NONCOOPERATIVE GAMES FOR AUTONOMOUS CONSUMER LOAD BALANCING OVER SMART GRID

A Thesis

by

TARUN AGARWAL

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

August 2010

Major Subject: Electrical Engineering

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Approved by:

Chair of Committee,	Shuguang (Robert) Cui
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ABSTRACT

Noncooperative Games for Autonomous Consumer Load Balancing Over Smart Grid. (August 2010)

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Chair of Advisory Committee: Dr. Shuguang (Robert) Cui

Traditionally, most consumers of electricity pay for their consumption according to a fixed-rate. The few existing implementations of real time pricing have been restricted to large industrial consumers, where the benefits could justify the high implementation cost. With the advancement of Smart Grid technologies, large scale implementation of variable-rate metering will be more practical. Consumers will be able to control their electricity consumption in an automated fashion, where one possible scheme is to have each individual maximize their own utility as a noncooperative game.

In this thesis, noncooperative games are formulated among the consumers of Smart Grid with two real-time pricing schemes, where the Nash equilibrium operation points are investigated for their uniqueness and load balancing properties. The first pricing scheme charges a price according to the average cost of electricity borne by the retailer and the second charges according to a time-variant increasing-block price. The zero revenue model and the constant revenue rate model, are the two revenue models being considered.

The relationship between these games and certain congestion games, known as atomic flow games from the computer networking community, is demonstrated. It is shown that the proposed noncooperative game formulation falls under the class of atomic splittable flow games. It is shown that the Nash equilibrium exists for four different cases, with different pricing schemes and revenue models, and is shown to be unique for three of the cases, under certain conditions. It is shown that both pricing schemes lead to similar electricity loading patterns when consumers are interested only in the minimization of electricity costs. Finally, the conditions under which the increasing-block pricing scheme is preferred over the average cost based pricing scheme are discussed.

To my family, friends, and to those whose work made this thesis possible

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Last but not the least, I would like to extend my thanks to everyone, who supported me in any way during the completion of this thesis.

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CHAPTER I

INTRODUCTION

Electricity consumers in most parts of the world, pay a fixed-rate retail price for their electricity usage, which changes on a seasonal or yearly basis. However, it has been long recognized in the economics community that charging consumers a flat rate for electricity creates allocative inefficiencies, i.e., consumers are not charged equilibrium prices for their consumption (see [1] for a complete reference). This is shown through an example in [2], which illustrates how flat pricing causes deadweight loss at off-peak times and excessive demand above the economic equilibrium at the peak times. The latter leads to blackouts in short run and excessive capacity buildup over long run. Variable-rate metering that reflects the real-time cost of generation can influence consumers to defer their power usage from the peak times. The reduced peak-load can significantly reduce the need for expensive generation during peak times and excessive capacity requirements. An additional benefit of real-time pricing and demand elasticity is that producer/generation firms may not be able to charge the retail providers with unreasonable rates during periods of high consumer demand, thus exerting undue market power [2, 3].

The main technical hurdle in implementing the real-time pricing has been the lack of cost-effective smart metering, which can communicate real-time prices to consumers and their consumption levels back to the energy provider. The claim of social benefits from real-time pricing also assumes that the consumer demand is elastic and responds to price changes. But traditional consumers do not possess the equipments that enable them to quickly alter their demands with changing prices, and significant research

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This thesis follows the style of *IEEE Transactions on Smart Grid*.

efforts on real-time pricing have involved estimating the consumer demand elasticity and the level of benefits that real time pricing can achieve [1, 4, 5]. Both these requirements, smart metering and consumer adaptability, have been unfulfilled till now. However, with the technological advancements in power generation and cyber-enabled metering in the framework of Smart Grid, real-time pricing and autonomous consumer load balancing becomes more practical [6].

In this thesis we formulate noncooperative games [7, 8] among the consumers of Smart Grid with two real-time pricing schemes. The first pricing scheme charges a price according to the average cost of electricity production and the second one charges according to a time-variant increasing-block price [9]. We investigate consumers' demand at the Nash equilibrium operation points for their uniqueness and load balancing properties. Two revenue models are considered and we show that both pricing schemes lead to similar electricity loading patterns when consumers are interested only in the minimization of electricity costs.

We demonstrate the relationship between these games and certain congestion games [10], known as atomic flow games [11] from the computer networking community. We show that the proposed noncooperative game formulation falls under the class of atomic splittable flow games [12]. Specifically, we show that the noncooperative game among the consumer has the same structure as in the atomic splittable flow game over a two node network with multiple parallel links between them. Finally we discuss the conditions under which the increasing-block pricing scheme is preferred over the average cost based pricing scheme.

The thesis is organized as follows. The system model, formulation of the noncooperative game, and its relationship to atomic flow games is presented in Chapter II. The game is analyzed with different real-time pricing schemes under different revenue models in Chapter III, where the Nash equilibrium properties are investigated. We conclude the thesis in Chapter IV.

