ASYMMETRIC INFORMATION IN COMMON-VALUE AUCTIONS AND CONTESTS: THEORY AND EXPERIMENTS

A Dissertation

by

LUCAS AAREN RENTSCHLER

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2010

Major Subject: Economics

Asymmetric Information in Common-Value

Auctions and Contests: Theory and Experiments

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ABSTRACT

Asymmetric Information in Common-Value

Auctions and Contests: Theory and Experiments. (August 2010)

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Chair of Advisory Committee: Dr. Rajiv Sarin

In common-value auctions and contests economic agents often have varying levels of information regarding the value of the good to be allocated. Using theoretical and experimental analysis, I examine the effect of such information asymmetry on behavior.

Chapter II considers a model in which players compete in two sequential contests. The winner of the first contest (the incumbent) privately observes the value of the prize, which provides private information if the prizes are related. Relative to the case where the prizes are independent, the incumbent is strictly better off, and the other contestants (the challengers) are strictly worse off. This increases the incentive to win the first contest such that the sum of expected effort over both contests increases relative to the case of independent prizes.

Chapter III experimentally considers the role of asymmetric information in first-price, sealed-bid, common-value auctions. Bidders who observe a private

signal tend to overbid relative to Nash equilibrium predictions. Uninformed bidders, however, tend to underbid relative to the Nash equilibrium.

Chapter IV examines asymmetric information in one-shot common-value all-pay auctions and lottery contests from both experimental and theoretical perspectives As predicted by theory, asymmetric information yields information rents for the informed bidder in both all-pay auctions and lottery contests.

DEDICATION

This dissertation / would not be complete without / an "I love you Anne."

ACKNOWLEDGEMENTS

I would like to thank my committee chair, Dr. Sarin, and my committee members, Dr. Grosskopf, Dr. Gronberg, and Dr. Woodward for their guidance and support throughout the course of this research.

I would like to thank my mother, for her continual support, Peter Amons, for teaching by example, Cliff Nowell, whose wonderful introduction to economics led me to this point, and Anne Decker, for always being there for me. I would also like to thank Martha Gukeisen, Chris Dawe and Charlie Webb.

Financial support from the Department of Economics at Texas A&M University and the NSF is gratefully acknowledged.

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INTRODUCTION

When economic agents compete for a good, they often hold different levels of information regarding it's value. Modeling such information asymmetries complicates the analysis, so attention in the literature has largely focused on (ex ante) symmetric information environments. As such, there are many open questions regarding the effects of information asymmetries. In this dissertation I analyze asymmetric information environments in common-value auctions and contests. I focus on a particular kind of information asymmetry; one of the economic agents privately observes a noisy signal regarding the common, but uncertain, value of the good. The other agents hold no private information. I utilize both theoretical and experimental methods in my analysis.

In Chapter II, I consider a model in which contestants compete in two sequential imperfectly discriminating contests where the prize in each contest has a common but uncertain value, and the value of the prize in the first contest is positively related to that in the second. The contestant who obtains the prize in the first contest (the incumbent) privately observes its value, so that information in the second contest is asymmetric.

Relative to the case where the prizes are independent random variables (so that the incumbent's private information does not provide a useful estimate of the value of the prize in the second contest), the incumbent is strictly better off, the other contestants (the challengers) are strictly worse off, and aggregate effort expenditures in the second contest strictly decrease.

This dissertation follows the style of the Journal of Economic Theory.

Further, aggregate effort expenditures in the first contest increases such that total effort expenditures over the two contests increase, relative to the case of independent prizes. Counterintuitively, the incumbent's ex ante probability of winning is strictly less than that of a challenger, despite expending (weakly) more effort than a challenger in expectation. In the second (terminal) contest, expected effort expenditure of an individual contestant is decreasing in the number of contestants, the expected utility of a contestant is decreasing in the number of contestants, and the aggregate expected effort expenditure is increasing in the number of contestants. I also consider the effects of a incumbency cost advantage and a "status quo bias."

Chapter III considers the role of information in first-price, sealed-bid, common-value auctions from an experimental perspective. We consider three information structures in such auctions: (1) symmetric information in which bidders hold no private information; (2) asymmetric information in which only one bidder observes a private signal; (3) symmetric information in which each bidder observes a private signal.

We find that bidders who observe a private signal tend to overbid relative to Nash equilibrium predictions. Uninformed bidders, however, tend to underbid relative to the Nash equilibrium. When both bidders observe a private signal, bidders overbid such that they often fall victim to the winner's curse. When neither bidder observes a private signal, the winner's curse is much less prevalent. This suggests that the prevalence of the winner's curse in previous studies may be an artifact of private information. The information rent of informed bidders facing uninformed bidders is greater than predicted by theory despite overbidding relative to the Nash equilibrium bid function.

Chapter IV examines common-value contests with incomplete and asymmetric information. In our experimental design one bidder observes an informative signal as to the realized common value of the good. The other bidder holds only public information; she knows only the distribution from which the value of the prize is drawn. This asymmetric information environment is compared to a symmetric information environment in which neither bidder observes a signal; both bidders know only the distribution from which the value is drawn. We characterize the equilibrium in a common value all-pay auction with this type of information asymmetry.

Consistent with theory, such asymmetric information yields information rents for the informed bidder in both all-pay auctions and lottery contests. Also consistent with theory, asymmetric information reduces the expected revenue in all-pay auctions. In lottery contests, information asymmetry has no effect on revenue. We also observe that bidders who observe a signal are much more prone to bid above a break-even bidding strategy than are bidders who do not observe a signal.

CHAPTER II
INCUMBENCY IN IMPERFECTLY DISCRIMINATING CONTESTS

OVERVIEW

Consider a group of workers who compete for a job with a particular firm. One of these workers prevails and begins working for the firm. Suppose that at some later date the firm seeks to fill a job opening which would be a promotion for the worker who got the job in the earlier competition. The group of workers is now in a position to compete for a second time. However, the second competition may be significantly different than the first.

In particular, it is natural to think that the worker who obtained the job in the first round will have more information than the other workers regarding the value of the second job. That is, she observes the intangible benefits of working for the firm, such as the corporate culture and how employees are treated. Further, such asymmetric information in the second competition may affect the incentives in the first competition by increasing the value of winning.

This chapter considers a model in which contestants compete in two sequential contests where the prize in each contest has a common but uncertain value, and the value of the prize in the first contest is positively related to that in the second contest. The contestant who obtains the prize in the first contest (the incumbent) privately observes its value, which provides a noisy estimate of the value in the second contest, thereby introducing asymmetric information. The contestants who do not obtain the prize in the first contest (the challengers) do not hold any private information in the second contest. Since contestants do not interact after the second contest, this framework allows me to examine the effect of information asymmetry on behavior in a one-shot game, as well as the effect on behavior when information asymmetry arises due to an incumbency advantage.

I utilize the well known model of imperfectly discriminating contests introduced in Tullock [46]. The associated literature is vast. Such a contest is a game in which economic agents expend unrecoverable effort in order to increase the probability of winning a prize. The contestant with the highest effort level does not win with certainty, but has the highest probability of winning.

Interestingly, I find that in the second contest, ex ante, the incumbent will expend weakly more effort than a challenger, but wins with a strictly lower probability. The intuition behind this result is that the incumbent expends little or no effort when she believes the value of the prize is low. As a result, the incumbent obtains the prize with low probability when its value is low. However, when the incumbent believes the value of the prize is high, she expends more effort than the challengers such that in expectation, the incumbent expends weakly more effort than a challenger. The incumbent's low effort expenditures when she believes the value of the prize is low dominates the higher effort expenditures when she believes the value of the prize is high, such that, ex ante, the incumbent's probability of obtaining the prize is strictly lower than that of a challenger.

I also find that, relative to the case where the value of the prizes in the two contests are independent (rendering the incumbent's private information strategically irrelevant), aggregate effort expenditures fall in the second contest, but increase in the first contest such that total effort expenditures summed over the two contests weakly increases.[1] This implies that, ex ante, contestants are worse off when there is an informational incumbency advantage. That is, the private incentive to acquire

[1] Aggregate effort over the two periods strictly increases relative to the case of independent prizes if the support of distribution from which prizes are drawn includes zero.

information relevant to the second contest is sufficiently high that contestants will increase their first period effort expenditures relative to the case of independent prizes such that they are, ex ante, worse off. The intuition behind this result is that challengers are strictly worse off than in the case of independent prizes, while the incumbent is strictly better off. Thus, contestants in the first contest stand to gain in the second contest by obtaining the prize in the first contest, and, conversely, stand to lose in the second contest by not obtaining the prize in the first contest. This added incentive is sufficient to increase aggregate effort expenditures over the two contests relative to the case of independent prizes. By way of contrast, in analogous twice-repeated all-pay and first-price auctions, expected revenue summed over both periods is unchanged between the case of an informational incumbency advantage and the case of independent values.

In the second (terminal) contest, expected effort expenditure of an individual contestant is decreasing in the number of contestants, the expected utility of a contestant is decreasing in the number of contestants, and the aggregate expected effort expenditure is increasing in the number of contestants. Interestingly, in analogous one-shot all-pay and first-price auctions, revenue and profit predictions are invariant to the number of bidders.

The second period of my model, in which the incumbent has an informational advantage, is a generalization of Wärneryd [48], which examines a one-shot, two-player imperfectly discriminating contest where the prize is of common and uncertain value. My model differs in that there are $n \geq 2$ contestants, and I allow the incumbent's information to be imperfectly informative. Indeed, I assume that the value of the prize in period two is positively regression dependent on the value in period one, a weaker assumption of positive dependence than the notion of affiliated random variables used extensively throughout the auction literature.

The asymmetric information structure studied in the second contest of my model has been studied in one-shot first-price auctions by Engelbrecht-Wiggans et al. [16] and Milgrom and Weber [37]. They find that this asymmetric information structure guarantees that the uninformed bidders have an expected payoff of zero. Further the informed bidder earns a positive information rent. Expected revenue is less than in a symmetric information structure due to the informed bidder's information rent. Chapter III considers this information structure in the context of an all-pay auction, and finds that expected revenue and the expected payoff of bidders are identical to those in a first-price auction.

This type of asymmetric information structure has also been examined in repeated games. Hörner and Jamison [23] study an infinitely repeated first-price auction with the information structure of Engelbrecht-Wiggans et al. [16]. In their model, bids are observed at the end of each auction, such that uninformed bidders update their beliefs regarding the value of the good by observing the behavior of the informed bidder. Consequently, uninformed bidders are able, in finite time, to infer the informed bidder's private information.

In a paper closely related to this one, Virág [47] examines a twice repeated first-price auction with an initial information structure as in Engelbrecht-Wiggans et al. [16]. There are two bidders, and one of them holds private information in the first period. Bids are not observed at the end of the period. If the uninformed bidder loses the first period auction, then asymmetric information still exists in the second period. If the uninformed bidder wins the first period auction she observes the value of the good, and information is symmetric in the second period. Virág finds that bidders bid more aggressively in the first period, because the uninformed bidder has more to gain in the first period, and the informed bidder has a higher incentive to win, in order to maintain the asymmetric information in the second period. My model differs in that contestants are symmetricly uninformed in the first period, and I consider an imperfectly discriminating contest rather than a first-price auction. However, my results are similar to his in that contestants expend more effort in response to the information asymmetry.

In Appendix B I consider an incumbency advantage in which the incumbent has a strictly greater probability of obtaining the prize for any vector of effort levels. Interestingly, I find that the effect on aggregate effort expenditures over the two periods is not monotonic in the magnitude of this "status quo bias." Indeed, regardless of whether effort expenditures are a social good or bad, the optimal status quo bias is positive. This approach has not been considered in the literature. The closest paper is Baik and Lee [3], which considers a contest where contestants can carry a portion of their effort in an early contest on to a final contest. They find that total effort levels increase in response to this carry-over. Their findings were generalized in Lee [33]. Schmitt et al. [44] show that this kind of carry-over will not change aggregate effort in a repeated contest, although it will shift effort towards early rounds.

Also in Appendix B, I consider a model in which the incumbent enjoys a cost advantage. I find that aggregate effort expenditures increase as a result. In a closely related paper, Mehlum and Moene [36] show that, if the incumbent in an infinitely repeated imperfectly discriminating contest has an inheritable cost advantage over its rival, the effort level of both contestants rises in any given period. In their model, information is complete.

MODEL

There are two periods $t = 1, 2$. In each of these periods a set of risk neutral contestants $\mathbf{N} = \{1, 2, ..., n\}$ compete for a prize with a common value. The value in period t is a realization of the random variable V_t, where V_1 and V_2 are both distributed according to the absolutely continuous distribution function F_V, with support contained in $[\underline{v}, \infty)$ with $\underline{v} > 0$. The expected value of $V_t = E(V)$. This distribution function is commonly known. In period t each contestant $i \in \mathbf{N}$ expends unrecoverable effort, $x_{it} \in \mathbb{R}_+$ at a cost of $C_i(x_{it}) = x_{it}$ in an effort to obtain the prize, v_t. These effort levels are chosen simultaneously. Contestants are not budget constrained; the strategy space of each player is $\mathbb{R}^2_+$. The vector of effort levels in period t is $\mathbf{x}_t \equiv \{x_{1t}, x_{2t}, ..., x_{nt}\}$. Further, $\mathbf{x}_{-it} \equiv \mathbf{x}_t \backslash x_{it}$ and $\mathbf{N}_{-i} \equiv \mathbf{N} \backslash i$.

The function $p_{it} : \mathbb{R}^n_+ \rightarrow [0, 1]$ maps $\mathbf{x}_t$ into the probability that contestant i will receive the good in period t. This function, which is typically called the contest success function, is given by

$$p_{it}(x_{it}, \mathbf{x}_{-it}) = \begin{cases} \frac{x_{it}}{x_{it} + \sum\limits_{j \in \mathbf{N}_{-i}} x_{jt}} & \text{if} \quad \max \mathbf{x}_t \neq 0 \\ b_i & \text{if} \quad \max \mathbf{x}_t = 0, \end{cases}$$

where $b_i \in [0, 1]$ for any i and $\sum_{i \in \mathbf{N}} b_i \leq 1$. Note that b_i is the probability that player i receives v_t when none of the contestants expend positive effort in t. Different applications suggest different assumptions regarding $\mathbf{b} \equiv \{b_1, b_2, ..., b_n\}$. Two common assumptions are $b_i = \frac{1}{n}, \forall i \in \mathbf{N}$ or that $b_i = 0, \forall i \in \mathbf{N}$. However, the choice of $\mathbf{b}$ does not affect the following results. This contest success function is a special case of the class axiomized in Skaperdas [45] and defines what is sometimes called a lottery contest because the probability that a contestant obtains the good is her proportion of total effort, as in a lottery.

Contestants in period t do not observe the value of v_t before choosing x_{it}. At the conclusion of period t, one of the contestants receives the prize, and privately observes v_t. As such, before contestants choose their effort expenditures in $t = 2$ the contestant who received the prize in $t = 1$ (the incumbent) holds private information, while the remaining contestants (the challengers) hold only public information. The incumbent is denoted as contestant I. The set of contestants who did not obtain the prize in $t = 1$, the challengers, is $\mathbf{C} \equiv \mathbf{N}/I$. $\mathbf{C}_{-j} \equiv \mathbf{N}_{-j} \cap \mathbf{C}$ is the set of challengers that does not include contestant j and $\mathbf{x}_C \equiv \{x_{j2} : j \in \mathbf{C}\}$ is the vector of effort levels chosen by the challengers.

Intertemporal Independence of Values (IIV)

Consider the case in which v_1 and v_2 are independent draws from F_V. In this case $E(V_2 \mid v_1) = E(V)$; the incumbent's private observation of v_1 does not provide information of strategic importance in $t = 2$. Thus, this game is a twice repeated contest in which the outcome in $t = 1$ does not affect the symmetry of contestants in $t = 2$. This case provides a benchmark against which an incumbency advantage can be compared.

The analysis of the incumbents problem is identical to that of a challenger. The analysis begins in $t = 2$, where contestant i's expected utility is

$$U_{i2}^{IIV} \equiv \int_{\underline{v}}^{\infty} p_{i2}(x_{i2}, \mathbf{x}_{-i2}) v_2 dF_V(v_2) - x_{i2}.$$

This objective function is strictly concave in x_{i2} given $\mathbf{x}_{-it}$, so the first order condition defines a best response. This first order condition is

$$\frac{E(V) \sum\limits_{j \in \mathbf{N}_{-i}} x_{j2}}{\left(x_{i2} + \sum\limits_{j \in \mathbf{N}_{-i}} x_{j2}\right)^2} - 1 = 0.$$

Note that there is no best response to $\sum_{j \in \mathbf{N}_{-i}} x_{j2} = 0$; for any $x_{i2} > 0$ contestant i obtains the prize with certainty, but has an incentive to reduce x_{i2} to a smaller positive number. As such, the best response function of contestant i is well defined on the interval $(0,\infty)$, and is given by

$$x_{i2}\left(\mathbf{x}_{-i2}\right)=\begin{cases}\sqrt{\sum\limits_{j\in\mathbf{N}_{-i}} x_{j2}E\left(V\right)}-\sum\limits_{j\in\mathbf{N}_{-i}} x_{j2} & \text{if} \quad \sum\limits_{j\in\mathbf{N}_{-i}} x_{j2}\in\left(0,E\left(V\right)\right] \\ 0 & \text{if} \quad \sum\limits_{j\in\mathbf{N}_{-i}} x_{j2}\in\left(E\left(V\right),\infty\right).\end{cases}$$

The well-known, unique[2] equilibrium is symmetric, and $\forall i\in\mathbf{N}$ expends

$$x_{i2}^{IIV}\equiv\frac{E\left(V\right)\left(n-1\right)}{n^{2}}.$$

Note that x_{i2}^{IIV} is decreasing in n, and limits to zero. Denoting equilibrium aggregate effort expenditures in period t of the IIV case as R_{t}^{IIV}, $R_{2}^{IIV}=\sum_{i\in\mathbf{N}}x_{i2}^{IIV}=\frac{E(V)(n-1)}{n}$ which is strictly less than $E\left(V\right)$ and increasing in n. Note that $\lim_{n\to\infty}R_{2}^{IIV}=E\left(V\right)$. Aggregate effort expenditures in imperfectly discriminating contests are often referred to as rent dissipation, a reference to rent seeking applications in which effort expenditures are a social bad.

The equilibrium expected utility of contestant i in $t=2$ is

$$E\left(U_{i2}^{IIV}\right)=\int\limits_{\underline{v}}^{\infty}\frac{x_{i2}^{IIV}}{\sum\limits_{j\in\mathbf{N}}x_{j2}^{IIV}}v_{2}dF_{V}\left(v_{2}\right)-x_{i2}^{IIV}=\frac{E\left(V\right)}{n^{2}}.$$

Note that $E\left(U_{i2}^{IIV}\right)$ is decreasing in n and that $\lim_{n\to\infty}E\left(U_{i2}^{IIV}\right)=0$. Contestants have positive expected utility, despite not holding any private information. This is attributable to the functional form of the contest success function, in which the highest effort level does not win with certainty, which induces contestants to expend less effort than $E\left(V\right)$. Since this equilibrium is symmetric, each of the contestants

[2] See, for example, Yamazaki [49].

has an equal chance of obtaining the prize.

In $t = 1$ contestant i's expected utility is

$$U_{i1}^{IIV} \equiv \int_{\underline{v}}^{\infty} p_{i1}\left(x_{i1}, \mathbf{x}_{-i1}\right) v_1 dF_V(v_1) - x_{i1} + E\left(U_{i2}^{IIV}\right).$$

Since $E\left(U_{i2}^{IIV}\right)$ does not depend on $\mathbf{x}_1$ or v_1, strategic considerations in $t = 1$ are identical to those in $t = 2$, and the equilibrium effort of contestant, x_{i1}^{IIV} is identical to that found in $t = 2$. That is, $x_{i1}^{IIV} = x_{i2}^{IIV}$, which also implies that $R_1^{IIV} = R_2^{IIV}$ and that $E\left(U_{i1}^{IIV}\right) = E\left(U_{i2}^{IIV}\right)$. Further, each of the contestants has an equal chance of obtaining the prize.

The sum of equilibrium effort expenditures across $t = 1, 2$, is

$$R^{IIV} \equiv \sum_{t=1}^{2} R^{IIV} = \frac{2E\left(V\right)\left(n-1\right)}{n}. \tag{1}$$

Note that R^{IIV} is increasing in n. Further $\lim_{n\to\infty} E\left(U_{i2}^{IIV}\right) = 2E\left(V\right)$.

Notice that if contestants observe the value of the prize in either or both contests prior to choosing their effort expenditures, the ex ante results are unchanged. In particular, if all contestants observe the realization of v_t, it is easy to show that the equilibrium effort level, x_{it}^{INF}, is

$$x_{it}^{INF} = \frac{v_t\left(n-1\right)}{n^2}.$$

Since $E\left(x_{it}^{INF}\right) = x_{it}^{IIV}$, ex ante, the equilibrium predictions of the IIV case are identical to the case in which contestants are symmetrically informed.

Intertemporal Dependence of Values (IDV)

Consider the case in which V_2 is positively regression dependent on V_1. Positive regression dependence dictates that $P(V_2 \leq v_2 \mid V_1 = v_1)$ be non-increasing in v_1 for all v_2.[3] Intuitively, positive regression dependence implies that as v_1 increases, the probability that V_2 will be large increases. Positive regression dependence is a strictly weaker concept of positive dependence than affiliated random variables, which is used extensively in auction theory; affiliation implies positive regression dependence.[4] Thus, the following results are also implied by affiliation between V_1 and V_2. Recall that the marginal distributions of V_1 and V_2 are identical, and equal to F_V. V_1 and V_2 are jointly distributed with the joint density function $f(v_1, v_2)$. The absolutely continuous joint distribution function of these random variables is $F(v_1, v_2)$. The distribution function of V_2, conditional on V_1, is $F(v_2 \mid v_1)$. Since V_2 is positively regression dependent on V_1, $F(v_2 \mid v_1)$ is non-increasing in v_1 for any v_2. To ensure that $E(V_2 \mid v_1)$ is strictly increasing in v_1, I assume that for $v_1' > v_1$, $F(v_2 \mid v_1') < F(v_2 \mid v_1)$ for at least one $v_2 \in [\underline{v}, \infty)$.

In $t = 2$ the incumbent has observed v_1, which provides information regarding v_2 in the form of $E(V_2 \mid v_1)$. This introduces asymmetric information into the contest in $t = 2$; the incumbent holds private information which allows her to form an updated expectation regarding v_2, while the challengers hold only public information. The information structure of the subgame in $t = 2$ is studied in Wärneryd [48], with $n = 2$ and a perfectly informed contestant. What follows generalizes those results since the informed contestant (the incumbent) need not be perfectly informed of v_2 and there are $n \geq 2$ contestants.

As above, the incumbent is denoted as contestant I. The set of contestants who

[3] See Lehmann [34].

[4] For proof of this implication, see Yanagimoto [50]. This is also shown in de Castro [13].

did not win the prize in $t = 1$, the challengers, is $\mathbf{C} \equiv \mathbf{N}/I$. $\mathbf{C}_{-j} \equiv \mathbf{N}_{-j} \cap \mathbf{C}$ is the set of challengers that does not include contestant j and $\mathbf{x}_C \equiv \{x_{j2} : j \in \mathbf{C}\}$ is the vector of effort levels chosen by the challengers. The incumbent's expected utility now depends on the privately observed v_1, and is given by

$$U_{I2}^{IDV}(v_1) \equiv \int_{\underline{v}}^{\infty} p_{I2}(x_{I2}(v_1), \mathbf{x}_C) v_2 dF(v_2 \mid v_1) - x_{I2}(v_1).$$

This expected utility is strictly concave in $x_{I2}(v_1)$, given $\mathbf{x}_C$ such that the first order condition is sufficient to establish a maximum. The partial derivative with respect to $x_{I2}(v_1)$ is

$$\frac{\sum\limits_{j\in\mathbf{C}} x_{j2}}{\left(x_{I2}(v_1) + \sum\limits_{j\in\mathbf{C}} x_{j2}\right)^2} E(V_2 \mid v_1) - 1.$$

Any $x_{I2}(v_1) \geqslant 0$ renders this expression negative if $\sum_{j\in\mathbf{C}} x_{j2} > E(V_2 \mid v_1)$. Thus, if the summed effort of the challengers is greater than the incumbent believes the prize is worth, the incumbent's best response is to expend no effort.. If $\sum_{j\in\mathbf{C}} x_{j2} \leq E(V_2 \mid v_1)$ then there exists a $x_{I2}(v_1) \geqslant 0$ for which the partial derivative is equal to zero. Since $E(V_2 \mid v_1)$ is strictly monotonically increasing in v_1, $\sum_{j\in\mathbf{C}} x_{j2} \leq E(V_2 \mid v_1)$ will hold with equality for exactly one v_1 if $\sum_{j\in\mathbf{C}} x_{j2} \geq E(V_2 \mid \underline{v})$. Thus, if $\sum_{j\in\mathbf{C}} x_{j2} \geq E(V_2 \mid \underline{v})$, then the expression $\sum_{j\in\mathbf{C}} x_{j2} = E(V_2 \mid v_1)$ defines a threshold value of v_1 above which the incumbent will expend positive effort. Since of $E(V_2 \mid v_1)$ is monotonic in v_1, its inverse, $s(\cdot)$, is well defined on $[E(V_2 \mid \underline{v}), \infty)$, and the threshold value of v_1 that the challenger must observe in order for $x_{I2}(v_1) \geq 0$

to be a best response to $\sum_{j\in\mathbf{C}} x_{j2}$ is

$$q\left(\sum_{j\in\mathbf{C}} x_{j2}\right) \equiv \begin{cases} s\left(\sum\limits_{j\in\mathbf{C}} x_{j2}\right) & \text{if} \quad \sum\limits_{j\in\mathbf{C}} x_{j2} \geq E\left(V_2 \mid \underline{v}\right) \\ \underline{v} & \text{if} \quad \sum\limits_{j\in\mathbf{C}} x_{j2} < E\left(V_2 \mid \underline{v}\right). \end{cases}$$

The best response function of the incumbent, which is defined on the domain $(0, \infty)$, can then be expressed as

$$x_{I2}\left(v_1\right) = \begin{cases} \sqrt{\sum\limits_{j\in\mathbf{C}} x_{j2} E\left(V_2 \mid v_1\right)} - \sum\limits_{j\in\mathbf{C}} x_{j2} & \text{if} \quad q\left(\sum\limits_{j\in\mathbf{C}} x_{j2}\right) \leq v_1 \\ 0 & \text{if} \quad q\left(\sum\limits_{j\in\mathbf{C}} x_{j2}\right) > v_1. \end{cases}$$

In equilibrium, the ex ante expected effort expenditure of the IDV incumbent is denoted as $E\left(x_{I2}^{IDV}\left(V_1\right)\right)$.

The expected utility of contestant $j \in \mathbf{C}$ is

$$U_{j2}^{IDV} \equiv E\left(\int\limits_{\underline{v}}^{\infty} \frac{x_{j2} V_2}{x_{I2}\left(V_1\right) + x_{j2} + \sum\limits_{k\in\mathbf{C}_{-j}} x_{k2}}\right) - x_{j2}.$$

As before, the strict concavity of this objective function in x_{j2} given $\mathbf{x}_{-i2}$ implies that the first order condition yields a maximum. This first order condition is

$$E\left(\frac{\left(x_{I2}\left(V_1\right) + \sum\limits_{k\in\mathbf{C}_{-j}} x_{k2}\right) V_2}{\left(x_{I2}\left(V_1\right) + x_{j2} + \sum\limits_{k\in\mathbf{C}_{-j}} x_{k2}\right)^2}\right) - 1 = 0.$$

The $(n-1)$ challengers each expend the same quantity of effort in equilibrium. To see this, consider the case in which contestant $m \in \mathbf{C}$ optimally expends $x_{m2} > 0$ while contestant $l \in \mathbf{C}$ optimally expends $x_{l2} > x_{m2}$ Since $x_{l2} > x_{m2} > 0$ the first order conditions for contestants l and m hold with equality such that

$$E\left(\frac{\left(x_{I2}\left(V_1\right) + \sum_{k\in\mathbf{C}/\{l,m\}} x_{k2} + x_{l2}\right) V_2}{\left(x_{I2}\left(V_1\right) + \sum_{k\in\mathbf{C}} x_{k2}\right)^2}\right) = E\left(\frac{\left(x_{I2}\left(V_1\right) + \sum_{k\in\mathbf{C}/\{l,m\}} x_{k2} + x_{m2}\right) V_2}{\left(x_{I2}\left(V_1\right) + \sum_{k\in\mathbf{C}} x_{k2}\right)^2}\right).$$

But this is a contradiction since $x_{l2} > x_{m2}$. Thus, if challengers are optimally expending a positive amount of effort, they each expend the same amount of effort. Likewise, the case in which one of the challengers is optimally expending zero effort implies that this is the optimal choice for the remaining challengers as well. The $(n-1)$ challengers can not expend zero effort in an equilibrium, since the best response of the incumbent does not exist when $\sum_{j\in\mathbf{C}} x_{j2} = 0$.

The equilibrium effort of a challenger in the IDV case is denoted by x_{C2}^{IDV}, and the sum of the challengers' effort expenditures is equal to $x_{C2}^{IDV}(n-1)$. Utilizing the incumbent's best response function simplifies the first order condition of a challenger. The resulting equation relates the equilibrium effort level of a challenger to the expected equilibrium effort level of the incumbent, where $\mathbf{1}_B$ is the indicator function that is equal to one if B is true, and zero otherwise,

$$\begin{aligned} x_{C2}^{IDV} &= \left(\frac{1}{\left(1 + F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right)(n-2)\right)}\right) E\left(x_{I2}^{IDV}\left(V_1\right)\right) \\ &\quad + \left(\frac{n-2}{(n-1)\left(1 + F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right)(n-2)\right)}\right) E\left(V_2 \mathbf{1}_{V_1 \le q\left(x_{C2}^{IDV}(n-1)\right)}\right). \end{aligned} \tag{2}$$

Note that this is not a closed form solution for x_{C2}^{IDV}, as it appears on both sides of the equation. Plugging in the best response function of the incumbent and simplifying

(2) further yields the following equation, which characterizes equilibrium in $t = 2$

$$\begin{aligned} & n - F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right) \\ = & \left(\frac{(n-2)}{x_{C2}^{IDV}(n-1)}\right) E\left(V_2 \mathbf{1}_{V_1 \leq q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\ & + \sqrt{\frac{(n-1)}{x_{C2}^{IDV}}} E\left(\sqrt{E(V_2 \mid V_1)} \mathbf{1}_{V_1 \geq q\left(x_{C2}^{IDV}(n-1)\right)}\right). \end{aligned} \tag{3}$$

Consider the special case where $x_{C2}^{IDV}(n-1) < E(V_2 \mid \underline{v})$. In this case there is no v_1 for which the incumbent believes the challengers are expending more effort than the prize is worth and $x_{I2}(v_1) > 0$ for any v_1. Following Wärneryd (2003), I call this an interior equilibrium. In such a situation (2) and (3) become

$$E\left(x_{I2}^{IDV}(V_1)\right) = x_{C2}^{IDV},$$

$$x_{C2}^{IDV} = \frac{(n-1)\left(E\left(\sqrt{E(V_2 \mid V_1)}\right)\right)^2}{n^2}.$$

Thus, if $x_{C2}^{IDV}(n-1) \leq E(V_2 \mid \underline{v})$, then there is an explicit solution for the equilibrium of this subgame. A sufficient condition for the existence of such an interior equilibrium is

$$\left(\frac{(n-1) E\left(\sqrt{E(V_2 \mid V_1)}\right)}{n}\right)^2 \leq E(V_2 \mid \underline{v}).$$

This sufficient condition restricts attention to a narrow set of distribution functions, and a more general result is desirable.

If $x_{C2}^{IDV}(n-1) > E(V_2 \mid \underline{v})$ the incumbent does not expend positive effort for some realizations of v_1. Consequently, there is no closed form solution for equilibrium. Furthermore, since the best response function of the incumbent is not defined at $\sum_{j \in \mathbf{C}} x_{j2} = 0$, the Banach fixed point theorem cannot be utilized to guarantee the existence or uniqueness of equilibrium in this subgame. However, the following result holds.

Proposition 1 *There is a unique Nash equilibrium in $t = 2$ of the IDV case.*

Proof. See Appendix A. ■

If the unique equilibrium is interior, then $E\left(x_{I2}^{IDV}(V_1)\right) = x_{C2}^{IDV}$, and $x_{I2}^{IDV}(v_1) > 0$, for all v_1. When the equilibrium is not interior, there are values of v_1 for which the incumbent will not expend any effort, which might suggest that a lack of an interior equilibrium would depress $E\left(x_{I2}^{IDV}(V_1)\right)$ relative to x_{C2}^{IDV}. Accordingly, the expected effort expenditure of the incumbent relative to a challenger is of interest. The following result refutes the line of thinking outlined above.

Proposition 2 *In the IDV case, the ex ante expected effort expenditure of the incumbent is weakly greater than that of a challenger. If $n = 2$, or there is an interior equilibrium, the incumbent's ex ante expected effort level is equal to that of a challenger, otherwise the inequality is strict.*

Proof. See Appendix A. ■

The intuition behind this result relies on the fact that the incumbent's best response function is increasing in v_1; she expends less effort than a challenger when v_1 is low, and more when v_1 is high. Consequently, a challenger is more likely to obtain the prize when v_1 is low, so that the expected value of the prize conditional on having been obtained by a challenger is lower than $E(V)$. Challengers reduce their effort expenditures relative to the incumbent to account for this. When the equilibrium is not interior incumbents do not expend any effort for low values of v_1 so that a challenger obtains the prize with certainty, providing challengers a stronger incentive to reduce their effort expenditures than in an interior equilibrium. That is, the presence of asymmetric information introduces a winner's curse for challengers, in which obtaining the prize depresses a challengers beliefs regarding its worth. A similar winner's curse arises in a first-price, sealed-bid auction with the $t = 2$ IDV information structure.[5]

The lottery contest success function utilized in this model awards the prize to a contestant with probability equal to her proportion of aggregate effort expenditures in the contest. Since $E\left(x_{I2}^{IDV}\left(V_1\right)\right) \geq x_{C2}^{IDV}$, the incumbent has, ex ante, the (weakly) highest proportion of aggregate effort. Recall that $E\left(x_{I2}^{IDV}\left(V_1\right)\right) > x_{C2}^{IDV}$ when the equilibrium in not interior and $n = 2$. As such, the following result is somewhat counterintuitive.

In equilibrium the incumbent will expend more effort than a challenger when v_1 is high, such that, ex ante, she is expected to expend more than a challenger, despite choosing $x_{I2}\left(v_1\right) = 0$ if $v_1 \leq q\left(x_{C2}^{IDV}\left(n-1\right)\right)$. Further, in equilibrium the incumbent obtains the prize with positive probability only when $v_1 > q\left(x_{C2}^{IDV}\left(n-1\right)\right)$. Thus, there are two effects influencing the ex ante probability of the incumbent obtaining the good in $t = 2$. I find the following, which holds in an interior equilibrium as well.

Proposition 3 *In the IDV case the incumbent's ex ante expected probability of obtaining the prize is strictly less than that of a challenger.*

Proof. See Appendix A. ■

[5] See Engelbrecht-Wiggans et al. [16] and Milgrom and Weber [38].

This result yields an interesting insight into the effect of an informed incumbent. In particular, incumbents are less entrenched under this informational asymmetry than in the IIV case; the incumbent is ex ante less likely to obtain the prize in $t = 2$. The intuition is that, in equilibrium the incumbent obtains the prize with positive probability only when $v_1 > q\left(x_{C2}^{IDV}(n-1)\right)$. Further, since the incumbent only expends more effort than a challenger when $x_{C2}^{IDV} \leq (n-1) E(V_2 \mid v_1)/n^2$, a challenger will obtain the prize with high probability when v_1 is low.

Contrasting this result with the analogous findings in standard auction formats is worthwhile. As mentioned above, the information structure in $t = 2$ of the IDV case has been studied in the context of first-price sealed-bid auctions in Engelbrecht-Wiggans et al. [16] and in the context of all-pay auctions in Chapter IV. In both of these auction formats, the ex ante probability that the informed bidder wins the auction is 50%, regardless of the number of bidders.

To ascertain the effect of the assumption that V_2 is positive regression dependant on V_1, consider the equilibrium effort expenditure of challengers in the IDV case to that of contestants in $t = 2$ of the IIV case. If the equilibrium is interior, notice that Jensen's Inequality yields

$$
\begin{aligned}
E\left(x_{I2}^{IDV}(V_1)\right) &= x_{C2}^{IDV} \\
&= \frac{(n-1)\left(E\left(\sqrt{E(V_2 \mid V_1)}\right)\right)^2}{n^2} \\
&< \frac{(n-1)E(V)}{n^2} \\
&= x_{i2}^{IIV}.
\end{aligned}
$$

Since $E\left(x_{I2}^{IDV}(V_1)\right) = x_{C2}^{IDV} < x_{i2}^{IIV}$, the expected revenue of such an interior equilibrium, R_2^{IDV}, is strictly less than R_2^{IIV}. Since, in general, there is not a closed form solution for the equilibrium in t=2 of the IDV case, comparisons between the IDV and IIV cases are not straightforward. However, a necessary and sufficient condition under which an IDV challenger will expend less effort than an IIV challenger exists. A more general result follows.

Proposition 4 *The equilibrium effort expenditure of a contestant in $t=2$ of the IIV case is strictly greater than the equilibrium effort expenditure of a challenger in $t=2$ of the IDV case if and only if*

$$\begin{aligned} & \frac{(n-2)}{(n-1)(n-F_V(q(B)))} E\left(V_2 1_{V_1 \leq q(B)}\right) \\ & + \frac{(n-1)\sqrt{E(V)}}{n(n-F_V(q(B)))} E\left(\sqrt{V_2} 1_{V_1 \geq q(B)}\right) \\ < \; & \frac{E(V)(n-1)}{n^2}, \end{aligned} \tag{4}$$

where $B \equiv \frac{E(V)(n-1)^2}{n^2} = x_{i2}^{IIV}(n-1)$.

