View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by Texas A&M Repository

CARTEL MECHANISM DESIGN:

NONRATIFIABLE CONDITIONS OF COLLUSIVE BEHAVIOR

A Dissertation

by

SHAO-CHIEH HSUEH

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

December 2011

Major Subject: Economics

CARTEL MECHANISM DESIGN:
NONRATIFIABLE CONDITIONS OF COLLUSIVE BEHAVIOR

A Dissertation

by

SHAO-CHIEH HSUEH

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Approved by:

Chair of Committee,	Guoqiang Tian
Committee Members,	Rajiv Sarin
	Ximing Wu
	Silvana Krasteva
Head of Department,	Timothy Gronberg

December 2011

Major Subject: Economics

ABSTRACT

Cartel Mechanism Design:

Nonratifiable Conditions of Collusive Behavior. (December 2011)

Shao-Chieh Hsueh, B.S., National Taipei University

Chair of Advisory Committee: Dr. Guoqiang Tian

This dissertation is about an open question of cartel ratifiable conditions. My research goal is to establish a mechanism which is able to detect and explain cartels' activities.

My research question in the second chapter is whether or not an efficient cartel mechanism is ratifiable in the first-price sealed-bid auction format with participation costs. R. McAfee and J. McMillan study the coordinated bidding strategies in a strong cartel, which is incentive-compatible and efficient. This chapter investigates this efficient collusive mechanism with two important conditions (1) bidders can update their information through a cartel's prior auction and (2) they have to pay participation costs to participate in seller's auction. The concept of ratifiability, introduced by P. Cramton and T. Palfrey, is applied to explore the first-price sealed-bid auction with participation costs. I discovered that the efficient cartel mechanism, such as pre-auction knockout, would be ratified when either of the two conditions exists. However, this mechanism is no longer ratifiable when both conditions exist. The bidder with the highest value in the cartel would have incentive to betray, since doing so sends a credible signal of high value. Hence, the other bidders will be discouraged from participating in the seller's auction and the highest-value bidder maximizes his revenue.

In the third chapter, I studied the seller's strategy when she faces a cartel in an auction mechanism. An active seller's optimal strategy is to raise the reserve

price to a level that is higher than her own valuation. The collusive mechanism is sustainable even though its revenue is extracted by the higher reserve price. If the seller is authorized to change the auction mechanism, she can receive the expected payoff, prevent the formation of a ring and keep the auction efficient. Further, I presented two methods that could deter a cartel under specific conditions. One is the residual claimants method as proposed by Y. Che and J. Kim and the other is to set a positive participation cost as outlined in the first chapter. The residual claimants method can inhibit a ring in many cases, but it may have some trouble in preventing an efficient cartel mechanism when there is only one participant in the seller's auction.

In the fourth chapter, I investigated how to achieve external efficiency in a repeated game. In particular, I looked into the allocation of the budgeted that allows an authority, such as the government, to differentiate collusive behavior and to expose agents to external threats. A threshold level of the budget payment is found in an incentive compatible collusive mechanism for which the government can prevent an agent from participating. In a two-stage model, I showed that if the government can boost exemption or have more budget to subsidize agents, it is less likely that a ring will be formed.

To my family and Hellen

ACKNOWLEDGMENTS

First, I am heartily grateful to my advisory committee. Professor Guoqiang Tian, chair of my advisory committee, recommended reference papers and suggested alternative methods whenever I met difficulties. Without his continuous encouragement and guidance, I would not have finished my dissertation smoothly. I really appreciate his kindness and patience. Professor Rajiv Sarin, Professor Silvana Krasteva, and Professor Ximing Wu gave me many constructive suggestions on my dissertation.

Thanks go to the professors in the Economics Department. I took classes and learned economics from them. I would like to express my thanks to the staff for their administrative and computer support. I would like to express my thanks to all the friends who supported me, especially for Xiaoyong Cao, and Meng-Shiuh Chang, and the financial support from the Economics Department.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

An efficient cartel mechanism is not freely implemented. Osborne [20] claims that a cartel faces one external and four internal problems. A cartel has to anticipate and prevent outside production in order to avoid external threat. The four internal problems are designing the rule, dividing the profit, detecting, and deterring cheating. When an illegal cartel is formed, it eliminates competition in the marketplace, makes its own secret decision to boost price and then takes steps to stick to high prices. It is against social welfare. In reality, it is hard to detect collusive behavior. When a cartel is formed, it usually comes with huge amount of financial benefit. And the cartel could earn excess profit from consumers' surplus. This is an unfair and illegitimate phenomena. In this dissertation, I investigate the ratifiable and nonratifiable conditions of a cartel mechanism with auction format.

An auction is an effective way to extract private information by increasing the competition of potential buyers and thus can increase allocation efficiency from the perspectives of both sellers and the social optimum when we do not have complete information about bidders' types. The private information also makes it more difficult for the bidders to profitably collude and share the generated collusive surplus, which may destabilize the collusive ring.

This dissertation is mainly based on McAfee and McMillan [17] paper. They establish an efficient cartel mechanism, which is ratifiable in the first-price sealed-bid auction format. The model consists of one seller and n bidders. The seller's behavior

---

This dissertation follows the style of Journal of Economic Theory.

is passive. She sets a reserve price to the object she wants to sell, and then she holds a legit auction to sell the object. She does not know whether she faces a cartel or not. There are two strong assumptions in this mechanism. One is that all bidders are in the cartel at the beginning. The other is that members in the cartel can make transfer payment. This transfer payment can make everyone in the cartel better off compared to the case if they are not in the cartel. Why is this true? The cartel would hold a prior auction before the seller's legit auction. All bidders submit a bid to the cartel. The cartel would choose the bidder with the highest bid to participate in the legit auction. Since all bidders are in the cartel at first but there is only one bidder in the legit auction, he can submit the lowest bid which is equal to the seller's reserve price, and win the auction for sure. The transfer payment is the difference between the winning bid in the prior auction and seller's reserve price. This transfer payment is distributed equally to all members in the cartel. In first-price auction, when the cartel does not exist, the winner is the bidder who places the highest bid in the auction, and he pays what he bids in the auction, and others get nothing. In efficient cartel mechanism, the winner also pays what he bids in the prior auction and gets the object. But he can earn extra profit, which is the transfer payment. All other losers in the cartel can also earn this extra profit. Everyone in the cartel is better than the noncollusive case, so nobody has incentive to betray the cartel. Thus, the cartel is ratifiable.

An auction with participation costs is a mechanism in which buyers must incur some costs, like entry fees, in order to participate in the auction. After the costs are incurred, a bidder can submit a bid. Sometimes pre-bid costs are also required to involve in auctions. If a bidder's expected profit from an auction is less than the participation costs, he will not attend the auction. Even if the bidder's valuation is greater than the participation costs, he may not have the same bidding strategy as that of without participation costs. Thus, I should consider the efficient cartel mechanism with participation costs as a more realistic model.

In Tan and Yilankaya [24] paper, they study McAfee and McMillan's model with participation costs, and they show that the cartel mechanism cannot be ratifiable in the second-price sealed-bid auction format. My question is whether the strong cartel mechanism is ratifiable in the first-price sealed-bid auction format with participation costs. Chapter II indicates the conditions that the cartel cannot be ratifiable. The cartel mechanism designed in McAfee and McMillan [17] is self-enforcement. When all members are better than the noncollusive case, no one has incentive to veto for the cartel. Even if the winner in the prior auction betrays the cartel, others who could bid in the auction for free, already know the winner's value. They can bid as much as possible to make the betrayer earn less profit than he stays in the cartel. So, the winner in the prior auction would not have incentives to betray the cartel. When participation costs exist, bidders cannot bid in the legit auction for free. After the prior auction, members in the cartel know who has the highest value in cartel's auction. If the winner vetoes for the cartel, others know that they will lose for sure if they participate in the legit auction. Due to the participation costs, they would not participate in the legit auction; otherwise, they have to pay the nonrefundable costs, which end up with negative profits. After the prior auction, if the winner chooses to betray the cartel, he would also be the only bidder in the legit auction. He can bid the lowest value and win for sure. With the above facts, the winner in the cartel has incentive to betray the cartel, hence the cartel is not ratifiable in the economy.

There is a group of papers investigating how to minimize the loss resulted from the cartel. In Laffont and Martimort [14, 15] papers, they find that the principal can achieve the optimal outcome at no cost when the agents are uncorrelated. When agents' values are correlated, a large cost is incurred to the principal in order to prevent collusive behavior. Che and Kim [3] design a new mechanism which provides the same expected revenue to the principal as that in noncollusive case of both uncorrelated and correlated types. Bidders in this mechanism become residual claimants of the entire surplus.

In Chapter III, I consider all possible values of buyers' that the seller may face, and try to analyze how the seller maximizes her expected revenue and minimizes the cartel's revenue when the efficient cartel mechanism exists. Given imperfect information between the seller and buyers, the seller's problem is to design an auction that provides her with the highest expected profit at Nash equilibrium. I investigate the question: what is the seller's optimal strategy to maximize her revenue when she is allowed to detect collusive behavior. My conclusion is that the seller's best strategy is to raise the reserve price to a higher level than her own value of the object. When she raise the reserve price, the object may not be sold even if a bidder's value is higher than her value. Another result in this chapter is that when bidders can update their information through their participating decisions, the cartel mechanism is sustainable regardless of the seller's type.

Additionally, I consider the case when the seller is authorized to change the mechanism (for example, the payment mechanism), and find that if the seller cannot influence nonparticipating bidders, she cannot prevent the cartel, even with the residual claimants method in Che and Kim [3]. I suggest that the seller can set up a positive participation cost for the bidders in the seller's auction to forbid the formation of a cartel as my result in Chapter II.

Although a cartel is prohibited by law in many countries, it is difficult to detect collusive behavior in the real world. If the government could not detect collusive behavior, is there any method that can help the government differentiate collusive behavior? As mentioned in Osborne [20] and McAfee and McMillan [17], if the effects of the obstacles that the cartel has to overcome can be enhanced by the government, it is easier for the government to prevent the formation of a ring.

Chapter IV investigates: (1) the difference in the behavior of heterogeneous firms in different economic states, (2) how the government chooses to give them a premium or punishment, in reputation or in finance, in order to increase consumers' welfare and induce the government to differentiate collusive mechanism. I specify the government's role in the efficient cartel mechanism, and explore the allocation of government's budget that entitles to regulate collusive behavior in a repeated game. The result is that allocating the budget among firms equally may lead to a collapse of collusive mechanism in some cases. In others, however, symmetric allocation may induce the agents to form a ring.

I also have a concern on whether the government should choose exemption or subsidy to destabilize the collusive behavior. The difference between exemption and subsidy is that exemption is usually restrained by law and it is costly to change, while subsidy is more applicable in reality. My conclusion is that if the exemption (it could be a premium or punishment) is large enough, members in a cartel may have incentives to veto, and outsiders may not participate in the collusive mechanism. On the other hand, an individual firm that receives subsidy from the government, is likely to earn more profit and thus not has incentives to participate in a collusive mechanism.

CHAPTER II

NONRATIFIABILITY OF THE CARTEL MECHANISM IN FIRST-PRICE SEALED-BID AUCTION WITH PARTICIPATION COSTS

2.1. Introduction

Asymmetric information between bidders makes them difficult to profitably collude and share the generated collusive surplus, which may destabilize a collusive ring. As pointed out by Cooper [4], vindictive strategies are used to enforce collusion. When a member betrays the ring, it would increase the seller's revenue as revengeful competition leading to crazy prices.

Under different assumptions, the requirements of a successful collusive mechanism are different. Roberts [23] shows if firms are similar, the cooperative organization cannot be achieved without side payments. If side payments are allowed, it is a dominant strategy for every member to join the cartel. The expected revenue of the mechanism is equal to the revenue in second-price auction. Graham and Marshall [8] analyze collusion in second-price sealed-bid and English auction with independent private values. In second-price auction with reserve price, they show that a knockout pre-auction mechanism organized by an outsider is efficient and sustainable. The knockout mechanism is also ex ante budget balanced among bidders. Mailath and Zemsky [16] consider second-price auction with reserve price and heterogeneous bidders. They discover that for any subset of bidders, there is an incentive compatible and individually rational collusive mechanism, which is ex post budget balanced and efficient. Cramton and Palfrey [5] find that, when private information exists, a cartel agreement is possible in the economy. Given incentives and participation constraints, they characterize a branch of applicable contracts. If a cartel is sustainable without unanimous members' agreements, the incentive problem can be avoided in a large cartel. When uncertainty exists, regardless of the ratification rule, a perfect collusion is possible in a large cartel.

As the synthesis of standard mechanism design literatures, McAfee and McMillan [17] explore the bidding strategies in first-price sealed-bid auction under two cases: weak cartel, in which bidders cannot make transfer payments, and strong cartel, in which the members can exclude new entrants and make transfer payments among themselves. In weak cartel, bidders can place identical bid in seller's legit auction to achieve the optimal outcome without colluding. When side-payments are prohibited, incentive compatibility condition requires that the object should be allocated with equal probability to anyone whose value is greater than the seller's price. Bidders treat the seller as their randomizing device by placing equal bids. In strong cartel, the optimal cartel mechanism can reach the efficiency by implementing a prior auction before the seller's legit auction. The cartel would choose the bidder with the highest value to participate in the seller's auction. Since all bidders are in the cartel at first but there is only one bidder in the seller's auction, he can submit the lowest bid which is equal to the seller's reserve price, and win the auction for sure. The transfer payment is the difference between the winning bid in the prior auction and seller's reserve price. This transfer payment is distributed equally to all members in the cartel. With this extra payment, everyone in the cartel is better than the noncollusive case, so nobody has incentive to betray the cartel.

The strong cartel is incentive-compatible and efficient. However, this mechanism ignores two important conditions in reality. One is that bidders can update their information (as called the information leakage problem) from their participation decisions in the prior auction. Cramton and Palfrey [6] show that the information leakage problem from bidders' participation decisions in the prior auction would disintegrate the optimal cartel mechanism. Bidders can observe a bidder's type from his choice of whether vetoing the collusive mechanism, which in turn may affect the seller's revenue. In order to specify this problem, they set up a two-stage game: all players simultaneously vote for or against the collusive mechanism. This vote is held at the interim stage. If the collusive mechanism is unanimously accepted, it is implemented; otherwise they participate in seller's auction with updated information.

The other one is that bidders may incur some costs, like entry fees, when they participate in an auction. After the costs are incurred, a bidder can submit a bid. Sometimes pre-bid costs are also required to involve in auctions. Mills [18] points out that the bidding costs in a government procurement auction often runs into millions of dollars. If a bidder's expected profit from an auction is less than the participation costs, he will not attend the auction. Even if the bidder's valuation is greater than the participation costs, he may not have the same bidding strategy as that of without participation costs. Thus, I should consider the efficient cartel mechanism with participation costs as a more realistic model.

Tan and Yilankaya [24] apply Cramton and Palfrey's [6] concept to show that the efficient collusive mechanism with independent private values is not ratifiable in second-price auction with positive participation costs. I investigate the efficient collusive mechanism in first-price sealed-bid auction with information leakage problem and positive participation costs because McAfee and McMillan examine the strong cartel primarily at sealed-bid first-price auctions. By constructing a veto set that if a bidder's value belongs in this set, he will choose to betray the cartel, and following the optimal inverse bidding function that is found in Cao and Tian [2] paper, I extend the strong cartel under the conditions if either the participation costs or information leakage problem does not exist. However, I get a negative result when both conditions exist. By vetoing the mechanism, a bidder sends a credible signal that he has relatively high value, which discourages the other bidders from joining a ring when there are positive participation costs. Thus, there may be a discontinuity between no participation costs and positive participation costs when information leakage problem exists.

The remainder of this chapter is structured as follows. Chapter 2.2 describes the model and investigate the nonratifiable condition of the efficient cartel mechanism. Chapter 2.3 concludes. All proofs are in Appendix A.

2.2. Model

There are one seller and n $(n \geq 2)$ bidders in the economy. Bidder i's value is $v_i \in V$, $i = (1, ..., n)$, which represents i's willingness to pay for an object in an auction, and $v = (v_1, ..., v_n)$ is the vector of n bidders' profile. v_i is private information which is a random draw from a population with a cumulative distribution function $F(\cdot)$ and density function $f(\cdot)$ supported on $[0, 1]$. The seller values her object at $r \in [0, 1]$. She is assumed to be passive in this model, i.e., she does not know whether she faces a cartel. She announces a reserve price and sells an indivisible object to the bidder at the highest bid in a first-price sealed-bid auction. I denote participation costs, $c \in (0, 1)$, for which bidders must pay before they bid. Bidders do not know others' participation decisions when they make theirs. Only the winner in seller's auction has to pay the reserve price and all participants in seller's auction have to pay the participation costs.

2.2.1. Non-Collusive Auction

In order to simplify the calculation and notation, I assume that the reserve price r equals to 0. Suppose that Y_1 is the maximum value of other $n-1$ bidders' values, and denote $F(Y_1)^{n-1} = G(Y_1)$. The expected payment $\gamma(v_i)$ in first-price sealed-bid auction can be written as

$$\gamma(v_i) = v^* G(v^*) + \int_{v^*}^{v_i} y dG(y),$$

where v^* is the cutoff point, which is determined by $c = v^* F(v^*)^{n-1}$, $v^* > c$. If $v_i < v^*$, the bidder i does not participate in the auction. I can write bidder i's expected utility when $v_i \geq v^*$ as follows:

$$\pi_i^s(v_i) = G(v_i)v_i - \gamma(v_i).$$

Thus, after integration by parts, the noncollusive profit $\pi_i^s(v_i)$ for bidder i is as follows:

$$\pi_i^s(v_i) = \begin{cases} 0 & v_i < v^* \\ \int_{v^*}^{v_i} G(y)dy & v_i \geq v^*. \end{cases}$$

2.2.2. Efficient Cartel Mechanism

An efficient cartel designs an incentive-compatible mechanism in order to maximize the ex ante sum of bidders' expected profits with transfer payment, i.e., this cartel is self-enforcement. Bidder i's payoff function is $\pi_i^m(v_i)$, by reporting his value to the cartel mechanism before seller's auction and $\pi_i^m(0)$ is the transfer payment received by each cartel member.