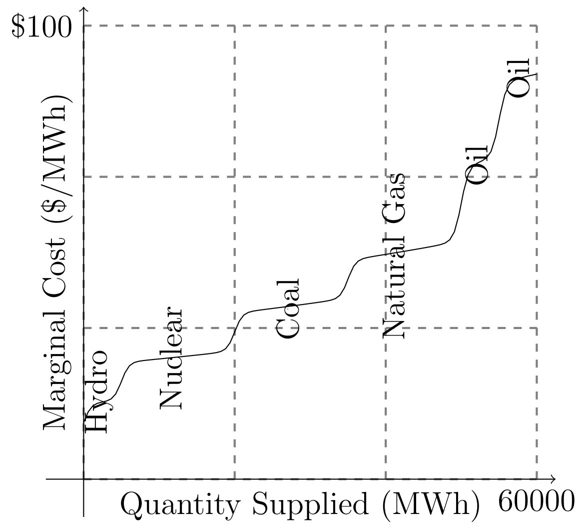
CHAPTER II

SYSTEM MODEL

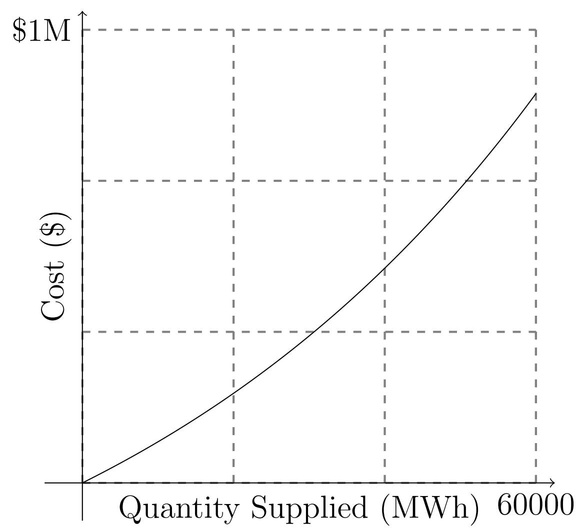
We study the transaction of energy between a single electricity retailer and multiple consumers. Each consumer has a demand for electrical energy (measured in Watt-hour, Wh) in a given time slot. The job of the retailer is to satisfy demands from all the consumers. The electricity supply of the retailer is purchased from a variety of sources over a wholesale electricity market. The retailer may possess some generation capacity as well. Each of these sources may use different technologies or fuels to generate electricity which leads to different marginal costs of electricity generation, where the marginal cost is the incremental cost incurred to produce an additional unit of output [13]. Mathematically, the marginal cost function (see an example in Fig. 1(a) based on data from [4]) is expressed as the first derivative of the total cost function (see an example in Fig. 1(b)). The retailer therefore attempts to satisfy demands by procuring the cheapest source first[1]. This results in a non-decreasing marginal cost of the supply curve, as illustrated through the example in Fig. 1(a). The retailer charges each consumer a certain price for its consumption in order to cover its costs. In real life the sum payments by all the consumers should be enough to cover the various costs incurred by the retailer as well as its profit margin. In our model we assume that all these components are incorporated within the marginal cost of electricity.

While the retailer procures sufficient supply to meet the sum demand of all its consumers in a given time slot, such supply may be limited by the sum generation

[1] In real life the base load, i.e., the regular power that is demanded by the consumers, is satisfied from sources such as hydro, coal, and nuclear as they are cheap. The fluctuating components of the demand are satisfied from sources such as oil, as the power-plants based on oil are more flexible to control.



(a) Marginal cost as a function of supply



(b) Total cost as a function of supply

Fig. 1. A hypothetical marginal cost of supply and the corresponding total cost curve as seen by the retailer in the wholesale market within a single time slot. Supply is from five different sources: hydroelectric, nuclear, coal, natural gas, and oil. Two different generators may use different technologies for power generation thus incurring different marginal costs with the same fuel (e.g., the two different cost levels for oil in Fig. 1(a)).

capacity available to the retailer from the multiple available sources. Thus, the maximum sum load that the retailer can service has an upper limit and we model this capacity limit by setting the marginal cost of electricity to infinity when sum load exceeds a predetermined threshold. Each consumer has an energy demand in a given time slot and it pays the retailer the corresponding price, where the price to consumer is set such that in each time slot the sum of payments made by all consumers meets the total cost of the retailer in that slot. A particular consumer's share of this bill depends on the retailer's pricing scheme, which is a function of the demands from all the consumers. Accordingly, as the total load varies over time, each consumer operates over a time-variant price with time-slotted granularity. Each consumer is assumed to have a certain total demand for electricity over each day, which it can distribute throughout the day in a time-slotted manner, to maximize certain utility function. We model such individual load balancing behaviors as a noncooperative game.

A. Noncooperative Load Balancing Game

The noncooperative game between these consumers is formulated as follows. We are given a group of N consumers, who submit their daily demands in a time-slotted pattern at the beginning of the day (which contains T time slots) to a retailer. The consumers are selfish, aiming to maximize their personal utility/payoff; hence do not cooperate with each other to manage their demands. Each consumer i has a minimum total daily requirement of energy, $\beta_i \geq 0$, which is split over the T time slots. We denote by x_t^i the ith consumer's demand in the tth time slot. A consumer can demand any value $x_t^i \geq 0$ (negativity constraint) with $\sum_t x_t^i \geq \beta_i$ (demand constraint). Another constraint $x_t^i \leq U$ is imposed on the demand x_t^i, to reflect the capacity constraint $C(x_t) = \infty, \forall x_t > U$. This constraint need not be imposed explicitly but we mention it in our formulations in order to make the feasible set of strategies compact. Let

$$\mathbf{x}^i = \{x_1^i, x_2^i, \ldots, x_t^i, \ldots, x_T^i\}$$

represent the ith consumer's demand vector, which is called the strategy for the ith consumer. Let

$$\mathbf{x}_t = \{x_t^1, \ldots, x_t^N\}$$

represent the demand vector from all consumers in time slot t. Let $\mathbf{x}$ represent the set $\{\mathbf{x}^1, \ldots, \mathbf{x}^N\}$. For ease of notation we use x_t to represent $\sum_i x_t^i$ and x^i to represent $\sum_t x_t^i$.

The payoff or utility for consumer i is denoted by π^i which is the total revenue it generates from the electricity that it purchases minus the total cost. In particular, let E_t^i, a function of x_t^i, represent the revenue generated by the ith consumer in the tth time slot and M_t^i, a function of $\mathbf{x}_t$, represent its payment to the retailer for purchasing x_t^i. Then the payoff π^i, to be maximized by consumer i, is given by

$$\pi^i = \sum_{t \in \{1,\ldots,T\}} \left[E_t^i - M_t^i\right].$$

Since M_t^i is a function of $\mathbf{x}_t$, we see that the consumer payoff is influenced by its load balancing strategy and that of other consumers. For ease of notation we use M^i to represent the total payments $(\sum_t M_t^i)$ made by consumer i.

To maximize the payoff at each consumer by designing the distributed load balancing strategy $\mathbf{x}^i$'s, we consider two real time pricing schemes. The first is the average cost based pricing scheme and the second is the increasing-block pricing scheme. Specifically, for the first one the retailer charges the consumers the average cost of electricity procurement. For the second one, the retailer charges according to a certain marginal cost function that depends on the vector of demands from all consumers, $\mathbf{x}_t$. Let us represent the wholesale cost of electricity by the function $C(x_t)$, where x_t is the total load posed to the retailer, with an example function plotted in Fig. 1(b).

Then under the average cost based pricing, the price per unit charged to the consumers is given by

$$A(x_t) = \frac{C(x_t)}{x_t}, \tag{2.1}$$

and at time t consumer i pays

$$M_t^i = x_t^i A(x_t) \tag{2.2}$$

for consuming x_t^i units of electricity. It is easy to see that $\sum_i M_t^i = C(x_t)$, i.e., with average cost based pricing the total payment made by the consumers covers the total cost to the retailer. In addition $C'(x_t)$ gives the marginal cost in the wholesale market and an example marginal cost curve is plotted in Fig. 1(a). In the context of electricity markets, as we discussed before, the marginal cost $C'(x_t)$ is always non-negative and non-decreasing such that $C(x_t)$ is always positive, non-decreasing and convex.

