THE ROLE OF PEDAGOGICAL CONTENT KNOWLEDGE IN THE LEARNING OF QUADRATIC FUNCTIONS

by

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ABSTRACT

This study investigates to what extent educators' pedagogical content knowledge affects learners' achievement in quadratic functions. The components of pedagogical content knowledge (PCK) examined are: (i) mathematical content knowledge (MCK), (ii) knowledge of learners' conceptions, and misconceptions, and (iii) knowledge of strategies. The participants were seventeen mathematics educators and ten learners from each educator's class. The sample of educators was a convenient sample, while the sample of learners was selected by means of random sampling. A mixed method design was used to execute the study. Data about educators' MCK, and knowledge of learners' misconceptions were collected by means of a questionnaire. An interview was used to gather data about educators' knowledge of strategies. Data on learners' achievements and misconceptions was collected by means of a questionnaire. Descriptive statistics were used to describe the effect of each component of the educators' PCK on learners' achievements. The result indicates that the achievement of learners who are taught by educators who have strong PCK is higher than the achievement of learners who are taught by educators who have weak PCK.

KEY TERMS

Pedagogical content knowledge, mathematical content knowledge, knowledge of learners’ conceptions and misconceptions, knowledge of strategies, learners’ achievement, quadratic functions, learners’ conceptions and misconceptions, educators’ knowledge.

DECLARATION

I declare that THE ROLE OF PEDAGOGICAL CONTENT KNOWLEDGE IN THE LEARNING OF QUADRATIC FUNCTIONS is my own work and that all the sources I have used or quoted are indicated and acknowledged by means of complete references.

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SIGNATURE DATE

(Ibeawuchi, Emmanuel Ositadinma)

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LIST OF ABBREVIATIONS

ACE	-	Advanced Certificate in Education
DoE	-	Department of Education.
Matric	-	Matriculation
MCK	-	Mathematical Content Knowledge
PCK	-	Pedagogical Content Knowledge
TIMSS	-	Trends in International Mathematics and Science Study

CHAPTER ONE

BACKGROUND AND OVERVIEW OF THE STUDY

1.1 Background information

Educators play a crucial role in the education of learners. This suggests that the quality of education in any country depends largely on the quality of its educators. In recognition of this fact, the National Education Policy Investigation (1993, p. 235), asserts that “educators are primary agents in education; the development of a quality teaching corps is thus a primary condition for education transformation”. This assertion holds that no educational system can rise above the quality of its educators regardless of how enlightened the aims might be, or how up-to-date and generous the equipment, or even how efficient the administrators, since the educators are the final interpreters of the curriculum (Remillard, 1999). In the light of the above, the mathematics education of any country can only be transformed to the extent that the knowledge possessed by its mathematics educators can achieve the desired transformation. This transformation is often measured by what the learners are able to do at the completion of a given course of study. In this regard, the educators’ role is to utilise the knowledge at their disposal to facilitate learners’ achievement in mathematics.

Educators’ knowledge is best described in terms of the pedagogical content knowledge. The term pedagogical content knowledge was first proposed by Shulman in 1986. This concept refers to educators’ interpretations and transformations of subject matter knowledge in the context of facilitating learners’ learning. In other words, it is the type of knowledge that is unique to educators and is based on the manner in which educators relate their subject matter knowledge to their pedagogical knowledge (Shulman, 1987; Marks, 1990; Cochram, DeRuiter & King, 1993; Griffin, Dodds & Rovengo, 1996). Pedagogical content knowledge is subject specific and concerns the teaching of specific topics. Shulman (1986) posited that expertise in teaching be evaluated in terms of the educators’ pedagogical content knowledge.

There is no universal agreement among scholars about what constitute educators' PCK. However, all scholars in mathematics education seem to agree on Shulman's two key elements, that is, knowledge of representations of subject matter, and understanding of specific learning difficulties and learners' misconceptions (Driel, Verloop, & De Vos, 1998). In this study, the mathematics educators' pedagogical content knowledge as conceptualised by the researcher is composed of:

- Educators' mathematical content knowledge.
- Educators' knowledge of learners' misconceptions and difficulties.
- Educators' knowledge of instructional strategies.

The importance of understanding the mathematics educators' PCK in the teaching of mathematics cannot be overemphasised. This is because the educators' PCK combines elements of the content (what is to be taught) with the pedagogy (which describes how it is to be taught).

Over the years, there has been the controversy between conceptualising educators' knowledge using subject matter knowledge, and using pedagogy. Since the 1970s, the pendulum swung to the other extreme when pedagogical knowledge, i.e. theories and methods of teaching were considered to be of paramount importance in the training and accreditation of educators. No wonder then that research studies on teaching around the 1970s and 1980s focused on generic skills and techniques rather than on content (Tsang & Rowland, 2005; McNamara, 1991). Meanwhile, the need to emphasise subject matter in the curriculum for educator training was well articulated by Dewey (1904/1964). Understanding educators' PCK can help in conceptualising how mathematics educators should be trained both in teacher education institutes and any other in-service programmes aimed at improving educators' expertise.

The widespread interest in and concern about educators' knowledge, what counts as mathematics educators' subject matter knowledge for teaching, and how this relates to learners' achievement have attracted a large number of research studies. Previous researchers (Begle, 1979; General Accounting Office, 1984; Goldhaber & Brewer, 2001;

Monk, 1994) on educators' knowledge did not find a strong relationship between educators' knowledge of mathematics and learners' achievement. These studies were characterised by the use of proxy variables such as courses taken, degrees attained or basic skills, as direct measures of educators' knowledge in models predicting learners' achievements. The limitation of these studies is that the researchers working in this tradition used proxy measures and no attempt was made to measure the complexity of educators' knowledge or the relationship between the formal mathematics the educator knew and what they taught in the classrooms (Hill, Rowan, & Ball, 2005). However, guided by Shulman's PCK, and employing strategies like interviews, educator tests and direct observations, researchers (Ball, Hill, & Brass, 2005; Ball, 1990; Even, 1993; Fennema, Franke, Carpenter & Carey, 1993; Hill, Rowan & Ball, 2005; Ma, 1999) started uncovering problems with educators' basic mathematical knowledge and arguing that there is a strong relationship between educators' knowledge and learners' achievement. Other scholars (Fennema & Franke, 1992; Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992) have identified the impact of educators' knowledge on instruction. From the above discussion, there is evidence that strong educator knowledge yields benefits for classroom instruction and learners' achievement (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008).

The present study, which investigates the role which the mathematics educators' pedagogical content knowledge plays in the teaching and learning of quadratic functions, was carried out in the context of South Africa, which, like any other developing country, is still battling with the problem of under-qualified and unqualified mathematics educators. According to Edusource (1993 in Adler, 1994), as at 1993, 72% of mathematics educators in African schools and 70% of science educators were under-qualified (having either less than Grade 12 or less than 3-years training). As at 2003, 40% of mathematics educators have not specialised in mathematics (Howie, 2003). This situation has been blamed on the apartheid policy in education which was characterised by low investment in human resource development particularly among blacks. Among other things, the policy included a racially fragmented teacher education system in which the majority of educators were trained in racially segregated colleges of education, most of which were isolated, small, poorly equipped and ineffective in provision of quality teacher education (National Education Policy Investigation, 1993). This circumstance has caused more "harm to mathematics education than to any other discipline" (Adler, 1994, p. 103). One of the reasons why mathematics was so badly affected could be that during the struggle, it was considered politically unacceptable for a black learner to study mathematics at school or college. Rather, one was expected study history and other subjects considered important in the struggle against apartheid (Graven, 2002).

1.2 Statement of problem

In South Africa there is continuous poor achievement of learners in mathematics in both national and international examinations. Nationally, for over ten years, mathematics has remained the only subject in the National School Certificate Examination where more than half of the candidates are unable to achieve the minimum score to pass. The performance of candidates in the last two years, 2008 and 2009 is shown in Table 1.1

Table 1.1 National Senior Certificate Examination results (2008 &2009)

Year	Number of candidates that sat for mathematics	Percentage that passed at 30% and above	Percentage that passed at 40% and above
2008	300008	45.4	29.7
2009	290630	46.0	29.4

(Department of Education, Matric Result 2008 & 2009).

Internationally, South African's performance in the Trends in Mathematics and Science Study (TIMSS) carried out in 1995, 1999, and 2003 was very poor. In each of the three assessment periods, South Africa was in the last position. In the 2003 assessment period, South Africa was outperformed by every country including the all African countries that participated in the assessment (TIMSS, 2003; Human Resources Research Council, 1998; Howie, 2003).

See Table 1.2 for a comparison between South Africa's performance and the performances of other African countries in the 2003 assessment period.

Table 1.2 Comparison of South Africa's average scale score with other African countries in TIMSS 2003

Country	Average age of learners	Average scale score
Tunisia	14.8	410
Egypt	14.4	406
Morocco	15.2	387
Botswana	15.1	366
Ghana	15.5	276
South Africa	15.1	264

(Reddy, 2006).

Table 1.2 above shows that South Africa was outperformed by other African countries securing only 264 points out of a total of 800 points. South Africa's score was also significantly lower than the international average score of 467 points. As a result of the underachievement of learners, South Africa is experiencing a shortage of professionals like accountants, engineers, financial experts, and mathematics educators, to mention but a few. This situation necessitates that the government supplement the shortages with expatriate workers; a step which has huge social and economic implications.

The situation highlighted above points to the need to improve learners' achievement in mathematics in South Africa. Numerous variables interact in determining learners' achievement, the quality of educator teaching being just one. In recognition of the significance of educators' knowledge in learners' achievement in mathematics, the government and other stake holders have tried several strategies to improve the quality of educators. The strategies include organising workshops for mathematics educators and retraining some mathematics educators through the ACE (Advanced Certificate in Education) programme. However, the continued under-achievement of learners in both national and international examinations indicates that this problem is far from solved. The need to improve the mathematics achievement of learners' in South Africa demands a thorough investigation into how the educators' knowledge, in particular the educators' pedagogical content knowledge influences the learning of each concept or topic in mathematics.

Taylor and Vinjevold, (1999) argue that South African educators need to develop conceptual understanding at a level higher than what they posses. In this perspective, what the educators need is to develop adequate pedagogical content knowledge. Research (Viri, 2003; Halim & Meerah, 2002) indicates that educators' knowledge especially educators' pedagogical content knowledge influences learners' achievement in mathematics. The studies show that the extent to which an educator possesses each component of pedagogical content knowledge determines the quality of instruction and subsequently affects what learners learn.

Although research has shown that what educators' know determines what is done in the classroom and hence learners' achievement in mathematics, what is yet to be clarified is how educators' PCK influences learners' understanding of the various mathematical concepts in the South Africa context. It is this vacuum that this research intends to fill with reference to quadratic functions. The researcher investigates the educators' pedagogical content knowledge on quadratic functions because of the central role the concept of functions (and specifically quadratic functions) plays in the secondary school curriculum. Quadratic functions connect a wide range of other concepts including the concept of graphs and all the concepts involved in graphing; modelling; quadratic equations, and the concepts involved in solving quadratic equations (Leinhardt, Zaslavsky & Stein, 1990).

1.3 Purpose of the study

The purpose of the study is to investigate to what extent educators' pedagogical content knowledge affects learners' achievement in quadratic functions. Educators' PCK will be investigated in terms of the following three components of PCK: the educators' subject matter knowledge of quadratic functions, educators' knowledge of learners' conceptions and misconceptions in quadratic functions, and the educators' knowledge of strategies for the teaching of quadratic functions. The educators' PCK is the independent variable while the three components of PCK are the sub-variables of the independent variable. This research will test the effect of the independent variable (PCK) on the dependent variable (learners' achievements in quadratic functions). The study therefore extends the knowledge of pedagogical content knowledge to the teaching and learning of quadratic functions.

1.4 Research questions

Based on the purpose of the study in section 1.3, the main research question is: How does educators' PCK affect learners' achievement in quadratic functions?

In order to explore the main question in depth, the following sub-research questions will be explored:

1. What are the learners' main misconceptions in quadratic functions?

2. Does educators' MCK affect learners' achievement in quadratic functions?

3. Does educators' knowledge of learners' conceptions and misconceptions affect learners' achievement in quadratic functions?

4. Does educators' knowledge of strategies for teaching quadratic functions affect learners' achievement in quadratic functions?

1.5 Significance of study

The study will contribute to the mathematics education literature by extending the concept of PCK to the teaching and learning of quadratic functions. It will open up new possibilities for improving the teaching of quadratic functions and learners' achievement in quadratic functions. One of the strategies used in the study whereby educators were asked to predict both their learners' responses to question items and learners' reason(s) for their answers as a means of investigating educators' PCK of their understanding of their learners' thinking, may prove to be useful in both in-service and pre-service educators' training. When educators think alone or in groups about possible learners' responses to a question, they tend to better understand learners' conceptions about the concept. The methodology employed in the study where a special kind of multiple-choice items were used to assess learners' achievements, suggests a means whereby higher order thinking and conceptual understanding can be assessed. This may be profitable to both researchers and educators. The non-standard tasks used in this research might be useful instructional activities that promote deep mathematical understanding. These tasks may be useful in guiding professional development of mathematics educators with reference to quadratic functions.

1.6 Delineation

There are many ways to measure educators' PCK. In this study, the researcher does not employ classroom observation as a means of investigating the educators' PCK. Educators' qualifications are neither taken as an indication of their mathematical content knowledge or measure of their PCK, nor does the researcher use an instrument where the educators give their own opinions of their PCK. Rather, the researcher uses the learners' achievement test to investigate the educators' MCK of quadratic function. A more direct way of investigating the MCK of mathematics education would be to probe into their knowledge of mathematics in the context of the school mathematics curriculum that they deliberate in their course of teaching (Tsang & Rowland, 2005). The same questions in the learners' test are also used to investigate the educators' understanding of their learners' conceptions and misconceptions of quadratic functions; and the educators' knowledge of strategies in teaching quadratic functions.

1.7 Assumptions

The research was carried out on the assumption that what educators know determines what they do in their classrooms and consequently what their learners learn.

1.8 Layout of study

Chapter 1. In this chapter the researcher discusses the problem that gave rise to the study including the research questions that guided the research. Also presented is an overview of what the reader will expect in the entire study.

Chapter 2. This chapter focuses on a specific kind of educator knowledge namely the mathematics educators' pedagogical content knowledge and the theoretical framework of the study. It includes discussions on the meaning and representations of quadratic functions, and strategies that can improve learners' understanding of quadratic functions.

Chapter 3. The methodology followed in the research is discussed in this chapter. This includes discussions on research design, sample selection method, instruments for data collection, procedure followed, data analyses technique and ethical considerations.

Chapter 4. Data collected from the educators' questionnaire, the learners' questionnaire and the educators' interviews are analysed and interpreted in this chapter. The result is used to answer the research questions.

Chapter 5. A summary of the research and the findings made are presented, followed by the implications that emanate from the findings. Recommendations are made, including suggestions for further investigation.

1.9 Conclusion

In this chapter, the background of the study was established. The problem that led to the research was discussed, and the purpose and significance of the study briefly stated. The research questions which the study intends to answer were posed. Assumptions made were discussed, and finally, the layout of the study was presented. In view of the role of educators' knowledge in the education of learners and in recognition of the urgent need to improve on the achievement of learners in mathematics; this study explores the effect of educators' PCK on the achievement of learners in quadratic functions. It is the researchers' belief that the findings from the study will contribute to improving the teaching of quadratic functions if the recommendations which emanate from the study are implemented.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

In this chapter the review of related literature is presented with the aim of gaining insight into the role of pedagogical content knowledge in the learning of quadratic functions. The researcher identifies the pedagogical content knowledge as a special kind of educators' knowledge which is needed for effective teaching of mathematics. An analysis of several scholars' conceptions of pedagogical content knowledge is done, from which the researcher draws the theoretical framework on which the study is based. The literature reviewed includes among other concepts: educators' subject matter knowledge, learners' misconceptions in quadratic functions, educators' knowledge of learners' misconceptions in quadratic functions, and educators' knowledge of strategies for teaching quadratic functions. Also discussed is the function concept, knowledge components of which understanding of quadratic functions can be measured, and ways of improving the teaching and learning of quadratic functions.

2.2 Educators' knowledge

"Teachers' knowledge is a large, integrated, functioning system which is comprised of many parts with each part difficult to isolate" (Fennema & Franke, 1992, p. 142). Many scholars have speculated on the components of educators' knowledge, including Lee Shulman's (1987) who lists seven categories of knowledge that constitutes the knowledge base for teaching. The seven categories are: content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge (PCK), and knowledge of learners, knowledge of contexts, and knowledge of educational ends.

2.2.1 Shulman's conception of PCK

The pedagogical content knowledge introduced by Shulman (1986) draws attention to a special kind of educators' knowledge that is specific to teaching and integrates the subject matter knowledge and knowledge of pedagogy; it refers to educators' interpretation of subject matter knowledge in the context of facilitating learners' learning. "Pedagogical content knowledge is therefore the amalgam of content and pedagogy that is the province of educators" (Shulman, 1987, p. 8). In other words, it is the type of knowledge that is unique to educators and is based on the manner in which educators relate their subject matter knowledge (what they know about what they teach) to their pedagogical knowledge (what they know about teaching). (Shulman, 1987 Marks, 1991; Cochram, et al., 1993; Griffin, et al, 1996). Since PCK is specific to teaching it therefore differentiates expert educators in a subject area from subject area experts. For instance, mathematics educators differ from mathematicians not necessarily in quantity or quality of subject matter knowledge, but in how that knowledge is organised and used. The differentiation is reflected in the

> capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by learners (Shuman, 1987, p. 15).

In other words, an experienced mathematics educator's knowledge of mathematics is organised from a teaching perspective and is used as a basis for helping learners to understand specific concepts. A mathematician's knowledge on the other hand, is organised from a research perspective and is used as a basis for developing new knowledge in the field (Fennema & Franke, 1992). The key elements in Shulman's conception of PCK are:

- Knowledge of ways of representing and formulating the subject matter that makes it comprehensible to the learners.
- Knowledge of strategies most likely to be fruitful in reorganising the understanding of learners.
- Knowledge of learners' preconceptions and misconceptions about the topic.

Shulman (1986 & 1987) suggests that teaching expertise be described and evaluated in terms of pedagogical content knowledge. Educators who lack highly developed PCK have trouble in designing appropriate learning tasks and progressions, presenting explanations and posing higher order questions, recognising common performance errors, and providing appropriate feed back. On the other hand, educators with rich PCK are capable of accommodating learners with diverse levels of skills and experience, transforming and presenting content in multiple fashions, varying instructional strategies, detecting and correcting performance errors (Griffery & Husner, 1991; Rovegno, 1994 & O'Sullivan 1996).

2.2.2 Other conceptions of PCK

Since Shulman introduced the concept of PCK about two decades ago, there have been a number of studies on the subject. Various scholars have elaborated on Shulman's conceptualisation and of pedagogical content knowledge in terms of what they include or emphasise. All of them seem to have adopted the two key elements of Schulman's PCK namely, knowledge of comprehensible representations of subject matter, and understanding of content-related learning difficulties and each of them has extended the concept by including in PCK some categories of knowledge distinct in Shulman's knowledge base for teaching (Driel, et al., 1998). For instance, Grossman (1990) developed an expanded definition of pedagogical content knowledge. Her definition is based on four central components: knowledge of learners' understanding, knowledge of curriculum, knowledge of instructional strategies, and knowledge of context.

Marks (1990), offers a conception of pedagogical content knowledge that consists of four components: subject matter for instructional purposes, learners' understanding of the subject matter, media for instruction in the subject matter, and instructional processes for the subject matter. These four areas are highly integrated. With this conception and model, Marks broadens the usual interpretation of pedagogical content knowledge (as an adaptation of subject matter knowledge for pedagogical process) and how it is generated. The emphasis is on the application of general pedagogical principles to particular subject matter areas. From this broadened interpretation, PCK is derived from the following: roots from subject matter knowledge, primarily from general pedagogy, and emanates more or less equally from subject matter knowledge and general pedagogical knowledge or from previous construction of pedagogical content knowledge (Marks, 1990).

Based on an explicit constructivist view of teaching, Cochran et al., (1993) proposed a modification of Shulman's concept of PCK. They renamed pedagogical content knowledge pedagogical context *knowing* (PCKg) which they defined as educators' integrated understanding of four components of pedagogy, subject matter, content, learners' characteristics, and mental context of learning. In their model, PCKg is generated as a synthesis from the simultaneous development of these four components. A strong PCKg therefore enables educators to use their understanding to create teaching strategies for teaching particular content in a discipline in a way that enables particular learners to construct useful understandings in a given context.

Table 2.1 Summary of conceptualisations of PCK of some scholars

	Knowledge of:						
Scholars	Subject Matter	Representations and strategies	Learner learning and conceptions	General pedagogy	Curriculum and media	Context	Purpose
Shulman (1987)	a	PCK	PCK	a	a	a	a
Grossman (1990)	a	PCK	PCK	a	PCK	a	PCK
Marks (1990)	PCK	PCK	PCK	b	PCK	b	b
Cochran, et al. (1993)	PCKg	b	PCKg	PCKg	b	PCKg	b
Fernandez-Balboa & Stiehl (1995)	PCK	PCK	PCK	b	b	PCK	PCK

[a] Distinct category in knowledge base for teaching
[b] Not explicitly discussed

Driel, et al., (1998:674).

In summary, the above discussion is not exhaustive; however, it shows that there is no universally accepted conceptualisation of PCK.

According to Gess-Newsome in Jong (2003:374) all the different views of PCK can be categorised as either integrative or transformative. In the integrative view, knowledge of teaching is merely the integration between other forms of educator knowledge; hence, PCK is a mixture. In other words, PCK does not really exist as its own domain and teaching is seen as an act of integrating knowledge of subject, pedagogy, and context. When teaching in the classroom, knowledge from all these domains are integrated by the educator to create effective learning opportunities. Traditional teacher education programmes organised in separate courses of subject matter, pedagogy, and practice, often follow this model of educator knowledge. The transformative view holds that different forms of educator knowledge (subject matter knowledge, pedagogical and contextual knowledge) are transformed into a new form of knowledge (PCK is a compound). In the transformative model PCK is the synthesis of the knowledge needed in order to be an effective educator. This model supports teacher education programs containing purposefully integrated courses in which the prospective educators quickly develop needed skills and knowledge. "Both views can be considered as the two ends of the PCK spectrum" (Jong, 2003, p. 374).

2.3 Components of PCK as used in the study

Although there are different conceptions of PCK found in the literature, the researcher, after analysing the views of different scholars on the subject, toes the line that in mathematics education the pedagogical content knowledge of the educator is primarily composed of:

- Educators' subject matter knowledge of mathematics (mathematical content knowledge);
- Educators' knowledge of learners conceptions, and misconceptions; and
- Educators' knowledge of strategies for teaching the subject matter.

This study investigates the educators' pedagogical content knowledge based on these components. The constituents of the mathematics educators' pedagogical content knowledge as conceptualised in this study is shown in the diagram below.

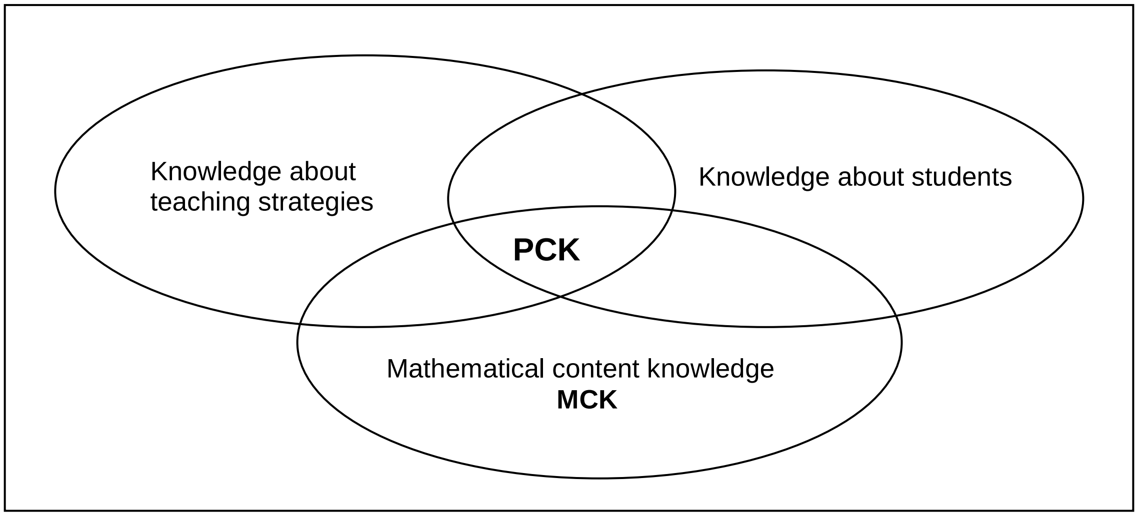


Figure 2.1 Components of PCK used in the study.

What follows is a discussion of the components of pedagogical content knowledge identified in 2.3.

2.3.1 Subject Matter Knowledge (Mathematical Content Knowledge)

"Subject matter content knowledge consists of an explanatory framework in the discipline and the rules of evidence and proof within the discipline" (Manouchehri 1997, p. 199). Subject matter knowledge is structured into substantive and syntactic areas where substantive content knowledge refers to the concepts, principles, laws, and models in a particular content area of science. Syntactic content knowledge of a discipline is the set of ways in which truth or falsehood, validity or invalidity is established (Schwab in Shulman, 1986). An educator with both synthetic and substantive knowledge will not only be capable of defining for learners the acceptable truths in a domain but will also be able to explain why it is worth knowing and how it relates to other proposition both within the discipline and without, both in theory and in practice. Both kinds of subject matter knowledge are needed for educators' development of PCK. The subject matter content knowledge of prospective mathematics educators is primarily acquired during disciplinary education (Jong, 2003).

Many scholars stress the need for educators to possess rich subject matter knowledge. Ball, Lubienski and Mewborn (2001) maintain that reform ideas, managing the challenges of change, using new curriculum materials, enacting new practices, and teaching new content all depended on educators' knowledge of mathematics. Wu, (2004, p. 6) posits that "good mathematics instruction requires good educators, and good teachers are those with good pedagogical content knowledge" who, most often, are predominately those educators with good subject matter content knowledge. In other words, it is not possible for an educator to have pedagogical content knowledge without a firm command of subject matter knowledge. Ma (1999) asserts that effective educators must have a profound understanding of mathematics. A profound understanding according to Ma is deep, vast, and thorough. Ma also highlights that for one to generate a representation, one should first know what to represent.

The research of Ma (1999) supports the idea that mathematical content knowledge (MCK) determines or at least influences what is done in the classroom. In her study of elementary mathematics educators in both China and the United States, Ma (1999) indicated that Chinese educators were able to generate multiple representations and also use variety of models of division by fractions which were pedagogically effective. These seemed to have emanated from the Chinese educators' firm knowledge of the subject matter. In contradistinction, USA mathematics educators were unable to clearly represent the operation and did not explain its meaning correctly. The study above suggests that for educators to have pedagogically powerful representation of a topic, they must first have deep understanding of the subject, and this cannot be substituted by anything else. Kahan, Copper, and Bethea (2003, p. 235) indicate that "educators' MCK play a significant role in the quality of teaching, the lesson planning process, and in other teaching processes as well".

Capraro, Capraro, Parker, Kulm and Raulerson (2005) indicate that mathematically competent pre-service educators exhibit progressive pedagogical content knowledge as they are exposed to mathematics pedagogy during their mathematics method course. Other studies (Even, 1993; Halim & Meerah, 2002; Driel, Veelop & De Vos 1998;

Tsangaridou, 2002; Manouchehri, 1997; Viiri, 2003; Carpenter, Fennema, Petterson and Carey, 1993) on the influence of subject matter knowledge on the PCK of pre-service, novice, and expert educators, reveal that educators' subject matter content knowledge goes a long way to determining the level of educators' PCK.