The efficient cartel mechanism works as follows. While the bidders reporting their values to the cartel mechanism, the ith bidder is awarded the good with probability $p_i(w_i, w_{-i})$, where w_i is the value reported from bidder i and w_{-i} is the vector of other bidders'. Then, bidder i's expected profit $\pi_i^m(w_i, v_i)$, if he has value v_i and reports w_i, is

$$\pi_i^m(w_i, v_i) = E_{-i}[v_i p_i(w_i, w_{-i}) - Z_i(w_i, w_{-i}) - c],$$

where E_{-i} is the expectation over w_{-i}, and $Z_i(\cdot)$ is the total transfer payment in the cartel. After dropping the bidder indices to simplify the notation, $\pi^m(w,v)$ becomes:

$$\begin{aligned}
\pi^m(w,v) &= [v - Z(w) - c]F(w)^{n-1} \\
&+ [1 - F(w)^{n-1}]\int_w^1 [\frac{Z(u)}{n-1}]\frac{(n-1)F(u)^{n-2}f(u)}{1-F(w)^{n-1}}du \\
&= [v - Z(w) - c]F(w)^{n-1} \\
&+ \int_w^1 [Z(u)]F(u)^{n-2}f(u)du.
\end{aligned}$$

$\frac{\partial \pi^m(w,v)}{\partial v} = F(w)^{n-1}$. Thus, we can write

$$\pi^m(v) = \begin{cases} \pi^m(0) & if \quad v < c. \\ \pi^m(0) + \int_c^v G(u)du & if \quad v \geq c, \end{cases} \tag{1}$$

where $\pi^m(0)$ is the transfer payment received by each cartel member. The cartel's total revenue is the expected difference between the winner's value and the participation costs. The density function of the winner's value is $nF(v)^{n-1}f(v)$. The total revenue for the cartel, $\pi^c(v)$, is:

$$\begin{aligned}
\pi^c(v) &= \frac{1}{n}\int_c^1 (v-c)nF(v)^{n-1}f(v)dv \\
&= \int_0^1 \pi^m(v)f(v)dv \\
&= \pi^m(0) + \int_c^1 \int_c^v F(u)^{n-1}duf(v)dv \\
&= \pi^m(0) + \int_c^1 [1 - F(v)]F(v)^{n-1}dv.
\end{aligned}$$

Thus, the transfer payment $\pi^m(0)$ is

$$\begin{aligned}\pi^m(0) &= \frac{1}{n}\int_c^1 (v-c)nF(v)^{n-1}f(v)dv \\ &- \int_c^1 [1-F(v)]F(v)^{n-1}dv \\ &= \int_c^1 [y - \frac{1-F(y)}{f(y)} - c]G(y)dF(y).\end{aligned}$$

Since $v^* \geq c$, and $\pi^m(0) \geq 0$, we obtain

$$\pi^m(v) \geq \pi^s(v).$$

This means that the efficient cartel mechanism satisfies bidders' individual rational condition. Bidders in the cartel earn more profit, whether they win the object or not.

2.2.3. Nonratifying Condition of the Cartel Mechanism

In this part, I investigate the efficient cartel mechanism when $c \geq 0$, and bidders can update their beliefs through the cartel's prior auction.

First, I explore a knockout cartel mechanism with $c > 0$. The cartel mechanism is as follows. The cartel holds a prior auction before the seller's auction. After bidders report their bids to the cartel, the cartel chooses the bidder with the highest bid to represent the cartel in the seller's auction. Specifically, the timing of possible cartel formation between date 0 and date 1 works as follows:

- At date 0, a cartel's mechanism exists and all bidders are in the cartel.
- At date $\frac{1}{4}$, nature draws a private valuation for each bidder.
- At date $\frac{2}{4}$, each bidder reports his bid in cartel's prior auction.
- At date $\frac{3}{4}$, bidders update their beliefs through the cartel's auction and vote for or against the collusive mechanism.

- At date 1, if all bidders accepted the collusive mechanism at date $\frac{3}{4}$, the winner in the prior auction represents the cartel to bid in the seller's auction, and he will compensate the losers with transfer payment. If at least one bidder rejects the collusive mechanism, no collusion occurs. Bidders bid in the seller's auction at date 1 noncooperatively.

Definition 1 *The information leakage problem is that all bidders can update their information through the cartel's prior auction before they participate in the seller's auction.*

I assume that when a bidder is indifferent between staying in and vetoing for the cartel, he would choose to stay in the cartel. In order to show that bidders may have incentive to jump out the cartel, I define a veto set A. If the vetoer's value is in this set, he will veto for the cartel, that is, vetoing for the cartel brings higher profit to the vetoer than in collusive case. $\pi_i^v(v_i)$ denotes the vetoer's payoff.

Definition 2 $A \subseteq [0,1]$ *is said to be a credible veto set if the individual rational (IR) condition,* $\pi_i^m(v_i) > \pi_i^v(v_i)$*, is not satisfied, i.e.,*

(1) $A \neq \emptyset$,

(2) $\pi_i^v(v_i) > \pi_i^m(v_i) \Rightarrow v_i \in A$,

(3) $\pi_i^v(v_i) < \pi_i^m(v_i) \Rightarrow v_i \notin A$.

Definition 3 *The cartel is "nonratifiable" if other bidders will update their beliefs about the vetoer* i*'s value, and identify that the vetoer's value belongs to the credible veto set* A *after bidder* i *chooses to jump out of the cartel.*

Definition 4 *The cartel mechanism is ratifiable, if for each bidder* i *either*

(1) there is no credible veto set for i*, or*

(2) there is a credible veto set A and a corresponding equilibrium bid b^* such that $\pi_i^m(v_i) = \pi_i^v(v_i)$ for all $v_i \in A$.

Suppose when one of the bidders vetoes the cartel, others believe that his value is in $[v_N, 1]$, so $A = [v_N, 1]$ is a credible veto set for any bidder i. I will present that, there is an asymmetric equilibrium of the auction with these updated beliefs, such that the vetoer's payoff in this equilibrium is larger than his payoff in the cartel if his value is larger than v_N.

Consider the first-price auction where the bidders in the cartel with values are distributed on $[0, 1]$, and the vetoer's value is distributed on $[v_N, 1]$. The asymmetric equilibrium I consider in this auction, denoted by b^*, is given as follows. For the vetoer,

$$b_i^*(v_i) = \lambda(v_i) \quad \forall v_i \in [v_N, 1],$$

where $\lambda(v_i)$ is a contingent bidding function when bidder i participates in the auction. For bidders in the cartel,

$$b_j^*(v_j) = \begin{cases} N_0 & v_j \leq v_Y \\ \lambda(v_j) & v_j > v_Y \end{cases} \quad \forall j \neq i.$$

N_0 indicates that the bidder does not participate in seller's auction. In this equilibrium, all bidders staying in the cartel use the same cutoff point v_Y, which is determined by the indifferent participation condition. Following Cao and Tian [2] paper, I assume that bidding function is monotonic increasing. Thus the optimal inverse bidding functions are uniquely determined by

$$v_i(b) = b + \frac{F(v_j(b))}{(n-1)f(v_j(b))v_j'(b)}, \tag{2}$$

and

$$v_j(b) = b + \frac{F(v_j(b))[F(v_i(b)) - F(v_N)]}{(n-2)f(v_j(b))v_j'(b)[F(v_i(b)) - F(v_N)] + F(v_j(b))f(v_i(b))v_i'(b)}, \quad (3)$$

with boundary conditions $v_i(\underline{b}) = v_N, v_j(\underline{b}) = \underline{b}$,[1] and $v_i(\overline{b}) = v_j(\overline{b}) = 1$. $\underline{b}$ and $\overline{b}$ are the lower and upper bounds that bidders would bid. From the optimal inverse bidding function, we have to find the relationship between the bidders' bids and their values.

From $v_i(b)$, I have

$$v_j'(b) = \frac{F(v_j(b))}{(v_i(b) - b)(n-1)f(v_j(b))}.$$

Substitute in $v_j(b)$.

$$\begin{aligned} v_j(b) &= b + \frac{F(v_j(b))[F(v_i(b)) - F(v_N)]}{(n-2)f(v_j(b))v_j'(b)[F(v_i(b)) - F(v_N)] + F(v_j(b))f(v_i(b))v_i'(b)} \\ &= b + \frac{F(v_i(b)) - F(v_N)}{(\frac{n-2}{n-1})(\frac{F(v_i(b)) - F(v_N)}{v_i(b) - b}) + f(v_i(b))v_i'(b)}. \end{aligned} \quad (4)$$

Because the inverse bidding function is monotonic increasing according to b, the relationship is uniquely determined. Define $Q(v_i(b)) \equiv v_j(v_i)$, where $Q(v_i(b))$ is the

[1] From Cao and Tian [2], $\underline{b} = \max \arg \max_b F(b)^{n-1}(v_N - b)$.

relationship between v_j and v_i when $b = b_j = b_i$.

$$\begin{aligned}
k(v_i) &= p(b_i > b_j | v_i > v_N) \\
&= p(v_j < Q(v_i) | v_i > v_N) \\
&= \frac{P(v_j < Q(v_i), v_i > v_N)}{P(v_i > v_N)} \\
&= \frac{\int_{v_N}^{1} f(v_i) \int_0^{Q(v_i)} f(v_j) dv_j dv_i}{1 - F(v_N)} \\
&= \frac{[1 - F(v_N)]F(Q(v_i))}{1 - F(v_N)} \\
&= F(Q(v_i)).
\end{aligned}$$

Let $H(v_i) = k(v_i)^{n-1}$ be the probability that all other bidders' bids are less than vetoer i's. For any given bidder in the cartel, the maximum of others' bids is distributed according to $\hat{H}(y) \equiv k(y)F(y)^{n-2}$. Let $\tilde{v}_Y$ be the solution to

$$[\tilde{v}_Y - b_j^*(\tilde{v}_Y)]\hat{H}(\tilde{v}_Y) = c.$$

The payoff of a $\tilde{v}_Y$ type bidder is equal to his participation costs, whenever $\tilde{v}_Y \leq 1$. I have $v_Y = min\{1, \tilde{v}_Y\}$ and $v_N < v_Y \leq 1$ because

$$[v_N - b_j^*(v_N)]\hat{H}(v_N) = 0.$$

Notice that v_Y is the cutoff point where bidders are indifferent between staying in and vetoing for the cartel. An increase in v_N leads to a higher v_Y. Thus we have that v_Y is a strictly increasing function of v_N until it reaches 1 for some value of v_N and stays there for higher value of v_N. The payoff of vetoer i is

$$\pi_i^v(v_i) = max\{[v_i - \lambda(v_i)]H(v_i) - c, 0\}.$$

Comparing this payoff with $\pi_i^m(v_i)$ from the cartel leads to the following lemma. All proofs are in Appendix A.

Lemma 1 *There exists a* $v_N \in (c, 1)$ *such that* $\pi_i^v(v_i) \geq \pi_i^m(v_i)$ *if and only if* $v_i \geq v_N$.

Lemma 1 shows that once bidder i chooses to veto for the cartel (and the equilibrium b_i^* is played) and others believe that his value exceeds v_N, i would benefit from vetoing if and only if his value exceeds v_N. Thus, $A = [v_N, 1]$ is a credible veto set for player i. Furthermore, there is no credible veto set for which all bidders in this set are indifferent between vetoing and ratifying, which means that

$$\pi_i^v(v_i) = \pi_i^m(v_i) \tag{5}$$

does not hold. This is stated in the following lemma.

Lemma 2 *There is no credible veto set* A *(for any* i*) and corresponding equilibrium* b_i^* *in the post-veto auction such that* $\pi_i^v(v_i) = \pi_i^m(v_i)$ $\forall v_i \in A$.

Proposition 1 follows from Definition 4 and Lemma 1 and 2.

Proposition 1 *In first-price sealed-bid auction, when* c *is positive, and the information leakage problem exists, the cartel mechanism is not ratifiable.*

Having updated their beliefs that the vetoer's value belongs to A, other bidders do not participate in seller's auction since they have to pay the non-refundable participation costs and earn a negative profit. The cartel mechanism is not ratifiable.

Then I turn to the case when $c = 0$, and bidders can update their beliefs from cartel's auction. With their information through the cartel's auction, if there is any bidder vetoing for the cartel after the cartel's auction, other bidders can enter the seller's auction for free and bid as much as possible to make the vetoer earn less or equal profit than his cartel revenue. The game becomes an auction without collusion. $\pi_i^m(v_i) \geq \pi_i^v(v_i) = \pi_i^s(v_i)$. This means that there is a discontinuity at $c = 0$.

Proposition 2 *The efficient cartel mechanism is sustainable without participation costs, even if the information leakage problem exists. Thus, there is a discontinuity at* $c = 0$.

I discover that the efficient cartel mechanism cannot be ratifiable when both information leakage problem and positive participation costs exist. When participation costs exist without information leakage problem, the efficient cartel mechanism is ratifiable. This efficient cartel mechanism is designed to maximize bidders' ex ante expected profit. Without information leakage problem, bidders cannot update their beliefs through cartel's auction, so nobody has incentive to jump out of the cartel. On the other hand, when information leakage problem exists without participation costs, this cartel mechanism is still ratifiable. Because bidders can submit a bid in the seller's auction for free, the vetoer cannot earn extra profit from betraying the cartel. Thus, I extend the efficient cartel mechanism in two cases: either the participation cost or the information leakage problem exists, the efficient cartel mechanism is ratifiable. However, when both exist at the same time, under the conditions in Lemma 1, the efficient cartel is not ratifiable.

2.2.4. Different Participation Costs

In this part, I consider the case when bidders have different participation costs. Assume that a monotonic equilibrium (v_i^*, v_j^*) exists, i.e., $v_j^* < v_i^*$ and $c_j < c_i$, when the distribution function is concave. Another assumption is that $v_i \geq v_i^* > v_j^* = v_j$ (we only consider the case that bidders with higher values would have higher costs). If the higher-values bidders have lower participation costs, the result is obviously the same as in Chapter 2.2.3. When information leakage problem exists, the bidder with the highest value would veto for the cartel. Note that the participation costs now become private information in this economic environment.

In cartel's auction, bidders would submit their bids to the cartel when their values are higher than the sum of seller's reserve price and their own participation costs. If bidders have to report the level at their values minus the participation costs, the type i bidders would use a higher cutoff point and submit a equilibrium bid b^* to convince the others that their values belong to a credible veto set A. thus, everyone in the cartel would believe that type i bidders' net values are larger than that of type j's, and the type i bidders have incentive to betray the cartel. If the losers (type j bidders) in cartel's auction participate in seller's auction, they will lose for sure. They do not want to pay the non-refundable participation costs. The bidder with the largest net value would have incentive to betray the cartel to maximize his revenue if his value is in A. This result is only held when $F(\cdot)$ is concave. Otherwise, the monotonic cutoff vector may not be the only one equilibrium. Accordingly, bidders with higher net values may not have a credible veto set to convince other bidders in the cartel. Thus, the cartel is ratifiable in this case.

2.3. Conclusion

As McAfee and McMillan [17] point out, a cartel agreement is worthless without some way to enforce it. Since contracts to fix prices usually cannot be written, any collusive agreement must be designed to be self enforcing. If individual rational condition and incentive compatible constraint could not be supported, the efficient colluding mechanism is not freely implemented. When bidders' actions are strategically interactive and their outcome is affected by bidders' beliefs about others' values, the standard mechanism design approach may suffer from information leakage problem. This observation is important when bidders are making interim decisions whether to participate in the collusive mechanism.

When collusion forms, it usually brings some losses to the economy, like reducing market competition or compressing consumers' welfare. In order to reduce the possibility of collusion, I want to investigate members' behavior in a cartel mechanism. This may provide important suggestions for the government to differentiate cartel or to prevent the formation of a cartel. In this chapter, the efficient cartel mechanism is no longer sustainable in the presence of positive participation costs and information leakage problem. The bidder with the highest value would choose to veto for the collusive mechanism. This behavior could maximize his benefit and leave all others with zero profit.

In the next chapter, I would like to check seller's behavior if she is not passive, and the third-party authority's strategies that can destabilize the cartel in Chapter IV.

CHAPTER III

EFFICIENT SELLING MECHANISM WITH BUYERS' CARTEL AND ACTIVE SELLER

3.1. Introduction

Asymmetric information usually results in collusive behavior among bidders. In reality, the collusive behavior is often found in an industry with heterogeneous firms and private information. In order to reduce the possible inefficiency caused by collusion, an auction mechanism is usually used to enhance efficiency. Myerson [19] considers seller's optimal auction design problem in which she has an indivisible object to sell to one of several buyers. Given there is imperfect information between the seller and buyers, the seller's problem is to design an auction that provides her with the highest expected profit at Nash equilibrium. Thus, the seller must consider all possible value of buyers and try to choose an auction mechanism that maximizes her expected revenue.

Is this optimal auction mechanism able to prevent collusive behavior? The answer is that it depends. The well-known advantage of auction will diminish when bidders have ample incentives to collude. However, it is difficult for bidders to profitably collude and share the generated collusive surplus because of private information. As the synthesis of standard mechanism design literatures, McAfee and McMillan [17] point out that the efficient collusive mechanism cannot be implemented with no cost. They explore the bidding strategies in first-price sealed-bid auction and show that when members can exclude new entrants and make transfer payments among themselves, the optimal cartel mechanism can reach the efficiency by implementing a prior auction. Cramton and Palfrey [6] find that the information leakage problem from bidders' participation decisions in the prior auction would disintegrate the optimal cartel mechanism. Bidders can observe a bidder's type from his choice of whether vetoing the collusive mechanism, which in turn may affect the seller's revenue. In order to specify this problem, they set up a two-stage game: all players simultaneously vote for or against the collusive mechanism. This vote is held at the interim stage. If the collusive mechanism is unanimously accepted, it is implemented; otherwise they participate in seller's auction with updated information.

There is a group of papers investigating how to minimize the loss resulted from the cartel. Laffont and Martimort [14, 15] find that if the agents are uncorrelated, the optimal outcome can be achieved at no cost for the principal. When agents are correlated, in order to prevent collusion, the principal has to pay a large cost. Che and Kim [3] construct a new mechanism that provides the principal with the same expected revenue as that in noncollusive case with risk neutral agents for both uncorrelated and correlated types. This mechanism forces the bidders in a collusion to be a residual claimant of the entire surplus.

In an efficient selling mechanism, the seller's behavior is usually assumed to be passive.[1] She announces a reserve price and sells an indivisible object to the bidder with the highest bid in an auction, and she does not know if she faces a cartel. Instead, I assume that the seller is active in this chapter. That is, the seller is allowed to detect collusive behavior. I establish a general auction mechanism without a particular auction format, and explore the seller's optimal strategy. The result is that the seller's best strategy is to raise the reserve price to a higher level than her own value of the object. This method may not satisfy the efficient condition, because the object may not be sold even if a bidder's value is higher than the seller's

[1]For example, the mechanism in McAfee and McMillan [17], and Tan and Yilankaya [24].

value. Moreover, when information leakage problem exists, the cartel mechanism is sustainable regardless of the seller's type.