The second scheme is a time-variant version of the increasing-block pricing scheme. With a typical increasing-block pricing scheme, consumer i is charged a certain rate b_1 for its first z_1 units consumed, then charged rate b_2 ($> b_1$) for additional z_2 units, and charged rate b_3 ($> b_2$) for additional z_3 units, and so on. In our scheme, the values for the above b's and z's depend on $\mathbf{x}_t$ and the function $C(.)$. We formulate the increasing-block pricing scheme as follows. Consumer i pays an amount determined by the marginal cost function $\mathcal{B}(x, \mathbf{x}_t)$, which is the same for all consumers at any instant t but depends on $\mathbf{x}_t$, the demand vector of all the consumers. In particular consumer i pays

$$M_t^i = \int_0^{x_t^i} \mathcal{B}(x, \mathbf{x}_t) dx \tag{2.3}$$

for consuming x_t^i units of electricity. $\mathcal{B}(.)$ is chosen as

$$\mathcal{B}(x, \mathbf{x}_t) = C' \left(\sum_j \min\left(x, x_t^j\right) \right),$$

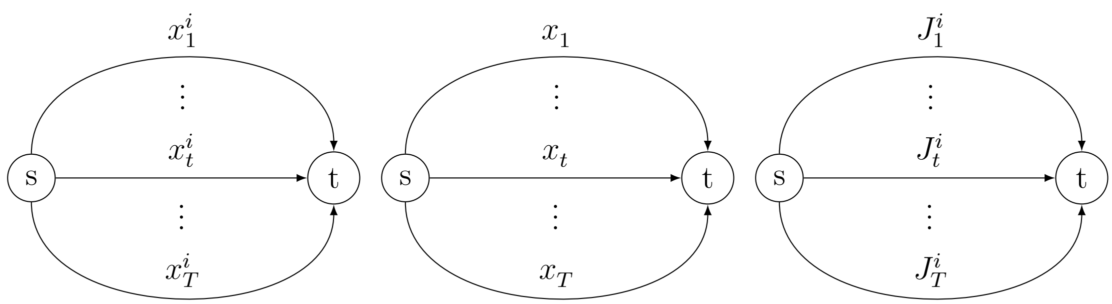
such that $\sum_i M_t^i = C(x_t)$ is satisfied.

For each of the two pricing schemes, we study two different revenue models. For the first case we set E_t^i as zero for all consumers over all time slots, which leads to payoff maximization being the same as cost minimization from the point of view of all the consumers. For the second case we assign consumer i a constant revenue rate ϕ_t^i at each time slot t, which gives $E_t^i = \phi_t^i x_t^i$, and leads to payoff maximization being the same as profit maximization.

B. Atomic Flow Games with Splittable Flows

The noncooperative game that we have formulated in the previous section is related to the following problem in the network routing literature [10, 11]. Consider several agents each of whom wish to establish paths from a specific source node to some destination node in order to transport a fixed amount of traffic. In the context of Internet, the agent can be viewed as a manager of packet routing. In the context of transportation, the agent is a company routing its fleet vehicles across the network of roads. The problem here is of competitive routing between agents, where each agent needs to deliver a given amount of flow over the network from its designated origin node to the corresponding destination node. An agent can choose how to divide its flow amongst the available routes. On each link the agents experience a certain delay. In the case of computer networks, if many agents collectively route a large number of packets through a particular link, the packets will experience larger delays; and beyond a certain level, the link may even start dropping packets, resulting in infinite delay. Such delay can be referred as cost, which is a function of the link congestion or the total flow through the link. The cost of a path is the sum of the link costs along the route.

To show the relationship between our noncooperative consumer load balancing problem and the above routing problem, we can reformulate the load balancing problem into the following routing game over two node multiple link network, first described in [10]. We have used notation similar to [10] in the interest of readability. Let there be N agents with throughput demands who share a common source node and a common destination over a two node network connected by T parallel links (see Fig. 2). It is assumed that the agents do not cooperate. Each agent



(a) Flows from the ith agent. (b) Sum of flows from all the agents. (c) Cost for different links for the ith agent.

Fig. 2. A two node network with T links between source s and destination t.

$i \in \{1, \ldots, N\}$ has a throughput demand β_i, which can be split among the T links as chosen by the agent. Let $x_t^i \geq 0$ denote the flow that agent i sends through link $t \in \{1, \ldots, T\}$. The sum of x_t^i should add upto β_i, i.e., $\beta_i = \sum_t x_t^i$. Let $x_t = \sum_i x_t^i$, and $\mathbf{x}_t = \{x_t^1, \ldots, x_t^i, \ldots, x_t^N\}$. The flow vector for agent i is denoted by the vector $\mathbf{x}^i = \{x_1^i, \ldots, x_t^i, \ldots, x_T^i\}$. The system flow vector is the collection of all agent flow vectors, denoted by $\mathbf{x} = \{\mathbf{x}^1, \ldots, \mathbf{x}^i, \ldots, \mathbf{x}^N\}$. A given $\mathbf{x}^i$ is feasible if its components obey the non-negativity constraint and the demand constraints. Let $\mathcal{X}^i$ be the set of all feasible choices of $\mathbf{x}^i$ for agent i, and $\mathcal{X}$ be the set of all feasible choices of $\mathbf{x}$.

Let $J^i(\mathbf{x})$ denote the *cost* for each agent i, that it wishes to minimize. As $J^i(\mathbf{x})$ is a function of the flow vector of all the agents, the best response of a given agent is a function of the responses of all the agents; and hence we can have a noncooperative game formulation. The Nash solution of the game is defined as the system flow vector such that none of the agents can unilaterally improve their performance. Formally, $\hat{\mathbf{x}} \in \mathcal{X}^i$ is a Nash Equilibrium Point (NEP) if the following condition holds for all agents

$$J^i(\hat{\mathbf{x}}) = \min_{\mathbf{x}^i \in \mathcal{X}^i} J^i(\hat{\mathbf{x}}^1, \ldots, \hat{\mathbf{x}}^{i-1}, \mathbf{x}^i, \hat{\mathbf{x}}^{i+1}, \ldots, \hat{\mathbf{x}}^N).$$

The above noncooperative game is known as an *atomic splittable flow game* [12, 14]. In [10], the existence of NEP is proved for atomic splittable flow game over the two node network with parallel links if the following five assumptions (G1-G5) are satisfied for the cost function.