Many studies have shown the role of educators' MCK in instruction. Shroyer (1981 in Ball 2001) studied how junior high school mathematics educators cope with learners' difficulties or unusual responses and found that the educators with weaker mathematics background have more difficulties generating alternative responses to these critical moments. Hill, Rowan and Ball (2005) found that educators' mathematical knowledge significantly relate to learners' achievement gains in both first and third grades. Their study was conducted with educators and learners from 115 elementary schools during the 2000 – 2001 and 2003 – 2004 school years in the USA. From the above, it is evident that the MCK that a teacher possesses plays a vital role in their teaching of the subject matter; however, subject matter alone does not guarantee effectiveness of teaching.

Borko, et al., (1992) indicate that a pre-service educator, who had strong knowledge of mathematics and was well prepared, had significant difficulties in explaining the division of a fraction when the educator was asked to do so by a learner. This instance and other literature already cited suggest that there is other knowledge used in teaching of which MCK is one.

2.3.2 Knowledge of learners' misconceptions

For the purpose of this study, misconception is defined as an incorrect feature of learners' knowledge that is repeatable and explicit. Some of these features may or may not be as a result of earlier learning. In most cases, the source of the misconception is the tendency of learners to over-generalise pervious knowledge that was essentially correct in an earlier domain, to an extended domain where it is no longer correct (Leinhardt, et al., 1990; Alwyn, 1989). This is because children not only interpret knowledge but also group together concepts that are interrelated. This is called schema. Schemas that were acquired early in the learners' learning and are well developed and highly resistant to change. This seems to be the reason why different curriculum sequences produce different misconceptions (Alwyn, 1989). Some misconceptions stem from intuitions; features of learners' knowledge that arise most commonly from everyday experience. In the more advanced learner they may involve a mixture of everyday and deeply understood formal knowledge. In general, intuitions seem to exist prior to specific formal instruction. For example, learners' tendencies to interpret graphs iconically may be traced to their intuitions regarding picture reading. Other misconceptions can be interpreted as the result of incomplete formal learning. An example of this type of misconception is learners' tendencies to readily see a one-to-one correspondence as a function but not also recognising a many-to-one correspondence as a function (Leinhardt, et al 1990). It is important to note that when there is a difficulty, it does not necessarily mean that a misconception is the reason for the difficulty. Rather, difficulties imply that there is something about the task that makes it especially difficult (Leinhardt, et al 1990).

Knowledge of learners' thinking and misconceptions of a particular topic helps educators to explain learners' actions and also to plan effective instruction. Educators need to work with learners' existing conceptions and prior knowledge. Learners' mistakes can provide valuable insights into the learners' knowledge. Researchers (Hope and Townsend, 1983; & Jong, 1992 in Halim et al., 2002) show that experienced educators who have rich subject matter knowledge but fail to take into account their learners' thinking about the subject matter often have difficulty in teaching the content. Educators' experience can be a source of difficulties in teaching if they do not consider learners' views of the topic (Halim et al. 2002). These findings support the assertion generally found in literature that good knowledge of subject matter alone is not enough for effective teaching. A lack of knowledge of learners' misconceptions could be the result of poor subject matter knowledge of the educator. Educators who have the same misconceptions as the learners are unlikely to be aware of the learners' misconceptions (Berg & Brouwer, 1991; Smith & Neale, 1991).

By getting acquainted with the learners' specific conceptions and ways learners' reason, prospective educators may start to restructure their content knowledge into a form that enables productive communication with their learners. In addition to field-based experiences, Jong (2003) noted that prospective educators may benefit from studying learners' preconceptions with respect to specific topics during educators' education courses, and comparing these preconceptions in relation to their own conceptions. Such activities may stimulate prospective teachers to generate transformations of content knowledge and topic-specific teaching strategies.

2.3.3 Knowledge of representations and instructional strategies

We use different kinds of instructional strategies, representations, and activities in teaching mathematics. Knowledge of specific strategies involves knowledge of ways of representing specific concepts to facilitate learning. Representations include illustrations, examples, models, and analogies. "Each analogy for instance has conceptual advantage and disadvantage with respect to others" (Treagust, 2007. p. 379). PCK in this area includes knowledge of the relative strengths and weaknesses of particular representations. Activities can be used to help learners understand specific concepts or relationships. Examples of such activities are demonstrations, simulations, investigations or even experimentations. For a representation to be powerful or more comprehensible to the learners the educator must know the learners' conceptions about a particular topic and also the possible difficulties the learners will come across with the topic. Representations must be clearly linked and the relationship among concepts must also be clear.

Many studies have highlighted the nature of this component of PCK. Hashweh (1987) found that educators who teach outside their field of expertise provide incorrect and misleading representations such as analogies and examples which depict the educators' misconceptions. A similar finding by Tobin, Tippins, and Gallard (1994) indicated that educators gave explanations and analogies which reinforced the misconceptions that learners were having when they were teaching outside their areas of specialisation. Magnusson, Krajcik, & Borko, (1999) argue that the knowledge of strategies is dependent on the educator's subject matter knowledge about that particular concept. This may not always be true as subject matter knowledge does not guarantee that it will become transformed into representations that will help learners' understand targeted concepts or that educators will be able to decide when it is pedagogically best to use particular representations. In a particular topic, the representation component of PCK seems also to depend on previous planning, teaching, and reflecting both in action and on action (Halim et al., 2002:217). Clermont, Borko, and Krajcik (1994) show that experienced educators know more variations of a demonstration for teaching than novice educators. However being experienced does not guarantee that one possesses effective teaching strategies. After studying eight elementary school mathematics educators, Marks (1991) also reported that two of the educators who have taught for 30 and 18 years respectively, demonstrated little evidence of PCK. This shows that experience alone does not guarantee that an educator will possess rich PCK.

In conclusion, I agree with Griffin, Doods, and Rovengno (1996) that developing rich PCK is a task for highly committed professionals who are willing to address their teaching practice thoughtfully and make substantial changes over time. These educators should be able to reflect both in action and on action. In addition they should strive to integrate everything they know in order to help learners learn. In-service educators as well as pre-service educators can accomplish these challenging professional tasks by using comprehensive teaching models (Mohr & Townsend, 2002, p. 2). Comprehensive teaching models are tools for teaching that require educators to consider learning theory, learning strategies, and assessment (Mohr & Townsend, 2002).

Having gained understanding on the meaning of PCK, the researcher now discusses the function concept in general and quadratic functions in particular since in this study, the concept of quadratic function was used to show how educators' pedagogical content knowledge relates to teaching and learning situations.

2.4 The function concept

2.4.1 Evolution of the function concept

According to Kleiner (1987) the evolution of the concept of function goes back 4000 years: 3700 of these consist in anticipation. The idea evolved over close to 300 years in intimate connection with problems in calculus and analysis. The notion of function in explicit form did not emerge until the beginning of the 18th century, although implicit manifestations of the concept date back to 200 B.C. The main reasons why the function concept did not emerge earlier were:

- Lack of algebraic prerequisite – the coming to terms with the continuum of real numbers, and the development of symbolic notations;
- Lack of motivation. Ancient mathematicians saw no need to define an abstract notion of function unless one has many examples from which to abstract.

In the course of about two hundred years (1450 – 1650), there occurred a number of developments that were fundamental to the rise of the function concept. These include:

- Extension of the concept of number to include real and (to some extent) even complex numbers,
- The creation of symbolic algebra,
- The study of motion as a central problem of science, and
- The wedding of algebra and geometry (Kleiner 1987).

This evolution of the function concept above can be interpreted as a tug of war between two elements, two mental images: the geometric (expressed in the form of curve) and the algebraic (expressed as formula – first finite and later allowing infinitely many terms, the so called analytic expression). Subsequently, a third element emerged on the scene namely the logical definition of function as a correspondence (with a mental image of an input–output machine). In the wake of this development, the geometric concept of function was gradually abandoned and a new tug of war ensued (and is, in one form or another still with us today). This is between this novel logical (abstract, synthetic, postulational) concept of function and the algebraic (concrete, analytic, constructive) conception (Kliener, 1987).

According to Markovits, Eyton & Bruckheiner (1986), the function concept has been included in school mathematics texts for some hundred years and has undergone considerable change over time. Each of the definitions was a reflection of the function concept in use in higher mathematics. The textbooks from the end of the 19th century until the middle of the 20th century perceived function as a change or as a variable depending on other variables. For instance, in the National Council of Teachers of Mathematics NCTM (1932), a function is defined as "any mathematical expression containing a variable x, that has a definite value when a number is substituted for *x*, is a function of *x*" (Height 1968 in Markovits, Eyton & Bruckheiner, 1986:18). From the above definition, the function concept is associated with numbers only.

However, with the modern conception of function (Dirichlet-Bourbaki's concept of function) a function is no longer associated with numbers, nor is the (dependant) variable identifiable with functions. In higher mathematics a function is now perceived as a special type of subset of the Cartesian product of two sets (Markovits, et al., 1986). Kieran (1992, p. 408) defines a function as "a relation between two sets (not necessarily numerical) or members of he same set, such that each member of the domain has only one image". The modern definition (set theoretic) though abstract, also has an influence on how function is now defined in school mathematics. In most school curricula, function is now defined by two sets A and B (not necessarily numerical), with a rule which assigns exactly one member of B to each member of A (Leindardt, et al, 1990). In South Africa for instance, the Grade 12 Classroom Mathematics (2007:42) defines a function as follows:

> A function *f* is a relationship between two sets A and B where every element of A (the input set) is mapped to only one element of B (the output set). (Laridon, Barnes, Jawurek, Kitto, Pike, Myburgh, Rhodes-Houghton, Scheiber, Sigabi, 2007:42).

From the above, a function is composed of the domain (the input set), the range (the output set) and the rule of correspondence. The value of *f* when the input variable is "*a*" is indicated by the symbol $f(a)$. $f(a)$ is read as *f* of 'a' or "the value of *f* at 'a'. Functions can also be denoted with any letters for example *g, h, j, k.*

From the modern conception of function, it follows that a one-to-many correspondence is not accepted as a function while a many-to-one correspondence could be a function. The modern conception of function is more abstract than the old conception, and the relation between variables which was emphasised in the old conception is not explicitly stated in the new definition (the rule of correspondence may be completely arbitrary).

For a proper understanding of the concept of function, first, learners should realise that a function is composed of three sub-concepts: the domain, range, and rule of correspondence. Secondly, they should realise that function can be represented in several forms, such as arrow diagrams, verbal, graphical, and algebraic representations. They should also learn that the function can be represented by each of the above representations. In so doing, they should learn to translate a given function from one representation to another.

2.4.2 Notion of the function concept in children

According to Leinhardt, et al., (1990) children possess intuitive ideas about functional relationships that have been developed from their perceptions of the natural phenomena that occur around them, for example change of temperature over time. However, unlike the formal definition of function which is algebraic in spirit, these childhood notions of function are based on an implicit sense of variables that are characterised by being concrete, dynamic, and continuous.

On a daily basis, people analyse information using algebraic thinking, often unaware of doing so. Variables and functions are constantly used by young children. For instance, when a second grader tells his parents the prices at the school store, he/she will say something like the following, ‘The small notebooks are R3.50, the large notebook are R6.00, pencils are R2 and erasers are R1.00 each”. The boy/girl is spontaneously interpreting price as a function to be evaluated on the product. For the function p above, we have:

- Domain : The set of products available at the school store
- Range : The set of prices
- Definition : P (product) = price, and
- Example : P (eraser) = R1.00

If we ask the learner whether there is anything at the school store that costs R200, he/she might tell us, "No nothing at the school store costs more than R50.00. By saying so, he/she is suggesting an upper bound for the range of the function (Devidenko, 1997:145).

2.4.3 Pre-algebraic notion of functions in children

Although learners are formally introduced to functions in their algebra classes, they already have considerable intuitive experience with function machines and other input-output representations in their arithmetic classes. Their earlier work with simple formulas such as $P = 4xS$ for the perimeter of a square, also provides a basis for understanding functions in their algebra classes. Algebraic thinking begins to develop in primary grades when children become aware of general relationships in arithmetic procedures, spatial patterns and number sequence. The role played by pre-algebra is to develop the more primitive concrete pre-concepts that are necessary for the development of the higher, more abstract concepts. Because the pre-concepts do not become redundant later on but continue to support the formal concepts, special care must be taken to develop the notions in a correct and special way (Linchevski, 1995).

2.4.4 Multiple representations of functions

"Representations are useful tools for learning and doing mathematics as well as communicating and making connections" (NCTM, 2000, p. 81). Multiple representations involve the representation of concepts or procedures in more than one format. Data or concepts displayed differently communicate differently. Each representation emphasises and suppresses various aspect of a concept; different representations add connections and perspectives that others miss. (Piez, & Voxman, 1997). Representing a concept in different forms helps in the development of better insight and understanding of the problem situation and increases comprehension about mathematics. Multiple representations therefore extend learners' understanding of the concept of quadratic function, and shed light on an idea not fully understood in another form.

Verstappen (1992) distinguishes three categories of recording functional relationships using mathematical language:

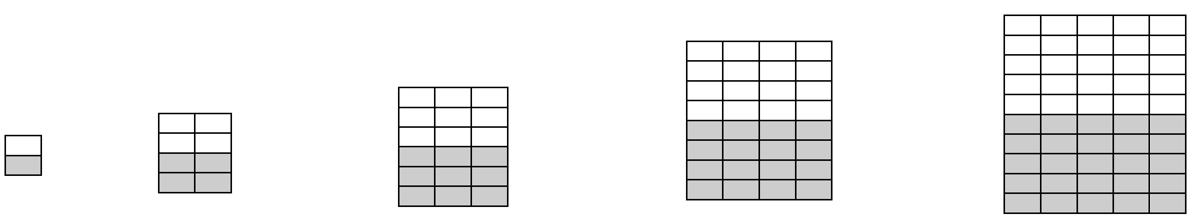
- Geometric – schemes, diagram, histogram, graphs, drawings
- Arithmetic – numbers, tables, ordered pairs, and
- Algebraic – letter symbols, formulas, mappings.

Kieran (1992) asserts that since functions are usually introduced in algebra classes by means of a formal set – theoretic definition, that is, as a many to-one correspondence between elements of domain and range, the representations that are initially generally involved are mapping, diagrams, equations, and ordered pairs. These representations are then usually extended to include tables of values and Cartesian graphs.
The researcher will use the function defined in the growing pattern in figure 2.2 to explain the multiple representations of functions which include:

- Verbal representation (in words)
- Tabular representation (numerical)
- Symbolic representation (formula) and
- Graphical representation

2.4.4.1 The context

Below is a growing pattern.



1 **2** **3** **4** **5**
Figure 2.2 Function in the context of a growing pattern

2.4.4.2 In words (verbal representation)

We can describe the growing patterns above in words as follows: *the number of blocks is always twice the square of the figure number*. Verbalising a given situation or relationship correctly shows good understanding of the context. Our field (mathematics) is a language in itself hence the ability to use words to accurately describe a formula, a graph or a table is highly recommended.

2.4.4 3 In a table (numerical)

A table is another way to describe a function. The growing pattern in figure 2.2 above can be represented numerically (i.e. in tabular form) as in table 2.2 below.

Table 2.2 Tabular representation of the growing pattern

Figure number (n)	1	2	3	4	5
Number of blocks (N_b)	2	8	18	32	50

2.4.4.4 In Symbolic form (formula**)**

Mathematics is a language spoken in symbols. Mathematical terms are carefully defined, and symbols are used to interpret mathematical statements. Symbols are thus a powerful way of communicating mathematics. Symbols communicate ideas in shorthand. The growing pattern can be represented in symbolic form as

$$N_b = 2n^2$$

(where N_b is the number of blocks, and “n” is the figure number)

2.4.4.5 Graphical form

Graphs and charts represent data and other mathematical relationships visually. Graphs are used to describe the trends in data. The ongoing pattern can be represented graphically as:

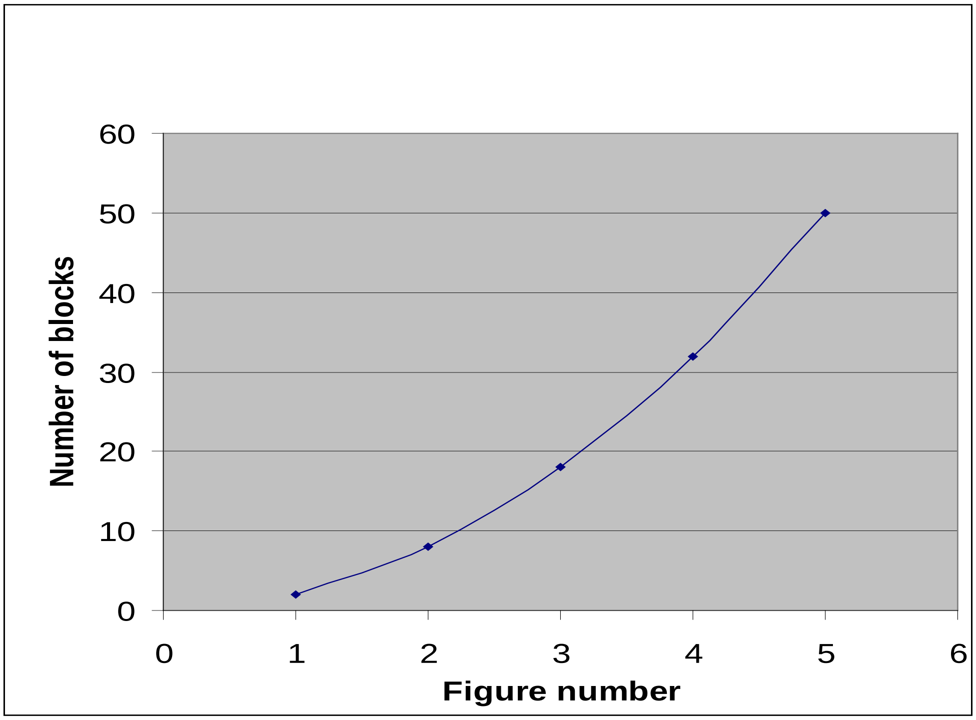


Figure 2.3 Graphical representation of the growing pattern

Learners often need to see many different representations before they can make proper sense of the concept, and build up a relational understanding of the concept. Taking into consideration the individual differences, some learners will learn better when they see pictures or graphs (visual learners). Some learners may need concrete models (tactile learners). Some may prefer algebraic or symbolic representations (abstract learners), while other learners need more than one form of representation for a clearer understanding. This has pedagogical implications as every mathematics classroom consists of different individuals with different ways of looking at things.

Multiple representations (verbal, symbolic, tabular, and graphical) of quadratic functions enable learners to have a thorough understanding of the concept. The different representations emphasise various facets of a quadratic function as shown in figure 2.4 below. Learners therefore have to learn to translate from one representation to another in order to have a comprehensive view of quadratic function. As they move from one representation to another, they discover new aspects of the concept. Also, as they analyse the different representations, they stand a better chance to decide which representation provides better and more useful information. Essentially, they can see how these modes of representation enhance each other. For instance, after using graphical and symbolic representations to solve quadratic functions, learners may discover that symbolic representations are more accurate.

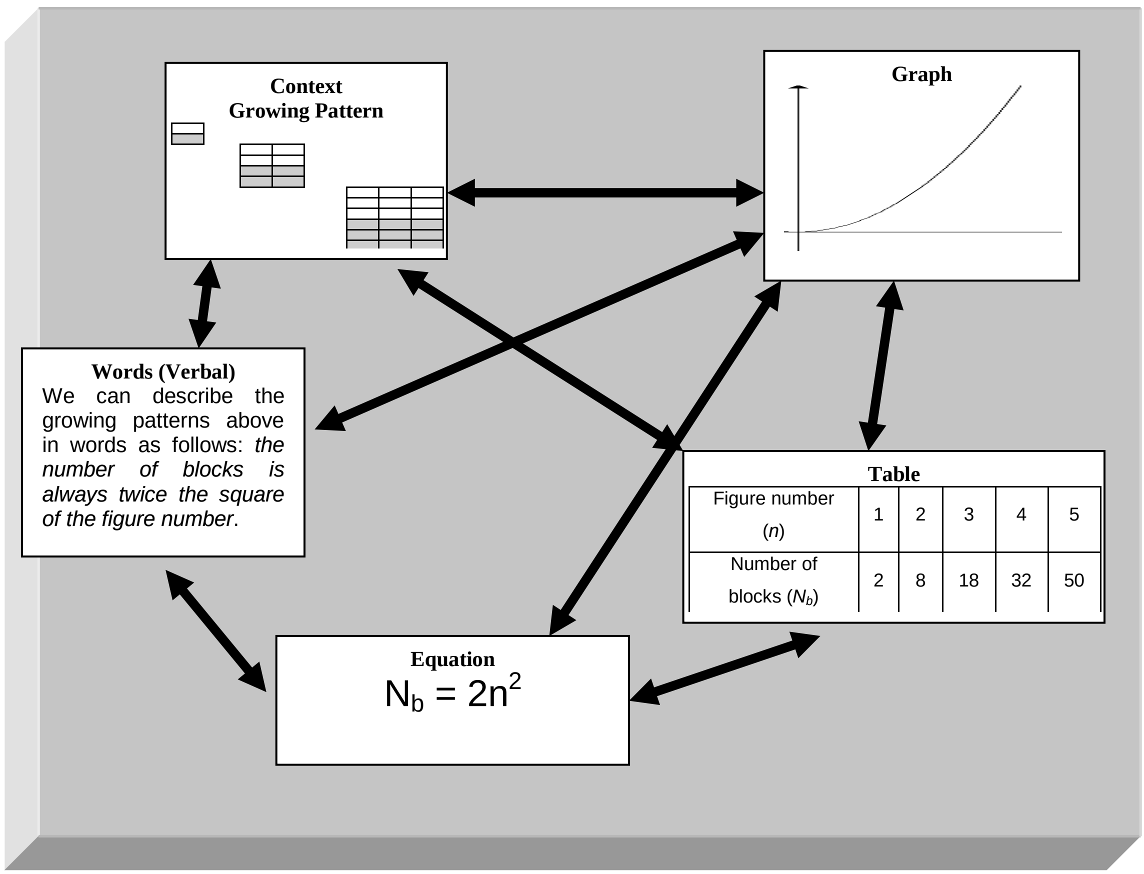


Figure 2.4 Multiple representations of the growing pattern.

2.5 Functions in the South African curriculum

Although the explicit study of the function concept is dealt with in grade 12, grades 10 and 11 learners are required to study various families of functions including:

- Linear functions
- Hyperbolic functions

- Quadratic functions
- Exponential functions, and
- Trigonometric functions

(DoE, 2007, National Curriculum Statement Grade 10 – 12. 2007).

The learners are required to recognise the relationships between variables in terms of numerical, graphical, verbal and symbolic representations and to convert flexibly between these representations (tables, graphs, words and formula). They are also required to generate as many graphs as necessary, initially by means of point-by point plotting supported by available technology, to make conjectures and hence to generalise the effects of the given parameters in the functions. In the process, they identify the characteristics listed below and hence use applicable characteristics to sketch graphs of functions.

- Domain and range;
- Intercepts with axes;
- Turning points, minima and maxima;
- Asymptotes;
- Shape and symmetry;
- Periodicity and amplitude;
- Average gradient (average rate of change)
- Intervals on which the function is increasing/decreasing;
- The discrete or continuous nature of graph.

(DoE, 2007, National Curriculum Statement Grade 10 – 12. 2007).

2.6 The quadratic function

Although there are many families of functions that are required to be studied in the curriculum, this research was carried out by focusing on the quadratic function. The standard form of a quadratic function is the form $y = ax^2 + bx + c$ (where $a \neq 0$ and a, b and c are constants). Other forms of representations (symbolic) are the canonical form: $y = a(x - p)^2 + q$, and the multiplicative form: $y = a(x - x_1)(x - x_2)$. Each form directly reveals some graphical information related to the location of special points of the parabola. The standard form indicates the location of the *y*-intercept (0, c), the canonical form indicates the location of the parabola's vertex (turning point) $V(p, q)$, while the multiplicative form discloses the location of the *x*-intercept (x_1; 0) and (x_2; 0) (Zaslasky, 1997).

2.6.1. The effects of the parameters *a*, *b* and *c* in the parabola

- **Effects of varying 'a'**

The graphs associated with quadratic functions are parabolas. Now, let us consider the function $y = ax^2$ to discuss this effect. Changing the value of *a* results in a vertical stretch of the graph of the function $y = ax^2$ (and of course the function $y = ax^2 + bx + c$) (Chazan, 1992). The bigger the value of *a*, the thinner (steeper) the graph becomes. Also, the smaller the value of value of *a*, the fatter (the more shallow) the graph becomes as in figure 2.5 below.

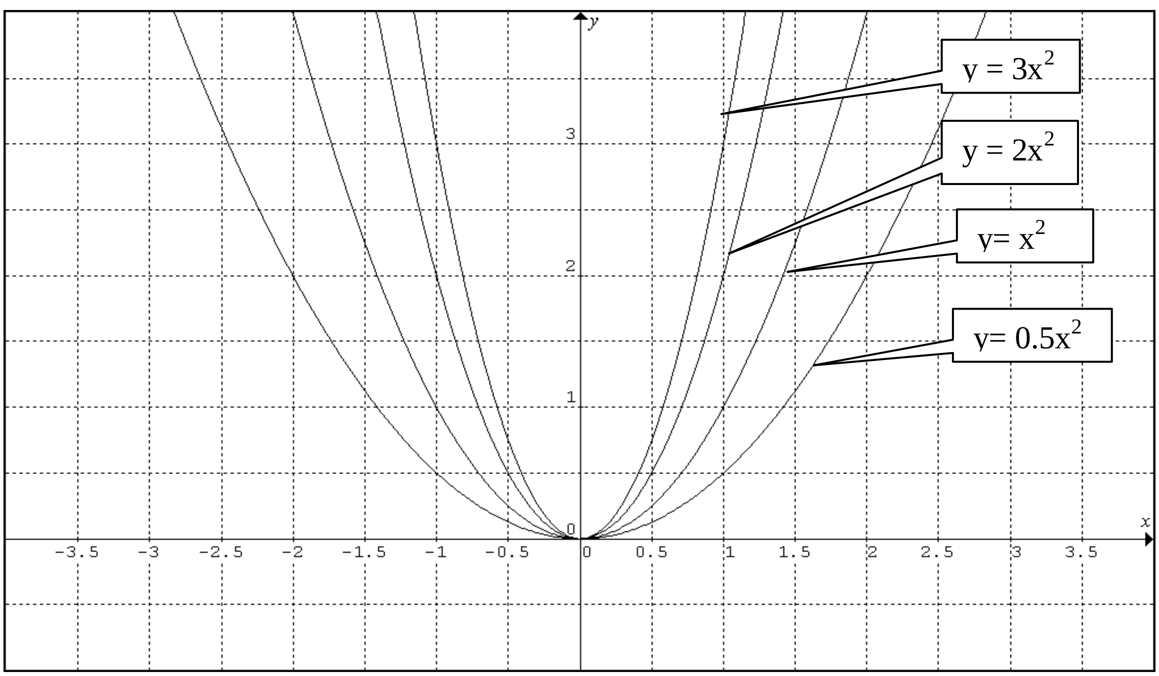


Figure 2.5 Effects of changing the value of 'a'

If "*a*" is negative, the graph is also reflected about the *x*-axis as seen in the figure below:

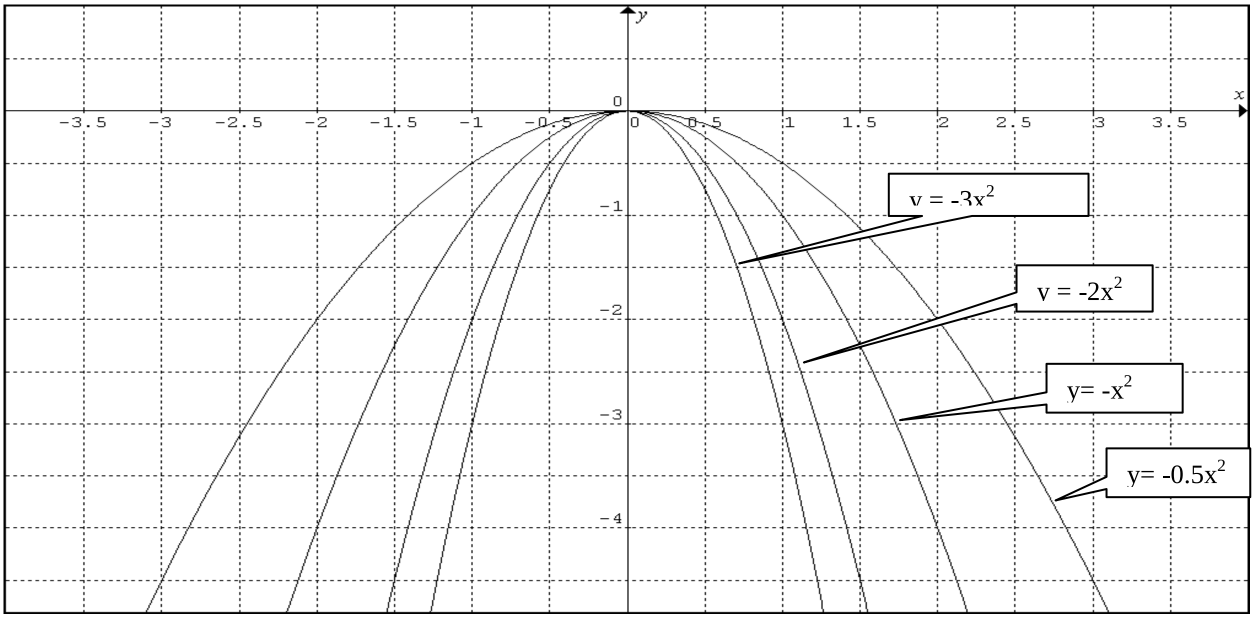


Figure 2.6 The effect of changing the sign ‘a’ to negative.