Additionally, I consider the possibility when the seller is authorized to change the mechanism (for example, the payment mechanism). The residual claimants method (Che and Kim [3]) guarantees the seller to receive her expected payoff by raising reserve price and the object sold to the bidder with the highest value. In our model, this method may not be able to prevent bidders from joining a cartel. When the efficient cartel exists, there is only one bidder representing the cartel to bid in the seller's auction. If the seller cannot influence nonparticipating bidders, she cannot prevent the cartel. I suggest another method to forbid the formation of a cartel. That is, the seller can set up a positive participation cost for the bidders in the seller's auction. In Hsueh and Tian [12], I show that when both information leakage problem and participation costs exist, the cartel mechanism cannot be sustained. The bidder with the highest value would have incentive to jump out of the cartel, which promises him a higher profit.

The remainder of this chapter is structured as follows. Chapter 3.2 describes the model. In Chapter 3.3, I demonstrate the bidders' and the seller's optimal strategies. Chapter 3.4 investigates the case when the seller is allowed to change the original auction mechanism. Chapter 3.5 concludes. All proofs are in Appendix B.

3.2. Model

There are one seller and n ($n \geq 2$) bidders in the economy. Bidder i's value for the object is $v_i \in V$, and $v = (v_1, ..., v_n)$ represents the vector of n bidders' value. v_i is private information which is a random draw from a population with cumulative distribution function $F(\cdot)$ and density function $f(\cdot)$ supported on $[0, 1]$. v_h is the highest value among bidders. The seller values her single indivisible object at v_0 and sets a reserve price $r \geq v_0$. I assume that the seller has no private information about the object, so v_0 is known to all bidders. The seller cannot force every bidder to join her auction. Bidders would participate in seller's auction if their values are higher than r, and they do not know others' participation decisions when they make theirs.

3.2.1. Direct Revelation Mechanism

I apply the concept "revelation principle", which shows that the outcomes generated from any equilibrium of any mechanism can be sustained by a truthful equilibrium of some direct mechanism. There is no loss of generality in restricting attention to direct mechanism. The direct revelation mechanism is that, bidders would simultaneously report their values to the seller, and the seller decides who is the winner and how much the winner should pay. The auction is efficient when the object is allocated to the bidder with the highest value.

This mechanism can be summarized in a pair of outcome functions (h, x) such that, $h_i(v)$ is the probability that bidder i wins the auction and $x_i(v)$ is bidder i's expected payment, then bidder i's expected utility function is[2]

$$U_i(v_i, v_{-i}) = E_{-i}[v_i h_i(v_i, v_{-i}) - x_i(v_i, v_{-i})]. \tag{6}$$

E_{-i} is the expectation over v_{-i}, and v_{-i} is the vector of other bidders' values. The seller's expected utility function is[3]

$$U_0 = E\{v_0[1 - \sum_{i \in n} h_i(v)] + \sum_{i \in n} x_i(v)\}. \tag{7}$$

[2]I prove this utility function is equivalent to the utility function in first-price and second-price sealed-bid auction. The proof is in Appendix B.

[3]This utility function is based on Myerson [19].

The direct revelation mechanism has to satisfy three following conditions.

The first condition is that the probability must satisfy:

$$\sum_{i\in n} h_i(v) \leq 1 \qquad and \qquad h_i(v) \geq 0 \quad \forall i \in n, \;\; \forall v \in V. \tag{8}$$

The second condition is to guarantee that bidders will have incentive to participate seller's auction. The individual rational (IR) condition is:

$$U_i(v_i, v_{-i}) \geq 0 \quad \forall i \in n. \tag{9}$$

Because bidders may have incentives misreporting their valuation, the third condition is to elicit the true values from bidders. The following incentive compatible (IC) constraint should be satisfied:

$$U_i(v_i, v_{-i}) \geq U_i(z_i, v_{-i}) \quad \forall i \in n, \tag{10}$$

where z_i is the value bidder i reports to the seller.

An auction mechanism (h, x) is *feasible* if (8), (9), and (10) are all satisfied. This means that the seller could allocate the object according to h and get a payment according to x with all bidders honestly reporting their value.

Let $\mathcal{M}$ denote the set of feasible mechanisms. For expressional simplicity, I define

$$M_i(h_i, v_i) \equiv E_{-i} h_i(v_i, v_{-i}). \tag{11}$$

3.2.2. Collusive Auction

In this part, I first describe a pre-auction knockout cartel mechanism. Then, I specify the seller's behavior.

The seller may face a knockout cartel which runs as follows. The cartel holds a prior auction before the seller's auction. After the bidders report their bids to the cartel, the cartel chooses the bidder with the highest bid to represent the cartel to bid in the seller's auction. Specifically, considering possible cartel formation between date 0 and date 1, the timing is as follows:

- At date 0, a cartel's mechanism exists and all bidders are in the cartel.
- At date $\frac{1}{4}$, nature draws a private valuation for each bidder.
- At date $\frac{2}{4}$, each bidder reports his bid in cartel's prior auction.
- At date $\frac{3}{4}$, bidders update their beliefs through the cartel's auction and vote for or against the collusive mechanism.
- At date 1, if all bidders accepted the collusive mechanism at date $\frac{3}{4}$, the winner in the prior auction represents the cartel to bid in the seller's auction, and he will compensate the losers with transfer payment. If at least one bidder rejects the collusive mechanism, no collusion occurs. Bidders bid in the seller's auction at date 1 noncooperatively.

Definition 5 *The information leakage problem is that all bidders can update their information through the cartel's prior auction before they participate in the seller's auction.*

I assume that the efficient cartel designs a mechanism in order to maximize the ex ante (before the valuations are known) sum of bidders' expected profits, which means that this cartel is self-enforcing without the monitor from a third party. I also assume that when a bidder is indifferent between staying in and vetoing for the cartel, he would choose to stay in the cartel.

Definition 6 *The seller's behavior is active, if she knows that she may face a collusive mechanism.*

If the seller knows that she may face a cartel, she will have incentives to change the reserve price to maximize her revenue. The reserve price is announced by the seller at the beginning of the game, and she cannot change her price after announcement. When the seller announces this reserve price, she is not sure if she faces a cartel as the general setting.

In order to show that bidders may have incentive to jump out the cartel, I define a vetoing condition. $U_i^M(v_i)$ and $x_i^M(v_i)$ are the expected utility and payment in the cartel.

Definition 7 *Bidder i will choose to veto for the cartel if the condition, $U_i^M(v_i) \geq U_i(v_i)$, is not satisfied, i.e., the expected payment $x_i^M(v_i) \geq x_i(v_i)$.*

The cartel mechanism is not sustainable if this vetoing condition is satisfied for one of all bidders, that is, the noncollusive behavior can bring higher revenue to him.

3.2.3. Noncollusive Auction

In this part, I specify a benchmark case that bidders' optimal strategies under non-cooperative case. For any vector v_{-i}, bidder i's expected payment is:

$$x_i(z_i, v_i) = v_i h_i(z_i, v_{-i}) - \int_{z_i}^{v_i} h_i(s_i, v_{-i}) ds_i.$$

$\int_{z_i}^{v_i} h_i(s_i, v_{-i}) ds_i$ is the expected profit for the bidder.[4] The optimal reporting value is:

$$k(v_{-i}) = inf\{z_i | z_i \geq v_0 \quad and \quad z_i \geq v_j, \quad \forall j \neq i\}.$$

[4]The probability of winning is $F(z_i)^{n-1}$. In other words, $\int_{z_i}^{v_i} h_i(s_i, v_{-i}) ds_i = \int_{z_i}^{v_i} F(s_i)^{n-1} ds_i$ is the expected profit. $\int_{z_i}^{1} \int_{z_i}^{v_i} F(s_i)^{n-1} ds_i f(t_i) dt_i = \int_{z_i}^{1} \int_{v_i}^{1} f(t_i) dt_i F(s_i)^{n-1} ds_i = \int_{z_i}^{1} [1 - F(s_i)] F(s_i)^{n-1} ds_i$ is bidder i's ex ante expected payoff.

$k(v_{-i})$ is the upper bound of other bidders' possible bids. I can obtain

$$h(z_i, v_{-i}) = \begin{cases} 1 \quad if \quad z_i > k(v_{-i}) \\ 0 \quad if \quad z_i < k(v_{-i}), \end{cases}$$

and

$$x_i(z_i, v_i) = \begin{cases} k(v_{-i}) & if \quad h(z_i, v_{-i}) = 1 \\ 0 & if \quad h(z_i, v_{-i}) = 0. \end{cases} \tag{12}$$

The winner's expected utility becomes:

$$U_i(z_i, v_i) = E_{-i}[v_i h(z_i, v_{-i}) - k(v_{-i})]. \tag{13}$$

3.3. Direct Revelation Mechanism with Reserve Price

The seller would not sell the object if bidders' highest bid is less than v_0, thus I assume the reserve price $r = v_0$ in Chapter 3.3.1, and release this assumption in Chapter 3.3.2.

3.3.1. Bidders' Optimal Strategy

In this part, I explore the optimal strategy for bidders when a cartel exists. Consider bidders' payoff in seller's auction. The total transfer payment is[5]

$$T(v_i) = F(v_i)^{-n} \int_r^{v_i} (s_i - r)(n-1)F(s_i)^{n-1} f(s_i) ds_i, \tag{14}$$

[5]As in McAfee and McMillan [17], $T(\cdot)$ is the total net transfer weighted by the probability of winning. The probability that v_i is the highest value among all bidders is $F(v_i)^n$. The expected difference between the winner's possible bid and the reserve price r is $\int_r^{v_i} (s_i - r)F(s_i)^{n-1} f(s_i) ds_i$. There are $(n-1)$ losers in the cartel.

which is the expected difference between the winner's reporting value and the reserve price. If winner i's value is v_i but he reports z_i, his expected profit in the cartel is

$$U_i^M(z_i, v_i) = E_{-i}[v_i h(z_i, v_{-i}) - T(z_i) - r]. \tag{15}$$

Since there is only one bidder in seller's auction, he could bid the lowest price r and win for sure. Note that if no one's value is higher than the seller's reserve price, the cartel cannot be held, and no one will join the seller's auction.

In this cartel mechanism, the expected payment function becomes

$$x_i^M(z_i, v_i) = \begin{cases} T(z_i) + r & if \quad h_i(z_i, v_{-i}) = 1 \\ \frac{-T(z_i)}{n-1} & if \quad h_i(z_i, v_{-i}) = 0. \end{cases} \tag{16}$$

Rewrite bidder i's utility function:

$$\begin{aligned} U_i^M(z_i, v_i) &= [v_i - T(z_i) - r]F(z_i)^{n-1} \\ &\quad + [1 - F(z_i)^{n-1}] \int_{z_i}^{1} \frac{T(s_i)}{n-1} \frac{(n-1)F(s_i)^{n-2} f(s_i)}{1 - F(z_i)^{n-1}} ds_i \\ &= [v_i - T(z_i) - r]F(z_i)^{n-1} + \int_{z_i}^{1} T(s_i)F(s_i)^{n-2} f(s_i) ds_i. \end{aligned} \tag{17}$$

The following lemma shows that the (IC) condition could be satisfied in the cartel.

Lemma 3 *Given $T(v_i)$ the prior auction is incentive compatible and efficient.*

From the lemma, I obtain $\frac{\partial U_i^M(z_i, v_i)}{\partial z_i}|_{z_i = v_i} = 0$, which means that in equilibrium, everyone in the cartel would report their value truthfully.

For any given vector v_{-i}, winner i's expected profit in the cartel becomes:

$$U_i^M(v_i) = E_{-i}[v_i h(v_i, v_{-i}) - T(v_i) - r]. \tag{18}$$

Compared (13), (15) with (18), I come up with the following proposition.[6]

Proposition 3 *Regardless of information leakage problem, if bidders report their true values to the cartel, $T(v_i) + r - k(v_{-i})$ is always less than zero. Hence, bidders would choose to stay in the cartel, and the bidding ring is formed.*

If $x_i(z_i, v_i)$ in (16) is less than that in (12), winner i would have incentive to participate in the cartel mechanism with $U_i^M(v_i) \geq U_i(v_i)$, because the winner is paying less than in non-cartel case.

3.3.2. Seller's Optimal Strategy

The seller's problem is to consider all possible values of buyers and try to maximize her expected revenue. The seller's objective function in (7) can be rewritten as:

$$U_0 = \int_V rf(v)dv + \sum_{i \in n} \int_V h_i(v)(v_i - r)f(v)dv + \sum_{i \in n} \int_V (x_i(v) - v_i h_i(v))f(v)dv. \quad (19)$$

I can simplify the seller's objective function with the following lemma.

Lemma 4 *With condition $U_i(v_i) = U_i(z_i) + \int_{z_i}^{v_i} M_i(s_i)ds_i$, seller's problem can be simplified to be*

$$\max \int_V [\sum_{i \in n} (v_i - r - \frac{1 - F(v_i)}{f(v_i)})h_i(v)]f(v)dv. \quad (20)$$

Based on this lemma, let

$$d(v_i) = v_i - \frac{1 - F(v_i)}{f(v_i)}, \quad (21)$$

[6]The proof is in Appendix B.

where $\frac{1-F(v_i)}{f(v_i)}$ is the expected information rent earned by the winner i.[7] $d(v_i)$ is the virtual valuation of a buyer with value v_i.[8]

There are two possibilities. One is when seller's reserve price is too high to sell the object, that is, $r > \max_{i\in n} v_i$. In this case, the seller keeps the object because the seller's utility is less than zero if she sells. While the other case is that, at least one bidder's value is higher than seller's reserve price, i.e., $r \leq \max_{i\in n} v_i$, and she gives the object to the bidder with the highest value (thus the highest $v_i - \frac{1-F(v_i)}{f(v_i)}$). The seller's problem becomes

$$\max \int_V (\sum_{i\in n}(d(v_i) - r)h_i(v))f(v)dv. \tag{22}$$

According to Proposition 3, bidders would choose to join the ring. When the seller announces the reserve price at v_0, she earns v_0. Thus, from (22), the optimal strategy for her becomes:[9]

$$r = d^{-1}(v_0), \tag{23}$$

where $d^{-1}(\cdot)$ is an inverse function of $d(\cdot)$.

In noncollusive auction case, the best price for the seller to sell the object is at $k(v_{-i}) = max_{j\neq i}v_j$. When cartel exists, there is only one bidder joining the seller's auction and he submits his bid at the seller's reserve price. For the seller, the only strategy to maximize her revenue is to raise the reserve price according to $d^{-1}(v_0)$.

---

[7] $\frac{1-F(v_i)}{f(v_i)}$ is an index of the winning bidder's payoff. $\frac{\partial}{\partial v_i}\frac{1-F(v_i)}{f(v_i)} > 0$ means that the bidder with higher value earns relatively higher payoff, and $\frac{\partial}{\partial v_i}\frac{1-F(v_i)}{f(v_i)} \leq 0$ implies that the higher value bidder receives relatively less payoff.

[8] As in Krishna [13], for all i, $E[d(v_i)] = 0$.

[9] This result is similar as in Myerson [19], when the assumption: "when the seller announces this reserve price, she is not sure if she faces a cartel." holds. If the cartel exists before the seller's announcement, the optimal reserve price for her is $r = z_i + (n-1)\int_r^{z_i} F(s_i)^n ds_i$, which is shown in the proof of proposition 3.

Due to asymmetric information, she does not know the true values of the bidders'. It is impossible for her to set up the reserve price exactly at either the highest or the second highest value among bidders. If no one's value is higher than the reserve price, she will keep the object. This implies that the efficiency may not always be held. I use the following proposition to summarize this result.[10]

Proposition 4 *If the seller is active,*

(a) she can set the reserve price r according to $d^{-1}(v_0) \geq v_0$ to maximize her expected payoff.

(b) The cartel mechanism could still be held when $d^{-1}(v_0) \leq v_h$, i.e., $T(v_i) + r - k(v_{-i}) \leq 0$. If $d^{-1}(v_0) > v_h$, I have $T(v_i) + r - k(v_{-i}) > 0$. Thus, the winner i has no incentive to stay in the cartel. The ring breaks down.

(c) The efficient condition may not be held, i.e., $d^{-1}(v_0) > v_h$. The object is not sold to the bidder with the highest value, which is less than the reserve price.

From this proposition, the cartel mechanism is sustainable when the seller is active. I use an example to illustrate the main idea. Suppose there are one seller and two bidders. Agent $i = 1, 2$ can bid the object at their own value v_i, which is drawn uniformly from $[0, 1]$. If the agents are not allowed to collude, it is optimal for the seller to use an auction, such as a second-price auction. The bidder with the higher value wins and gets the object with a payment that equals to the lower bid. In our model, if the seller's value is $v_0 = 0$, I have $d(v_i) = v_i - \frac{1-F(v_i)}{f(v_i)} = 2v_i - 1$, which is an increasing function of v_i. Suppose now the agents can collude, the second-price auction would be affected by the collusion. Before the seller's auction, bidders

[10]The proof is in Appendix B.

can organize a knockout auction in which agents bid for the right to participate in seller's auction; i.e., the winner bids the seller's reserve price and he would win the auction for sure. With collusion, the seller can only receive the reserve price from the winner. Thus, the optimal strategy for the seller is to set the reserve price equal to $d^{-1}(v_0) = 1/2$, and there is a probability of $(1/2)^2$ that the object will not be sold even if someone's value is higher than the seller's original value 0. Her expected payoff becomes 3/8, which is greater than her value 0. This is why the efficiency may not hold, yet the cartel may be sustained, and the seller's optimal strategy helps to maximize her expected payoff.

3.4. Direct Revelation Mechanism with Residual Claimants Method and Participation Costs

In Chapter 3.3, the only tool that the seller can use is the reserve price. When the seller raises the price, it may lead to inefficiency of the auction. In Chapter 3.4, I allow the seller to change the auction mechanism with two methods: residual claimants method and participation costs, in which the seller is able to earn expected profit and an auction is always efficient.

3.4.1. Residual Claimants Method

I use "*robustly collusion-proof* (RCP)" to specify our idea.