G1: J^i is the sum of link cost functions, i.e., $J^i(\mathbf{x}) = \sum_t J^i_t(\mathbf{x}_t)$.

G2: $J^i_t : [0, \infty)^N \to [0, \infty]$ is a continuous function.

G3: J^i_t is convex over x^i_t.

G4: Wherever finite, J^i_t is continuously differentiable over x^i_t.

G5: For every system flow configuration $\mathbf{x}$, if not all cost function values are finite then at least one agent with infinite cost ($J^i(\mathbf{x}) = +\infty$) can change its own flow configuration to make its cost finite.

In the context of two node network with parallel links (Fig. 2), G5 is equivalent to the assumption that sum capacities of all links is greater than the sum of agent's demands [10].

As a side-note, in [10], the uniqueness of NEP is further imposed for two node network if the cost function J_t^i additionally complies with the following assumptions:

A1: J_t^i is a function of two arguments, namely agent i's flow on link t and the total flow on that link, i.e., $J_t^i(\mathbf{x}_t) = \bar{J}_t^i(x_t^i, x_t)$.

A2: $\bar{J}_t^i$ is increasing over each of its two arguments.

A3: Let $K_j^i = \frac{\partial \bar{J}_t^i}{\partial x_t^i}$. Wherever J_t^i is finite, $K_t^i = K_t^i(x_t^i, x_t)$ is strictly increasing in each of its two arguments.

In particular, functions that comply with the assumptions G1-G5 and A1-A3 are referred to as *type-A* functions in [10]. In the following chapters we will apply some of the results in [10] to facilitate our analysis over the noncooperative consumer load balancing game. The cost functions in our formulation do not satisfy one or more of the assumptions A1-A3, and hence we use other means to prove uniqueness of NEP.

CHAPTER III

NASH EQUILIBRIUM WITH DIURNAL STRATEGIES

For each of the two pricing schemes, discussed previously two different revenue models are studied to provide more design insights, which leads to two different payoff structures. In the first case the revenue is set to zero, such that payoff maximization is the same as cost minimization. In the second case, the rate of revenue generation at each consumer is set as a non-zero constant, such that payoff maximization is profit maximization.

A. Average Cost based Pricing

For this scheme, the payment to the retailer in slot t by consumer i is given by (2.2).

Case 1: Zero revenue model

In this case the revenue is set to zero as

$$E_t^i = 0,$$

which results in payoff maximization being the same as cost minimization for each consumer. Specifically, the payoff for consumer i is given by

$$\pi^i = -\sum_t M_t^i.$$

The consumer level optimization problem for each consumer i is given by the following optimization problem.

$$
\begin{aligned}
\text{maximize} \quad & \pi^i(\mathbf{x}^i) = -\sum_t M_t^i \\
\text{subject to} \quad & M_t^i = x_t^i A(x_t), \quad \forall t \\
& \sum_t x_t^i \geq \beta_i \\
& x_t = \sum_j x_t^j, \quad \forall t \\
& 0 \leq x_t^i \leq U \quad \forall t.
\end{aligned}
$$

We have already shown that this game is similar to the routing game described in [10]. With the average cost based pricing and the zero revenue model, the effective cost function for agent i to minimize in the routing game is

$$J_t^i = M_t^i = x_t^i A(x_t) = \frac{x_t^i}{x_t} C(x_t).$$

This cost function satisfies the assumptions G1-G5 given earlier. In particular, G1 holds as the total payment made by the consumers satisfies

$$M^i = \sum_t M_t^i,$$

which is the cost to the agents in the routing formulation. In addition, G2 trivially holds by the definition of M_t^i. In order to satisfy G3, i.e., to show that J_t^i is convex over x_t^i, we show that $\frac{\partial^2 J_t^i}{\partial {x_t^i}^2} \geq 0$. First we evaluate

$$
\begin{aligned}
\frac{\partial J_t^i}{\partial x_t^i} &= \frac{\partial \left(\frac{x_t^i}{x_t} C(x_t) \right)}{\partial x_t^i} \\
&= \frac{C(x_t)}{x_t} + x_t^i \left[\frac{1}{x_t} \frac{\partial C(x_t)}{\partial x_t^i} - \frac{C(x_t)}{{x_t}^2} \frac{\partial x_t}{\partial x_t^i} \right] \\
&= \frac{C(x_t)}{x_t} + x_t^i \left[\frac{1}{x_t} \frac{\partial \left(\int_0^{x_t - x_t^i} C'(z).dz + \int_{x_t - x_t^i}^{x_t} C'(z).dz \right)}{\partial x_t^i} - \frac{C(x_t)}{{x_t}^2} \right] \\
&= \frac{C(x_t)}{x_t} + x_t^i \left[\frac{1}{x_t} \frac{\partial \left(\int_{x_t - x_t^i}^{x_t} C'(z).dz \right)}{\partial x_t^i} - \frac{C(x_t)}{{x_t}^2} \right] \\
&= \frac{C(x_t)}{x_t} + x_t^i \left[\frac{1}{x_t} C'(x_t) - \frac{C(x_t)}{{x_t}^2} \right] \\
&= \frac{C(x_t)}{x_t} + x_t^i \left[\frac{x_t C'(x_t) - C(x_t)}{{x_t}^2} \right] \\
&= \frac{C(x_t)}{x_t} + x_t^i \left[\frac{x_t C'(x_t) - x_t A(x_t)}{{x_t}^2} \right] \\
&= A(x_t) + x_t^i \left[\frac{C'(x_t) - A(x_t)}{x_t} \right] \\
&= \frac{x_t A(x_t) + x_t^i C'(x_t) - x_t^i A(x_t)}{x_t} \\
&= \frac{(x_t - x_t^i) A(x_t) + x_t^i C'(x_t)}{x_t}.
\end{aligned}
\tag{3.1}
$$