Proof. See Appendix A. ■

Note that (4), which holds trivially when $n=2$, states that if the IDV incumbent were to best respond to the equilibrium strategy of the challengers in the IIV case ($\sum_{j \in \mathbf{C}} x_{j2} = x_{i2}^{IIV}(n-1)$), then the best response of the IDV challengers is to reduce their effort expenditures relative to the IIV case. Suppose the IDV challengers expend $\sum_{j \in \mathbf{C}} x_{j2} = x_{i2}^{IIV}(n-1)$. Since, in this scenario, the incumbent's equilibrium effort expenditure is monotonically increasing in v_1 when $v_1 \geq q\left(x_{i2}^{IIV}(n-1)\right)$, and she expends more effort than x_{i2}^{IIV} only when $x_{i2}^{IIV} \leq (n-1)E(V_2 \mid v_1)/n^2$, a challenger who expends x_{i2}^{IIV} is more likely to obtain the prize when it has a low value. As discussed above, the expected value of the prize, conditional on a challenger having obtained it, is then less than $E(V)$. As such, it is reasonable to assume that risk-neutral challengers shade their effort levels below x_{i2}^{IIV}, as required by (4). It is important to note that (4) is not a restrictive assumption; for example the Pareto, Gamma, Uniform and Triangular distributions all satisfy if for a broad range of parameterizations. In what follows, I assume that (4) is satisfied.

Interestingly, the comparison between $E\left(x_{I2}^{IDV}(V_1)\right)$ and x_{i2}^{IIV} depends on n and the distribution function F_V. If there is an interior equilibrium or if $n = 2$, then $E\left(x_{I2}^{IDV}(V_1)\right) = x_{C2}^{IDV} < x_{i2}^{IIV}$. When the equilibrium in not interior and $n > 2$, the incumbent expends $E\left(x_{I2}^{IDV}(V_1)\right) > x_{i2}^{IIV}$ when

$$\begin{aligned} x_{i2}^{IIV} - x_{C2}^{IDV} &< (n-2)\, x_{C2}^{IDV} F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right) \\ &\quad - \frac{(n-2)}{(n-1)} E\left(V_2 1_{V_1 \leq q\left(x_{C2}^{IDV}(n-1)\right)}\right). \end{aligned}$$

Since the equilibrium is not interior if $E\left(x_{I2}^{IDV}(V_1)\right) > x_{i2}^{IIV}$, and there is no closed form solution for such an equilibrium, I am unable to give further conditions. However, examples demonstrate that $E\left(x_{I2}^{IDV}(V_1)\right) > x_{i2}^{IIV}$ in many cases. For example, if $V_1 = V_2 \sim U(1, 11)$, and $n = 200$, then $E\left(x_{I2}^{IDV}(V_1)\right) = 0.79$, while $x_{i2}^{IIV} = 0.03$.

As mentioned above, the information structure in $t = 2$ of the IDV case has been studied in a variety of auction formats. Engelbrecht-Wiggans et al. [16] finds that this asymmetric information structure guarantees that the uninformed bidders have expected payoff of zero in any equilibrium of any standard auction format. Further, in all-pay and first-price auctions, the informed bidder earns a positive information rent. Since the expected payoff off bidders in the symmetric information structure in which no bidders hold private information is zero (as in the IIV case), this information rent is extracted from the seller.[6]

Comparing the ex ante expected utility of contestants in $t = 2$ of the IIV and IDV

[6] See Chapter IV and Milgrom and Weber [38].

cases is of interest as it reveals the effect of information asymmetry. Additionally, comparing these results to those found in all-pay and first-price auctions yields insight into the effect of utilizing an imperfectly discriminating contest success function. Note that the expected utility of a contestant in $t = 2$ of the IIV case is $E\left(U_{i2}^{IIV}\right) = E\left(V\right)/n^2 > 0$, whereas in the analogous first-price or all-pay auction her expected utility would be zero.[7] This is attributable to the imperfectly discriminating nature of the lottery contest considered.

Notice that, in equilibrium, the expected utility of a challenger in the IDV case can be written as

$$\begin{aligned} E\left(U_{C2}^{IDV}\right) &= \frac{1}{(n-1)^2} E\left(V_2 \mathbf{1}_{V_1 \le q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\ &\quad + \frac{x_{C2}^{IDV}\left(1 - F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right)\right)}{(n-1)}. \end{aligned}$$

Since $x_{C2}^{IDV} > 0$ in equilibrium, $E\left(U_{C2}^{IDV}\right) > 0$. Since the expected utility of uninformed bidders in all-pay auctions is zero, the imperfectly discriminating contest success function allows IDV challengers to earn a positive expected utility, despite the information asymmetry. While the presence of asymmetric information does not reduce $E\left(U_{C2}^{IDV}\right)$ to zero, I have the following result.

Proposition 5 *If (4) is satisfied, then the ex ante expected utility of a challenger is strictly less in the IDV case than the IIV case.*

Proof. See Appendix A. ■

In contrast to the aforementioned results in all-pay and first-price auctions, an information asymmetry makes the challengers worse off. Notice that while bidders

[7] See Baye et al. [5] for an analysis of all-pay auctions under complete information.

who do not observe a signal regarding the value of the good in an all-pay or first price auction are indifferent between the information structures in $t = 2$ of the IDV and IIV case, the same is not true in the lottery contest.

Next, I look at the expected utility of the incumbent. Utilizing (3) and the best response function of the incumbent, the ex ante equilibrium expected utility of the incumbent can be written as

$$\begin{aligned} E\left(U_{I2}^{IDV}\right) &= E\left(V\right) + \frac{(n-3)}{(n-1)} E\left(V_2 \mathbf{1}_{V_1 \leq q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\ &\quad - x_{C2}^{IDV}\left((n+1) + F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right)(n-3)\right). \end{aligned}$$

I can now say the following.

Proposition 6 *If (4) is satisfied the ex ante expected utility of the incumbent is strictly greater in the IDV case than in the IIV case.*

Proof. See Appendix A. ■

The IDV incumbent earns a positive information rent. Since the challengers are ex ante worse off in the IDV case, at least some of this information rent is extracted from them. The effect of the information asymmetry on aggregate effort expenditures in $t = 2$ of the IDV case is closely related since for any v_1, it must be the case that the sum of effort expenditures and realized payoffs of the contestants equal $E(V_2 \mid v_1)$. In expectation, $E\left(U_{I2}^{IDV}\right) + E\left(U_{C2}^{IDV}\right)(n-1) + R_2^{IDV} = E\left(V\right)$. As such, $E\left(U_{I2}^{IDV}\right) + E\left(U_{C2}^{IDV}\right) > 2E\left(U_{i2}^{IIV}\right)$, would indicate that $R_2^{IDV} < R_2^{IIV}$. The following result establishes this.

Proposition 7 *If (4) is satisfied, ex ante expected effort expenditures are strictly lower in $t = 2$ of the IDV case than in $t = 2$ of the IIV case.*

Proof. See Appendix A. ■

This result shows that the information rent earned by the IDV incumbent is extracted from the challengers, and by reducing aggregate effort expenditures in $t = 2$. The ex ante expected value of obtaining the prize in $t = 1$ is then $E(V) + E\left(U_{I2}^{IDV}\right) - E\left(U_{C2}^{IDV}\right) > E(V)$. Thus, contestants in $t = 1$ of the IDV case have an increased incentive to obtain the prize.

In $t = 1$ the n contestants are symmetric. None of them hold private information, although they are aware that privately observing v_1 will, in expectation earn them an information rent. The expected utility of contestant i in $t = 1$ is

$$\begin{aligned} U_{i1}^{IDV} \equiv\ & p_{i1}\left(x_{I2}\left(v_1\right), \mathbf{x}_C\right)\left(E(V) + E\left(U_{I2}^{IDV}\right) - E\left(U_{C2}^{IDV}\right)\right) \\ & -x_{i1} + E\left(U_{C2}^{IDV}\right) \end{aligned}$$

This problem is strategically equivalent to a complete information contest with a prize of $E(V) + E\left(U_{I2}^{IDV}\right) - E\left(U_{C2}^{IDV}\right)$. As in the IIV case, there is a unique equilibrium which is symmetric. The equilibrium effort expenditure of contestant i in $t = 1$ is

$$x_{i1}^{IDV} \equiv \frac{\left(E(V) + \left(E\left(U_{I2}^{IDV}\right) - E\left(U_{C2}^{IDV}\right)\right)\right)(n-1)}{n^2}$$

$\forall i \in \mathbf{N}$.

Since $E\left(U_{I2}^{IDV}\right) - E\left(U_{C2}^{IDV}\right) > 0$, $x_{i1}^{IDV} > x_{i1}^{IIV}$, which implies that $R_1^{IDV} > R_1^{IIV}$. The sum of ex ante expected effort expenditures across both periods is $R^{IDV} \equiv \sum_{t=1}^{2} R_t^{IDV}$. Since, $R_2^{IDV} < R_2^{IIV}$, the effect of the information asymmetry on total effort expenditures across the two periods is of interest.

Proposition 8 *When the equilibrium in $t = 2$ of the IDV case is not interior , total* *effort expenditures in the IDV case, R^{IDV}, strictly exceed those of the IIV case. If the equilibrium in $t = 2$ of the IDV case is interior then $R^{IDV} = R^{IIV}$.*

Proof. See Appendix A. ∎

It is worth noting that if the game were modified such that in $t = 1$, contestants were to compete for the chance to privately observe v_1 without obtaining it, that this result holds. That is, if the contest in $t = 1$ is over the acquisition of information, the result is the same. Notice that if $\underline{v} = 0$, then there can not be an interior equilibrium, and $R^{IDV} > R^{IIV}$. Since $R^{IDV} \geq R^{IIV}$, the reduction of effort expenditures in $t = 2$ of the IDV case, are at least offset by the increase in effort expenditures in $t = 1$. Interestingly, in a twice repeated first-price or all-pay auction, analogous to the IIV and IDV cases studied here, revenue summed across the two periods is, ex ante, unchanged between the two information structures. The intuition is that in $t = 2$ of an IDV information structure the uninformed bidders earn an expected payoff of zero, while the informed bidder earns a positive information rent. In $t = 1$ the value of winning the auction is this information rent plus $E(V)$. The revenue $t = 1$ is equal to this value, because the game in $t = 1$ is a complete information auction in which the equilibrium expected utility is equal to zero.

Further, $R^{IDV} \geq R^{IIV}$ implies that the ex ante expected utility of a contestant in $t = 1$ of the IDV case is (weakly) less than in the IIV case. As such, if a contestant were offered the choice between the information structures in the IDV and IIV case, she would weakly prefer the IIV case.

Recall that as n increases in the IIV case, aggregate effort expenditures increase in both periods, and so, overall. Likewise, the equilibrium expected utility of contestants is decreasing in n in both periods and overall. Also, the equilibrium probability of obtaining the prize in each period, $1/n$, is decreasing in n as well. Consider the effect of an increase in n on behavior in the IDV case. If the equilibrium in $t = 2$ of the IDV case is not interior, then the equilibrium is characterized by the implicit function (3). Totally differentiating (3) yields the following result.

Proposition 9 *The equilibrium effort expenditure of a challenger and of the incumbent in the IDV case is decreasing in n. The ex ante expected aggregate effort expenditures in $t = 2$ is increasing in n.*

Proof. See Appendix A. ■

In $t = 1$ of the IDV case the equilibrium is analogous to that of the IIV case, except with an expected value of obtaining the prize equal to $E(V) + \left(E\left(U_{I2}^{IDV}\right) - E\left(U_{C2}^{IDV}\right)\right)$. It is therefore straightforward to show that the comparative statics in $t = 1$ of the IDV case are consistent with those of the IIV case.

Contrasting this result with all-pay, first-price and second-price auctions reveals significant differences. In an asymmetric information structure as in IDV case, equilibrium bidding strategies and revenue predictions are invariant to the number of bidders in first-price, all-pay and second-price auctions. In an imperfectly discriminating contest, this is not the case.

Another interesting exercise is to vary the level of positive dependence between V_1 and V_2. The value of information has garnered considerable attention in the literature, mostly in the context of decision problems.[8] These results do not generalize to games, although the value of information in zero sum games has been, dealing with a finite partition of the state space has been studied. Unfortunately, this setup does not directly apply to this model.

[8] See Blackwell.[7].

However, the fact that there is a unique Nash equilibrium for the contest with asymmetric information suggests that comparing equilibrium payoffs under different information structures may yield results. Consider two information structures, defined by their joint distribution functions: $F(v_1, v_2)$ and $G(v_1, v_2)$ where these two distribution functions have identical marginals, namely F_V. Kimeldorf and Sampson [31] say that $G(v_1, v_2)$ is more positively quadrant dependent than $F(v_1, v_2)$ if $G(v_1, v_2) \geq F(v_1, v_2)$ for all $(v_1, v_2) \in \mathbb{R}^2$. In this positive dependence ordering, V_1, V_2 are more positively dependent under $G(v_1, v_2)$ than $F(v_1, v_2)$. Since the equilibrium need not be interior, comparing the equilibria under $G(v_1, v_2)$ than $F(v_1, v_2)$ yields ambiguous results. As such I am unable to give a general result regarding the effect of changes in the quality of signal.

I next introduce an example in which there is a particularly tractable way to vary the informativeness of v_1. In this example, $n = 2$, and the value of the prize in period two is uniformly distributed on $[\underline{v}, \overline{v}]$. It is also assumed that $\overline{v} < 7\underline{v}$. Let a second random variable, E, be uniformly distributed on $[-\delta, \delta]$, with $\delta > 0$. To ensure that δ is not so high as to render the signal devoid of information, it is also assumed that $\delta < \overline{v} - \underline{v}$. The signal that the incumbent receives is then $V_1 = V_2 + E$. Thus, the signal received by the incumbent must be within δ of the actual value of the prize. Examining how equilibrium effort changes in response to changes in δ is equivalent to observing the effect of changes in signal quality on equilibrium effort. Note that this example is not consistent with the model outlined above in that the distribution of V_1 is not the same as the distribution of V_2. However, it does yield some insight into how the quality of information affects equilibrium effort levels. Since $\overline{v} < 7\underline{v}$, the equilibrium is interior. The closed form of this equilibrium is

$$\begin{aligned} x_{C2}^{IDV} &= \frac{\left(E\left(\sqrt{E\left(V_2 \mid v_1\right)}\right)\right)^2}{4} \\ &= \frac{2\left(4\underline{v}^{\frac{5}{2}}+4\overline{v}^{\frac{5}{2}}-4\overline{v}^2\sqrt{\overline{v}-\delta}+3\overline{v}\sqrt{\overline{v}-\delta}\delta\right)}{15\left(\overline{v}-\underline{v}\right)\delta} \\ &\quad+\frac{2\left(4\underline{v}^2\sqrt{\underline{v}+\delta}-3\underline{v}\delta\sqrt{\underline{v}+\delta}+\delta^2\left(\sqrt{\overline{v}-\delta}+\sqrt{\underline{v}+\delta}\right)\right)}{15\left(\overline{v}-\underline{v}\right)\delta}. \end{aligned}$$

The partial derivative of this expression with respect to δ yields

$$\begin{aligned} \frac{\partial x_{C2}^{IDV}}{\partial\delta} &= \frac{8\overline{v}^2\sqrt{\overline{v}-\delta}-8\overline{v}^{\frac{5}{2}}+4\overline{v}\delta\sqrt{\overline{v}-\delta}-4\underline{v}\delta\sqrt{\underline{v}+\delta}}{15\left(\overline{v}-\underline{v}\right)\delta} \\ &\quad+\frac{8\underline{v}^2\sqrt{\underline{v}+\delta}-8\underline{v}^{\frac{5}{2}}+3\delta^2\left(\sqrt{\overline{v}-\delta}+\sqrt{\underline{v}+\delta}\right)}{15\left(\overline{v}-\underline{v}\right)\delta} \end{aligned}$$

This partial derivative is negative, so equilibrium effort levels increase as the quality of the signal decreases. Further, x_{C2}^{IDV} converges to x_{i2}^{IIV} as δ increases. Since $n = 2$, $x_{C2}^{IDV} = E\left(x_{I2}^{IDV}\left(V_1\right)\right)$; aggregate effort expenditures converge to R_2^{IIV}. This is consistent with the result that the presence of an information asymmetry decreases effort in the second period. As the value of this signal decreases, equilibrium effort levels get closer and closer to x_{i2}^{IIV}.

Next, consider the problem faced by the contest designer. Suppose that this contest designer can choose between two types of information revelation policies. First, she can publicly announce the value of the prize in contest, either before or after contestants have chosen their effort levels. Notice that, ex ante, both of these policies will result in expected equilibrium effort expenditures as in the IIV case. Second, she can privately reveal this value to the contestant who obtained it (the IDV case). If the contest designer seeks to minimize effort expenditures, then Proposition 8 implies that she will adopt a policy of publicly revealing the value of the prize before or after the contestants choose their effort levels. Adopting such a policy ensures that, ex ante, effort expenditures are expected to correspond to the IIV case. If the contest designer seeks to maximize effort expenditures she will choose to adopt a policy of privately revealing the value of the prize to the contestant who obtains the prize. Interestingly, this is the opposite of the predictions in a one-shot asymmetric information contest. As such, taking account of the incentives to acquire information is important when considering optimal information revelation policy. In rent seeking applications, this result offers support for the view that there is social benefit to public disclosure of information.

CHAPTER III

AN EXPERIMENTAL INVESTIGATION OF ASYMMETRIC INFORMATION IN COMMON-VALUE AUCTIONS

OVERVIEW

In much of the auction literature, bidders are assumed to be ex ante symmetrically informed. However, in many situations such an assumption is problematic. For example, in many auctions experienced dealers bid against non-dealers. In such an auction, it is natural to assume that dealers have more information than non-dealers; that is, bidders are asymmetrically informed.

One of the earliest and well known models analyzing auctions with asymmetrically informed bidders is found in Engelbrecht-Wiggans, Milgrom and Weber [16] (hereafter EMW). EMW derive the unique equilibrium of a first-price, common-value auction in which one of the bidders observes an informative signal regarding the realized common value of the object for sale.[9] The other bidders know only the joint distribution from which the signal and realized value are drawn, which is common knowledge. Thus, the uninformed bidders hold only public information, while the informed bidder holds private information. This information structure guarantees that, in equilibrium, the uninformed bidders have expected profits of zero, and the informed bidder has a positive expected profit. Further, this information asymmetry reduces the expected revenue of the auction relative to a symmetric information framework.

Several papers model information asymmetry in common-value auctions by varying the quality of information while allowing each bidder to hold private information.

[9] A correction to their proof of uniqueness is found in Dupra [14].

Hausch [22] and Campbell and Levin [8] show that less informed bidders earn positive expected profit in equilibrium, provided they hold some private information. Campbell and Levin [8] also demonstrate that a seller's expected revenue can benefit from an information asymmetry between the bidders.

This paper experimentally investigates the role of asymmetric information in two-bidder, first-price, sealed-bid, common-value auctions by varying the number of bidders who receive a signal regarding the value of the good prior to bidding. In one treatment bidders know only the distribution from which the value of the good is drawn; no bidder holds any private information. In another, each bidder observes a conditionally independent signal of the common value of the good. In the asymmetric information treatment, only one of the bidders receives such a signal; the other bidder holds no private information. Our asymmetric information treatment is theoretically analyzed in EMW.

We find several interesting results. First, bidders who observe a signal overbid relative to the Nash equilibrium prediction on average, regardless of whether or not the other bidder observes a signal. Conversely, bidders who do not observe a signal underbid relative to Nash equilibrium predictions on average. Indeed, when neither bidder observes a signal, the average bid is 42% below the predicted bid.

This result cannot be explained by risk aversion, since the degree of risk aversion required to induce such behavior is unreasonably large. Further, limited liability of losses does not explain this behavior, since the balance held by bidders is much more than the value of the good is able to be, even in later rounds. We interpret this result in terms of overconfidence. We suggest that providing bidders with a signal induces overconfidence. That is, bidders who observe a private signal become overconfident regarding the value of their signal and overbid accordingly. This exemplifies the hypothesis that "a little knowledge is a dangerous thing."[10]

The effect of an information asymmetry among bidders is ambiguous, because of the systematic overbidding of informed bidders, and underbidding of uninformed bidders. In particular, the effect of an information asymmetry depends on the symmetric information structure against which it is compared. We find that the revenue generated by an auction in which both bidders observe a signal is higher than when only one bidder observes a signal; the informed bidder earns a substantial information rent. However, the dramatic underbidding when no bidder is informed results in much lower revenue than predicted. This result is surprising, since this treatment is predicted to generate the highest revenue. That is, revenue is lowest when bidders hold no private information.

Observed bidder payoffs also deviate from theoretical predictions in interesting ways. In particular, when neither bidder observes a signal both bidders underbid significantly and, on average, earn a substantial payoff as a result.[11] Conversely, when both bidders observe a signal, bidders overbid relative to the Nash equilibrium, such that they earn less than theoretical predictions. Lastly, when only one bidder observes a private signal, the informed bidder earns a substantial information rent, despite overbidding relative to Nash predictions. This is because the uninformed bidder, on average, bids less than the expected value of the good. When the informed bidder observes a signal above this expected value, she can still win the auction by bidding substantially less than the expected value, and earn a significant payoff as a

[10] Alexander Pope first addressed this hypothesis by writing: "A little learning is a dangerous thing; drink deep, or taste not the Pierian spring: there shallow draughts intoxicate the brain, and drinking largely sobers us again."

[11] The only difference between auctions in which both bidders observe a private signal and auctions in which neither bidder observes a private signal is the information structure. Since bidders overbid when both bidders observe a signal, and underbid when both bidder do not observe a signal, the fact that bidders earn such a large payoff when neither bidder observe private signals is unlikely to be the result of collusion.

result.

This bidding behavior has dramatic implications regarding the winner's curse. Experimental investigations of common-value auctions with symmetrically informed bidders are numerous.[12] Inexperienced bidders consistently fall victim to the winner's curse.[13] However, throughout the literature, each bidder is provided with a private signal as to the value of the good.[14] Our results suggest that the persistent winner's curse observed throughout the literature may be an artifact of this private signal.

In research of particular relevance to this Chapter, Kagel and Levin [27] report the results of an experiment in which one of the bidders in a first-price auction observes a more precise estimate of the common value of the good than other bidders, but all bidders hold some private information. Our design differs in that our uninformed bidders do not hold private information. Interestingly, the predicted results of these models differ considerably. For the parameters employed in Kagel and Levin [27], seller revenue is expected to be higher than in a symmetric information environment where all bidders have equally precise estimates. Our design that predicts seller revenue will fall relative to both our symmetric information treatments. Note that Kagel and Levin [27] compare their asymmetric information treatment to a single

[12] Kagel and Levin [28] provides an overview of this literature.

[13] Once bidders gain sufficient experience, bidders fall victim much less frequently. However, they continue to overbid relative to the Nash equilibrium bid function. Further, this phenomenon is not driven by a small subset of aggressive bidders who overbid such that the average bid is greater than the value of the good conditional on winning. While this overbidding varies across bidders, most inexperienced bidders fall victim to the winner's curse, and earn negative profits as a result.

[14] An exception is Bazerman and Samuelson [6] which reports the result of classroom experiments in which bidders were asked to guess the value of a commodity (either an unknown quantity of coins or paper clips) and place a bid. Each participant bid in four different auctions. A winner's curse is observed. Hovever, the number of bidders per auction was high (between 34 and 54), and an increase in the number of bidders has typically increases how aggresively participants bid. Also, bidders only particpated in four auctions; they did not have an opprotunity to learn. Lastly, the pool of participants was made up of MBA students. Casari et al. [9] finds that business majors are much more susceptible to the winner's curse than other majors.

symmetric information structure.

Further, the theoretical predictions against which Kagel and Levin compare their experimental data employ different assumptions regarding bidders. In particular, in their asymmetric information treatment they test a model which assumes that bidders are boundedly rational such that they employ an affine bid function. In their symmetric information treatment (every bidder observes an equally precise estimate of the value of the good) bidders are assumed to be unboundedly rational. Indeed, the bid function against which the data is compared is nonlinear. In our design, we test Nash equilibrium predictions with unboundedly rational bidders; we have closed form solutions of the Nash equilibrium in each treatment.

Harrison and List [21] test the same model used in Kagel and Levin [27], but change the population from which the participants are drawn. They perform the same laboratory experiments as Kagel and Levin [27], but recruit participants from attendees (dealers and non-dealers) of a sport-card show. They also run a field experiment testing the asymmetric information structure using unopened packs of sport cards as the good for sale. They find the winner's curse is much less prevalent among dealers.

EXPERIMENTAL DESIGN

Within a group of ten, participants are randomly and anonymously matched into pairs. Each pair participates in a two-bidder, first-price, sealed-bid auction. Each bidder submits a bid. The bidder who submits the highest bid wins the auction and receives the good (in the event of equal bids, both bidders have a 50% chance of obtaining the good) and pays her bid. Only the winner pays her bid. Participants are randomly and anonymously rematched after each round. This process is repeated for thirty rounds.[15]

In each auction a good with a common but uncertain value is available. The common value, x, is a realization of the random variable X, which is uniformly distributed with support $[25, 225]$. The realized value of the good is not observed by bidders before placing their bids. The distribution of X is common knowledge. We employ a 3×1 between-subject design which varies the information observed by bidders prior to placing their bids.

1. *Symmetric information with only public information (SPUB).*—Neither bidder observes any information regarding x beyond the distribution of X. As such, no bidder holds any private information, and information is symmetric.

2. *Symmetric information with private signals (SPRIV).*—Each bidder privately observes a signal. These signals, z_1 and z_2, are independently drawn from a uniform distribution with support $[x - 8, x + 8]$. In this treatment both bidders hold private information in the form of their signal. Information is symmetric in that each signal is an equally precise estimate of x.

3. *Asymmetric information (ASYM).*—One of the bidders is randomly chosen to be the informed bidder, who privately observes a signal. This signal, z_I, is drawn from a uniform distribution with support $[x - 8, x + 8]$. The other bidder does not observe a signal; all the information available to them was common knowledge. Since the informed bidder is randomly determined in each auction, bidders change roles throughout each session.

[15] Since matching of participants occured within groups of ten, and thirty rounds were conducted, participants were inevitably matched together more than once. However, participants were anonymously matched such that they were unable to build a reputation. Further, each session was ususally run with twenty or thirty participants, and participants were not informed that they would only interact within a group of ten.

Table 1: Experimental design summary for first-price auctions

	First-price auctions.
Symmetric information with only public information	5 groups of 10 participants
Symmetric information with private signals	5 groups of 10 participants
Asymmetric Information	5 groups of 10 participants

In each of these three treatments, the information structure of the auction is common knowledge. That is, if a bidder observes a signal, this fact, as well as the distribution from which the signal is drawn, is common knowledge. At the conclusion of each auction each bidder observes both bids, the earnings of both bidders, their own balance and, if applicable, the private signal(s) (participant numbers are suppressed).[16] This design is illustrated in Table 1.

Examining two-bidder auctions makes sense for several reasons. First, in ASYM auctions the equilibrium bid function of the informed bidders does not depend on the number of bidders. The expected payoffs of ASYM bidders (and hence, expected revenue) also do not depend on the number of bidders either. In SPUB auctions Nash equilibrium bids and expected revenue are invariant to the number of bidders. Since we are interested in the role of information, we leave the test of these comparative statics to future research. Second, SPRIV auctions have been extensively examine in the experimental literature, but we are unaware of any study which examines this information structure in a two-bidder context. Thus, our SPRIV treatment provides insight not already found in the literature.

All sessions were run at the Economic Research Laboratory (ERL) at Texas A&M University, and our participants were matriculated undergraduates of the institution. The sessions were computerized using z-Tree (Fischbacher [18]). Participants were

[16] Armantier [1] finds that the ex post observation of bids, earnings and signals "homogenizes behavior, and accelerates learning toward the Nash equilibrium" in common-value auctions with the SPRIV information structure. Futher, this level of ex post observation has been widely used throughout the literature, so this increases the comparability of our results with previous studies

separated by dividers such that they can not interact outside of the computerized interface. They were provided with instructions, which were read aloud by an experimenter.[17] After they instructions were read, questions were answered privately. Each participant then individually answered a set of questions to ensure understanding of the experimental procedure; their answers were checked by an experimenter who also answered any remaining questions. Participants were provided with a history sheet which allowed them to keep track of bids, earnings and. if applicable, signal(s) in each round. Each session lasted approximately two hours. Each participant began with a starting balance of \$20 to cover any losses; no participant went bankrupt. At the end of all thirty rounds, each participant was paid their balance, as well as a show-up fee of \$5. The bids, signals and values were all denominated in Experimental Dollars (ED), which were exchanged for cash at a rate of 160ED/\$1. The average payoff was \$26.91, with a range of \$23.31 and \$32.33.

THEORETICAL PREDICTIONS

Symmetric Information With Only Public Information

If both bidders hold only public information, the distribution of X is the only information regarding x available to bidders before placing their bids. Assuming risk-neutral bidders, the unique Nash equilibrium of this auction is for both bidders to bid $E(X) = 125$. To see this, note that if either bidder were to bid above 125, they would earn negative expected profits upon winning. For any bid $b < 125$, the other bidder would have an incentive to bid $b+\epsilon < 125$, and earn a positive expected profit. As only the bidder to whom the good is allocated pays her bid, the expected revenue generated by an auction, $E\left(R^{SPUB}\right) = 125$ and the expected profit of bidder

[17] The instructions for the ASYM treatment are found in Appendix D. Instructions for the remaining treatments are available upon request.

i, $E\left(\Pi_i^{SPUB}\right) = 0$.

Note that the Nash equilibrium in a SPUB auction also represents a break-even bidding strategy. That is, conditional upon winning, bidding less than 125 guarantees an expected profit greater than zero whereas bidding above 125 yields a negative expected profit. Bidding above a break-even biding threshold is widely referred to as the winner's curse.[18] We adopt this terminology, although this threshold is not constant across information structures.

Symmetric Information With Private Signals

Each bidder i receives a private signal z_i. The signals are independently drawn from a uniform distribution on $[x-8, x+8]$.[19] The symmetric equilibrium of this game can be obtained by suitably specializing the results in Milgrom and Weber [37][20] This gives the symmetric risk neutral Nash equilibrium bid function to be

$$\gamma(z_i) = \begin{cases} \frac{1}{3}(z_i - 58) & \text{if} \quad z_i \in [17, 33) \\ \\ z_i - 8 + g(z_i) & \text{if} \quad z_i \in [33, 217) \\ \\ \frac{z_i}{3} + 142 + h(z_i) & \text{if} \quad z_i \in [217, 233] \end{cases}$$

where $g(z_i) = \frac{16}{3}\exp\left[\frac{1}{8}(33 - z_i)\right]$ is the nonlinear portion of the bid function when $z_i \in [33, 217)$, and $h(z_i) = \frac{4096}{3(z_i-201)^2\exp(23)} - \frac{4096}{3(z_i-201)^2}$ is the nonlinear part of the bid

[18] See, e.g., Kagel and Levin [28].

[19] These assumptions are widely used throughout the experimental literature on first-price, common-value auctions. Examples include Casari et al. [9], Kagel and Richard [30] and Kagel and Levin [27]. Our setup differs in the parameter choice as well as in the number of bidders.

[20] The derivations of the symmetric Nash equilibrium bid function, are found in Appendix A. Similar derivations can be found in Kagel and Levin [28] (Appendix to Chapter 6), and in Kagel and Richard [30]. Derivations of expected revenue and bidders' expected payoffs are also in Appendix C.

function when $z_i \in [217, 233]$.

Notice that the equilibrium bid function is monotonically increasing. Bidders shade their bids in equilibrium. Intuitively, this can be seen as arising for two reasons. First, in first-price auctions, bidders shade their bids to what they expect the second highest signal holder to bid, conditional on their own signal being the highest signal. Second, in common-value auctions, bidders take into account that the bidder with the highest signal will win the auction. Although z_i is an unbiased estimate of x, in equilibrium bidder i uses z_i as a first order statistic because conditional on winning bidder i has the highest signal.

The expected payoff of bidder i who observes z_i is:

$$\Pi_i^{SPRIV}(z_i) = \begin{cases} 0 & \text{if} \quad z_i \in [17, 33) \\ \frac{8}{3}\left(1 - \exp\left(\frac{33 - z_i}{8}\right)\right) & \text{if} \quad z_i \in [33, 217) \\ \frac{z_i}{3} - \frac{217}{3} - \frac{128}{3(z_i - 201)\exp(23)} + \frac{128}{3(z_i - 201)} & \text{if} \quad z_i \in [217, 233]. \end{cases}$$

Bidder i enjoys a positive expected payoff when $z_i > 33$. This is the private information rent to the bidder. The ex ante expected payoff of bidder i, $E\left(\Pi_i^{SPRIV}\right)$, is found by integrating over $\Pi_i^{SPRIV}(z_i)$ with respect to F_{Z_i}, which yields: $E\left(\Pi_i^{SPRIV}\right) = 2.5$.[21] We refer to this as the information rent a bidder earns in a SPRIV auction.

The expected revenue of this auction is, $E\left(R^{SPRIV}\right) = 2\ E\left(\Pi_i^{SPRIV}\right) = 120$. SPRIV auctions generate lower expected revenue that SPUB auctions due to the private information held by the bidders in the former.

Bidders fall victim to the winner's curse when they bid more than the expected

[21] Decimal numbers are rounded off to two decimal places.

value of the good conditional on having won the auction (the break even bidding strategy). In an SPRIV auction, each bidder receives a signal regarding x. Since the equilibrium bid function is monotonically increasing in the signal, the winner's curse is found when bids exceed the expected value of the good conditional on having the highest signal. That is, bidder i falls victim to the winner's curse when she bids more than $E(X \mid z_i > z_j)$.[22] This threshold is:

$$E(X \mid z_i > z_j) = \begin{cases} \frac{1}{3}(z_i + 58) & \text{if} \quad z_i \in [17, 33) \\ z_i - \frac{8}{3} & \text{if} \quad z_i \in [33, 217) \\ \frac{z_i(z_i+257)-92570}{3(z_i-201)} & \text{if} \quad z_i \in [217, 233]. \end{cases}$$

Asymmetric Information

One bidder observes a signal before placing her bid. We refer to this bidder as the informed bidder. The signal is a realization of Z_I which is uniformly distributed on $[x-8, x+8]$. The distribution function of Z_I is F_{Z_I}. The other bidder holds no private information. We refer to this bidder as the uninformed bidder. Engelbrecht-Wiggans et al. [16] provide the unique, risk neutral Nash equilibrium of this game.[23]

[22] The derivation of $E(X \mid z_i > z_j)$ can be found in Appendix C.

[23] The derivations of the bidding strategy, equilibrium payoffs and expected revenue for the distributions we use are found in Appendix C.

The risk neutral Nash equilibrium bid function of an informed bidder is given by

$$\beta(z_I) = \begin{cases} \frac{z_I}{3} + \frac{58}{3} & \text{if} \quad z_I \in [17, 33) \\ \\ \frac{z_I}{2} + \frac{75}{6} + m(z_I) & \text{if} \quad z_I \in [33, 217) \\ \\ \frac{z_I}{3} + \frac{442}{3} + n(z_I) & \text{if} \quad z_I \in [217, 233] \end{cases}$$

where $m(z_I) = \frac{32}{3(z_I - 25)}$ is the nonlinear portion of the equilibrium bid function when $z_I \in [33, 217)$ and $n(z_I) = \frac{1}{3}\left(\frac{15200}{z_I - 313} - \frac{8800}{z_I - 153}\right)$ is the nonlinear portion of the equilibrium bid function when $z_I \in [217, 233]$.

In equilibrium, the uninformed bidder employs a mixed strategy with the distribution function Q, with support on $[25, 125]$. The probability that the uninformed bidder will bid no more than b is given by:

$$\begin{aligned} Q(b) &= \text{Prob}\left[\beta(Z_I) \leq b\right] \\ &= F_{Z_I}\left(\beta^{-1}(b)\right). \end{aligned}$$

The uninformed bidder will not bid more than $E(X)$, because this would ensure negative expected profits upon winning the auction.

Since, in equilibrium, the uninformed bidder employs a mixed strategy, it must be the case that the expected payoff of any bid in the support of this strategy yields the same expected payoff. Engelbrecht-Wiggans et al. [16] demonstrate that the uninformed bidder wins only when the informed bidder's signal indicates that x is low, such that the expected payoff of an uninformed bidder is zero, conditioned on winning the auction. This implies that the ex ante expected payoff of the uninformed bidder, $E\left(\Pi_U^{ASYM}\right)$, is zero.

Let $q(z_I) \equiv E(X \mid z_I)$. Since $q(z_I)$ is monotonically increasing in z_I, the distribution function of this random variable is just $F_{Z_I}(q^{-1}(\cdot))$, where $q^{-1}(\cdot)$ is the inverse of $q(\cdot)$. The expected payoff of the informed bidder, when z_I is observed, is $\Pi_I^{ASYM}(z_1) = \int_{25}^{q(z_I)} F_{Z_I}(q^{-1}(s))\, ds$. This yields

$$\Pi_I^{ASYM}(z_I) = \begin{cases} \frac{(z_I-17)^3}{38400} & \text{if} \quad z_I \in [17,33) \\ \frac{1811+3z_I(z_I-50)}{1200} & \text{if} \quad z_I \in [33,217) \\ \frac{12015737-143667z_I+699z_I^2-z_I^3}{38400} & \text{if} \quad z_I \in [217,233]. \end{cases}$$

Integrating over $\Pi_I^{ASYM}(z_I)$ with respect to F_{Z_I} yields the ex ante expected profit of the informed bidder, $E\left(\Pi_I^{ASYM}\right) = 33.23$. We refer to this as the informed bidder's information rent in an ASYM auction. This large information rent is largely due to the fact that the upper bound of the support of the uninformed bidder's equilibrium mixed strategy is 125. The ex ante expected revenue of an ASYM auction, $E\left(R^{ASYM}\right)$, is equal to $E(X) - E\left(\Pi_I^{ASYM}\right) - E\left(\Pi_U^{ASYM}\right) = 91.77$.