Definition 8 *A mechanism $M \in \mathcal{M}$ is RCP, if every mechanism $\tilde{M}$ in the subset of $\mathcal{M}_M$ can give the same expected payoff to the principal as the original mechanism M.*

$\mathcal{M}_M$ is the set of reallocational manipulation that satisfies (IR) and (IC) conditons. This RCP is presented in Che and Kim [3](from now on, CK). Their idea is that, when a RCP mechanism manipulates the outcome of the original mechanism, the principal could receive an ex post constant surplus at first, which guarantees the same expected payoff for the seller, and the bidders become residual claimants of the entire surplus.

Bidders in the cartel could receive a higher payoff because of the distortion resulted from asymmetric information between the bidders and the seller. This asymmetric information induces an information rent from the seller which must be given to the members of the collusion. When the seller can be paid first to extract the information rent, any other allocation designed by the collusion will not affect seller's surplus. Based on the CK model, when the seller can use other mechanism to replace the original mechanism, this new mechanism could still bring the same expected payoff to the seller if it is RCP mechanism. A new mechanism is likely to prevent the ring from being formed, and guarantee that the efficient condition holds. Thus, I have the following proposition:[11]

Proposition 5 *Based on the residual claimants method, if the optimal expected payoff level is implementable, there exists a RCP mechanism that implements the non-collusive optimal payoff for the seller, and bidders have no incentive to form a ring.*

Making bidders the residual claimant is a strategy to prevent collusion. I continue to use the example from Chapter 3.3.2 to illustrate the idea. The seller's optimal strategy is to set the reserve price equal to 1/2 and earn an expected payoff 3/8. If the seller can replace the auction mechanism with a different payment arrangement, the result is different. First, the seller received her optimal reserve price 1/2 from the loser after the auction, then the winner pays his bid to the loser. In this case, the winner still pays his bid in the auction, but the loser has incentives to make the winner pay as much as possible. In other words, the loser's payoff becomes the difference

[11]The proof is in Appendix B.

between $b_i - 1/2$, where b_i is winner i's bid; thus the loser would like to maximize his payoff by maximizing winner's bid. If bidders know this new payment mechanism, they have no incentive to collude. Since the bidder (loser) who is appointed to pay the reserve price to the seller, acting like a third party authority, always has incentive to betray the cartel. He monitors the other bidder's bid in order to maximize his own payoff.

However, this residual claimants method may not be able to prevent the formation of a ring if there is only one bidder in the seller's auction. According to our setting, when the ring is formed, there is only one bidder joining the seller's auction. Even if she can extract her expected revenue from this single participant, bidders may still have incentive to form a ring.[12]

Proposition 6 *The residual claimants method in CK model is not able to prevent the cartel if the seller cannot extract her expected payoff from non-participating bidders.*

When at least two bidders' values are higher than seller's reserve price, the cartel would be held. In the cartel, bidders can make transfer payments among themselves. The winner in the cartel could earn more profit than that of not in the cartel, because the transfer payment is positive. When there is only one bidder's value higher than the reserve price, he is indifferent between staying in and vetoing for the cartel because his transfer payment is zero. According to our assumption, in equal profit case, the winner would choose to stay in the cartel. Therefore, when there is information leakage problem with no participation costs, the cartel mechanism is sustainable.

Consider the case that the seller is allowed to extract her expected payoff from other bidders. The appointed bidder pays seller's expected payoff to the seller first. If he chooses to jump out of the cartel, he could place a bid in the seller's auction

[12]The proof is in Appendix B.

to threat the winner to bid as much as possible because he has already updated his information through the cartel's auction.[13] Based on this condition, I consider the following two cases. First, there is only one bidder's value higher than the seller's reserve price. In this case, this bidder would bid the reserve price to the seller in the second-price auction, and the loser's payoff becomes zero. The other case is that there are at least two bidders' values higher than seller's reserve price. The loser's payoff of jumping out of the cartel is higher than that of staying in the cartel. Once he jumps out of the cartel, the cartel collapses. Bidders will bid in seller's auction noncooperatively. Hence, if the seller can influence nonparticipants, the residual claimants method can prevent the cartel.

3.4.2. Participation Costs

The residual claimants method is not the only method to prevent the collusion. In Hsueh and Tian [12] paper, I show that if both information leakage problem and participation costs exist, bidders do not have incentives to form a ring. If the seller can set a positive participation cost, no matter how small it is, the highest-value bidder does not have incentive to stay in the cartel. There are two reasons for the winner to veto for the cartel.

(1) After the prior auction, the winner knows that no one has higher value than his. If he is the only one bidder participating in the auction, he can pay the reserve price plus the participation costs and win this auction for sure.

(2) All other bidders know who has the highest bid in the cartel after

[13]Note that bidders do not bid higher than their own values, which can be erased easily by a trembling hand equilibrium.

> prior auction. If the winner of the prior auction places his bid in seller's auction, other bidders will lose. After updating their belief, no one wants to join the auction because they would lose the auction for sure and have to pay the non-refundable participation cost, which ends up with negative profits.

If the seller is allowed to change the auction mechanism, either the residual claimants method or positive participation costs, she can prevent a ring. Note that in residual claimant method, the loser may have incentives to jump out of the cartel; while with a positive participation cost, it is the winner who has incentives to veto for the cartel. If the seller can give higher revenue to cartel members, no matter it is the winner or the loser, the cartel is no longer sustainable.

3.5. Conclusion

In this chapter, I explore two assumptions about the seller's behavior in an efficient cartel mechanism. When the seller is active, she may change the result of the cartel mechanism in many cases.

When the seller's type is passive, I find that the cartel mechanism is sustainable, regardless of information leakage problem. Suppose that the seller can detect collusive bidders, the only tool to increase her revenue is to raise the reserve price. When the information leakage problem exists, the cartel mechanism is still sustainable.

Additionally, I examine two methods that the seller can use to prevent the cartel. When I focus on the sustainability of the cartel mechanism, I discover that the residual claimants method defined in Che and Kim [3] is not effective in decomposing cartel, since the seller cannot force non-participants. However, if the seller can set up participation costs as Hsueh and Tian [12], the winner will have incentive to veto when information leakage problem exists. To sum up, when the seller is authorized to change the auction mechanism, she may deter the collusive behavior.

In next chapter, I study whether a third-party authority helps to prevent a cartel, when the seller's behavior is passive and no participation cost exists. From the residual claimants method, if there is a third-party in the economy, the cartel mechanism may collapse. The cartel mechanism described in this chapter, is designed to be self-enforcement and with no punishment. If there is a third party authority who can help to monitor and punish, the result may be different. It would be interesting to investigate the optimal strategy for the authority to prevent a cartel.

CHAPTER IV

DIFFERENTIATE COLLUSIVE MECHANISM WITH THE GOVERNMENT

4.1. Introduction

When a ring is formed, it usually comes with huge amount of financial benefit for the members but brings loss to the economy. A cartel makes secret decisions to boost prices, share production information and then make high prices sticky. In reality, the collusive problem often takes place in an industry with heterogeneous firms and private information. When there is collusion, it usually reduces market competition and erodes consumers' welfare.

Although a cartel is prohibited by law in many countries, it is difficult to detect collusive behavior in the real world. If the government could not detect collusive behavior, is there a mechanism that can give the members enough incentives to veto for the cartel? Or, how does the government differentiate collusive behavior? Osborne [20] and McAfee and McMillan [17] point out that a successful cartel has to overcome many obstacles. If I can enhance the effects of these obstacles, it is easier for the government to prevent the formation of a ring. In the literatures on organization structure, the external threats might distort the internal efficient structure. In the literatures of efficient selling mechanism, like McAfee and McMillan [17] and Tan and Yilankaya [24], they establish a set of conditions for the formation of a ring. However, they consider about buyers' and seller's behavior in an auction without a third party authority. If this authority, like the government, is allowed to exist in the economy, the bidders' collusive decisions may be different.

Osborne [20] claims that a cartel faces one external and four internal problems. The external problem is to predict and discourage production by outsiders. The four internal problems are designing the contract, solving the sharing problem, detecting, and deterring cheating. He mentions that even if a cartel can solve these problems, it does not guarantee a stable cartel. A cartel breaks down because it cannot control external production or detect cheating. McAfee and McMillan [17] also indicate that a successful cartel must overcome four obstacles. First, the conspirators must design some mechanism to partition the revenue, because members have incentives to contend for more revenue. Second, all collusive contracts should be self enforcement. Third, the high profits earned in a cartel provide incentives for new firms to join in. The competition resulted from these new firms would distort the revenue allocation. Fourth, the losers of the cartel may try to destabilize it. Due to the above conditions, an efficient colluding mechanism is not freely implemented.

Cramton and Palfrey [5] find that, when private information exists, a cartel agreement is possible in the economy. Given incentives and participation constraints, they characterize a series of applicable contracts. If a cartel is sustainable without unanimous agreements, the incentive problem can be avoided in a large cartel. When uncertainty exists, regardless of the ratification rule, a perfect collusion is possible in a large cartel. Porter [21] empirically tests cartel's stability with weekly time series data on the Joint Executive Committee railroad cartel; he proposes that observed prices reflected switches from collusive to noncollusive behavior from 1880 to 1886. The hypothesis that no change in the price, is rejected in the paper.

In this chapter, I use the methodology in the literatures of organization structure to specify the government's role in efficient cartel mechanism. As the synthesis of standard mechanism design literatures, McAfee and McMillan [17] point out that the efficient collusive mechanism cannot be implemented with no cost. They explore the bidding strategies in first-price sealed-bid auction and show that when members can exclude new entrants and make transfer payments among themselves, the optimal cartel mechanism can reach the efficiency by implementing a prior auction. Tan and Yilankaya [24] show that the efficient collusive mechanism with independent private values is not ratifiable in second-price auction with positive participation costs and information leakage problem.

In organization structure literatures, there is a group of papers dealing with the impact of a third party authority on the cartel's behavior. Ben-Porath and Kahneman [1] study a repeated game. If only a subset of the agents can observe all others' behavior, they show that, "two" agents are sufficient to achieve efficient payoff with the discount factor closing to 1. Garupa [7] investigates the organizational problem. He focuses on the optimal size of the illegal organization given its internal structure. He shows the trade off between increasing internal productivity and leaving members exposed to be detected. Waldman [25] and Ricart-I-Costa [22] consider what affects internal efficiency. They show that the external considerations might distort the most internal efficient structure. Harrington [9, 10, 11] examines the effect of the external shock from a third party authority on the cartel's optimal pricing behavior.

This chapter investigates the government's strategy in destabilizing a cartel and hindering the formation of a cartel. I explore the allocation of government's budget that entitles to regulate collusive behavior in a repeated game. In some cases, allocating the budget among firms equally may lead to a collapse of collusive mechanism. In other cases, however, symmetric allocation may induce the agents to form a ring. How the government optimizes its budget allocation to prevent collusive behavior becomes an important question.

I also have a concern on whether the government should choose exemption or subsidy. If the exemption (it could be a premium or punishment) is large enough, members in a cartel may have incentives to veto, and outsiders may not participate in the collusive mechanism. On the other hand, an individual firm that receives subsidy from the government, is likely to earn more profit and thus not to participate in a collusive mechanism. Exemption is usually restrained by law and it is costly to change. Compared with exemption, subsidy is more applicable in reality.

The rest of this chapter proceeds as follows. In Chapter 4.2, I present a general setting of economic environment and introduce the stage game. In Chapter 4.3, I investigate different reactions of members under different conditions and the optimal strategy for the government to detect the collusive behavior. In Chapter 4.4, I illustrate our main idea with a two stage model. Concluding remarks are provided in Chapter 4.5. All proofs are in the Appendix C.

4.2. Model

Suppose there are $N \geq 2$ risk-neutral players, and a third party authority, that is, the government. The N players have the probability of forming a collusive mechanism by exchanging information. Players do not know others' exchanging decisions when they make theirs. After forming a ring, the N agents play an infinitely repeated stage game.

4.2.1. Detection

The government announces a monitor strategy at beginning. Each agent could be detected by the government at each period of the stage game. If one agent is detected, he has to pay a fine $e > 0$. Agents can be detected by two ways: a direct way and an indirect way. At each period, the probability that agent i is detected directly by the government is α_{it}. Agents being detected at each period is uncorrelated with others' detection and previous history. α_{it} depends on whether or not the agent colludes with others at time t and on the monitor strategy of the government's action β_i. The government has a budget $B \in [0, N]$ to allocate to the N members of the collusion and devotes $\beta_i \in [0, 1]$ to detecting member i, where $\sum_{i=1}^{N} \beta_i \leq B$. The probability that agent i is detected directly at t is $\alpha_{it} = \beta_i$ if the agent colludes in t.

The government can also detect collusive agents indirectly. I assume that when the government is able to detect an agent with information about other members in the cartel, the probability that the government can detect other members is 1. If agent i has information about agent j within information structure $V_t \in V$ and i is detected, agent j will be surely detected. Given information structure V_t and directly detected probabilities $\alpha_1, ..., \alpha_N$, agent i is detected if and only if at least one agent in V_t is detected in period t. V_t can be perfectly observed by each players in the collusion but the government.

Based on the assumption above, agent i is detected by the government with probability $1 - \prod_{j \in V_t} (1 - \alpha_j)$.

4.2.2. Stage Game

After exchanging information, agents play an infinitely repeated stage game. Agents can choose to collude (C) or not collude (NC) at beginning of every period. The choice set of the stage game for player i is $A_i = [C, NC]$ for $i = 1, ..., N$. Collusion increases the sum of the revenue to the agents, but it costs c to the agent who colludes. In particular, if $N - 1$ other agents collude, the payoff of an agent is $\lambda N - c$ if he cooperates, and $\lambda(N - 1)$ if he does not, with $\lambda, c > 0$.

I assume $c < \lambda$, which implies that colluding is a dominant strategy for player i in the stage game, and that $\lambda N > c$, which implies that full colluding is the most efficient outcome of the collusion. Suppose that player i has revealed his information to player j in V_t, which means that the player i is colluding with player j, this revelation makes player i vulnerable to player j. The timing of the game is as follows:

(1) The government allocates $[\beta_1, ..., \beta_N]$, which is common knowledge to each agent.

(2) Each agent chooses to collude or not with the others. The information set $V_t \in V$ arises.

(3) Given the set $V_1, V_2, ...$, the stage game described above is played infinite times at periods $t = 1, 2, ...$ At every period agents simultaneously choose whether to collude or not collude. And each agent is likely to be directly detected (with probability $\alpha_1, ..., \alpha_N$) by the government if the agent chooses to collude. If detected (either directly or indirectly), he has to pay a fine e.

h_t^i denotes the history in period t for agent i in the repeated game. Let $\mathcal{H}^i$ denote the set of histories for individual i. If agent i participates in the cartel at t, he observes V_t, where $V_t \in h_t^i$, and his payoff would be affected by other members' behavior. If he does not join the ring at t, $V_t \notin h_t^i$, and his payoff at t would not be affected by others. Player $i's$ strategy is denoted as $s_i : \mathcal{H}^i \to A^i$. Given the agents' behavior $s(h_t^i)$ at period t, player $i's$ payoff in t is:

$$\pi_t^i(s(h_t^i)) = \lambda n(s(h_t^i)) - c1_t^i - e[1 - \prod_{j \in V_t}(1 - \alpha_j(s(h_t^i)))], \tag{24}$$

where $n(s(h_t^i))$ denotes the number of players colluding at time t under $s(h_t^i)$. 1_t^i is a bernoulli variable that takes the value 1 if agent i colludes at t and 0 otherwise. Thus, from (24), if there is no government in this economy, the maximum revenue for the player i is $\lambda N - c$.

$\pi_t^i(s(h_t^i))$ can be divided into an internal part and an external part. $\lambda n(s(h_t^i)) - c1_t^i$ is the revenue from the interaction among the N agents in the stage game. It is independent of the fine upon detection. $e[1 - \prod_{j \in V_t}(1 - \alpha_j(s(h_t^i)))]$ is the per-period information leakage cost for agent i with the information set V_t, and agent i will choose to collude with agent j, given j's probability of detection β_j, if

$$\lambda n(s(h_t^i)) - c > max_i[\beta_i e + (1 - \beta_i)\beta_j e] \tag{25}$$

stands, which means the revenue for agent i to collude should be higher than his maximum information leakage cost.

Suppose discount factor $\delta \in (0, 1)$, $\pi^i(s) = \sum_{t=0}^{\infty} \delta^t \pi_t^i(s(h_t^i))$. Finally, the overall payoff of the cartel in period t is $\Pi_t(s) = \sum_{i=1}^{N} \pi_t^i(s)$.

4.3. Strategies

Following the model, we investigate the government's and players' strategies.

4.3.1. Government's Strategy

The government's goal is to prevent the collusion among the N agents. At each period t, the total revenue of all cartel members is $\Pi_t(s)$. The government aims to minimize $\sum_{t=0}^{\infty} \delta^t \Pi_t(s)$. For simplicity, I assume that the government receives no utility from saving part of the budget B. Also, the government does not benefit from the payment e.

I focus on the government's strategy in how to allocate $[\beta_1, ...\beta_N]$ before the exchange of information in order to minimize $\sum_{t=0}^{\infty} \delta^t \Pi_t(s)$. Under our setting, a larger number of colluding agents with agent i would lead to a higher probability of agent i's indirect detection. If agent i colludes, his information leakage cost has a lower bound at $e\beta_i$, where $e\beta_i$ is the expected fine for which agent i has to pay under the case of direct detection. Thus, the government could take advantage of this fact, as in the following lemma:[1]

Lemma 5 *By allocation* $\beta_i = \frac{N\lambda - c}{e}$*, the government can always prevent agent* i *from colluding.*

Using this result, I can characterize the optimal strategy for the government.[2]

Proposition 7 *The optimal strategy for the government is to allocate the budget* $\beta_i = \frac{N\lambda - c}{e}$ *among* $\lfloor B\frac{e}{N\lambda - c}\rfloor$ *agents; if there is excess budget, the government should use the remaining budget to one of the other players, and to spend nothing on all others.*

Suppose $B \geq 2\frac{N\lambda - c}{e}$. In this case, it is possible to allocate $\beta_i = \beta_j = \frac{N\lambda - c}{e}$ to at least two agents i and j and prevent them from colluding. If there is some agent l with $\beta_l < \frac{N\lambda - c}{e}$, agent l will choose to collude with other agents. Therefore, allocating $\frac{N\lambda - c}{e}$ to as many agents as possible is the optimal strategy for the government.

This proposition states the optimal strategy for the government to monitor a cartel with no more than $\lfloor B\frac{e}{N\lambda - c}\rfloor$ agents. This means that the government can discourage these agents from colluding by allocating $\frac{N\lambda - c}{e}$ to as many agents as possible. Thus, the higher level of B and e and the lower $N\lambda - c$, the less likely is collusive behavior.