Then we evaluate

$$\begin{aligned}
\frac{\partial^2 J_t^i}{\partial {x_t^i}^2} &= \frac{1}{x_t}\left[(x_t - x_t^i)\left(\frac{C'(x_t)}{x_t} - \frac{C(x_t)}{{x_t}^2}\right) + C'(x_t) + x_t^i C''(x_t)\right] \\
&\quad - \frac{(x_t - x_t^i)A(x_t) + x_t^i C'(x_t)}{{x_t}^2} \\
&= \frac{1}{{x_t}^2}\left[(x_t - x_t^i)\left(C'(x_t) - \frac{C(x_t)}{x_t}\right) + x_t C'(x_t) + x_t x_t^i C''(x_t)\right. \\
&\quad \left. - (x_t - x_t^i)A(x_t) - x_t^i C'(x_t)\right] \\
&= \frac{1}{{x_t}^2}\left[(x_t - x_t^i)\left(C'(x_t) - \frac{C(x_t)}{x_t}\right) + x_t C'(x_t) + x_t x_t^i C''(x_t)\right. \\
&\quad \left. - (x_t - x_t^i)\frac{C(x_t)}{x_t} - x_t^i C'(x_t)\right] \\
&= \frac{1}{{x_t}^2}\left[2(x_t - x_t^i)C'(x_t) + x_t x_t^i C''(x_t) - 2(x_t - x_t^i)\frac{C(x_t)}{x_t}\right] \\
&= \frac{1}{{x_t}^2}\left[2(x_t - x_t^i)\left(C'(x_t) - \frac{C(x_t)}{x_t}\right) + x_t x_t^i C''(x_t)\right].
\end{aligned} \tag{3.2}$$

Given $C(x)$ is convex, both $\left(C'(x_t) - \frac{C(x_t)}{x_t}\right) \geq 0$ and $C''(x_t) \geq 0$; and therefore $\frac{\partial^2 J_t^i}{\partial {x_t^i}^2} \geq 0$. Thus J_t^i is convex over x_t^i and G3 holds. The above also shows that J_t^i is continuously differentiable over x_t^i and hence G4 holds. Finally, as mentioned earlier, in the context of our special case of two node network with all parallel links (Fig. 2), G5 is equivalent to the assumption that sum capacities of all links is greater than the sum of agent's demands. Thus, as we assume that the sum capacity of the retail provider over the T time slots is enough to satisfy the total demand $\sum_i \beta_i$, G5 holds by construction.

By the proof in [10] we know that if the cost function satisfies assumption G2 and G3, there exists an NEP strategy for all agents. Therefore the NEP solution exists for the noncooperative consumer load balancing game.

The cost function J_t^i, does not satisfy the assumption A3, so it does not qualify as a type-A function from [10] and hence the corresponding uniqueness result cannot be extended to our formulation. We prove the uniqueness of the NEP solution by extending the result in [12], where it is proved that for more than two types of players playing a routing game over a network with the per unit cost function for each link belonging to the class of strictly semi-convex, non-negative, and non-decreasing cost functions, the Nash equilibrium is unique if and only if the graph is generalized nearly-parallel. The number of types of players in a particular game refers to the number of different values for β_i's. Thus a single type of players implies that all players have the same value for β_i. Next, we first introduce some definitions and show that the above atomic flow problem (and hence the load balancing problem) satisfies the conditions for NEP uniqueness as described in [12]. We now begin with some definitions from [12].

Definition 1. *The* component-join *operation for two given graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ consists of* merging *any two vertices $v_1 \in V_1$ and $v_2 \in V_2$ into a single vertex v.*

Definition 2. *A* hub-component *is a graph consisting of a set of vertex-disjoint paths connecting two nodes called* hubs. *The solid circles shown in Fig. 3 are the* hubs.

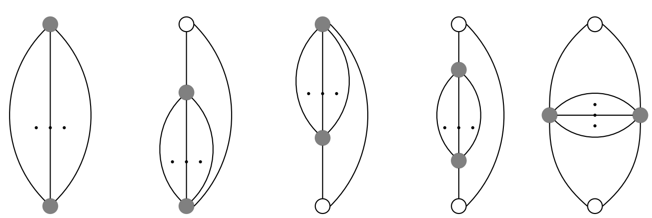


Fig. 3. Hub-components, the five basic units of nearly-parallel graphs.

Definition 3. *A* generalized nearly-parallel graph *is any graph that can be constructed from hub-components applying component-join operations.*

Two node network with parallel links of Fig. 2, is the first of the five basic units (as drawn in Fig. 3) of nearly-parallel graphs [12, 15], and by definitions a hub-component, hence is also a generalized nearly-parallel graph.

Definition 4. *A cost function $f(x)$ is (strictly)* semi-convex *if $xf(x)$ is (strictly) convex.*

For the load balancing game with average cost based pricing and zero revenue, the cost of consumer i is given by (2.2) where the cost per unit $A(x_t)$ is given by (2.1), with $A(x_t)$ a non-negative and non-decreasing function. For the function $A(x_t)$ to be strictly semi-convex, $x_tA(x_t)$ needs to be strictly convex. Since $C(x_t) = x_tA(x_t)$ is the total cost of electricity to the retailer, and as we assume the marginal cost price $C'(x)$ is a monotonically increasing function, $C(x)$ is strictly convex.

Thus our problem can be converted into an atomic flow game with splittable flows and different *player types* (i.e., each player controls a different amount of total flow) over a generalized nearly-parallel graph with strictly semi-convex, non-negative, and non-decreasing functions for cost per unit flow through links. By Theorem 3.11 of [12] the NEP solution for the load balancing game is unique.

In the following, the value of the unique NEP is evaluated.

Lemma 1. *With the average cost based pricing and zero revenue, at the Nash equilibrium the unit price of electricity faced by all consumers will be same over all time slots.*

Proof. Consider two arbitrary time slots t_1 and t_2. At the Nash equilibrium the sum demands on the system are either the same over the two slots or different. If the sum demands are equal over the two time slots, by (2.1), we know that the unit price of electricity will be same for the two slots. If the sum demands are not equal, without losing generality, let us assume $x_{t_1} < x_{t_2}$ such that

$$A(x_{t_1}) < A(x_{t_2}) \tag{3.3}$$

holds. Then any consumer j with $x_{t_2}^j > 0$ can reduce cost by reducing $x_{t_2}^j$ and increasing $x_{t_1}^j$ by the same small quantity. This contradicts our assumption that the system is in equilibrium. Hence $A(x_{t_1}) = A(x_{t_2})$. □

In order to prove the following lemma we need $C(.)$ to be strictly convex as without it, $C(x_t) = kx_t$ is an admissible cost function and with it $A(x_{t_1}) = A(x_{t_2})$ holds while $x_{t_1} < x_{t_2}$.