- **Effects of varying ‘b’**

For the parabola $y = ax^2 + bx + c$, changing the values of b and keeping the values of a and c constant, results in translations for the range of values implemented. The parabola maintains its shape and direction. This can be seen in the graphs of $y = x^2 + bx + 4$ with b = 3, 2, 1, -1, -2 and -3 drawn in figure 2.7 below

We can use the parametric equation ($x = -b/2a$; $y = -b^2/4a + c$) to describe the locus of points where the vertices of each of the graphs will fall irrespective of the value of b. To do this, we solve for b in $x = -b/2a$ and substitute this value ($b = -2ax$) in the equation $y = -b^2/4a + c$. We get $y = -(-2ax)^2/ 4a + c = -ax^2 + c$, the equation of the parabola with vertex (0, c) (Owens, 1992). This is shown in the figure 2.7 below.

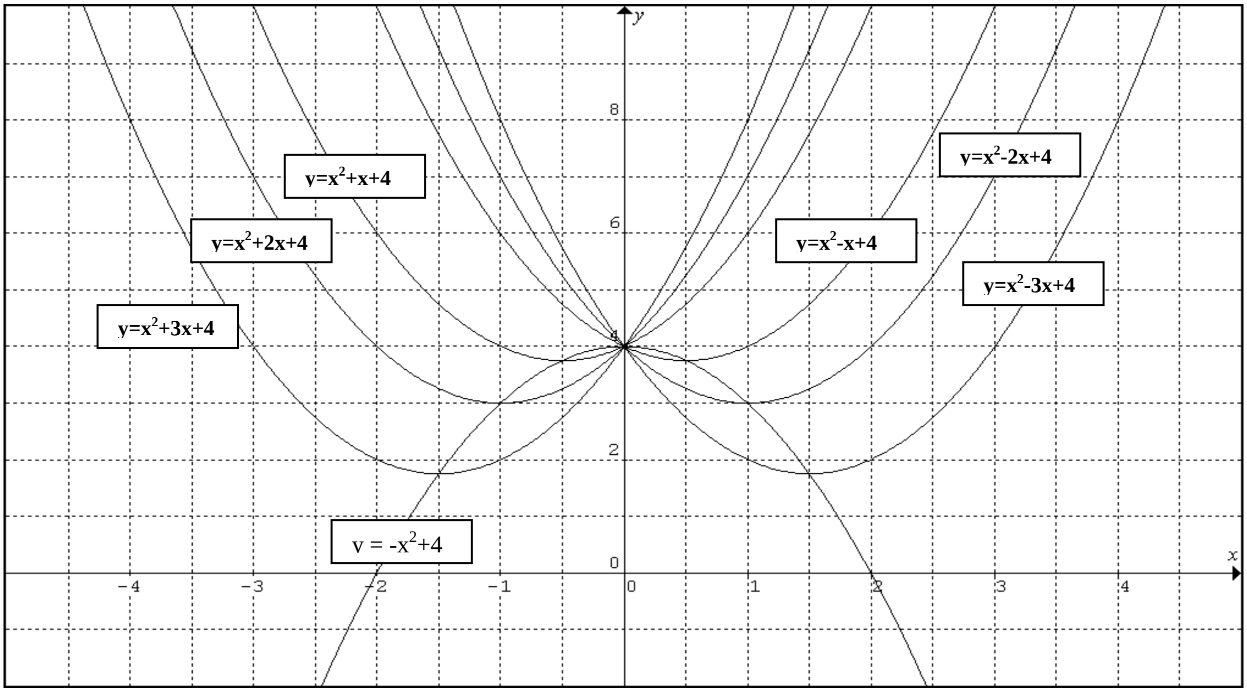


Figure 2.7 the effect of varying 'b'.

- **Effect of varying 'c'**

For the quadratic function in the standard form: $y = ax^2 + bx + c$, changing the value of c results in a vertical translation of the graph of the function by "c" units (up if $c > 0$ and down if $c < 0$. Changing c moves the locus of the vertex along the line $x = -b/2a$. The parabola maintains its shape and directions for the range implemented. The equation $x = -b/2a$ is very useful, more so, since it explains the effects of changing c without including c. The line $x = -b/2a$ is the axis of symmetry irrespective of the value of c. We can find the value of y at $x = -b/2a$ (the vertex) by substituting $x = -b/2a$ in the equation $y = ax^2 + bx + c$ to have; $y = a(-b/2a)^2 + b(-b/2a) + c$. For any parabola in the standard form, the vertex is $(-b/2a; -b^2/4a + c)$ (Owens 1992).

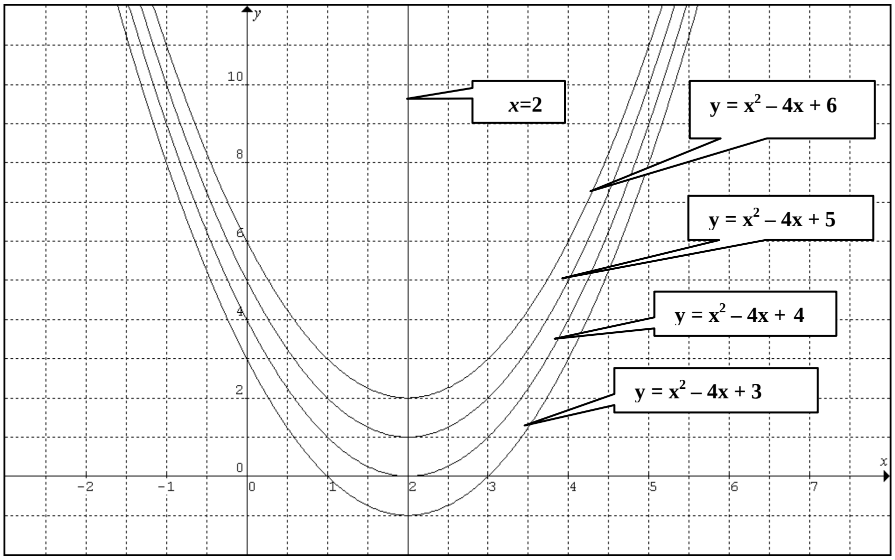


Figure 2.8 The effects of varying ‘c’

2.6.2 Horizontal shifting of the parabola

Addition of a number ‘p’ to x in a quadratic function $y = x^2$ results in a horizontal shifting (translation) of the parabola by *p* units. If *p* is less than 0 ($P < 0$), the graph is translated to the right as shown in figure 2.9 (below). If $p > 0$, the graph is translated to the left. The translation does not change the shape or size of the graph. Every point is translated by *p* units. This is shown in the two graphs below in figure 2.9.

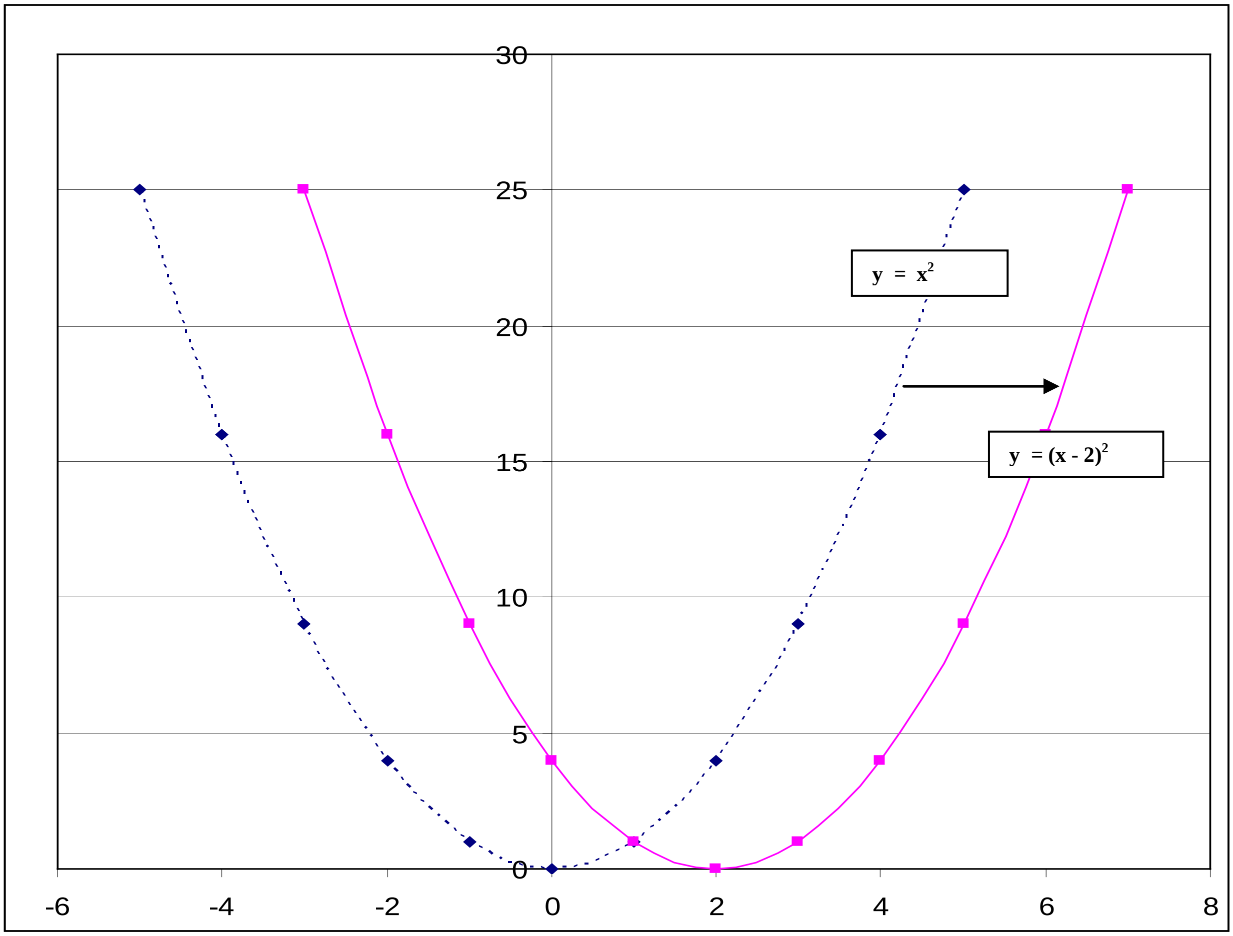


Figure 2.9 The horizontal shifting of the parabola

2.6.3 Mathematical modeling using quadratic functions

A mathematical model is any mathematical relationship (equations, table of data, graphs, diagrams etc) that closely fits real-life data, or describes a real life problem. It can be used to verify, or make predictions for that real-life situation (Meyer, 1994). Certain real life situations can be modelled using quadratic functions. For instance, we can use quadratic function to model projectile motion. A projectile emotion is obtained when an object is thrown or forced into the air. The object that is being thrown into the air is called the projectile. When an object is thrown (or forced) into the air, its height depends on three factors:

- The starting position or initial height of the object
- The speed or velocity at which the object is thrown into the air

- The force of acceleration due to gravity. The force of gravity makes an object accelerates towards the earth. We use $g = 9.8m/s^2$ (Bennie, Black, & Fitton, 2006).

In general, the height of a projectile in metres at time t (seconds) is given by the equation $h = -1/2gt^2 + v_0t + h_0$, where g is the acceleration due to gravity ($g = 9.8m/s^2$), v_0 is the initial speed or velocity (in m/s), that is, the velocity at which the object is released into the air; h_0 is the initial height (in m) – the height of the object when it is released into the air. The shape of the graph is a parabola (Bennie, et al., 2006).

2.7 Learners' misconceptions and difficulties in dealing with quadratic functions

Learners encounter many obstacles which impede their understanding of quadratic functions. Some of these obstacles are conceptual in nature while others are not. Conceptual obstacles are those that have a cognitive nature and can be explained in terms of mathematical structures and concept which can be traced to learners' earlier learning. Some difficulties are caused by misconceptions which may have been developed as a result of over-generalising an essential correct conception, or may be due to interferences from everyday knowledge (Leinhardt, et al., 1990). However, when there is a difficulty, it does not necessarily mean that there is a misconception that is responsible for it. The obstacles which may impede learners' understanding of quadratic functions include:

- Limiting the graph of quadratic function to the visible part of the graph (Zaslavsky, 1999).
- Attributing asymptotic bebaviour to the graph of the quadratic functions (Zaslavsky, 1999).
- Determining a point on the graph by using only eye "measurement" (Zaslavsky, 1999; Kerslake, 1981).
- Treating two quadratic functions where $a_1 = ka_2$, $b_1 = kb_2$, $c_1 = kc_{2;}$ as if they are equivalent (e.g., treating $x^2 + 3x - 4$ to be same as $2x^2 + 6x - 8$) (Zaslavsky, 1999).
- Adhering excessively to linearity. This is why it may seem to learners that a

parabola passes through three collinear points, even though there are not three points on the parabola which are collinear (Dreyfus & Eisenberg, 1983; Karplus, 1979; Leinhardt et a., 1990; Markovits et al., 1986).

- Believing the seeming change in form of quadratic function whose parameter is zero (i.e., learners think that if one of the parameters of an equation of quadratic function is zero, then that equation is not an element of $y = ax^2 + bx + c, a \neq 0$) (Zaslavsky, 1999).
- Over-emphasising only one of the coordinates of special points (Zaslavsky, 1999).
- Lacking ability to use the implicit information related to the line symmetry when not directly cued to focus on symmetry (Zaslavsky, 1999).
- Constructing axes and scales (Kerslake, 1981).
- Transitioning to continuous graph: learners' find it difficult to conceive of points on the graph other than those they plotted (Kerslake, 1981),

2.8 How quadratic function is taught in the traditional classroom

In the traditional mathematics classroom, the teacher is seen as knowing everything and that the learner is almost blank (Onwuka, 1980). Accordingly, it is the teacher's role to transmit his knowledge to his pupils. That is to say that the pupils who are usually supposed to be ignorant and bare, acquire the knowledge from the teacher. The traditional classroom was seen as a silent working place where children are passive listeners and where narrow views and misconception are born. The lesson does not allow for experiences where children are able to discover, invent or apply mathematics to problems that are meaningful to them (Cangelosi, 1996). A good mathematics classroom where meaningful teaching and learning takes place provides a powerful means of communication between the teacher and students or between the students themselves but the traditional mathematics classroom is ironically a place where the children's opinion are never heard. This teaching approach does not take much account of differences between pupils in a particular class with respect to their speed of learning or their previous knowledge (Bell, 1973). This method of learning mathematics involves the whole class instruction, recitation and individual seat work. Success in traditional approach, however, does not provide a thorough understanding of mathematical concept as more classroom time is spent on routine computation skills than on understanding mathematical concepts.

Traditionally linear functions and quadratic equations are considered prerequisites for quadratic functions. Thus the solution of quadratic equations is taught in isolation from the study of quadratic functions (Craine, 1996). In a traditional mathematics classroom, quadratic function is introduced by writing the standard form of quadratic function ($y = ax^2 + bx + c$). The students are then given a quadratic function (in standard form) for which the students are required to complete the table of values and hence plot the graph form the table. No activities are given to enable students to investigate the properties of the quadratic functions. The students are then required to solve problems using the graph (including symmetry, maximum and minimum value, the roots of the given equation etc). Students' difficulties in dealing with a function given in graphical form or connecting functions with their graph is as a result of the traditional instructional methods, that do little in the way of the reverse – from graph to algebraic formulas or tables (Leinhardt et al. 1990).

2.9 The role of technology in promoting learners' understanding of quadratic functions

Technology provides learners with opportunities to investigate and manipulate mathematical situations, to observe, experiment with, and make conjectures about patterns, relationships, tendencies, and generalisations (Erbas, 2004). Thus, it empowers learners who may have limited symbolic and numeric skills to investigate problem situations by freeing them from tedious and repetitive computations (Cuoco, Goldenberg, & Mark, 1995).

The availability of graphical technologies affects issues of concern to educators and curriculum developers as well. With these technologies learners have access much earlier in the curriculum to more complex notions related to graphs. For example, lack of familiarity or competence with algebraic techniques does not prevent a learner from exploring complicated computer-generated graphs and from being engaged in the problem-solving activities that follow (Leinhardt, et al, 1990; Cuoco, Goldenberg & Mark, 1995). Properties of a graph, such as continuity, may need to be addressed earlier and in a manner different from the traditional approach by using these technologies. Issues of scale become more fundamental when using graphical technologies. In a traditional classroom, it is common to sketch only one graph for each function, restricting oneself to the boundaries of the paper and choosing a reasonable scale to fit these limits. With graphical technologies however learners are able to look at a graph of one particular function through many different windows (Leinhardt, et al, 1990). For instance, the graphing window of the Function Probe allows one to display more than one graph at the same time, while keeping the history of the function plotted. The Function Probe is a multi-representational software package that combines graphs, tables, algebra, and a calculator, to enable learners to explore the idea of functions and relations in diverse forms (Confrey, 1991). Also, graphing calculators (TI-83 & TI-92) enable learners to solve problems visually and to make connections (Embse & Yoder, 1988). With these calculators, learners can explore families of functions, determine solutions to equations, and calculate the minimum and maximum values.

Using the function plotter's ability to depict a series of graphs quickly and accurately allows learners to make conjectures about emerging patterns (Owens 1992). By taking advantage of these technologies, we can pursue the effects of varying *'a'*, *'b'*, and *'c'* and the horizontal shifting of the parabola, and also describe the locus of the vertices of the resultant parabolas which was discussed in sections 2.6.1 and 2.6.2.

Learning to perform translation tasks (between graphical and algebraic representations) is easier with computer technologies. This is because graphs can be generated quickly by the computer, freeing the learners from the burden of calculating, plotting and drawing (Owens, 1992). Thus learners are provided with the opportunity to view many graphs and their corresponding equations and can begin to examine the relationship between graphical entailment and algebraic parameters (e.g. the steeping and direction of the graph is related to the magnitude and the sign of the leading coefficient of the equation of the graph).

Technology can be very useful in allowing learners to explore multiple representations and mathematical situation and relationships. The issue of multiple representations may become more salient in the context of technology. Most graphical technologies provide graphical representation and can display simultaneously at least two representations, such as an equation and graph, or an ordered pair of numbers and a point on a graph. Working simultaneously with at least two linked representations is more manageable with these media.

Generally, these packages were designed to encourage learners to invest a multiplicity of methods to attack a problem, to pursue those methods in the company of other learners, and then to discuss and negotiate a variety of acceptable and often alternative approaches to the possible solution (Confrey 1991). Finally, I agree with Cuoco et al., (1995:236) that "technology should be tightly interwoven into the educational experience, used as a tool and a means for creating new teaching strategies" in mathematics education.

2.10 Using the problem solving approach to promote learners'understanding of quadratic functions

Problem solving has been used to express multiple meanings that range from working rote exercises to doing mathematics as a professional (Shoenfeld, 1992). However, we shall use the problem solving approach to teaching and learning to mean a situation in which learners are expected to solve problems for which no well-defined routine or procedures exist. That is, the problem solver does not know any clear path to the solution and has no algorithm which can be directly applied to guarantee a solution. (Erickson, 1999). This does not mean that such algorithm does not exist, rather, it is not known to the problem solver at that particular point in time. The learners must therefore draw on their knowledge, and through this process, will often develop new mathematical understanding.

The educator can approach the teaching of quadratic functions in line with the problem solving approach in which the learners are given opportunities to make reasoned conjectures about a problem solving task, justify their thinking, and listen to and consider other learners' ideas. In so doing the learners have the opportunity of using multiple solutions strategies, and multiple representations of the concept (Hiebert, Carpenter, Fennema, Fussion, Human, Murray, Oliver, & Warne, 1996).

This approach is compatible with the constructivist view that mathematics learning is a process in which learners reorganise their activities to resolve situations that they find problematising. As a consequence, all instructional activities are designed to be problematising. In addition, the class activities reflect the view that conceptual and procedural developments should, ideally, go hand in hand.

Treating mathematics to be problematising allows us to view the classroom activity from both functional and structural perspectives. From the functional perspective, understanding means participating in a community of people who practice mathematics. The functional view focuses on the activities in the classroom. Understanding is defined in terms of the ways in which learners contribute to and share in the meaning making process. The structural view focuses on the understanding and skills that individual learners take with them from the classroom experiences. Either perspective can be used to link the problematising of the subjects with development of understanding (Hiebert, et al, 1996).

2.11 Using the theory of constructivism in the teaching and learning of quadratic functions

Constructivism stems from the theory of cognitive development in the philosophy of mathematics education through the work of Piaget. Central to the constructivist theories of learning is that learners arrive at meaning making by actively selecting and cumulatively constructing their own knowledge through both individual and social activity. The learner therefore brings an accumulation of assumptions, motives, intentions and previous knowledge about the learning / teaching situation and determines the course and quality of the learning that may take place. This view of learning places a lot of emphasis on understanding instead of memorising and reproducing information (Tynjala, 1999) in every learning situation.

The constructivist view of learning is in sharp contrast to the notion implicit in the traditional approach that learners come to understand by taking clear explanation (Simon & Schiffer, 1993). Teaching in the constructivist perspective is not transmitting of knowledge but helping learners to actively construct knowledge by assigning to the learners tasks that enhance this process. The constructivists' assumptions about how learners learn changes the assumptions about what kind of educator actions or behaviours might be desirable.

One who views teaching as the transmission of knowledge would follow a teaching-by-imposition model, and those who view teaching as the facilitation of the construction of knowledge would follow a teaching-by-negotiation model. The educators' role in initiating and guiding mathematical negotiations is a highly complex activity that includes highlighting conflicts between alterative interpretations or solutions, helping learners to develop productive small-group collaborative relationships, facilitating mathematical dialogue between learners, implicitly legitimising selected aspects of contributions to discussion in light of their potential fruitfulness for further mathematical constructions, re-describing learners' explanations in more sophisticated terms that are nonetheless comprehensible to learners, and guiding the development of taken-to-be-shared interpretations when particular representational systems are established (Simon & Schiffer, 1993). With the principle of constructivism in view, for the educator to engage the learners to learn in a meaningful way, he/she should do the following as suggested by Wood (1995).

1 Provide instructional situations that elicit subject appropriate activities.
2 View learners' conceptions from their (the learners') perspectives.
3 See errors as reflecting their learners' current level of development.

4 Recognise that substantive learning occurs in periods of conflict, surprise over periods of time, and through social interaction.

The above having been said, it is explicit that using the view of constructivism in the quadratic function classroom will go a long way to resolving some of the difficulties which impede learners' understanding of quadratic function.

In Australia (1990), New Zealand (1992), South Wales (1998), and later in South Africa's Curriculum 2005, it was advocated that active learning processes should replace traditional passive learning.

2.12 The use of context questions to promote learners' understanding of quadratic functions

The point of departure in the Realistic Mathematics Education (RME) is that context problems can function as anchoring points for the reinvention of mathematics by the learners themselves. "Context problems are defined as problems of which the problem situation is experimentally real to the learners" (Gravemeijer & Doorman, 1991, p. 111). The most important strategy here is to take some everyday experiences that learners are familiar with and mathematise it, by constructing mathematical models around them. It is necessary to point out that the process of mathematisation should at the same time be in line with learning objectives or the critical outcomes (Adendorff, & Heerden, 2001).

In the Realistic Mathematics Education everyday situations should be used and adapted to develop the concept of 'parabola' with its distinctive properties. The focus here is on a more practical way of dealing with the parabolic function. During this process learners develop properties of the parabola. Other skills that are being developed would include some of the following: approximation, pattern recognition, predicting, generalisation, manipulation, application in daily contexts, deduction, and problem-solving. Such everyday experiences can include a question like:

A drunken driver lost control of her car, which resulted in the car grazing, and ripping off a section of a parked car. The traffic policeman who examines the accident knows that the impact of the collision is a function of the speed at which the collision occurs. The impact of the collision of the car is the result of the force experienced when it collided with another object. Suppose the following function rule (equation) is used to determine the collision impact: $l = 2v^2$

(l ~ collision impact, and v ~ speed in kilometre per hour).

Complete the following table:

Table 2.3 Collusion impact task: $l = 2v^2$

v	0	10	20	30	40	50	60
l	0	200	800	1800			7200

Now draw a graph by using the above table. Let the vertical axis represent the collision impact *(l)*. (Adendorff & Van Heerden, 2001, p. 60).

Questions such as the one mentioned above, can be effectively used to stimulate learners interest and enthusiasm for mathematics. Not only can such questions be used to highlight meaningful and contextual applications of $y = ax^2 + bx + c$ but also to reinforce basic concepts and terminology. Furthermore, such problems can be used to develop independent working strategies, or can be adapted to be used optimally in a group context.

Using contexts exhibiting parabolic properties allow learners to identify more readily with the content. It also allows learners to interact much more meaningfully and intimately with the mathematics material.

2.13 Conclusion

The literature review presented in this chapter was aimed at linking research findings and theory about educators' pedagogical content knowledge within the context of quadratic functions. Research and literature reviewed indicated that:

- Different conceptions of PCK exist. These differences are the result of what one includes or subtracts from Shulman's conceptions.
- PCK is domain specific and involves the integration or synthesis of subject matter content knowledge and pedagogical knowledge.
- Educators with rich subject matter content knowledge are more likely to develop rich PCK than those with low content knowledge.
- Learners have misconceptions that may impede their understanding of quadratic functions. These misconceptions are often lodged in the areas of interpretation of graphical information, the relationship between quadratic function and quadratic equation, the seeming change in the form of a quadratic function whose parameter is zero, misunderstanding concerning the concept of line symmetry, the over-emphasis on only one coordinate of special points, misconception of axes and scales, difficulties in the general notion of functions, amongst others.
- The traditional approach to teaching quadratic function is blamed as one of the major reasons why learners find the learning of quadratic functions difficult.
- Technology can be used to promote learners' understanding of quadratic functions.
- The constructivist perspective to learning and teaching can be used to overcome some of the learners' misconceptions about quadratic functions.
- Using a realistic approach to the teaching and learning of quadratic functions can be useful in stimulating learners' interest and also reducing their misconceptions in learning quadratic functions.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1. Introduction

In this chapter the method used for the empirical investigation of the role of mathematics educators' pedagogical content knowledge in the learning of quadratic functions is discussed. The research method used in this study is informed by the purpose of the study, which is to investigate to what extent educators' pedagogical content knowledge affects learners' achievement in quadratic functions, and the type of data needed to answer the research questions outlined in section 1.3. The questions are:

1. What are the learners' main misconceptions in quadratic functions?
2. Does educators' MCK affect the learners' achievement in quadratic functions?
3. Does educators' knowledge of learners' conceptions and misconceptions affect learners' achievement in quadratic functions?
4. Does educators' knowledge of strategies for teaching quadratic functions affect learners' achievement in quadratic functions?