Since the uninformed bidder has an expected payoff of zero for any bid $b \in [25,125]$, 125 is a break-even strategy for uninformed ASYM bidders. Bidding above 125 ensures negative expected profit upon winning, while bidding below 125 yields an expected payoff of zero conditional on winning the auction. That is, if an uninformed bidder bids above 125, she is said to fall victim to the winner's curse.

The expected value of the good conditional on z_I is the same as the expected value of the good conditional on z_I *and* having won the auction. Winning the auction does not provide the informed bidder additional information regarding x.

Therefore, the break-even bidding strategy for an informed ASYM bidder is to bid:

$$E\left(X \mid z_I\right)=\begin{cases}\frac{z_I+33}{2} & \text{if} \quad z_I \in [17,33) \\ \\ z_I & \text{if} \quad z_I \in [33,217) \\ \\ \frac{z_I+217}{2} & \text{if} \quad z_I \in [217,233]\,.\end{cases}$$

So, if an informed bidder bidder bids above $E\left(X \mid z_I\right)$, she is said to fall victim to the winner's curse.

Testable Hypotheses

The revenue generated by auctions has garnered significant interest in the literature. Much of this attention has focused on the revenue ranking of auction formats, holding the information structure constant. Since the revenue predictions of an auction format are not invariant to the information structure, we test the predicted revenue ranking of different information structures within a single auction format. The ex ante expected revenue of each treatment is found above. Notice that $E\left(R^{ASYM}\right) < E\left(R^{SPRIV}\right) < E\left(R^{SPUB}\right)$. If both bidders observe a private signal, they are predicted to earn a positive payoff which reduces expected revenue relative to a SPUB auction. Additionally, the introduction of asymmetric information sharply reduces expected revenue in a ASYM auction below that of a SPRIV auction.

Since auctions are constant sum games between the seller and the bidders, revenue and bidder payoffs are closely related. When there is an information asymmetry as modeled in an ASYM auction, the decrease in revenue relative to either symmetric information structure must improve the expected payoffs of at least one bidder.

Table 2: Revenue ranking of information structures in first price auctions

Information structure	**Ex ante expected revenue**
SPUB	125
SPRIV	120
ASYM	91.77

Table 3: Ranking of ex ante expected bidder payoffs in first-price auctions

Bidders	**Ex ante expected payoffs**
ASYM-Informed	32.23
SPRIV	2.5
SPUB	0
ASYM-Uninformed	0

Who gets this decrease in revenue, the informed bidder, the uninformed bidder or both? There are a number of predictions with regards to bidder payoffs which we test. The ex ante expected payoffs of bidders are found above. Notice that, $E\left(\Pi_U^{ASYM}\right) = E\left(\Pi_i^{SPUB}\right) < E\left(\Pi_i^{SPRIV}\right) < E\left(\Pi_I^{ASYM}\right)$. These hypotheses are summarized in Table 2 and Table 3.

Since $E\left(\Pi_U^{ASYM}\right) = E\left(\Pi_i^{SPUB}\right)$, a bidder who does not observe a private signal has an expected profit of zero, regardless of whether or not the other bidder observes a signal. This implies that, in equilibrium, the ex ante expected payoff of a bidder who observes a signal is a measure of the value of that signal, given the information structure of the game. That is, an informed bidder's ex ante expected payoff represents the expected information rent associated with the signal. Since $E\left(\Pi_i^{SPRIV}\right) < E\left(\Pi_1^{ASYM}\right)$, the information rent associated with a signal is greater if the other bidder is uninformed.

EXPERIMENTAL RESULTS

Revenue

Table 4 reports summary statistics of revenue. Average predicted revenue was

Table 4: Revenue in first-price auctions aggregated over all rounds and sessions

Treatment	**Average observed revenue (standard deviation)**	**Average predicted revenue (standard deviation)**
SPUB	84.06	125
	(21.87)	(0)
SPRIV	112.36	110.67
	(55.94)	(55.01)
ASYM	88.96	88.24
	(37.33)	(21.87)

calculated using the realized value of the signal(s) and x.

There are three revenue ranking predictions, which we test using the nonparametric robust rank order test on session-level data.[24][25] Predictions are borne out between SPRIV and ASYM auctions, where at least one bidder holds private information; we find strong support for the prediction that $E\left(R^{ASYM}\right) < E\left(R^{SPRIV}\right)$ (robust rank-order test, $\acute{U} = n.d.$, $p = 0.004$).[26] Predictions regarding SPUB auctions, however, are off. We find that $E\left(R^{SPRIV}\right) > E\left(R^{SPUB}\right)$ (robust rank-order test, $\acute{U} = n.d.$, $p = 0.004$). Further, our data does not support the prediction that $E\left(R^{ASYM}\right) < E\left(R^{SPUB}\right)$. Rather, we are unable to reject revenue equivalence between these treatments (robust rank order test, $\grave{U} = -0.473$, n.s.).

Clearly, the observed effect on revenue of an asymmetry as modeled in ASYM auctions depends on the symmetric information structure. While theory predicts that the information asymmetry will reduce revenue relative to both SPUB and SPRIV information structures, we find that this only holds true relative to the SPRIV structure.

[24] See Castellan [10] for a description of the tests used in our analysis.

[25] The critical values of the robust rank order test are found in Feltovich [17].

[26] The highest average revenue observed within a group of ten participants in any SPUB session is lower than the lowest average reveune observed within a group of ten participants any SPRIV session. As such, the test statistic of the robust rank order test is not defined. We denote such a test statistic as $\acute{U} = n.d.$.

This is in contrast to the results reported in Kagel and Levin [27] and Harrison and List [21]. They employed a design in which each bidder observed a private signal, and one bidder observed a perfectly precise signal. This was compared to a symmetric information structure as in our SPRIV treatment. Theory predicts that such an information asymmetry will increase the expected revenue relative to the SPRIV case, and their experimental results are consistent with that prediction. Our results suggest that this type of information asymmetry would increase revenue relative to a SPUB information structure as well.

Bidder Payoffs

Table 5 reports summary statistics of bidder payoffs per auction. Note that uninformed bidders in ASYM auctions are losing money on average. Despite this, 96.4% of these bidders bid positive amounts. Indeed, the percentage of uninformed ASYM bids below twenty is lower in the last ten periods than in the first ten.

We find, in keeping with theoretical predictions, that the average payoff of informed ASYM bidders is significantly greater that the average payoff of SPUB bidders (robust rank order test, $\acute{U} = 7.19$, $p = 0.008$) and SPRIV bidders (robust rank order test, $\acute{U} = n.d.$, $p = 0.004$). Thus, informed ASYM bidders earn a significant information rent on average and are significantly better off than in either symmetric information structure.

SPUB bidders earn more than SPRIV bidders (robust rank order test, $\acute{U} = n.d.$, $p = 0.004$) and uninformed ASYM bidders (robust rank order test, $\acute{U} = n.d.$, $p = 0.004$). Additionally, we are unable to reject that uninformed ASYM bidders and SPRIV bidders obtain the same payoffs on average (robust rank order test, $\grave{U} = 1.136$, $n.s.$). That is, we find that a SPRIV bidder would not be significantly worse off than if she did not observe a signal, and would be significantly better off if both bidders

Table 5: Bidder payoffs in first-price auctions aggregated over all rounds and sessions

Bidders	**Average observed payoffs (standard deviation)**	**Average predicted payoffs (standard deviation)**
SPUB	15.74	0
	(45.71)	(0)
ASYM-Informed	28.37	27.29
	(37.39)	(27.7)
ASYM-Uninformed	−1.81	0
	(23.63)	(0)
SPRIV	1.59	2.43
	(5.66)	(0.69)

Table 6: Information rents in first-price auctions aggregated across all rounds and sessions

Bidders	**Average observed information rent (standard deviation)**	**Average predicted information rent (standard deviation)**
ASYM-Informed	12.63	27.29
	(37.39)	(27.7)
SPRIV	3.40	2.43
	(5.66)	(0.69)

did not observe a signal.

Uninformed ASYM bidders earn less than informed ASYM bidders (Wilcoxon matched pairs test, $z = -6.13$, $p = 0.000$).[27]

Since bidders who do not observe a signal are, on average, not earning zero payoffs, the value of an observed signal is not accurately measured by the expected payoff of the bidder who observes said signal. The ex ante expected value of an informed ASYM bidder's signal is the difference between the ex ante expected payoff of an informed ASYM bidder and that of a SPUB bidder. Likewise, the ex ante expected

[27] In the ASYM treatment, participants switched roles throughout the experiment. To test the prediction that $E\left(\Pi_U^{ASYM}\right) < E\left(\Pi_I^{ASYM}\right)$, the average payoff of a participant when she was informed was matched with the average payoff of a participant when she was uninformed, for a total of 50 matched pairs.

value of a SPRIV bidder's signal is the difference between the ex ante expected payoff of a SPRIV bidder and that of an uninformed ASYM bidder. That is, the ex ante expected information rent associated with a signal is the difference between the ex ante expected payoff of a bidder who observes the signal and that of a bidder who does not observe the signal, given whether or not the other bidder observes a signal.

Table 6 reports summary statistics of this measure of information rent, aggregated over all rounds and sessions. While the average payoffs of uninformed ASYM bidders and SPRIV bidders are not significantly different, the average value of a SPRIV bidder's signal is positive. The positive average payoff of SPUB bidders drives the value of an informed ASYM bidder's signal down, but on average it is positive and larger than that of a SPRIV bidder.

Winner's Curse

A bidder is said to fall victim to the winner's curse regardless of whether she actually won the auction in which they bid. That is, the winner's curse is defined for all bidders; a victim of the winner's curse has negative expected profits if they were to win the auction.

Table 7 contains summary statistics of the winner's curse where the winner's curse is defined as the observed bid less the break-even bid. Thus, a positive winner's curse indicates that the observed bid is above the break-even bid.

There are several things worth noting. First, on average, bidders in all information structures do not fall victim to the winner's curse. In the symmetric treatment with private signals the percentage of bidders who are cursed is significantly lower in our experiment than in other studies. Table 8 summarizes the frequency with which inexperienced bidders fall victim to the winner's curse in the literature. This difference is attributable to the fact that we examine two bidder auctions, while the

Table 7: Winner's curse in first-price auctions aggregated across all rounds and sessions

Bidders	Frequency of winner's curse: All bidders	Frequency of winner's curse: Winning bidders	Frequency the high (or only) signal holder wins
SPUB	1.6%	3.2%	NA
	(24/1500)	(24/750)	NA
ASYM-Informed	6%	6.9%	65.5%
	(45/750)	(34/491)	(491/750)
ASYM-Uninformed	3.3%	9.7%	NA
	(28/750)	(25/259)	NA
SPRIV	30.9%	45.3%	72.3%
	(464/1500)	(340/750)	(542/750)

NA = not applicable.
The decimal numbers in parentheses are standard deviations.
The fractions in parentheses are relative frequencies.

rest of the literature has examined auctions with a larger number of bidders.[28] As number of bidders increases the adverse selection problem increases; in order to win the auction a bidder's estimate must be the largest of a larger number of signals, driving the break-even bidding strategy down. Thus a bidding strategy which may not lead to being cursed with a small number of bidders may do so with a larger number of bidders. Further, bidders tend to bid more aggressively when there is a larger number of bidders.[29] Second, the frequency with which SPRIV bidders fall victim to the winner's curse is dramatically different than that of SPUB bidders.

Figure 1 illustrates how the bidders' susceptibility to the winner's curse changes as they gain experience. Note that the frequency with which bidders fall victim to the winner's curse decreases as bidders gain experience. However, even in the last periods, many SPRIV bidders are cursed. In contrast, very few SPUB bidders

[28] $n \in \{4, 6, 7\}$ are typical. Frequently, n is varied. Examples include Kagel et al. [29] and Kagel and Levin [26].

[29] This behavior has been observed in many studies. See Kagel and Levin [28].

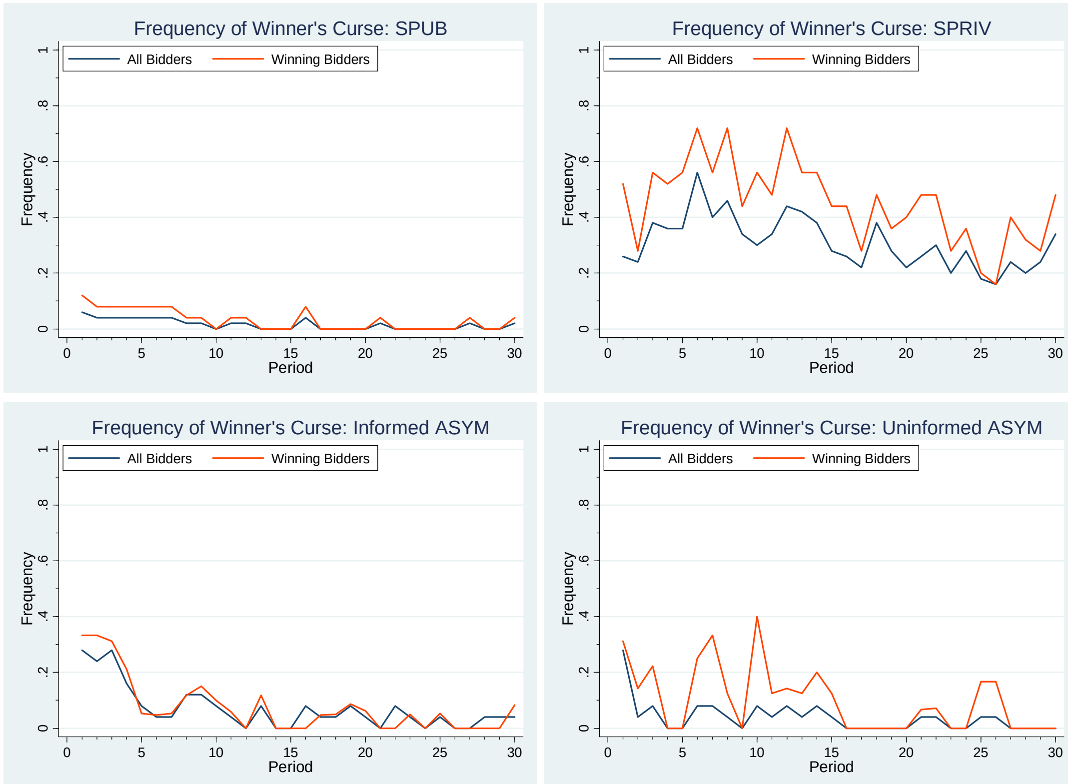

Figure 1: Frequency of the winner's curse in first-price auctions depending on the period

fall victim to the winner's curse in later periods. This is also true of informed and uninformed ASYM bidders.

Figure 2 contains box plots which illustrate how the magnitude of the winner's curse is related to the signals observed by SPRIV and informed ASYM bidders. As we can see, the magnitude of a SPRIV bidder's signal has little effect on the magnitude of the winner's curse for all bidder's or the winning bidders. This is not surprising. since each bidder knows that x is within $8ED$'s of their signal. Bidding $x - 8ED$ guarantees a payoff of at least zero conditional on winning the auction; this implies that the break-even bid is within $8ED$ of their signal. Interestingly, the magnitude of the winner's curse for informed ASYM bidders is decreasing in the observed signal. This is because uninformed ASYM bidders typically bid a low

Figure 2: Magnitude of the winner's curse in first-price auctions depending on the signal

Table 8: Frequency of the winner's curse in the existing literature

Paper	Journal	Information Structure	Frequency of winner's curse: All Bidders	Frequency of winner's curse: Winning Bidders	Number of bidders
Casari et al. [9]	AER	SPRIV	43.9	66.2	6
Kagel and Levin [26]	AER	SPRIV	–	71.4	7
Kagel and Levin [26]	AER	SPRIV	–	31.9	4
Kagel and Levin [27]	Econometrica	SPRIV	60.5	70.9	4
Kagel and Levin [27]	Econometrica	SPRIV	52.3	76.6	7
Kagel and Levin [27]	Econometrica	INSIDER[a]	57.5	78.7	4
Kagel and Levin [27]	Econometrica	INSIDER[a]	83.9	92.9	7
Kagel et al. [29]	EI	SPRIV	59.4	81.8	5-10[b]
Dyer et al. [15]	EJ	SPRIV	55	66	4
Kagel and Garvin [25]	JEBO	SPRIV	56.3	75.3	4
Kagel and Garvin [25]	JEBO	SPRIV	49.8	75.4	6,7
Lind and Plott [35]	AER	SPRIV	–	59.5[c]	–

[a]One bidder is perfectly informed, while the remaining bidders observe noisy signals.
[b]The number of bidders decreased as participants went bankrupt.
[c]This is the percentage of winning bidders who realized a negative payoff.

amount, allowing informed ASYM bidders to bid far below their break-even bid for high values of the good, and still obtain it.

The most significant result regarding the winner's curse is the stark difference between the two symmetric information structures studied: SPUB and SPRIV. SPRIV bidders, who observe a signal, are much more susceptible to the winner's curse than SPUB bidders, who do not observe a signal.

Bidding

We next turn to the question of how bidders bid relative to the Nash equilibrium predictions. Table 9 gives summary statistics on bidding aggregated across all rounds and sessions. We find that SPUB bidders underbid relative to Nash predictions (sign test, $w = 50$, $p < 0.001$).[30] Further, SPRIV bidders overbid relative to Nash

[30]The unit of observation in this test is the individual participant. That is, the averge bid of

Table 9: Bids in first-price auctions relative to the Nash equilibrium aggregated over all rounds and sessions

Bidders	Average bid	Average Nash equilibrium bid	Average percent over Nash	Frequency of positive bids
SPUB	72.57	125	−42%	100%
	(23.62)	(0.00)	(0.11)	(1500/1500)
SPRIV	108.34	105.93	3%	99.9%
	(55.99)	(55.03)	(0.11)	(1498/1500)
ASYM-Informed	77.94	69.54	10%	100%
	(41.84)	(27.65)	(0.35)	(750/750)
ASYM-Uninformed	57.81	75.23[a]	−23%	96.4%
	(30.99)	(28.64)	(0.42)	(723/750)

[a]This is the expected value of the equilibrium mixed strategy.
The decimal numbers in parentheses are standard deviations.
The fractions in parentheses are relative frequencies.

predictions (sign test, $w = 39$, $p < 0.001$). Informed ASYM bidders also overbid relative to Nash predictions (sign test, $w = 31$, $p = 0.0595$).[31] Figure 3 plots the equilibrium bid functions of SPRIV and informed ASYM bidders over a scatterplot of the respective experimental data from all periods and sessions. The SPRIV data closely tracks the equilibrium bid function. The informed ASYM data does not follow as closely, which is largely a result of the increased variance in overbidding as the signal increases. Notice that there are a substantial number of bids at or just below the 45° line, meaning that many bidders bid close to their signal.

Uninformed ASYM bidders are predicted to play a mixed strategy with support $[25, 125]$. As such, we do not have a point prediction for Nash bidding. However, comparing the expected value of the equilibrium mixed strategy with the observed

a participant averaged over all periods is compared with the average Nash equilibrium bid. This unit of observation was used for all tests regarding bidding.

[31]The Wilcoxon signed-rank test assumes that the underlying distribution is symmetric, and is more powerful than the sign test as a result. Consequently, the Wilcoxon signed-rank test finds that informed ASYM bidders overbid relative to Nash predictions with a higher degree of confidence ($z = 2.891$, $p = 0.0038$).

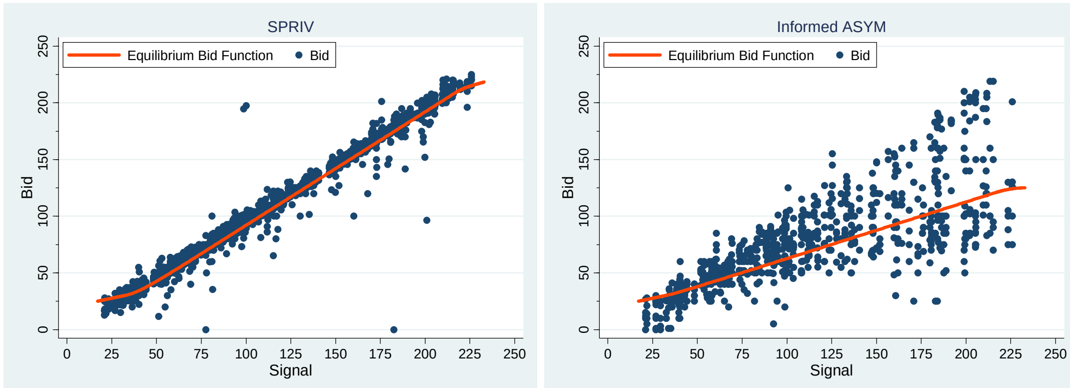

Figure 3: Equilibrium bid functions and observed bids in first-price auctions

average bid demonstrates that, on average, uninformed ASYM bidders are bidding below the expected value of the equilibrium mixed strategy. To test whether the observed distribution of uniformed ASYM bids conforms to the predicted mixed strategy, we employ the nonparametric Kolmogorov–Smirnov test, which strongly rejects the null (Kolmogorov–Smirnov test, $D = 0.6323$, $p < 0.001$). Figures 4 and 5 provide further insight. Figure 4 provides the observed cumulative distribution of uninformed ASYM bids (aggregated over all periods and sessions) relative to the distribution function of the equilibrium mixed strategy. Notice that the observed distribution is almost entirely to the left of the Nash equilibrium mixed strategy. Figure 5 gives these observed distributions, but restricts attention to the first and last ten periods. Notice that there are fewer bids of zero, and fewer bids above the break-even bid of 125 in the last ten periods.

The above analysis of uninformed ASYM bidding uses aggregate data. At the individual participant level, are uninformed ASYM bidders mixing at all? Analyzing the individual data clearly demonstrates that they are not. Individual participants tend to choose the same bid in consecutive instances of being uninformed. Individual participants of the ASYM treatment chose their modal uninformed bid an average of

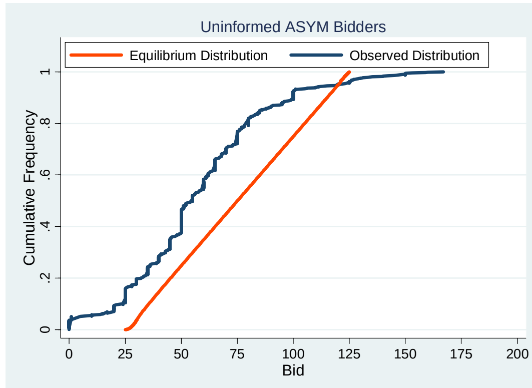

Figure 4: Uninformed ASYM cumulative distribution (all periods)

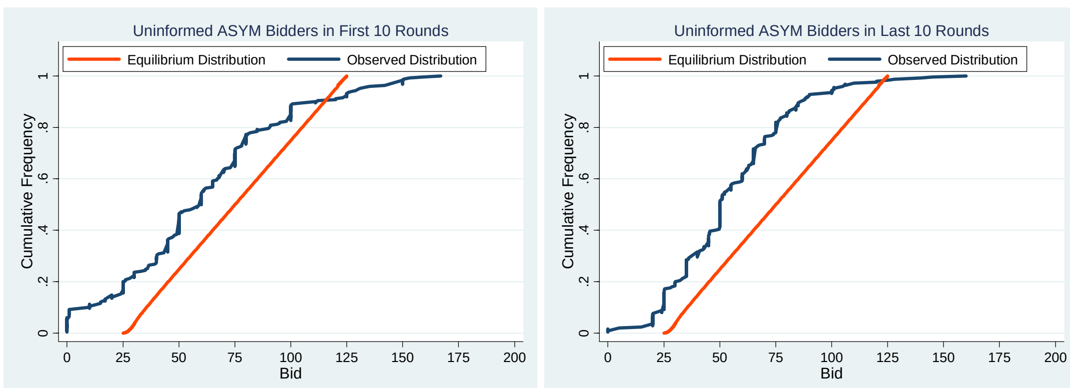

Figure 5: Uninformed ASYM cumulative distribution (first and last ten periods)

29% of the instances in which they are uninformed. Additionally, 82% of uninformed ASYM bids are integers, and 59.47% of uninformed ASYM bids are multiples of five. This is strong evidence against the prediction that uninformed ASYM bidders are mixing continuously on the interval $[25, 225]$, much less mixing according to $Q(b)$.

To summarize, in line with previous experimental findings, bidders who observe a signal overbid relative to the Nash equilibrium on average. However bidders who do not observe a signal bid below the expected Nash equilibrium bid, on average. Indeed, the magnitude by which uninformed bidders bid below Nash predictions is stunning. SPUB bidders bid a full 42% below Nash predictions. While underbidding has been observed in independent private value auctions when bidders have low

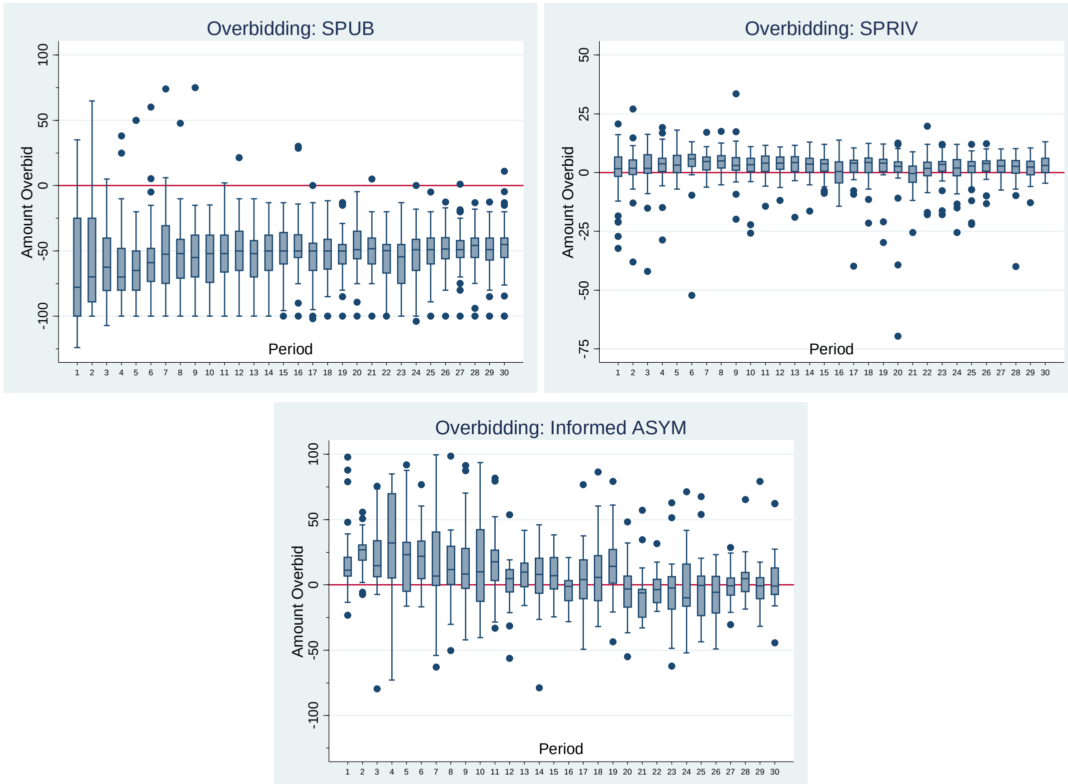

Figure 6: Overbidding in first-price auctions depending on the period

valuations, this is, as far as we know, the first observed underbidding in single-unit common-value auctions.

Figure 6 illustrates how overbidding relative to the Nash equilibrium evolves over time for bidders whose equilibrium bidding strategy is pure. Median overbidding of informed ASYM bidders declines as bidders gain experience. It is important to note, however, that substantial overbidding persists throughout the experiment. In stark contrast, SPUB bidders bid dramatically less than the Nash predictions. While this underbidding decreases in early rounds, median underbidding does not dramatically change in later rounds.

Figure 7 yields insight into how the signal a SPRIV or an informed ASYM bidder observes is related to overbidding. The variance of overbidding by informed ASYM

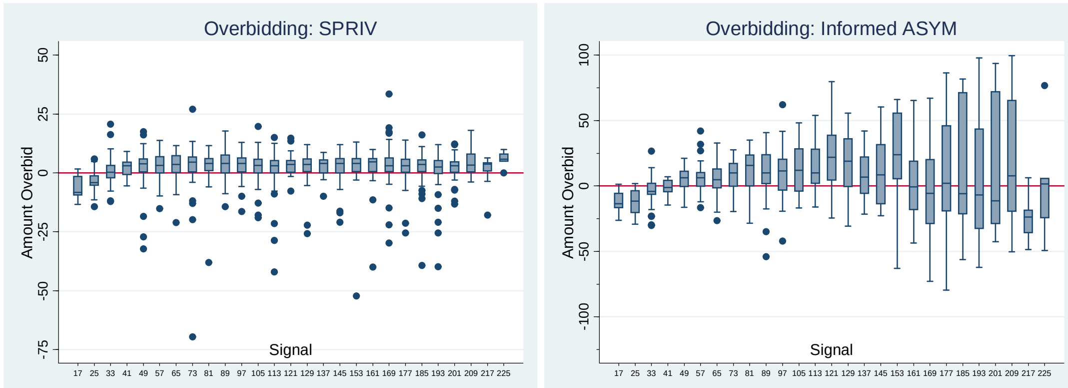

Figure 7: Overbidding in first-price auctions depending on the signal

bidders is clearly positively related to the signal the bidder observes. The same does not hold for SPRIV bidders.

Estimating Bid Functions

In estimating bid functions, we employ a random effects Tobit estimation to control for correlation of participant behavior over time, and the fact that bids were restricted to be within the interval $[0, 225]$. In estimating bid functions, we restrict our attention to observations in which the observed signal (or the signal that a bidder would have observed had she been informed) is in the interval $[33, 217)$, where the majority of observations lie. Following Casari et al. [9], we employ specifications with and without gender and learning interaction. For the SPUB treatment, the specification without gender interaction is given by

$$bid_{it} = \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) + \alpha_i + \epsilon_{it},$$

where z_{it} is the (unobserved) signal, M_i is equal to one if the participant is a male, and $\ln(1+t)$ captures learning. We include z_{it} as a test of whether or not the signal which is observed by a bidder in the corresponding SPRIV auction has any explanatory value in the SPUB auction. The specification which included gender interaction is given by

$$bid_{it} = \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) + \beta_4 M_i \ln(1+t) + \alpha_i + \epsilon_{it}.$$

For the SPRIV treatment, the specification without gender interaction is given by

$$bid_{it} = \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) + \beta_4 g(z_{it}) + \alpha_i + \epsilon_{it},$$

where $g(z_{it})$ is the nonlinear portion of the SPRIV equilibrium bid function when $z_{it} \in [33, 217)$. Likewise the SPRIV specification with the gender and learning interaction is given by

$$bid_{it} = \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) + \beta_4 M_i \ln(1+t) + \beta_5 g(z_{it}) + \alpha_i + \epsilon_{it}.$$

When estimating bid functions for uninformed ASYM bidders, the specification without the gender and learning interaction is

$$bid_{it} = \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) + \alpha_i + \epsilon_{it},$$

where $m(z_{it})$ is the nonlinear portion of the informed ASYM equilibrium bid function when $z_{it} \in [33, 217)$. With the gender and learning interaction the specification is

$$bid_{it} = \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) + \beta_4 M_i \ln(1+t) + \alpha_i + \epsilon_{it}.$$

These specifications for uninformed ASYM bidders allows us to test whether or not the (unobserved) signal that is observed by the analogous bidder in the SPRIV treatment has any explanatory value. $+\beta_4 m(z_{it})$

When estimating bid functions for informed ASYM bidders, the specification without the gender and learning interaction is

$$bid_{it} = \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) + \beta_4 m(z_{it}) + \alpha_i + \epsilon_{it},$$

where $m(z_{it})$ is the nonlinear portion of the informed ASYM equilibrium bid function when $z_{it} \in [33, 217)$. With the gender and learning interaction the specification is

$$bid_{it} = \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) + \beta_4 M_i \ln(1+t) + \beta_5 m(z_{it}) + \alpha_i + \epsilon_{it}.$$

Lastly, we jointly estimate the bid function with and without the gender and learning interaction. Without this interaction the specification is

$$\begin{aligned}
bid_{it} &= \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) \\
&\quad + \beta_4 SPRIV_{it} + \beta_5 AINF_{it} + \beta_6 AUNF_{it} \\
&\quad + \beta_7 SPRIV_{it} z_{it} + \beta_8 AINF_{it} z_{it} + \beta_9 AUNF_{it} z_{it} \\
&\quad + \beta_{10} SPRIV_{it} M_i + \beta_{11} AINF_{it} M_i + \beta_{12} AUNF_{it} M_i \\
&\quad + \beta_{13} SPRIV_{it} \ln(1+t) + \beta_{14} AINF_{it} \ln(1+t) + \beta_{15} AUNF_{it} \ln(1+t) \\
&\quad + \beta_{16} SPRIV_{it} g(z_{it}) + \beta_{17} AINF_{it} m(z_{it}) + \alpha_i + \epsilon_{it},
\end{aligned}$$

where $SPRIV_{it}$ is a dummy variable for the SPRIV bidders, $AINF_{it}$ is a dummy for informed ASYM bidders and $AUNF_{it}$ is a dummy for uninformed ASYM bidders.

With the gender and learning interaction, the specification is

$$
\begin{aligned}
bid_{it} &= \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) + \beta_4 M_i \ln(1+t) \\
&\quad + \beta_5 SPRIV_{it} + \beta_6 AINF_{it} + \beta_7 AUNF_{it} \\
&\quad + \beta_8 SPRIV_{it} z_{it} + \beta_9 AINF_{it} z_{it} + \beta_{10} AUNF_{it} z_{it} \\
&\quad + \beta_{11} SPRIV_{it} M_i + \beta_{12} AINF_{it} M_i + \beta_{13} AUNF_{it} M_i \\
&\quad + \beta_{14} SPRIV_{it} \ln(1+t) + \beta_{15} AINF_{it} \ln(1+t) + \beta_{16} AUNF_{it} \ln(1+t) \\
&\quad + \beta_{17} SPRIV_{it} M_i \ln(1+t) + \beta_{18} AINF_{it} M_i \ln(1+t) + \beta_{19} AUNF_{it} M_i \ln(1+t) \\
&\quad + \beta_{20} SPRIV_{it} g(z_{it}) + \beta_{21} AINF_{it} m(z_{it}) + \alpha_i + \epsilon_{it}.
\end{aligned}
$$

Tables10 contains estimated bid functions without the gender and learning interaction, and Table 11 contains estimated bid functions with the gender and learning interaction.

Notice that, as expected, the (unobserved) signal is not significant in the estimated SPUB and uninformed ASYM bid functions. Conversely, the (observed) signal is highly significant in the estimated bid function for SPRIV bidders. Indeed, the coefficient of the signal is only slightly less than one for SPRIV bidders. Further, the nonlinear part of the bid function ($g(z_{it})$) is not significant. A similar result is found for informed ASYM bidders; the coefficient of the signal is positive and highly significant, and the nonlinear portion of the bid function ($m(z_{it})$) in not significant. The magnitude of the coefficient for informed ASYM bidders is less than for SPRIV bidders; while bidders are not bidding according to the equilibrium bid functions, informed ASYM bidders do reduce their bids relative to the signal to account for uninformed ASYM bidders' bidding below 125, on average.

Interestingly, the results regarding learning differ substantially across treatments.

Table 10: Estimated bid functions for first-price auctions without gender interaction (standard errors in parentheses)

	SPUB	**SPRIV**	**Informed ASYM**	**Uninformed ASYM**	**Joint**
z_{it}	-0.014	0.989^{***}	0.568^{***}	-0.022	-0.014
	(0.011)	(0.005)	(0.018)	(0.021)	(0.010)
$\ln(1+t)$	4.400^{***}	-0.084	-12.072^{***}	-5.113^{***}	4.400^{***}
	(0.846)	(0.356)	(1.420)	(1.627)	(0.793)
M_i	-0.734	-2.137^{***}	-5.260^{***}	-3.317	-0.734
	(1.211)	(0.474)	(1.915)	(2.384)	(1.135)
$g(z_{it})$	–	0.093	–	–	–
		(0.089)			
$m(z_{it})$	–	–	-0.199	–	–
			(0.208)		
$SPRIV_{it}$	–	–	–	–	-65.359^{***}
					(3.543)
$AINF_{it}$	–	–	–	–	-14.645^{***}
					(4.368)
$AUNF_{it}$	–	–	–	–	12.477^{***}
					(4.346)
$SPRIV_{it}z_{it}$	–	–	–	–	1.000^{***}
					(0.015)
$AINF_{it}z_{it}$	–	–	–	–	0.582^{***}
					(0.018)
$AUNF_{it}z_{it}$	–	–	–	–	-0.008
					(0.018)
$SPRIV_{it}\ln(1+t)$	–	–	–	–	-4.491^{***}
					(1.127)
$AINF_{it}\ln(1+t)$	–	–	–	–	-16.470^{***}
					(1.375)
$AUNF_{it}\ln(1+t)$	–	–	–	–	-9.644^{***}
					(1.376)
$SPRIV_{it}M_i$	–	–	–	–	-1.411
					(1.613)
$AINF_{it}M_i$	–	–	–	–	-4.560^{**}
					(2.004)
$AUNF_{it}M_i$	–	–	–	–	-2.318
					(2.001)
$AINF_{it}m(z_{it})$	–	–	–	–	-0.209
					(0.177)
$SPRIV_{it}g(z_{it})$	–	–	–	–	0.101
					(0.211)
$Constant$	63.100^{***}	-2.254^{**}	48.496^{***}	75.194^{***}	63.100^{***}
	(2.652)	(1.105)	(4.448)	(5.160)	(2.485)

*Significant at the 0.10 level.
**Significant at the 0.05 level.
***Significant at the 0.01 level.