Lemma 2. *If $C(.)$ is strictly convex, at the Nash equilibrium, the sum of demands on the system, x_t, keeps the same across each time slot t.*

Proof. If $C(.)$ is strictly convex then $C''(.) > 0$. At Nash equilibrium we have $A(x_{t_1}) = A(x_{t_2})$ from Lemma 1 for all possible t_1 and t_2. The two conditions together imply

$$A(x_{t_1}) = A(x_{t_2}) \Leftrightarrow x_{t_1} = x_{t_2}.$$

□

Lemma 3. *If $C(.)$ is strictly convex, at Nash equilibrium, each consumer will distribute its demands equally over the T time slots.*

Proof. As the Nash equilibrium is unique, by symmetry over all the time slots, for consumer i we shall have

$$x_{t_1}^i = x_{t_2}^i, \quad \forall t_1, t_2,$$

as otherwise we could swap the demand vectors $\mathbf{x}_{t_1}$ and $\mathbf{x}_{t_2}$ in time slot t_1 and t_2 without altering the Nash equilibrium conditions and get another distinct NEP, thus contradicting uniqueness. Thus, with $\sum_t x_t^i = \beta_i$, we have the solution $x_t^i = \beta_i/T$ for all consumers i and time slots t. $\square$

Under the average cost based pricing scheme with zero revenue, if one particular consumer increases its demand of electricity, the per unit price $A(.)$ will increase, which increases the payments for all other consumers as well. Theoretically one consumer may cause indefinite increases in the payments of all others, and in this sense this scheme does not protect the group from reckless action of some consumer(s). This issue will be addressed by our second pricing scheme as we will show later.

Case 2: Constant Revenue Rate Model

In this case, the rate of revenue generation for each consumer at each time slot is taken as a non-negative constant ϕ_t^i. Thus,

$$E_t^i = \phi_t^i \times x_t^i.$$

The consumer level optimization problem for each consumer i is given by the following optimization problem.

$$\begin{aligned} \text{maximize} \quad & \pi^i(\mathbf{x}^i) = \sum_t \left[E_t^i - M_t^i\right] \\ \text{subject to} \quad & E_t^i = \phi_t^i x_t^i, \quad \forall t \\ & M_t^i = x_t^i A(x_t), \quad \forall t \\ & \sum_t x_t^i \geq \beta_i \\ & x_t = \sum_j x_t^j, \quad \forall t \\ & 0 \leq x_t^i \leq U \quad \forall t. \end{aligned}$$

We set $\beta_i = 0$ for all consumers, as $\beta_i > 0$ will cause those consumers with rate of revenue less than the per unit price of electricity, to incur a negative payoff. As under this formulation, $E_t^i \neq 0$, hence payoff maximization for each consumer is equivalent to profit maximization and it will not result in the trivial solution $x_t^i = 0$, for all i and t. The side-effect of this however is that the constraint $\sum_t x_t^i \geq \beta_i = 0$ vanishes as $\sum_t x_t^i \geq 0$ is redundant and there is no more a common constraint on demand vectors from different time slots $\mathbf{x}_{t_1}$ and $\mathbf{x}_{t_2}$; hence the payoff for the overall noncooperative game is a sum of the payoffs for T, single time slot, noncooperative games.

We briefly show that under these assumptions there exists an NEP for this game. The effective cost function for the corresponding routing game is given as

$$J_t^i = M_t^i - \phi_t^i \times x_t^i. \tag{3.4}$$

As M_t^i is continuous in $\mathbf{x}_t$, J_t^i is continuous in $\mathbf{x}_t$ as well and satisfies assumption G2. We have already shown that M_t^i under the average cost based pricing scheme is convex in x_t^i through (3.2). The function $-\phi_t^i x_t^i$ is linear and hence convex in x_t^i. Thus, by the property that the summation of two convex functions is convex, J_t^i from (3.4) is convex in x_t^i and hence satisfies assumption G3. Following the proof in [10], we consider the point-to-set mapping $\mathbf{x} \in \mathcal{X} \to \Gamma(\mathbf{x}) \subset \mathcal{X}$ defined as

$$\Gamma(\mathbf{x}) = \{\hat{\mathbf{x}} \in \mathcal{X} : \hat{\mathbf{x}}^i \in \arg\min_{\mathbf{z}^i \in \mathcal{X}^i} J^i(\mathbf{x}^1, \ldots, \mathbf{z}^i, \ldots, \mathbf{x}^N)\},$$

where Γ is an upper semicontinuous mapping (by the continuity assumption G2) that maps each point of the convex compact set $\mathcal{X}$ into a closed (by G2) convex (by G3) subset of $\mathcal{X}$. By the Kakutani Fixed Point Theorem [16], there exists a fixed point $\mathbf{x} \in \Gamma(\mathbf{x})$ and such a point is NEP [17].

Lemma 4. *At the Nash equilibrium, the consumer(s) with the highest revenue rate (ϕ_t^i) within the time slot, may be the only one(s) buying the power in that time slot.*

Proof. For a given time slot t consumer i has an incentive to increase its demand x_t^i

as long as the payoff can increase, i.e., as long as

$$\frac{\partial \pi_i}{\partial x_t^i} > 0.$$

Therefore at the equilibrium the following holds for all consumers.

$$\begin{aligned} \frac{\partial \pi^i}{\partial x_t^i} &\leq 0 \\ \Rightarrow \frac{\partial \left[E_t^i - M_t^i\right]}{\partial x_t^i} &\leq 0 \\ \Rightarrow \frac{\partial E_t^i}{\partial x_t^i} - \frac{\partial M_t^i}{\partial x_t^i} &\leq 0 \\ \Rightarrow \phi_t^i \leq \frac{\partial M_t^i}{\partial x_t^i} = \frac{C(x_t)}{x_t} &= A(x_t) \end{aligned}$$

For the consumers with a strict inequality $\phi_t^i < A(x_t)$, the rate of revenue is less than the price per unit of electricity at time t; hence the revenue is less than the cost, $E_t^i < M_t^i$, such that buying electricity will incur them a negative payoff and hence all such consumers, with $\phi_t^i < A(x_t)$, will not buy any power in that time slot, i.e., $x_t^i = 0$. Therefore only the set of consumers $\{\arg\max_k \phi_t^k\}$, i.e., the consumers who enjoy the maximum rate of revenue may be able to purchase electricity. $\square$

Thus if consumer i has the maximum rate of revenue, either it is the only consumer buying non-zero power x_t^i such that $\phi_t^i = A(x_t^i)$ or $\phi_t^i < C'(0)$ and hence $x_t^i = 0$ in that time slot, which leads to a unique Nash equilibrium for the sub-game. If in a given time slot multiple consumers experience the same maximum rate of revenue, then the sub-game will turn into a Nash Demand Game [18] between the set of consumers, $\{\arg\max_k \phi_t^k\}$, and will lead to multiple Nash equilibriums. Thus the overall noncooperative game has a unique Nash equilibrium if and only if, in each time slot, at most one consumer experiences the maximum rate of revenue.