This chapter gives a description of the research design, the research sample, the procedure of the study, the instruments employed in collecting the data, the method used in analysing the data, validity of instruments, reliability of instruments, and the ethical issues considered in the study.

3.2 Research design

The research design describes the major procedure to be followed in carrying out the research. It is a specification of the most adequate operations suitable to the specific research goal (Bless & Higson-Smith, 1995). To answer the research questions and also achieve the purpose of the study, the mix method design, in which both qualitative and quantitative methods were implemented concurrently in a single study in order to provide a better understanding of the research problem than either type by itself, was adopted (Creswell, 2008). This design was employed to enable the researcher to interpret and describe how each component of mathematics educators' pedagogical content knowledge affects learners' achievement in quadratic functions. The mix method design was also utilised to investigate the educators' mathematics content knowledge and how it influences learners' achievement in quadratic functions; the educators' knowledge of their learners' conceptions and misconceptions, and how it relates to learners' achievement in quadratic functions; and the educators' knowledge of strategies for teaching quadratic functions and how it influences learners' achievement in quadratic functions.

Quantitative and qualitative data were collected simultaneously in which the researcher "further explored findings from one method by the use of the other" (Creswell, 2008, p. 557), thereby combining the "best" of both quantitative and qualitative research. Quantitative data was gathered about educators' mathematical content knowledge, and learners' achievement in quadratic functions; qualitative data were gathered about learners' conceptions and misconceptions in quadratic functions. Quantitative and qualitative data were gathered about educators' knowledge of learners' conceptions and misconceptions, and about educator knowledge of strategies for teaching quadratic functions.

3.3 The research sample

The study was conducted with seventeen mathematics educators and ten grade eleven learners taught by each educator in eight secondary schools in the North West Province of South Africa. The sample of educators that participated in the study was a convenient sample. The educators were selected because of the proximity of their schools to the researcher. From the seventeen educators who agreed to participate in the research, fourteen teach in seven schools, while three teach in one school. Two educators from each of the seven schools agreed to participate in the study, while three educators from school A (see Table 3.2) agreed to participate. The highest qualification of the educators was a degree in mathematics education; this accounted for 17.6 percent of the sample. 64.7 percent of the educators have a diploma, and 17.6 percent were professionally qualified although not qualified to teach mathematics at the secondary school level. The information about the educators' qualifications, gender and experience in teaching is shown in Table 3.1 below.

Table 3.1 Background information of sampled educators (N = 17)

	Number of educators
Gender	
Male	11
Female	6
Teaching Experiences	
0 – 5 years	5
6 – 10 years	7
11 – 15 years	3
16 – 20 years	2
More than 20 years	-
Qualifications	
Degree in mathematics education or mathematics.	3
Diploma or Certificate in mathematics education.	11
Any other qualification.	3

The background information of the participants in Table 3.1 provides reasonable evidence about the participants' knowledge in teaching. This sample can make reasonable statements about the contribution of educators' knowledge to learners' achievement in mathematics in grade 11.

From each educator's class, ten learners were selected by random sampling making a total of hundred and seventy learners. The reason for using the random sampling technique was to avoid selection bias. The reason for selecting learners who were taught by educators who agreed to participate in the research was to afford the researcher the opportunity to investigate the effects of educators' pedagogical content knowledge on the achievement of their learners'. The learners were black learners of predominantly low-income family backgrounds. The average age of the learners was 17 years. All the learners had completed the study of quadratic functions at most three months before the investigation was carried out.

3.3.1 Research site

The research site consisted of eight township secondary schools in the North West province of South Africa. Township schools are schools located in communities that were historically disadvantaged. Township is a context of interest insofar as the condition in township schools is typical of the conditions in most secondary schools across the country. In terms of resources for mathematics teaching and learning, there are no graph boards, and neither projectors nor computers for teaching and learning mathematics in the eight schools. However, each learner has a calculator, and at least one mathematics textbook.

3.3.2 Access to the research site

The researcher contacted educators and solicited their participation, having explained the purpose of the study and the level of participation that was needed from them. The School Governing Boards of the educators who agreed to participate were approached for permission to conduct the study on their premises (see Appendix iii). This was done through the principals of the schools. The researcher explained the purpose of the study and the level of participation needed from both the educators and the learners to the governing boards of the schools. Because the learners were minors, the consent of their parents was also obtained (see Appendix ii).

3.3.3 Performance of the sample schools in the Matric examination**.**

The performance in mathematics in the 2008 and 2009 Matriculation (Matric) examinations for the sample schools is shown in Table 3.2.

Table 3.2 Performance in mathematics in 2008 and 2009 Matric examination

School	A	B	C	D	E	F	G	H	Mean
2008	51	38	15	36	18	40	14	48	32
2009	68	31	68	71	06	42	09	41	42

(Department of Education, Matric Result 2008 & 2009)

A comparison between the performance of the schools that participated in the study with the national pass rate for the 2 year period (see section 1.2 Table 1.1), indicates that the mean performance of the eight schools was slightly below the national pass rate for each year. However, school A, and school H performed above the national pass rate in 2008, while in 2009, schools A, C, and D performed at a rate higher than the national pass rate.

3.4 Procedure for the study

The empirical investigation was conducted in phases as follows:

Phase one:

Based on the theoretical framework for assessing educators' pedagogical content knowledge, the South African curriculum statements and available mathematics text books, the researcher constructed the questionnaire for the learners, the questionnaire for the educators, and the interview items for the educators.

Phase two

The instruments constructed in phase one was used for a pilot study from which also the reliability of the instruments was established.

Phase three

The learners' questionnaire developed in phase one, was administered to the learners with the aim of investigating their achievement in quadratic functions, and identifying their conceptions and misconceptions.

Phase four

The educators' questionnaire developed in phase 1 was administered to seventeen mathematics educators that participated in the study. This was aimed at gathering data about the educators' MCK in quadratic functions, and the educators' knowledge of their learners' conceptions and misconceptions about quadratic functions.

Phase five

Educators who participated in the study were interviewed in order to collect data about their knowledge of strategies for teaching quadratic functions.

Phase six:

With the data obtained on each of the components of the educators' PCK, the educators were categorised as either possessing adequate PCK or not. The achievement of learners taught by educators who possess adequate PCK was compared with the achievement of learners who were taught by educators who do not possess adequate PCK. These analyses exposed the effect of the educators' PCK in the learning of quadratic functions. With these analyses, the role of the educators' pedagogical content knowledge in the learning of quadratic functions was established

3.5 Instruments

As indicated in section 3.4 above, the following instruments were used to collect the data in this study:

- Learners' questionnaire;
- Educators' questionnaire; and
- Interview for educators.

3.5.1 Learners' questionnaire

The learners' questionnaire was used to gather data about learners' achievements, and learners' conceptions and misconceptions about the concept of quadratic functions.

3.5.1.1 Development of learners' questionnaire

The learners' questionnaire was constructed by the researcher. To ensure that the questionnaire adequately covered the concept of quadratic functions, the researcher used the South African curriculum statements on quadratic functions alongside with the knowledge components indicative of understanding of quadratic functions as identified by Zaslavsky (1997) as a guide. The knowledge component indicative of understanding of quadratic functions as identified by Zaslavsky (1997) in the study of the conceptual obstacles on the learning of quadratic functions in Israel include: common algebraic forms of quadratic functions, connections between the *x*–intercepts of a parabola, condition determining the location of the *x*-intercepts of a parabola, conditions determining the number of *x*-intercepts, conditions determining the location of the *y*-intercepts of a parabola, condition determining the type of concavity of a parabola, symmetrical properties of a parabola, extreme values of a quadratic function, connection to a linear function, and special cases of pairs of quadratic functions. The following knowledge components indicative of understanding of quadratic functions were used in constructing both the learners' and educators' questionnaires.

1. Effects of varying the value of 'a' in a parabola;
2. Infinite nature of the parabola and its relationship to the *x*–axis (infinite domain);
3. Infinite nature of the parabola and its relationship to the *y*–intercept;
4. Symmetrical properties of the parabola and its connections to the vertex, and the *x*– intercepts;
5. Effects of varying the value of 'c' on the line symmetry, the vertex, the *x*-intercepts, and *y*–intercepts;
6. Horizontal shifting of the parabola;
7. Turning point of the parabola and its relation to symmetry and the axes;
8. Special cases of 2 parabolas where $a_1 = ka_2$, $b_1 = kb_2$, $c_1 = kc_2$;
9. Modelling real life situations using quadratic functions;
10. Connections to linearity;
11. Identifying a corresponding parabola if the symbolic form of the function is

given;

12. Identifying corresponding symbolic representations of functions if the parabola was given; and
13. The use of the tabular representation of quadratic functions to determine the *x*–intercepts, the turning point, and the region of increase, and decrease.

A question was constructed around each knowledge component although knowledge component 13 has question 13a; 13b, and 13c in order to cover the three aspects of knowledge component 13.

The learners' questionnaire consisted of a variety of non-standard problems, most of which provided an opportunity for qualitative considerations and reasoning. The reason was to enable the researcher investigate the learners' achievement as well as the learners' conceptions and misconceptions about quadratic functions. The majority of the tasks in the questionnaire can be classified as translation tasks (between graphical and algebraic representations). The learners' questionnaire was designed with the following considerations:

i. Most of the questions were not found in the textbooks used by the learners. The aim was to reduce learners' over reliance on computational skills or rote memorisations. It was the researcher's opinion that if the learners were not familiar with the problem, they would focus much on a variety of considerations to solve the problem rather than rely on already known algorithm.
ii. The amount of quantitative information provided in most questions was reduced to the barest minimum. The aim was to make learners focus their attention on the qualitative properties of the questions.
iii. The questions required several considerations hence demanded learners' logical reasoning.
iv. Seven of the tasks were designed to be multiple choice questions three; were "yes" or "no" questions and four were short answer questions. The reason was to maintain objectivity in scoring.

Learners were required to give the right answer in each question. They were also required to give reason(s) for their answers.

The researcher constructed the learners' questionnaire with the assistance of mathematics education specialists at the University of South Africa (UNISA). Questions 5, 8, and 11 were adopted from Zaslavsky (1997) while the researcher constructed ten questions. The questionnaire was then discussed with fifteen Grade 11 learners to determine if the learners' understanding of the questions was as intended. The feedback was used in drafting the final copy of the questionnaire. The questionnaire was given to three experienced heads of department (HOD) of mathematics in secondary schools to verify if the questionnaire covered the content of quadratic functions in Grade 11 in the South African curriculum and if the questions were within the scope of Grade 11 learners. Each of the three heads of the department affirmed that the knowledge required for the solution of each task could reasonably be assumed to be possessed by their own learners upon completion of the study of quadratic functions in Grade 11. The three heads of the department of mathematics also confirmed that the questionnaire reasonably covered the content of quadratic functions in grade 11. The questionnaire was given to the mathematics subject specialist at the North West Department of Education (Letlhabile Area Project Office) who confirmed that the questions can probe learners' knowledge of quadratic functions and explore their conceptions and misconceptions. These reports confirm the validity of the instrument. The reports agree with the assertion that content evidence is "a matter of determining whether the samples are representative of the larger domain of tasks it is supposed to represent" (Gronlund, 1998, p. 202).

3.5.1.2 Administration of learners' questionnaire

The questionnaire for learners was administered to the learners in their respective schools between 7th and 21st May 2009. The mathematics educators who teach the learners who participated in the study administered the questionnaire to them. This was done to reduce examination anxiety. The learners who participated in the study in each school completed the questionnaire in the same venue. There was no time limit for the learners to complete the questionnaire since the researchers interest was not on how fast the learners could answer the questions, rather the researcher's intent was to tap into the learners' conceptions and misconceptions, and the learners' achievement in quadratic functions. The questionnaire took the learners about one to two hours to complete. The researcher was present in each of the venues when the learners completed their questionnaire and collected the questionnaire after the learners had completed it.

3.5.2 Educators' questionnaire

The questionnaire for educators was developed by the researcher for the purpose of collecting data on mathematics educators' subject matter knowledge of quadratic functions, and educators' knowledge of their learners' conceptions and misconceptions in quadratic functions.

3.5.2.1 Development of educators' questionnaire

The development of the educators' questionnaire was based on the following considerations:

(i) the idea that a more direct way of investigating the mathematical content knowledge of mathematics educators is to probe into the educators' knowledge of mathematics in the context of the school mathematics curriculum that they deliberate in their course of teaching. This idea was used by Rowland, Martyn, and Barber (2001) to investigate the mathematical content knowledge of pre-service primary educators in England, and

(ii) the strategy used by Viri (2003) to investigate the engineering educators' pedagogical content knowledge. In Viri's (2003) strategy, educators were given the same question as their learners, were asked to give correct answers to the questions, and to describe their expectations of the learners' reasons for their answers. This strategy was used to investigate the educators' knowledge of their learners' conceptions, and how the educators' knowledge of the learners' conception influenced the learners' achievement. Viri's strategy is the same as the strategy used by Halim et al., (2002) to study science trainee educators' pedagogical content knowledge and its influence on physics teaching. The strategy used by Viri, and Halim and Meerah is consonant with the idea of Rowland et al. (2001), above.

The educators' questionnaire used in this study consists of the same tasks (13 questions) as in the learners' questionnaire. In each of the tasks, the educators were required to

i. describe in writing their expectations of their learners answers;
ii. describe in writing their expectations of the learners' reasons for the answers given by the learners, and
iii. give answers to the questions as educators.

The educators' ability to provide the correct answer to the questions depicts the educators' subject matter knowledge of quadratic function while the educators' abilities to predict their learners' reason(s) for their answer shows the educators' knowledge of their learners' conceptions and misconceptions.

The educators' questionnaire was also used to collect data about educators' qualifications, experiences, gender, and about the nature of the in-service training on quadratic functions, which they attended.

Since the tasks in the educators' questionnaire were the same tasks as in the learners' questionnaire, the process for the development of the educators' questionnaire included the processes and considerations for the development of the learners' questionnaire (see section 3.5.1). The process of the development of the educators' questionnaire also included that the questionnaire was given to a mathematics specialist at the North West Department of Education, and two lectures in the department of mathematics education at UNISA for validation. The specialist and each of the lecturers affirmed that any educator who possesses the knowledge demanded for answering the questions in the educators' questionnaire has sufficient subject matter knowledge for effective teaching of quadratic functions in grade 11. The subject specialist and each of the lecturers also confirmed that the educators' questionnaire could effectively investigate the educators' understanding of the learners' conceptions and misconceptions in quadratic functions.

3.5.2.2 Administration of educators' questionnaire

The researcher administered the educators' questionnaire to the educators who participated in the research. This was after the learners had responded to their questionnaire. Due to time constraints on the part of the educators, the researcher allowed them to complete their questionnaire at their convenience but not exceeding three days. However, some educators took a week to complete their questionnaire. Non-supervision of the educators was a limitation to the study as the educators could have shared their answers. To reduce this possibility, the researcher explained the need to respond to the questionnaire individually (see Appendix v). Also, the researcher explained to the educators that the research was not aimed at evaluating their knowledge, but to investigate how their knowledge can influence their learners' achievement. The responses provided by the educators' in the questionnaire do not portray that they collaborated with each other when responding to the questionnaire.

3 5.3 Interview for educators

Educators' knowledge of strategies (ability to formulate analogies, explanations, examples and demonstrations) to teach the topic was investigated by means of an interview. The interview items are open-ended questions as shown below:

"If you were asked to teach the concept involved in question……, how will you teach it so that the learners can understand the mathematical concept involved in it".

The interview item was structured from Halim et al. (2002).

The interview items were given to the subject specialist and two lecturers for evaluation. The subject specialist and each of the two lecturers testified that the interview items seek information about educators' knowledge of strategies for teaching quadratic functions.

3.5.3.1 The interview section

The researcher conducted the interview on a one-to-tone basis with each participating educator. The venue was the educator's school. In the interview, the researcher posed the interview item to the educator as follows:

"If you were asked to teach the concept involved in question……, how will you teach it so that the learners can understand the mathematical concept involved in it".
(Depending on the educator's response, questions for clarifications may be posed to the educator where need be).

After the educator has responded to the question, the researcher posed the same interview item for the next question in the learners' questionnaire. This pattern was followed until the last question was attended to. The interviews lasted between thirty minutes and an hour and half.

3.6 Validity of the instruments

When an instrument is valid, it means that it measures adequately what it is expected to measure. Thus, the validity of an assessment instrument refers to the extent to which the instrument measures what it is supposed to measure (De Vos, 2002, p. 166). Four categories of validity exist: content, face, criterion, and construct validity. Content validity in this study refers to the extent to which the questionnaire measures a representative sample of the subject matter treated. Face validity does not refer to what an instrument (questionnaire or interview) "actually" measures but rather to what it "appears" to measure (i.e., it seeks to find if the instrument appears relevant to those who will complete it). Criterion validity refers to the degree which the result of an instrument is consistent with external or independent criteria. Construct validity refers to the consistency between the instrument and accepted theoretical constructs related to the subject matter being studied (Babbie, 2001). Content, construct, and face validities of the questionnaires were achieved in the process of constructing the instruments as discussed in sections 3.5.1.1 and 3.5.2.1, and 3.5.3.

Based on this, the researcher believes that the questionnaire and interview sufficiently assessed the pedagogical content knowledge specific to quadratic functions.

3.7 Reliability of instruments

Reliability of an instrument refers to the degree to which independent administration of the same instrument (or highly similar instruments) consistently yield the same result under comparable conditions (De Vos, 2002). The methods for establishing the reliability of an instrument include: the test-retest method, alternative form method, split half method, or calculation of the Chronbach's alpha coefficient (De Vos, 2002). As indicated in section 3.5, the questionnaires used in this study were designed by the researcher, hence the items in the questionnaire were not standardised. The reliability of the instruments was achieved using the test-retest method. The questionnaire was pre-tested on three Grade 11 mathematics educators and fifteen Grade 11 learners. After about four weeks, the educators' and learners' questionnaires were administered to the respective pilot samples. The reliability of both educators' and learners' questionnaires were calculated using the Pearson product-moment correlation coefficient r. The correlation coefficient of the learners' questionnaire was 0.94, while that of the educators was 0.91. The two correlation coefficients were high and therefore depict that the instruments were reliable.

3.8 Data analysis method

To determine the effects of each component of the educators' PCK on the achievement of learners; the following analyses will be carried out.

Educators' knowledge in each component of PCK will be analysed. From the result of the analyses, the educators will be grouped according to the level of their PCK. Learners will be grouped according to the PCK of their educators. Achievements of the groups of learners will be compared to determine the effects of educators' knowledge on learners' achievement.

3.8.1 Analyses of learners' questionnaire

Two sets of data were collected from the learners' questionnaire – quantitative data and qualitative data.

Quantitative analyses:

The quantitative data was used to measure the learners' achievement in quadratic functions. In each question, a learner was scored 1 if he/she got the right answer and 0 if the answer was wrong. Descriptive statistics were used to organise this data.

Qualitative analyses:

Content analyses of the learners' reasons for their responses in each question was made from which the researcher uncovered the learners' misconceptions in quadratic functions.

3.8.2 Analyses of educators' questionnaire

Quantitative analyses:

The MCK of educators was determined through the educators' scores in the content part of the educators' questionnaire. In each question, an educator was scored 1 if he/she gave the right response to the question and 0 if the answer was wrong. The total score for each educator was recorded. This data was presented using descriptive statistics (see Table 4.1). Each educator's score was a description of his/her MCK. An educator's MCK in a question is deemed strong if the response to the question was correct but weak if the response was incorrect.

Qualitative analyses:

Content analyses of the educators' expectations of the reasons given by their learners were done to obtain data on educators' knowledge of their learners' conceptions and misconceptions in quadratic functions. In the analysis, the researcher compared the educators' expectations of their learners' reasons with the reasons given by the learners themselves. This approach was used by Halim, et al. (2002) and Viri, (2003).

3.8.3 Analyses of educators' interviews

To investigate educators' knowledge of mathematics specific teaching strategies for teaching the concept of quadratic functions, educators' responses in the interviews were analysed. The analysis was based on the type of representations, analogies, illustrations, or examples formulated by the educators for teaching the concept during the interview.

3.9 Ethical issues considered in the study

Ethical consideration involves a set of moral principles that should guide the behaviour of a researcher towards respondents and other researchers (De Vos, 2002). To obtain the participants' informed consent, the researcher provided adequate and necessary information to the learners, the educators, and the school governing board of the schools that participated in the study. The information included: the purpose of the study, the extent of information required from each participant, and the credibility of the researcher. The participants willingly decided to get involved. The educators' written consent was obtained. Because the learners were minors, their parents' written consent were obtained as well (see Appendix i & ii).

The right to privacy and confidentiality of the respondents was maintained throughout the research, for instance, numbers were used to represent educators' names while alphabets were used to represent the names of the schools. To maintain anonymity, the learners were not asked to write their names on the questionnaire; rather they were informed to indicate their educators' names at the back of their questionnaire. Identifying the educator who taught particular learners was necessary for the analyses. Educators' responses in the questionnaire and interview sessions were kept confidential. The study did not expose the respondents to harm of any kind as the learners and educators responded to the questionnaires at a time considered convenient to them.

The research findings were accurately and objectively reported to the best of the researcher's abilities. The shortcomings of the research have been stated in no unclear terms. Subjects were informed about the findings in an objective manner, and without offering any details that revealed the confidentiality of the respondents. The reason for sharing the research findings with the educators was to encourage them to change their practice in line with the findings and to maintain good relationship in case the researcher decides to conduct a follow-up study.

3.10 Summary

This chapter gave the description of the research design, the research sample, the instruments employed in collecting the data, the procedure followed in data collection, and the method used in analysing the data. The ethical issues considered in the study were also discussed.

CHAPTER FOUR

PRESENTATION AND INTERPRETATION OF RESULTS

4.1 Introduction

This chapter presents data obtained from the scientific investigation of the role of pedagogical content knowledge in the teaching and learning of quadratic functions, the analyses of the data and the interpretation of the results. Data presented will be used to answer the research questions, which are:

1. What are the main learners' misconceptions in quadratic functions?
2. Does the educators' MCK affect learners' achievement in quadratic functions?
3. Does educators' knowledge of learners' conceptions and misconceptions affect learners' achievement in quadratic functions?
4. Does educators' knowledge of strategies for teaching quadratic functions affect learners' achievements in quadratic functions?

To study the role of PCK, the researcher first investigated the learners' misconceptions and achievements in quadratic functions. The result of learners' misconceptions answers question 1. To answer the subsequent research questions, results of the learners' achievement, the learners' misconceptions, and the result of the educators' knowledge in the research question, will be used. Data on learners' achievement will be used in all the research questions. As a result, the achievement of learners will be presented first.

4.2 Analyses of learners' questionnaire

4.2.1 Learners' achievement in quadratic functions

A learner was scored 1 if he/she got the answer right but 0 if the answer was wrong. Learners' achievement in the quadratic function is presented such that it shows the performance of the learners taught by each educator. It is necessary to present the learners' achievement in this form because it enables the researcher to compare the achievement of learners taught by different educators.

Table 4.1 Achievements of learners in quadratic functions (N = 170)

Educators	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13		
													A	B	C
1	90	60	40	60	50	90	90	30	20	20	30	30	70	70	30
2	80	70	50	70	40	80	80	30	10	30	20	30	80	80	30
3	90	50	30	90	30	100	80	20	20	20	30	30	60	80	30
4	80	40	40	60	40	90	90	30	20	20	30	20	60	70	20
5	100	40	30	70	50	80	90	20	00	10	30	40	70	70	20
6	80	40	30	70	30	80	80	20	30	30	30	20	70	80	30
7	90	40	40	80	40	100	90	40	20	20	50	40	80	60	40
8	100	40	60	80	30	100	100	40	20	30	40	30	80	70	30
9	100	30	50	50	50	100	100	20	00	20	50	40	90	90	40
10	80	40	40	90	40	80	80	30	10	00	40	30	60	80	30
11	80	30	30	80	40	100	70	20	00	00	40	30	70	70	30
12	70	30	30	70	40	90	80	40	20	10	40	20	80	70	20
13	70	30	20	80	20	90	90	20	00	00	30	20	80	80	10
14	70	20	30	80	20	80	80	00	00	00	20	10	70	60	40
15	80	20	20	70	10	70	60	10	00	10	30	10	60	80	30
16	60	20	20	60	20	90	80	00	00	00	20	30	80	70	20
17	60	10	10	60	30	70	70	10	00	00	30	10	70	70	40
Mean	81	36	34	72	34	88	83	22	10	13	33	26	72	74	29

The result shows that on the average, the learners' achievements in questions 1, 4, 6, and 7 are high, while the learners' achievements in question 8, 9, 10, 11, and 12 are poor. The highest achievement of learners was in question 6 – horizontal shifting of the parabola, while the lowest mark was in question 9 - modelling real life situations using quadratic functions. In question 13, learners' achievements in 13 and 13b were high, while learners' achievements in 13c were poor.

4.2.2 Result of research question 1:

What are the main learners' misconceptions in quadratic functions?

Content analyses of the reasons given by the learners' for their choice of answers were done from which the researcher obtained the main learners' misconceptions reported in this study. To aid the analyses of the learners' misconceptions, a table indicating the type of responses made by the learners in each question is presented.

Table 4.2 Learners responses (N = 170)

Question number	Percentage of learners that chose the highest occurring wrong answer	Percentage of learners that chose other wrong answers	Percentage of learners that chose the correct answer
1	08	11	81
2	64	nil	36
3	66	nil	34
4	13	15	72
5	66	Nil	34
6	12	Nil	88
7	Not applicable	17	83
8	68	10	22
9	Not applicable	90	10
10	87	Nil	13
11	29	38	33
12	28	46	26
13a	10	18	72
13b	11	15	74
13c	28	43	29

In Table 4.2 above, the "highest occurring wrong response" represents the wrong option chosen by more learners. In question 1 for instance, eight percent of the learners chose a particular option that was wrong, eleven percent of the learners chose the remaining three wrong options, while eighty-one percent of the learners got the right answer. The table shows that a greater number of learners chose the same wrong options in questions 2, 3, 5, 8, and 10. In questions 7 and 9, no options were provided to the learners. They were short answer questions hence the learners that failed questions 7 and 9 gave different wrong answers.

The main learners misconceptions reported in this study are as follows:

- *Limiting the graph of the quadratic functions only to the visible region*

This misconception was observed in question 2 and question 3. In question 2, the following task was presented to the learners:

The parabola below represents the graph of a function of the form, $y = ax^2 + bx + c$. Is it possible to have a point on the graph when x is 20?

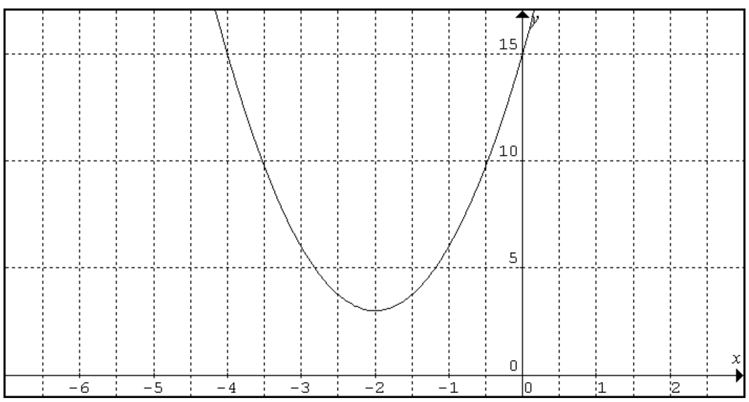


Figure 4.1 Representation of the parabola in question 2.

a. Yes
b No.