Table 11: Estimated bid functions for first-price auctions with gender interaction (standard errors in parentheses)

	SPUB	**SPRIV**	**Informed ASYM**	**Uninformed ASYM**	**Joint**
z_{it}	−0.014	0.989***	0.568***	−0.022	−0.014
	(0.011)	(0.005)	(0.018)	(0.021)	(0.010)
$\ln(1+t)$	3.350***	−0.288	−12.929***	−6.262**	3.349***
	(1.247)	(0.524)	(2.197)	(2.615)	(1.169)
M_i	−5.797	−3.054*	−8.815	−8.212	−5.797
	(4.580)	(1.791)	(7.218)	(9.044)	(4.292)
$M_i \ln(1+t)$	1.945	0.352	1.371	1.873	1.945
	(1.697)	(0.663)	(2.684)	(3.339)	(1.590)
$g(z_{it})$	–	0.093	–	–	–
		(0.089)			
$m(z_{it})$	–	–	−0.202	–	–
			(0.208)		
$SPRIV_{it}$	–	–	–	–	−67.574***
					(4.847)
$AINF_{it}$	–	–	–	–	−15.148**
					(6.166)
$AUNF_{it}$	–	–	–	–	12.547**
					(6.139)
$SPRIV_{it} z_{it}$	–	–	–	–	1.000***
					(0.015)
$AINF_{it} z_{it}$	–	–	–	–	0.583***
					(0.018)
$AUNF_{it} z_{it}$	–	–	–	–	−0.018
					(0.018)
$SPRIV_{it} \ln(1+t)$	–	–	–	–	−3.641**
					(1.695)
$AINF_{it} \ln(1+t)$	–	–	–	–	−16.287***
					(2.173)
$AUNF_{it} \ln(1+t)$	–	–	–	–	−9.666***
					(2.153)
$SPRIV_{it} M_i$	–	–	–	–	−2.753
					(6.100)
$AINF_{it} M_i$	–	–	–	–	−3.096
					(7.566)
$AUNF_{it} M_i$	–	–	–	–	−1.814
					(7.582)
$SPRIV_{it} M_i \ln(1+t)$	–	–	–	–	−1.600
					(2.260)
$AINF_{it} M_i \ln(1+t)$	–	–	–	–	−0.557
					(2.810)
$AUNF_{it} M_i \ln(1+t)$	–	–	–	–	−0.200
					(2.802)
$AINF_{it} m(z_{it})$	–	–	–	–	−0.211
					(0.177)
$SPRIV_{it} g(z_{it})$	–	–	–	–	0.100
					(0.211)
Constant	65.839***	−1.719	50.703***	78.207***	65.839***
	(3.569)	(1.495)	(6.199)	(7.446)	(3.344)

*Significant at the 0.10 level.
**Significant at the 0.05 level.
***Significant at the 0.01 level.

In SPUB auction, participants are learning to bid closer to equilibrium as they gain experience. Since they are, on average, underbidding relative to the Nash equilibrium, this means that they are increasing their bid as they gain experience. In the ASYM treatment, both informed and uninformed bidders are reducing their bids as they gain experience. In the case of informed ASYM bidders this corresponds to bidding closer to the Nash equilibrium, but for uninformed ASYM bidders this means that as they gain experience they increase how much they underbid relative to the Nash equilibrium. Given that uninformed ASYM bidders are losing money on average, this is not surprising. In the case of SPRIV bidders, learning is not significant. This is in contrast to previous studies, which typically find that bidders in this information structure learn to bid closer to equilibrium as they gain experience.[32]

We find that when bidder's hold private information there is a significant gender difference, but that when they do not hold private information, this difference is not significant. Namely, males bid less than females when they hold private information. We find that the interaction between gender and learning is not significant for any type of bidders. Casari et al. [9] examine gender differences in an SPRIV information structure and find that males bid less than females, but that females learn faster than males. Since we do not find evidence of learning in SPIRIV auctions, the fact that there is not a significant gender difference in learning is not surprising.

Notice that the dummy variables for types of bidders are all highly significant when the bid functions are estimated jointly. Also, gender differences are largely insignificant between types of bidders.

[32]See e.g., Casari et al. [9].

CHAPTER IV
ASYMMETRIC INFORMATION IN CONTESTS: THEORY AND EXPERIMENTS

OVERVIEW

In a contest a set of economic agents expend unrecoverable effort to increase the probability of obtaining a good. One of the contestants wins the contest and obtains the good. In a perfectly discriminating contest, also known as an all-pay auction, the contestant who expends the most effort wins the contest with certainty. In an imperfectly discriminating contest, an increase in effort relative to the other contestants increases the probability of winning, but no contestant wins the contest with certainty. The applications of such games are abundant and diverse. Contests are used to model research and development, elections, sports, labor markets and many more.

The theoretical analysis of contests is a vast and burgeoning literature which traces its roots to Tullock.[46] A survey of this literature can be found in Konrad [32]. An important topic in this literature is the role of asymmetric information. However, the literature concerning asymmetric information in contests is quite small. Wärneryd [48] analyses a two player imperfectly discriminating contest in which one contestant is informed of the common but uncertain value of the good prior to bidding, while the other contestant knows only the distribution from which this value was drawn. In this framework, revenue decreases relative to the cases in which neither, or both, contestants are informed regarding the realized value of the good.. The informed contestant is better off in expectation than in either of these symmetric information cases, and the uninformed contestant is worse off in expectation. Chapter II extends these results in a two period model with more than two contestants. In the first round information is symmetric; no contestant holds information regarding the common and uncertain value of the good, beyond the distribution from which it is drawn. The winner of the first contest privately observes the value of the good in the first contest, and this value serves as a noisy signal regarding the value of the good in the second contest. The results in Wärneryd [48] extent to this generalized case. Further, the increased incentive to win the first contest is sufficient to increase aggregate effort relative to the case in which information is symmetric in both contests.

In a related paper, Hurley and Shogren [24] analyze a two player contest in which one contestant knows the other's valuation of the good, while the informed contestant's valuation is private information. They find that such an information asymmetry reduces the uninformed contestant's probability of winning. Fu [19] considers a model in which contestants are asymmetrically informed and endogenously choose the order in which they choose their respective bids. In this model the uninformed contestant chooses to move first, and effort expenditures are reduced relative to a simultaneous move game. Prior to this paper, the role of asymmetric information in perfectly discriminating contests has not been analyzed theoretically.

This Chapter experimentally examines the role of asymmetric information in incomplete information contests, both perfectly and imperfectly discriminating. In our experimental design two contestants, or bidders, simultaneously submit bids, in an effort to obtain a good.[33] This good has a common but uncertain value. We vary the contest success function between perfectly discriminating (all-pay auction), and imperfectly discriminating (lottery contest). We also vary the information structure

[33] In the contest literature players are typically called contestants. In the all-pay auction literature, players are typically called bidders, and their effort expenditures are refered to as bids. Throughout the body of the paper we refer to players as bidders, and effort expenditures as bids. Our experimental instructions also used this terminology to frame the game.

of the game. In the symmetric information structure, neither bidder holds any private information regarding the value of the good. In the asymmetric information structure one bidder observes a noisy signal regarding the value of this good, while the other does not. We also examine an all-pay auction in which each bidder observes a private signal, which allows us to compare our results to those found in Chapter III.

We also characterize the Nash equilibrium in an asymmetric information all-pay auction; one contestant receives a noisy estimate regarding the common and uncertain value of the good, while the other contestant does not. We find that aggregate effort falls in expectation relative to the case in which neither bidder observes a signal. Further, the informed contestant is better off relative to this symmetric information case, while the uninformed contestant has an expected payoff of zero.

Our experimental analysis yields several interesting results. First, information asymmetry reduces revenue in all-pay auctions. However, this in not the case in lottery contests; we are unable to reject revenue equivalence. We also find that the symmetric information all-pay auctions yields higher revenue that the symmetric information lottery contest. Interestingly, when there is asymmetric information, this does not hold. That is, we are unable to reject revenue equivalence between all-pay auctions and lottery contests when there is asymmetric information.

We also find, in both all-pay auctions and lottery contests, that the informed bidder is better off than the uninformed bidder. Additionally in both all-pay auctions and lottery contests, the informed bidder in the asymmetric information environment is better off than bidders in the symmetric information environment; the informed bidder earns a positive information rent. In accordance with theory, the uninformed bidder in the asymmetric information lottery contest is worse off than bidders in the symmetric information lottery contest. Also in accordance with theory, the uninformed bidder in the asymmetric information all-pay auction is not worse off than bidders in the symmetric information all-pay auction; we are unable to reject payoff equivalence between these two types of bidders. We also find that bidders in the symmetric information lottery contest are better off than bidders in the symmetric information all-pay auction. Additionally, we are unable to reject payoff equivalence between uninformed bidders in all-pay auctions and lottery contests, as well as payoff equivalence between informed bidders in all-pay auctions and lottery contests. This observation provides additional insight into the revenue equivalence between the two asymmetric information environments

We also compare bidding behavior to a strategy above which a bidder is guaranteed to earn negative payoffs, provided the other bidder is bidding according to the Nash equilibrium. We call such a bidding strategy a break-even bidding strategy. Such a threshold is of interest, since experimentalists have observed that bidders in contests often overbid relative to Nash predictions and go bankrupt as a result. Bidding above a break-even bidding strategy is analogous to falling victim to the winner's curse, which has been widely observed in the experimental auction literature.[34] We observe that informed bidders in the asymmetric information environments are much more prone to bid above this break-even bidding strategy than are uninformed bidders in the asymmetric or symmetric information environments. This is consistent with the findings of Chapter III, which experimentally analyses the effect of asymmetric information in first-price, sealed-bid, common-value auctions. We also ran sessions in which bidders participate in a series of all-pay auctions and both bidders privately observe a signal (the signals are independent, conditional on the realized value of the good). While we do not have theoretical predictions for this game, bidding above a break-even bidding strategy is much more prevalent than in the

[34] For an overview of this literature see Kagel and Levin.[28].

symmetric information all-pay auctions in which neither bidder observed a signal.[35] As such, we can confidently say that asymmetric information is not the determining factor in informed bidders bidding above the break-even bidding strategy.

We also find evidence that men bid less than women regardless of the contest success function or the information structure of the game. In asymmetric information lottery contests, women learned to decrease their bids faster than men, such that by the final periods behavior had converged. This accelerated learning of women was not significant for bidders with symmetric information, or bidders in all-pay auctions with symmetric or asymmetric information.

Most of the existing experimental literature regarding contests and all-pay auctions study complete information environments. That is, each bidder's valuation of the good is common knowledge. Miller and Pratt [39], examines lottery contests with complete information and find significant overbidding. Miller and Pratt [40] finds that bidding is decreasing in risk aversion in complete information, common-value lottery contests. Davis and Reilly [12] and Potters et al. [42] both examine lottery contests and all-pay auctions in a complete information and common value context, and find that the all-pay auction generates more revenue than lottery contests. Rapoport and Amaldoss [43] experimentally examine all-pay auctions with complete information, a common-value good, and binding budget constraints. They find that behavior is consistent with equilibrium predictions at the aggregate, but not individual, level. Gneezy and Smorodininsky [20] study common-value all-pay auctions with complete information and find dramatic overbidding relative to Nash predictions.

The experimental literature regarding contests with incomplete information is

[35] The break even bidding-strategy in the all-pay auction in which each bidder observes a private signal is defined under the assumption that bidders employ a monotonically increasing bid function.

surprisingly small. Noussair and Silver [41] study all-pay auctions in an independent private value environment. They find that this all-pay auction yields more revenue than predicted by theory, as well as yielding more revenue than the analogous first-price, sealed-bid auction. Barut et al. [4] examines an independent private value all-pay auction with multiple units of the good, and find that bidder's over-bid relative to the Baysian equilibrium. To the best of our knowledge, this is the first experimental analysis of perfectly or imperfectly discriminating contests with asymmetric information.

EXPERIMENTAL DESIGN

We employ a between-subject design which varies the game between an all-pay auction (perfectly discriminating contest) and a lottery contest (imperfectly discriminating contest) and varies the information observed by bidders prior to placing their bids. This design is summarized in Table 12. Participants engage in either a series of common-value, two-player all-pay auctions or lottery contests. Within a group of ten, participants are randomly and anonymously matched into pairs at the beginning of each session. Each bidder submits a bid, which must be paid. In all-pay auction sessions the bidder who submits the highest bid wins the all-pay auction and receives the good (in the event of equal bids, both bidders have a 50% chance of obtaining the good). In lottery contest sessions the probability that a bidder obtains the good is her proportion of the sum of bids. Participants are randomly and anonymously rematched after each round. This process is repeated for thirty rounds.[36][37]

[36] Since matching of participants occured within groups of ten, and thirty rounds were conducted, participants were inevitably matched together more than once. However, participants were anonymously matched such that they were unable to build a reputation. Further, each session was ususally run with twenty or thirty participants, and participants were not informed that they would only interact within a group of ten.

[37] In one of the contest sessions, there are only 29 rounds.

In each all-pay auction or lottery contest a good with a common but uncertain value is available. The common value, x, is a realization of the random variable X, which is uniformly distributed with support $[25, 225]$. The realized value of the good is not observed by bidders before placing their bids. The distribution of X is common knowledge. Prior to placing their bid, bidders may privately observe a signal, which is drawn from a uniform distribution with support $[x - 8, x + 8]$. The treatments of our experimental design are as follows.

1. *Symmetric information all-pay auction (SAP).*—Participants engage in 30 all-pay auctions. In each of these all-pay auctions neither bidder observes any information regarding x beyond the distribution of X. As such, no bidder holds any private information, and information is symmetric.

2. *Asymmetric information all-pay auction (AAP).*—Participants engage in 30 all-pay auctions. In each of these all-pay auctions one of the bidders is randomly chosen to be the informed bidder, who privately observes a signal. This signal, z_I, is drawn from a uniform distribution with support $[x - 8, x + 8]$. The other bidder does not observe a signal; all the information available to them was common knowledge. Since the informed bidder is randomly determined in each auction, bidders change roles throughout each session.

3. *Symmetric information lottery contest (SLC).*—Participants engage in 30 lottery contests auctions. Neither bidder observes any information regarding x beyond the distribution of X. As such, no bidder holds any private information, and information is symmetric.

4. *Asymmetric information lottery contest (ALC).*—Participants engage in 30 lottery contests auctions. One of the bidders is randomly chosen to be the in-

Table 12: Experimental design summary for contests

Between-subject design		
	All-pay auctions.	Lottery contests
Symmetric information	5 groups of 10	5 groups of 10
Asymmetric information	5 groups of 10	5 groups of 10

formed bidder, who privately observes a signal. This signal, z_I, is drawn from a uniform distribution with support $[x-8, x+8]$. The other bidder does not observe a signal; all the information available to them was common knowledge. Since the informed bidder is randomly determined in each auction, bidders change roles throughout each session.

In each of these treatments, the information structure is common knowledge. That is, if a bidder observes a signal, this fact, as well as the distribution from which the signal is drawn, is common knowledge. At the conclusion of each auction each bidder observes both bids, the earnings of both bidders, their own balance and, if applicable, the private signal(s) (participant numbers are suppressed).

Examining two-bidder games makes sense because in all-pay auctions with asymmetric information the equilibrium bid function of the informed bidder does not depend on the number of bidders. The expected payoffs of these bidders (and hence, expected revenue) also do not depend on the number of bidders. Since we are interested in the role of information, we leave the test of these comparative statics to future research. Second, existing experimental analysis on all-pay auctions with symmetric information examines games with more than two bidders. Thus, our SAP treatment provides insight not already found in the literature.

All sessions were run at the Economic Research Laboratory (ERL) at Texas A&M University, and our participants were matriculated undergraduates of the institution. The sessions were computerized using z-Tree (Fischbacher [18]). Participants were separated by dividers such that they can not interact outside of the computerized interface. They were provided with instructions, which were read aloud by an experimenter.[38] After they instructions were read, questions were answered privately. Each participant then individually answered a set of questions to ensure understanding of the experimental procedure; their answers were checked by an experimenter who also answered any remaining questions. Participants were provided with a history sheet which allowed them to keep track of bids, earnings and. if applicable, signal(s) in each round. Each session lasted approximately two hours. Each participant began with a starting balance of \$20 to cover any losses; no participant went bankrupt. At the end of all rounds, each participant was paid their balance, as well as a show-up fee of \$5. The bids, signals and values were all denominated in Experimental Dollars (ED), which were exchanged for cash at a rate of 160ED/\$1. The average payoff was \$25.57, with a range of \$9.44 to \$33.62.

THEORETICAL PREDICTIONS

A set of risk neutral players $\mathbf{N} \equiv \{1, 2\}$ compete for a good with a common but uncertain value. The value of the good is a realization of the random variable X, which is uniformly distributed on $[25, 225]$. This distribution function is commonly known. The expected value of $X = E(X) = 125$. Player $i \in \mathbf{N}$ chooses an unrecoverable bid, $b_i \in \mathbb{R}_+$ at a cost of $C_i(b_i) = b_i$ in an effort to obtain the good. These bids are chosen simultaneously, and players do not observe the value of x before choosing b_i. Players are not budget constrained; the strategy space of each player is $\mathbb{R}_+$. The vector of bids is $\mathbf{b} \equiv \{b_1, b_2\}$. Further, $\mathbf{b}_{-i} \equiv \mathbf{b} \backslash b_i$ and $\mathbf{N}_{-i} \equiv \mathbf{N} \backslash i$.

The function $p_i : \mathbb{R}_+ \rightarrow [0, 1]$ maps $\mathbf{b}$ into the probability that contestant i will receive the good. This function is typically called the contest success function in

[38] The instructions for the ALC treatment are found in Appendix I. Instructions for the remaining treatments are available upon request.

the contest literature. Different functional forms of p_i have been studied in the literature. Depending on the functional form of p_i a contest may be characterized as either a perfectly discriminating contest or an imperfectly discriminating contest. In a perfectly discriminating contest, p_i as is given by

$$p_i = \begin{cases} 1 & \text{if} \quad b_i = \max\{b_1, b_2\} \\ \\ 0 & \text{if} \quad b_i = \max\{b_1, b_2\} \\ \\ \frac{1}{2} & \text{if} \quad b_1 = b_2. \end{cases}$$

Note that in such a perfectly discriminating contest the bidder with the highest bid obtains the good with certainty. Since bids are unrecoverable, this perfectly discriminating contest is equivalent to a first-price, sealed-bid, all-pay auction. Indeed, this game is typically referred to as an all-pay auction. As this terminology is prevalent throughout the literature, we adopt it.

In an imperfectly discriminating contest the bidder with the highest bid does not obtain the good with certainty. Skaperdas [45] axiomises a class of imperfectly discriminating contest success functions. A special case of this class is

$$p_i = \begin{cases} \frac{b_i}{b_1+b_2} & \text{if} \quad \max\{b_1, b_2\} > 0 \\ \\ \frac{1}{2} & \text{if} \quad \max\{b_1, b_2\} = 0, \end{cases}$$

which characterizes a lottery contest. Notice that each bidder's probability of obtaining the good is proportional to the revenue generated by the contest. Also, when $b_i = b_j = 0$ then each bidder has an equal probability of obtaining the good.

However if both bidders were to bid nothing, there is an incentive to bid an arbitrarily small amount and win the good with certainty. Thus this boundary case does not arise in equilibrium. As such, any assumption regarding this case would serve equally well. This particular contest success function is widely utilized throughout the experimental literature regarding contests. To aid in the comparability of our result with this literature we utilize it as well.

Symmetric Information All-Pay Auctions (SAP)

In a SAP auction, neither bidder holds private information. The distribution from which the value of the good is drawn is common knowledge. Assuming risk neutral bidders, this is strategically equivalent to an all-pay auction with complete information in which $E(X)$ is the common value of the good. The equilibria of all-pay auctions with complete information are completely characterized in Baye et al. [5]. In a two-bidder all-pay common-value auction with complete information, there is a unique, symmetric, risk neutral Nash equilibrium. In this equilibrium, both bidders employ a mixed strategy with support on $[0, 125]$. The distribution function of this equilibrium mixed strategy is given by

$$K(b_i) = \frac{b_i}{125}.$$

where b_i is the bid of bidder i.

Notice that zero is an element of the support of this mixed strategy, which implies that the bidders have an expected payoff of zero for every bid in that support. That is $E\left(\Pi^{SAP}\right) = 0$. The expected revenue generated by this equilibrium is $E\left(R^{SAP}\right) = E(X) = 125$.

Suppose that bidder j were to employ the equilibrium mixed strategy described above. Bidder i then has an expected payoff of zero for any $b_i \in [0, 125]$. For any $b_i > 125$, bidder i has a negative expected payoff in expectation. As such, $\rho^{SAP} = 125$ is a break-even bidding strategy; any bid above 125 guarantees a negative payoff in expectation.

Asymmetric Information All-Pay Auctions (AAP)

One of the bidders observes a signal, z_I, prior to bidding; we refer to this bidder as the informed bidder. This signal is a realization of the random variable Z_I which is uniformly distributed on $[x - 8, x + 8]$. The distribution function of Z_I is denoted as F_{Z_I}. The other bidder, who we refer to as the uninformed bidder, does not observe a signal. She only knows the distribution of X, Z_I and the fact that the informed bidder will observe a realization of Z_I.

This model is similar to the one in Engelbrecht-Wiggans et al. [16], which studies this information structure in the context of a first-price, sealed-bid auction. The primary difference is that the low bidder must also pay her bid. The model found in Engelbrecht-Wiggans et al. [16] is experimentally tested in Chapter III.

The equilibrium for this model is derived for general joint distribution of X and Z_I in Appendix E. For the distributions and parameters employed in our experimental design the risk neutral Nash equilibrium bid function for the informed bidder is given by

$$\beta(z_I) = \begin{cases} \frac{(z_I+58)(z_I-17)^2}{19200} & \text{if} \quad z_I \in [17, 33) \\ \\ z_I + g(z_I) & \text{if} \quad z_I \in [33, 217) \\ \\ \frac{151683 z_I - z_I^3 + 24 z_I^2 - 21595738}{19200} & \text{if} \quad z_I \in [217, 233], \end{cases}$$

where $g(z_I) = \frac{3z_I^2 - 1200 z_I - 1811}{1200}$ is the nonlinear portion of the informed AAP bidder's equilibrium bid function when $z_I \in [33, 217)$.[39]

The uninformed bidder mixes on the interval $[0, 125]$, where the probability that she bids b is

$$\begin{aligned} J(b) &= \text{Prob}\left[\beta(Z_I) \le b\right] \\ &= F_{Z_I}\left(\beta^{-1}(b)\right). \end{aligned}$$

The derivation of $J(b)$ can be found in Appendix F. Note that the uninformed bidder will not bid more than 125 in equilibrium, because this would ensure negative expected profits upon winning the auction. Further, note that $J(b)$ indicates that the distribution of bids of the uninformed bidder is identical to that of the informed bidder. As such, the ex ante probability that the uninformed bidder will obtain the good is equal to the ex ante probability that the informed bidder will obtain the good.

Since, in equilibrium, the uninformed bidder employs a mixed strategy, it must be the case that the expected payoff of any bid in the support of this strategy yields

[39] This definition of $g(z_I)$ is for notational convenience; we utilize this notation when estimating bid functions.

the same expected payoff. As above, the fact that zero is in the support of the uninformed bidder's equilibrium bidding strategy implies that the ex ante expected payoff of the uninformed bidder, $E\left(\Pi_U^{AAP}\right)$, is zero.

Let $q(z_I) \equiv E(X \mid z_I)$. Since $q(z_I)$ is monotonically increasing in z_I, the distribution function of this random variable is $F_{Z_I}(q^{-1}(\cdot))$, where $q^{-1}(\cdot)$ is the inverse of $q(\cdot)$. The expected payoff of the informed bidder, when z_I is observed, is $\Pi_I^{AAP}(z_1) = \int_{25}^{q(z_I)} F_{Z_I}(q^{-1}(s))\, ds$. This yields

$$\Pi_I^{AAP}(z_I) = \begin{cases} \frac{(z_I-17)^3}{38400} & \text{if} \quad z_I \in [17, 33) \\ \\ \frac{1811+3z_I(z_I-50)}{1200} & \text{if} \quad z_I \in [33, 217) \\ \\ \frac{12015737-143667z_I+699z_I^2-z_I^3}{38400} & \text{if} \quad z_I \in [217, 233]. \end{cases}$$

Integrating over $\Pi_I^{AAP}(z_I)$ with respect to F_{Z_I} yields the ex ante expected profit of the informed bidder, $E\left(\Pi_I^{AAP}\right) = 33.23$. We refer to this as the informed bidder's information rent in an AAP auction. This large information rent is largely due to the fact that the upper bound of the support of the uninformed bidder's equilibrium mixed strategy is 125. The ex ante expected revenue of an AAP auction, $E\left(R^{AAP}\right)$, is equal to $E(X) - E\left(\Pi_I^{AAP}\right) - E\left(\Pi_U^{AAP}\right) = 91.77$.

Interestingly, the expected payoffs of both bidders in this AAP auction are exactly the same as in the analogous first-price sealed-bid auction. These results extend to a more general model, the proof of which is found in Appendix E.

For the informed bidder a break-even bidding strategy is a bid which satisfies

$$F_{Z_I}\left(\beta^{-1}(b)\right) E(X \mid z_I) - b = 0.$$

Since the uninformed bidder will never bid above $E(X) = 125$ in equilibrium, when $z_I \geq 125$, $b = E(X \mid z_I)$ is the break-even bid. For brevities sake, we do not include the derivations of the break-even bidding strategy when $z_I < 125$. These derivations can be found in Appendix F.

For the uninformed bidder, the break-even bidding strategy is $\rho_U^{AAP} = 125$. The reasoning behind this is similar to that of SAP bidders. Namely, for any bid less or equal to 125, the expected payoff is zero. To obtain a negative expected payoff, the uninformed bidder must bid more than 125.

Symmetric Information Lottery Contests (SLC)

If both bidders hold only public information, the distribution of X is the only information regarding x available to bidders before placing their bids. Assuming risk neutral bidders, the well known unique equilibrium of this game is for each bidder to bid $\frac{E(X)}{4} = 31.25.$[40]. The revenue generated by this equilibrium, $E\left(R^{SLC}\right)$, is simply the sum of the bids, which is 62.5. The expected payoff of each bidder is $E\left(\Pi^{SLC}\right) = 31.25$, which is equal to the equilibrium bid.

Notice that bidders earn a positive payoff in equilibrium, despite holding no private information. Further the $E\left(R^{SLC}\right)$ is half of $E(X)$. Contrasting this with the revenue prediction of the analogous all-pay auction, $E\left(R^{SAP}\right) = 125$, we see that a SLC generates half the revenue of a SAP, in equilibrium.

The break-even bidding strategy of bidder i in a SLC is the b_i which satisfies

$$\frac{b_i}{b_i + 31.25} E(X) - b_i = 0.$$

That is, the break-even bidding strategy of a SLC bidder is $\rho^{SLC} = 93.75$. This

[40] This well known result can be found in Cornes and Hartly [11]. The derivations of this equilibrium is found in Appendix F.

break-even bidding strategy is defined assuming the other bidder is bidding according to the Nash equilibrium. Notice that if the other bidder were to bid more than the Nash equilibrium bid, as is often observed, the bid which ensures an expected payoff of zero is lower than 93.75. As such, this measure of overbidding is conservative, given the behavior typically observed in lottery contest experiments.

Asymmetric Information Lottery Contests (ALC)

One bidder observes a private signal before placing her bid. We refer to this bidder as the informed bidder. The signal is a realization of Z_I which is uniformly distributed on $[x-8, x+8]$ The distribution of Z_I is F_{Z_I}. The other bidder holds no private information, and we refer to this bidder as the uninformed bidder. Chapter II provides the unique, risk neutral Nash of this game.[41] The equilibrium bid function of the informed bidder is:

$$\zeta(z_I) = \begin{cases} 0 & \text{if } \; z_I \in [17, 25.74) \\ \sqrt{14.68(z_I+33)} - 29.37 & \text{if } \; z_I \in [25.74, 33) \\ m(z_I) & \text{if } \; z_I \in [33, 217) \\ \sqrt{14.68(z_I+217)} - 29.37 & \text{if } \; z_I \in [217, 233], \end{cases}$$

where $m(z_I) = \sqrt{29.37 z_I} - 29.37$ is the nonlinear portion of $\zeta(z_I)$ when $z_I \in [33, 217)$.[42]

The equilibrium bid of the uninformed bidder, rounded to the nearest cent, is $b_U = 29.37$. Integrating $\zeta(z_I)$ over Z_I yields the ex ante expected bid of the informed bidder, $E(\zeta(z_I)) = 29.37$.

[41] The derivations of this Nash equilibrium bidding strategy, as well as the equilibrium payoff and expected revenue predictions for the distributions used in our experimental design are found in Appendix F.

[42] This definition of $m(z_I)$ is done for notational convenience. We will utilize this notation when estimating bid functions.

Notice that, in expectation, the informed bidder and the uniformed bidder bid the same amount. Also, notice that if the informed bidder observes a value of z_I such that $E(X \mid z_I) < 29.37$, the informed bidder will bid zero. When $E(X \mid z_I) < 29.37$, the informed bidder has no incentive to bid; submitting a positive bid in such a circumstance yields negative expected profits. An interesting consequence of this observation is that, ex ante, the uniformed bidder is expected to obtain the good with a higher probability than the informed bidder.

The expected payoff of the informed bidder, when he observes z_I, is given by

$$\Pi_I^{ALC}(z_I) = \begin{cases} 0 & \text{if} \quad z_I \in [17, 25.74) \\ \frac{z_I + 91.74}{2} - 2\sqrt{14.685(z_I + 33)} & \text{if} \quad z_I \in [25.74, 33) \\ z_I + 29.3663 - 2\sqrt{29.37 z_I} & \text{if} \quad z_I \in [33, 217) \\ \frac{z_I + 275.74}{2} - 2\sqrt{14.685(z_I + 217)} & \text{if} \quad z_I \in [217, 233]. \end{cases}$$

The ex ante expected payoff of the informed bidder is $E\left(\Pi_I^{ALC}\right) = 36.92$. The expected payoff of the uninformed bidder is $E\left(\Pi_U^{ALC}\right) = 29.72$. The ex ante expected revenue of an ALC is $E\left(R^{ALC}\right) = 58.74$.

Note that $E\left(\Pi_I^{ALC}\right) > E\left(\Pi^{SLC}\right)$. This is a result of the private information held by the informed bidder. As such, we refer to $E\left(\Pi_I^{ALC}\right) - E\left(\Pi^{SLC}\right) > 0$ as the informed bidder's information rent in an ALC. This is a measure of the value of observing a private signal in a lottery contest.

The break-even bidding strategy of an informed ALC bidder, when she observes z_I is the largest b_I that satisfies

$$\frac{b_I}{b_I + 29.37} E(X \mid z_I) - b_I = 0.$$

That is, the break-even bidding strategy of the informed ALC bidder is

$$\rho_I^{ALC}(z_I) = \begin{cases} 0 & \text{if} \quad z_I \in [17, 25.74) \\ \frac{z_I+33}{2} - 29.37 & \text{if} \quad z_I \in [25.74, 33) \\ z_I - 29.37 & \text{if} \quad z_I \in [33, 217) \\ \frac{z_I+217}{2} - 29.37 & \text{if} \quad z_I \in [217, 233]. \end{cases}$$

For the uninformed bidder in an ALC, the break-even bidding strategy is the bid that satisfies

$$E\left(\frac{b_U}{b_U + \zeta^{ALC}(z_I)} X\right) - b_U = 0.$$

That is, the break-even bidding strategy for the uninformed bidder in an ALC is $\rho_U^{ALC} = 90.17$.

Testable Hypotheses

Revenue predictions of all-pay auctions and lottery contests are not invariant to the information structure. The ex ante expected revenue predictions of each treatment where we have theoretical predictions are found above. Notice that $E\left(R^{ALC}\right) < E\left(R^{SLC}\right) < E\left(R^{AAP}\right) < E\left(R^{SAP}\right)$. When one bidder observes a signal, she is expected to earn an information rent which reduces expected revenue relative to the case where neither bidder observes a signal. Also, holding the information structure constant, all-pay auctions are expected to generate more revenue than lottery contests. These hypotheses are summarized in Table 13.

Since all-pay auctions and lottery contests are constant sum games between the seller and the bidders, revenue and bidder payoffs are closely related. When there is an information asymmetry as in our experimental design, the decrease in revenue

Table 13: Revenue ranking of contests in decreasing order

Information structure	**Ex ante expected revenue**
SAP	125
AAP	91.77
SLC	62.50
ALC	58.74

Table 14: Ranking of ex ante expected bidder payoffs in contests in decreasing order

Bidders	**Ex ante expected payoffs**
ALC-Informed	36.92
AAP-Informed	33.23
SLC	31.25
ALC-Uninformed	29.72
SAP	0
AAP-Uninformed	0

relative to the symmetric information structure in which neither bidder observes a signal must improve the expected payoffs of at least one bidder. Who gets this decrease in revenue, the informed bidder, the uninformed bidder or both? There are a number of predictions with regards to bidder payoffs which we test. The ex ante expected payoffs of bidders are found above. Notice that, $E\left(\Pi_U^{AAP}\right) = E\left(\Pi_i^{SAP}\right) < E\left(\Pi_U^{ALC}\right) < E\left(\Pi_i^{SLC}\right) < E\left(\Pi_I^{AAP}\right) < E\left(\Pi_I^{ALC}\right)$. These hypotheses are summarized in Table 14.

Since $E\left(\Pi_U^{AAP}\right) = E\left(\Pi_i^{SAP}\right)$, a bidder who does not observe a private signal in an all-pay auction has an expected profit of zero, regardless of whether or not the other bidder observes a signal. This implies that, in equilibrium, the ex ante expected payoff of a bidder who observes a signal in an all-pay auction is a measure of the value of that signal, given the information structure. That is, an informed bidder's ex ante expected payoff represents the expected information rent associated with the signal in an all-pay auction.

Since $E\left(\Pi_U^{ALC}\right) > 0$, $E\left(\Pi_I^{ALC}\right)$ is not the expected value of observing a signal in a lottery contest. This value, or information rent, is given by $E\left(\Pi_I^{ALC}\right) - E\left(\Pi_i^{SLC}\right)$. Notice that the expected information rent obtained by an informed bidder is greater in an all-pay auction than in a lottery contest.

EXPERIMENTAL RESULTS

Revenue

Table 15 reports summary statistics of revenue. Average predicted revenue was calculated using the realized value of the signal(s) and x. As a result, the predictions where there is an informed bidder differs slightly from the ex ante revenue predictions. Note, however, that the revenue ranking remains the same.

There are six revenue ranking predictions, which we test using the nonparametric robust rank order test on session-level data.[43][44] Predictions are borne out between SAP and AAP auctions; we find support for the prediction that $E\left(R^{SAP}\right) > E\left(R^{AAP}\right)$ (robust rank-order test, $\acute{U} = 2.36$, $p < 0.048$). Further, we find strong support for the predictions that $E\left(R^{SAP}\right) > E\left(R^{SLC}\right)$ (robust rank-order test, $\acute{U} = 7.19$, $p = 0.008$) and $E\left(R^{SAP}\right) > E\left(R^{ALC}\right)$ (robust rank-order test, $\acute{U} = n.d.$, $p = 0.004$).[45]

We are, however, unable to reject equivalence between $E\left(R^{SLC}\right)$ and $E\left(R^{ALC}\right)$ (robust rank order test, $\grave{U} = -0.09$, $n.s.$). That is, our data indicates that the presence of asymmetric information does not reduce revenue in lottery contests, contrary to theory.

Interestingly, we are also unable to reject equivalence between $E\left(R^{AAP}\right)$ and

[43] See Castellan [10] for descriptions of the tests used in our analysis.

[44] The critical values of the robust rank order test are found in Feltovich [17].

[45] The highest average revenue observed within a group of ten participants in any ALC session is lower than the lowest average reveune observed within a group of ten participants any SAP session. As such, the test statistic of the robust rank order test is not defined. We denote such a test statistic as $\acute{U} = n.d.$.

Table 15: Revenue in contests aggregated across all rounds and sessions

Treatment	**Average observed revenue (standard deviation)**	**Average predicted revenue (standard deviation)**
SAP	119.09	125.00
	(65.77)	(0.00)
AAP	95.23	88.24
	(69.31)	(29.80)
SLC	96.76	62.50
	(44.44)	(0.00)
ALC	95.97	56.13
	(56.83)	(14.65)

$E\left(R^{SLC}\right)$ (robust rank order test, $\grave{U} = -0.09$, $n.s.$). Likewise, we are unable to reject equivalence between $E\left(R^{AAP}\right)$ and $E\left(R^{ALC}\right)$ (robust rank order test, $\grave{U} = -0.09$, $n.s.$). This observed revenue equivalence between the asymmetric information all-pay auction and the asymmetric information lottery contest is surprising, given the magnitude of the difference in the theoretical predictions. The revenue in lottery contests, regardless of the information structure is much higher than predicted. As such, the observed revenue equivalence between the ALC, SLC and AAP treatments is largely the result of significant overbidding on the part of bidders in lottery contests.

Bidder Payoffs

Table 16 provides summary statistics regarding bidder payoffs. Average predicted payoffs are calculated using the signals observed by participants. Notice that, on average, the only bidders who have positive payoffs when not observing a signal are bidders in symmetric information lottery contests.

We find, in keeping with theoretical predictions, that informed AAP bidders earn significantly more than uninformed AAP bidders (sign test, $w = 46$, $p < 0.001$)[46] and

[46] In the asymmetric information treatments (AAP and ALC), participants switched roles

Table 16: Bidder payoffs in contests aggregated over all rounds and sessions

Bidders	**Average observed payoffs (standard deviation)**	**Average predicted payoffs (standard deviation)**
SAP	-1.72	0
	(62.77)	(0)
AAP-Informed	26.38	27.29
	(59.50)	(27.70)
AAP-Uninformed	-6.08	0
	(44.06)	(0)
SLC	9.39	31.25
	(68.58)	(0)
ALC-Informed	22.72	31.20
	(60.96)	(26.85)
ALC-Uninformed	-3.16	29.72
	(54.68)	(0)

SAP bidders (robust rank-order test, $\acute{U} = n.d.$, $p = 0.004$). That is, informed AAP bidders earn a significant information rent by virtue of holding private information. As predicted by theory, we are unable to reject that SAP bidders and uninformed AAP bidders have equal payoffs (robust rank-order test, $\acute{U} = 0.669$, $n.s.$). So, a bidder who does not observe a signal is not made worse off when the other bidder does. This implies that the positive information rent obtained on average by informed AAP bidders is extracted from the seller.

