics Education in Science and Technology. Vol.24, No. 4, p.569-574.

46. Olayi, G.A. 1990. Mathematics curriculum in third world Universities for prospective mathematics teachers. International Journal of Mathematics Education in Science and Technology. Vol. 21, No. 5, p.695-700.

47. Orton, T. 1986. Introducing calculus: an accumulation of teaching ideas? International Journal of Mathematics Education in Science and Technology. Vol. 17, No. 6, p.659-668.

48. O'Shea, D. 1992. Teaching conceptions and strategies for calculus. Proceeding of Working Group 3 on students difficulties in calculus. Seventh International Congress on Mathematical Education. Quebec 1992. College de Sherbrooke Canada. p.49-52.

49. Picciarelli, M.T. et al. 1990. Identifying high-risk students in Mathematics and Physics. International Journal of Mathematics Education in Science and Technology. Vol. 21, No. 3, p.369-373.

50. Pittendrigh, A. 1988. Technikons in South Africa. An evaluation analysis of the sectors influencing the development of Technikons. Halfway House : Building Industries Federation of South Africa.

51. Protter, M.H. 1991. The Self-Paced Calculus Program at Berkely. The American Mathematical Monthly. Vol. 98, No. 3, p.245-249.

52. Rees, W. 1957. The Natal Technical College 1907-1957. Durban : University of Natal Press.

53. Rhodes, S. 1988. Peoples Education in Mathematics. A developing South African revolution. Unpublished lecture given to the 6th International Congress on Mathematical Education, Budapest, 1988.

54. Roder, P. 1992. Graphic calculators versus sophisticated software in the classroom. Proceeding of Working Group 3 on students difficulties in calculus. Seventh International Congress on Mathematical Education. Quebec 1992. College de Sherbrooke Canada. p.87-89.

55. Saunders, S.J. 1992. Report on tertiary education in South America, the U.S.A, Europe, Sub-Saharan African and South Africa. Cape Town : U.C.T. Publishing.

56. Schwingendorf, 1992. Calculus Reform in the U.S. A closer look at the Purdue Project. Proceeding of Working Group 3 on students difficulties in calculus. Seventh International Congress on Mathematical Education. Quebec 1992. College de Sherbrooke Canada. p.96-99.

57. Searle, 1987. Tutorial Class in Mathematics. International Journal of Mathematics Education in Science and Technology. Vol. 10, No. 4, p..553-555.

58. Selden, J., Mason, J. & Selden, A. 1989. Can Average calculus students work non-routine problems? The Journal of Mathematical Behaviour, U.S.A., No. 8, p.45-50.

59. Selden, A. & Selden, J. 1991. Calculus for non-specialists. Proceeding of Working Group 3 on students difficulties in calculus. Seventh International Congress on Mathematical Education. Quebec 1992. College de Sherbrooke Canada. p.63-65.

60. Simon, B. 1990. The new world of higher Math. P C Magazine. Vol. 9, no. 10, p.323-336.

61. Smith, J.C. 1992 Calculus for non-specialists. Proceeding of Working Group 3 on students difficulties in calculus. Seventh International Congress on Mathematical Education. Quebec 1992. College de Sherbrooke Canada. p.66-68.

62. Smith, R.M. 1991. Mastering Mathematics. How to be a great Mathematics student. London : Wadsworth.

63. Standler, R.B. 1990. Mathematics for engineers. Undergraduate Mathematics Applications. Vol. 11, No. 1, p.1-6.

64. Steen, L.A. 1992. 20 Questions that Deans should ask their Mathematics Department. America Association for Higher Education Bulletin, Vol. 44, No. 9, p.3-6.

65. Straesser, R. & Thiering, J. 1986. Action Group 7: Mathematics in Adult, Technical and Vocational Education. M. Carss (Ed). Proceedings of the Fifth International Congress on Mathematical Education. Boston: Birkhauser, p.124-132.

66. Strauss, J. & Diab, R. 1988. Mathematical Requirements of technical engineers. South African Journal for Education. Vol. 8, No. 2, p.133-141.

67. Sullivan, D. 1991. On Jargon: Toward a clean and lively Calculus. Undergraduate Mathematics Applications. Vol. 12, No. 2, p.165-174.

68. Tall, D. 1985. Understanding of Calculus. Mathematics Teaching, No. 110, p.49-53.

69. Tall, D. 1992. Students difficulties in calculus. Proceeding of Working Group 3 on students difficulties in calculus. Seventh International Congress on Mathematical Education. Quebec 1992. College de Sherbrooke Canada. p.13-28.

70. Tall, D. & Vinner, S. 1981. Concept ways and concept definition in mathematics with particular reference to limits and continuity. Educational Studies in Mathematics, Vol. 12, p.151-169.

71. Tall D. & Razali, H.R. 1993. Diagnosing students' difficulties in learning mathematics. International Journal of Mathematics Education in Science and Technology. Vol. 24, No. 2, p.209-222.

72. Toumassis, C. 1993. Fostering auto-learning abilities in mathematics. A Greek experience. International Journal of Mathematics Education in Science and Technology. Vol. 24, No. 4, p.553-567.

73. Van Pletzen, J. 1981. The supply and demand for engineers in 1987. Institute of Manpower Research. No. MM-82. Pretoria: Human Science Research Council.

74. Van Rooy, M.P. 1984. Die onderrig van wiskunde aan die Technikon met verwysing na die moontlikhede van 'n bemeesteringsleerstrategie in die onderrig van ingenieurswiskunde. Unpublished M. Ed. Thesis. Pretoria: Unisa.

75. Van Rooy, M.P. 1987. Doelstelling as komponent van die kurrikulumsiklus met verwysing na wiskunde aan die Technikon. Unpublished D.Ed. Thesis. Pretoria: Unisa.

76. Van Rooy, T. 1988. The aims of teaching Mathematics - A technical and vocational education perspective on curriculum development in Mathematics. Pythagoras. No. 20, p.29-31.

77. Van Zyl, A.J. 1971. Optimum use of manpower. Pretoria College for Advanced Technical Education.

78. Waks, S.G. 1987. Incorporating technological aspects in a mathematical course. A case study. International Journal of Mathematics Education.in Science and Technology. Vol. 18, No. 3, p.381.

79. Wilson, D.B. 1978. Report on the use of structural tutorials as a means to creative thinking. International Journal of Mathematics Education in Science and Technology. Vol. 9, No. 4, p.423-431.

80. Winkel, B.J. 1990. First year calculus students as in class consultants. International Journal of Mathematics Education in Science and Technology. Vol. 21, No. 3, p.363-368.

81. Woods, R. 1989. How can interest in calculus be increased? American Mathematical Monthly. Vol. 36, p.28-32.