Reason(s)………………………………………………………………………………...

There was a tendency of learning to read graphs like a picture. In question 2, sixty four percent of the learners chose option 'b'- the wrong answer (see Table 4.2). That is, they responded that it was not possible to have a point in the graph when *x* is 20. The reasons given by the learners were based on the reading of the parabola like a picture, while ignoring the general characteristics of the parabola. Three of the reasons given by the learners, which reflected the reasons given by sixty four percent of the learners, are presented below:

a. Yes
b No.

Reason(s) 20 is too far from the graph, so the graph will not reach there.

a. Yes
b No

Reason(s) Only Y is 20 it is possible to have a point on the graph when Y is 20

a. Yes
b No.

Reason(s) Because the graph is on the negetive side of the x axis

Each learner's reason was based on reading the graph like a picture. The learners seem to have rejected the fact that for every parabola, any value of *x* has a corresponding value of *y*. The learners seem also to have rejected that the parabola has an infinite domain.

In question 3, the learners were given the task:

If the parabola below is extended indefinitely, do you think that it will ever cut the y-axis?

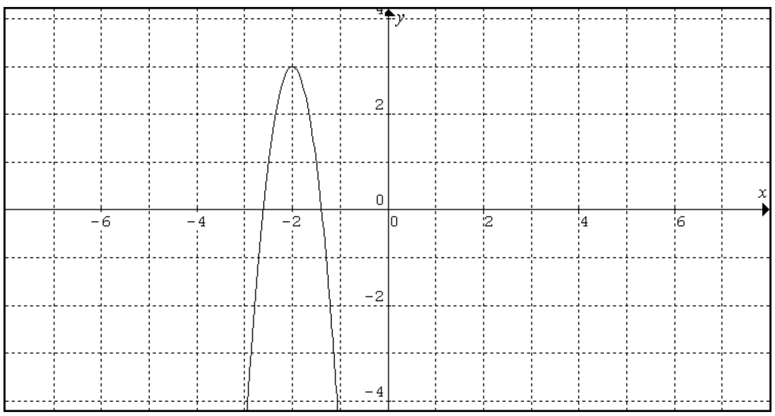
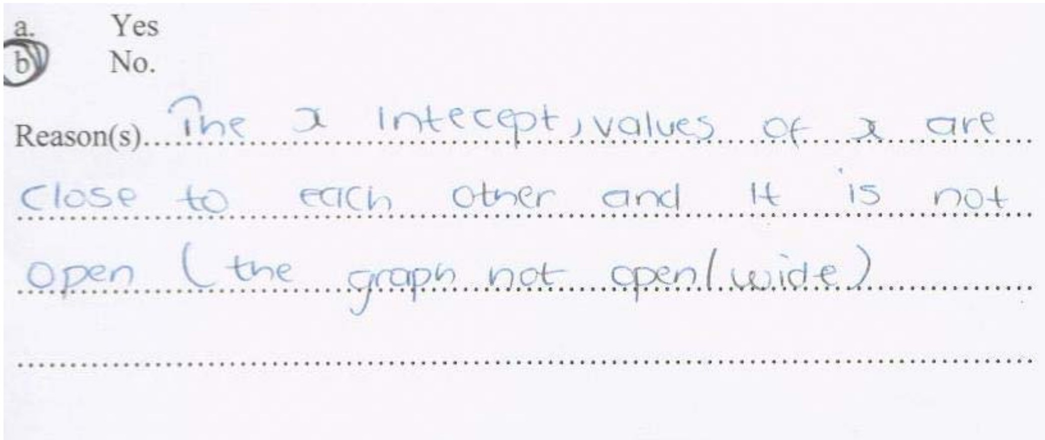


Figure 4.2 Representation of the parabola in question 3.

a. yes
b. No

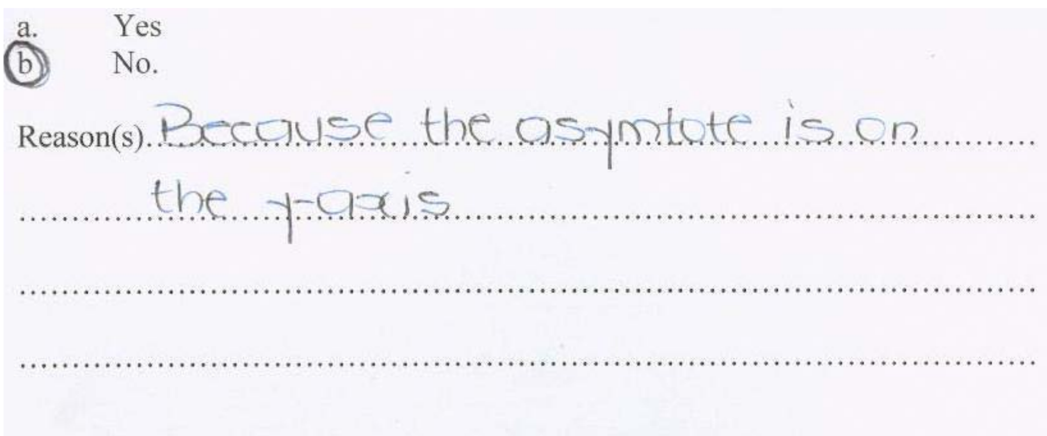
Reason(s)
……………………………………………………………………………………………

In question 3, sixty six percent of learners chose option 'b' - the wrong answer (see Table 4.2). These learners relied on the picture before them and neglected the characteristics of a parabola having an infinite domain. The picture reading of the graph was the reason why the sixty six percent answered that the graph cannot cut the *y*-axis, although for every quadratic function there is a y-intercept. Presented below are two learners' reasons which are a reflection of the reasons given by the 66 percent of the learners who answered that the parabola will not cut the *y*-axis.



a. Yes
b. No.

Reason(s) The x intecept, values of x are close to each other and it is not open (the graph not open (wide)



a. Yes
b. No.

Reason(s) Because the asymtote is on the y-axis

Based on the reading of the graph like a picture, the learners concluded that the graph cannot cut *y*-axis, and even overtly attributed asymptotic to the parabola. The iconic interpretation of the graph which could be traced to the learners' intuition regarding picture reading has been reported in numerous studies including Leinhardt, et al (1990); and Zaslavsky (1997).

- *Determining a special point by only one coordinate of the special point*

This misconception was observed in question 5. The task below was given to the learners in question 5:

The equations of two are given below:

$y = ax^2 + bx + 3$
$y = ax^2 + bx + 7$

Complete the table below. Mark X in the correct column, and justify your choice in the last column.

Table 4.3 Table for question 5

Do the two parabolas have:	Yes	No	Why?
The same line of symmetry?			
The same vertex?			

Sixty six percent of the learners stated that the two parabolas have the same vertex (see Table 4.2). This sixty six percent defined the vertex of a parabola based only on one of its coordinates (x-coordinate) when in fact, the vertex is defined by both *x* and *y* coordinates. Below are the reasons given by two learners which are a reflection of the reasons given by the sixty six percent.

Do the two parabolas have:	Yes	No	Why?
The same line of symmetry	X		a and b are equal in the two equations ∴ turning point is the same
The same vertex?	X		a and b are equal in the two equations ∴ turning point is the same

Do the two parabolas have:	Yes	No	Why?
The same line of symmetry	X		They have the Same value For a and b, So when you use $x = \frac{-b}{2a}$, the answer is equal
The same vertex?	X		They have the Same value For a and b, So when you Use $x = \frac{-b}{2a}$, the answer is equal

The sixty six percent of learners who stated that “the two functions have the same axis of symmetry and so they have the same vertex” made no consideration regarding the y-axis in determining the vertex. A similar finding was reported by Zaslavsky, (1997). This misconception may be as a result of the traditional way in which learners determine the *y*-intercept and *x*-intercept by simply finding the missing coordinate. They seem to have forgotten that one of the coordinates is fixed –zero.

- *Treating quadratic function as if it were just a quadratic equation*

The above misconception was found in question 8. Learners were given the following question:

The parabola below is the graph of $f(x) = x^2 - 3x - 4$

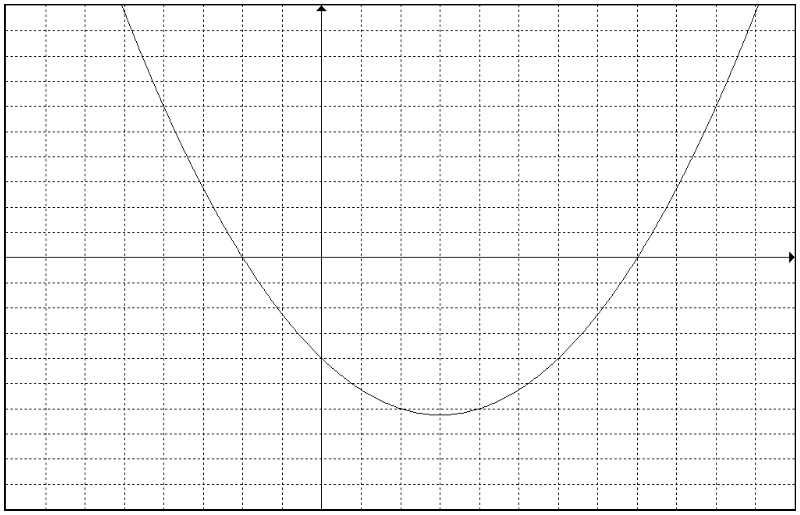


Figure 4.3 Representation of the parabola in question 8.

Which of the following has the same graph as $f(x) = x^2 -3x - 4$ above

a. $f(x) = 2x^2 - 6x - 8$

b. $f(x) = 3x^2 - 9x - 12$

c. $f(x) = 4x^2 - 12x - 16$

d. All of the above

e. None of the above

68 percent of the learners chose option'd' – the wrong answer (see Table 4.2). That is, they chose that $f(x) = 2x^2 - 6x - 8$; $f(x) = 3x^2 - 9x - 12$; and $f(x) = 4x^2 - 12x - 16$ has the same parabola as $x^2 - 3x - 4$. Three learners' reasons which reflect the reasons advanced by 68 percent of the learners for choosing option 'd' are presented below.

d. I chose this answer because

When you divide by 2 at a, 3 at b and 4 at c it will be the same because it will be $x^2 - 3x - 4$

d. I chose this answer because

If we do a division of a co-efficient of x^2 to the whole equation we get $x^2 - 3x - 4$

d. I chose this answer because

they are all equal if we divide all equestions we will get the the equation above

The learners' reasons was that after dividing by the common factor in each case you will get $f(x) = x^2 - 3x - 4$. However, two quadratic functions which have different values in their leading coefficients are totally two different functions. The learners' reasons show that they treated symbolic representations of quadratic functions as if they were treating quadratic equations. This misconception emanates from the fact that learners had already completed working with quadratic equations in which they were taught equivalent equations. In this concept, they were taught that equivalent equations are the same hence they could operate on the simpler equation. In this sense, when working with the equations; $2x^2 – 6x – 8 = 0$; $3x^2 – 9x – 12 = 0$; and $4x^2 – 12x – 16 = 0$, the learners can divide through by 2, 3, and 4 respectively to obtain $x^2 – 3x – 4 = 0$. However, a similar operation of dividing for instance a quadratic function $f(x) = 2x^2 - 6x – 8$ by 2 to obtain $y = x^2 – 3x - 4$ results in two different functions which differ in all values of *x* except for *x*-coordinates of their points of intersection. However, with quadratic equations, the two equations $2x^2 – 6x - 8 = 0$ and $x^2 – 3x – 4 = 0$ are equivalent equations which have the same truth value for any value of *x*. A similar finding was made by Zaslavsky, (1997). The above misconception may have been developed as a result of the learning sequence in which the learners were only exposed to quadratic functions after they had completed extensive work on the quadratic equations earlier. Such misconceptions according to Alwyn, (1989) are a result of learners' "over generalisation of previous knowledge (that was correct in an earlier domain), to an extended domain (where it is not valid)".

- *Over-emphasis of linearity*

In question 10, the learners were given the task:

In the figure below, a parabola is graphed.

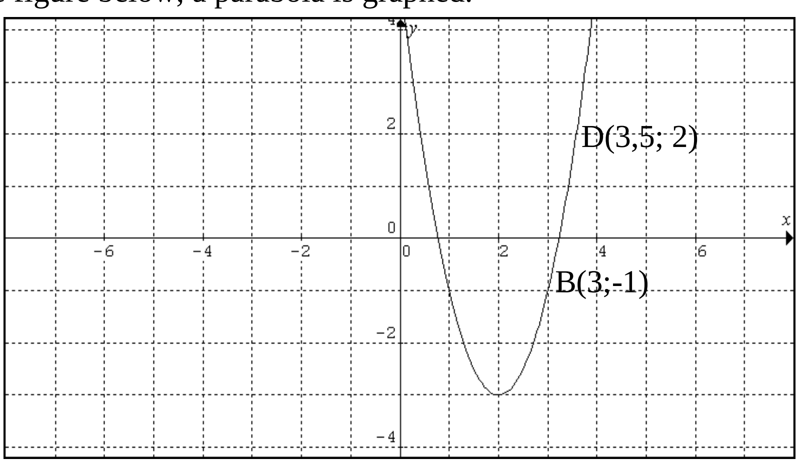


Figure 4.4 Representation of the parabola in question 10.

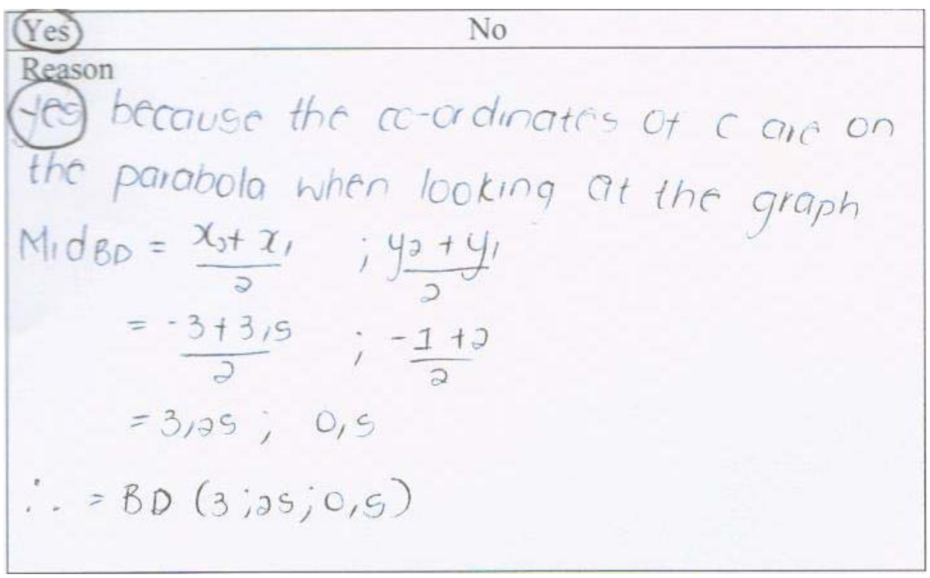
Points B (3; -1) and D (3.5; 2) are two points on the parabola. C (3.25; 0.5) is the mid point of line BD. Is point C on the parabola?

(a) Yes

(b) No

Give reason(s) to justify your answer.

Eighty seven percent of learners chose option ‘a’, which is a wrong answer (see Table 4.2). That is, they chose that a third point C (3.25; 0.5) which is the midpoint of points B (3; -1) and D (3.5; 2) was also on the parabola. The learners’ based their reason on eye measurement. Below is how the eighty seven percent of the learners answered the question.



Yes	No

Reason

yes because the co-ordinates of C are on the parabola when looking at the graph

$\text{Mid}_{BD} = \frac{x_2 + x_1}{2} ; \frac{y_2 + y_1}{2}$

$= \frac{-3 + 3,5}{2} ; \frac{-1 + 2}{2}$

$= 3,25 ; 0,5$

$\therefore = BD\ (3;25; 0,5)$

However, a parabola cannot pass through any three collinear points, but an eye-measurement of some parts of the parabola may make it seem as if the parabola can pass through three collinear points. Over-attachment to linearity has been reported in many studies including Zaslavsky (1997) and Dreyfus & Eisenberg (1983). Reasons for learners’ over-attachment to linearity may be that learners are only introduced to quadratic function in Grade 10 while they are exposed to the linear graphs as early as their preschool days (Leinhardt, Zaslavsky, & Stein, 1990). In addition, since the linear function is the first family of functions which the learners are exposed to, they tend to over-generalise the conjectures which they make when they learn the linear function.

4.2.3 Summary of the content analyses of the learners' questionnaire

Content analyses of the learners' reasons also indicate that there were some concepts in quadratic functions that the learners found difficult. The difficulties were not traced to learners' misconceptions since the reason given by the learners did not portray any misconceptions; rather they found the mathematical reasoning required to answer the questions difficult (Leinhardt, et al., 1990). The following learners' difficulties were recorded from the study: difficulties in translating from graphical to symbolic representation of a quadratic function – question 13; difficulties in interpreting the information in the tabular representation of quadratic functions – questions 11, and 12**;** and difficulties in modelling real life problems using quadratic functions – question 9.

Finally, the analyses of the learners' questionnaire revealed that:

i. Learners had misconceptions in the following concepts:
   Infinite nature of the parabola and its relationship to the *x*–axis (infinite domain) - question 2; infinite nature of the parabola and its relationship to the *y*–intercept – question 3; effects of varying the value of 'c' on the line symmetry, the vertex, the *x*-intercepts, and *y*–intercepts – question 5; special cases of 2 parabolas where $a_1 = ka_2$, $b_1 = kb_2$, $c_1 = kc_2$ – question 8, and connections to linearity – question 10.

ii. Learners had difficulties in the following concepts:
   Modelling real life situations using quadratic functions – question 9; identifying a corresponding parabola if the symbolic form of the function is given question 11; identifying corresponding symbolic representations of functions if the parabola was given – question 12; and the use of the tabular representation of quadratic functions to determine the *x*–intercepts, the turning point, and the region of increase, and decrease - question 13.

iii. Learners had no difficulties or misconceptions in the following concepts:

Effects of varying the value of 'a' in the parabola - question 1; symmetrical properties of the parabola and its connections to the vertex, and the *x*– intercepts question - 4; horizontal shifting of the parabola – question 6; and turning point of the parabola and its relation to symmetry and the axes - question7.

4.3 Result of research question 2

Does educators' MCK affect learners' achievements in quadratic functions?

Data about the educators' MCK collected from the educators' questionnaire, and data about learners' achievement will be used to investigate the effects of educators' MCK on learners' achievement in quadratic functions.

In order to investigate the effect of the educators' MCK on learners' achievement, the researcher, will:

(i) present the result of the educators' MCK, and

(ii) compare learners' performance based on the type of MCK possessed by the educators.

4.3.1 Result of educators' mathematical content knowledge

Educators' MCK was obtained from the responses made by the educators in the content part of the educators' questionnaire. In each question, an educator was scored 1 if he/she gave the correct answer but 0 if the answer was incorrect. The scores are presented in the table below.

Table 4.4 Educators' MCK (N = 17)

Serial number of educators	Number of questions that educator answered correctly	Questions that educator answered incorrectly	Percentage score of educators
1	13	None	100
2	13	None	100
3	13	None	100
4	13	None	100
5	13	None	100
6	13	None	100
7	13	None	100
8	13	None	100
9	13	None	100
10	13	None	100
11	13	None	100
12	13	None	100
13	11	9, 10	85
14	11	8, 9	85
15	11	8, 9	85
16	11	8, 10	85
17	10	8, 9, 10	77
Mean	**12**		**95**

Table 4.4 shows that 12 out of the 17 educators gave right responses to all the questions. Based on the educators' responses as shown in Table 4.4 above, educators with serial numbers 1, 2, 3, to 12, answered all the questions correctly hence they possess strong MCK in all the concepts indicated in the knowledge component indicative of understanding of quadratic functions (see section 3.5.1.1). Educator number 13 has weak MCK in modelling real life situations using quadratic functions - question 9, and connections to linearity – question 10; educator number 14 has weak MCK in special cases of 2 parabolas where $a_1 = ka_2$, $b_1 = kb_2$, $c_1 = kc_2$ – question 8 and in modelling real life situations using quadratic functions - question 9; educator number 15 has weak MCK in special cases of 2 parabolas where $a_1 = ka_2$, $b_1 = kb_2$, $c_1 = kc_2$ – question 8 and in modelling real life situations using quadratic functions - question 9; educator number 16 has weak MCK in special cases of 2 parabolas where $a_1 = ka_2$, $b_1 = kb_2$, $c_1 = kc_2$ – question 8 and in connections to linearity – question 10 while educator number 17 possesses weak MCK in special cases of 2 parabolas where $a_1 = ka_2$, $b_1 = kb_2$, $c_1 = kc_2$ –

question 8, in modelling real life situations using quadratic functions - question 9 and in connections to linearity – question 10.

The reason why educators number 13, 14, 15, 16, and 17 did badly in questions 8, 9, and 10 could be that they did not understand the question or because it was one that they hadn't seen before.

Although educators' experience and qualifications were not used as indicators of educators' knowledge in this study, it is however important to note that the five educators (educator number 13, 14, 15, 16, and 17) who gave wrong responses to some of the questions were newly recruited. Educators 15 and 17 have qualifications but not in teaching (see Table 3.1 for the educator backgrounds).

4.3.2 The effect of educators' MCK on learners' achievement in questions 8, 9, and 10

Some educators gave wrong responses to question(s) 8, 9, and 10. The concepts being tested were: special cases of 2 parabolas where $a_1 = ka_2$, $b_1 = kb_2$, $c_1 = kc_{2;}$ modelling real life situations using quadratic functions; and connections to linearity. These concepts were tested in questions 8, 9, and 10 respectively. To investigate the effect of educators' MCK on learners' achievement, the achievement of learners who were taught by the educators who had strong MCK in the concepts in questions 8, 9, and 10 was compared with the achievement of learners who were taught by educators who had weak MCK in the concepts in each of questions 8, 9, and 10 as shown in Table 4.5 below.

Table 4.5 The effect of' MCK on learners' achievement in questions 8, 9, and 10.

Question	Mean score (%) of learners taught by educators with strong MCK	Mean score (%) of learners taught by educators with weak MCK
8	28	05
9	13	00
10	16	00
Mean	**19**	**1.7**

Table 4.5 shows that the mean score of learners who were taught by educators who have weak MCK in the concepts in questions 8, 9 and 10 was 1.7 percent, while the mean score of learners that were taught by educators that have strong MCK in the three questions was 19 percent. Although the achievement of the learners in questions 8, 9, and 10 was generally poor, the achievement of learners taught by educators that possess strong MCK was higher than the achievement of learners that were taught by educators that have weak MCK.

4.3.3 The effect of the different MCK of educators on learners' achievement

Educators number 13, 14, 15, 16, and 17 had weak MCK in question(s) 8, 9, and 10. The achievement of the learners in the questions where these educators possess strong MCK was compared with the achievement of the learners in the questions where the educators possess weak MCK.

Table 4.6 The effects of the educators' different MCK on learners' achievement

Educator	Learners' score (%) in questions in which educator has strong MCK	Learners' scores (%) in questions in which educator has weak MCK
13	48	00
14	42	00
15	39	05
16	42	00
17	41	03
Mean	**42**	**1.6**

Table 4.6 shows that the mean score of learners in the questions in which educators (13, 14, 15, 16, & 17) have strong MCK is 42 percent, while the mean score of learners in the questions in which the educators have weak MCK is 1.6 percent. Table 4.6 also indicates that the achievement of the learners of each educator was higher in the questions in which the educator has strong MCK than in the questions in which the educator has weak MCK. Caution must be taken when comparing the learners' achievement in the concepts in which educators' have strong MCK with the learners' achievements in the concepts in which the educators show weak MCK. The reason is that the questions in which some educators have weak MCK are more difficult than the questions in which the educators have strong MCK.

4.3.4 Discussion of findings in research question 2

It might be imperative to note that the same results of learners' achievement used as evidence to draw conclusions about the effect of educators' MCK on the achievement of learners, was also used to draw conclusions about the effect of other components of the educators' PCK. Strong claims can therefore not be made in the conclusion since the learners' achievement was influenced by many variables.

The results in Tables 4.5 and 4.6 indicate that the achievement of learners whose educators have strong MCK in quadratic functions was higher than the achievement of learners who were taught by educators who have weak MCK. The result suggests that the learners' achievement in quadratic functions may have been influenced by the educators' MCK, however, there were other factors which could also affect the learners' achievement in quadratic functions but which were not considered in this analysis. From this result, it is evident that for effective teaching and learning to take place, an educator must have a firm grasp of the concept that is deliberated upon in the classroom. The result indicated above corroborates with Capraro, et al., (2005) who found that lack of MCK leads to ineffective instruction. It also supports the assertion generally found in the literature especially Ball's (2001) contention that to teach mathematics effectively, educators must have knowledge of mathematics and possess explicit conceptual understanding of the principles and meaning underlying the mathematical topics that they teach in the classroom. Table 4.6 indicates the poor achievement of learners (1.6%) in the concepts in which their educators have weak MCK. This result shows that it may not be possible for learners to have meaningful learning or conceptual understanding in quadratic functions if the educator lacks conceptual understanding of quadratic functions. In a situation like this, educators' knowledge or rather educators' lack of knowledge of the concept becomes a stumbling block in the learners' road to success in quadratic functions. It must be indicated that the questions in which educators 13, 14, 15, 16, and 17 showed weak MCK (see 4.3.1) were more difficult than the other questions. This may well be the reason why the learners did not do well in these questions

4.4 Result of research question 3

Does the educators' knowledge of learners' conceptions and misconceptions affect learners' achievement in quadratic functions?

Data obtained from the educators' questionnaire about educators' knowledge of the learners' conceptions and misconceptions, and data on learners' achievement was used to investigate the effects of educators' knowledge of learners' conceptions and misconceptions, on learners' achievement. To do so, the researcher will:

(i) present the result of the educators' knowledge of learners' conceptions and misconceptions in quadratic functions;

(ii)determine the effect of educators' knowledge of learners' misconceptions on learners' achievement;

(iii) determine the effect of educators' misconceptions on learners' achievement; and

(iv) determine the effect of educators' knowledge of learner' conceptions and learners' achievement.

4.4.1 The result of educators' knowledge of learners' conceptions and misconceptions

Content analysis of the educators' written expectations of the learners' reasons was used to investigate the educators' knowledge of learners' conceptions and misconceptions. In the content analyses, the researcher compared the educators' written expectations of their learners' reasons with the reasons given by the learners themselves in each question. From the content analyses, the educators were categorised into three categories according to their knowledge of learners' conceptions and misconceptions. The three categories are:

(i) Educators who were aware of learners' conceptions and misconceptions. These educators were able to predict the learners' reasons for their answers hence they were able to predict their learners' likely misconceptions.

(ii) Educators who were unaware of their learners' conceptions and misconceptions. The educators in this category never anticipated their learners to have misconceptions in the questions and so they were unable to predict the learners' reasons for their choice of answer.

(iii) Educators who showed the same misconceptions as their learners. These educators accepted as correct the incorrect reasons given by the learners.

In Table 4.7 that follows, the educators' knowledge of their learners' conceptions and misconceptions in each question is presented. Table 4.7 indicates the question(s) which educator was aware of learners' conceptions or misconceptions; unaware of learners' conceptions or misconceptions; and the questions educators had the same misconceptions.