Informed ALC bidders have higher payoffs than uninformed ALC bidders (sign test, $w = 45$, $p < 0.001$) and SLC bidders (robust rank-order test, $\acute{U} = 7.188$, $p = 0.008$). Uninformed ALC bidders earn less than SLC bidders (robust rank-order test, $\acute{U} = 2.859$, $p = 0.028$). So informed ALC bidders earn a significant information rent. Unlike all-pay auctions, uninformed bidders in asymmetric information lottery

throughout the experiment. To test the prediction that informed bidders in asymmetric information structures have greater expected profits than their uninformed counterparts, the average payoff of a participant when she was informed was matched with the average payoff of that same participant when she was uninformed, for a total of 50 matched pairs. As such, the test of these predictions are within subject.

contests are worse off than if neither bidder were informed. That is, the information rent that accrues to informed ALC bidders is extracted, at least in part, from the uninformed bidder.

These results have interesting implications in terms of the value of information in contests, and are in line with theoretical predictions. In particular, a bidder in a SAP auction is not worse off if the other bidder were to observe a signal, and would have no incentive to expend resources to prevent such an information asymmetry. The same does not hold true in lottery contests. An interesting question for further research would be whether or not an uninformed bidder would be willing to pay to observe a signal that has been observed by the other bidder. Theory predicts that a bidder in an all-pay auction would be indifferent, while a bidder in a lottery contest would be willing to expend resources to eliminate the information asymmetry.

As predicted by theory, SLC bidders have higher payoffs than SAP bidders (robust rank-order test, $\acute{U} = 7.188$, $p = 0.008$). Interestingly, we are unable to reject that informed ALC bidders and informed AAP bidders have equal payoffs (robust rank-order test, $\acute{U} = 0.435$, $n.s.$). Likewise, we are unable to reject that uninformed ALC bidders and uninformed AAP bidders have equal payoffs (robust rank-order test, $\acute{U} = 0.473$, $n.s.$). This yields additional insight into the observed revenue equivalence between the ALC and AAP treatments. In particular, it seems that the observed revenue equivalence between the AAP and ALC treatments is simply because bidders, both informed and uninformed, are equally well off under all-pay auctions and lottery contests; the change in contest success function does not change the welfare of bidders in an asymmetric information structure. Note that this does not hold when neither bidder observes a signal. The imperfectly discriminating contest success function actually makes bidders better off than the perfectly discriminating contest success function.

Lastly, we find that SAP bidders have lower payoffs than informed ALC bidders (robust rank-order test, $\acute{U} = n.d.$, $p = 0.004$), and are unable to reject that SAP bidders and uninformed ALC bidders have equal payoffs (robust rank-order test, $\acute{U} = 0.473$, $n.s.$). We find that SLC bidders have higher payoffs than uninformed AAP bidders (robust rank-order test, $\acute{U} = 7.188$, $p = 0.008$), and that SLC bidders have lower payoffs than informed AAP bidders (robust rank-order test, $\acute{U} = 4.20$, $p < 0.028$).

Break-even Bidding

In standard auctions, the bidders who do not win the auction do not expend any money; their payoff from losing the auction is zero. As such, a bid above the break-even bidding strategy is a bid above the expected value of the good, conditional on winning the auction. In the experimental auction literature it is widely observed that inexperienced bidders bid above the break-even bidding strategy when they observe a private signal. Such bidders are said to fall victim to the winner's curse. This finding is very robust, and has been observed in many different auction formats. However, Chapter III finds that bidders who do not observe a private signal in a first-price, sealed-bid auction are much less prone to fall victim to the winner's curse than bidders who do observe a private signal. This finding is true of informed bidders who face informed opponents, and bidders who do not.

In contests, bidders must pay their bid whether or not they obtain the good. As a result, the break-even bidding strategy in a contest (the bid above which a bidder has a negative expected payoff, given that the other bidder is bidding according to equilibrium) is substantially less than the expected value of the good, conditional on obtaining the good. Prior to this paper, experimental analysis of contests have often observed significant overbidding, even in very simple environments. The benchmark against which this overbidding has been measured is the Nash equilibrium predictions. While we do compare behavior to Nash predictions, we are interested in whether bidders in common-value contests with incomplete information overbid such that they guarantee themselves negative expected payoffs, as bidders in standard auctions do. We are also interested in the role of observing a private signal on this overbidding. Does observation of such a signal make bidders more prone to bid above the break-even bidding strategy?

Table 17 contains summary statistics regarding when bidders bid above the break-even bidding strategy, aggregated across all rounds and sessions. There are several things worth noting. First, on average, bidders who observe a signal (i.e. informed bidders in the asymmetric information treatments) bid above the break-even bidding threshold much more frequently than bidders who do not observe a signal. Second, the proportion of informed AAP and informed ALC bidders who bid above the break-even bidding threshold is actually greater than the proportion such winning bids that fall above the break-even threshold. This is largely due to the fact that for low signal values, the break-even bidding strategy for informed bidders is quite low. As such, for low signal values a bidder may bid above the break-even strategy, and still be unlikely to obtain the good. Third, notice that informed AAP bidders win almost 70% of the time. Theory predicts that the informed and uninformed AAP bidders have an equal probability of obtaining the good. Further, the informed ALC bidder wins just over 50% of the time, while theory predicts that the uninformed ALC bidder has a higher ex ante probability of obtaining the good.

Figure 8 illustrates how the bidders' propensity to bid above the break-even bidding strategy varies as they gain experience. Note that as bidders gain experience the frequency with which they bid more than their break-even bidding strategy decreases. This is most pronounced for bidders who do not observe a signal. Also,

Table 17: Bidding above the break-even bidding strategy in contests aggregated across all rounds and sessions

Bidders	**Frequency bid exceeds break-even bid: All bidders**	**Frequency bid exceeds break-even bid: Winning bidders**	**Frequency the informed bidder wins**
SAP	6.2%	12.1%	NA
	(93/1490)	(90/745)	NA
AAP-Informed	32.7%	30.4%	69.2%
	(245/750)	(158/519)	(519/750)
AAP-Uninformed	4%	11.3%	NA
	(30/750)	(26/205)	NA
SLC	8.1%	12.1%	NA
	(122/1500)	(91/750)	NA
ALC-Informed	34.3%	32.8%	50.7%
	(257/750)	(168/512)	(380/750)
ALC-Uninformed	8.3%	16%	NA
	(62/750)	(38/238)	NA

NA = not applicable.

The decimal numbers in parentheses are standard deviations.

The fractions in parentheses are relative frequencies.

Figure 8: Frequency of bids above the break-even bidding strategy in contests by period

the bidders who do observe a signal are much more likely to bid more than their break-even bidding strategy than uninformed bidders. Indeed, in the last periods, many informed bidders bid continue to bid above this break-even bidding threshold. In contrast, uninformed bidders, regardless of whether or not they face an informed bidder, have stopped bidding above this threshold almost entirely.

This interesting result is consistent with the behavior observed in Chapter III in the context of first-price, sealed-bid auctions; informed bidders are much more likely to bid above a break-even bidding strategy than are uninformed bidders. We have now observed this bidding behavior in three separate games: first-price auctions, all-pay auctions and lottery contests. As before, we interpret this behavior as overconfidence on the part of informed bidders; informed bidders are overconfident regarding the value of observing a private signal, and bid accordingly. In Appendix G, behavior when both bidders in an all-pay auction observe a signal is analyzed. The same pattern emerges; these informed bidders are much more prone to bid above the break-even bidding threshold than are bidders in an all-pay auction who do not observe a signal.

This behavior is particularly interesting in the context of contests, because a bidder who loses must still pay her bid. As a result, there are two ways in which a bid may result in negative payoffs. First, an informed bidder may bid more than the value of the good, and end up with a negative payoff despite obtaining the good. Second, the informed bidder may not obtain the good, and still be forced to pay her bid. This is in contrast to first-price auctions, in which the only way a bidder may end up with a negative payoff is by obtaining the good by bidding more than its value.

Figure 9 illustrates how the frequency with which winning bidders bid more than the break-even bidding strategy changes as bidders gain experience. Here, the analysis is less clear. This is largely attributable to the fact that uninformed bidders who won when facing an informed bidder were likely to have bid more than the break-even bidding threshold in order to do so, while the other uninformed bidders typically bid conservatively and lost as a result. Spikes in the proportion of winning bids of uninformed AAP or ALC bidders who bid above the break-even bidding threshold reflect this. However, in later periods is it clear that informed winning bidders are much more prone to bid above the break-even bidding threshold.

Figure 10 contains box plots which illustrate how the magnitude of the difference between observed bids and the break-even bidding threshold depends on the signal observed by informed bidders. Interestingly, for small signal values, this magnitude is larger than for large signals. This is true of all informed bids, as well as winning

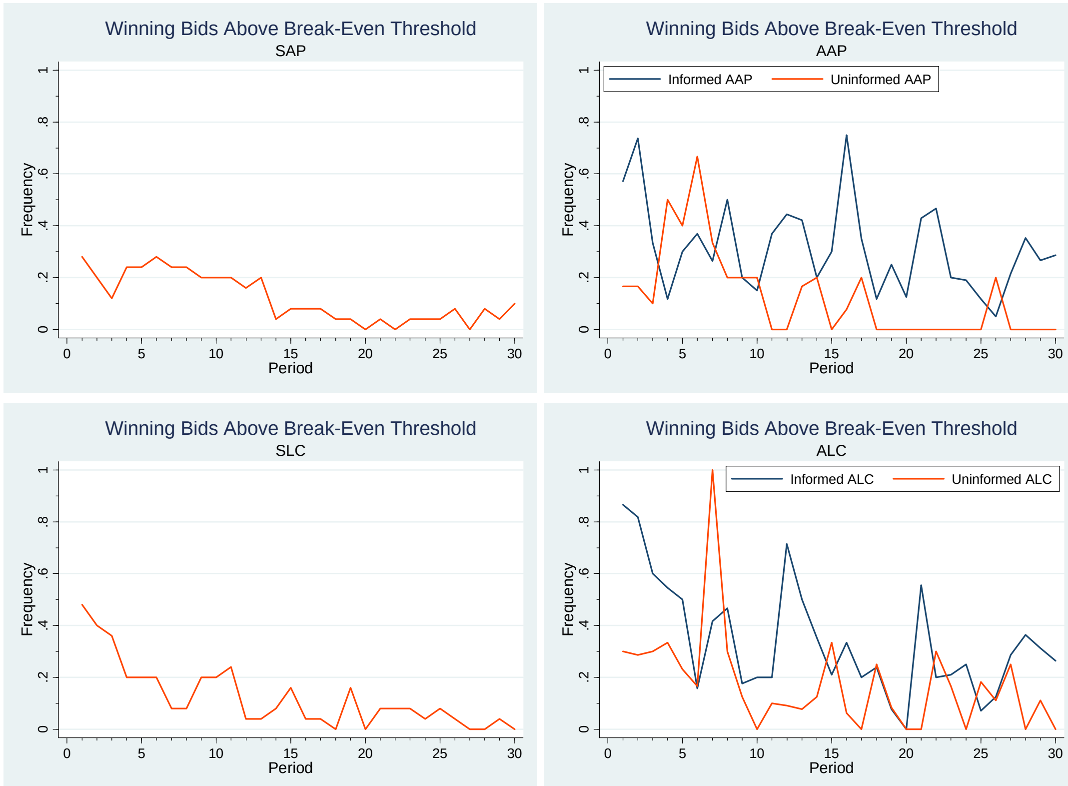

Figure 9: Frequency of winning bids above the break-even bidding strategy in contests by period

informed bids. This is a consequence of the fact that these bidders are facing uninformed opponents. Since an uninformed bidder is unlikely to bid a large amount, an informed bidder who observes a high signal is likely to win the contest, even if she bids much less than the value of the good. Taking this into account reduces her bid relative to the break-even bidding threshold.

Notice that the range of the difference between observed bids and the break-even bidding threshold increase with signal size. This is a result of the fact that for low signal values, the range of rationalizable bids is smaller than when the observed signal is high. An informed bidder knows that the value of the good will never exceed her signal by more than eight. Further, she cannot bid less than zero. These bounds, or course, expand in the signal size, and the range of bidding behavior expands as

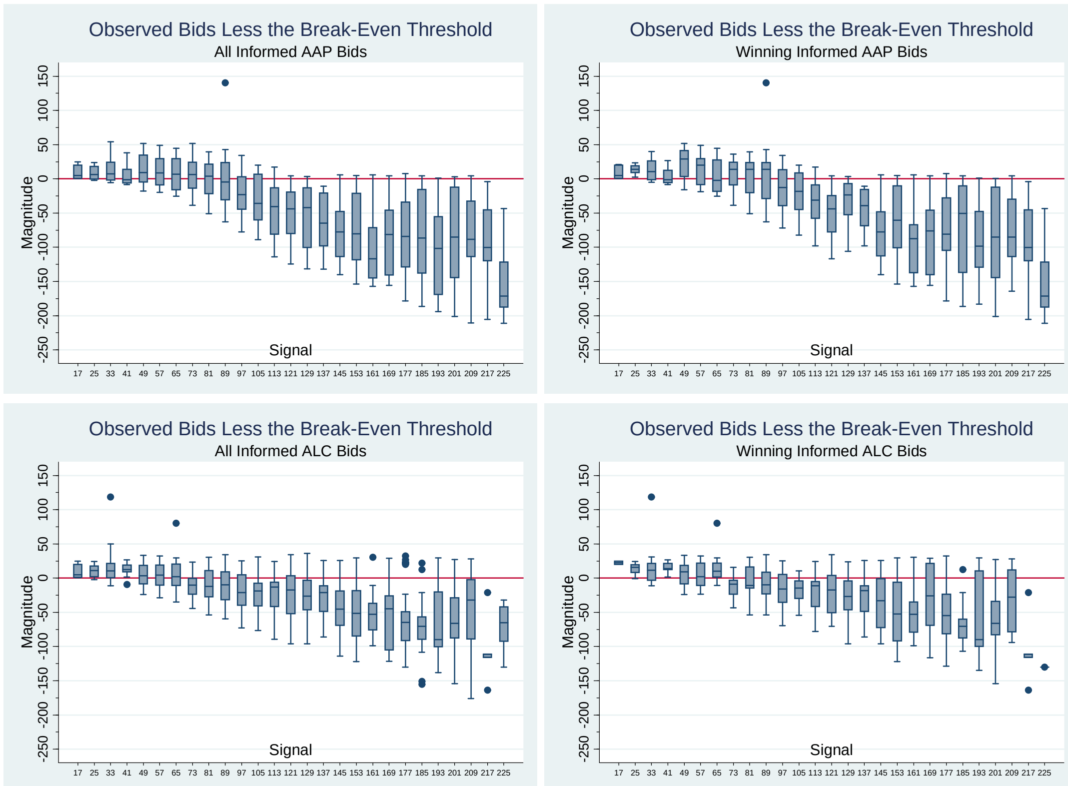

Figure 10: The difference between observed bids and break-even bids in contests depending on the signal

well.

Lastly, notice that for large signal values very few informed AAP bidders bid more than the break-even bidding threshold. In contrast, a non-trivial number of informed ALC bids fall above this threshold, for all but the highest signals. In spite of this, recall that we are unable to reject payoff equivalence between informed AAP and informed ALC bidders.

Bidding

We now compare the bidding behavior of participants across bidder types. Several interesting observations arise. First, we find that informed AAP bidders bid more than uninformed AAP bidders (sign test, $w = 45$, $p < 0.001$).[47] This result is contrary to theory; the distribution of Nash equilibrium bids for the informed AAP bidder is the same as that of the uninformed AAP bidder. In lottery contests. we find that, contrary to theory, informed ALC bidders are bidding more than uninformed ALC bidders (sign test, $w = 45$, $p < 0.001$). Theory predicts that, ex ante, the expected bid of an uninformed ALC bidder is equal to that of an informed ALC bidder (recall that the realized signals in our design reduce the average predicted bid of informed ALC bidders slightly). These two results, of course, are consistent with the hypothesis that the observation of a private signal induces a bidder to increase her bid, on average.

Comparing the behavior of bidders who do not observe signals yields interesting results. SAP bidders bid more than uninformed AAP bidders (robust rank-order test, $\acute{U} = n.d.$, $p = 0.004$).[48] Likewise, SLC bidders bid more that uninformed ALC bidders (robust rank-order test, $\acute{U} = n.d.$, $p = 0.004$). That is, in all-pay auctions and lottery contests, uninformed bidders bid less if their opponent observes a signal than if they do not. This is interesting, in light of the fact that a SAP bidder is not significantly worse off than if her opponent were to observe a signal, while a SLC bidder is significantly better off than if her opponent were to observe a signal. While uninformed AAP bidders are able to reduce their bids relative to SAP bids such that they avoid a reduced payoff, uninformed ALC bidders are not. This is largely due to the fact that bidders in lottery contests have a positive expected payoff regardless of the whether they, or their opponent, observe a signal. In all-

[47]The average uninformed bid of a participant is paired with the average informed bid of the same participant. As such, there are 50 observations for this test.

[48]Since both SAP and uninformed AAP bidders are predicted to employ a mixed strategy in equilibrium, we also employ a two sample Kolmogorov-Smirnov equaltiy of distributions test, in which the average uninformed bid of an individual participant is the unit of observation. The null is strongly rejected (Kolmogorov-Smirnov test, $D = 0.400$, $p = 0.001$).

pay auctions, uninformed bidders have an expected payoff of zero, regardless of the information structure. As such, SLC bidders have something to lose if their opponent were to observe a signal; SAP bidders do not.

We are unable to reject the hypothesis that SAP and informed AAP bidders bid the same amount (robust rank-order test, $\acute{U} = 0.341$, $n.s.$). This result runs contrary to theory, because informed AAP bidders are expected to bid less in equilibrium than SAP bidders. Similarly, in lottery contests we find that informed ALC bidders bid more than SLC bidders (robust rank-order test, $\acute{U} = 2.064$, $p = 0.048$), which is also contrary to theory; informed ALC bidders are, ex ante, predicted to reduce their bids relative to SLC bids. That informed bidders do not bid less than their symmetric information counterparts suggests that informed bidders are not taking advantage of the fact that their uninformed opponents are predicted to reduce their bids in response to the asymmetric information, and may be overbidding relative to Nash predictions as a result. This assertion is tested explicitly below.

In addition, we are unable to reject the hypothesis that informed AAP bidders and informed ALC bidders bid the same amount (robust rank-order test, $\acute{U} = 0.088$, $n.s.$). Likewise, we are unable to reject the hypothesis that uninformed AAP bidders and uninformed ALC bidders bid the same amount (robust rank-order test, $\acute{U} = -0.258$, $n.s.$). Recall that we are also unable to reject payoff equivalence between informed AAP and informed ALC bidders, as well as between uninformed AAP and uninformed ALC bidders. Furthermore, we are unable to reject revenue equivalence between these two asymmetric information treatments. Consequently, these results are not surprising.

Lastly, SAP bidders bid more than SLC bidders (robust rank-order test, $\acute{U} = 7.188$, $p = 0.008$). This result is consistent with theory. Likewise, it is consistent with the existing literature. For example, Potters et al. [42] find that bidders in all-pay auctions bid more than bidders in lottery contests.

Nash Equilibrium

We now turn to the question of how bidders bid relative to the Nash equilibrium predictions. Table 18 contains summary statistics regarding observed and predicted bids, using data aggregated across all rounds and sessions. Average Nash equilibrium bids are calculated using realized signals, rather than ex ante predictions. When Nash predictions involve mixed strategies, the expected value and standard deviation of the mixed strategy are reported. Notice that in the case of all-pay auctions, both SAP and uninformed AAP bidders bid below Nash predictions, on average. In stark contrast, informed AAP bidders bid a staggering 385.48% above Nash predictions, on average. Furthermore, informed ALC bidders overbid relative to Nash predictions much more than SLC or uninformed ALC bidders on average, although all bidders in lottery contests overbid.

Also of interest is the fact that bidders do bid positive amounts, even when uninformed. This is of particular interest for uninformed bidders in all-pay auctions because for every bid in the support of their respective mixed strategies, they have an expected payoff of zero. As such, uninformed bidders are, in equilibrium,.indifferent between the Nash equilibrium mixed strategy, and bidding zero with probability one. Indeed, uninformed AAP bidders had negative payoffs on average, but submitted positive bids 73.86% of the time.

Figure 11 plots the equilibrium bid functions of informed bidders against a scatterplot of the observed bids. Notice that a great many bids lie on the 45° line, for both informed AAP and informed ALC bidders. This indicates that some bidders are naive, in that they simply bid their signal. Further, most bids lie above the equilibrium bid function, indicating that informed bidders tend to overbid relative

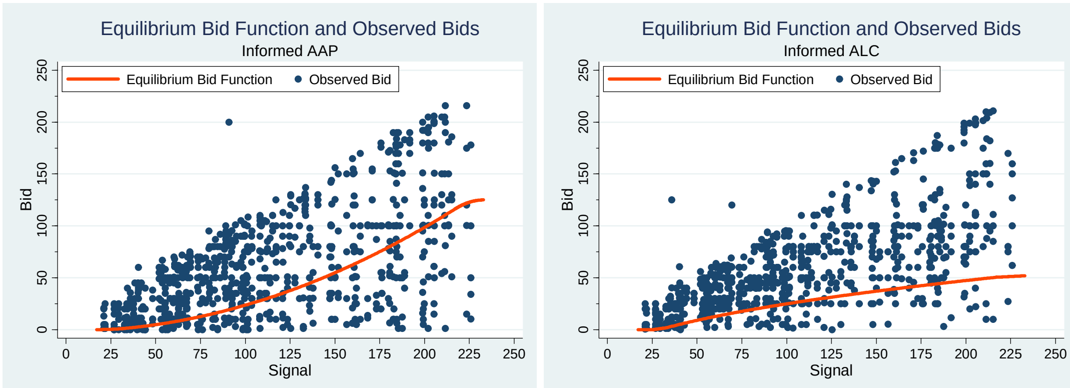

Figure 11: Equilibrium bid functions and observed bids for contests

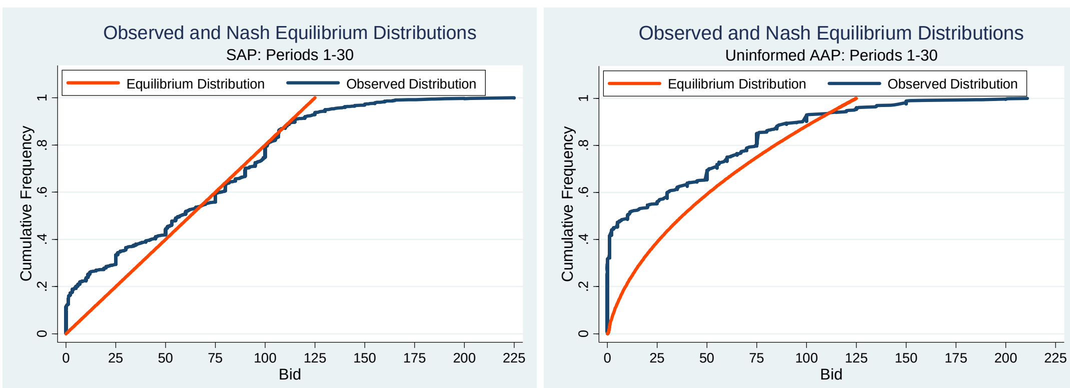

Figure 12: SAP and Uninforrmed AAP cumulative distribution (all periods)

to the Nash equilibrium.

For bidders whose Nash equilibrium bidding strategy is pure, we compare bidding behavior using the nonparametric sign test. Accordingly, we find that informed AAP bidders overbid relative to Nash predictions (sign test, $w = 41$, $p < 0.001$).[49] Further, informed ALC bidders overbid relative to Nash predictions (sign test, $w = 48$, $p < 0.001$). As described above, these informed bidders are prone to bidding in excess of the break-even bidding strategy. This measure of overbidding is looser than Nash equilibrium predictions. As such, it is hardly surprising to find that informed

[49]The unit of observation in this and subsequent sign tests is the average bid of an individual participant. That is, the bid of an individual participant averaged over all periods relative to the Nash equilibrium bid averaged over all periods. There are then 50 observations.

Table 18: Bids relative to the Nash equilibrium in contests aggregated over all rounds and sessions

Bidders	Average bid	Average Nash equilibrium bid	Average percent over Nash	Frequency of positive bids
SAP	59.54	62.5[a]	−4.73%	90.13%
	(46.50)	(36.08)	(0.74)	(1353/1490)
AAP-Informed	61.11	38.49	385.48%	98.40%
	(50.54)	(34.55)	(20.54)	(738/750)
AAP-Uninformed	34.13	45.89[a]	−25.63%	73.86%
	(42.99)	(36.85)	(0.94)	(554/750)
SLC	48.38	31.25	54.81%	94.20%
	(30.38)	(0.00)	(0.97)	(1413/1500)
ALC-Informed	61.02	26.53	229.95%	99.73%
	(44.30)	(14.59)	(6.83)	(748/750)
ALC-Uninformed	34.95	29.37	19.00%	89.47%
	(33.69)	(0.00)	(1.15)	(671/750)

[a]This is the expected value of the equilibrium mixed strategy.
The decimal numbers in parentheses are standard deviations.
The fractions in parentheses are relative frequencies.

bidders overbid relative to equilibrium. However, we also find that SLC bidders overbid relative to Nash predictions (sign test, $w = 41$, $p < 0.001$). This is in contrast to the results of Chapter III, which found bidding in first-price auctions was significantly below Nash predictions when neither bidder observed a signal. In lottery contests, then, observation of a signal increases the magnitude of overbidding, rather than swinging a bidder to overbidding from underbidding as in first-price auctions.

We are unable to reject that uninformed ALC bidders bid according to the Nash equilibrium (two-tailed sign test, $w = 27$, $p = 0.6718$).[50] That is, the only bidders in lottery contests who bid according to Nash predictions are uninformed ALC bidders.

Next, recall that there are two types of bidders whose Nash equilibrium involves a mixed strategy: SAP and uninformed AAP bidders. The support for both of these equilibrium mixed strategies is $[0, 125]$. As such, we do not have point predictions for these bidders. Comparing the expected value of the equilibrium mixed strategy with the average bid tells us that, on average, uninformed AAP bidders are underbidding. The same is true of SAP bidders, although the difference is small. To test whether observed distribution of bids is consistent with the CDF of the equilibrium mixed strategies, we employ the nonparametric Kolmogorov–Smirnov test. We reject the hypothesis that the observed distribution of uninformed AAP bids is equal to that of the equilibrium mixed strategy (Kolmogorov–Smirnov test, $D = 0.1943$, $p = 0.0459$).[51] However, we are unable to reject the hypothesis that the observed distribution of SAP bids is equal to that of the equilibrium mixed strategy (Kolmogorov–Smirnov test, $D = 0.1030$, $p = 0.6630$).[52]

[50] If we assume that participant's bids are independent over time, such that there are 750 observations, we find that uninformed ALC bidders underbid relative to Nash predictions, although this result is only marginally significant (sign test, $w = 397$, $p = 0.0582$).

[51] The unit of observation is the average uninformed AAP bid of an individual participant.

[52] If we assume that an individual participant's bids are independent over time, such that there are 1490 independent observations, then the Null is strongly rejected (Kolmogorov–Smirnov test, $D = 0.8013$, $p < 0.001$).

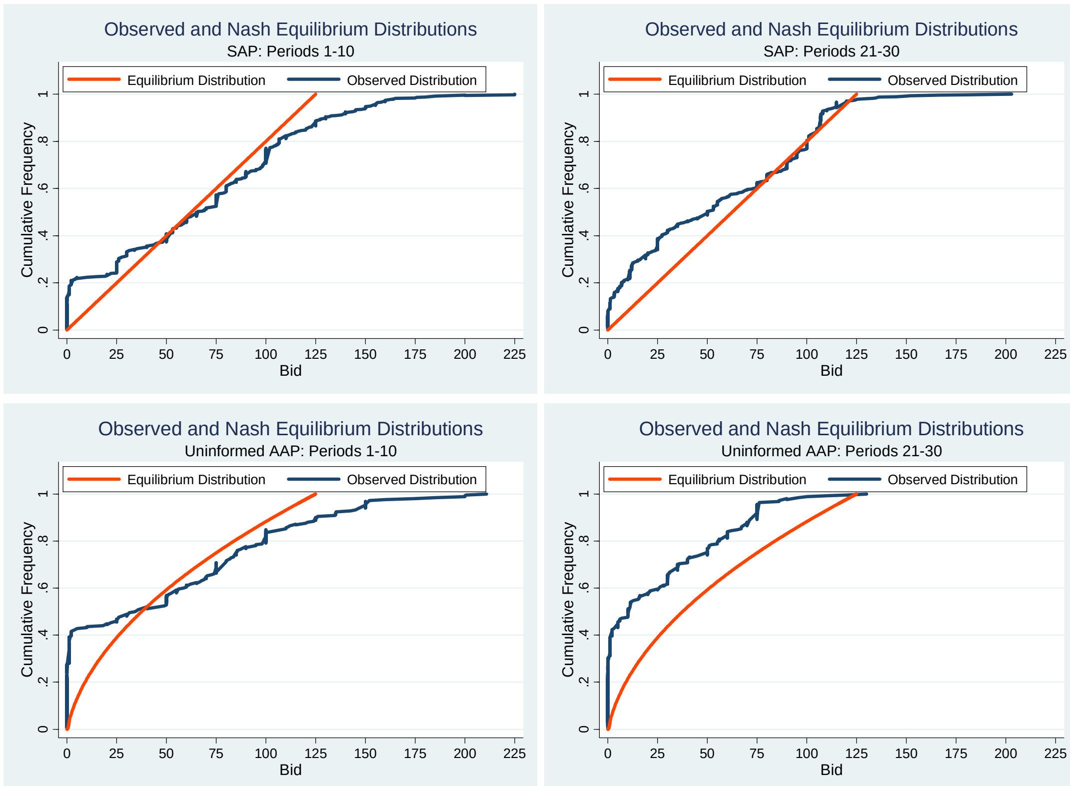

Figure 13: SAP and Uninforrmed AAP cumulative distribution (periods 1-10 and 21-30)

Figures 12 and 13 yield additional insight. Figure 12 plots the empirical cumulative distribution of bids in all periods against the distribution function of the equilibrium mixed strategy for both SAP and uninformed AAP bidders. For SAP bidders, there are more bids at both tails than predicted by theory. However, for uninformed AAP bidders, the empirical distribution is almost entirely to the left of the Nash distribution, save for several bids in at the right tail. Figure 13 restricts attention to the first and last ten periods. In the first ten periods, both uninformed AAP and SAP bidders have more bids on the right tail than predicted. However, in the last ten periods the empirical distribution of SAP bids has shifted dramatically to the left, such that the equilibrium mixed strategy lies almost entirely to the right of the empirical distribution. The change is even more dramatic for uninformed

AAP bidders. In the last ten rounds the empirical distribution is far to the left of the equilibrium distribution. Clearly, as SAP and uninformed AAP bidders gain experience they reduce their bids such that, on average, they are underbidding.

The above analysis of uninformed AAP and SAP bids relies on aggregated data. Of interest is whether or not an individual participant is mixing at all, regardless of the distribution. Examining the behavior of bidders over time clearly demonstrates that they are not. A participant in a SAP session bids her modal bid 32.48% of the time. While the equilibrium distribution function for SAP bidders is continuous on $[0, 125]$, SAP bids are integers 69.93% of the time, and are multiples of five 49.4% of the time. For uninformed AAP bidders, an individual bids her modal uninformed AAP bid 44.00% of the time. Uninformed AAP bids are integers 81.07% of the time, and multiples of five 61.73% of the time. Clearly, these bidders are not mixing continuously. The fact that the modal bids are submitted so frequently suggests that they are not mixing at all.

Estimating Bid Functions

In estimating bid functions, we employ a random effects Tobit estimation to control for correlation of participant behavior over time, and the fact that bids were restricted to be within the interval $[0, 225]$. We restrict our attention to observations in which the observed signal (or the signal that a bidder would have observed had she been informed) is in the interval $[33, 217)$, where the majority of observations lie.

The specification for bidders who do not observe a signal (SAP, SLC, uninformed AAP, and uninformed ALC bidders) is given by

$$b_{it} = \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1 + t) + \alpha_i + \epsilon_{it},$$

where b_{it} is participant i's bid in period t, z_{it} is the (unobserved) signal of participant i in period t, M_i is equal to one if participant i is a male, and $\ln(1+t)$ captures learning.[53] This specification is estimated separately for each type of uninformed bidder, for a total of four such estimations. Recall that SAP and uninformed AAP bidders are predicted to employ a mixed strategy in equilibrium. We justify our estimation of bid functions for these bidders by noting that the data demonstrates that these bidders are not mixing. We include z_{it} as a test of whether or not the signal which would have been observed by the bidder if she were informed has any explanatory value. In each contest (there are 150 contests in each group of ten contestants) a realization of the good was drawn, as well as two signals, which are independent conditional on the realized value of the good. These same realizations were used for each group of ten participants, for all treatments (these are the same realizations used in Chapter III). As such, in SAP and SLC sessions, neither bidder in any given contest observed the signal that was "assigned" to them. In AAP and ALC treatments, one of the bidders was randomly chosen to observe one of the signals. The other bidder did not observe one, although there was one "assigned" to them. We also ran sessions with all-pay auction in which both bidders observed the signal that was "assigned" to them. For SAP, SLC, uninformed AAP and uninformed ALC bidders, the (unobserved) signal that was assigned to them should not have any predictive power concerning bidding behavior. Inclusion of this signal as a covariate tests this assertion.

Following Casari et al. [9], we also employ specifications which interact gender and learning. Casari et al. [9] finds that women initially bid more than men, but that they learn faster than men such that bidding behavior quickly converges. We are

[53] Since period defines the panel, it cannot be included as a covariate. The inclusion of $\ln(1+t)$ captures learning. Moreover, since $\ln(1+t)$ is nonlinear in t, it takes account of diminishing returns to learning.

interested in whether or not this observation holds in the context of contests. The specification for uninformed bidders which includes gender and learning interaction is given by

$$b_{it} = \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) + \beta_4 M_i \cdot \ln(1+t) + \alpha_i + \epsilon_{it}.$$

For informed AAP bidders, the specification without the gender and learning interaction is given by

$$b_{it} = \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) + \beta_4 g(z_{it}) + \alpha_i + \epsilon_{it},$$

where $g(z_{it})$ is the nonlinear portion of the informed AAP equilibrium bid function when $z_{it} \in [33, 217)$. Furthermore, the informed AAP specification with the gender and learning interaction is given by

$$b_{it} = \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) + \beta_4 M_i \cdot \ln(1+t) + \beta_5 g(z_{it}) + \alpha_i + \epsilon_{it}.$$

Similarly, when estimating bid functions for informed ALC bidders, the specification without the gender and learning interaction is

$$b_{it} = \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) + \beta_4 m(z_{it}) + \alpha_i + \epsilon_{it},$$

where $m(z_{it})$ is the nonlinear equilibrium bid function of informed ALC bidders when $z_{it} \in [33, 217)$. By including both z_{it} and $m(z_{it})$, we are testing whether informed ALC bidders bid according to a linear function of their signal, or whether they bid according to the nonlinear bid function, as predicted by theory. With the gender and learning interaction the specification is

$$b_{it} = \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) + \beta_4 M_i \cdot \ln(1+t) + \beta_5 m(z_{it}) + \alpha_i + \epsilon_{it}.$$

Lastly, we jointly estimate bid functions. Without the gender and learning interaction the specification is

$$\begin{aligned}
b_{it} &= \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) \\
&\quad + \beta_4 IAAP_{it} + \beta_5 UAAP_{it} + \beta_6 SLC_{it} + \beta_7 IALC_{it} + \beta_8 UALC_{it} \\
&\quad + \beta_9 IAAP_{it} \cdot z_{it} + \beta_{10} UAAP_{it} \cdot z_{it} + \beta_{11} SLC_{it} \cdot z_{it} \\
&\quad + \beta_{12} IALC_{it} \cdot z_{it} + \beta_{13} UALC_{it} \cdot z_{it} + \beta_{14} IAAP_{it} \cdot M_i \\
&\quad + \beta_{15} UAAP_{it} \cdot M_i + \beta_{16} SLC_{it} \cdot M_i + \beta_{17} IALC_{it} \cdot M_i \\
&\quad + \beta_{18} UALC_{it} \cdot M_i + \beta_{19} IAAP_{it} \cdot \ln(1+t) + \beta_{20} UAAP_{it} \cdot \ln(1+t) \\
&\quad + \beta_{21} SLC_{it} \cdot \ln(1+t) + \beta_{22} IALC_{it} \cdot \ln(1+t) + \beta_{23} UALC_{it} \cdot \ln(1+t) \\
&\quad + \beta_{24} IAAP_{it} \cdot g(z_{it}) + \beta_{25} IALC_{it} \cdot m(z_{it}) + \alpha_i + \epsilon_{it},
\end{aligned}$$

where $IAAP_{it}$ is a dummy variable for informed AAP bidders, $UAAP_{it}$ is a dummy for uninformed AAP bidders, SLC_{it} is a dummy for SLC bidders, $IALC_{it}$ is a dummy variable for informed ALC bidders, and $UALC_{it}$ is a dummy variable for uninformed

ALC bidders. When the gender and learning interaction, the joint specification is

$$
\begin{aligned}
b_{it} &= \beta_0 + \beta_1 z_{it} + \beta_2 M_i + \beta_3 \ln(1+t) \\
&+\beta_4 IAAP_{it} + \beta_5 UAAP_{it} + \beta_6 SLC_{it} + \beta_7 IALC_{it} + \beta_8 UALC_{it} \\
&+\beta_9 IAAP_{it} \cdot z_{it} + \beta_{10} UAAP_{it} \cdot z_{it} + \beta_{11} SLC_{it} \cdot z_{it} + \beta_{12} IALC_{it} \cdot z_{it} \\
&+\beta_{13} UALC_{it} \cdot z_{it} + \beta_{14} IAAP_{it} \cdot M_i + \beta_{15} UAAP_{it} \cdot M_i \\
&+\beta_{16} SLC_{it} \cdot M_i + \beta_{17} IALC_{it} \cdot M_i + \beta_{18} UALC_{it} \cdot M_i \\
&+\beta_{19} IAAP_{it} \cdot \ln(1+t) + \beta_{20} UAAP_{it} \cdot \ln(1+t) + \beta_{21} SLC_{it} \cdot \ln(1+t) \\
&+\beta_{22} IALC_{it} \cdot \ln(1+t) + \beta_{23} UALC_{it} \cdot \ln(1+t) \\
&+\beta_{24} IAAP_{it} \cdot M_i \cdot \ln(1+t) + \beta_{25} UAAP_{it} \cdot M_i \cdot \ln(1+t) \\
&+\beta_{26} SLC_{it} \cdot M_i \cdot \ln(1+t) + \beta_{27} IALC_{it} \cdot M_i \cdot \ln(1+t) \\
&+\beta_{28} UALC_{it} \cdot M_i \cdot \ln(1+t) + \beta_{29} IAAP_{it} \cdot g(z_{it}) \\
&+\beta_{30} IALC_{it} \cdot m(z_{it}) + \alpha_i + \epsilon_{it},
\end{aligned}
$$

Table 19 contains estimated bid functions for all-pay auction without the gender and learning interaction (as well as a joint estimation with all tretments). Table 20 contains estimated bid functions for lottery contests without the gender and learning interaction (as well as a joint estimation with all treatments. Tables 21 and 22 contain the analogous estimated bid functions, but with the gender and learning interaction included.