B. Increasing-Block Pricing

In this section we study the load balancing game with the time-variant increasing block pricing scheme. Under this scheme consumer i pays M_t^i for x_t^i units of electricity, which is given by (2.3) where $\mathcal{B}(x, \mathbf{x}_t)$ is the marginal cost function posed to the consumer. Thus,

$$\mathcal{B}(x, \mathbf{x}_t) = C'\left(\sum_j \min\left(x, x_t^j\right)\right).$$

As an example, if the demand from all consumer's at time slot t is identical, i.e., if $x_t^i = x_t^j$, for all i and j, then we have,

$$\mathcal{B}(x, \mathbf{x}_t) = C'(Nx).$$

Case 1: Zero revenue model

In this case the payment by consumer i is given by (2.3)

$$M_t^i = \int_0^{x_t^i} \mathcal{B}(x, \mathbf{x}_t) dx.$$

The consumer level optimization problem for each consumer i is given by the following optimization problem.

$$\begin{aligned}
\text{maximize} \quad & \pi^i(\mathbf{x}^i) = -\sum_t M_t^i \\
\text{subject to} \quad & M_t^i = \int_0^{x_t^i} \mathcal{B}(x, \mathbf{x}_t) dx, \quad \forall t \\
& \sum_t x_t^i \geq \beta_i \\
& 0 \leq x_t^i \leq U \quad \forall t.
\end{aligned}$$

As M_t^i is continuous in $\mathbf{x}_t$, therefore in the corresponding routing game

$$J_t^i = M_t^i \tag{3.5}$$

is continuous in $\mathbf{x}_t$ and satisfies assumption G2. In addition, M_t^i is convex in x_t^i as its derivative, the marginal cost function $\mathcal{B}(x, \mathbf{x}_t)$, is non-decreasing. Thus, J_t^i is convex in x_t^i and hence satisfies assumption G3. Following the proof in [10], we consider the point-to-set mapping $\mathbf{x} \in \mathcal{X} \to \Gamma(\mathbf{x}) \subset \mathcal{X}$ defined by

$$\Gamma(\mathbf{x}) = \{\hat{\mathbf{x}} \in \mathcal{X} : \hat{\mathbf{x}}^i \in \arg\min_{\mathbf{z}^i \in \mathcal{X}^i} J^i(\mathbf{x}^1, \ldots, \mathbf{z}^i, \ldots, \mathbf{x}^N)\},$$

where Γ is an upper semicontinuous mapping (by the continuity assumption G2) that maps each point of the convex compact set $\mathcal{X}$ into a closed (by G2) convex (by G3) subset of $\mathcal{X}$. By the Kakutani Fixed Point Theorem, there exists a fixed point $\mathbf{x} \in \Gamma(\mathbf{x})$ and such a point is NEP.

When each consumer tries to minimize its total cost while satisfying its minimum daily energy requirement β_i, we have the following result.

Lemma 5. *If $C(.)$ is strictly convex, the Nash equilibrium is unique and each consumer distributes its demand uniformly over all time slots.*

Proof. For the equilibrium conditions to be satisfied,

$$\mathcal{B}(x_{t_1}^i, \mathbf{x}_{t_1}) = \mathcal{B}(x_{t_2}^i, \mathbf{x}_{t_2}), \quad \forall i, t_1, t_2,$$

should hold; otherwise consumer i can increase payoff by varying $x_{t_1}^i$ and $x_{t_2}^i$. The condition can be rewritten after expanding $\mathcal{B}(.)$ as

$$C'\left(\sum_j \min\left(x_{t_1}^i, x_{t_1}^j\right)\right) = C'\left(\sum_j \min\left(x_{t_2}^i, x_{t_2}^j\right)\right), \quad \forall i, t_1, t_2. \tag{3.6}$$

Given $C(.)$ is strictly convex and then we have $C'(.)$ monotonically increasing, which gives

$$C'(z_1) = C'(z_2) \Leftrightarrow z_1 = z_2. \tag{3.7}$$

Therefore, (3.6) implies

$$\sum_j \min{(x^i_{t_1}, x^j_{t_1})} = \sum_j \min{(x^i_{t_2}, x^j_{t_2})}, \quad \forall i, t_1, t_2. \tag{3.8}$$

Now lets assume that there exist an NEP $\mathbf{x}$ with demand vectors $\mathbf{x}_{t_1} \neq \mathbf{x}_{t_2}$. Let $\mathcal{P}$ represent the subset of consumers with unequal demands in time slots t_1 and t_2,

$$\mathcal{P} = \{k | x^k_{t_1} \neq x^k_{t_2}, k \in \{1, 2, \ldots, N\}\}.$$

Then let a represent the consumer from subset $\mathcal{P}$ with the highest value of demand in time slot t_1

$$a = \arg\max_{k \in \mathcal{P}} x^k_{t_1}, \tag{3.9}$$

and let b represent the consumer from subset $\mathcal{P}$ with the highest value of demand in time slots t_2

$$b = \arg\max_{k \in \mathcal{P}} x^k_{t_2}. \tag{3.10}$$

From (3.8) we have

$$\sum_j \min{(x^a_{t_1}, x^j_{t_1})} = \sum_j \min{(x^a_{t_2}, x^j_{t_2})}, \quad \forall i, t_1, t_2, \tag{3.11}$$

and

$$\sum_j \min{(x^b_{t_1}, x^j_{t_1})} = \sum_j \min{(x^b_{t_2}, x^j_{t_2})}, \quad \forall i, t_1, t_2. \tag{3.12}$$

Combining (3.8) and (3.9) leads to

$$\sum_j \min{(x^a_{t_1}, x^j_{t_1})} \geq \sum_j \min{(x^b_{t_1}, x^j_{t_1})}; \tag{3.13}$$

combining (3.8) and (3.10) leads to

$$\sum_j \min\left(x^a_{t_2}, x^j_{t_2}\right) \leq \sum_j \min\left(x^b_{t_2}, x^j_{t_2}\right). \tag{3.14}$$

If $x^a_{t_1} \neq x^b_{t_1}$ or $x^a_{t_2} \neq x^b_{t_2}$, (3.13) holds with strict inequality. With (3.11), (3.12), and (3.13), we have

$$\sum_j \min\left(x^a_{t_2}, x^j_{t_2}\right) > \sum_j \min\left(x^b_{t_2}, x^j_{t_2}\right),$$

which contradicts (3.14). If $x^a_{t_1} = x^b_{t_1}$ and $x^a_{t_2} = x^b_{t_2}$ then (3.11) and (3.12) imply $x^a_{t_1} = x^a_{t_2}$ and $x^b_{t_1} = x^b_{t_2}$, respectively, which contradicts that $a, b \in \mathcal{P}$. This implies that the set $\mathcal{P}$ is empty, which contradicts that $\mathbf{x}_{t_1} \neq \mathbf{x}_{t_2}$.