Corollary 1 *In an efficient all-inclusive cartel mechanism, which is sustainable with unanimous agreement among all agents, when* $B \geq \frac{N\lambda - c}{e}$*, the government can prevent the cartel by allocating* $\beta_i = \frac{N\lambda - c}{e}$ *to one of the agents.*

---

[1]The proof is in Appendix C.

[2]The proof is in Appendix C.

4.3.2. Player's Strategy

Given the government's strategy, when $B = \frac{N\lambda - c}{e}$, the government will allocate its budget fully to agent i and $\beta_i = \frac{N\lambda - c}{e}$. The optimal strategy for player i is not to collude with others, and also other players have no incentive to collude with player i. In this case, I have $N\lambda - c < e\beta_i + (1-\beta_i)e\beta_j$.

For other player $j \neq i$, because the government spends all budget on player i, $[\beta_1, ..., \beta_{i-1}, \beta_{i+1}, ..., \beta_N] = [0, ..., 0, 0, ...0]$. From (24), player j's revenue equal to $\lambda(N-1)-c$ if he colludes, and $\lambda(N-2)$ if not collude. Because $c < \lambda$, $\lambda(N-1)-c \geq \lambda(N-2)$. Therefore, All bidders other than agent i choose to collude.

When $B < \frac{N\lambda - c}{e}$, from Proposition 1, the government's strategy is to spend all budget on player i. From (25), the information leakage cost for player i is $e\beta_i < N\lambda - c$. Thus, if $\lambda n(s(h_t^i)) > e\beta_i$, all other bidders still have incentive to collude with player i. If all bidders choose to collude, the revenue for each bidder is $N\lambda - c$, which is higher than $e\beta_i$. The collusion is sustainable in this case. I use the following proposition to specify this result.[3]

Proposition 8 *The optimal strategy for the players is to collude with player j when $0 \leq \beta_j < \frac{N\lambda - c}{e}, \forall j \in N$, and not to collude with player i when $\beta_i = \frac{N\lambda - c}{e}, \forall i \in N$.*

In this part, I investigate how to achieve external efficiency in a repeated game. In particular, I look into the allocation of the budget that allows the authority, like the government, to differentiate collusive behavior and to expose agents to external threat. In some cases, allocating the budget symmetrically gives no incentive to the agents to exchange information. However, in other cases a symmetric allocation induces the agents to form a ring. I find out a threshold level of payment for an

[3] The proof is in Appendix C.

incentive compatible collusive mechanism for which the government can prevent an agent from participating in the cartel.

4.4. An Application: A Two-Stage Model

In this part, I use a two-stage model to show that the government's action can help to differentiate the collusion. Consider a standard, two-period decision-making framework, where agents know the present but face an uncertain future. In the first period, there is one decision node. In the second period, one of three possible states occurs; a good state, indexed g, a bad state, indexed b, and a financial crisis state, indexed f. Each state corresponds to a decision node, and the probability of each state is θ_g, θ_b, and θ_f, respectively, with $\theta_g + \theta_b + \theta_f = 1$.

There are one government and N firms in this economy. m denotes the number of firms in the collusive mechanism. The government can provide subsidy to individual firm, and the limit is controlled by the government. The government has two tools to prevent collusive behavior. One is exemption, specified by e, the other is subsidy d_i to firm i, $i = 1, ..., N$, and $d_i \leq \bar{d} \ \ \forall i$. A firm i in a collusive mechanism has to decide her bid at each node; her bidding strategy is indexed $\lambda_{i0}, \lambda_{ig}, \lambda_{ib}$, and λ_{if}. The more she bids, the more she earns. Moreover, subsidy is available for her at a one-period, risk-adjusted market and the interest rate is denoted by r. As usual, a single bidder takes interest rate as given. Her endowment in bidding units at each node is denoted by w_{i0}, w_{ig}, w_{ib}, and w_{if}. (For convenience, suppose $w_{i0} = 0$, and $0 < w_{if} < w_{ib} < w_{ig}$.) Moreover, she has to decide how much subsidy, d_i, to take, subjected to some exogenously specified subsidy limit. Her twice continuously differentiable profit function is denoted by $\pi_i(\lambda_i)$ with $\pi_i'(\lambda_i) > 0, \pi_i''(\lambda_i) < 0, \lim_{\lambda_i \to 0} \pi_i'(\lambda_i) = \infty, \lim_{\lambda_i \to \infty} \pi_i(\lambda_i) = \infty$. Her expected profit is $\pi_i(\lambda_i) = \frac{1}{m} \sum_{i=1}^{m} \{\pi_i(\lambda_{i0}) + \delta[\theta_g \pi_i(\lambda_{ig}) +$

$\theta_b \pi_i(\lambda_{ib}) + \theta_f \pi_i(\lambda_{if})]\}$.

Any firm is a rational bidder who includes veto as an option in her profit-maximizing problem. In period 1, she plans for states g, b, and f. Suppose $0 < w_{if} < w_{ib} \leq e < w_{ig}$. That is, exemptions are sufficiently high to have non-negative financial benefit from vetoing in bad and crisis states, but not necessarily in a good state. I specify the assumption about the exemption $0 < w_{if} < w_{ib} \leq e < w_{ig}$. If e is larger than w_{ig}, no one wants to join the ring because the non-negative financial benefit from vetoing for the ring is high enough. If e is between w_{ib} and w_{if}, the exemption is too small to for the individual to stay out the ring unless a crisis state occurs. The government should be able to provide enough incentive to betray the cartel at least in bad state, which is more reasonable.

In each state of the second period, the bidder has an option to veto for the cartel, and solves the following problem.

$$\max_{d_i, \lambda_{i0}, \lambda_{ig}, \lambda_{ib}, \lambda_{if}} \pi_i(\lambda_i) = \frac{1}{m} \sum_{i=1}^{m} \{\pi_i(\lambda_{i0}) + \delta[\theta_g \pi_i(\lambda_{ig}) + \theta_b \pi_i(\lambda_{ib}) + \theta_f \pi_i(\lambda_{if})]\}$$

$$\begin{aligned}
subject \quad to \quad \lambda_{i0} &= d_i \\
\lambda_{ig} &= \max(w_{ig} - (1+r)d_i, min(w_{ig}, e)) \\
\lambda_{ib} &= \max(w_{ib} - (1+r)d_i, min(w_{ib}, e)) \\
\lambda_{it} &= \max(w_{if} - (1+r)d_i, min(w_{if}, e)) \\
d_i &\leq \bar{d}
\end{aligned}$$

In order to simplify calculation, I elide the subscript i. [4] For firms, the maximum operator for decision nodes is the second period corresponds to the vetoing decision. For example, if a member decides not to veto in g, her constraint is $w_g - (1+r)d$,

[4] After eliding the subscript i, m becomes a constant term for the optimizing problem. Thus, we also elide m.

and if she decides to veto, her constraint is e. In order to show the optimization problem, I have to define a point d^* and to show that there exists a unique d^*, such that the firm can maximize her profit at this point.[5] $MP(d)$ is the marginal profit of individual firm.

Lemma 6 *There is a unique* $d \equiv d^* > 0$ *such that* $MP(d^*) = 0$.

Now I want to solve the optimization problem for a firm. In state g, the optimal decision of a firm can be characterized as follows.

$$\pi(veto, d) = \pi(d) + \delta[\theta_g \pi(e) + \theta_b \pi(w_b) + \theta_f \pi(w_f)]$$

$$\pi(not, d) = \pi(d) + \delta[\theta_g \pi(w_g - (1+r)d) + \theta_b \pi(w_b) + \theta_f \pi(w_f)]$$

where $\pi(veto, d)$ is the profit for her to veto for the cartel, and $\pi(not, d)$ is the profit she stays in the cartel.

Consider the action of not vetoing. After comparing d^* with the subsidy limit $\bar{d}$, I can get the following proposition:[6]

Proposition 9 *If a cartel member considers not to veto, maximum possible profit when subsidy limit is* $\bar{d}$ *is as follows:*

- *if* $\bar{d} \leq d^*$, *maximum profit is* $\pi(not, \bar{d}) = \pi(\bar{d}) + \delta[\theta_g \pi(w_g - (1+r)\bar{d}) + \theta_b \pi(w_b) + \theta_f \pi(w_f)]$,
- *if* $\bar{d} > d^*$, *maximum profit is* $\pi(not, d^*) = \pi(d^*) + \delta[\theta_g \pi(w_g - (1+r)d^*) + \theta_b \pi(w_b) + \theta_f \pi(w_f)]$.

I can see that different $\bar{d}$ may influence the profit level of the firm, and this can also influence the optimal decision for her to veto or not to veto.

[5]The proof is in Appendix C.

[6]The proof is in Appendix C.

Consider the action of vetoing. Here I have to define a $\hat{d}$ which solves $w_g - (1 + r)\hat{d} = e$.

Definition 9 $\hat{d} = \frac{w_g - e}{1+r}$ *is a threshold value, which means that at* $\hat{d}$*, the bidder is financially indifferent between vetoing and not vetoing for the cartel.*

Now, I can consider the level of $\bar{d}$, $\hat{d}$, and d^*. The difference between these three variables would influence the member's decision. After comparison, I get the following proposition:[7]

Proposition 10 *Consider cartel member's action of optimal decision:*

1. *Suppose* $\bar{d} \leq \hat{d}$. $e = w_g - (1 + r)\hat{d} \leq w_g - (1 + r)\bar{d}$. *Then the firm's optimal decision is not to veto in g, which means* $\pi(not, d^*) \geq \pi(not, \bar{d}) \geq \pi(veto, \bar{d})$.
2. *Suppose the case* $\bar{d} > \hat{d}$. $e = w_g - (1 + r)\hat{d} > w_g - (1 + r)\bar{d}$. *Then the optimal vetoing decision depends on the tradeoff between exemptions and net wealth after paying off endogenously determined subsidy use.*
   - *If* $\hat{d} \leq d^*$*, the optimal decision is to veto.*
   - *If* $\hat{d} > d^*$*, there is a unique* $\bar{d}^*$*, such that* $\hat{d} < \bar{d}^*$*. If* $\hat{d} < \bar{d} < \bar{d}^*$*, optimal decision is to not veto; and if* $\bar{d}^* < \bar{d}$*, optimal decision is to veto.*

In this part, I specify the individual firm's behavior. In two-stage model, members include betraying the cartel as an option in all cases in order to maximize their own profit. They can choose different actions at different time nodes. At each time node, members in the mechanism place a bid, like an all-pay auction. Everyone pays her own bid to the mechanism. Then the mechanism distributes its revenue to all

[7] The proof is in Appendix C.

members according to their share of total bids. The more they bid, the more they earn.

The government can give more offer to cartel members to achieve the goal of differentiating the collusive mechanism. The intuition is that a cartel member could choose to receive more subsidy from the government to enhance her profit in the market. Furthermore, in our model, there are two tools to differentiate the collusion. One is exemption e and the other is subsidy limit $\bar{d}$. Both tools can be used to achieve the goal. If the government can boost exemption or have more budget to subsidize agents, it is less likely that a ring will be formed. Subsidizing the agents is a good strategy for the government with more budget to reduce collusive behavior. Or, if the exemption is high enough, no one will choose to participate in a ring, and thus collusive behavior will never happen. e, the punishment or the premium, is defined by law. If I need to use e to achieve the goal, the cost to legislate or amend a law is huge. In reality, $\bar{d}$ is controlled by the government. It is less costly to change $\bar{d}$ than e. Given the budget, the government can determine the subsidy limit $\bar{d}$ to decide the subsidy amount to each firm.

4.5. Conclusion

Collusive behavior often comes with asymmetric information. When there is collusion, the market usually suffers from huge economic loss. In recent antitrust case of TFT-LCD industry, AUO, Chi-Mei, LG, Samsung, and Sharp have around 75 percentage market share in TFT-LCD industry 2006. The defendants and their co-conspirators control over 90 percent of this multi-billion dollar market. These firms earn billions in profit due to the fact that conspirators suppress and eliminate competition by fixing the prices of TFT-LCD panels. It is difficult to detect collusive mechanism in the real world. Thus, I need to investigate some mechanism for the government to detect, differentiate, and prohibit collusive behavior.

I investigate how to achieve external efficiency in a repeated game. In particular, I look into the allocation of the budget that allows the government, to differentiate collusive behavior and to expose agents to external threat. I define members' behavior in a collusive mechanism, and find out the government's strategy to prevent the collusion. The government's goal is to optimize budget allocation to lessen collusive behavior. In some cases, allocating the budget symmetrically gives no incentive to the agents to exchange information. However, in other cases, a symmetric allocation induces the agents to form a ring. I find out a threshold level of payment for an incentive compatible collusive mechanism for which the government can prevent an agent from participating the mechanism. If the government can boost premium or have more budget to subsidize agents, it is less likely that the ring will be formed. Subsidizing the agents is a good strategy for the government with more budget to reduce collusive behavior. Or, if the premium is high enough, no one will choose to participate in a ring and thus collusive behavior will never happen.

CHAPTER V

SUMMARY AND CONCLUSION

As mentioned in McAfee and McMillan [17], an illegal cartel's agreement is worthless without some way to enforce it, since contracts to fix prices usually cannot be written. When members' individual rational condition and incentive compatible constraint could not be satisfied, the efficient colluding mechanism is not freely implemented. When bidders' actions are strategically interactive and their outcome is affected by bidders' belief about others' values, the standard mechanism design approach may suffer from information leakage problem. In Chapter II, the efficient cartel mechanism is no longer sustainable in the presence of positive participation costs and information leakage problem. The bidder with the highest value would choose to veto for the collusive mechanism. This behavior could maximize his benefit and leave all others with zero profit.

In Chapter III, I investigate the seller's behavior in an efficient cartel mechanism. When the seller is active, the only tool to increase her revenue is to raise the reserve price. When the information leakage problem exists, the cartel mechanism is still sustainable. Additionally, I examine two methods that the seller can use to prevent the cartel. The residual claimants method defined in Che and Kim [3] is not effective, since the seller cannot force non-participants. However, if the seller can set up a positive participation cost as in Chapter II, the winner will have incentive to veto when information leakage problem exists.

In Chapter IV, I investigate how to achieve external efficiency in a repeated game, and specify the government's role in the efficient cartel mechanism. In particular, I look into the allocation of the budget that allows the government to differentiate collusive behavior and to expose agents to external threat. The result is that allocating the budget among firms equally may lead to a collapse of collusive mechanism in some cases. While in others, a symmetric allocation induces the agents to form a ring. I find out a threshold level of payment for an incentive compatible collusive mechanism for which the government can prevent an agent from participating the mechanism. I also have concern on whether the government should choose exemption or subsidy to destabilize the collusive behavior. My conclusion is that if the exemption is high enough, members in a cartel would veto for the cartel, and outsiders may not participate in the collusive mechanism. On the other hand, an individual firm that receives subsidy from the government, is likely to earn more profit and thus not has incentives to participate in a collusive mechanism.

For future research, there are two possibilities worth considering. One is different information structure setting. In this dissertation, the information structure is perfect observed by members. With a different information structure setting, the optimal outcome for the cartel may change. It may also affect members' decision whether staying in or jumping out of the collusive mechanism. The other is to introduce bribe in the model. If the cartel can bribe the bank or the government (politician), I may have to pay more social cost to differentiate the collusive behavior, and it would be interesting to see the result.

REFERENCES

[1] E. Ben-Porath, M. Kahneman, Communication in repeated games with private moniotoring, J. Econ. Theory 70 (1996) 281-297.

[2] X. Cao, G. Tian, Equilibria in first price auctions with participation costs, Games Econ. Behav. 2 (2010) 258-273.

[3] Y. Che, J. Kim, Robustly collusion-proof implementation, Econometrica 74 (2006) 1063-1107.

[4] C. Cooper, Under the hammer: the auctions and auctioneers of London, London: Constable, 1979.

[5] P. Cramton, T. Palfrey, Cartel enforcement with uncertainty about costs, Int. Econ. Rev. 31(1) (1990) 17-47.

[6] P. Cramton, T. Palfrey, Ratifiable mechanisms: learning from disagressment, Games Econ. Behav. 10(2) (1995) 255-283.

[7] N. Garoupa, Optimal law enforcement and griminal organization, J. Econ. Behav. & Organ. 63 (2007) 461-474.

[8] D. Graham, R. Marshall, Collusive bidder behavior at single-object second-price and Engish auctions, J. Political Economy. 95 (1987) 1217-1239.

[9] J. Harrington, Some implications of antirust laws for cartel pricing, Econ. Letter 79 (2003) 377-383.

[10] J. Harrington, Cartel pricing dynamics in the presence of an antitrust authority, RAND J. Econ. 35 (2004) 651-673.

[11] J. Harrington, Optimal cartel pricing in the presence of an antitrust authority, Int. Econ. Rev. 46 (2005) 145-169.

[12] S. Hsueh, G. Tian, Nonratifiability of the cartel mechanism in first-price sealed-bid auction with participation costs, Texas A&M University, Working Paper, 2011.

[13] V. Krishna, Auction Theory (2nd edition), Academic Press, 2010.

[14] J. Laffont, D. Martimort, Collusion under asymmetric information, Econometrica 65 (1997) 875-911.

[15] J. Laffont, D. Martimort, Collusion mechanism design with collusion and correlation, Econometrica 68 (2000) 309-342.

[16] G. Mailath, P. Zemsky, Collusion in second price auctions with heterogeneous bidders, Games and Econ. Behav. 3 (1991) 467-503.

[17] R. McAfee, J. McMillan, Bidding rings, Amer. Econ. Rev. 67 (1992) 579-599.

[18] M. Mills, Giving contractors a break, Cong. Quart. 51 (1993) 2947.

[19] R. Myerson, Optimal auction design, Math. Oper. Res. 6 (1981) 58-73.

[20] D. Osborne, Cartel problems, Amer. Econ. Rev. 66 (1976) 835-844.

[21] R. Porter, A study of cartel stability: the joint executive committee, 1880-1886, Bell J. Economics 14 (1983) 301-314.

[22] J. Ricart-I-Costa, Managerial task assignment and promotions, Econometrica 56 (1988) 449-466.

[23] K. Roberts, Cartel behavior and adverse selection, J. Ind. Economics 33 (1985) 401-413.

[24] G. Tan, O. Yilankaya, Ratifiability of efficient collusive mechanisms in second-price auctions with participation costs, Games Econ. Behav. 59 (2007) 383-396.