Notice that, as expected, the (unobserved) signal is not significant in the estimated bid functions of SAP, SLC, uninformed AAP and uninformed ALC bidders. Conversely, the (observed) signal is highly significant in the estimated bid function of informed AAP and informed ALC bidders. Interestingly, in the joint specifications, the coefficient on signal is larger for informed ALC bidders than for informed AAP

Table 19: Estimated bid functions for all-pay auctions without gender interaction (standard errors in parentheses)

	SAP	Informed AAP	Uninformed AAP	Joint
z_{it}	0.015	0.415***	−0.015	0.014
	(0.024)	(0.092)	(0.035)	(0.019)
$\ln(1+t)$	−5.535***	−22.824***	−20.490***	−5.672***
	(1.842)	(1.964)	(2.720)	(1.479)
M_i	−17.982***	−7.340***	−2.030	−17.323***
	(2.646)	(2.941)	(4.064)	(2.215)
$g(z_{it})$	–	−0.379	–	–
		(0.230)		
$m(z_{it})$	–	–	–	–
$IAAP_{it}$	–	–	–	−30.837***
				(10.913)
$UAAP_{it}$	–	–	–	2.734
				(8.080)
SLC_{it}	–	–	–	−17.079***
				(6.532)
$IALC_{it}$	–	–	–	−39.189***
				(8.617)
$UALC_{it}$	–	–	–	−29.935***
				(7.993)
$IAAP_{it} \cdot z_{it}$	–	–	–	0.400***
				(0.100)
$UAAP_{it} \cdot z_{it}$	–	–	–	−0.029
				(0.033)
$SLC_{it} \cdot z_{it}$	–	–	–	−0.027
				(0.027)
$IALC_{it} \cdot z_{it}$	–	–	–	0.598**
				(0.253)
$UALC_{it} \cdot z_{it}$	–	–	–	−0.004
				(0.033)
$IAAP_{it} \cdot \ln(1+t)$	–	–	–	−17.139***
				(2.564)
$UAAP_{it} \cdot \ln(1+t)$	–	–	–	−14.253***
				(2.565)
$SLC_{it} \cdot \ln(1+t)$	–	–	–	−2.300
				(2.084)
$IALC_{it} \cdot \ln(1+t)$	–	–	–	−7.311***
				(2.557)
$UALC_{it} \cdot \ln(1+t)$	–	–	–	0.767
				(2.556)
$IAAP_{it} \cdot M_i$	–	–	–	9.944***
				(3.788)
$UAAP_{it} \cdot M_i$	–	–	–	15.588***
				(3.804)
$SLC_{it} \cdot M_i$	–	–	–	28.465***
				(3.045)
$IALC_{it} \cdot M_i$	–	–	–	5.986
				(3.789)
$UALC_{it} \cdot M_i$	–	–	–	8.686**
				(3.752)
$IAAP_{it} \cdot g(z_{it})$	–	–	–	−0.381
				(0.245)
$IALC_{it} \cdot m(z_{it})$	–	–	–	−0.294
				(0.976)
Constant	78.789***	49.037***	81.644***	79.658***
	(5.728)	(9.280)	(8.632)	(4.599)
Observations	1450	710	750	5830
Left Censored	143	9	196	514
Right Censored	2	0	0	3
Log Likelihood	−7116.064	−3546.939	−3140.289	−27703.290

*Significant at the 0.10 level.
*Significant at the 0.05 level.
*Significant at the 0.01 level.

Table 20: Estimated bid functions for lottery contests without gender interaction (standard errors in parentheses)

	SLC	Informed ALC	Uninformed ALC	Joint
z_{it}	-0.012	0.613^{***}	0.016	0.014
	(0.015)	(0.199)	(0.025)	(0.019)
$\ln(1+t)$	-7.893^{***}	-12.980^{***}	-4.975^{**}	-5.672^{***}
	(1.138)	(1.648)	(2.126)	(1.479)
M_i	10.729^{***}	-11.351^{***}	-8.687^{***}	-17.323^{***}
	(1.690)	(2.479)	(2.789)	(2.215)
$g(z_{it})$	–	–	–	–
$m(z_{it})$	–	-0.301	–	–
		(0.772)		
$IAAP_{it}$	–	–	–	-30.837^{***}
				(10.913)
$UAAP_{it}$	–	–	–	2.734
				(8.080)
SLC_{it}	–	–	–	-17.079^{***}
				(6.532)
$IALC_{it}$	–	–	–	-39.189^{***}
				(8.617)
$UALC_{it}$	–	–	–	-29.935^{***}
				(7.993)
$IAAP_{it} \cdot z_{it}$	–	–	–	0.400^{***}
				(0.100)
$UAAP_{it} \cdot z_{it}$	–	–	–	-0.029
				(0.033)
$SLC_{it} \cdot z_{it}$	–	–	–	-0.027
				(0.027)
$IALC_{it} \cdot z_{it}$	–	–	–	0.598^{**}
				(0.253)
$UALC_{it} \cdot z_{it}$	–	–	–	-0.004
				(0.033)
$IAAP_{it} \cdot \ln(1+t)$	–	–	–	-17.139^{***}
				(2.564)
$UAAP_{it} \cdot \ln(1+t)$	–	–	–	-14.253^{***}
				(2.565)
$SLC_{it} \cdot \ln(1+t)$	–	–	–	-2.300
				(2.084)
$IALC_{it} \cdot \ln(1+t)$	–	–	–	-7.311^{***}
				(2.557)
$UALC_{it} \cdot \ln(1+t)$	–	–	–	0.767
				(2.556)
$IAAP_{it} \cdot M_i$	–	–	–	9.944^{***}
				(3.788)
$UAAP_{it} \cdot M_i$	–	–	–	15.588^{***}
				(3.804)
$SLC_{it} \cdot M_i$	–	–	–	28.465^{***}
				(3.045)
$IALC_{it} \cdot M_i$	–	–	–	5.986
				(3.789)
$UALC_{it} \cdot M_i$	–	–	–	8.686^{**}
				(3.752)
$IAAP_{it} \cdot g(z_{it})$	–	–	–	-0.381
				(0.245)
$IALC_{it} \cdot m(z_{it})$	–	–	–	-0.294
				(0.976)
Constant	62.947^{***}	40.498^{***}	49.629^{***}	79.658^{***}
	(3.595)	(5.758)	(6.538)	(4.599)
Observations	1460	710	750	5830
Left Censored	85	2	79	514
Right Censored	0	0	1	3
Log Likelihood	-6782.575	-3454.378	-3441.625	-27703.290

*Significant at the 0.10 level.
*Significant at the 0.05 level.
*Significant at the 0.01 level.

Table 21: Estimated bid functions for all-pay auctions with gender interaction (standard errors in parentheses)

	SAP	**Informed AAP**	**Uninformed AAP**	**Joint**
z_{it}	0.016	0.418^{***}	-0.017	0.014
	(0.024)	(0.092)	(0.035)	(0.019)
$\ln(1+t)$	-4.106	-26.202^{***}	-23.722^{***}	-4.002^{**}
	(2.591)	(3.282)	(4.441)	(2.080)
M_i	-10.462	-20.805^{*}	-15.349	-8.606
	(9.884)	(10.895)	(15.085)	(7.930)
$M_i \cdot \ln(1+t)$	-2.885	5.233	5.135	-3.368
	(3.681)	(4.078)	(5.600)	(2.954)
$g(z_{it})$	–	-0.371	–	–
		(0.230)		
$m(z_{it})$	–	–	–	–
$IAAP_{it}$	–	–	–	-17.630
				(13.667)
$UAAP_{it}$	–	–	–	13.976
				(11.512)
SLC_{it}	–	–	–	-8.031
				(8.964)
$IALC_{it}$	–	–	–	-18.186
				(11.866)
$UALC_{it}$	–	–	–	-14.653
				(11.307)
$IAAP_{it} \cdot z_{it}$	–	–	–	0.403^{***}
				(0.100)
$UAAP_{it} \cdot z_{it}$	–	–	–	-0.031
				(0.033)
$SLC_{it} \cdot z_{it}$	–	–	–	-0.027
				(0.027)
$IALC_{it} \cdot z_{it}$	–	–	–	0.622^{**}
				(0.253)
$UALC_{it} \cdot z_{it}$	–	–	–	-0.004
				(0.033)
$IAAP_{it} \cdot \ln(1+t)$	–	–	–	-22.204^{***}
				(4.071)
$UAAP_{it} \cdot \ln(1+t)$	–	–	–	-18.505^{***}
				(4.021)
$SLC_{it} \cdot \ln(1+t)$	–	–	–	-5.807^{*}
				(3.169)
$IALC_{it} \cdot \ln(1+t)$	–	–	–	-15.386^{***}
				(4.047)
$UALC_{it} \cdot \ln(1+t)$	–	–	–	-5.136
				(4.013)
$IAAP_{it} \cdot M_i$	–	–	–	-12.312
				(14.066)
$UAAP_{it} \cdot M_i$	–	–	–	-3.782
				(14.125)
$SLC_{it} \cdot M_i$	–	–	–	12.120
				(11.356)
$IALC_{it} \cdot M_i$	–	–	–	-28.695^{**}
				(14.122)
$UALC_{it} \cdot M_i$	–	–	–	-17.367
				(14.046)
$IAAP_{it} \cdot M_i \cdot \ln(1+t)$	–	–	–	8.631
				(5.256)
$UAAP_{it} \cdot M_i \cdot \ln(1+t)$	–	–	–	7.476
				(5.250)
$SLC_{it} \cdot M_i \cdot \ln(1+t)$	–	–	–	6.315
				(4.229)
$IALC_{it} \cdot M_i \cdot \ln(1+t)$	–	–	–	13.372
				(5.247)
$UALC_{it} \cdot M_i \cdot \ln(1+t)$	–	–	–	10.066^{*}
				(5.231)
$IAAP_{it} \cdot g(z_{it})$	–	–	–	-0.375
				(0.245)
$IALC_{it} \cdot m(z_{it})$	–	–	–	-0.395
				(0.976)
Constant	75.050^{***}	57.826^{***}	90.265^{***}	75.288^{***}
	(7.453)	(11.524)	(12.725)	(5.985)
Observations	1450	710	750	5830
Left Censored	143	9	196	514
Right Censored	2	0	0	3
Log Likelihood	-7115.757	-3546.116	-3139.862	-27697.128

*Significant at the 0.10 level.
*Significant at the 0.05 level.
*Significant at the 0.01 level.

Table 22: Estimated bid functions for lottery contests with gender interaction (standard errors in parentheses)

	SLC	Informed ALC	Uninformed ALC	Joint
z_{it}	-0.012	0.638^{***}	0.016	0.014
	(0.015)	(0.198)	(0.025)	(0.019)
$\ln(1+t)$	-9.549^{***}	-19.347^{***}	-9.205^{***}	-4.002^{**}
	(1.854)	(2.731)	(3.258)	(2.080)
M_i	3.854	-37.162^{***}	-26.001^{**}	-8.606
	(6.305)	(9.191)	(10.443)	(7.930)
$M_i \cdot \ln(1+t)$	2.656	9.944^{***}	6.687^{*}	-3.368
	(2.347)	(3.411)	(3.887)	(2.954)
$g(z_{it})$	–	–	–	–
$m(z_{it})$	–	-0.401	–	–
		(0.768)		
$IAAP_{it}$	–	–	–	-17.630
				(13.667)
$UAAP_{it}$	–	–	–	13.976
				(11.512)
SLC_{it}	–	–	–	-8.031
				(8.964)
$IALC_{it}$	–	–	–	-18.186
				(11.866)
$UALC_{it}$	–	–	–	-14.653
				(11.307)
$IAAP_{it} \cdot z_{it}$	–	–	–	0.403^{***}
				(0.100)
$UAAP_{it} \cdot z_{it}$	–	–	–	-0.031
				(0.033)
$SLC_{it} \cdot z_{it}$	–	–	–	-0.027
				(0.027)
$IALC_{it} \cdot z_{it}$	–	–	–	0.622^{**}
				(0.253)
$UALC_{it} \cdot z_{it}$	–	–	–	-0.004
				(0.033)
$IAAP_{it} \cdot \ln(1+t)$	–	–	–	-22.204^{***}
				(4.071)
$UAAP_{it} \cdot \ln(1+t)$	–	–	–	-18.505^{***}
				(4.021)
$SLC_{it} \cdot \ln(1+t)$	–	–	–	-5.807^{*}
				(3.169)
$IALC_{it} \cdot \ln(1+t)$	–	–	–	-15.386^{***}
				(4.047)
$UALC_{it} \cdot \ln(1+t)$	–	–	–	-5.136
				(4.013)
$IAAP_{it} \cdot M_i$	–	–	–	-12.312
				(14.066)
$UAAP_{it} \cdot M_i$	–	–	–	-3.782
				(14.125)
$SLC_{it} \cdot M_i$	–	–	–	12.120
				(11.356)
$IALC_{it} \cdot M_i$	–	–	–	-28.695^{**}
				(14.122)
$UALC_{it} \cdot M_i$	–	–	–	-17.367
				(14.046)
$IAAP_{it} \cdot M_i \cdot \ln(1+t)$	–	–	–	8.631
				(5.256)
$UAAP_{it} \cdot M_i \cdot \ln(1+t)$	–	–	–	7.476
				(5.250)
$SLC_{it} \cdot M_i \cdot \ln(1+t)$	–	–	–	6.315
				(4.229)
$IALC_{it} \cdot M_i \cdot \ln(1+t)$	–	–	–	13.372
				(5.247)
$UALC_{it} \cdot M_i \cdot \ln(1+t)$	–	–	–	10.066^{*}
				(5.231)
$IAAP_{it} \cdot g(z_{it})$	–	–	–	-0.375
				(0.245)
$IALC_{it} \cdot m(z_{it})$	–	–	–	-0.395
				(0.976)
Constant	67.162^{***}	57.033^{***}	60.523^{***}	75.288^{***}
	(5.176)	(8.059)	(9.102)	(5.985)
Observations	1460	710	750	5830
Left Censored	85	2	79	514
Right Censored	0	0	1	3
Log Likelihood	-6781.935	-3450.154	-3440.149	-27697.128

*Significant at the 0.10 level.
*Significant at the 0.05 level.
*Significant at the 0.01 level.

bidders. Also of interest is the fact that the nonlinear part of the informed AAP bidder's bid function $(g(z_{it}))$ is not significant. A similar result is found for informed ALC bidders; the coefficient of the signal is positive and highly significant, and the nonlinear informed ALC bidder's bid function $(m(z_{it}))$ in not significant. As such, it is clear that informed bidder's bid function is linear in their signals, contrary to theory.

Interestingly, the results regarding learning differ substantially across treatments, when we do not include the gender and learning interaction. In SAP auctions, participants learn relatively slowly to reduce their bids as they gain experience. The same holds for SLC bidders. The fact that SLC bidders learn slowly is surprising, since they are, on average, bidding more than equilibrium predictions. However, as discussed above, SLC bidders are typically not bidding more than the break-even bidding strategy. As such, most SLC bidders are earning positive payoffs on average. These average positive payoffs are less likely to reduce bidding behavior than negative payoffs.

In stark contrast, informed AAP and informed ALC bidders learn to reduce their bids much faster than SAP and SLC bidders. We attribute this to the fact that these informed bidders are much more prone to bid above the break-even bidding strategy than are uninformed bidders. The resulting negative payoffs provides a strong incentive for these bidders to reduce their bids. It is important to recall that when informed bidders observe a high signal, they bid above the break-even bidding strategy infrequently. When they observe a small signal, the probability of obtaining the good is small, because the uninformed bidder cannot take the low value of the good into account when choosing her bid. If the informed bidder does not take this into account by, in some sense, ceding the contest to the uninformed bidder she is likely to bid such that she loses the contest and still must pay her bid. This process is, for the most part, the mechanism through which informed bidders learn to reduce their bids. Notice that this allows the average payoff of the informed bidders to be quite high (since they are likely to earn a substantial payoff for high signal values), while still facing negative payoffs which induce learning that is quicker than that of uninformed bidders.

Also, notice that uninformed AAP bidders learn to reduce their bids faster than SAP bidders, but uninformed ALC bidders do not. This is attributable to the fact that, on average, uninformed AAP bidders quickly learn that when they obtain the good, it is because the informed AAP bidder has observed that it is low valued. This induces the uninformed AAP bidders to reduce their bids faster than SAP bidders, who do not face this "winner's curse." On the other hand, an uninformed ALC bidder has a positive probability of obtaining the good, regardless of the informed ALC bidder's bid, provided she has submitted a positive bid of her own.[54] As such, uninformed ALC bidders often obtain the good, and earn a substantial payoff in the process. Consequently, they have less incentive to reduce their bids than the uninformed AAP bidders.

Interestingly, when we do not include the gender and learning interaction, there are significant gender differences. In particular, notice that women bid more than men everywhere except in symmetric information lottery contests (although the magnitude of this difference is quite small in the case of uninformed AAP bidders). Clearly this fact is not simply a consequence of the imperfectly discriminating contest success function; women bid more than men in asymmetric information lottery contests, regardless of whether or not they are informed.

Notice that when we include the gender and learning interaction, it is not signif-

[54]Note that this argument neglects the boundary case in which neither bidder submits a positve bid. As this case does not arise in our data, there is no need to consider it.

icant in all-pay auctions, regardless of the information structure. Indeed, inclusion of this interaction renders the gender dummy insignificant for SAP and uninformed AAP bidders, and only marginally significant for informed AAP bidders. Further, note that when we include the gender and learning interaction, the gender dummy in the SLC treatment is also no longer significant.

In contrast, note that inclusion of this gender and learning interaction does not render the gender dummy insignificant for ALC bidders, regardless of whether or not they are informed. Indeed, the magnitude of the coefficients has increased. Also, the gender and learning interaction itself is significant for informed and uninformed ALC bidders. That is, we find that in asymmetric information lottery contests, women bid more than men, but also learn faster. This result does not extend to other treatments.

CHAPTER V

CONCLUSION

In Chapter II I examine the case where the values of the prizes are positively related in a twice repeated imperfectly discriminating contest. If the incumbent privately observes the value of the prize in the first contest, then she is better informed than the challengers in the subsequent contest.

I find that in the second contest, the incumbent has a strictly lower ex ante probability of obtaining the prize than a challenger, despite expending (weakly) more effort than a challenger in expectation. The incumbent expends low effort for low values of the prize and high effort for high values of the prize; the incumbent's low probability of obtaining the prize when its value is low is such that the ex ante probability of obtaining the prize is lower than that of a challenger.

Since the incumbent expends low effort for low values of the prize, the challengers face an analogue of the winner's curse, and reduce their second period effort expenditures relative to the symmetric information case as a result. This is sufficient to reduce aggregate effort expenditure in the second contest relative to the IIV case, despite the fact that the incumbent's expected effort expenditures may have increased relative to the IIV case.

The incumbent's ex ante expected utility is strictly higher than in the IIV case; the incumbent obtains an information rent. This information rent creates an increased incentive to obtain the prize in the first contest, which increases aggregate effort expenditures in the first contest. This incentive is sufficiently high to increase total effort expenditure over both contests, offsetting the decrease in expected effort expenditure in $t = 2$ caused by the information asymmetry.

In Chapter III the role of asymmetric information in first-price common-value auctions is experimentally examined by varying the information available to bidders before placing their bids. We compare three information structures. In the first, no bidders hold any private information regarding the uncertain value of the good (SPUB). In the second, both bidders privately observe noisy signals regarding the value of the good (SPRIV). In the third, only one bidder observes a noisy signal; the other bidder does not hold private information (ASYM).

The most surprising result is that bidders who do not hold private information underbid relative to the Nash predictions, while bidders who hold private information overbid relative to the Nash predictions. Indeed the underbidding by uninformed bidders is dramatic. Bidders in the SPUB treatment bid 42% less than predicted by theory. Overbidding by informed bidders is a widely observed phenomenon in laboratory experiments, but the behavior of uninformed bidders has not been studied previous to this paper. Our results suggest that the overbidding typically observed may be an artifact of the private signal that is typically provided to subjects. As such, our result offer support for the hypothesis that “a little knowledge is a dangerous thing.” That is, people who have a little information become overconfident.

Our results have significant implications regarding the widespread observation of the winner’s curse in common-value auctions. In particular, we find that the winner’s curse is almost entirely eliminated when bidders are not given private information. In addition, the winner’s curse is largely eliminated when only one of the bidder’s holds private information. This is despite the fact that the informed bidder overbids.

The observed bidding behavior also has significant effects on bidder payoffs. In particular, when neither bidder holds private information, bidders earn a substantial payoff, on average. When bidders both hold private information, bidder payoffs are positive, but quite small as a result of informed overbidding relative to Nash predictions. Note that informed ASYM bidders earn, on average, more than predicted despite overbidding relative to the Nash predictions.

Additionally, the observed bidding behavior has significant effects on the revenue ranking of the three information structures studied. Namely, the SPUB auction, which is predicted to have the highest revenue, is observed to have the lowest revenue because the uninformed bidders underbid. However, when both bidders hold private information, revenue is higher than when only one bidder holds private information, as predicted.

In Chapter IV the role of asymmetric information in two types of contests is examined: all-pay auctions and lottery contests. In particular, we examine these contests in a common-value environment in which there is uncertainty regarding the value of the good. We employ a 2×2 between subject design which varies the information structure of the game and the contest success function. In the symmetric information structure, neither bidder observes a signal regarding the value of the good; both bidder know only the distribution from which the value is drawn. In the asymmetric information structure, one of the bidders is randomly chosen to privately observe a signal in the form of a noisy estimate of the value of the good. The other bidder does not observe a signal, and holds no private information. The two contest success functions we utilize in our design represent opposite extremes of discrimination. At one end, there is perfectly discriminating contest success function, which allocates the good to the bidder with the highest bid with certainty. At the other, there is the lottery contest success function which allocates the good to each bidder with probability equal to her proportion of the sum of bids.

In addition to the 2×2 design outlined above, we also ran sessions in which participants played a series of all-pay auctions where both bidders observe a private signal. While we do not have theoretical predictions for this game, behavior in this environment is of interest in light of the fact that bidders who observe a signal in first-price auctions are much more prone to bid above their break-even bid, regardless of whether or not their opponent observed a signal (Chapter III). As such, we ran these additional sessions to compare behavior in all-pay auctions to behavior in first-price auctions.

Perhaps the most interesting result is that bidders in asymmetric information treatments who observe a signal are much more prone to bid above their break-even bidding strategy than are bidders who do not observe a signal. Similarly, we find that when both bidders in an all-pay auction observe a signal, they are much more likely to bid above their break-even bidding strategy than are bidders who do not observe a signal. As such, the results of Chapter III do extend to all-pay auctions.

We also find that when neither bidder observes a signal, all-pay auctions generate more revenue than lottery contests. Consequently, bidders in such all-pay auctions earn more than bidders in lottery contests, on average. Interestingly the same does not hold when information is asymmetric. We are unable to reject revenue equivalence between asymmetric information all-pay auctions and asymmetric information lottery contests. Further, we are unable to reject payoff equivalence between uninformed bidders in these two asymmetric information games. Likewise, we are also unable to reject payoff equivalence between the informed bidders in these asymmetric information games.

Another interesting result we find is that, in asymmetric information lottery contests, women bid significantly more than men in early periods, but learn at a faster rate than men such that behavior converges in later periods. This result does not extend to the other treatments.

Our results suggest several questions which provide avenues for future research. First, what induces informed bidders to overbid so dramatically? Is it that the information is privately observed? Second,.what happens to behavior as the quality of the signal decreases? Third, does the observed revenue equivalence in the asymmetric information treatments extend to other games? Lastly, how much are bidders willing to pay for a signal? Could a seller increase revenue by selling signals?

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APPENDIX A

This appendix contains proof of the propositions in Chapter II.

Proof of Proposition 1

Define the function

$$
\begin{aligned}
g(x) \quad \equiv \quad & \frac{(n-2)}{x\,(n-1)} \int_{\underline{v}}^{q(x(n-1))} \int_{\underline{v}}^{\infty} v_2 f(v_1, v_2) dv_2 dv_1 \\
& + \sqrt{\frac{(n-1)}{x}} \int_{q(x(n-1))}^{\infty} \int_{\underline{v}}^{\infty} \sqrt{E\left(V_2 \mid v_1\right)} f(v_1, v_2) dv_2 dv_1 \\
& + F_V\left(q\left(x\left(n-1\right)\right)\right) - n.
\end{aligned}
$$

Notice that $g(x) = 0$ satisfies (3), which defines an equilibrium. Note that $q(\frac{\underline{v}}{n-1}) = \underline{v}$.

$$
\begin{aligned}
g\left(\frac{\underline{v}}{n-1}\right) \quad &= \quad \sqrt{\frac{(n-1)}{\left(\frac{\underline{v}}{n-1}\right)}} \int_{\underline{v}}^{\infty} \sqrt{E\left(V_2 \mid v_1\right)} dF_V(v_1) - n \\
&= \quad \frac{(n-1)}{\sqrt{\underline{v}}} \int_{\underline{v}}^{\infty} \sqrt{E\left(V_2 \mid v_1\right)} dF_V(v_1) - n
\end{aligned}
$$

Now, suppose that $g(\frac{\underline{v}}{n-1}) \leq 0$. In this case, $\left(\frac{(n-1)\left(E\left(\sqrt{E(V_2|V_1)}\right)\right)}{n}\right)^2 \leq \underline{v} \leq E\left(V_2 \mid \underline{v}\right)$. That is, there is interior equilibrium. If $g(\frac{\underline{v}}{n-1}) > 0$, there need not be an interior equilibrium. However,

$$
\lim_{x \to \infty} g(x) = 1 - n < 0.
$$

Thus, either there is an interior equilibrium, or the intermediate value theorem assures at least one finite value of x where $g(x) = 0$. If there is an interior equilibrium, then it has a unique closed form solution. To prove the uniqueness of a non-interior equilibrium note that:

$$\begin{aligned}\frac{\partial g(x)}{\partial x} = {} & \frac{(n-2)(n-1)\int_{\underline{v}}^{\infty} v_2 f(q(x(n-1)), v_2) dv_2 q'(x(n-1))}{x(n-1)} \\ & - f_V(q(x(n-1)))q'(x(n-1))(n-1)^2 \\ & + f_V(q(x(n-1)))q'(x(n-1))(n-1) \\ & - \frac{\sqrt{(n-1)} \int_{q(x(n-1))}^{\infty} \int_{\underline{v}}^{\infty} \sqrt{E(V_2 \mid v_1)} f(v_1, v_2) dv_2 dv_1}{2x^{\frac{3}{2}}} \\ & - \frac{(n-2) \int_{\underline{v}}^{q(x(n-1))} \int_{\underline{v}}^{\infty} v_2 f(v_1, v_2) dv_2 dv_1}{x^2 (n-1)}.\end{aligned}$$

If there is not an interior equilibrium, $E(V_2 \mid q(x(n-1))) = x(n-1)$. Using this to reduce the above expression yields:

$$\begin{aligned}\frac{\partial g(x)}{\partial x} = {} & - \frac{\sqrt{(n-1)} \int_{q(x(n-1))}^{\infty} \int_{\underline{v}}^{\infty} \sqrt{E(V_2 \mid v_1)} f(v_1, v_2) dv_2 dv_1}{2x^{\frac{3}{2}}} \\ & - \frac{(n-2) \int_{\underline{v}}^{q(x(n-1))} \int_{\underline{v}}^{\infty} v_2 f(v_1, v_2) dv_2 dv_1}{x^2 (n-1)}.\end{aligned}$$

Since this expression is negative, $g(x)$ is monotonically decreasing in x, which means that the equilibrium whose existence was shown above is unique. $\square$

Proof of Proposition 2

Rearranging (3), which characterizes equilibrium effort yields:

$$\begin{aligned} x_{C2}^{IDV}-E\left(x_{I2}^{IDV}\left(v_1\right)\right) &= \left(\frac{n-2}{n-1}\right)E\left(V_2\mathbf{1}_{V_1\leq q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\ &\quad -x_{C2}^{IDV}F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right)\left(n-2\right) \end{aligned}$$

Note that the right hand side of this equation is equal to zero if $n=2$, or if the equilibrium in interior ($q\left(x_{C2}^{IDV}\left(n-1\right)\right)=\underline{v}$), yielding the desired result. Now suppose the equilibrium is not interior, $n>2$, and that $x_{C2}^{IDV}\geq E\left(x_{I2}^{IDV}\left(v_1\right)\right)$. This implies that:

$$\left(\frac{n-2}{n-1}\right)E\left(V_2\mathbf{1}_{V_1\leq q\left(x_{C2}^{IDV}(n-1)\right)}\right)\geq x_{C2}^{IDV}F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right)\left(n-2\right).$$

This simplifies to

$$\begin{aligned} E\left(V_2\mid V_1\leq q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right) &\geq x_{C2}^{IDV}\left(n-1\right) \\ &= E\left(V_2\mid q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right). \end{aligned}$$

Since $E\left(V_2\mid v_1\right)$ is strictly increasing in v_1, this is a contradiction.□

Proof of Proposition 3

First consider the case where $n=2$, or there is an interior equilibrium. Recall that, when $n=2$, or there is an interior equilibrium, $x_{C2}^{IDV}=E\left(x_{I2}^{IDV}\left(v_1\right)\right)$. In this case, note that the probability contestant $j\in\mathbf{C}$ will obtain the prize, $p_{j2}\left(x_{I2}\left(v_1\right),\mathbf{x_C}\right)=\frac{x_{j2}}{\left(x_{i2}(v_1)+x_{j2}+\sum_{k\in\mathbf{C}_{-j}}x_{k2}\right)}$, is strictly convex in $x_{I2}\left(v_1\right)$. Jensen's Inequality yields:

$$E\left(p_{j2}\left(x_{I2}\left(v_1\right),\mathbf{x_C}\right)\right)>p_{j2}\left(Ex_{I2}^{IDV}\left(v_1\right),\mathbf{x_C}\right).$$

Further, since $p_{I2}\left(x_{i2}\left(v_1\right),\mathbf{x_C}\right) = \frac{x_{I2}(v_1)}{\left(x_{I2}(v_1)+\sum_{k\in\mathbf{C}} x_{k2}\right)}$ is strictly concave in $x_{I2}\left(v_1\right)$, Jensen's Inequality also tells us that:

$$E\left(p_{I2}\left(x_{I2}^{IDV}\left(v_1\right),\mathbf{x_C}\right)\right) < p_{I2}\left(E\left(x_{I2}^{IDV}\left(v_1\right)\right),\mathbf{x_C}\right).$$

Dividing both sides of $x_{C2}^{IDV} = E\left(x_{I2}^{IDV}\left(v_1\right)\right)$ by $E\left(x_{I2}^{IDV}\left(v_1\right)\right) + (n-1)\,x_{C2}^{IDV}$, and using the above inequalities yields:

$$\begin{aligned} E\left(p_{j2}\left(x_{I2}\left(v_1\right),\mathbf{x_C}\right)\right) &> p_{j2}\left(E\left(x_{I2}^{IDV}\left(v_1\right)\right),\mathbf{x_C}\right) \\ &= p_{I2}\left(E\left(x_{I2}^{IDV}\left(v_1\right)\right),\mathbf{x_C}\right) \\ &> E\left(p_{I2}\left(x_{I2}\left(v_1\right),\mathbf{x_C}\right)\right). \end{aligned}$$

When $n > 2$ and there is not an interior equilibrium $x_{C2}^{IDV} \leq E\left(x_{I2}^{IDV}\left(v_1\right)\right)$. The ex ante probability that the incumbent obtains the good is given by

$$\begin{aligned} & E\left(p_{I2}\left(x_{I2}\left(v_1\right),\mathbf{x_C}\right)\right) \\ = & \left(1 - F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right)\right) \\ & -\sqrt{x_{C2}^{IDV}\left(n-1\right)}E\left(\frac{1}{\sqrt{E\left(V_2 \mid V_1\right)}}\mathbf{1}_{V_1 \geq q\left(x_{C2}^{IDV}(n-1)\right)}\right). \end{aligned}$$

The ex ante probability that a challenger $j \in \mathbf{C}$ obtains the good is given by

$$\begin{aligned} & E\left(p_{j2}\left(x_{I2}\left(v_1\right),\mathbf{x_C}\right)\right) \\ = & \frac{F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right)}{(n-1)} \\ & +\sqrt{\frac{x_{C2}^{IDV}}{(n-1)}}E\left(\frac{1}{\sqrt{E\left(V_2 \mid V_1\right)}}\mathbf{1}_{V_1 \geq q\left(x_{C2}^{IDV}(n-1)\right)}\right). \end{aligned}$$

Suppose that $E\left(p_{j2}\left(x_{I2}\left(v_1\right),\mathbf{x_C}\right)\right) < E\left(p_{I2}\left(x_{I2}\left(v_1\right),\mathbf{x_C}\right)\right)$. This simplifies to

$$\begin{aligned}
& 1-\frac{1}{n\left(\left(1-F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right)\right)\right)} \\
> \; & \sqrt{x_{C2}^{IDV}\left(n-1\right)}E\left(\frac{1}{\sqrt{E\left(V_2\mid V_1\right)}}|V_1 \geq q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right) \\
> \; & 1.
\end{aligned}$$

This is a contradiction.□

Proof of Proposition 4

Notice that, when $n=2$, Jensen's inequality implies that (4) holds. Further, notice that (4) states that $g\left(\frac{E(V)(n-1)}{n^2}\right) < 0$ ($g\left(x\right)$ was defined in the proof of Proposition 1). Recall that in the proof of Proposition 1 it was shown that $g\left(x\right)$ is a monotonically decreasing function, and that $g\left(x\right)=0$ defines the unique equilibrium of the game. So if $x_{C2}^{IDV} < x_{i2}^{IIV} = \frac{E(V)(n-1)}{n^2}$, then $g\left(x_{C2}^{IDV}\right) > g\left(x_{i2}^{IIV}\right) = g\left(\frac{E(V)(n-1)}{n^2}\right)$. Since $g\left(x_{C2}^{IDV}\right)=0$ in equilibrium, $g\left(x_{i2}^{IIV}\right) = g\left(\frac{E(V)(n-1)}{n^2}\right) < 0$, which is the condition given in (4). To see that (4) implies $x_{C2}^{IDV} < x_{i2}^{IIV}$, consider $g\left(x_{i2}^{IIV}\right) = g\left(\frac{E(V)(n-1)}{n^2}\right) < 0$. Since $g\left(x_{C2}^{IDV}\right)=0$, and $g\left(x\right)$ is monotonically decreasing in x, it must be the case that $x_{C2}^{IDV} < x_{i2}^{IIV}$.□

Proof of Proposition 5

Define the following function:

$$h(x) \equiv \frac{1}{\left(n-1\right)^2}E\left(V_2\mathbf{1}_{V_1\leq q(x(n-1))}\right)+\frac{x\left(1-F_V(q\left(x\left(n-1\right)\right))\right)}{\left(n-1\right)}.$$

Note that:

$$
\begin{aligned}
h'(x) &= \frac{1}{(n-1)}\int_{\underline{v}}^{\infty} v_2 f\left(q\left(x\left(n-1\right)\right), v_2\right) q'\left(x\left(n-1\right)\right) dv_2 \\
&\quad + \frac{\left(1-F_V(q\left(x\left(n-1\right)\right))\right)}{(n-1)} - \int_{\underline{v}}^{\infty} x f\left(q\left(x\left(n-1\right)\right), v_2\right) q'\left(x\left(n-1\right)\right) dv_2
\end{aligned}
$$

But if $x\left(n-1\right) > \underline{v}$, then $x\left(n-1\right) = E\left(V_2 \mid q\left(x\left(n-1\right)\right)\right)$. Plugging this in simplifies this expression down to the following:

$$
h'(x) \geq \frac{\left(1-F_V(q\left(x\left(n-1\right)\right))\right)}{(n-1)} > 0
$$

Since $h'(x) > 0$, and (4) is satisfied my assumption, $x_{C2}^{IDV} < \frac{E(V)(n-1)}{n^2} = x_{i2}^{IIV}$. Thus, $h\left(x_{C2}^{IDV}\right) < h(\frac{E(V)(n-1)}{n^2})$. Note that (where the second line follows from the definition of conditional probability):

$$
\begin{aligned}
h\left(\frac{E\left(V\right)\left(n-1\right)}{n^2}\right) &= \frac{1}{\left(n-1\right)^2}\int_{\underline{v}}^{\infty}\int_{\underline{v}}^{q(B)} v_2 f(v_1, v_2) dv_1 dv_2 + \frac{E\left(V\right)}{n^2}\left(1-F_V\left(q\left(B\right)\right)\right) \\
&= \frac{F_V\left(q\left(B\right)\right)}{\left(n-1\right)^2} E\left(V_2 \mid V_1 \leq q\left(B\right)\right) + \frac{E\left(V_2\right)}{n^2}\left(1-F_V\left(q\left(B\right)\right)\right) \\
&\leq \frac{F_V\left(q\left(B\right)\right)}{\left(n-1\right)^2}\frac{E\left(V\right)\left(n-1\right)^2}{n^2} + \frac{E\left(V_2\right)}{n^2}\left(1-F_V\left(q\left(B\right)\right)\right) \\
&= \frac{E\left(V\right)}{n^2} F_V\left(q\left(B\right)\right) + \frac{E\left(V_2\right)}{n^2}\left(1-F_V\left(q\left(B\right)\right)\right) \\
&= \frac{E\left(V\right)}{n^2}\square.
\end{aligned}
$$