Table 4.7 Educators' knowledge of learners' conceptions and misconceptions

Questions	Educators who were aware of learners' conceptions or misconceptions	Educators who were unaware of learners' conceptions or misconceptions	Educators who have misconceptions about their learners
1	1,2,3,4,5,6,7,8,9,10,11. 13,15.	1,2,14,16,17.	nil
2	1,2,3,4,5,6,7.	8,9,10,11,12,13,14, 15,16,17.	nil
3	2,4,7,8,9.	1,3,5,6,10,11,12,13, 14,15,16,17.	nil
4	2,3,5,7,8,10,11,12,13, 14,15.	1,4,6,9,16,17.	nil
5	1,5,7,9,10.	2,3,4,6,8,11,12,13,14 15,16,17.	nil
6	1,2,3,4,5,6,7,8,9,11,12, 13,14,16.	10,15,17.	nil
7	1,2,3,4,5,6,7,8,9,10,12, 13,14,16.	11,15,17.	nil
8	7,8,12.	1,2,3,4,5,6,9,10,11,13.	14,15,16,17.
10	2,3,6,8.	1,4,5,7,9,10,11,12,14,15.	13,16,17.

Questions 9, 11, 12, and 13 were not included in Table 4.7. In questions 9, 11, and 12, there was no clarity in the reasons given by the learners. In question 13, there was evidence of learners' inability to use the tabular representations to answer the questions; learners resorted to using the symbolic representations in answering the question. The inability was not due to a misconception. As a result of the above, the researcher could not determine if the educators' were aware or unaware of the learners' conceptions and misconceptions in questions 9, 11, 12, and 13.

4.4.2 The effects of educators' knowledge of learners' misconceptions on learners' achievement

Learners had misconceptions in questions 2, 3, 5, 8, and 10 (see section 4.2.3). To determine the effect of educators' knowledge of learners' misconceptions on learners' achievement, the researcher compared the achievement of learners who were taught by educators who were aware of learners’ misconceptions with the achievement of learners’ who were taught by educators who were unaware of learners’ misconceptions in questions 2, 3, 5, 8, and 10.

Table 4.8 The effect of educators’ knowledge of learners’ misconceptions on learners’ achievement

Question	Number of educators who were aware of learners’ misconceptions	Number of educators who were unaware of learners’ misconceptions	Learners’ achievement taught by educators who were aware of learners’ misconceptions	Learners’ achievement taught by educators who were unaware of learners’ misconceptions
2	7	10	49	27
3	5	12	48	28
5	5	12	46	29
8	3	14	40	19
10	4	13	28	09
Mean	**5**	**12**	**42.2**	**22.4**

Table 4.8 indicates that most educators (12 out of 17) were unaware of their learners’ misconceptions. Table 4.8 also indicates that the achievement of the learners (42.2%) taught by educators who were aware of the learners’ misconceptions was higher than the achievement of learners (22.4%) taught by educators who were unaware of the learners’ misconceptions.

4.4.3 The effects of educators’ misconceptions on learners’ achievement

Educators numbers 13, 14, 15, 16, and 17 have misconceptions in question(s) 8 and 10 (see Table 4.7). To determine the effect of educators’ misconceptions on learners’ achievement, the researcher compared learners’ achievement in the questions where the educator was aware of learners’ likely misconceptions, with learners’ achievement in the questions in which the educator had the same misconceptions.

Table 4.9 The effects of educators' misconceptions on learners' achievement

Educator	Learners' achievement in the questions where educator was aware of learners' misconceptions	Learners' achievement in questions where educator had the same misconceptions as the learners
13	83	00
14	80	00
15	75	10
16	70	00
17	60	05
Mean	**74**	**03**

Table 4.9 shows that the achievement of the learners' was high (74%) in the questions in which the educators did not have misconceptions but low (03%) in the questions in which educators have misconceptions. For instance the achievement of learners taught by educator number 13 was 83 percent in the questions where educator number 13 has knowledge of learners' misconceptions, but was 0 percent in the questions where educator number 13 has misconceptions.

4.4.4 The effects of educators' knowledge of learners' conceptions, on learners' achievement

The learners had no misconceptions in questions 1, 4, 6, and 7 (see section 4.2.3). To determine the effect of educators' knowledge of learners' conceptions, on learners' achievement, the researcher compared the achievement of learners taught by educators who were aware of learners' conceptions in questions 1, 4, 6, and 7, with the achievement of learners taught by educators who were unaware of learners' conceptions in questions 1, 4, 6, and 7.

Table 4.10 The effects of educators' knowledge of learners' conceptions, on learners' achievement

Question	Number of educators who were aware of learners' conception	Number of educators who were unaware of learners' conceptions	Achievement of learners taught by educators who were aware of learners' conceptions	Achievement of learners taught by educators who were unaware of learners' conceptions
1	13	4	86	65
4	11	6	78	60
6	14	3	91	70
7	14	3	86	67
Mean	**13**	**5**	**85.3**	**65.5**

Table 4.10 indicates that in each of the questions where the learners' do not have misconceptions, the achievement of the learners (85.3%) whose educators were aware of their conceptions was higher than the achievement of learners (65.5%) whose educators were unaware of their conceptions.

4.4.5 Discussion of the findings in research question 3

It should be indicated that educators 13, 14, 15, 16 and 17 who had misconceptions in questions 8, 9, and 10 were the only educators who gave wrong responses to those questions (see sections 4.4 and 4.7). The above result shows an overlap of evidence. This overlap was as the result of the nature of the concept of PCK, which involves the transformation of the subject matter knowledge for teaching. On this Wu (2004) articulated that educators with good PCK most often are predominantly those educators with good subject matter knowledge. Also, learners' achievements in the same set of questions were used as evidence to draw different conclusions about the different components of the educators' PCK. As a result, the conclusions drawn about the effect of the educators' knowledge of learners' conceptions, and misconceptions cannot be strong claims. Tables 4.8, 4.9, and 4.10 indicate that the learners' achievement was better when the educators were aware of learners' conceptions and likely learners' misconceptions, than when the educators were unaware of learners' conceptions and misconceptions. The results in Tables 4.8, 4.9, and 4.10 suggest that for educators to be effective in their work of facilitating students' learning, they must possess knowledge of learners' conceptions and misconceptions about the concepts they teach in the classroom. If teaching and learning are viewed from the constructivist perspective, educators' knowledge of their learners' conceptions and misconceptions is an important aspect of the educators' PCK. When the educators are aware of their learners' thinking about the concept they deliberate in the classroom, it becomes easier for the educators to guide the learners' in the meaning making process. This could be the reason why the achievement of learners' taught by educators who were aware of their learners' conceptions was higher than the achievement of learners taught by educators who were unaware of their learners' conceptions. Educators' awareness of learners' misconceptions is very important because mistakes can provide valuable insights into the implicit knowledge of the problem solver (Martz, 1982). The poor achievement of learners ((22.4%) taught by educators who are unaware of students' misconceptions may suggest that it is unlikely that educators' who are unaware of learners' misconceptions can direct the classroom discourse for meaningful learning to take place. On the other hand, educators who are aware of learners' likely misconceptions are better equipped to address the misconceptions and achieve meaningful learning in the learners. The above results support the findings of Hill, Ball, and Schilling (2008) which indicate that familiarity with aspects of learners' mathematics thinking, such as common learners' errors is one element of teaching.

4.5 Result of research question 4

Does educators' knowledge of strategies for teaching quadratic functions affect learners' achievement in quadratic functions?

Data obtained from analyses of the educators' interviews, and from learners' achievement will be used to investigate the effects of educators' knowledge of strategies for teaching quadratic functions on learners' achievement in quadratic functions.

To determine the effects of educators' strategies on learners' achievement, the researcher will:

(i) present the result of the educators' knowledge of strategies for teaching quadratic functions obtained from the analyses of the interview;
(ii) determine the effects of educators' strategies on learners' achievement in questions 2, 3, 5, 8, and 10.
(iii) determine the effect of educators' strategies on learners' achievement in questions 1, 4, 6, and 7; and
(iv) determine the effects of educators' misleading strategies, on learners' achievement in questions 8, 9, and 10.

4.5.1 The result of the content analyses of the educators' interview

Content analyses of the educators' interviews revealed three categories of educators:

(i) Educators who gave detailed teaching strategies for explaining the mathematical concept in dealing with learners' misconceptions. Included in the detailed strategies are various activities such as providing explanations in which a series of linked ideas were expressed, providing analogies and suggesting mathematical experiments or investigations, and the use of technological instruments (e.g. calculators) in helping learners to reorganise their conceptions. The detailed strategies are good in helping learners develop conceptual understanding and in resolving learners' misconceptions.

(ii) Educators who gave brief teaching strategies as a way of teaching the mathematical concepts to the learners. Central to educators in this category was their suggestion of the use of verbal explanations which often restated the educators' understanding of the ideas in the concept. The brief strategies however are not good enough in guiding learners to develop conceptual understanding or resolving misconceptions.

(iii) Educators who gave explanations that reinforced the learners' misconceptions. Educators' in this category have the same misconceptions as their learners.

The type of strategies formulated by the educators in each question is presented below.

Table 4.11 Type of strategies formulated by the educators

Questions	Educators who gave detailed teaching strategies	Educators who gave brief teaching strategies	Educators who gave misleading strategies
1	1,2,3,4,5,7,8,9,10,11,15.	6,12,13,14,16,17.	none
2	1,2,3,6,7.	4,5,8,9,10,11,12, 13,14,15,16,17.	none
3	2,8,9.	1,3,4,5,6,7,10,11,12 13,14,15,16,17.	none
4	3,7,8,10,11,13,14,15.	1,2,4,5,6,9,12,16,17.	none
5	1,5,9.	2,3,4,6,7,8,10,11,12, 13,14,15,16,17.	none
6	1,3,4,7,8,9,11,12,13,16.	2,5,6,10,14,15,17.	none
7	1,3,4,5,7,8,9,12,13,14,16.	2,6,10,11,15,17.	none
8	8,12.	1,2,3,4,5,6,7,9,10, 11,13.	14,15,16,17.
9	2,6,8.	1,3,4,5,7,9,10,11, 12,16.	13,14,15,17.
10	2,6.	1,3,4,5,7,8,9,10,11, 12,14,15.	13,16,17
11	7,9,10.	1,2,3,4,5,6,8,11,12, 13,14,15,16,17.	none
12	5,7,9.	1,2,3,4,6,8,10,11, 12,13,14,15,16,17.	none

Question 13 was not presented in the result. Question 13 consists of three different questions namely 13a; 13b, and 13c, and educators gave different strategies to each of the questions. It was not possible to classify each educator’s strategies into categories.

The result indicated that in each question educators formulated different strategies for teaching the question. In question 1 for instance, eleven educators’ formulated detailed strategies, six educators gave brief strategies, and none of the educators formulated strategies that can impede learners’ understanding in question 1. The result also indicated that some educators formulated misleading strategies in questions 8, 9, and 10. In question 8, educators 14, 15, 16, and 17 gave misleading strategies; in question 9, educators 13, 14, 15, and 17 gave misleading strategies; and in question 10, educators 13, 16, and 17 formulated misleading strategies.

4.5.2 The effects of educators' strategies on learners' achievement in questions 2, 3, 5, 8 and 10

Learners had misconceptions in questions 2, 3, 5, 8, and 10 (see section 4.2.3). Achievement of learners taught by educators who formulated detailed strategies in questions 2, 3, 5, 8, and 10 was compared with the achievement of learners' taught by educators who gave brief teaching strategies in questions 2, 3, 5, 8, and 10.

Table 4.12 The effects of educators' strategies on learners' achievement in questions 2, 3, 5, 8, and 10.

Question	Number of educators who gave detailed strategies	Number of educators who gave brief strategies	Learners' achievement taught by educators who gave detailed strategies	Learners' achievement taught by educators who gave brief strategies
2	5	12	52	29
3	3	14	53	26
5	3	14	50	31
8	2	11	40	25
10	2	12	30	15
Mean	**4**	**13**	**45**	**25.2**

Table 4.12 shows that in the questions where learners had misconceptions, the achievement of learners (45%) taught by educators who formulated detailed strategies for teaching the concept in the questions was higher than the achievement of learners (25.2%) whose educators formulated brief strategies. Table 4.12 also indicates that most educators (13 out of 17) gave poor strategies as a means of teaching the mathematical concepts in the questions where learners' had misconceptions.

4.5.3 The effects of educators' strategies on learners' achievement in questions 1, 4, 6, and 7

Learners did not have misconceptions in questions 1, 4, 6, and 7 (see section 4.2.3). To determine the effects of educators' knowledge of strategies on learners' achievement in questions where learners did not have misconceptions, the achievement of learners taught by educators who formulated detailed strategies for teaching the concepts in questions 1, 4, 6, and 7 was compared with the achievement of learners taught by educators who formulated brief strategies for teaching the concepts in questions 1, 4, 6, and 7.

Table 4.13 The effects of educators' strategies on learners' achievement in questions 1, 4, 6, and 7.

Question	Number of educator who gave detailed strategies	Number of educators who gave brief strategies	Learners' achievement taught by educators who gave detailed strategies	Learners' achievement taught by educators who gave brief strategies
1	11	6	81	68
4	8	9	81	63
6	10	7	95	71
7	11	6	88	72
Mean	10	7	**86.3**	**68.5**

Table 4.13 shows that the majority of educators (10 out of the 17) formulated detailed strategies for teaching the mathematical concepts in the questions 1, 4, 6, and 7. This means that most educators had good PCK of knowledge of strategies for teaching the mathematical content knowledge in questions where learners do not have misconceptions. Table 4.13 also shows that the achievement of learners (86.5%) taught by educators who formulated detailed teaching strategies was higher than the achievement of learners (68.5%) taught by educators who gave brief teaching strategies.

4.5.4 The effects of educators' misleading strategies on learners' achievement

Comparison of the achievement of learners taught by educators who gave detailed teaching strategies questions 8, 9, and 10, and the achievement of learners taught by educators who gave strategies that can reinforce learners' misconceptions in questions 8, 9, and 10 is given in Table 4.14.

Table 4.14 The effects of educators' misleading strategies on learners' achievement in questions 8, 9, and 10.

Question	Achievement of learners taught by educators who gave detailed strategies	Achievement of learners taught by educators who gave strategies that are misleading
8	40	04
9	20	00
10	30	00
Mean	**30**	**1.3**

Table 4.14 indicates that the achievement of learners (30%) who were taught by educators who provided detailed strategies for teaching the concepts in questions 8, 9, and 10 was higher than the achievement of learners who were taught by educators' who gave strategies that were misleading.

4.5.5 Discussion of the findings in research question 4

The only questions where educators gave misleading strategies are questions 8, 9, and 10, and these were given by the same educators who gave wrong responses to the questions (see Table 4.4 & 4.11). This indicates an overlap of evidence. One cannot draw strong claims in the face of this overlap and the fact that the same achievements of learners were used to draw conclusion in the different components of pedagogical content knowledge. Results from Tables 4.12, 4.13, and 4.14, indicate that the achievement of learners taught by educators who have rich PCK of knowledge of strategies was higher than the achievement of learners who were taught by educators who have poor PCK of knowledge of teaching strategies. As indicated in Table 4.12 most educators (13 out of 17) have poor PCK of knowledge of strategies in the questions where learners have misconceptions. From this result, educators' non-knowledge of powerful strategies for communicating the mathematical concepts in the questions where learners have misconceptions may be a contributing reason why learners' misconceptions persist even after being taught by the educators. Educators' poor PCK of strategies may also be among the reasons why their learners' achievement was low (25.2%), compared with the achievement of learners (45%) taught by educators who possess rich PCK of knowledge of strategies in teaching the concepts where learners have misconceptions. The above result suggests that when an educator lacks the knowledge of strategies for effective communication of a concept, the achievement of the learners may be impeded. The PCK of knowledge of specific instructional strategies is important for every educator because learners' construction of knowledge is often only successful with appropriate instructional support and guidance.

4.6 Summary

In Chapter 4, the data collected from the learners' questionnaire, the educators' questionnaire, and educators' interviews were analysed. Analyses of the learners' questionnaire revealed the learners' achievement and misconceptions in quadratic functions reported in this study. Data from the educators' questionnaire and interviews were analysed in relation to learners' achievement. The analyses revealed the role of each component of educators' pedagogical content knowledge in the learning of quadratic functions.

CHAPTER 5

KEY FINDINGS, IMPLICATIONS, RECOMMENDATIONS AND CONCLUSSION

5.1 Introduction

This chapter presents the summary of key findings in relation to the research questions, implications of findings, some recommendations, and limitations of the study, suggestions for future study, and finally the conclusion.

5.2 Summary of key findings

The summary of key findings is presented according to the research questions.

5.2.1 What are the learners' main misconceptions in quadratic functions?

Content analyses of the reasons given by the learners for their choice of answers revealed that the learners had some misconceptions in quadratic functions. The main misconceptions include:

(i) Limiting the graph of the quadratic functions only to the visible region. The learners considered only the part of the parabola that they could see, thereby ignoring the general characteristics of the parabola such as its infinite domain. Iconic reading of the graph was the reason why learners attributed vertical asymptote to the parabola. A similar finding was made by Zaslavsky (1997).

(ii) Treating the quadratic function as if it were a quadratic equation. This misconception may have been developed as a result of the learning sequence in which the learners were only exposed to quadratic functions after they had completed extensive work on the quadratic equations. Such misconceptions according to Alwyn (1989, p. 11) was a result of “learners’ over-generalisation of previous knowledge (that was correct in an earlier domain), to an extended domain (where it is not valid)”.

(iii) Defining a special point by only one coordinate of the special point. Learners defined the vertex of a parabola based only on one of its coordinates (x-coordinate) when in fact, the vertex is defined by both x and y coordinates.

(iv) Over-attachment to linearity. The learners determined that three collinear points can pass through a parabola, although there are no three collinear points in a parabola. Reasons for learners over attachment to linearity may be that they were only introduced to quadratic functions in grade 10 while they were exposed to linear graphs as early as preschool days (Leinhardt, Zaslavsky, & Stein, 1990). In addition, since the linear function was the first family of functions which the learners were exposed to, they tended to over-generalise the conjectures which they made when they learnt the linear function.

The questionnaire also indicated that the learners had difficulties in solving some questions. The difficulties were not necessarily the result of misconceptions; rather, learners found the mathematical reasoning required to answer the questions difficult. The following learners’ difficulties were indicated: difficulty in translating the parabola to the symbolic representation of the function; inability to model real life problems using quadratic functions; difficulty in interpreting the information in the tabular representational mode.

5.2.2 Does educators’ MCK affect learners’ achievement in quadratic functions?

The study indicates that the achievement of learners whose educators have strong MCK in quadratic functions was higher than the achievement of learners who were taught by educators who have weak MCK. Indicated also was that learners’ performed well in questions in which their educator possess strong MCK but learners’ performance in questions where their educator have weak MCK was poor.

5.2.3 Does educators' knowledge of learners' conceptions and misconceptions affect learners' achievement in quadratic functions?

In terms of educators' knowledge of learners' conceptions and misconceptions, the study showed three categories of educators: (i) Educators who were aware of learners' conceptions and misconceptions. (ii) Educators who were unaware of their learners' conceptions and misconceptions. (iii) Educators who showed the same misconceptions as their learners.

The result of educators' knowledge of learners' conceptions and misconceptions showed that most educators (12 out of 17) were not aware of their learners' misconceptions. The achievement of these learners was poor compared with the achievement of learners taught by the five educators who were aware of the learners' misconceptions. Most of the educators (13 out of 17) were aware of their learners' conceptions in the questions where their learners did not have misconceptions. The achievement of these learners was higher than the achievement of the learners whose educators were unaware of their learners' conceptions. In general, the results indicate that the achievement of learners taught by educators who were aware of the learners' conceptions and misconceptions was higher than the achievement of learners taught by educators who were unaware of the learners' misconceptions. Results also indicate that when educators have the same misconceptions as the learners it becomes unlikely for learners to exhibit conceptual understanding.

5.2.4 Does educators' knowledge of strategies for teaching quadratic functions affect learners' achievement in quadratic functions?

The study indicated that the ability of educators to formulate strategies for teaching quadratic functions differs. There were three groups of educators in this regard. (i) There were educators who gave detailed teaching strategies for explaining the mathematical concept and dealing with learners' misconceptions. Included in the detailed strategies were various activities such as providing explanations in which a series of linked ideas were expressed, providing analogies and suggesting mathematical experiments or investigations, including the use of technological instruments (e.g. calculators) in helping learners to reorganise their conceptions. (ii) There were educators who gave brief teaching strategies as a way of teaching the mathematical concepts. Central to educators in this category was their suggestion of the use of verbal explanations which often restated the educators' understanding of the ideas in the concept. Brief strategies are not good enough to convey mathematical concepts and or dealing with learners' misconceptions (iii) Educators who gave explanations that reinforced the learners' misconceptions. In this case, educators had the same misconceptions as their learners.

The study indicated that educators' PCK of knowledge of strategies for teaching quadratic functions has influence on learners' achievement in quadratic functions. Achievement of learners who were taught by educators who formulated detailed strategies was higher than those taught by educators who formulated brief strategies for teaching quadratic functions. The achievement of learners in the questions where educators gave strategies that were misleading was very poor; their mean achievement was 1.3 percent. This result shows that meaningful learning cannot take place in the classrooms of educators who formulate strategies that are misleading.

5.3 Implications for teaching quadratic functions

The results revealed the learners' misconceptions and difficulties when they learn quadratic functions. The findings suggest that educators may plan their lessons in such a way that they can resolve the learners' likely misconceptions and identified difficulties. From this perspective, emphasis should be given to the infinite domain of the parabola, the relation between the quadratic functions and the quadratic equations, the analogy between the quadratic functions and the linear function, and the determination of the vertex of the parabola. Educators should also emphasise the concept of modelling using quadratic functions as the learners' found it to be overly difficult. The results also showed that learners exhibited much weakness in visual reasoning and an inability to translate between graphical and symbolic representations of quadratic functions. This points to the need for emphasis to be given to the concept of multiple representations of functions in the classroom.

The finding that learners' achievement in quadratic functions is influenced by the educators' PCK of knowledge of subject matter, points to the need to assign only qualified mathematics educators to teach mathematics in the grade 11.This finding also points to the need for the Heads of Mathematics Department in each secondary school to mentor their mathematics educators with the aim of developing educators' MCK in quadratic functions.

It is indicated in the study that most educators (12 out of the 17) who participated in the study were unaware of learners' likely misconceptions in quadratic functions. It was also highlighted that the educators' PCK of knowledge of representation has an effect on the achievement of learners. This implies that to improve on the learners' achievement in quadratic functions, educators need to be trained on possible learners' conceptions and misconceptions in quadratic functions. To tap into learners' thinking and discover their misconceptions, the educators can assess understanding by using tasks that are similar to the questions in the questionnaire used in this study whereby learners were required to give reasons for their answers. The questions are such that quantitative information was suppressed to the minimum in order to give the learners the opportunity to focus on the quantitative considerations about the question.

The study has shown that the educators' abilities to formulate strategies for teaching quadratic functions influences the achievement of their learners, and that most educators (13 out 17) cannot formulate strategies to deal with learners' misconceptions. This finding demands that educators should use detailed strategies which include that learners be allowed to investigate the concepts in quadratic functions, make conjectures and prove them in the presence of other learners, as against the "telling" method. The use of calculators was indicated to be very useful in carrying out such investigations. Educators need to be equipped with knowledge of strategies and powerful representations that can support and extend the understanding of learners in quadratic functions. To generate powerful representations, educators should look at the learners' existing conceptions and misconceptions and formulate representations that can reorganise their thinking.

5.4 Recommendations

The study indicates that educators' MCK, knowledge of the learners' conceptions and misconceptions, and knowledge of strategies for teaching quadratic functions, have effects on learners' achievement in quadratic functions. Therefore, there is need for well articulated support and professional development programmes for Grade 11 mathematics educators. To improve on educators' MCK of quadratic functions, mathematics subject specialists and the Heads of Department (H.O.Ds) of mathematics in each school should have continuous support programmes for newly appointed educators and educators whose area of specialisation is not mathematics**.** To improve learners' achievement in quadratic functions, educators with good knowledge of strategies for teaching quadratic functions need to be identified so that they can mentor the educators with weak PCK in knowledge of strategies. The mentors should help the educators to come up with conceptual explanations, models, analogies and powerful representational modes relevant to the teaching of the specific concepts in quadratic functions. In line with this suggestion, mathematics educators should not be assigned classes in the last two periods of every friday so that they can use these periods for meetings. In these meetings, educators can seek clarifications on concepts that are challenging to them. Lesson observation is also recommended as part of professional development programmes for newly appointed educators. Lesson observations and assigning mentors to educators has been part of very successful professional development programmes in Japan – a country reputed for excellence in mathematics achievement (Tendre, 1999).

Workshops organised for mathematics educators should not only deal with issues of educators' MCK. They should also address educators' PCK of the knowledge of learners' conceptions and misconceptions in specific topics in mathematics. Workshops aimed at improving mathematics educators' PCK of knowledge of learners can borrow from the strategy used in this study to investigate educators' PCK of knowledge of learner. The strategy involves asking educators to predict learners' answers as well as reasons for answers in given questions. This can help improve educators' knowledge of learners' conceptions and misconceptions. By thinking individually or in groups about learners' likely answers and misconceptions, educators are likely to tap into these conceptions and misconceptions.

Membership in professional bodies like AMESA (the Association for Mathematics Educators of South Africa) and SAMSTE (the Southern African Association for Research in Mathematics, Science and Technology Education) is recommended for all mathematics educators. Through these organisations educators can keep abreast with current researches that are relevant to their levels of practice. Schools should encourage mathematics educators to register with these bodies by paying the educators' subscription fees, and sponsoring their educators to attend conferences of these organisations.

Mathematics educators in each Area Project Office (APO) should be encouraged to form communities of practice (Wenger 1998). The community of practice should be coordinated by the mathematics subject specialist in each APO and should comprise of individual mathematics educators who make a coordinated effort to improve mathematics learning. A well functioning community of practice can function as an avenue where educators can share their experiences geared towards improving the teaching and learning of mathematics.

Finally, the department of education should provide modern technologies for use in the teaching and learning of quadratic functions in particular and mathematics in general. The use of these technologies (computers, graphing programmes, and calculators) helps learners to easily make conjectures and also reduces the tendency of learners to entertain misconceptions about quadratic functions.

5.5 Limitations of the study

The sample size used in this study was small; hence, care must be taken about the extent to which one generalises the results. Financial constraints and the researcher's inexperience did not permit of a large scale study. Although the study was conducted with the understanding that what educators know determine what is done in the classroom, not observing the educators in their classrooms seems to be a limitation of the study. Observations in the classroom could have revealed additional and useful information regarding the sample and could also have shown if the educators actually used the knowledge which they profess to possess. Time constraints were the reason why the researcher was unable to carry out such observations. The researcher teaches in a school that could not afford him time off. The Heads of Department of mathematics in the schools where the sample educators teach could have done the observations but they were also either too busy with their school work or inexperienced in doing such observations.

As indicated in section 3.5.2.2, the educators responded to the questionnaire without the researcher's supervision. Non supervision of the educators was a limitation as it may have given the educators the opportunity to seek assistance from other educators. To reduce this possibility, the researcher emphasised the need for the educators to attend to the questionnaire independently (see Appendix v). Also, the educators knew the purpose of the research and the extent to which participation was required from them before they agreed to participate in the study. They were moreover eager to learn how their knowledge influences their learners' achievement. They may therefore not have engaged in any activity that can distort the results. The above measures seem to have reduced the limitation of non-supervision since the responses given by the educators' do not portray that they shared their responses with one another.

There were some limitations that emanated from some question items in the instrument. In question 2, there were no arrowheads to show that the parabola extends indefinitely (see Appendix iv & v). This may have affected the reasoning of learners and the educators. In question 9, it was not explicitly stated that the charge paid by the customers depended only on the number of minutes spent on a call (there was no outstanding charge). This may have affected the performance of both the learners and the educators in question 9. In question 10, there was inconsistency in the use of notation for the decimal point. Both ‘,’ and ’.’ were used interchangeably to represent the position of the decimal. This may have confused the respondents. Although the coordinates of B and D in question 10 were indicated, perhaps dots indicating points B and D on the graph (see Appendix iv & v) and a line joining the two points would have strengthened learners’ and educators’ understanding of the question. These limitations may have impacted on the quality of the result.