[25] M. Waldman, Job assignments, signalling and efficiency, RAND J. Econ. 15 (1984) 255-267.

APPENDIX A

TECHNICAL PROOFS

Proof of Lemma 1. In this chapter, I assume homogeneous valuation among bidders.[1] With this assumption, I find the relationship between b_i and b_j when $v_i = v_j$. In order to avoid the confusion of the effect between v_i and v_j, I substitute this assumption in our model at the end of this proof. When vetoer i vetoes the cartel, there could be three possible cases. The first case is that $v_iG(v_Y) \leq c$, i.e., his expected payoff is less than his participation costs. No matter other bidders participate in seller's auction or not, the vetoer will not join the auction. The second case is that $c \leq v_iG(v_Y) \leq v_YG(v_Y)$. The vetoer chooses to participate in seller's auction because his expected payoff is larger than c. The third case is that $v_i \geq v_Y$. His expected profit is his expected payoff minus his participation costs. These three cases can be written as follows:

$$\pi_i^v(v_i) = \begin{cases} 0 & v_i \leq \frac{c}{G(v_Y)} \\ v_iG(v_Y) - c & \frac{c}{G(v_Y)} \leq v_i \leq v_Y \\ v_iH(v_i) - \lambda(v_i)H(v_i) - c & v_i > v_Y. \end{cases}$$

The expected profit in the cartel is:

$$\pi_i^m(v_i) = \begin{cases} \pi_i^m(0) & v_i < c. \\ \pi_i^m(0) + \int_c^{v_i} G(y)dy & v_i \geq c. \end{cases}$$

I want to show:

$$\pi_i^v(v_i) \geq \pi_i^m(v_i) \ \ \forall v_i \geq v_N.$$

[1] In Tan and Yilankaya [24] paper, they use this assumption to show that the credible veto set exists.

I will first find a v_N for which $\pi_i^v(v_N) = \pi_i^s(v_N)$, and then check the inquality.

Step 1 : To show $\exists v_N \in (c, 1)$, such that $\pi_i^v(v_N) = \pi_i^s(v_N)$. Because $\frac{c}{G(v_Y)} \geq c$, if $v_N \leq c$, I have $\pi_i^s(v_N) = \pi_i^v(v_N) = 0$, which is impossible. When $v_Y \geq v_N > c$, I have $\pi_i^v(v_N) = v_N G(v_Y(v_N)) - c$, and $\pi_i^m(v_N) = \pi_i^m(0) + \int_c^{v_N} G(y)dy$, i.e., I need $v_N G(v_Y(v_N)) - c - \pi_i^m(0) - \int_c^{v_N} G(y)dy = 0$. Let

$$\phi(v_i) = v_i G(v_Y(v_i)) - \int_c^{v_i} G(y)dy - c - \pi_i^m(0),$$

and

$$\begin{aligned}
\phi'(v_i) &= G(v_Y(v_i)) + v_i G'(v_Y(v_i))v_Y'(v_i) - G(v_i) \\
&= [G(v_Y(v_i)) - G(v_i)] + v_i G'(v_Y(v_i))v_Y'(v_i).
\end{aligned}$$

Because $G(\cdot)$ is an increasing function, and $v_Y(v_i) \geq v_i$, I have $G(v_Y(v_i)) - G(v_i) > 0$, and $v_Y'(v_i) > 0$. Therefore, $\phi'(v_i) > 0$ and

$$\phi(c) = cG(v_Y(c)) - c - \pi_i^m(0) = c[G(v_Y(c)) - 1] - \pi_i^m(0) < 0,$$

$$\begin{aligned}
\phi(1) &= G(v_Y(1)) - \int_c^1 G(y)dy - c - \pi_i^m(0) \\
&= 1 - c - \int_c^1 G(y)dy - \pi_i^m(0).
\end{aligned}$$

To prove $\int_c^1 [1 - G(y)]dy > \pi_i^m(0)$, where $\pi_i^m(0) = \int_c^1 [y - c - \frac{1-F(y)}{f(y)}]G(y)dF(y)$. I know

$$\begin{aligned}
\phi(1) &= 1G(v_Y(1)) - \int_c^1 G(y)dy - c - \pi_i^m(0) \\
&= 1 - c - \int_c^1 G(y)dy - \int_c^1 [y - \frac{1-F(y)}{f(y)} - c]G(y)dF(y) \\
&= \int_c^1 (1 - F(y)^n)dy - \frac{1}{n}[1 - \int_c^1 F(y)^n dy] + \frac{c}{n} \\
&= \frac{n-1}{n}[1 - c - \int_c^1 F(y)^n dy].
\end{aligned}$$

Because $F(y)^n \leq 1$, $\int_c^1 F(y)^n dy \leq \int_c^1 1dy = 1-c$. I have $\phi(1) > 0$.

For $v_i < 1$, and $v_Y(v_i) \geq v_i$, since $\phi(v_i)$ is continuous, $\phi(c) < 0$, and $\phi(1) > 0$, a unique solution to $\phi(v_i) = 0$ exists, and is our candidate for v_N.

Step 2: I want to show $\pi_i^v(v_i) > \pi_i^m(v_i) \;\; \forall v_i > v_N$. Fix c, and hence v_N and v_Y are fixed, I have $c \leq \frac{c}{G(v_Y)} \leq v_N \leq v_Y$. The payoff difference $\pi_i^v(v_i) - \pi_i^m(v_i) = \tau$ is continuous and given by:

$$\tau = \begin{cases} -\pi_i^m(0) & v_i < c & I \\ -\pi_i^m(0) - \int_c^{v_i} G(y)dy & c \leq v_i < \frac{c}{G(v_Y)} & II \\ v_i G(v_Y) - c - \pi_i^m(0) - \int_c^{v_i} G(y)dy & \frac{c}{G(v_Y)} \leq v_i \leq v_Y & III \\ v_i H(v_i) - \lambda(v_i)H(v_i) - c - \pi_i^m(0) - \int_c^{v_i} G(y)dy & v_i > v_Y & IV \end{cases}$$

I,II: $\pi_i^v(v_i) - \pi_i^m(v_i) < 0, \;\; when \;\; v_i < \frac{c}{G(v_Y)}$.

III: Let $\varphi(v_i) = v_i G(v_Y) - c - \int_c^{v_i} G(y)dy - \pi_i^m(0)$, because $\varphi(v_N) = \phi(v_N) = 0$, I have $\varphi'(v_i) = G(v_Y) - G(v_i) > 0$ when $v_i < v_Y$, and $\varphi'(v_Y) = 0$. Therefore,

$$\varphi(v_i) \begin{cases} < 0 & v_i < v_N \\ = 0 & v_i = v_N \\ > 0 & v_i > v_N. \end{cases}$$

Thus, $\varphi(v_Y)$ is strictly positive.

IV: $H(v_i) = F(Q(v_i))^{n-1}$,

$$\begin{aligned} \frac{d}{dv_i}[\pi_i^v(v_i) - \pi^m(v_i)] &= \frac{d}{dv_i}[v_i H(v_i) - \lambda(v_i)H(v_i) - c - \int_c^{v_i} G(y)dy - \pi_i^m(0)] \\ &= H(v_i) + v_i H'(v_i) - \lambda(v_i)H'(v_i)) - \lambda'(v_i)H(v_i) - G(v_i) \\ &= (v_i - \lambda(v_i))H'(v_i) + (1 - \lambda'(v_i))H(v_i) - G(v_i). \end{aligned}$$

Denote $X = (v_i - \lambda(v_i))H'(v_i) + (1 - \lambda'(v_i))H(v_i) - G(v_i)$. $\lambda(v_i) = v_i - \frac{F(v_j)}{(n-1)f(v_j)\frac{\partial v_j}{\partial b}}$.

$$
\begin{aligned}
X &= [\frac{F(v_j)}{(n-1)f(v_j)\frac{\partial v_j}{\partial b}}](n-1)F(Q(v_i))^{n-2}f(Q(v_i))Q'(v_i) \\
&+ F(Q(v_i))^{n-1}[\frac{f(v_j)^2\frac{\partial v_j}{\partial v_i}\frac{\partial v_j}{\partial b} - F(v_j)f'(v_j)\frac{\partial v_j}{\partial v_i}\frac{\partial v_j}{\partial b} - F(v_j)f(v_j)\frac{\partial^2 v_j}{\partial b \partial v_i}}{(n-1)f(v_j)^2(\frac{\partial v_j}{\partial b})^2}] - F(v_i)^{n-1} \\
&= \frac{F(v_j)^{n-1}\frac{\partial v_j}{\partial v_i}\frac{\partial v_j}{\partial b}[nf(v_j)^2 - F(v_j)f'(v_j)] - F(v_j)^n f(v_j)\frac{\partial^2 v_j}{\partial b \partial v_i}}{(n-1)f(v_j)^2(\frac{\partial v_j}{\partial b})^2} - F(v_i)^{n-1}.
\end{aligned}
$$

With

$$
\frac{\partial v_j}{\partial b} = \frac{F(v_j)}{(v_i - b)(n-1)f(v_j)},
$$

and

$$
\begin{aligned}
\frac{\partial^2 v_j}{\partial b \partial v_i} &= \frac{f^2(v_j)\frac{\partial v_j}{\partial v_i}(v_i - b)(n-1) - F(v_j)(1-b)(n-1)f(v_j)}{[(n-1)f(v_j)(v_i - b)]^2} \\
&- \frac{(v_i - b)(n-1)f'(v_j)\frac{\partial v_j}{\partial v_i}}{[(n-1)f(v_j)(v_i - b)]^2},
\end{aligned}
$$

$$
\begin{aligned}
X &= \frac{F(v_j)^{n-1}\frac{\partial v_j}{\partial v_i}\frac{\partial v_j}{\partial b}[nf(v_j)^2 - F(v_j)f'(v_j)] - \frac{f(v_j)^3 F(v_j)^n \frac{\partial v_j}{\partial v_i}(v_i - b)(n-1)}{[(n-1)f(v_j)(v_i - b)]^2}}{(n-1)f(v_j)^2(\frac{\partial v_j}{\partial b})^2} \\
&+ \frac{\frac{f(v_j)^2 F(v_j)^{n+1}(n-1)(1-b)}{[(n-1)f(v_j)(v_i - b)]^2} + \frac{f(v_j)F(v_j)^n(n-1)(v_i - b)f'(v_j)\frac{\partial v_j}{\partial v_i}}{[(n-1)f(v_j)(v_i - b)]^2}}{(n-1)f(v_j)^2(\frac{\partial v_j}{\partial b})^2} - F(v_i)^{n-1} \\
&= F(v_j)^{n-2}(v_i - b)\frac{\partial v_j}{\partial v_i}[f(v_j)(n-1) + \frac{f'(v_j)[1 - F(v_j)]}{f(v_j)}] \\
&+ F(v_j)^{n-1}(1-b) - F(v_i)^{n-1}.
\end{aligned}
$$

Because bidding function is monotonic increasing, I have $Q'(v_i) = \frac{\partial v_j}{\partial v_i} \geq 0$.[2]

When $X \geq 0$, I have $[\pi_i^v(v_i) - \pi^m(v_i)]$ is increasing when $v_i > v_Y$. To guarantee

---

[2] $v_j(v_i) = Q(v_i)$ is the the relationship when $v_i(b_i) = v_j(b_j)$. When v_i increases, b_i increases, which means that b_j increases with a larger v_j.

that this condition holds, I have to check if $X \geq 0$:

$$
\begin{aligned}
X &= F(v_j)^{n-2}(v_i - b)Q'(v_i)[f(v_j)(n-1) + \frac{f'(v_j)[1-F(v_j)]}{f(v_j)}] \\
&+ F(v_j)^{n-1}(1-b) - F(v_i)^{n-1} \\
&= F(v_j)^{n-1}[\frac{(v_i - b)Q'(v_i)}{F(v_j)}[f(v_j)(n-1) + \frac{f'(v_j)[1-F(v_j)]}{f(v_j)}] + (1-b) - \frac{G(v_i)}{G(v_j)} \\
&= F(v_j)^{n-1}[\frac{(v_i - b)Q'(v_i)f(v_i)(n-1)}{F(v_j)} + \frac{(v_i - b)Q'(v_i)f'(v_j)}{F(v_j)f(v_j)} \\
&+ 1 - b - \frac{G(v_i)}{G(v_j)} - \frac{(v_i - b)Q'(v_i)f'(v_j)}{f(v_j)}].
\end{aligned}
$$

If $f'(v_j) \geq 0$, $\frac{(v_i-b)Q'(v_i)f'(v_j)}{F(v_j)f(v_j)} \geq 0$. Because $F(v_j) \leq 1$, $\frac{(v_i-b)Q'(v_i)f'(v_j)}{F(v_j)f(v_j)} \geq \frac{(v_i-b)f'(v_j)Q'(v_i)}{f(v_j)}$. Check $f'(v_j)$. With $v_i = v_j$, the assumption I mentioned in the beginning of this proof, from (2), I have

$$
v_j'(b) = \frac{F(v_i)}{(n-1)(v_i - b)f(v_i)}.
$$

Substitute it into (4),

$$
\begin{aligned}
v_i'(b) &= \frac{F(v_i) - F(v_N)}{(v_i - b)f(v_i)} - \frac{(n-2)[F(v_i) - F(v_N)]}{(n-1)(v_i - b)f(v_i)} \\
&= \frac{F(v_i) - F(v_N)}{(n-1)(v_i - b)f(v_i)}.
\end{aligned}
$$

Comparing the second part of the righthand sides in (2) and (3) with $v_i = v_j$, I have

$$
\begin{aligned}
\frac{F(v_i)}{(n-1)f(v_i)v_j'(b)} &= \frac{F(v_i)}{(n-1)f(v_i)\frac{F(v_i)}{(n-1)(v_i-b)f(v_i)}} \\
&= v_i - b.
\end{aligned}
$$

Denote $Z = \frac{F(v_i)[F(v_i)-F(v_N)]}{(n-2)f(v_i)v_j'(b)[F(v_i)-F(v_N)]+F(v_i)f(v_i)v_i'(b)}$.

$$
\begin{aligned}
Z &= \frac{F(v_i)[F(v_i) - F(v_N)]}{(n-2)f(v_i)\frac{F(v_i)}{(n-1)(v_i-b)f(v_i)}[F(v_i) - F(v_N)] + F(v_i)f(v_i)\frac{F(v_i)-F(v_N)}{(n-1)(v_i-b)f(v_i)}} \\
&= v_i - b.
\end{aligned}
$$

The second parts of the righthand sides in (2) and (3) are the same. Thus, I can get when $v_i = v_j$, $v_i(b) = v_j(b)$.

$$
\begin{aligned}
\pi_i^v(v_i) &= (v_i - \lambda(v_i))H(v_i) - c \\
&= v_i - [v_i - \frac{F(v_i)}{(n-1)f(v_i)v_j'(b)}]F(Q(v_i))^{n-1} - c \\
&= \frac{F(v_i)}{\frac{(n-1)f(v_i)F(v_i)}{(n-1)(v_i-b)f(v_i)}}F(v_i)^{n-1} - c \\
&= (v_i - b)F(v_i)^{n-1} - c.
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \pi_i^v(v_i)}{\partial b} &= F(v_i)^{n-1}v_i'(b) + v_i(n-1)F(v_i)^{n-2}f(v_i)v_i'(b) - F(v_i)^{n-1} \\
&- b(n-1)F(v_i)^{n-2}f(v_i)v_i'(b) \\
&= F(v_i)^{n-2}\{\frac{F(v_i) - F(v_N)}{(n-1)(v_i-b)f(v_i)}[F(v_i) + v_i(n-1)f(v_i) - b(n-1)f(v_i)] \\
&- F(v_i)\}.
\end{aligned}
$$

I need $[F(v_i) - F(v_N)][F(v_i) + (v_i - b)(n-1)f(v_i)] - F(v_i)(n-1)(v_i - b)f(v_i) = 0$.

$$
\begin{aligned}
0 &= F(v_i)^2 + F(v_i)(v_i - b)(n-1)f(v_i) - F(v_i)F(v_N) \\
&- F(v_N)(v_i - b)(n-1)f(v_i) - F(v_i)(n-1)(v_i - b)f(v_i) \\
&= F(v_i)[F(v_i) - F(v_N)] - F(v_N)v_i(n-1)f(v_i) + bF(v_N)(n-1)f(v_i).
\end{aligned}
$$

$$
b^* = v_i - \frac{F(v_i)[F(v_i) - F(v_N)]}{(n-1)F(v_N)f(v_i)}.
$$

$$
\begin{aligned}
\frac{\partial b^*}{\partial v_i} &= 1 - \frac{[(n-1)F(v_N)f(v_i)][f(v_i)[F(v_i) - F(v_N)] + F(v_i)f(v_i)]}{[(n-1)F(v_N)f(v_i)]^2} \\
&+ \frac{F(v_i)[F(v_i) - F(v_N)][(n-1)F(v_N)f'(v_i)]}{[(n-1)F(v_N)f(v_i)]^2} \\
&= \frac{(n-1)F(v_N)f(v_i)^2[nF(v_N) - 2F(v_i)]}{[(n-1)F(v_N)f(v_i)]^2} \\
&+ \frac{(n-1)F(v_i)F(v_N)f'(v_i)[F(v_i) - F(v_N)]}{[(n-1)F(v_N)f(v_i)]^2}.
\end{aligned}
$$

Because $\frac{\partial b^*}{\partial v_i} \geq 0$,

$$\frac{F(v_N)f(v_i)^2[nF(v_N) - 2F(v_i)]}{(n-1)F(v_N)^2 f(v_i)^2} + \frac{F(v_i)F(v_N)f'(v_i)[F(v_i) - F(v_N)]}{(n-1)F(v_N)^2 f(v_i)^2}] \geq 0,$$

$$\frac{nF(v_N) - 2F(v_i)}{(n-1)F(v_N)} + \frac{F(v_i)f'(v_i)[F(v_i) - F(v_N)]}{(n-1)F(v_N)f(v_i)^2} \geq 0,$$

$$f'(v_i) \geq \frac{f(v_i)^2[2F(v_i) - nF(v_N)]}{F(v_i)[F(v_i) - F(v_N)]}.$$

When $n = 2$, I have $f'(v_i) \geq 0$.