Proof of Proposition 6

Notice that $E\left(U_{i2}^{IIV}\right) < E\left(U_{I2}^{IDV}\right)$ when

$$E\left(V\right) + \left(\frac{n-3}{n-1}\right) E\left(V_2 \mathbf{1}_{V_1 < q\left(x_{C2}^{IDV}(n-1)\right)}\right)$$
$$-x_{C2}^{IDV}\left(n+1\right) - xF_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right)\left(n-3\right) > \frac{E\left(V\right)}{n^2}.$$

This expression can be rewritten as

$$\left(E\left(U_{i2}^{IIV}\right) - E\left(U_{C2}^{IDV}\right)\right)\left(n-1\right) > R_2^{IDV} - R_2^{IIV}.$$

Similarly, $E\left(U_{i2}^{IIV}\right) > E\left(U_{I2}^{IDV}\right)$ when

$$\left(E\left(U_{i2}^{IIV}\right) - E\left(U_{C2}^{IDV}\right)\right)\left(n-1\right) < R_2^{IDV} - R_2^{IIV}.$$

Likewise, $E\left(U_{i2}^{IIV}\right) = E\left(U_{I2}^{IDV}\right)$ when

$$\left(E\left(U_{i2}^{IIV}\right) - E\left(U_{C2}^{IDV}\right)\right)\left(n-1\right) = R_2^{IDV} - R_2^{IIV}.$$

Define the function

$$r\left(x\right) \equiv \sqrt{x\left(n-1\right)}E\left(\sqrt{V_2 \mathbf{1}_{V_1 \geq q(x(n-1))}}\right) + x\left(n-1\right)F_V\left(q\left(x\left(n-1\right)\right)\right),$$

which corresponds to R_2^{IDV}, and

$$w\left(x\right) = \left(\frac{1}{n-1}\right) E\left(V_2 \mathbf{1}_{V_1 < q(x(n-1))}\right) + x\left(1 - F_V\left(q\left(x\left(n-1\right)\right)\right)\right),$$

which corresponds to $E\left(U_{C2}^{IDV}\right)\left(n-1\right)$. Note that

$$r'\left(x\right) = \left(n-1\right)F_V\left(q\left(x\left(n-1\right)\right)\right) + \frac{1}{2}\sqrt{\frac{n-1}{x}}E\left(\sqrt{V_2 \mathbf{1}_{V_1 \geq q(x(n-1))}}\right),$$

and that

$$w'(x) = 1 - F_V(q(x(n-1))).$$

Now notice that $r'(x) > w'(x) > 0$. Since $r(x)$ and $w(x)$ are both strictly monotonically increasing, and $r'(x) > w'(x)$, the expressions $E\left(U_{i2}^{IIV}\right)(n-1) - w(x)$ and $r(x) - R_2^{IIV}$ intersect only once. Let $\widetilde{x} \equiv \left\{x : r(x) - R_2^{IIV} = E\left(U_{i2}^{IIV}\right)(n-1) - w(x)\right\}$, which has a single element. Notice that if $x_{C2}^{IDV} = \widetilde{x}$, then the IDV incumbent's expected utility in the IDV case is the same as in the IIV case. It has been proven that $E\left(U_{C2}^{IDV}\right) < E\left(U_{i2}^{IIV}\right)$, which implies that $x_{C2}^{IDV} < \widetilde{x}$. Thus, $E\left(U_{i2}^{IIV}\right)(n-1) - w\left(x_{C2}^{IDV}\right) > E\left(U_{i2}^{IIV}\right)(n-1) - w(\widetilde{x})$. Also, $x_{C2}^{IDV} < \widetilde{x}$ implies that $r\left(x_{C2}^{IDV}\right) - R_2^{IIV} < r(\widetilde{x}) - R_2^{IIV}$. Since $r'(x) > w'(x) > 0$,

$$\begin{aligned} & E\left(U_{i2}^{IIV}\right)(n-1) - w\left(x_{C2}^{IDV}\right) - E\left(U_{i2}^{IIV}\right)(n-1) - w(\widetilde{x}) \\ < \;& r(\widetilde{x}) - R_2^{IIV} - \left(r\left(x_{C2}^{IDV}\right) - R_2^{IIV}\right). \end{aligned}$$

This simplifies to

$$\begin{aligned} r\left(x_{C2}^{IDV}\right) + w\left(x_{C2}^{IDV}\right) &< w(\widetilde{x}) + r(\widetilde{x}) \\ &= E\left(U_{i2}^{IIV}\right)(n-1) + R_2^{IIV}. \end{aligned}$$

That is, $\left(E\left(U_{i2}^{IIV}\right) - E\left(U_{C2}^{IDV}\right)\right)(n-1) > R_2^{IDV} - R_2^{IIV}$.□

Proof of Proposition 7

Suppose that $R_2^{IDV} > R_2^{IIV}$. Since $E\left(U_{I2}^{IDV}\right) > E\left(U_{i2}^{IIV}\right)$

$$R_2^{IIV} - R_2^{IDV} > \left(E\left(U_{C2}^{IDV}\right) - E\left(U_{i2}^{IIV}\right)\right)(n-1).$$

But this can be rewritten as

$$\begin{aligned} & \left(E\left(U_{i2}^{IIV}\right)-E\left(U_{C2}^{IDV}\right)\right)n\left(n-1\right)+2x_{C2}^{IDV}F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right) \\ & -2\left(\frac{1}{n-1}\right)E\left(V_2\mathbf{1}_{V_1\leq q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\ > \; & \left(E\left(U_{C2}^{IDV}\right)-E\left(U_{i2}^{IIV}\right)\right)\left(n-1\right). \end{aligned}$$

Since, $x_{C2}^{IDV}F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right) \geq \left(\frac{1}{n-1}\right)E\left(V_2\mathbf{1}_{V_1\leq q\left(x_{C2}^{IDV}(n-1)\right)}\right)$, and $E\left(U_{C2}^{IDV}\right) < E\left(U_{i2}^{IIV}\right)$, the LHS of this inequality is positive. $R_2^{IDV} > R_2^{IIV}$ implies that the LHS is negative, a contradiction.□

Proof of Proposition 8

In equilibrium, the difference between the IDV incumbent's ex ante expected utility and that of the challenger is:

$$\begin{aligned} E\left(U_{I2}^{IDV}\right)-E\left(U_{C2}^{IDV}\right) &= E\left(V\right)+\left(\frac{n-3}{n-1}\right)E\left(V_2\mathbf{1}_{V_1\leq q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\ & -\frac{1}{\left(n-1\right)^2}E\left(V_2\mathbf{1}_{V_1\leq q\left(x_{C2}^{IDV}(n-1)\right)}\right) \\ & -\frac{x_{C2}^{IDV}\left(1-F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right)\right)}{\left(n-1\right)}. \end{aligned}$$

Notice that total effort expenditure in the IPV case will increase relative to the IIV case if:

$$\begin{aligned} \frac{2E\left(V\right)\left(n-1\right)}{n} &\leq \frac{\left(E\left(V\right)+E\left(U_{I2}^{IDV}\right)-E\left(U_{C2}^{IDV}\right)\right)\left(n-1\right)}{n}+ \\ & nx_{C2}^{IDV}+x_{C2}^{IDV}F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right)\left(n-2\right) \\ & -\left(\frac{n-2}{n-1}\right)E\left(V_2\mathbf{1}_{V_1\leq q\left(x_{C2}^{IDV}(n-1)\right)}\right). \end{aligned}$$

This condition simplifies to:

$$E\left(V_2 \mathbf{1}_{V_1 \leq q\left(x_{C2}^{IDV}(n-1)\right)}\right) \leq F_V\left(q\left(x_{C2}^{IDV}(n-1)\right)\right) x_{C2}^{IDV}(n-1).$$

Since $E(V_2 \mid v_1)$ is strictly increasing in v_1 the inequality is strict if the equilibrium in $t=2$ is not interior. If the equilibrium is interior, then $R^{IDV} = R^{IIV}$.□

Proof of Proposition 9

Recall that $g(x)=0$ satisfies (3), which defines an equilibrium

$$\begin{aligned} g(x) &= \frac{(n-2)}{x(n-1)} \int_{\underline{v}}^{q(x(n-1))} \int_{\underline{v}}^{\infty} v_2 f(v_1,v_2) dv_2 dv_1 \\ &+ \sqrt{\frac{(n-1)}{x}} \int_{q(x(n-1))}^{\infty} \int_{\underline{v}}^{\infty} \sqrt{E(V_2 \mid v_1)} f(v_1,v_2) dv_2 dv_1 \\ &+ F_V(q(x(n-1))) - n. \end{aligned}$$

The partial derivative with respect to x is

$$\begin{aligned} \frac{\partial g}{\partial x} &= -\frac{\sqrt{(n-1)} \int_{q(x(n-1))}^{\infty} \int_{\underline{v}}^{\infty} \sqrt{E(V_2 \mid v_1)} f(v_1,v_2) dv_2 dv_1}{2x^{\frac{3}{2}}} \\ &- \frac{(n-2) \int_{\underline{v}}^{q(x(n-1))} \int_{\underline{v}}^{\infty} v_2 f(v_1,v_2) dv_2 dv_1}{x^2(n-1)} < 0. \end{aligned}$$

The partial derivative with respect to n is

$$\begin{aligned}
\frac{\partial g}{\partial n} &= \frac{1}{x\left(n-1^2\right)} \int\limits_{\underline{v}}^{q(x(n-1))} \int\limits_{\underline{v}}^{\infty} v_2 f(v_1, v_2) dv_2 dv_1 \\
&\quad + \frac{1}{2\sqrt{x\left(n-1\right)}} \int\limits_{q(x(n-1))}^{\infty} \int\limits_{\underline{v}}^{\infty} \sqrt{E\left(V_2 \mid v_1\right)} f(v_1, v_2) dv_2 dv_1 - 1.
\end{aligned}$$

(3) immediately demonstrates that this expression is negative. Since both of these partial derivatives are negative,

$$\frac{dx}{dn} = -\frac{\left(\frac{\partial g}{\partial n}\right)}{\left(\frac{\partial g}{\partial x}\right)} < 0.$$

That is $\frac{dx_{C2}^{IDV}}{dn} < 0$.

Next, note that

$$\begin{aligned}
\frac{\partial E\left(x_{I2}^{IDV}\left(V_1\right)\right)}{\partial n} &= x_{C2}^{IDV} F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right) - x \\
&\quad + \frac{1}{2}\sqrt{\frac{x_{C2}^{IDV}}{n-1}} \int\limits_{q(x(n-1))}^{\infty} \int\limits_{\underline{v}}^{\infty} \sqrt{E\left(V_2 \mid v_1\right)} f(v_1, v_2) dv_2 dv_1.
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E\left(x_{I2}^{IDV}\left(V_1\right)\right)}{\partial x_{C2}^{IDV}} &= \left(n-1\right) F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right) - n \\
&\quad + \frac{1}{2}\sqrt{\frac{n-1}{x_{C2}^{IDV}}} \int\limits_{q(x(n-1))}^{\infty} \int\limits_{\underline{v}}^{\infty} \sqrt{E\left(V_2 \mid v_1\right)} f(v_1, v_2) dv_2 dv_1.
\end{aligned}$$

Utilizing (3), it is straightforward to show that both of these are positive. Plugging these partial derivatives into

$$\frac{dE\left(x_{I2}^{IDV}\right)}{dn} = \frac{\partial E\left(x_{I2}^{IDV}\left(V_1\right)\right)}{\partial n} + \frac{\partial E\left(x_{I2}^{IDV}\left(V_1\right)\right)}{\partial x_{C2}^{IDV}} \frac{dx_{C2}^{IDV}}{dn},$$

and simplifying demonstrates that $\frac{dE\left(x_{I2}^{IDV}\right)}{dn} < 0$. Next, note that

$$\begin{aligned} \frac{\partial R_2^{IDV}}{\partial n} &= x_{C2}^{IDV} F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right) \\ &+ \frac{1}{2}\sqrt{\frac{x_{C2}^{IDV}}{n-1}} \int_{q(x(n-1))}^{\infty} \int_{\underline{v}}^{\infty} \sqrt{E\left(V_2 \mid v_1\right)} f(v_1, v_2) dv_2 dv_1. \end{aligned}$$

$$\begin{aligned} \frac{\partial R_2^{IDV}}{\partial x_{C2}^{IDV}} &= \left(n-1\right) F_V\left(q\left(x_{C2}^{IDV}\left(n-1\right)\right)\right) \\ &+ \frac{1}{2}\sqrt{\frac{n-1}{x_{C2}^{IDV}}} \int_{q(x(n-1))}^{\infty} \int_{\underline{v}}^{\infty} \sqrt{E\left(V_2 \mid v_1\right)} f(v_1, v_2) dv_2 dv_1. \end{aligned}$$

These partial derivatives are positive. Plugging them into

$$\frac{dR_2^{IDV}}{dn} = \frac{\partial R_2^{IDV}}{\partial n} + \frac{\partial R_2^{IDV}}{\partial x_{C2}^{IDV}} \frac{dx_{C2}^{IDV}}{dn},$$

and simplifying demonstrates that $\frac{dR_2^{IDV}}{dn} > 0$.$\square$

APPENDIX B

This appendix contains two alternative ways of modeling an incumbency advantage. Both of these maintain the information structure of the IIV case, such that information is complete in $t = 1, 2$.

Status Quo Bias (SQB)

One way in which an incumbent might have an advantage over a challenger is through an increased probability of winning the subsequent contest for any vector of effort $\mathbf{x}_2$. That is, by virtue of holding the high ground, the incumbent has an exogenously higher probability of winning than she would otherwise have. I call such an incumbency advantage a status quo bias.

Consider the case in which v_1 and v_2 are independent draws from the distribution F_V (the information structure found in the IIV case). To model a status quo bias, the contest success function is modified such that the probability that contestant i obtains the prize in $t = 2$ is now given by

$$\widetilde{p}_{i2}\left(x_{i2}, \mathbf{x}_{-i2}\right) = \frac{x_{i2} + \theta \mathbf{1}_{\{i=I\}}}{x_{i2} + \theta + \sum\limits_{j \in \mathbf{N}_{-i}} x_{j2}},$$

where $\theta > 0$ is added to the aggregate effort expenditures in $t = 2$, and the probability $\theta / \left(\theta + \sum_{i \in \mathbf{N}} x_{i2}\right) > 0$ represents the status quo bias. This is similar to the incumbent having a negative fixed cost of effort. However, it differs in that the incumbent is not awarded θ if she were to expend zero effort. Notice that $\theta / \left(\theta + \sum_{i \in \mathbf{N}} x_{i2}\right)$ is decreasing in $\sum_{i \in \mathbf{N}} x_{i2}$. This captures the idea that an incumbent has an increased probability of obtaining the prize in $t = 2$, but that challengers are at less of a disadvantage as they increase their effort. If $\sum_{i \in \mathbf{N}} x_{i2} = 0$, then the incumbent wins with certainty. As such there is no need to separately define the border case in which no contestant expends any effort. In $t = 1$, the contest success function is unchanged from that of the IIV and IDV cases.

I now turn attention to the incumbent's problem in the $t = 2$. (as before, player I is the incumbent). The incumbent's expected utility is

$$U_{I2}^{SQB} \equiv \int_{\underline{v}}^{\infty} \widetilde{p}_{i2}\left(x_{I2}, \mathbf{x}_C\right) v_2 dF_V(v_2) - x_{I2}.$$

The partial derivative is given by

$$\frac{E\left(V\right) \sum\limits_{j \in \mathbf{C}} x_{j2}}{\left(x_{I2} + \theta + \sum\limits_{j \in \mathbf{C}} x_{j2}\right)^2} - 1.$$

Similarly, the expected utility of contestant $j \in \mathbf{C}$ is

$$U_{j2}^{SQB} \equiv \int_{\underline{v}}^{\infty} \widetilde{p}_{j2}\left(x_{j2}, \mathbf{x}_{-j2}\right) v_2 dF_V(v_2) - x_{j2}$$

with partial derivative

$$\frac{E\left(V\right)\left(\sum\limits_{k \in \mathbf{C}_{-j}} x_{k2} + x_{I2} + \theta\right)}{\left(x_{I2} + \theta + \sum\limits_{j \in \mathbf{C}} x_{j2}\right)^2} - 1.$$

Reasoning identical to that used in the IDV case demonstrates that the challengers will exert the same amount of effort in equilibrium. In the SQB case, I denote equilibrium effort by the incumbent as x_{I2}^{SQB} and equilibrium effort of a challenger as x_{C2}^{SQB}. The magnitude of θ determines whether contestants will expend positive effort in equilibrium.

First, consider $\theta \geq E(V)$. Notice that when $\theta \geq E(V)$, a challenger's will optimally expend zero effort. Also, when $\sum_{j\in\mathbf{C}} x_{j2} = 0$, then the incumbent's best response is to expend zero effort because she will obtain the prize with certainty regardless of expenditure. Thus, when $\theta > E(V)$, $x_{I2}^{SQB} = x_{C2}^{SQB} = 0$. The intuition of this scenario is clear: when the incumbent has an advantage so significant that $x_{C2}^{SQB} \geq E(V)$ just to have an equal probability of winning the prize (even when the incumbent doesn't expend any effort), the challengers will not expend any effort. In this case, the incumbent obtains the prize with certainty. Thus, if $\theta \geq E(V)$, the ex ante value of obtaining the good in $t = 1$ is $2E(V)$.

Now consider $\theta \in [E(V)(n-1)/n^2, E(V))$. The status quo bias is significant enough that $x_{I2}^{SQB} = 0$. The first order condition of a challenger holds, and

$$x_{C2}^{SQB} = \frac{(n-2)E(V) - 2(n-1)\theta + \sqrt{(n-2)^2 E(V)^2 + 4E(V)(n-1)\theta}}{2(n-1)^2}.$$

In this case, the status quo bias is not so large that a challenger will not attempt to obtain the prize, but it is large enough that that the incumbent does not expend any effort. Notice that this is the case if $\theta \geq x_{i2}^{IIV}$.

Next, consider $\theta \in (0, E(V)(n-1)/n^2)$. Here every contestant's first order condition holds. Solving the set of n simultaneous equations yields equilibrium effort levels $x_{C2}^{SQB} = x_{i2}^{IIV}$, and $x_{I2}^{SQB} = x_{i2}^{IIV} - \theta$. Notice that $x_{I2}^{SQB} > 0$ only when $\theta < E(V)(n-1)/n^2$.

So, to summarize, the equilibrium effort levels of a challenger in $t = 2$ of the SQB case are given by

$$x_{C2}^{SQB} = \begin{cases} \frac{E(V_2)(n-1)}{n^2} & \text{if} \quad \theta \in \left(0, \frac{E(V)(n-1)}{n^2}\right) \\ \frac{(n-2)E(V)-2(n-1)\theta+\sqrt{(n-2)^2E(V)^2+4E(V)(n-1)\theta}}{2(n-1)^2} & \text{if} \quad \theta \in \left[\frac{E(V)(n-1)}{n^2}, E(V)\right) \\ 0 & \text{if} \quad \theta \in [E(V), \infty). \end{cases}$$

Notice that x_{C2}^{SQB} is decreasing in n when $\theta < E(V)$, and that $\lim_{n\to\infty} x_{C2}^{SQB} = 0$ as in the IIV case. The equilibrium effort expenditure of the incumbent is

$$x_{I2}^{SQB} = \begin{cases} \frac{E(V)(n-1)}{n^2} - \theta & \text{if} \quad \theta \in \left(0, \frac{E(V)(n-1)}{n^2}\right) \\ 0 & \text{if} \quad \theta \in \left[\frac{E(V)(n-1)}{n^2}, E(V)\right) \\ 0 & \text{if} \quad \theta \in [E(V), \infty). \end{cases}$$

Which is decreasing in θ and n when $\theta < E(V)(n-1)/n^2$. Since

$$\lim_{n\to\infty} E(V)(n-1)/n^2 = 0$$

for any $\theta > 0$ there exists some n large enough that $\theta > E(V)(n-1)/n^2$ and $x_{I2}^{SQB} = 0$ above this n. Therefore $\lim_{n\to\infty} x_{I2}^{SQB} = 0$.

The equilibrium aggregate effort expenditures in $t = 2$ of the SQB case, R_2^{SQB}, is given by

$$R_2^{SQB} = \begin{cases} \frac{E(V)(n-1)}{n} - \theta & \text{if} \quad \theta \in \left(0, \frac{E(V)(n-1)}{n^2}\right) \\ \frac{(n-2)E(V)-2(n-1)\theta+\sqrt{(n-2)^2E(V)^2+4E(V)(n-1)\theta}}{2(n-1)} & \text{if} \quad \theta \in \left[\frac{E(V)(n-1)}{n^2}, E(V)\right) \\ 0 & \text{if} \quad \theta \in [E(V), \infty). \end{cases}$$

Notice that R_2^{SQB} is decreasing in θ and n when $\theta < E(V)$. As such

$$\begin{aligned}\lim_{n\to\infty} R_2^{SQB} &= \lim_{n\to\infty} \frac{(n-2)E(V) - 2(n-1)\theta + \sqrt{(n-2)^2 E(V)^2 + 4E(V)(n-1)\theta}}{2(n-1)} \\ &= E(V) - \theta.\end{aligned}$$

The equilibrium expected utility of the incumbent in the SQB case is given by

$$U_{I2}^{SQB} = \begin{cases} \frac{E(V)}{n^2} + \theta & \text{if} \quad \theta \in \left(0, \frac{E(V)(n-1)}{n^2}\right) \\ \frac{\sqrt{(n-2)^2 E(V)^2 + 4E(V)(n-1)\theta} - (n-2)E(V)}{2} & \text{if} \quad \theta \in \left[\frac{E(V)(n-1)}{n^2}, E(V)\right) \\ E(V) & \text{if} \quad \theta \in [E(V), \infty). \end{cases}$$

When $\theta \in (0, E(V)(n-1)/n^2)$, the incumbent's expected utility has increased by exactly θ relative to the IIV case. For $\theta \in [E(V)(n-1)/n^2, E(V))$, the incumbent's expected utility is increasing at a decreasing rate in θ. Once $\theta \geq E(V)$, the status quo bias is so large that the incumbent wins the prize with certainty without expending any effort. As such increasing the magnitude of θ does not increase her expected utility. Likewise increasing n does not affect U_{I2}^{SQB} when $\theta \geq E(V)$. When $\theta < E(V)$, U_{I2}^{SQB} is decreasing in n. Because $\lim_{n\to\infty} E(V)(n-1)/n^2 = 0$, $\lim_{n\to\infty} U_{I2}^{SQB} = 0$.

The equilibrium expected utility of a challenger in the SQB case is given by

$$U_{C2}^{SQB} = \begin{cases} \frac{E(V)}{n^2} & \text{if} \quad \theta \in \left(0, \frac{E(V)(n-1)}{n^2}\right) \\ \frac{E(V)(n(n-2)+2) + 2\theta(n-1) - n\sqrt{(n-2)^2 E(V)^2 + 4E(V)(n-1)\theta}}{2(n-1)^2} & \text{if} \quad \theta \in \left[\frac{E(V)(n-1)}{n^2}, E(V)\right) \\ 0 & \text{if} \quad \theta \in [E(V), \infty). \end{cases}$$

Notice that when $\theta \in \left(0, E\left(V\right)\left(n-1\right)/n^2\right)$, $U_{C2}^{SQB} = U_{i2}^{IIV}$. For

$$\theta \in \left[E\left(V\right)\left(n-1\right)/n^2, E\left(V\right)\right)$$

the expected utility of a challenger is decreasing in θ. Once $\theta \geq E\left(V\right)$, a challenger does not obtain the prize with certainty, and has an expected utility of zero as a result. Notice that when $\theta < E\left(V\right)$, U_{C2}^{SQB} is decreasing in n and that $\lim_{n\to\infty} U_{C2}^{SQB} = 0$

An interesting result arises when $\theta \in \left(0, E\left(V\right)\left(n-1\right)/n^2\right]$. The expected utility of the incumbent has increased by θ relative to the IIV case, and the expected utility of a challenger remains unchanged relative to the benchmark case. Further, $R_2^{IIV} - R_2^{SQB} = \theta$. If a contest designer were concerned with the welfare of the contestants, and would also like to decrease total effort in $t = 2$, choosing $\theta = E\left(V\right)\left(n-1\right)/n^2$ reduces equilibrium effort expenditures, and strictly increases the expected utility of the incumbent without reducing the expected utility of the challengers. Put another way, in a one-shot game, where effort is a social bad, choosing $\theta = E\left(V\right)\left(n-1\right)/n^2$ Pareto dominates $\theta < E\left(V\right)\left(n-1\right)/n^2$.

Turning attention to $t = 1$, note that the incentives the contestants face in $t = 1$ will be different, depending on the magnitude of θ. Thus, each of the three cases outlined above must be considered individually. The expected utility of contestant i is

$$\begin{aligned} U_{i1}^{SQB} \equiv & \int_{\underline{v}}^{\infty} p\left(x_{i1}, \mathbf{x}_{-i1}\right) v_1 dF_V\left(v_1\right) \\ & -x_{i1} + \left(p\left(x_{i1}, \mathbf{x}_{-i1}\right)\left(U_{I2}^{SQB}\right)\right) \\ & + \left(1 - p\left(x_{i1}, \mathbf{x}_{-i1}\right)\right)\left(U_{C2}^{SQB}\right). \end{aligned}$$

The first period is, in essence, a contest in which the prize over which the contestants compete is $E(V) + \left(U_{I2}^{SQB} - U_{C2}^{SQB}\right)$. The unique and symmetric equilibrium involves every contestant $i \in \mathbf{N}$ expending

$$x_{i1}^{SQB} \equiv \frac{\left(E(V) + \left(U_{I2}^{SQB} - U_{C2}^{SQB}\right)\right)(n-1)}{n^2}$$

in $t = 1$. The equilibrium aggregate effort expenditures in $t = 1$ is then

$$R_1^{SQB} \equiv n x_{i1}^{SQB} = \frac{\left(E(V) + \left(U_{I2}^{SQB} - U_{C2}^{SQB}\right)\right)(n-1)}{n}$$

and the equilibrium expected utility of contestant i in $t = 1$ is

$$U_{i1}^{SQB} = \frac{\left(E(V) + \left(U_{I2}^{SQB} - U_{C2}^{SQB}\right)\right)}{n^2}.$$

Total equilibrium effort expenditures across both periods is given by

$$R^{SQB} \equiv \begin{cases} \frac{2E(V)(n-1)}{n} - \frac{\theta}{n} & \text{if} \quad \theta \in \left(0, \frac{E(V)(n-1)}{n^2}\right) \\ \frac{\left(E(V)+\left(U_{I2}^{SQB}-U_{C2}^{SQB}\right)\right)(n-1)}{n} + (n-1)\, x_{C2}^{SQB} + x_{I2}^{SQB} & \text{if} \quad \theta \in \left[\frac{E(V)(n-1)}{n^2}, E(V)\right) \\ \frac{E(V)(n-1)}{n} & \text{if} \quad \theta \in [E(V), \infty). \end{cases}$$

When $\theta \in [E(V)(n-1)/n^2, E(V))$, I have not simplified R^{SQB} due to space constraints. $R^{SQB} > R^{IIV}$ if

$$\frac{\left(E(V) + \left(U_{I2}^{SQB} - U_{C2}^{SQB}\right)\right)(n-1)}{n} + (n-1)\, x_{C2}^{SQB} + x_{I2}^{SQB} > \frac{2E(V)(n-1)}{n}.$$

When

$$\theta \in \left(0, E\left(V\right)\left(n-1\right)/n^2\right)$$

$R^{SQB} - R^{IIV} = -\theta/n$. When $\theta \in [E\left(V\right)\left(n-1\right)/n^2, E\left(V\right))$, R^{SQB} is concave, and has a maximum value such that $R^{SQB} > R^{IIV}$. Once $\theta \in [E\left(V\right), \infty)$, $R^{SQB} = R^{IIV}$. Indeed, $R^{SQB} = R_1^{SQB}$.

A contest designer who seeks to maximize R^{SQB}, would would choose

$$\theta \in \left[E\left(V\right)\left(n-1\right)/n^2, E\left(V\right)\right).$$

Doing so ensures the the incumbent will not expend any effort. Effort expenditures in $t=1$ more than make up for the decrease expenditures in $t=2$. Further, if a contest designer sought to minimize effort expenditures (that is, maximize the sum of the contestants expected utility) she would choose $\theta = E\left(V\right)\left(n-1\right)/n^2$. Notice that this is the largest θ which does not reduce the expected utility of the challengers relative to the IIV case. Of interest is the fact that the optimal level of θ is positive, regardless of whether or not effort expenditures are a social bad.

Cost Advantage (CST)

Another way to approach the concept of incumbency advantage is to allow the incumbent to have a cost advantage over the challenger. That is, allow the incumbent to have a lower marginal cost than the challenger. A model using this approach was introduced in Mehlum and Moene [36]. They model an infinitely repeated contest between two contestants in which a cost advantage is held by the contestant who obtained the prize in the previous period.

Below is a modified version of their model in which contestants compete in $t = 1$ with symmetric costs, and in $t = 2$, the incumbent has a lower marginal cost of effort than the challengers. Modeling it in this fashion allows me to examine the incentive to acquire this cost advantage when contestants are symmetric; and change in behavior in $t = 1$ relative to the IIV case is then attributable to the incumbents cost advantage. As such, the only difference between this model and the IIV case is that the incumbent has a cost of effort of $C_I(x_{I2}) = cx_{I2}$, where $c \in (0, 1)$.

In $t = 2$ the expected utility of the incumbent is

$$U_{I2}^{CST} \equiv \int_{\underline{v}}^{\infty} p_{I2}(x_{I2}, \mathbf{x}_C) v_2 dF_V(v_2) - cx_{I2}.$$

Similarly, the expected utility of contestant $j \in \mathbf{C}$ is

$$U_{j2}^{CST} \equiv \int_{\underline{v}}^{\infty} p_{j2}(x_{j2}, \mathbf{x}_{-i2}) v_2 dF_V(v_2) - x_{j2}.$$

This subgame has a unique equilibrium. I denote the equilibrium effort expenditure of the incumbent as x_{I2}^{CS} and that of a challenger as x_{C2}^{CST}. The equilibrium effort levels are given by

$$\begin{aligned} x_{I2}^{CS} &= \frac{E(V)(n-1)(n(1-c)+2c-1)}{(n-1+c)^2} \\ x_{C2}^{CST} &= \frac{c(n-1)E(V)}{(n-1+c)^2}. \end{aligned}$$

The equilibrium aggregate effort expenditures in $t = 2$ is

$$R_2^{CST} \equiv \frac{E(V)(n-1)}{(n-1+c)}.$$

Further, the equilibrium expected utility of the incumbent is

$$U_{I2}^{CST} = \frac{cE(V)}{(n+c-1)^2}$$

and the equilibrium expected utility of a challenger is

$$U_{C2}^{CST} = \frac{c^2 E(V)}{(n+c-1)^2}.$$

Next, consider contestant i's expected utility in $t = 1$.

$$\begin{aligned} U_{i1}^{CST} &\equiv \int_{\underline{v}}^{\infty} p_{i1}(x_{i1}, \mathbf{x}_{-i1}) v_1 dF_V(v_1) - x_{i1} \\ &\quad + p_{i1}(x_{i1}, \mathbf{x}_{-i1}) \left(\frac{c_I E(V_1)}{(n+c_I-1)^2} \right) \\ &\quad + (1 - p_{i1}(x_{i1}, \mathbf{x}_{-i1})) \frac{c_I^2 E(V_1)}{(n+c_I-1)^2}. \end{aligned}$$

Equilibrium effort expenditure in $t = 1$ is

$$x_{i1}^{CST} \equiv \frac{2(n-1)E(V)}{n^2(c+1)}.$$

Total equilibrium effort expenditures across $t = 1, 2$ is

$$R^{CST} \equiv \frac{2(n-1)E(V)}{n^2(c+1)} + \frac{E(V)(n-1)}{(n-1+c)}.$$

Notice that $R^{CST} > R^{IIV}$. This is because the reduced marginal cost causes the incumbent to increase her effort expenditures in $t = 2$ relative to the IIV case. In response, the challengers also increases their expenditures. Further, contestants in $t = 1$ increase their effort expenditures relative to the IIV case an attempt to obtain the incumbent cost advantage. Also, notice that R^{CST} is monotonically decreasing in c; as the incumbents cost advantage increases, so does R^{CST}. This is in contrast to the status quo bias model discussed above. In that model, there were two competing effects, one of which increased effort, while the other decreased effort. As such, the effect of an incumbency advantage is sensitive to how it is modeled.

APPENDIX C

This Appendix contains derivations for Chapter III.

The common value of the available good, x, is a realization of a random variable X with a uniform distribution with support $[\underline{x}, \overline{x}]$. The realization of this value, x, is not observed by the two bidders before placing their bids. However, the distribution from which it is drawn is common knowledge.

In a SPRIV auction, bidder $i \in \{1, 2\}$ observes an estimate of the realized value of the good. Each estimate is the realization of X plus an error term X_i. This error term is $U(-\delta, \delta)$, and is independent of X and X_{-i}. That is, each estimate is a realization of $Z_i = X + X_i$.(We denote the distribution function of Z_i as F_{Z_i}). Notice that Z_i is independent of Z_{-i}, conditional on the realization of X. Throughout, we use f_A to denote the density function of the random variable A. A joint density function will be denoted as $f(\mathbf{x})$ where the vector $\mathbf{x}$ indicates the random variables for which $f(\mathbf{x})$ pertains.