Another limitation of the study is that there were overlaps of evidence as the same evidence (learners’ achievement) was used to draw conclusion about the educators’ MCK, the educators’ knowledge of learners’ misconceptions, and the educators’ knowledge of strategies for teaching quadratic functions. In the face of these overlaps one cannot make strong claims about the findings of the investigation.

5.6 Suggestions for further research.

Further research should explore the relationship between educators’ pedagogical content knowledge and learners’ achievement in the learning of quadratic functions. In the study the researcher should observe educators in the classroom setting which may reveal additional and useful information about the topic.

Explore the criteria for integrating information technologies in the teaching and learning of quadratic functions.

Explore the possibility of using context questions to enhance learners’ achievement in quadratic functions.

5.7 Conclusion

The study has highlighted the effects of educators' PCK of quadratic functions on learners' achievement. In particular, the study has shed light on how educators' MCK, educators' knowledge of learners', and educators' knowledge of strategies affect learners' achievement in quadratic functions. In order to investigate the effect of the educators PCK on learners, the researcher first investigated learners' misconceptions and achievement. A learners' questionnaire was used to gather data on the learners' misconceptions and achievement. An educators' questionnaire was used to gather data about educators' knowledge of the subject matter, and educators' knowledge of learners. Interviews were used to investigate educators' knowledge of strategies for teaching quadratic functions. To determine the role of PCK in the learning of quadratic functions, the researcher determined the effects of each component of the educators' PCK on learners' achievement. The results indicate that the educators' PCK have an effect on learners' achievement in quadratic functions. The findings suggest that if learners' achievement in quadratic functions and mathematics are generally to improve, then the educators' PCK must be developed to bring about the desired improvement since educators are the primary agents in education transformation (National Education Policy Investigation, 1993).

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Appendix i

CONSENT LETTER TO THE EDUCATORS

Eletsa Secondary School
P,O. Box 1473
Letlhabile,
350
18/09/2008

Dear colleague,

Following our oral discussion, I hereby write to confirm your acceptance to participate in this study. As I informed you in our discussion, the investigation will be used for academic purposes only. The topic of my study is “the role of pedagogical content knowledge in the teaching and learning of quadratic functions. It is my belief that the information gathered in this research will be useful in improving the teaching and learning of mathematics and specifically, the teaching and learning of quadratic functions.

Rest assured that the information supplied by you will be treated with the confidentiality it demands.

The research is structured such that you will be required to respond to a questionnaire containing questions about quadratic functions, and your experience and understanding about your learners’ thinking and conceptions about quadratic functions. The questionnaire will take you about 1 to 2 hours to complete. At a later stage, you will be interviewed about the strategies that you adopt to ensure that your learners understand the concept of quadratic function very well.

Your learners will also respond to a questionnaire which will be used to collect data about their achievement and understanding of different components of quadratic functions.

Once more thank you for accepting to be part of this study

Yours truly,

Ibeawuchi, Emmanuel (Mr.)

(Please, fill the consent form below to confirm your acceptance)

CONSENT FORM

I,……………………………………………………………………………. hereby confirm that I have accepted to participate in the study as explained above.

Signature …………………………, Date ………………………………………

Appendix ii

CONSENT LETTER TO PARENTS/GUARDIAN

Eletsa Secondary School
P.O. Box 1473
Letlhabile,
350
15/10/2008

Dear parent/guardian

Your ward Master/Miss ………………………………………………………… has been selected to participate in a study aimed at shedding light on the impact of pedagogical content knowledge in the teaching and learning of quadratic functions. The research will be used for academic purposes only. However, your consent is required since your ward is a minor.

Your ward will only be required to respond to a questionnaire containing questions about his/her understanding /conceptions about quadratic functions. This questionnaire will take your ward about one hour to complete. Arrangements will be made to ensure that this does not disturb his/her school work.

Rest assured that the information supplied by your ward will be treated confidentially.

Thank you for your cooperation.

Yours truly,

Ibeawuchi, Emmanuel (Mr.).

(Please complete the acceptance form that follows to confirm your approval)

APPROVAL FORM

I, …………………………………………………………, the parent/guardian of Master/Miss…………………………………………….. hereby give my approval for his/her participation in the study described above.

Signature……………………………………………. Date…………………………….

Appendix iii

CONSENT LETTER TO SCHOOL GOVERNING BOARD

Eletsa Secondary School
P.O. Box 1473
Letlhabile,
350
15/10/2008

The Secretary,
…………………………………………
…………………………………………
Governing Board

Sir/madam,

Permission to conduct a research

I hereby request your permission to allow me to conduct a study in your school. The research is for academic purposes only. It is aimed at shedding light on the impact of the educators' pedagogical content knowledge on the teaching and learning of quadratic functions. The information obtained can be useful in improving the teaching and learning of mathematics, specifically the teaching and learning of quadratic functions.

The study will require the participation of two of your Grade 11 mathematics educators and ten learners of each educator. The educators will respond to a questionnaire, and at a later stage be interviewed about their strategies in helping their learners understand the concept of quadratic functions. The learners will also respond to a questionnaire which will last for about one hour. Meanwhile, arrangements will be made to ensure that the process does not disturb any academic activities in your school.

It might be important to inform you that I have already negotiated with the mathematics educators and they showed interest in participating in the research if you approve of it.

Your cooperation in this regard will be highly appreciated.

Yours truly,

Ibeawuchi, Emmanuel (Mr.)

(Please sign the consent form to confirm your acceptance)

APPROVAL FORM

We the School Governing Board of ………………………………………………………,
hereby approve your request to carry out the study described above.

……………………………………………. ………………………………………
Chairman Secretary

………………………………..
Principal

Appendix iv

LEARNERS' QUESTIONNAIRE

Eletsa Secondary School
P.O. Box 1742
Letlhabile
0264.

Dear Respondent,

Thank you for accepting to participate in this study. The information supplied by you will be used for academic purposes only. Specifically, the information will be used in research work for an M.Ed degree in mathematics education. The research is aimed at evaluating the role of pedagogical content knowledge in the teaching and learning of quadratic functions. It is my belief that the information obtained from this research will be profitable in improving the teaching and learning of mathematics, and specifically, the teaching and learning of quadratic functions.

Rest assured that the information supplied by you in this questionnaire will be treated confidentially.

May I thank you once more for the sacrifice you have made in completing this questionnaire despite numerous other engagements.

Yours,

Ibeawuchi, Emmanuel.

(Researcher).

INSTRUCTIONS

This questionnaire consists of **TWO** sections (A & B). Please, complete the two sections. In section **B,** only one option is correct.

- You are requested to circle the letter in the given option that bears the answer that you have chosen.
- You are requested also to give reason(s) for choosing your answer and the reason(s) for rejecting the other options where necessary.
- Reasons like “because it is the right answer” or “because it is not the answer” is not acceptable. Please give mathematical reason(s).

SECTION A.

Age ………………………………………………………………………………………

Sex ………………………………………………………………………………………

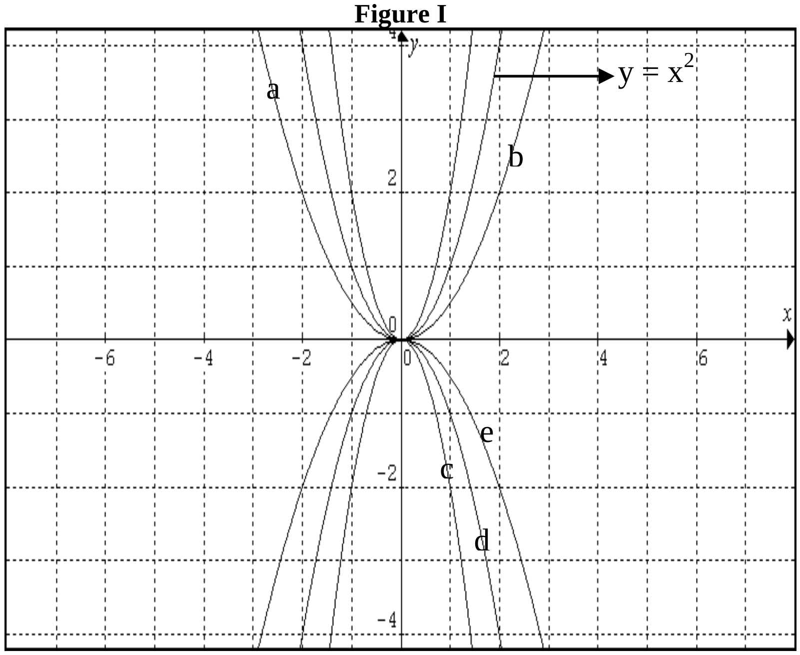
School ……………………………………………………………………………………..

SECTION B

QUESTION 1

In the diagram below, the graph of $y = x^2$ has been indicted. Other graphs are labeled "a" to "e". Which of the graphs labeled "a" to "e" could be the graph of - $2x^2$?

Figure I



a

b

c

d

e

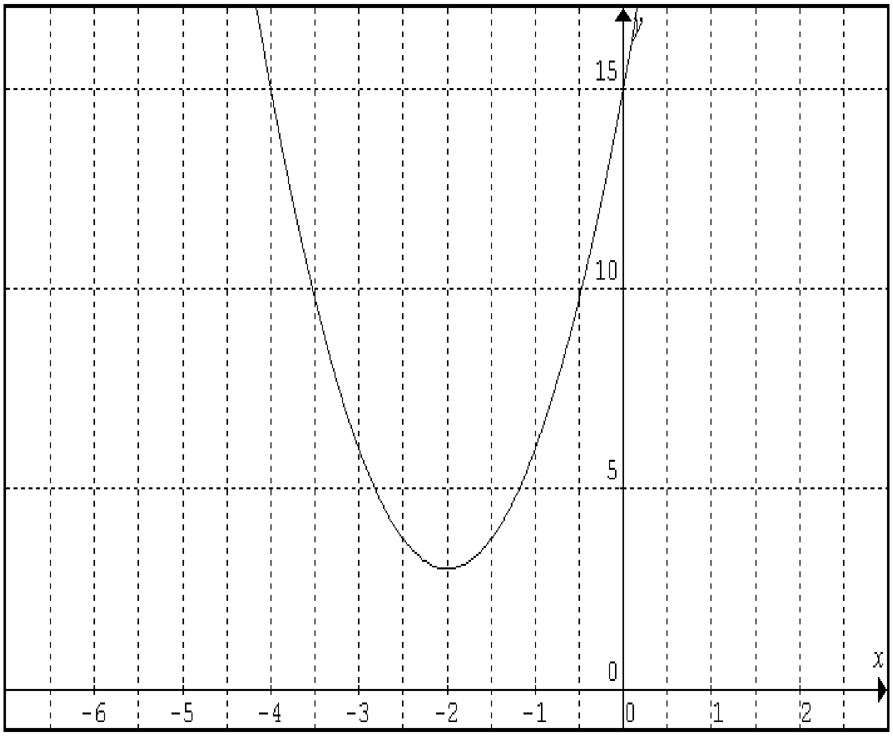
Give reason(s) for your answer…………………………………………………………...

…………………………………………………………………………………………………

…………………………………………………………………………………………………

QUESTION 2

Figure 2



The parabola above represents the graph of the function of the form, $y = ax^2 + bx + c$. Is it possible to have a point on the graph when x is 20?

a. Yes

b No.

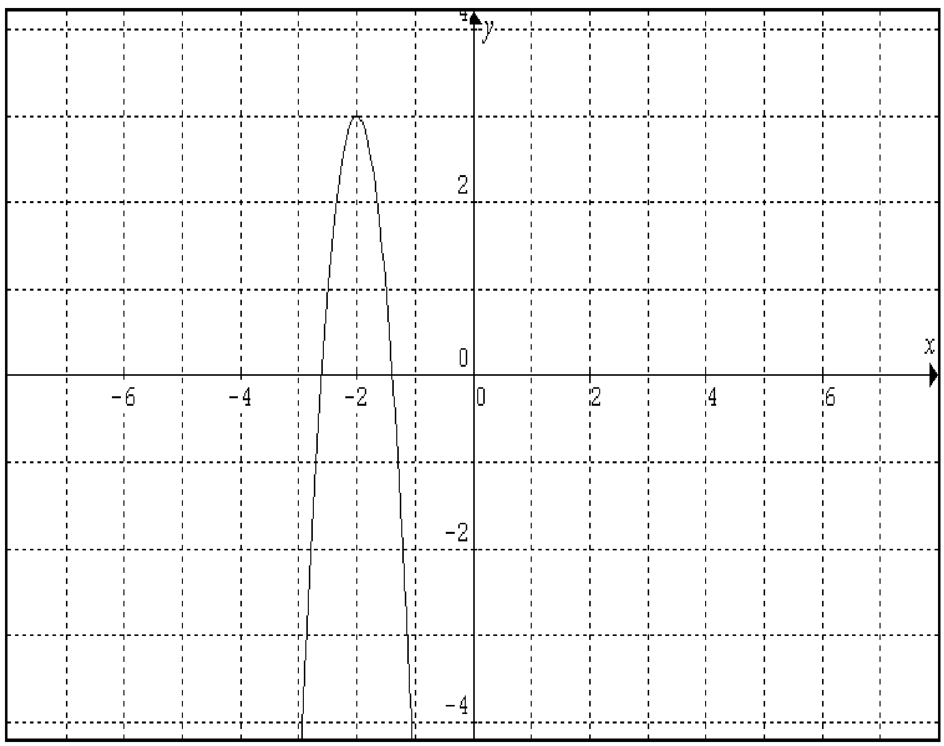
Reason(s)……………………………………………………………………………………...

…………………………………………………………………………………………

…………………………………………………………………………………………

QUESTION 3

If the parabola below is extended indefinitely, do you think that it will ever cut the y-axis?



a. Yes

b. No.

Reason(s) ……………………………………………………………………………..

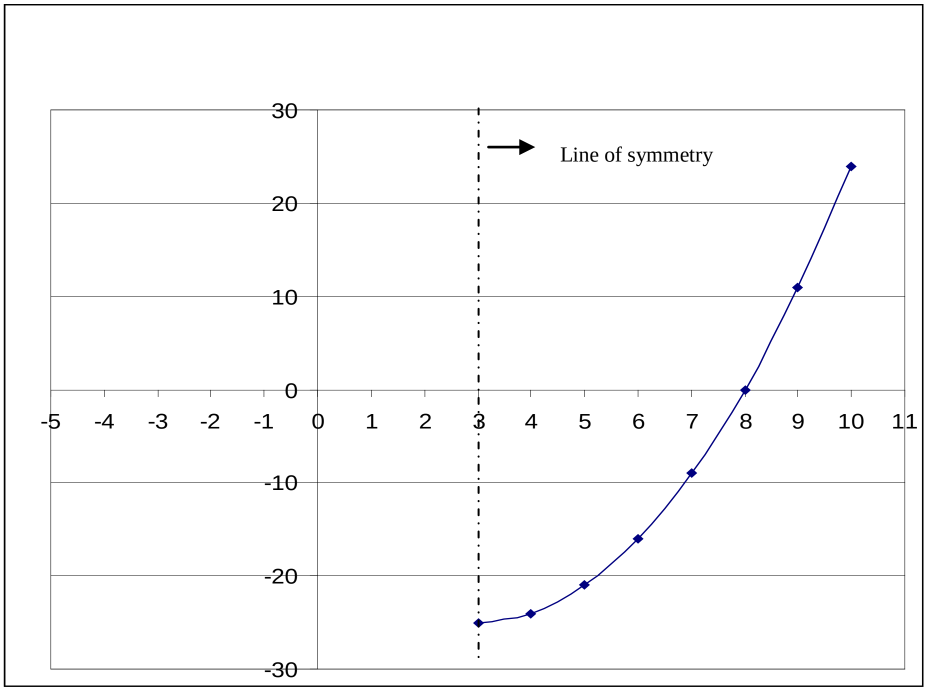
………………………………………………………………………………………………

……………………………………………………………………………………………….

…………………………………………………………………………………………………

QUESTION 4

A parabola of the form $y = ax^2 + bx + c$, is sketched below



If one of the x intercepts is 8 as shown in the sketch, what is the other x-intercept of the parabola?

a. 0 I chose this answer because……………………………………………………….

…………………………………………………………………………………………..

b. -1 .I chose this answer because……………………………….................................

……………………………………………………………………………………………

c. – 2. I choose this answer because………………………………………………………

…………………………………………………………………………………………..

d. – 3. I choose this answer because……………………………………………………..

e. -4 I chose this answer because………………………………………………………..

QUESTION 5

The equations of TWO parabolas are given below;

$y = ax^2 + bx + 3$
$y = ax^2 + bx + 7$

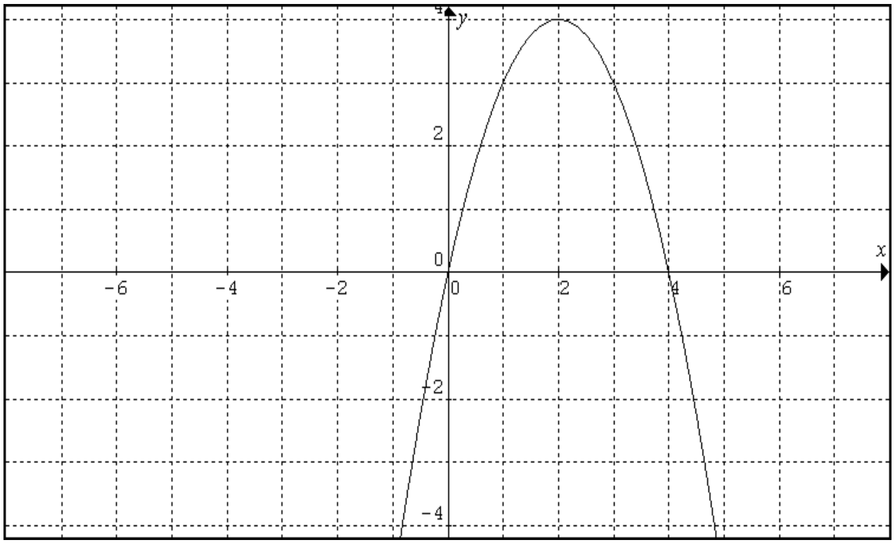
Complete the table below. Mark X in the correct column, and justify your choice in the last column.

Do the two parabolas have:	Yes	No	Why?
• The same line of symmetry?			
• The same vertex?			

Space for calculations

QUESTION 6

The parabola $y = -(x - 2)^2 + 4$ is shown in the diagram below



If this parabola is shifted 3 units to the right, use this information to complete the table that follows.

	Before the shift	After the shift
(i) The equation of the parabola	$y = -(x-2)^2 + 4$	
(ii) x coordinate of the turning point	x = 2	
(iii)y coordinate of the turning point	y = 4	
(iv)The x intercepts	x = 0, and x = 4	

Give reason(s) for your answer

(i)……………………………………………………………………………………………

……………………………………………………………………………………………

(ii)……………………………………………………………………………………………

……………………………………………………………………………………………

(iii)……………………………………………………………………………………………

(iv)……………………………………………………………………………………......

QUESTION 7

Given the quadratic function $y = 2x^2 - 3x + 4$.

(a). Determine the turning point.

Space for calculations

Give reason for your answer

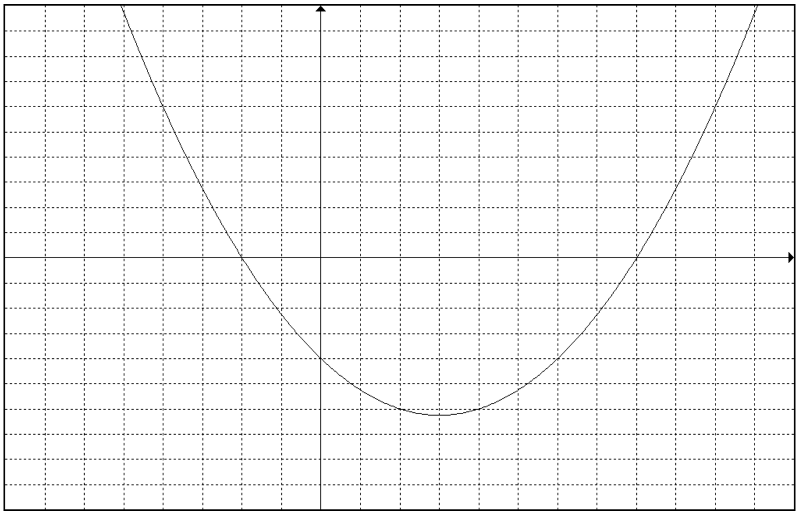
…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

QUESTION 8

The parabola below is the graph of $f(x) = x^2 – 3x - 4$



Which of the following has the same graph as $f(x) = x^2 -3x - 4$ above?

a. $f(x) = 2x^2 – 6x - 8$ I chose this answer because……………………………………..

…………………………………………………………………………………………………

b. $f(x) = 3x^2 - 9x - 12$ I chose this answer because………………….................................

…………………………………………………………………………………………………

c. $f(x) = 4x^2 - 12x – 16$. I chose this answer because……………………………………

…………………………………………………………………………………………………

d. All of the above. I chose this answer because…………………………………………

…………………………………………………………………………………………………

e. None of the above. I chose this answer because……………………………………………………………………………………..

QUESTION 9

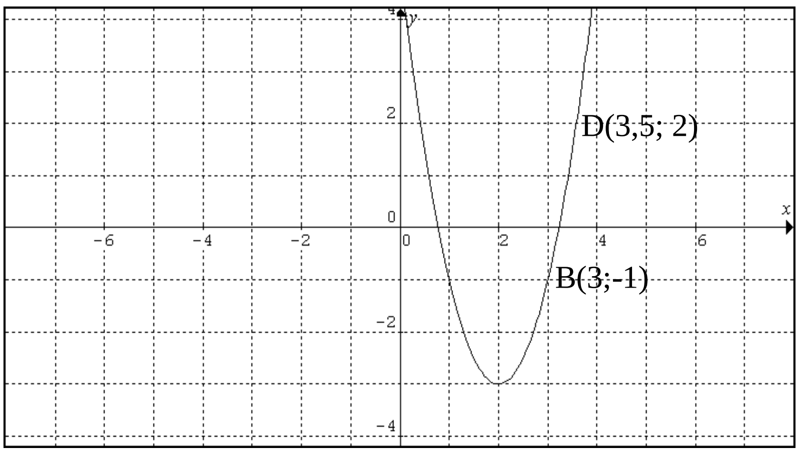
A cell phone company increased its charge per minute by 50 cents. After the increase, a R48 airtime calls for two and half minutes less than before. Write a model (equation) that you can use to determine the original price per minute.

……………………………………………………………………………………..

Space for calculations

QUESTION 10

In the figure below, a parabola is graphed.



Points B (3; -1) and D (3.5; 2) are two points on the parabola. C (3,25; 0.5) is the mid point of line BD. Is point C on the parabola? Give reason(s) to justify your answer.

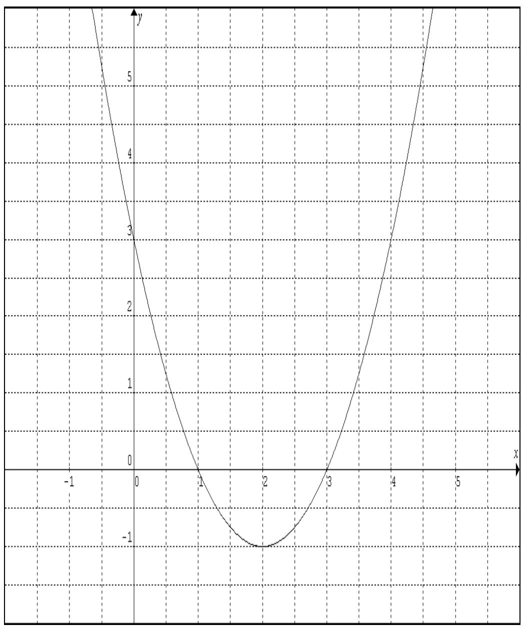
Space for calculations

QUESTION 11

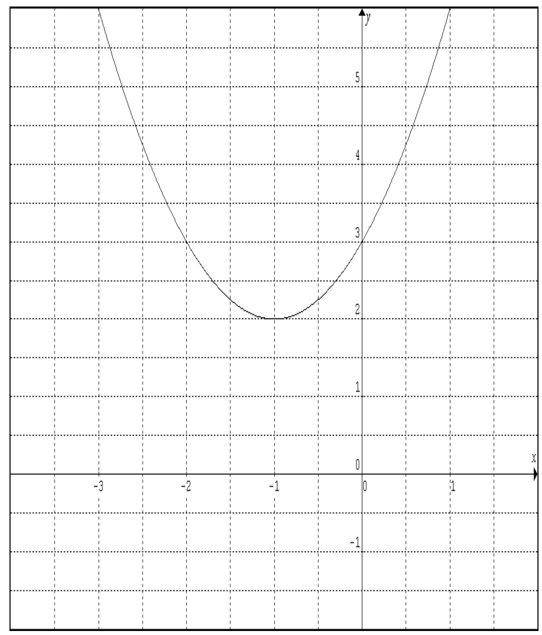
Which of the following parabolas matches with the function $f(x) = x^2 - 2x + 3$

You are requested to give reason(s) for rejecting any option and also the reason(s) for selecting the option that you did

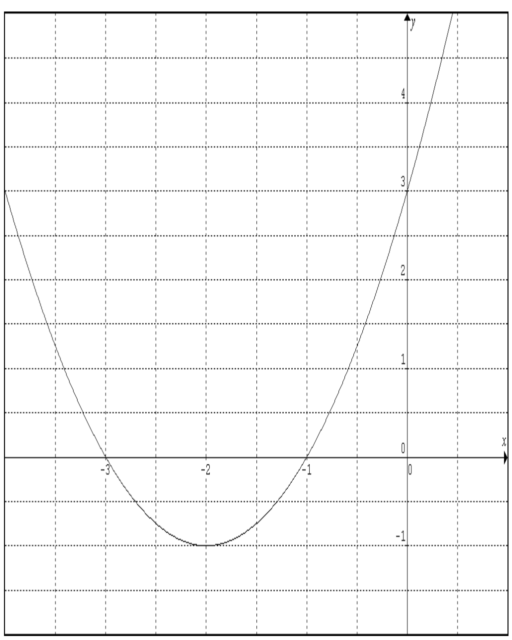
a



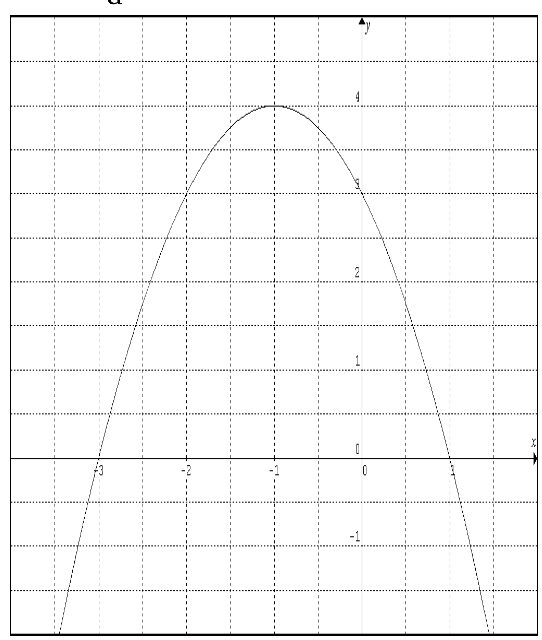
b



c



d



(e). None of the above.

Reasons for choosing your answer

(a)……………………………………………………………………………………...

…………………………………………………………………………………………

(b)……………………………………………………………………………………...

…………………………………………………………………………………………

(c)……………………………………………………………………………………...

…………………………………………………………………………………………

…………………………………………………………………………………………

(d)……………………………………………………………………………………..