In order to guarantee $X \geq 0$, I still need $\frac{(v_i-b)Q'(v_i)f(v_i)(n-1)}{F(v_j)} \geq \frac{G(v_i)}{G(v_j)}$. I know $v_j'(b) = \frac{F(v_j)}{(n-1)(v_i-b)f(v_j)}$, and $Q(v_i(b)) = v_j(v_i(b))$. Thus, $\frac{(v_i-b)Q'(v_i)f(v_i)(n-1)}{F(v_j)} = \frac{Q'(v_i)}{v_j'(b)}$.

$$Q'(v_i(b)) = \frac{\partial Q(v_i(b))}{\partial v_i} \geq \frac{G(v_i(b))}{G(v_j(b))} v_j'(b) = \frac{G(v_i(b))}{G(v_j(b))} \frac{\partial Q(v_i(b))}{\partial v_i} v_i'(b),$$

I need $v_i'(b) \leq \frac{G(v_j(b))}{G(v_i(b))} = 1$, with $v_i = v_j$. Substitute b^* into $v_i'(b) = \frac{F(v_i)-F(v_N)}{(n-1)(v_i-b)f(v_i)}$, I have $v_i'(b) = \frac{F(v_N)}{F(v_i)} \leq 1$. Thus $X \geq 0$. The cartel is not ratifiable.

Proof of Lemma 2. I prove by contradiction. Suppose there exists a credible veto set A and a equilibrium bid b^* such that

$$\pi_i^v(v_i) = \pi_i^m(v_i) \quad \forall v_i \in A. \tag{5}$$

Note that it must be the case that $\pi_i^v(v_i) > 0$ for all $v_i \in A$, because $\pi_i^m(v_i) > 0$ for all $v_i \in [0, 1]$.

Let $a^* = infA$. Since any vetoer type's payoff would be zero if she were not participating in the auction, the vetoer's equilibrium bid b^* must be $b^* \geq a^* \geq c$, i.e., all vetoers participate the auction with probability one. Plus, the continuity of payoff functions implies that $v_i = a^*$ satisfies (5). Denote $a_1 < a_2 < \cdots < a_J$ the cutoffs of the bidders in the cartel, where $a_j \in [0, 1]$ is used by n_j bidders and $\sum_{j=1}^{J} n_j = n - 1$.

Suppose that $a^* < 1$. It must be that $a_1 > a^*$, otherwise the bidder using a_1

loses in the auction, and earns a negative payoff. For the vetoer $v_i \in (a^*, a_1)$,

$$\pi_i^v(v_i) - \pi_i^m(v_i) = v_i \prod_{j=1}^{J} F(a_j)^{n_j} - c - \int_c^{v_i} G(y)dy - \pi_i^m(0),$$

which is strictly increasing in v_i, since $\prod_{j=1}^{J} F(a_j)^{n_j} \geq F(a_1)^{n-1} > G(v)$. So, bidders whose values in (a^*, a_1) have strict incentive to veto, and thus must belong to the veto set A, which contradicts (5). Suppose that $a^* = 1$, i.e., $A = 1$. Given that the vetoer is participating with probability one, none of the ratifiers will participate, and thus $\pi_i^v(1) = 1 - c > \pi_i^m(1)$. A contradiction.

Proof of Proposition 2. When the bidder's value is in $[v_N, 1]$ and participation costs exist, he would choose to jump out of the cartel because $\pi_i^v(v_i) > \pi_i^m(v_i)$. When information leakage problem exists, other bidders can update their beliefs about the vetoer's value through the equilibrium bid b^*. Thus, bidders know that if they submit a bid in seller's auction, they will lose for sure and have to pay the non-refundable participation costs $c > 0$, which ends up with negative profit. When $c = 0$, if bidders participate in seller's auction, they earn zero profit which is equivalent to the profit when they do not participate. In this case, the game becomes the non-collusive game without participation costs. Thus, the winner's value becomes

$$\pi_i^v(v_i) = \pi_i^s(v_i) = \begin{cases} 0 & v_i < v^* \\ \int_{v^*}^{v_i} G(y)dy & v_i \geq v^*, \end{cases}$$

which is less than

$$\pi_i^m(v_i) = \begin{cases} \pi_i^m(0) & v_i < c \\ \pi_i^m(0) + \int_c^{v_i} G(y)dy & v_i \geq c, \end{cases}$$

Thus, the bidder with value $v_i \in A$ does not veto for the cartel in this case.

APPENDIX B

TECHNICAL PROOFS

Proof of Equation (6).

$$U_i(v_i) = E_{-i}[v_i h_i(v_i, v_{-i}) - x_i(v_i, v_{-i})] = \int_{v_{-i}} [v_i h_i(v) - x_i(v)] f_{-i}(v_{-i}) dv_{-i}. \quad (1)$$

Suppose Y_1 is the maximum value among other $n-1$ bidders' values, and $F(Y_1)^{n-1} = G(Y_1)$, thus,

$$\begin{aligned} \int_{v_{-i}} [v_i h_i(v) - x_i(v)] f_{-i}(v_{-i}) dv_{-i} &= E_{Y_1}(v_i G(v_i) - Y_1) \\ &= v_i(G(v_i)) - E(Y_1), \end{aligned}$$

where $E(Y_1) = E(Y_1|Y_i < v_i)G(v_i)$. $E(Y_1) = \int_0^{v_i} Y_1 dG(Y_1)$, and $E(Y_1|Y_1 < v_i) = \int_0^{v_i} Y_1 dF(Y_1|Y_1 < v_i)$, so $F(Y_1|Y_1 < v_i) = \frac{G(Y_1)}{G(v_i)}$. I have

$$E(Y_1|Y_1 < v_i)G(v_i) = E(Y_1) = \int_0^{v_i} Y_1 dG(Y_1).$$

In second-price sealed-bid auction, the expected payment for a bidder with value v_i is:

$$x_i(v_i) = G(v_i)E[Y_1|Y_1 < v_i].$$

Bidder i's utility function can be written as:

$$U^{II}(v_i) = v_i G(v_i) - G(v_i)E[Y_1|Y_1 < v_i].$$

The bidding function in first-price sealed-bid auction is

$$\beta(v_i) = \frac{1}{G(v_i)} \int_0^{v_i} y dG(y) = E[Y_1|Y_1 < v_i].$$

Under the condition that the bidder cannot benefit by bidding anything other than $\beta(v_i)$, the bidder i's expected utility from bidding $\beta(v_i)$ is as follows:

$$\begin{aligned} U^I(v_i) &= G(v_i)[v_i - \beta(v_i)] \\ &= G(v_i)v_i - G(v_i)E[Y_1|Y_1 < v_i]. \end{aligned}$$

Thus,

$$U_i(v_i) = U^I(v_i) = U^{II}(v_i).$$

The proof is complete.

Proof of Lemma 3. From (17),

$$\begin{aligned} U_i^M(z_i, v_i) &= [v_i - T(z_i) - r]F(z_i)^{n-1} \\ &+ [1 - F(z_i)^{n-1}] \int_{z_i}^{v_h} \frac{T(s_i)}{n-1} \frac{(n-1)F(s_i)^{n-2}f(s_i)}{1 - F(z_i)^{n-1}} ds_i \\ &= [v_i - T(z_i) - r]F(z_i)^{n-1} \\ &+ \int_{z_i}^{v_h} [T(s_i)]F(s_i)^{n-2}f(s_i)ds_i. \end{aligned}$$

$$\begin{aligned} \frac{\partial U_i^M(z_i, v_i)}{\partial z_i} &= [v_i - T(z_i) - r](n-1)F(z_i)^{n-2}f(z_i) \\ &- T'(z_i)F(z_i)^{n-1} - T(z_i)F(z_i)^{n-2}f(z_i) \\ &= [(n-1)v_i - (n-1)T(z_i) - (n-1)r - T(z_i)]F(z_i)^{n-2}f(z_i) \\ &- T'(z_i)F(z_i)^{n-1} \\ &= [(n-1)v_i - nT(z_i) - (n-1)r]F(z_i)^{n-2}f(z_i) - T'(z_i)F(z_i)^{n-1}, \end{aligned}$$

and $\frac{\partial^2 U_i^M(z_i,v_i)}{\partial v_i \partial z_i} = (n-1)F(z_i)^{n-2}f(z_i) \geq 0$. Check the truth telling condition, $z_i = v_i$, with the transfer payment:

$$T(v_i) = F(v_i)^{-n} \int_r^{v_i} (s_i - r)(n-1)F(s_i)^{n-1}f(s_i)ds_i.$$

Given $z_i = v_i$,

$$\begin{aligned}
\frac{\partial U_i^M(v_i)}{\partial v_i} &= [(n-1)v_i - nT(v_i) - (n-1)r]F(v_i)^{n-2}f(v_i) - T'(v_i)F(v_i)^{n-1} \\
&= [(n-1)v_i - nF(v_i)^{-n}\int_r^{v_i}(s_i - r)(n-1)F(s_i)^{n-1}f(s_i)ds_i \\
&- (n-1)r]F(v_i)^{n-2}f(v_i) - F(v_i)^{n-1}\{-nF(v_i)^{-(n+1)}f(v_i) \\
&\times \int_r^{v_i}(s_i - r)(n-1)F(s_i)^{n-1}f(s_i)ds_i \\
&+ F(v_i)^{-n}[(v_i - r)(n-1)F(v_i)^{n-1}f(v_i)]\} \\
&= (n-1)v_iF(v_i)^{n-2}f(v_i) - nF(v_i)^{-2}f(v_i) \\
&\times \int_r^{v_i}(s_i - r)(n-1)F(s_i)^{n-1}f(s_i)ds_i \\
&- (n-1)rF(v_i)^{n-2}f(v_i) + nF(v_i)^{-2}f(v_i) \\
&\times \int_r^{v_i}(s_i - r)(n-1)F(s_i)^{n-1}f(s_i)ds_i - (n-1)(v_i - r)F(v_i)^{n-2}f(v_i) \\
&= (n-1)v_iF(v_i)^{n-2}f(v_i) - (n-1)rF(v_i)^{n-2}f(v_i) \\
&- (n-1)(v_i - r)F(v_i)^{n-2}f(v_i) \\
&= 0.
\end{aligned}$$

The proof is complete.

Proof of Lemma 4. The seller's objective function is

$$U_0 = \int_V rf(v)dv + \sum_{i\in n}\int_V h_i(v)(v_i - r)f(v)dv + \sum_{i\in n}\int_V (x_i(v) - v_ih_i(v))f(v)dv. \quad (19)$$

Here I do not consider the case $z_i > v_i$, which can be erased by trembling hand equilibrium. I can write

$$U_i(v_i) = U_i(z_i) + \int_{z_i}^{v_i} M_i(s_i)ds_i \quad \forall z_i \le v_i \in [0,1].$$

The third part of (19) becomes:

$$\begin{aligned}
\int_V (x_i(v) - v_i h_i(v)) f(v) dv &= -\int_0^1 U_i(v_i) f(v) dv \\
&= -\int_0^1 [U_i(z_i) + \int_{z_i}^{v_i} M_i(s_i) ds_i] f(v) dv \\
&= -U_i(z_i) - \int_0^1 \int_{s_i}^1 f(v) dv M_i(s_i) ds_i \\
&= -U_i(z_i) - \int_0^1 (1 - F(s_i)) M_i(s_i) ds_i \\
&= -U_i(z_i) - \int_V (1 - F(v_i)) h_i(v) f_{-i}(v_{-i}) dv. \qquad \text{(I)}
\end{aligned}$$

Substituting (I) into (19):

$$U_0 = \int_V (\sum_{i \in n} (v_i - r - \frac{1 - F(v_i)}{f(v_i)}) h_i(v)) f(v) dv + \int_V r f(v) dv - \sum_{i \in n} U_i(z_i). \qquad \text{(II)}$$

From (7), the bidder's utility function is

$$U_i(v_i) = \int_{v_{-i}} [v_i h_i(v) - x_i(v)] f_{-i}(v_{-i}) dv_{-i}.$$

I can write

$$U_i(z_i) = \int_{v_{-i}} [v_i h_i(v) - x_i(v) - \int_{z_i}^{v_i} h_i(s_i, v_{-i}) ds_i] f_{-i}(v_{-i}) dv_{-i}.$$

Thus, I have $\sum_{i \in n} U_i(z_i) = 0$, which means that the total distorting effect of all other values have zero expected value. The third part of the right hand side in (II) becomes zero. Furthermore, the second term of the right hand side is a constant. The seller's problem becomes

$$\max \int_V (\sum_{i \in n} (v_i - r - \frac{1 - F(v_i)}{f(v_i)}) h_i(v)) f(v) dv. \qquad (20)$$

The proof is complete.

Proof of Proposition 3.

$$U_i(z_i, v_i) = E_{-i}[v_i h_i(z_i, v_{-i}) - x_i(z_i, v_{-i})] = \int_{V_{-i}} [v_i h_i(z_i, v_{-i}) - x_i(z_i, v_{-i})] f_{-i}(v_{-i}) dv_{-i},$$

and

$$k(v_{-i}) = inf\{z_i | z_i \geq v_0 \quad and \quad z_i \geq v_j, \quad \forall j \neq i\},$$

which means that $k(v_{-i})$ is the upper bound of other bidders' bids. From (12), if a bidder does not stay in the cartel, his expected payment is:

$$x_i(z_i) = \begin{cases} k(v_{-i}) & if \quad h_i(z_i, v_{-i}) = 1 \\ 0 & if \quad h_i(z_i, v_{-i}) = 0. \end{cases} \tag{12}$$

The winner's utility function in noncollusive case becomes

$$U_i(v_i) = \int_{V_{-i}} (v_i h_i(z_i, v_{-i}) - k(v_{-i})) f_{-i}(v_{-i}) dv_{-i}.$$

and from (16)

$$x_i^M(z_i) = \begin{cases} T(z_i) + r & if \quad h_i(z_i, v_{-i}) = 1 \\ \frac{-T(z_i)}{n-1} & if \quad h_i(z_i, v_{-i}) = 0. \end{cases} \tag{16}$$

In collusive case, winner's utility function is

$$U_i^M(v_i) = \int_{V_{-i}} (v_i h_i(z_i, v_{-i}) - T(z_i) - r) f_{-i}(v_{-i}) dv_{-i}.$$

For losers in cartel's auction, they are better off to stay in the cartel, because they can receive a transfer payment from the cartel, whether they can update their belief or not. Thus, I only need to check the winner's payoff. When bidders cannot update their belief in noncollusive case, the payoff for the winner is the difference between his value and the second highest value among bidders. In collusive case, the payoff is this difference plus a transfer payment. If the transfer payment to other $n-1$ members plus the reserve price is higher than the second highest value, bidder i would have no incentive to stay in the cartel no matter the auction format is first-price or second-price auction. The total transfer payment in the cartel is

$$T(z_i) = F(z_i)^{-n} \int_r^{z_i} (s_i - r)(n-1)F(s_i)^{n-1} f(s_i) ds_i,$$

which is the expected difference between the winner's possible bid in the cartel and the reserve price.

$$\begin{aligned}
T(z_i) &= F(z_i)^{-n} \int_r^{z_i} (s_i - r)(n-1)F(s_i)^{n-1} f(s_i) ds_i \\
&= \frac{n-1}{n} F(z_i)^{-n} [\int_r^{z_i} (s_i - r) n F(s_i)^{n-1} f(s_i) ds_i] \\
&= \frac{n-1}{n} F(z_i)^{-n} [\int_r^{z_i} s_i n F(s_i)^{n-1} f(s_i) ds_i - r \int_r^{z_i} n F(s_i)^{n-1} f(s_i) ds_i] \\
&= \frac{n-1}{n} F(z_i)^{-n} [s_i F(s_i)^n |_r^{z_i} - \int_r^{z_i} F(s_i)^n ds_i - r F(s_i)^n |_r^{z_i}] \\
&= \frac{n-1}{n} F(z_i)^{-n} [z_i F(z_i)^n - r F(r)^n - \int_r^{z_i} F(s_i)^n ds_i - r F(z_i)^n + r F(r)^n] \\
&= \frac{n-1}{n} F(z_i)^{-n} [(z_i - r) F(z_i)^n - \int_r^{z_i} F(s_i)^n ds_i] \\
&= \frac{n-1}{n} (z_i - r) - \frac{n-1}{n} \int_r^{z_i} F(s_i)^n ds_i.
\end{aligned}$$

Thus, I have $T(z_i) + r \leq k(z_{-i}) = z_i$, where z_i is the possible reporting value of the winner. Therefore, the difference between (12) and (16) is

$$\begin{aligned}
T(z_i) + r - k(v_{-i}) &= \frac{n-1}{n}(z_i - r) - \frac{n-1}{n} \int_r^{z_i} F(s_i)^n ds_i + r - z_i \\
&= \frac{1}{n}(r - z_i) - \frac{n-1}{n} \int_r^{z_i} F(s_i)^n ds_i.
\end{aligned}$$

If $r \leq z_i + (n-1)\int_r^{z_i} F(s_i)^n ds_i$, $T(z_i) + r - k(v_{-i})$ is less or equal to zero. If bidders report their true value to the cartel, $T(v_i) + r - k(v_{-i})$ is always less than zero.

When information leakage problem exists, bidders can update their beliefs through the cartel's auction. After updating their information, if there is a bidder vetoing for the cartel, other bidders can enter the seller's auction for free and bid as much as possible to make the vetoer earn no more profit than that of staying in a cartel[3]. As the result with $U_i(z_i, v_i) \leq U_i^M(z_i, v_i)$, the cartel can still be supported. Moreover, if the cartel can generate a higher revenue for every bidder, whether the information leakage problem exists or not, bidders would be no worse than non-cartel case.

Proof of Proposition 4.

Proof (a):

From (22), the seller's problem is

$$\max \int_V (\sum_{i \in n} (d(v_i) - v_0) h_i(v)) f(v) dv. \tag{III}$$

When cartel exists, there is only one bidder joining the seller's auction and he submits his bid at the seller's reserve price. If the seller set the reserve price $r = v_0$, her utility is

$$\max \int_V (\sum_{i \in n} (r - v_0) h_i(v)) f(v) dv = 0, \tag{IV}$$

which means that she always earns her valuation when cartel exists. If she sets the reserve price according to $d^{-1}(v_0)$, where $d^{-1}(v_0) \geq v_0$, she can earn more than v_0. Comparing (III) and (IV), I find that the seller's optimal strategy is to set $r = d^{-1}(v_0) \geq v_0$.

Proof (b):

If $d^{-1}(v_0) > v_h$, no one has incentive to submit a bid in seller's auction and the cartel collapses. If $d^{-1}(v_0) \leq v_h$, I compare $U_i(z_i, v_i)$ with $U_i^M(z_i, v_i)$. From the proof of Proposition 1, the cartel's sustainable condition is $r \leq z_i + (n-1) \int_r^{z_i} F^n(s_i) ds_i$.