Hence we have

$$x^i_{t_1} = x^i_{t_2} \qquad \forall i, t_1, t_2,$$

and the solution is given by $x^i_t = \beta_i/T,\ \forall i, t$. Under the necessary conditions for NEP (3.6), this is the only solution for the set $\mathbf{x}$, hence NEP is unique. □

Notice that for zero revenue model, the resulting value of NEP is the same with both increasing-block pricing and average cost based pricing. For both the cases, at NEP, we have $x^i_t = \beta_i/T,\ \forall i, t$. However, even though the loading pattern is similar, the payments M^i_t made by the consumers will differ and, with increasing-block pricing, is likely to be lesser for consumers with relatively lesser consumption. Secondly, under this pricing scheme, with N consumers in the system, the maximum payment M^i_t made by the ith consumer given x^i_t demand will be $C(Nx^i_t)/N$, irrespective of what other consumers demand and consume. Thus this addresses the issue faced with average cost based pricing and zero revenue model, in which one consumer can increase their demand indefinitely and cause indefinite increase in the payments of all other consumers.

Case 2: Constant Revenue Rate Model

The consumer level optimization problem for each consumer i is given by the following optimization problem.

$$\begin{aligned}
\text{maximize} \quad & \pi^i(\mathbf{x}^i) = \sum_t \left[E_t^i - M_t^i\right] \\
\text{subject to} \quad & E_t^i = \phi_t^i x_t^i, \quad \forall t \\
& M_t^i = \int_0^{x_t^i} \mathcal{B}(x, \mathbf{x}_t) dx, \quad \forall t \\
& \sum_t x_t^i \geq \beta_i \\
& 0 \leq x_t^i \leq U \quad \forall t.
\end{aligned}$$

In this case, we briefly show that with increasing-block pricing and a constant revenue rate, there exists an NEP solution for this game. First, the cost function in the corresponding routing game is given by

$$J_t^i = M_t^i - \phi_t^i \times x_t^i, \tag{3.15}$$

where M_t^i is continuous in $\mathbf{x}_t$, and therefore J_t^i is continuous in $\mathbf{x}_t$ and satisfies assumption G2. We have already shown that M_t^i under the increasing-block pricing scheme is convex in x_t^i in previous subsection. The function $-\phi_t^i x_t^i$ is linear and hence convex in x_t^i as well. Thus, J_t^i from (3.15) is convex in x_t^i and hence satisfies assumption G3. Following the proof in [10], we consider the point-to-set mapping $\mathbf{x} \in \mathcal{X} \to \Gamma(\mathbf{x}) \subset \mathcal{X}$ defined by

$$\Gamma(\mathbf{x}) = \{\hat{\mathbf{x}} \in \mathcal{X} : \hat{\mathbf{x}}^i \in \arg\min_{\mathbf{z}^i \in \mathcal{X}^i} J^i(\mathbf{x}^1, \ldots, \mathbf{z}^i, \ldots, \mathbf{x}^N)\},$$

where Γ is an upper semicontinuous mapping (by the continuity assumption G2) that maps each point of the convex compact set $\mathcal{X}$ into a closed (by G2) convex (by G3) subset of $\mathcal{X}$. By the Kakutani Fixed Point Theorem, there exists a fixed point

$\mathbf{x} \in \Gamma(\mathbf{x})$ and such a point is NEP.

Within the average cost based pricing scheme and constant revenue rate model, we saw that in the given time slot, if a single consumer enjoys the maximum rate of revenue, then it will be the only consumer who may be able to purchase power. We show here that within increasing-block pricing scheme and constant revenue rate model, the consumer with highest earnings coefficient will not be able to dominate the time slot.

For a given time slot t a consumer i will have an incentive to increase their demand x_t^i as long as the payoff increases, i.e.,

$$\frac{\partial \pi^i}{\partial x_t^i} > 0.$$

Therefore at the equilibrium following holds for all consumers.

$$\begin{aligned} &\frac{\partial \pi^i}{\partial x_t^i} \leq 0 \\ &\Rightarrow \phi_t^i \leq \frac{\partial M_t^i}{\partial x_t^i} = \mathcal{B}(x_t^i, \mathbf{x}_t). \end{aligned}$$

As $\mathcal{B}(x_t^i, \mathbf{x}_t)$ differs with x_t^i for each consumer, it constraints each consumer's demand differently. This is unlike the constraint $\phi_t^i \leq A(x_t)$ which has the same value for $A(x_t)$, thus constraining all consumers similarly. Thus any consumer, irrespective of other's rate of revenue, will be able to procure a non-zero amount of energy as long as its rate of revenue is larger than $\mathcal{B}(0, \mathbf{x}_t)$ which is equal to $C'(0)$.

CHAPTER IV

CONCLUSION

In this thesis we formulated noncooperative games among the consumers of Smart Grid with two real-time pricing schemes to derive autonomous load balancing schemes. The first pricing scheme charges consumers a price that is equal to the average cost of electricity borne by the retailer and the second one charges consumers an amount determined by increasing-block pricing, which applies the same marginal cost function to all consumers. Two revenue models were considered for each of the pricing schemes and we investigated consumers' demand at the Nash equilibrium operation points for their uniqueness and load balancing properties. For the zero revenue model, we showed that when consumers are interested only in the minimization of electricity costs the Nash equilibrium point is unique over both the pricing schemes and leads to similar electricity loading patterns in either case. For the constant revenue rate model, we showed existence of Nash equilibrium for both the pricing schemes and showed the uniqueness results for the average cost based pricing scheme. Under the zero revenue model, consumers get a protection from an arbitrary increase in their payments with the increasing-block pricing scheme but not with the average cost based pricing. This is due to existence of an upper bound on the payment that the consumers maybe expected to make within the former scheme that is not present in the latter. Under the constant rate revenue model, consumers who enjoy the maximum rate of revenue may drive the prices up with the average cost based pricing till only they can profitably buy electricity, an issue that is not faced by the consumers with lower rate of revenue in the increasing-block pricing scheme.

We demonstrated the relationship between the load balancing games and the atomic splittable flow games from the computer networking community. We showed that the proposed noncooperative game formulation is related to the atomic splittable flow game over a two node network with parallel links, and used some of the results for atomic flow games to prove the properties of the load balancing games.

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