…………………………………………………………………………………………

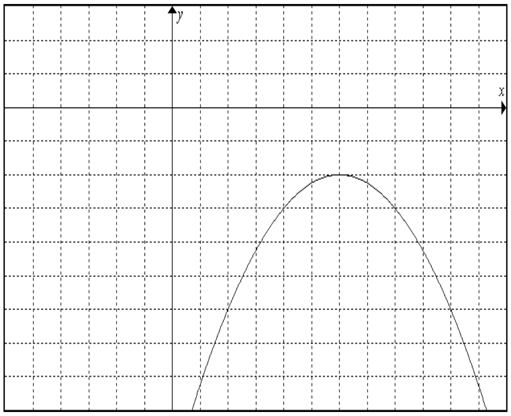
(e)……………………………………………………………………………………..

…………………………………………………………………………………………

Space for calculations

QUESTION 12

Which of the following functions could be the representation of the parabola in the figure



(a). $y = x^2 – 6x + 7$

(b). $y = -x^2 - 6x - 7$

(c). $y = -x^2 + 6x$

(d). $y = -x^2 + 8$

(e). None of the above

Reasons for choosing your answer

a. I chose this answer because…………………………………………………………….

…………………………………………………………………………………………………

---

b. I chose / did not choose this answer because…………………………………………….

…………………………………………………………………………………………………

c. I chose this answer because……………………………………………………………..

…………………………………………………………………………………………………

---

d. I chose this answer because…………………………………………………………….

…………………………………………………………………………………………………

e. None of the above equations can be the equation of the parabola in the figure

because……………………………………………………………………………

QUESTION 13a

The table below shows the tabular representation of the function $y = 2x^2 + 10x + 8$.

x	-6	-5	-4	-3	-2	-1	0	1	2	3
$y = 2x^2 + 10x + 8$	20	8	0	- 4	-4	0	8	20	36	56

Use the table to find the x-intercept of the function

a. x = 0 or x = -4 I chose this answer because……………………….........................

…………………………………………………………………………………………

b. . x = 0 or x = 8 I chose this answer because………………………………………

…………………………………………………………………………………………

c. x = -6 or x = 3 I chose this answer because………………………………………..

…………………………………………………………………………………………

d. x = -4 or x = -1 I chose this answer because……………………...........................

…………………………………………………………………………………………

e. x = -1 or x = 0. I chose this answer because………………………………………...

…………………………………………………………………………………………

QUESTION 13b

The table below is the tabular representation of the function $y = x^2 - 4x + 1$ within the interval $-5 \leq x \leq 6$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
$y = x^2 - 4x + 1$	45	33	22	13	6	1	-2	-3	-2	1	6	13

Use the table to answer the question that follows:

What is the value of x at the turning point?

a x = 0 I chose this answer because……………………………………………………………

………………………………………………………………………………………………

b x = 1. I chose this answer because……………………………………………………………

………………………………………………………………………………………………

c. x = 2 I chose this answer because……………………………………………………………

………………………………………………………………………………………………

d. x = -3 I chose this answer because……………………………………………………………

………………………………………………………………………………………………

e x = -5 I chose this answer because……………………………………………………………

………………………………………………………………………………………………

QUESTION 13c

The table below shows the tabular representation of f$(x) = x^2 - 2x + 3$ within the interval $x = -4$ *to* $x = +7$.

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$f(x) = x^2 - 2x + 3$	27	18	11	6	3	2	3	6	11	18	27	38

Use the table to answer the question that follows.

For which values of x is f(x) increasing?

a. $0 \leq x \leq 7$. I chose this answer because…………………………………………..............

…………………………………………………………………………………………………

b. $1 \leq x \leq 7$. I chose this answer because…………………………………………...............

…………………………………………………………………………………………………

c. $-5 \leq x \leq 0$. I chose this answer because…………………………………………..............

…………………………………………………………………………………………………

d. $-5 \leq x \leq 7$. I chose this answer because…………………………………………............

…………………………………………………………………………………………………

e. None of the above. I chose this answer because…………………………………….

…………………………………………………………………………………………………

Thank you for your co-operation

Appendix v

EDUCATORS' QUESTIONNAIRE

Eletsa Secondary School
P.O. Box 1742
Letlhabile
0264.
073 343 6688

Dear educator,

Thank you, for accepting to participate in this study. The information supplied by you will be used for academic purposes only. Specifically, the information will be used in a research work for M.Ed degree in mathematics education. The research is aimed at finding the roles of pedagogical content knowledge in the teaching and learning of quadratic functions. It is my belief that the information obtained from this research will be profitable in improving the teaching and learning of mathematics, and specifically, the teaching and learning of quadratic functions.

Rest assured that the information supplied by you in this questionnaire will be treated confidentially.

May I thank you once more for the sacrifice that you will make in completing this questionnaire despite your numerous engagements.

Please, note the following:

- 100 percent individual response is solicited
- Complete in detail

- Do your level best
- It makes no sense to seek for help in this questionnaire
- It is not an examination
- If you seek help you will distort the data

Yours,

Ibeawuchi, Emmanuel.

(Researcher).

INSTRUCTIONS

This questionnaire consists of **TWO** sections (A & B). Please, complete the two sections. For each question, you are requested to

- Circle the option which you think that most of your learners will choose
- Give reason(s) which you think that informed their choice
- Give the right answer in the space where you are required to do so.

SECTION A.

Sex ………………………………………………………………………………………

School …………………………………………………………………………………..

Qualification(s)

(i)………………………………………………………………………………………….

(ii)…………………………………………………………………………………………

(iii)……………………………………………………………………………………..

(iv)………………………………………………………………………………………….

(Please indicate your subject(s) of specialization in your last year of study)

How many years experience do you have in teaching quadratic functions?

....

Have attended any form of professional development programme about quadratic functions in the last five years?

yes

no

If your response to the above question is yes, please indicate the type of programme below

……………………………………………………………………………………..

………………………………………………………………………………………

Indicate the aspect covered by the professional development programme in quadratic functions that you attended (if any)

- Content
- Learners' misconceptions and difficulties about quadratic functions
- Strategies for resolving learners' misconceptions and difficulties about quadratic functions
- Others (specify)……………………………………………………………………..

…………………………………………………………………………………………..

…………………………………………………………………………………………..

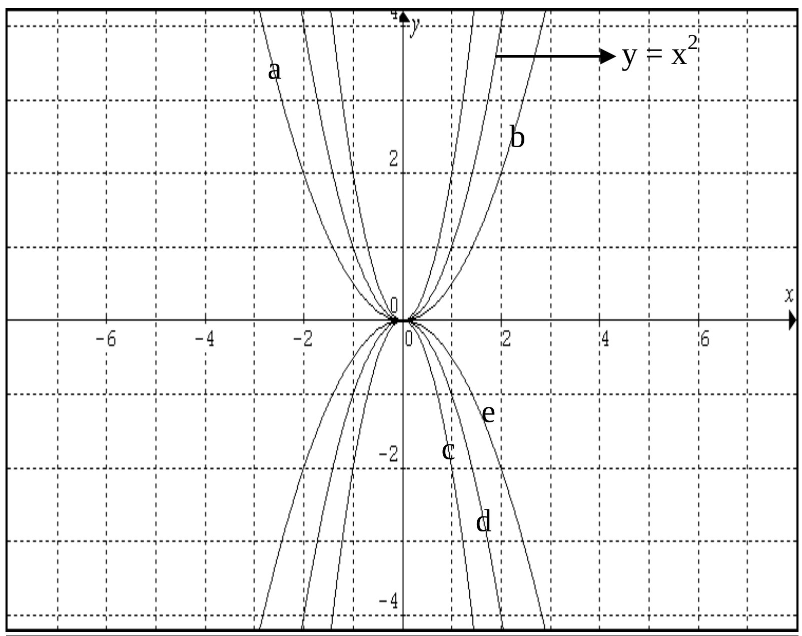
SECTION B

QUESTION 1

Your learners' were given the question below:

In the diagram below, the graph of $y = x^2$ has been indicted. Other graphs are labeled "a" to "e". Which of the graphs labeled "a" to "e" could be the graph of $-2x^2$

Figure II



Which option do you think most of your learners would choose?

a. **b.** **c.** **d.** **e.**

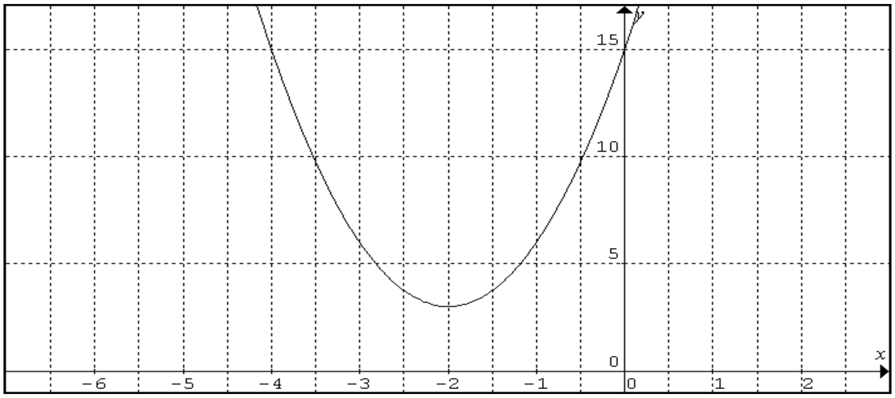
What reason(s) do you think that informed their choice?

…………………………………………………………………………………………………

…………………………………………………………………………………………………

What is your answer to the question? ___

QUESTION 2



The parabola above represents the graph of the function of the form, $y = ax^2 + bx + c$.

Your learners were asked the question below

"Is it possible to have a point on the graph when x is 20?"

Which option below do you think that most of them will choose?

a. Yes

b. No

What reason(s) do you think informed their choice?

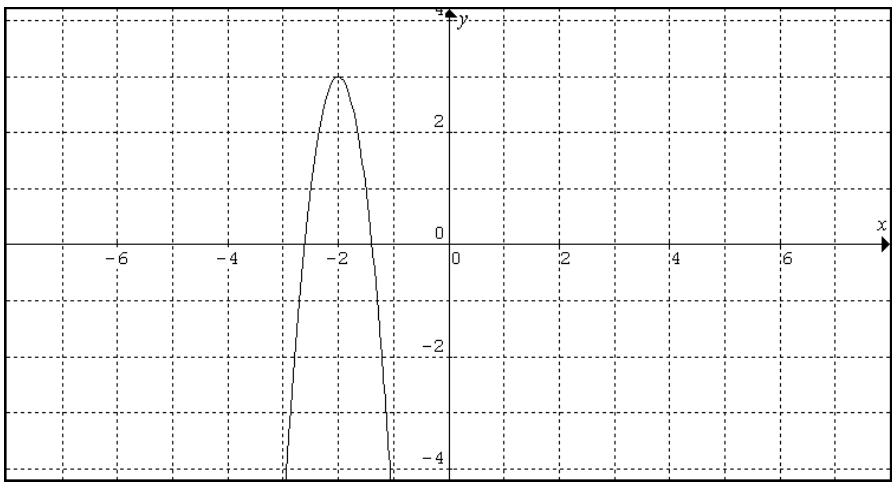
…………………………………………………………………………………………………

……………………………………………………………………………………………….

What is your answer? ___

QUESTION 3

Your learners were asked “If the parabola in the graph below is extended indefinitely, will it ever cut the y-axis? “



Which option below do you think that most of them will choose?

a. Yes.

b. No.

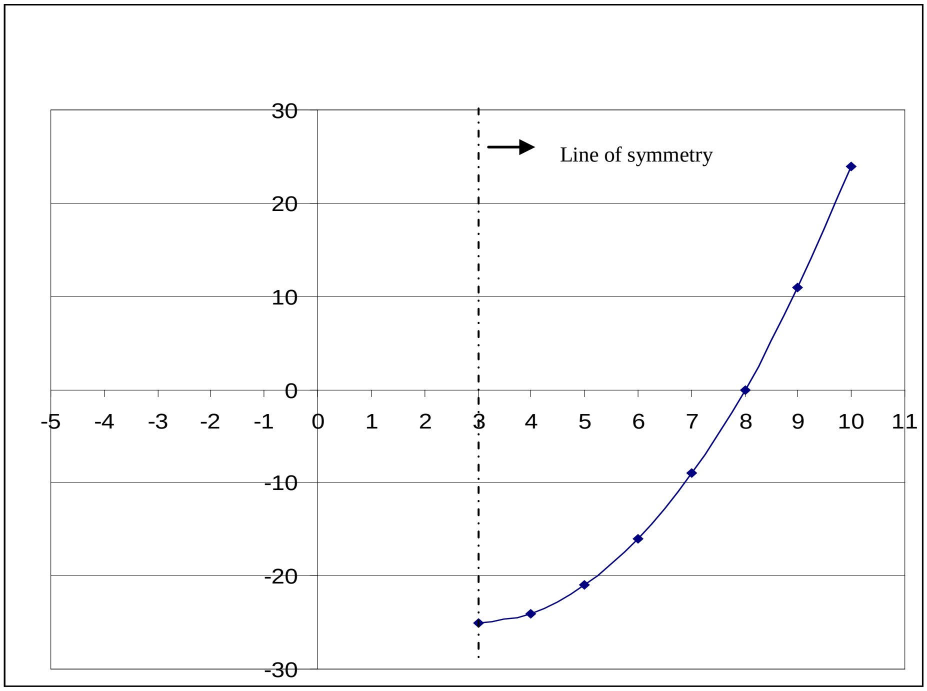
What reason(s) do you think informed their choice?

…………………………………………………………………………………………………

What is your own answer? ____

QUESTION 4

A parabola of the form $y = ax^2 + bx + c$, is sketched below



One of the x-intercepts is 8 as shown in the sketch. If your learners were asked to determine the other x-intercept of the parabola, which option below do you think most of them will choose?

I

a. 0 b. -1 c. -2 d.. -3 e. -4

What reason do you think informed their answer? ....

....

What is your answer to the question?

QUESTION 5

Your learners were given the question below

The equations of two parabolas are given below;

$y = ax^2 + bx + 3$

$y = ax^2 + bx + 7$

They were asked to complete the table below.

Write the answer which most of your learners will give in the spaces provided in above.

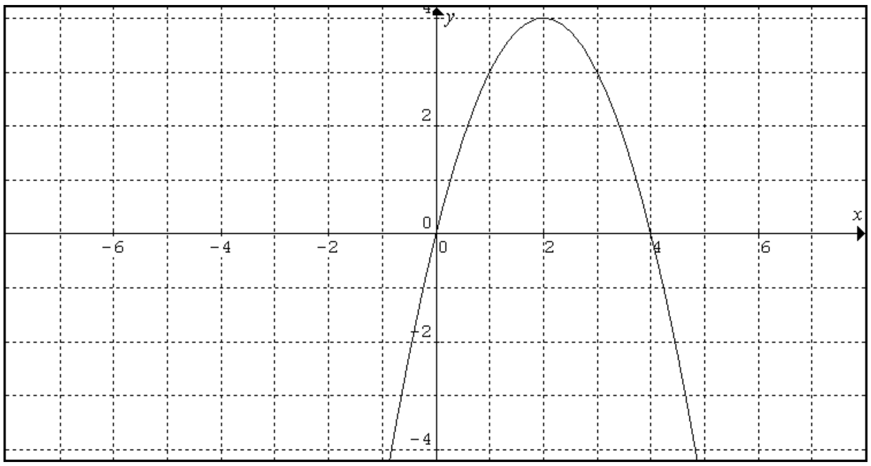
Do the two parabolas have:	Yes	No	Why?
• The same line of symmetry?			
• The same vertex?			

Write your own answer in the table below

Do the two parabolas have:	Yes	No	Why?
• The same line of symmetry?			
• The same vertex?			

QUESTION 6

The parabola $y = -(x - 2)^2 + 4$ is shown in the diagram below. If this parabola is shifted 3 units to the right



You learners were asked to use the above information to complete the table that follows. Write what you think most of them will give as the answer to each.

	Before the shift	After the shift
(i) The equation of the parabola	$y = -(x - 2)^2 + 4$	
(ii) x coordinate of the turning point	x = 2	
(iii) y coordinate of the turning point	y = 4	
(iv)The x intercepts	x = 0, and x = 4	

What reason(s) do you think informed their choice?

(i)…………………………………………………………………………………………

(ii)……………………………………………………………………………………………

(iii)……………………………………………………………………………………………

(iv)…………………………………………………………………………………………….

Write your own answer to the question in the table below

	Before the shift	After the shift
(i) The equation of the parabola	$y = -(x-2)^2 + 4$	
(ii) x coordinate of the turning point	x = 2	
(iii) y coordinate of the turning point	y = 4	
(iv)The x intercepts	x = 0, and x = 4	

QUESTION 7

$y = 2x^2 - 3x + 4$ is a quadratic function.

If your learners were asked to determine the turning point.

What answer do you think that most of them will give?

(a) ………………………………………………………………………………………

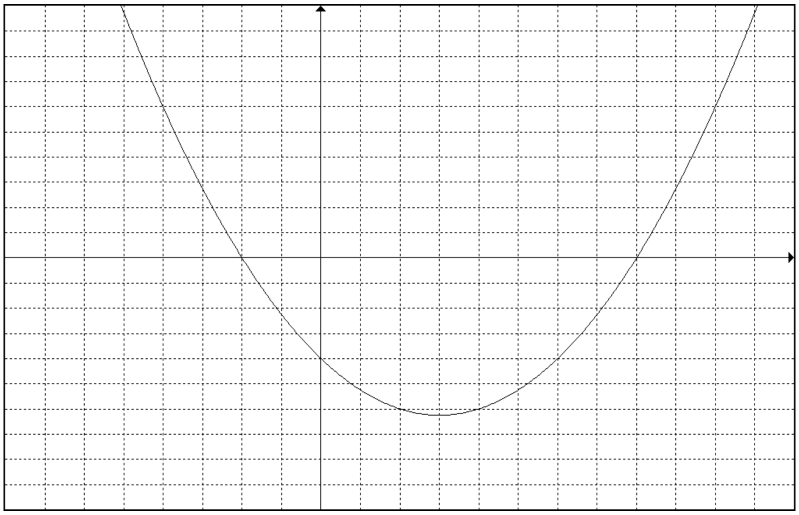
What is your own answer to the question**?**

……………………………………………………………………………………….. ……

Space for calculations

QUESTION 8

The parabola below represents the graph of $f(x) = x^2 – 3x - 4$



Which of the following has the same graph as $f(x) = x^2 -3x - 4$ above?

(a) $f(x) = 2x^2 – 6x – 8$

(a) $f(x) = 3x^2 – 9x – 12$

(c) $f(x) = 4x^2 – 12x – 16$

(d) All of the above

(e) None of the above

What is the learners’ reason(s) for their answer?

……………………………………………………………………………………………...

…………………………………………………………………………………………

What is your answer?

QUESTION 9

A cell phone company increased its charge per minute by 50 cents. After the increase, a R48 airtime calls for two and half minutes less than before. If your learners were asked to write a model (equation) that can used to determine the original price per minute.

Write the equation that you think most of them will write?

……………………………………………………………………………………..

What reason(s) do you think that informed their answer?

……………………………………………………………………………………..

……………………………………………………………………………………..

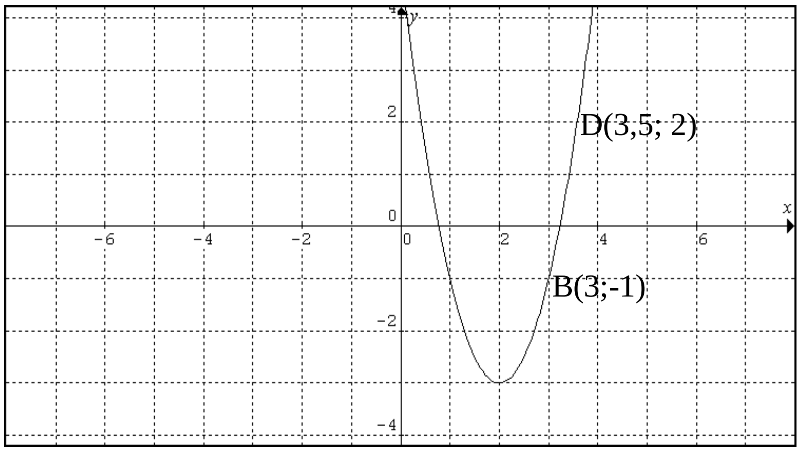
What is your own answer?

Space for calculations

QUESTION 10

Your learners were given the question below:

In the figure below, a parabola is graphed.



Points B (3; -1) and D (3.5; 2) are two points on the parabola. C (3,25; 0.5) is the mid point of line BD. Assuming your learners were asked to determine if point C is on the parabola, which option below do you think that most of them will choose?

(a) yes   b. no

What reason(s) do you think that informed their choice? **...**

……………………………………………………………………………….. ………..

What option is your own answer?

Space for calculations

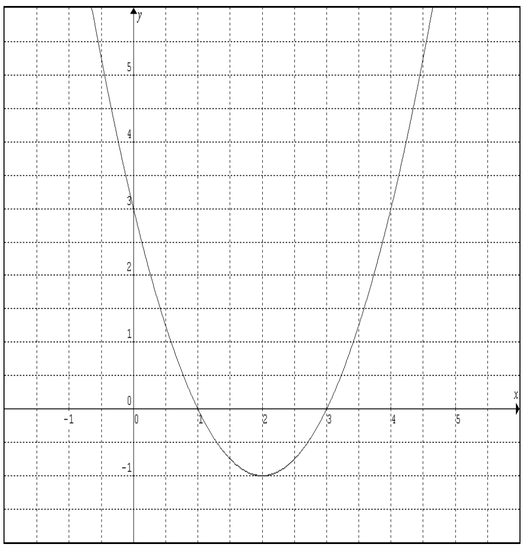
QUESTION 11

Your learners were given the question below:

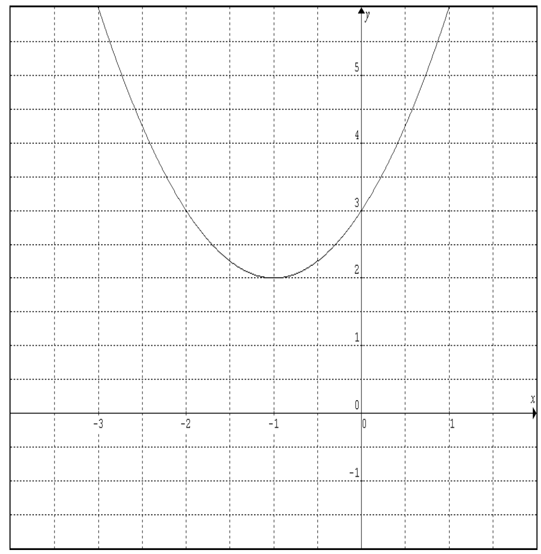
Which of the following parabolas matches with the function $f(x) = x^2 - 2x + 3$

You are requested to give reason(s) for rejecting any option and also the reason(s) for selecting the option that you did

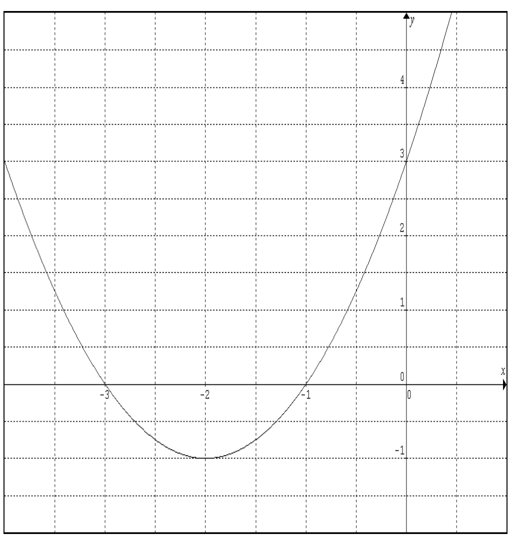
a



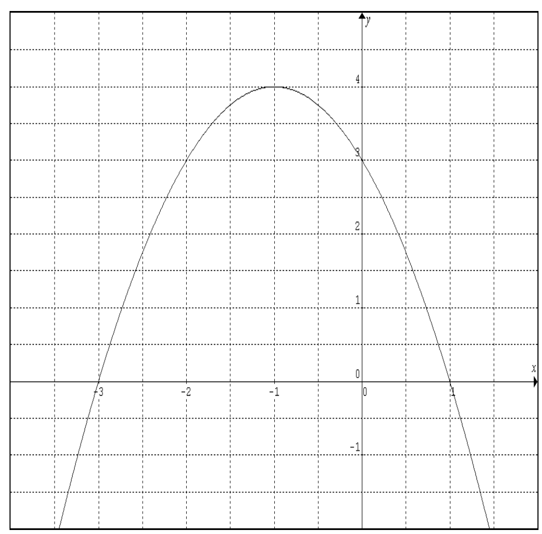
b



c



d



(e). None of the above.

What is the learners' reason(s) for their answer? ....

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

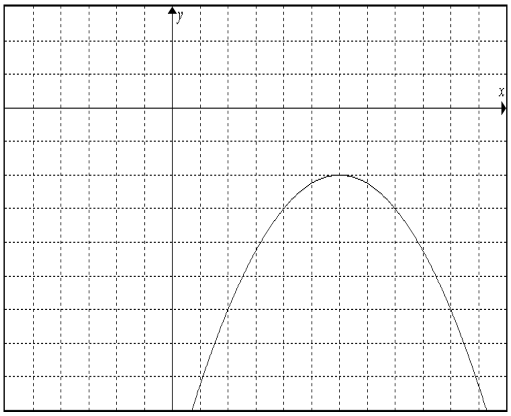
…………………………………………………………………………………………

What is your answer to the question?

QUESTION 12

Your learners were given the question below:

Which of the following functions could be the representation of the parabola in the figure



(a). $y = x^2 – 6x + 7$

(b). $y = -x^2 - 6x - 7$

(c). $y = -x^2 + 6x$

(d). $y = -x^2 + 8$

(e). None of the above

Which of the option do you think that most of your learners will choose?

What is your learners' reason for their answer? …………………………………………...

………………………………………………………………………………………………

………………………………………………………………………………………………

What is your answer?

Space for calculations

QUESTION 13a

The table below shows the tabular representation of the function $y = 2x^2 + 10x + 8$.

x	-6	-5	-4	-3	-2	-1	0	1	2	3
$y = 2x^2 + 10x + 8$	20	8	0	- 4	-4	0	8	20	36	56

If your learners were asked to use the table to find the x-intercept of the function, which option below do you think that most of them will choose?

a. x = 0 or x = -4.

b. . x = 0 or x = 8

c. x = -6 or x = 3.

d. x = -4 or x = -1

e. x = -1 or x = 0.

What reason(s) do you think informed their choice?

…………………………………………………………………………………………………
.
…………………………………………………………………………………………………

Which option is your own answer?

Space for calculations

QUESTION 13b

The table below is the tabular representation of the function $y = x^2 - 4x + 1$ within the interval $-5 \leq x \leq 6$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
$y = x^2 - 4x + 1$	45	33	22	13	6	1	-2	-3	-2	1	6	13

If your learners were asked to use the table to find the value of x at the turning point, which of the options below do you think that most of them will choose?

a x = 0
b x = 1.
c. x = 2
d. x = -3
e x = -5

What reason(s) do you think informed their choice?

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

Which option is your own answer?

Space for calculations

QUESTION 13c

The table below shows the tabular representation of $f(x) = x^2 – 2x + 3$ within the interval $x = -4$ *to* $x = + 7$.

x	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$f(x) = x^2 - 2x + 3$	27	18	11	6	3	2	3	6	11	18	27	38

If your learners were asked to use the table above to determine the values of x for which f(x) is increasing, which option below do you think that most of them will choose?

a. $0 \leq x \leq 7$.

b. $1 \leq x \leq 7$.

c. $-5 \leq x \leq 0$.

d. $-5 \leq x \leq 7$.

e. None of the above.

What reason(s) do you think informed their choice?

…………………………………………………………………………………………………

…………………………………………………………………………………………………

…………………………………………………………………………………………………

Which option is your own answer? ___

Thank you for your co-operation