[3] Similar cases in Tan and Yilankaya [24], Hsueh and Tian [12].

If $d^{-1}(v_0) > z_i$, I have $r > z_i + (n-1)\int_{z_i}^{z_i} F^n(s_i)ds_i$. Thus, bidders have no incentive to stay in the cartel. When $d^{-1}(v_0) \leq z_i$, $T(z_i) + r - k(v_{-i}) \leq 0$ would always holds. Bidders would choose to stay in the cartel. Therefore, the cartel is sustainable in this case.

Proof (c):

From (b), when $d^{-1}(v_0) > v_h$, no one's value is higher than the seller's reserve price and the seller keeps the object. Even if some bidders' value is higher than seller's value, the object is not sold to the bidder with the highest value, which means the efficient condition is not satisfied.

Proof of Proposition 5. Prove by contradiction. Suppose the residual claimants method cannot prevent a cartel. I want to show the loser's utility $U_j(v_j) \leq U_j^M(v_j)$. When winner i stays in the cartel, his expected utility function is:

$$U_i^M(z_i, v_i) = \int_{V_{-i}} (v_i h_i(z_i, v_{-i}) - T(z_i) - r) f_{-i}(v_{-i}) dv_{-i}.$$

When cartel exists, for the loser j in cartel's auction, he receives a transfer payment equal to $\frac{T(z_i)}{n-1}$. In non-cartel case, if the seller extracts the optimal expected payoff directly from one of the losers, and the winner pays his bid to the loser j, the loser j's utility function becomes $z_i - r$, where z_i is winner i's bid in seller's auction.

The total transfer payment is

$$T(z_i) = \frac{n-1}{n}(z_i - r) - \frac{n-1}{n}\int_r^{z_i} F(s_i)^n ds_i.$$

Thus, the loser's utility function becomes:

$$U_j^M(v_j) = \frac{1}{n}(z_i - r) - \frac{1}{n}\int_r^{z_i} F(s_i)^n ds_i.$$

When cartel exists, which is less than

$$U_j(v_j) = z_i - r.$$

A contradiction.

Proof of Proposition 6. Prove by contradiction. Suppose the residual claimants method can prevent bidders from staying in the cartel. I want to show the winner's utility $U_i(z_i, v_i) \leq U_i^M(z_i, v_i)$. Because the seller cannot force everyone to participate in her auction, she cannot extract her payoff $r = d^{-1}(v_0)$ from non-participants.

The utility function for the winner in cartel's auction is:

$$U_i^M(z_i, v_i) = \int_{V_{-i}} (v_i h_i(z_i, v_{-i}) - T(z_i) - d^{-1}(v_0)) f_{-i}(v_{-i}) dv_{-i},$$

which can be maximized by bidding the reserve price. The seller can only extract her expected profit from the single participant in her auction. In cartel case, if there are at least two bidders' values higher than the seller's reserve price, the winner can earn extra profit, $\frac{1}{n}(z_i - r) - \frac{1}{n}\int_r^{z_i} F(s_i)^n ds_i \geq 0$, where z_i is the winner's bid equal to second highest value among cartel members. The winner's utility is

$$U_i^M(z_i, v_i) = v_i - z_i + \frac{1}{n}(z_i - r) - \frac{1}{n}\int_r^{z_i} F(s_i)^n ds_i > v_i - z_i = U_i(z_i, v_i).$$

Thus, he has incentive to stay in the cartel. If there is only one bidder's value higher than r, he will be indifferent between staying in or jumping out of the cartel. According to our assumption, he would stay in the cartel. Contradiction.

APPENDIX C

TECHNICAL PROOFS

Proof of Lemma 5. For the collusion, the per-period maximum benefit of the collusion is $N\lambda - c$. If agent i colludes, his information leakage cost has a lower bound at $e\beta_i$, where $e\beta_i$ is the expected fine for which agent i chooses to collude. Thus, an agent will not collude if $e\beta_i$ is higher than $N\lambda - c$. Therefore, by allocating a budget of $\frac{N\lambda-c}{e}$ to monitor an agent, the government can prevent this agent from colluding.

Proof of Proposition 7. Consider the case $B \geq 2\frac{N\lambda-c}{e}$. I can allocate $\beta_i, \beta_j = \frac{N\lambda-c}{e}$ to at least two agents i and j and prevent them from colluding. If agent i does not collude, the threshold value of β_j at which j stops colluding is equal to $\frac{N\lambda-c}{e}$. This is the lower bound for the threshold. Similarly, since j does not collude, $\frac{N\lambda-c}{e}$ is also the threshold for i. Therefore, with some agent not colluding, if there is some other agent l with $\beta_l < \frac{N\lambda-c}{e}$, then agent l can collude with a non-colluding agent and can be elicited to collude in equilibrium. Therefore, allocating $\frac{N\lambda-c}{e}$ to as many agents as possible is the optimal strategy for the government.

When $\beta < 2\frac{N\lambda-c}{e}$, I consider two cases: (1)$\beta < \frac{N\lambda-c}{e}$. (2)$\frac{N\lambda-c}{e} \leq \beta \leq 2\frac{N\lambda-c}{e}$. In $\beta < \frac{N\lambda-c}{e}$ case, the government's optimal strategy cannot discourage any agent. Thus, the government's strategy is useless because all budget allocations are lower than the optimal lower bound. When $\frac{N\lambda-c}{e} \leq \beta \leq 2\frac{N\lambda-c}{e}$, the government can monitor directly to one agent i. Other agents cannot be monitored directly from colluding. Consider the case: colluding with other agents. The collusion does not want agent j to collude when the collusive revenue from the collusion $N\lambda - c$ is lower than the minimum information leakage cost $min[e\beta_i + (1 - \beta_j)\beta_e]$. Thus, the strategy, allocating all resources to monitor one agent, rather than any others, will make it most difficult to support the collusion, if the collusion prefers all agents to join.

Proof of Proposition 8. Proof by contradiction. I want to show player i would choose to collude when $\beta_i = \frac{N\lambda - c}{e}$. When $\beta_i = \frac{N\lambda - c}{e}$. From

$$\lambda n(s(h_t^i)) - c > max_i[\beta_i e + (1-\beta_i)\beta_j e], \tag{25}$$

I have $\beta_i e + (1-\beta_i)\beta_j e \geq N\lambda - c$. Thus player i's information leakage cost is higher than his collusive revenue.

For other player $j \neq i$, because the government spends all budget on player i, $[\beta_1, ..., \beta_{i-1}, \beta_{i+1}, ..., \beta_n] = [0, ..., 0, 0, ...0]$. From

$$\pi_t^i(s(h_t^i)) = \lambda n(s(h_t^i)) - c1_t^i - e[1 - \prod_{j \in V_t}(1 - \alpha_j(s(h_t^i)))], \tag{24}$$

player j's revenue equal to $\lambda(n-1) - c$ if he colludes, and $\lambda(n-2)$ if not collude. Because $c < \lambda$, $\lambda(n-1) - c \geq \lambda(n-2)$. Therefore, all bidders other than agent i choose to collude. When they collude with player i, the player j's revenue is $N\lambda - c$, which is equal to i's minimum information leakage cost: $e\beta_i = N\lambda - c$. Thus, other players do not collude with player i because the lower bound of their information leakage costs are equal to their collusive revenue. A contradiction.

In $B < \frac{N\lambda - c}{e}$ case, I want to show other bidders have no incentive to collude with player i. When $B < \frac{N\lambda - c}{e}$, from Proposition 1, the government's strategy is to spend all budget on player i. From (25), the information leakage cost for player i is $e\beta_i < N\lambda - c$. Thus, if $\lambda n(s(h_t^i)) > e\beta_i$, all other bidders still have incentive to collude with player i. If all bidders choose to collude, the revenue for each bidder is $N\lambda - c$, which is higher than $e\beta_i$. A contradiction.

Proof of Lemma 6. I have to show that d^* exists. Notice that the first-order condition for the bidder's stage I problem is:

$$\begin{aligned} MP(d) &= \pi'(d) - \delta(1+r)\theta_g \pi'(w_g - (1+r)d) \\ &- \delta(1+r)(\theta_b)\pi'(w_b - (1+r)d) \\ &- \delta(1+r)(\theta_f)\pi'(w_f - (1+r)d), \end{aligned}$$

where $MP(d)$ is the marginal profit. I can find when $\lim_{d\to 0} \pi'(d) = \infty$, for d small enough, $MP(d) > 0$. If d is sufficiently large, $w_g - (1+r)d$, $w_g - (1+r)d$, and $w_f - (1+r)d$ are sufficiently small. Then, I can find that $MP(d) < 0$. According to these two conditions, I could say there is an unique $d \equiv d^* > 0$ which makes $MP(d^*) = 0$.

It is easily to check the first-order condition for $MP(d)$ is less than zero.

$$\begin{aligned} \frac{\partial MP(d)}{\partial d} &= \pi''(d) + \delta(1+r)^2 d\theta_g \pi''(w_g - (1+r)d) \\ &+ \delta(1+r)^2 d(\theta_b)\pi''(w_b - (1+r)d) \\ &+ \delta(1+r)^2 d(\theta_f)\pi''(w_f - (1+r)d) \\ &< 0. \end{aligned}$$

When $d \le d^*$, then $MP(d) \ge MP(d^*)$, and if $d > d^*$, then $MP(d) < MP(d^*)$.

Proof of Proposition 9. In order to prove Proposition 9, I want to find the solution for the optimization problem for a member. In the state g, the optimal decision of a member can be characterized as follows: When debt is d, the profit of vetoing in g is

$$\pi(veto, d) = \pi(d) + \delta[\theta_g \pi(e) + \theta_b \pi(w_b) + \theta_f \pi(w_f)],$$

and the profit of not vetoing in g is

$$\pi(not, d) = \pi(d) + \delta[\theta_g \pi(w_g - (1+r)d) + \theta_b \pi(w_b) + \theta_f \pi(w_f)].$$

Consider the action of not vetoing. The marginal profit of not vetoing is:

$$MP(not, d) = \pi'(d) - \delta(1+r)\theta_g \pi'(\lambda_g),$$

where MP denoted marginal profit of the member, and the marginal profit is decreasing in subsidy.

$$\frac{\partial MP(not, d)}{\partial d} = \pi''(d) + \delta(1+r)^2\theta_g \pi''(\lambda_g) < 0.$$

Besides, $\lim_{d \to 0} \pi'(d) = \infty$ implies when d small enough, $MP(not, d) > 0$, and for d sufficiently large, $w_g - (1+r)d$ is sufficiently small, and hence, $MP(not, d) < 0$. Therefore, there is a unique $d^* > 0$ such that $MP(not, d^*) = 0$. Furthermore, $\frac{\partial MP(not,d)}{\partial d} < 0$ implies that if $d < d^*$, then $MP(not, d) \geq MP(not, d^*)$, and if $d > d^*$, then $MP(not, d) < MP(not, d^*)$.

Proof of Proposition 10. Consider the action of vetoing. The marginal profit for the member is

$$MP(veto, d) = \pi'(d) > 0,$$

and the optimal subsidy choice is $d = \bar{d}$. Therefore, if a bidder considers vetoing for the collusion, the maximum profit when subsidy limit is $\bar{d}$ is:

$$\pi(veto, \bar{d}) = \pi(\bar{d}) + \delta[\theta_g \pi(e) + \theta_b \pi(w_b) + \theta_f \pi(w_f)].$$

In order to specify the optimal decision, I have to define the level of subsidy at which the bidder is financially indifferent between vetoing or not. Let $\hat{d}$ solve $w_g - (1+r)\hat{d} = e$. In other words, let

$$\hat{d} = \frac{w_g - e}{1+r}.$$

Suppose $\bar{d} \leq \hat{d}$, subsidy limit is small relative to $\hat{d}$ ($e = w_g - (1+r)\hat{d} \leq w_g - (1+$

$r)\bar{d}$. That is, exemptions are small relative to net wealth after maximizing possible subsidy payoff). Thus, the member's optimal decision is not to veto in g. This can be shown by considering two cases: $\bar{d} \leq d^*$, and $\bar{d} > d^*$. When $\bar{d} \leq d^*$, the maximum profit from not vetoing is

$$\pi(not, \bar{d}) = \pi(\bar{d}) + \delta[\theta_g \pi(w_g - (1+r)\bar{d}) + \theta_b \pi(w_b) + \theta_f \pi(w_f)],$$

and maximum profit from vetoing is

$$\pi(veto, \bar{d}) = u(\bar{d}) + \delta[\theta_g \pi(e) + \theta_b \pi(w_b) + \theta_f \pi(w_f)].$$

$\bar{d} \leq \hat{d}$ implies that $e = w_g - (1+r)\hat{d} \leq w_g - (1+r)\bar{d}$, and hence, $\pi(not, \bar{d}) \geq \pi(veto, \bar{d})$. If $\bar{d} > d^*$, then maximum profit from not vetoing is:

$$\pi(not, d^*) = \pi(d^*) + \delta[\theta_g \pi(w_g - (1+r)d^*) + \theta_b \pi(w_b) + \theta_f \pi(w_f)],$$

and maximum profit from vetoing is

$$\pi(veto, \bar{d}) = \pi(\bar{d}) + \delta[\theta_g \pi(e) + \theta_b \pi(w_b) + \theta_f \pi(w_f)].$$

Moreover, $\bar{d} \leq \hat{d}$, and the optimality of d^* implies that

$$\pi(d^*) + \delta\theta_g \pi(w_g - (1+r)d^*) > \pi(\bar{d}) + \delta\theta_g \pi(w_g) - (1+r)\bar{d} \geq \pi(\bar{d}) + \delta\theta_g \pi(e),$$

and thus, $\pi(not, d^*) \geq \pi(not, \bar{d}) \geq \pi(veto, \bar{d})$.

Consider the case $\bar{d} > \hat{d}$, subsidy limit is large relative to $\hat{d}$ ($e = w_g - (1+r)\hat{d} > w_g - (1+r)\bar{d}$, exemptions are large relative to net wealth after maximizing possible subsidy payoff). Then the vetoing decision is a little more difficult to make, and it depends on the tradeoff between exemptions and net wealth after paying off endogenously determined subsidy amount. This can be seen by considering the following cases: when $\hat{d} \leq d^*$ (that is, exemptions are large relative to net wealth after paying off optimal subsidy in the case of not vetoing) and when $\hat{d} > d^*$ (that is, exemptions are small relative to net wealth after paying off optimal subsidy in the case of not vetoing).

Suppose $\hat{d} \leq d^*$ (Exemptions are large relative to wealth after paying off d^*). Then the optimal decision is to veto, and it can be seen by considering the following two cases. If $\bar{d} \leq d^*$, then maximum profit from not vetoing is:

$$\pi(not, \bar{d}) = \pi(\bar{d}) + \delta[\theta_g \pi(w_g - (1+r)\bar{d}) + \theta_b \pi(w_b) + \theta_f \pi(w_f)],$$

and maximum profit from vetoing is

$$\pi(veto, \bar{d}) = \pi(\bar{d}) + \delta[\theta_g \pi(e) + \theta_b \pi(w_b) + \theta_f \pi(w_f)].$$

Moreover, $\bar{d} > \hat{d}$ implies that $e = w_g - (1+r)\hat{d} > w_g - (1+r)\bar{d}$, and hence, $\pi(veto, \bar{d}) > \pi(not, \bar{d})$. If $\bar{d} > d^*$, then maximum profit from not vetoing is

$$\pi(not, d^*) = \pi(d^*) + \delta[\theta_g \pi(w_g - (1+r)d^*) + \theta_b \pi(w_b) + \theta_f \pi(w_f)],$$

and maximum profit from vetoing is

$$\pi(veto, \bar{d}) = \pi(\bar{d}) + \delta[\theta_g \pi(e) + \theta_b \pi(w_b) + \theta_f \pi(w_f)].$$

Moreover, $\hat{d} \leq d^*$ implies that $e = w_g - (1+r)\hat{d} > w_g - (1+r)\bar{d}$, and thus, $\pi(veto, \bar{d}) > \pi(not, \bar{d})$.

Suppose $\hat{d} > d^*$ (Exemptions are small relative to d*). Then there is a unique $\bar{d}^*$, $\hat{d} < \bar{d}^*$, such that if $\hat{d} < \bar{d} < \bar{d}^*$, then optimal decision is not to veto, and if $\bar{d}^* < \bar{d}$, then optimal decision is to veto. This case highlights interesting dynamics. In this case, relatively high subsidy limits additionally affect a bidder's decision to veto. That is, even when exemptions are relatively small as compared to a bidder's desired subsidy (when not vetoing), she may decide to veto, if her exemptions are sufficiently high to make the intertemporal bid tradeoff valuable. This sufficiently high threshold is characterized by $\bar{d}^*$. Recall from a previous case, if $\hat{d} = \bar{d}$ and $\hat{d} > d^*$, then $\pi(not, d^*) > \pi(veto, \bar{d})$. In other words, if exemptions are the same as net wealth after maximum subsidy payoff, but bidder's optimal use of subsidy is smaller than maximum subsidy allowed, then it is beneficial for the bidder not to veto. Essentially because the additional bid in period 1 from additional subsidy does not compensate for the decrease in bidding in state g that results from vetoing. Therefore, for $\bar{d}$ slightly larger than $\hat{d}$, $\pi(not, d^*) > \pi(veto, \bar{d})$. However, in the region $[d^*, \infty), \frac{\partial \pi(not,d)}{\partial d^*} = 0$, and, $\frac{\partial \pi(veto,\bar{d})}{\partial \bar{d}} = \pi'(\bar{d}) > 0$. In other words, maximum profit from vetoing is strictly increasing in $\bar{d}$, while maximum profit from not vetoing is constant. Moreover, π is unbounded above. Hence, there is a unique $\bar{d}^*$, $\hat{d} < \bar{d}^*$, such that for each $\bar{d}$, if $\hat{d} < \bar{d} < \bar{d}^*$, then optimal choice is not to veto; and if $\bar{d}^* < \bar{d}$, then optimal choice is to veto.

VITA

- Name: Shao-Chieh Hsueh
- Address: Department of Economics, Texas A&M University, 4228 TAMU, College Station, TX 77843
- Email address: waju0927@tamu.edu
- Education:

  Ph.D., Economics, Texas A&M University, December 2011

  B.S., Economics, National Taipei University, June 2001
- Fields of Specialization: Microeconomic Theory, Mechanism Design, Auction Theory, Public Economics, Industrial Organization

The typist for this dissertation was SHAO-CHIEH HSUEH.