Since Z_i is simply the sum of independent random variables, it's density function is easily calculated. To do so, we use the following, well known, formula:

$$\begin{aligned} f_{Z_i}(z_i) &= \int_{-\infty}^{\infty} f_X(z_i - x_i) f_{X_i}(x_i)\, dx_i \\ &= \int_{-\delta}^{\delta} f_X(z_i - x_i) f_{Xi}(x_i)\, dx_i \end{aligned}$$

This becomes a piecewise linear function:

$$f_{Z_i}(z_i) = \begin{cases} \int\limits_{-\delta}^{z_i-\underline{x}} \left(\frac{1}{2\delta(\overline{x}-\underline{x})}\right) dx_i = \frac{z_i+\delta-\underline{x}}{2\delta(\overline{x}-\underline{x})} & \text{if} \quad z_i \in [\underline{x}-\delta, \underline{x}+\delta) \\ \int\limits_{-\delta}^{\delta} \left(\frac{1}{2\delta(\overline{x}-\underline{x})}\right) dx_i = \frac{1}{(\overline{x}-\underline{x})} & \text{if} \quad z_i \in [\underline{x}+\delta, \overline{x}-\delta) \\ \int\limits_{z_i-\overline{x}}^{\delta} \left(\frac{1}{2\delta(\overline{x}-\underline{x})}\right) dx_i = \frac{\delta-z_i+\overline{x}}{2\delta(\overline{x}-\underline{x})} & \text{if} \quad z_i \in [\overline{x}-\delta, \overline{x}+\delta]. \end{cases}.$$

The distribution function of Z_i is

$$F_{Z_i}(c) = \begin{cases} \frac{(c-\underline{x}+\delta)^2}{4\delta(\overline{x}-\underline{x})} & \text{if} \quad c \in [\underline{x}-\delta, \underline{x}+\delta) \\ \frac{c-\underline{x}}{(\overline{x}-\underline{x})} & \text{if} \quad c \in [\underline{x}+\delta, \overline{x}-\delta) \\ \frac{\overline{x}-\underline{x}-\delta}{(\overline{x}-\underline{x})} + \frac{(\overline{x}+3\delta-c)(c-\overline{x}+\delta)}{4\delta(\overline{x}-\underline{x})} & \text{if} \quad c \in [\overline{x}-\delta, \overline{x}+\delta]. \end{cases}.$$

In a SPRIV auction, both bidders receive a signal. The joint density function of X, Z_1, and Z_2 is given by:

$$f(x, z_1, z_2) = \frac{1}{4\delta^2(\overline{x}-\underline{x})}.$$

In an ASYM auction, only one of the bidders observes a signal. Thus the joint distribution of X and Z_i is of interest. Integrating Z_j out of $f(x, z_1, z_2)$ yields:

$$f(x, z_i) = \int_{x-\delta}^{x+\delta} \frac{1}{4\delta^2(\overline{x} - \underline{x})} dz_j = \frac{1}{2\delta(\overline{x} - \underline{x})}.$$

The density function of x given the realized value of a bidders signal is:

$$f_X(x \mid z_i) = \begin{cases} \frac{1}{z_i + \delta - \underline{x}} & \text{if} \quad z_i \in [\underline{x} - \delta, \underline{x} + \delta) \\ \\ \frac{1}{2\delta} & \text{if} \quad z_i \in [\underline{x} + \delta, \overline{x} - \delta) \\ \\ \frac{1}{\delta - z_i + \overline{x}} & \text{if} \quad z_i \in [\overline{x} - \delta, \overline{x} + \delta]. \end{cases}$$

The joint density function of X and Z_j given that $Z_i = z_i$ is:

$$f(x, z_j \mid z_i) = \begin{cases} \frac{1}{2\delta(z_i + \delta - \underline{x})} & \text{if} \quad z_i \in [\underline{x} - \delta, \underline{x} + \delta) \\ \\ \frac{1}{4\delta^2} & \text{if} \quad z_i \in [\underline{x} + \delta, \overline{x} - \delta) \\ \\ \frac{1}{2\delta(\delta - z_i + \overline{x})} & \text{if} \quad z_i \in [\overline{x} - \delta, \overline{x} + \delta]. \end{cases}$$

The Prob$(z_i > z_j)$ is

$$F_{Z_j|Z_i}(z_j \mid z_i) = \begin{cases} \int_{\underline{x}-\delta}^{z_i} \frac{z_j + \delta - \underline{x}}{2\delta(z_i + \delta - \underline{x})} dz_j = \frac{z_i - \underline{x} + \delta}{4\delta} & \text{if} \quad z_i \in [\underline{x} - \delta, \underline{x} + \delta) \\ \\ \int_{z_i - 2\delta}^{z_i} \frac{z_j - z_i + 2\delta}{4\delta^2} dz_j = \frac{1}{2} & \text{if} \quad z_i \in [\underline{x} + \delta, \overline{x} - \delta) \\ \\ \int_{z_i - 2\delta}^{\overline{x} - \delta} \frac{z_j - z_i + 2\delta}{2\delta(\overline{x} - z_i + \delta)} dz_j + \int_{\overline{x} - \delta}^{z_i} \frac{1}{2\delta} dz_j = \frac{z_i - \overline{x} + 3\delta}{4\delta} & \text{if} \quad z_i \in [\overline{x} - \delta, \overline{x} + \delta]. \end{cases}$$

Symmetric Information With Private Signals

The derivations to find the symmetric Nash equilibrium bid function can be found in Kagel and Levin [28] and Kagel and Richard [30]. Assume that bidder $j \neq i$ bids according to the symmetric Nash equilibrium bid function, $\gamma(z_j)$. Consider bidder i who observes a signal z_i but bids as though he/she observed y. If $a(z_i) = \max(\underline{x}, z_i - \delta)$ and $b(z_i) = \min(\overline{x}, z_i + \delta)$, then the expected payoff of such a bidder is as follows:

$$\begin{aligned}\Pi(z_i, y) &= \int_{a(z_i)}^{b(z_i)} (x - \gamma(y)) F(y \mid x) f_X(x \mid z_i)\, dx \\ &= \int_{a(z_i)}^{b(z_i)} (x - \gamma(y)) \left(\frac{y - x + \delta}{2\delta}\right) \left(\frac{1}{b(z_i) - a(z_i)}\right) dx.\end{aligned}$$

The revelation principle tells us that:

$$\frac{d\Pi(z_i, y)}{dy} \Big|_{y=z_i} = 0.$$

Using the initial condition $\gamma(\underline{x} - \delta) = \underline{x}$ and assuming continuity of the equilibrium bid function yields the solution:

$$\gamma(z_i) = \begin{cases} \underline{x} + \frac{1}{3}(z_i - \underline{x} + \delta) & \text{if} \quad z_i \in [\underline{x} - \delta, \underline{x} + \delta) \\ z_i - \delta + \frac{2\delta}{3} \exp\left[\frac{1}{\delta}(\underline{x} + \delta - z_i)\right] & \text{if} \quad z_i \in [\underline{x} + \delta, \overline{x} - \delta) \\ \frac{2\overline{x}^3 + z_i^3 + 3\delta z_i^2 - 9\delta^2 z_i + 12\delta\overline{x}(z_i + 3\delta) - 3\overline{x}^2(z_i + 5\delta) + \delta^3\left(8\exp\left[\frac{2\delta + \underline{x} - \overline{x}}{\delta}\right] - 35\right)}{3(z_i - \overline{x} + 3\delta)^2} & \text{if} \quad z_i \in [\overline{x} - \delta, \overline{x} + \delta]. \end{cases}$$

The expected payoff of bidder i when she observes a private signal z_i is

$$\begin{aligned}
\Pi_i^{SPRIV}(z_i) &= \int_{a(z_i)}^{b(z_i)} (x-\gamma(z_i))\, F(z_i \mid x)\, f_X(x \mid z_i)\, dx \\
&= \int_{a(z_i)}^{b(z_i)} (x-\gamma(z_i)) \left(\frac{z_i - x + \delta}{2\delta}\right) \left(\frac{1}{b(z_i)-a(z_i)}\right) dx.
\end{aligned}$$

This simplifies to

$$\Pi_i^{SPRIV}(z_i) = \begin{cases} 0 & \text{if} \quad z_i \in [\underline{x}-\delta, \underline{x}+\delta) \\[2em] \frac{\delta}{3}\left(1-\exp\left(\frac{\underline{x}-z_i+\delta}{\delta}\right)\right) & \text{if} \quad z_i \in [\underline{x}+\delta, \overline{x}-\delta) \\[2em] \frac{\overline{x}^2+z_i^2+4z_i\delta+\delta^2\left(5-2\exp\left(2-\frac{(\overline{x}-\underline{x})}{\delta}\right)-2\overline{x}(z_i+2\delta)\right)}{3(z_i-\overline{x}+3\delta)} & \text{if} \quad z_i \in [\overline{x}-\delta, \overline{x}+\delta]. \end{cases}$$

Bidder i's ex ante expected payoff is obtained by integrating over z_i. This yields

$$\begin{aligned}
E\left(\Pi_i^{SPRIV}\right) &= \int_{\underline{x}-\delta}^{\overline{x}+\delta} \Pi_i^{SPRIV}(z_i)\, f_{Z_i}(z_i) dz_i \\
&= \frac{\delta\left(3\underline{x}-3\overline{x}+\delta\left(13-12\ln(2)\right)\right)+3\delta^2\exp\left(\frac{2\delta+\underline{x}-\overline{x}}{\delta}\right)\left(\ln(16)-3\right)}{9\left(\underline{x}-\overline{x}\right)}.
\end{aligned}$$

For the parameter's employed in our design, $E\left(\Pi_i^{SPRIV}\right) = 2.50019$. Since the ex ante expected revenue in an auction is the expected value of the good, minus the ex ante expected payoff's of the bidders, the ex ante expected revenue of a SPRIV auction, $E\left(R^{SPRIV}\right)$, is

$$E\left(R^{SPRIV}\right) = \left(\frac{\overline{x}+\underline{x}}{2}\right) - \frac{2\delta\left(3\underline{x}-3\overline{x}+\delta\left(13-12\ln\left(2\right)\right)\right)+6\delta^{2}\exp\left(\frac{2\delta+\underline{x}-\overline{x}}{\delta}\right)\left(\ln\left(16\right)-3\right)}{9\left(\underline{x}-\overline{x}\right)}.$$

For the parameters in our design, this is $E\left(R^{SPRIV}\right) = 119.99962$.

Winner's Curse in SPRIV Auctions

In a SPRIV auction a bidder is said to fall victim to the winner's curse if she bids more than the expected value of the good conditional on winning the auction, which defines a break-even bidding strategy. If all bidder's bid according to a monotonically increasing bid function, the bidder with the highest signal wins the auction. Therefore, if bidders are bidding according to monotonically increasing bid function, bidders are said to fall victim to the winner's curse if they bid more that the expected value of the good conditional on having the largest signal. If bidder's do not use their signal as an order statistic for the value of the good, they will overestimate it, and will have negative expected profits upon winning the auction. In our design, if bidder i observes a signal z_i and bids more than $E\left(X \mid Z_i = z_i > z_j\right)$, then she is a victim of the winner's curse. When $z_i \in [\underline{x}-\delta, \underline{x}+\delta)$,

$$\begin{aligned}
E\left(X \mid Z_i = z_i > z_j\right) &= \frac{1}{F_{Z_j|Z_i}\left(z_i \mid z_i\right)} \int_{\underline{x}-\delta}^{z_i} \int_{\underline{x}}^{z_j+\delta} x f_X\left(x, z_j \mid z_i\right) dx dz_j \\
&= \left(\frac{4\delta}{z_i - \underline{x} + \delta}\right) \int_{\underline{x}-\delta}^{z_i} \int_{\underline{x}}^{z_j+\delta} x \frac{1}{2\delta\left(z_i + \delta - \underline{x}\right)} dx dz_j \\
&= \frac{1}{3}\left(z_i + 2\underline{x} + \delta\right).
\end{aligned}$$

When $z_i \in [\underline{x}+\delta, \overline{x}-\delta)$,

$$\begin{aligned}
E(X \mid Z_i = z_i > z_j) &= \frac{1}{F_{Z_j|Z_i}(z_i \mid z_i)} \int_{z_i-2\delta}^{z_i} \int_{z_i-\delta}^{z_j+\delta} x f_X(x, z_j \mid z_i)\, dx dz_j \\
&= 2 \int_{\underline{x}-\delta}^{z_i} \int_{\underline{x}}^{z_j+\delta} x \frac{1}{4\delta^2} dx dz_j \\
&= z_i - \frac{\delta}{3}.
\end{aligned}$$

When $z_i \in [\overline{x}-\delta, \overline{x}+\delta]$

$$\begin{aligned}
E(X \mid Z_i = z_i > z_j) &= \frac{1}{F_{Z_j|Z_i}(z_i \mid z_i)} \int_{z_i-2\delta}^{z_i} \int_{z_i-\delta}^{z_j+\delta} x f_X(x, z_j \mid z_i)\, dx dz_j \\
&= \left(\frac{4\delta}{z_i - \overline{x} + 3\delta}\right) \int_{z_i-2\delta}^{\overline{x}-\delta} \int_{z_i-\delta}^{z_j+\delta} x \frac{1}{2\delta(\overline{x}+\delta-z_i)} dx dz_j + \\
&\quad \left(\frac{4\delta}{z_i - \overline{x} + 3\delta}\right) \int_{\overline{x}-\delta}^{z_i} \int_{z_i-\delta}^{\overline{x}} x \frac{1}{2\delta(\overline{x}+\delta-z_i)} dx dz_j \\
&= \frac{(z_i+5\delta)(z_i-\delta) + \overline{x}(z_i+5\delta) - 2\overline{x}^2}{3(z_i - \overline{x} + 3\delta)}.
\end{aligned}$$

That is,

$$E(X \mid Z_i = z_i > z_j) = \begin{cases} \frac{1}{3}(z_i + 2\underline{x} + \delta) & \text{if} \quad z_i \in [\underline{x}-\delta, \underline{x}+\delta) \\ z_i - \frac{\delta}{3} & \text{if} \quad z_i \in [\underline{x}+\delta, \overline{x}-\delta) \\ \frac{(z_i+5\delta)(z_i-\delta)+\overline{x}(z_i+5\delta)-2\overline{x}^2}{3(z_i-\overline{x}+3\delta)} & \text{if} \quad z_i \in [\overline{x}-\delta, \overline{x}+\delta]. \end{cases}$$

This is the threshold that defines the winner's curse in a SPRIV auction.

Asymmetric Information

Engelbrecht-Wiggans et. al. [16] provides the unique equilibrium of this game. We denote the informed bidder as bidder I. In this equilibrium, when the informed bidder observes z_I she bids according to the function

$$\begin{aligned}\beta\left(z_I\right) &= E\left(E\left(X \mid Z_I\right) \mid Z_I \leq z_I\right)\\ &= \frac{1}{F_{Z_I}\left(z_I\right)}\int_{\underline{x}-\delta}^{z_I} E\left(X \mid Z_I = s\right) f_{Z_I}\left(s\right) ds.\end{aligned}$$

When $z_I \in [\underline{x}-\delta, \underline{x}+\delta)$, this is

$$\begin{aligned}\beta\left(z_I\right) &= \frac{4\delta\left(\overline{x}-\underline{x}\right)}{\left(c-\underline{x}+\delta\right)^2}\int_{\underline{x}-\delta}^{z_I}\left(\frac{\underline{x}+s+\delta}{2}\right)\left(\frac{s+\delta-\underline{x}}{2\delta\left(\overline{x}-\underline{x}\right)}\right) ds\\ &= \frac{2\underline{x}+z_I+\delta}{3}.\end{aligned}$$

When $z_I \in [\underline{x}+\delta, \overline{x}-\delta)$, this is

$$\begin{aligned}\beta\left(z_I\right) &= \frac{\left(\overline{x}-\underline{x}\right)}{z_I-\underline{x}}\left(\int_{\underline{x}-\delta}^{\underline{x}+\delta}\left(\frac{\underline{x}+s+\delta}{2}\right)\left(\frac{s+\delta-\underline{x}}{2\delta\left(\overline{x}-\underline{x}\right)}\right) ds+\int_{\underline{x}+\delta}^{z_I} s\left(\frac{1}{\left(\overline{x}-\underline{x}\right)}\right) ds\right)\\ &= \frac{z_I+\underline{x}}{2}+\frac{\delta^2}{6\left(z_I-\underline{x}\right)}.\end{aligned}$$

When $z_I \in [\overline{x}-\delta, \overline{x}+\delta]$ this is

$$\begin{aligned}
\beta(z_I) &= \left(\frac{4\delta(\overline{x}-\underline{x})}{4\delta(\overline{x}-\underline{x}-\delta)+(\overline{x}+3\delta-z_I)(z_I-\overline{x}+\delta)}\right)\int_{\underline{x}-\delta}^{\underline{x}+\delta}\left(\frac{\underline{x}+s+\delta}{2}\right)\left(\frac{s+\delta-\underline{x}}{2\delta(\overline{x}-\underline{x})}\right)ds+ \\
&\quad \left(\frac{4\delta(\overline{x}-\underline{x})}{4\delta(\overline{x}-\underline{x}-\delta)+(\overline{x}+3\delta-z_I)(z_I-\overline{x}+\delta)}\right)\int_{\underline{x}+\delta}^{\overline{x}-\delta} s\left(\frac{1}{(\overline{x}-\underline{x})}\right)ds+ \\
&\quad \left(\frac{4\delta(\overline{x}-\underline{x})}{4\delta(\overline{x}-\underline{x}-\delta)+(\overline{x}+3\delta-z_I)(z_I-\overline{x}+\delta)}\right)\int_{\overline{x}-\delta}^{z_I}\left(\frac{\overline{x}+s-\delta}{2}\right)\left(\frac{\overline{x}+\delta-s}{2\delta(\overline{x}-\underline{x})}\right)ds. \\
&= \frac{2\overline{x}^3+(z_I-\delta)^3+6\underline{x}^2\delta-3\overline{x}^2(z_I+\delta)}{3\left(\overline{x}^2+(z_I-\delta)^2+4\underline{x}\delta-2\overline{x}(z_I+\delta)\right)}.
\end{aligned}$$

That is, the equilibrium bid function for the informed bidder in an ASYM auction is

$$\beta(z_I) = \begin{cases} \frac{2\underline{x}+z_I+\delta}{3} & \text{if} \quad z_I \in [\underline{x}-\delta, \underline{x}+\delta) \\[2em] \frac{z_I+\underline{x}}{2}+\frac{\delta^2}{6(z_I-\underline{x})} & \text{if} \quad z_I \in [\underline{x}+\delta, \overline{x}-\delta) \\[2em] \frac{2\overline{x}^3+(z_I-\delta)^3+6\underline{x}^2\delta-3\overline{x}^2(z_I+\delta)}{3\left(\overline{x}^2+(z_I-\delta)^2+4\underline{x}\delta-2\overline{x}(z_I+\delta)\right)} & \text{if} \quad z_I \in [\overline{x}-\delta, \overline{x}+\delta]. \end{cases}$$

In equilibrium, the uninformed bidder will mix on the interval $[\underline{x}, E(X)]$ according to the following distribution function:

$$\begin{aligned}
Q(b) &= \text{Prob}[\beta(Z_I) \leq b] \\
&= F_{Z_I}\left(\beta^{-1}(b)\right).
\end{aligned}$$

So, the uninformed bidder will mix according using this distribution function:

$$Q(b)=\begin{cases}\frac{\left(\beta^{-1}(b)-\underline{x}+\delta\right)^{2}}{4\delta(\overline{x}-\underline{x})} & \text{if}\quad b\in\left[\beta\left(\underline{x}-\delta\right),\beta\left(\underline{x}+\delta\right)\right)\\ \\ \frac{\beta^{-1}(b)-\underline{x}}{(\overline{x}-\underline{x})} & \text{if}\quad b\in\left[\beta\left(\underline{x}+\delta\right),\beta\left(\overline{x}-\delta\right)\right)\\ \\ \frac{4\delta(\overline{x}-\underline{x}-\delta)+\left(\overline{x}+3\delta-\beta^{-1}(b)\right)\left(\beta^{-1}(b)-\overline{x}+\delta\right)}{4\delta(\overline{x}-\underline{x})} & \text{if}\quad b\in\left[\beta\left(\overline{x}-\delta\right),\beta\left(\overline{x}+\delta\right)\right].\end{cases}$$

Engelbrecht-Wiggans et al [16] shows that, in equilibrium, the uninformed bidder obtains an expected payoff of zero for any bid in the support of $Q(b)$. Let $q(z_I):=E(X\mid z_I)$. Since $q(z_I)$ is monotonically increasing in z_I, the distribution function of this random variable is just $F_{Z_I}(q^{-1}(\cdot))$, where $q^{-1}(\cdot)$ is the inverse of $q(\cdot)$. Engelbrecht-Wiggans et al [16] shows that when the informed bidder observes z_I his/her expected payoff is

$$\Pi_I^{ASYM}(z_I)=\int_{\underline{x}}^{q(z_I)}F_{Z_I}\left(q^{-1}(s)\right)ds.$$

When $z_I\in[\underline{x}-\delta,\underline{x}+\delta)$ this is

$$\Pi_I^{ASYM}(z_I)=\int_{\underline{x}}^{q(z_I)}\frac{\left(q^{-1}(s)-\underline{x}+\delta\right)^2}{4\delta\left(\overline{x}-\underline{x}\right)}ds=\frac{\left(z_I-\underline{x}+\delta\right)^3}{12\delta\left(\overline{x}-\underline{x}\right)}.$$

When $z_I\in[\underline{x}+\delta,\overline{x}-\delta)$ this is

$$\begin{aligned}\Pi_I^{ASYM}(z_I) &= \int_{\underline{x}-\delta}^{\underline{x}+\delta}\frac{\left(q^{-1}(s)-\underline{x}+\delta\right)^2}{4\delta\left(\overline{x}-\underline{x}\right)}ds+\int_{\underline{x}+\delta}^{q(z_I)}\frac{q^{-1}(s)-\underline{x}}{\left(\overline{x}-\underline{x}\right)}ds\\ &= \frac{3\left(\underline{x}-z_I\right)^3-\delta^2}{6\left(\overline{x}-\underline{x}\right)}.\end{aligned}$$

If $z_I \in [\overline{x}-\delta, \overline{x}+\delta]$ this is

$$\begin{aligned}
\Pi_I^{ASYM}(z_I) &= \int_{\underline{x}-\delta}^{\underline{x}+\delta} \frac{\left(q^{-1}(s)-\underline{x}+\delta\right)^2}{4\delta\left(\overline{x}-\underline{x}\right)} ds + \int_{\underline{x}+\delta}^{\overline{x}-\delta} \frac{q^{-1}(s)-\underline{x}}{\left(\overline{x}-\underline{x}\right)} ds \\
&\quad + \int_{\overline{x}-\delta}^{q(z_I)} \frac{4\delta\left(\overline{x}-\underline{x}-\delta\right)+\left(\overline{x}+3\delta-q^{-1}(s)\right)\left(q^{-1}(s)-\overline{x}+\delta\right)}{4\delta\left(\overline{x}-\underline{x}\right)} ds \\
&= \frac{\left(\overline{x}-z_I+\delta\right)^3}{24\delta\left(\overline{x}-\underline{x}\right)} + \frac{\left(\overline{x}+z_I-\delta\right)}{2} - \frac{\left(\overline{x}-\underline{x}\right)}{2}.
\end{aligned}$$

That is, the expected payoff of an informed bidder is

$$\Pi_I^{ASYM}(z_I) = \begin{cases}
\frac{\left(z_I-\underline{x}+\delta\right)^3}{12\delta\left(\overline{x}-\underline{x}\right)} & \text{if} \quad z_I \in [\underline{x}-\delta, \underline{x}+\delta) \\
\frac{3\left(\underline{x}-z_I\right)^3-\delta^2}{6\left(\overline{x}-\underline{x}\right)} & \text{if} \quad z_I \in [\underline{x}+\delta, \overline{x}-\delta) \\
\frac{\left(\overline{x}-z_I+\delta\right)^3}{24\delta\left(\overline{x}-\underline{x}\right)} + \frac{\left(\overline{x}+z_I-\delta\right)}{2} - \frac{\left(\overline{x}-\underline{x}\right)}{2} & \text{if} \quad z_I \in [\overline{x}-\delta, \overline{x}+\delta].
\end{cases}$$

The ex ante expected payoff of the informed bidder can be found by integrating over z_1. This yields

$$\begin{aligned}
E\left(\Pi_I^{ASYM}\right) &= \int_{\underline{x}-\delta}^{\overline{x}+\delta} \Pi_I^{ASYM}(z_I)\, dz_I \\
&= \frac{5\left(\overline{x}-\underline{x}\right)^3 - 10\delta^2\left(\overline{x}-\underline{x}\right)+8\delta^3}{30\left(\overline{x}-\underline{x}\right)^2}.
\end{aligned}$$

For the parameters employed in our design, $E\left(\Pi_I^{ASYM}\right) = 33.2301$. The ex ante expected revenue for the seller is found by subtracting the ex ante expected payoff of the informed bidder from the expected value of X. This yields

$$E\left(R^{ASYM}\right) = \left(\frac{\overline{x}+\underline{x}}{2}\right) - \frac{5\left(\overline{x}-\underline{x}\right)^3 - 10\delta^2\left(\overline{x}-\underline{x}\right) + 8\delta^3}{30\left(\overline{x}-\underline{x}\right)^2}.$$

For the parameter values used in our design $E\left(R^{ASYM}\right) = 91.7699$.

Winner's Curse in ASYM Auctions

Since the uninformed bidder has an expected payoff of zero for any bid $b \in [\underline{x}, E(X)]$, $E(X)$ is a break-even strategy for uninformed ASYM bidders. Bidding above $E(X)$ ensures negative expected profit upon winning, while bidding below $E(X)$ yields an expected payoff of zero conditional on winning the auction. That is, if an uninformed bidder bids above $E(X)$, she is said to fall victim to the winner's curse.

The expected value of the good conditional on z_I is the same as the expected value of the good conditional on z_I *and* having won the auction. Winning the auction does not provide the informed bidder additional information regarding x. Therefore, the break-even bidding strategy for an informed ASYM bidder is to bid:

$$E\left(X \mid z_I\right) = \begin{cases} \frac{z_I+\delta+\underline{x}}{2} & \text{if} \quad z_I \in [\underline{x}-\delta, \underline{x}+\delta) \\ \\ z_I & \text{if} \quad z_I \in [\underline{x}+\delta, \overline{x}-\delta) \\ \\ \frac{z_I-\delta+\overline{x}}{2} & \text{if} \quad z_I \in [\overline{x}-\delta, \overline{x}+\delta]. \end{cases}$$

So, if an informed bidder bidder bids above $E\left(X \mid z_I\right)$, she is said to fall victim to the winner's curse.

APPENDIX D

This Appendix contains the experimental instructions for the ASYM treatment in Chapter III.

Introduction

Welcome. This experiment is about decision making in markets. The following instructions describe the markets you will be in and the rules that you will face. The decisions you make during this experiment will determine how much money you earn. If you make good decisions, you can earn a substantial amount of money. You will be paid in cash privately at the end of our experiment.

It is important that you remain silent and do not look at other people's work. If you have any questions, or need assistance of any kind, please raise your hand and an experimenter will come to you. If you talk, laugh, exclaim out loud, etc., you will be asked to leave and you will not be paid. We expect and appreciate your cooperation.

We will go over these instructions with you. After we have read the instructions, there will be time to ask clarifying questions. When we are done going through the instructions, each of you will have to answer a few brief questions to ensure everyone understands.

Overview

Our experiment will consist of 30 rounds. In each of these rounds, you will be randomly paired with another participant in today's experiment. Both of you will be buyers in a market. In each market, there will be a single unit of an indivisible good for sale. As a buyer, your task is to submit a bid for the purchase of the good. You will receive earnings based on the outcome of the market. This process will be repeated until all 30 rounds have been completed.

Determination of Your Earnings

Each participant will receive a show-up fee of $5. In addition, each participant in this experiment will start with a balance of $3,200 "experimental dollars" (EDs). EDs will be traded in for cash at the end of the experiment at a rate of $\$160ED = \$1$. Your starting balance can increase or decrease depending on your payoffs in each round. That is, if you have a negative payoff in a round, this loss will be deducted from your balance. If you earn a positive payoff, this is added to your balance. You are permitted to bid more than your remaining balance. However, if after a round is completed your balance is less than or equal to zero, you will not be able to participate in any future rounds.

In each round, you and the other buyer in the market will submit a bid. The higher bid will have to be paid, and the buyer with the higher bid will receive the good. The buyer who submits the lower bid does not get the good, but does not pay his/her bid. That is, for each market, the buyer who submits the higher bid will receive:

$$(\text{Value of the good}) - (\text{Own bid})$$

The person who submits the lower bid will receive:

$$0$$

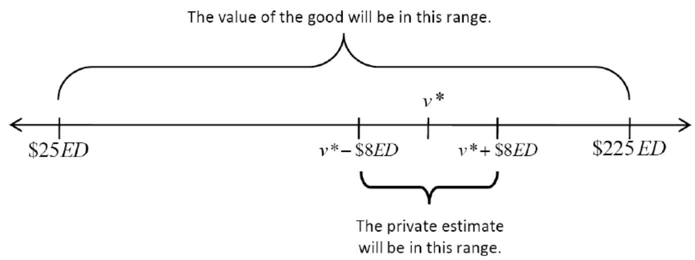
If both buyers bid the same amount, then the winner is determined randomly, with both buyers having equal probability of receiving the good. You can think of this as a flip of a fair coin, which determines the winner in the event of a tie. Only the bidder who receives the good must pay his/her bid.

Notice that the buyer who submits the highest bid can end up with a negative payoff, if he/she bids more than the good is worth. No buyer is permitted to submit a bid that is lower than zero.

In each round, the value of the good, which we will denote as v^*, will not be known to the buyers. The value of this good will be between \$25ED and \$225ED. Any value between \$25ED and \$225ED is equally likely to be chosen as v^*. The value of the good in any given round is independent of the value in any other round. That is, the value of the good in one round will not have *any* effect on the value of the good in a different round.

Private Information

In each market, one of the two buyers will be randomly chosen to receive some private information about the value of the good (you can think of this as flipping a coin to determine which of the buyers will receive this information, where the probability of the coin landing on each side is 50%). The person who receives the private information will be given an estimate of the value of the good. The estimate will be a randomly chosen number that is within \$8ED above or below the real value of v^* (see the illustration below). Any number between $v^* - \$8ED$, and $v^* + \$8ED$ is equally likely to be chosen as the estimate. For example, if you receive an estimate of \$125ED, then you know that v^* is between \$117ED and \$133ED, inclusive. It is possible for the estimate to be a value below \$25ED or above \$225ED, but the real value of v^* will always be between \$25ED and \$225ED.



Rounds

As mentioned before, there will be 30 rounds in this experiment. In each round there will be several markets going on simultaneously, with two buyers in each market. After each round you will be randomly paired with another participant in today's experiment. This random assignment is done *every round* so that two buyers will probably not be in the same market together for two consecutive rounds. Further, this pairing is anonymous. That is, if you are a buyer in a given market, you do not know which of the other participants in the experiment is the other buyer in that market. Remember that these different markets and rounds are independent from all others, and from one another. The bids and the value of the good and the estimate in one market or round do not have any effect on other markets or rounds. Markets and rounds operate independently.

Summary

1. Each participant has a starting balance of $\$3,200ED$.

2. In every round, each participant will be a buyer in one market. Two participants are randomly assigned to a market in each round.

3. The value of the good, v^*, is unknown. It is known that it is somewhere between $\$25ED$ and $\$225ED$. Every value between $\$25ED$ and $\$225ED$ is

equally likely to be v^*.

4. One buyer in a market is randomly chosen to receive an estimate of v^*. A buyer's estimate is not observed by the other buyer in the market. These estimates are randomly and independently drawn from the interval between $v^* - \$8ED$ and $v^* + \$8ED$, inclusive. Any number from this interval is equally likely to be chosen as the estimate.

5. In each market the high bidder gets $v^* -$ (Own bid), and the low bidder gets 0. This payoff is added to the balance of each bidder (a bidder's balance will go down if the value is negative, up if this value is positive, and remain unchanged if this value is zero).

6. Every participant will receive the show-up fee of \$5. Additionally, each participant will receive his/her balance at the end of all 30 rounds, based on the $\$3,200ED$ beginning balance and earnings in each market.

7. If a participant's balance should become negative at any point during this experiment, he/she will not be permitted to participate in future rounds.

If you have any questions, raise your hand and one of us will come help you. Please do not ask any questions out loud.

Questions

Before we begin the experiment, we would like you to answer a few questions that are meant to review the rules of today's experiment. Please raise your hand once you are done, and an experimenter will attend to you.

1. How many buyers are in each market? ____________

2. Who pays their bid in each market, the high bidder, the low bidder, or both?

   ____

3. Each estimate must be within what range of v^*? ____________

4. Are you allowed to bid more than your current balance? ____________

5. For each market, how many buyers get to see an estimate of v^*? ____________

6. If the highest bid in a market is \$152.10ED, and the value of the good is revealed to be \$200.90ED, what is the winner's payoff for that market? ____________

7. What would the earnings from question six have been if the value of the good had been \$25.90ED? ____________

8. If Buyer 1 bids \$150.00ED, and Buyer 2 bids \$200.00ED, and the value of the good is revealed to be \$220.75ED, what are the payoffs for Buyer 1 and Buyer 2? ____________

APPENDIX E

This appendix supplies a general proof of the equilibrium in an AAP auction as defined in Chapter IV.

In a first price sealed bid auction, each bidder submits a bid, and the highest bid wins with certainty. In the first price all-pay auction, every bidder must pay his/her bid.

Consider the first-price all-pay auction where the value of the prize has a common, but uncertain, value. This value, X, has the distribution function $H(x)$, with support contained in $[0, \infty)$ It is assumed that $E(X) < \infty$. Let there be two risk neutral bidders, one of whom observes an informative signal, Z, regarding the value of the good prior to bidding. The other bidder knows only the distributions from which both these random variables are drawn. Let $V = E(X \mid Z)$, and let $F(v)$ denote the distribution function of V, which is assumed to be absolutely continuous. Let the informed bidder be bidder one, and the uninformed bidder be bidder two.

Proposition 10 *The following strategies characterize an equilibrium in this game: Bidder one bids:*

$$\zeta(v) = F(v) E(V \mid V \leq v).$$

Bidder two mixes on the interval $[0, E(V)]$*, where the probability that she bids* x *is:*

$$G(x) = Prob[F(v) E(V \mid V \leq v) \leq x].$$

Proof. *Note that if both bidders bid according to the strategy outlined above, and* *bidder two bids* $x \in [0, E(V)]$, *and wins, her expected payoff will be:*

$$\begin{aligned} & E(V \mid \zeta(V) < x) - x \\ = \; & \frac{\zeta\left(\zeta^{-1}(x)\right)}{F\left(\zeta^{-1}(x)\right)} - x \\ = \; & \frac{x}{F\left(\zeta^{-1}(x)\right)} - x. \end{aligned}$$

Further, if bidder two bids x *and loses, her payoff is* $-x$. *Thus, the expected payoff of bidding* x *is:*

$$\begin{aligned} E(U_2) &= \left(\frac{x}{F\left(\zeta^{-1}(x)\right)} - x\right) \mathit{Prob}(x \text{ wins}) - x\left(1 - \mathit{Prob}(x \text{ wins})\right) \\ &= \left(\frac{x}{F\left(\zeta^{-1}(x)\right)} - x\right) \mathit{Prob}(\zeta(V) < x) - x\left(1 - \mathit{Prob}(\zeta(V) < x)\right) \\ &= \left(\frac{x\mathit{Prob}(\zeta(V) < x)}{F\left(\zeta^{-1}(x)\right)} - x\right) \\ &= 0. \end{aligned}$$

Thus, the uninformed bidder is indifferent over the interval $[0, E(X_0)]$. *Now consider the case in which the informed bidder bids* $\zeta(z)$ *when he observes* v. *If the uninformed bidder is following the equilibrium strategy outlined above, then the expected payoff for the informed bidder is:*

$$\begin{aligned} E(U_1) &= G(\zeta(z))\, v - \zeta(z) \\ &= \mathit{Prob}(\zeta(V) \leq \zeta(z))\, v - \zeta(z) \\ &= \mathit{Prob}(V \leq z)\, v - \zeta(z) \\ &= F(z)\, v - \zeta(z) \end{aligned}$$

Differentiating this with respect to z yields:

$$\begin{aligned}
& f(z)\,v-\frac{d}{dz}\zeta(z) \\
= \;& f(z)\,v-\frac{d}{dz}F(z)\,E(V\mid V\leq z) \\
= \;& f(z)\,v-\frac{d}{dz}\int_0^z t\,dF(t) \\
= \;& f(z)\,v-zf(z) \\
= \;& f(z)\,(v-z)
\end{aligned}$$

Notice that bidding where $v\neq z$ diminishes the expected payoff of the informed agent, and so he should bid $\zeta(v)$. ■

Proposition 11 *In equilibrium, the informed bidder"s ex ante expected payoff is*

$$\int_0^\infty (1-F(z))\,F(z)\,dz.$$

Proof. When an informed bidder bids z, he wins with probability $F(z)$. His payoff is thus

$$\begin{aligned}
\Pi_1(z) &= F(z)\,v-\zeta(z) \\
&= F(z)\,v-F(v)\,E(V\mid V\leq v) \\
&= F(z)\,v-F(z)\,v+\int_0^z F(t)\,dt \\
&= \int_0^z F(t)\,dt.
\end{aligned}$$

Integrating this over z gives us

$$\begin{aligned} E(\Pi_1) &= \int_0^\infty \int_0^z F(t)\,dt f(z)\,dz \\ &= \int_0^\infty F(z)\left(\int_z^\infty f(t)\,dt\right)dz \\ &= \int_0^\infty (1 - F(z))\,F(z)\,dz. \end{aligned}$$

■

APPENDIX F

This Appendix contains derivations of theoretical predictions for Chapter IV.

The common value of the available good, X, is drawn from a uniform distribution on the interval $[\underline{x}, \overline{x}]$. The realization of this value, x, is not observed by the two bidders before placing their bids. However, the distribution from which it is drawn is common knowledge.

In asymmetric information treatments, the informed bidder observes an estimate of the realized value of the good. This estimate is the realization of X plus an error term X_I. This error term is $U(-\delta, \delta)$, and is independent of X. That is, the estimate is a realization of $Z_I = X + X_I$.(We denote the distribution function of Z_I as F_{Z_I}). Throughout, we use f_A to denote the density function of the random variable A. A joint density function will be denoted as $f(\mathbf{x})$ where the vector $\mathbf{x}$ indicates the random variables to which $f(\mathbf{x})$ pertains.

Since Z_I is simply the sum of independent random variables, it's density function is easily calculated. To do so, we use the following, well known, formula:

$$
\begin{aligned}
f_{Z_I}(z_I) &= \int_{-\infty}^{\infty} f_X(z_I - x) f_X(x)\, dx \\
&= \int_{-\delta}^{\delta} f_X(z_I - x) f_X(x)\, dx.
\end{aligned}
$$

This becomes a piecewise linear function:

$$f_{Z_I}(z_I) = \begin{cases} \int\limits_{-\delta}^{z_I-\underline{x}} \left(\frac{1}{2\delta(\overline{x}-\underline{x})}\right) dx = \frac{z_I+\delta-\underline{x}}{2\delta(\overline{x}-\underline{x})} & \text{if} \quad z_I \in [\underline{x}-\delta, \underline{x}+\delta) \\ \int\limits_{-\delta}^{\delta} \left(\frac{1}{2\delta(\overline{x}-\underline{x})}\right) dx = \frac{1}{(\overline{x}-\underline{x})} & \text{if} \quad z_I \in [\underline{x}+\delta, \overline{x}-\delta) \\ \int\limits_{z_I-\overline{x}}^{\delta} \left(\frac{1}{2\delta(\overline{x}-\underline{x})}\right) dx = \frac{\delta-z_I+\overline{x}}{2\delta(\overline{x}-\underline{x})} & \text{if} \quad z_I \in [\overline{x}-\delta, \overline{x}+\delta]\,. \end{cases}$$

The distribution function of Z_I is

$$F_{Z_I}(c) = \begin{cases} \int\limits_{\underline{x}-\delta}^{c} \frac{z+\delta-\underline{x}}{2\delta(\overline{x}-\underline{x})} dz & \text{if} \quad c \in [\underline{x}-\delta, \underline{x}+\delta) \\ \int\limits_{\underline{x}-\delta}^{\underline{x}+\delta} \frac{z+\delta-\underline{x}}{2\delta(\overline{x}-\underline{x})} dz + \int\limits_{\underline{x}+\delta}^{c} \frac{1}{(\overline{x}-\underline{x})} dz & \text{if} \quad c \in [\underline{x}+\delta, \overline{x}-\delta) \\ \int\limits_{\underline{x}-\delta}^{\underline{x}+\delta} \frac{z+\delta-\underline{x}}{2\delta(\overline{x}-\underline{x})} dz + \int\limits_{\underline{x}+\delta}^{\overline{x}-\delta} \frac{1}{(\overline{x}-\underline{x})} dz + \int\limits_{\overline{x}-\delta}^{c} \frac{\delta-z+\overline{x}}{2\delta(\overline{x}-\underline{x})} dz & \text{if} \quad c \in [\overline{x}-\delta, \overline{x}+\delta]\,. \end{cases}$$

This reduces to:

$$F_{Z_I}(c) = \begin{cases} \frac{(c-\underline{x}+\delta)^2}{4\delta(\overline{x}-\underline{x})} & \text{if} \quad c \in [\underline{x}-\delta, \underline{x}+\delta) \\ \\ \frac{c-\underline{x}}{(\overline{x}-\underline{x})} & \text{if} \quad c \in [\underline{x}+\delta, \overline{x}-\delta) \\ \\ \frac{\overline{x}-\underline{x}-\delta}{(\overline{x}-\underline{x})} + \frac{(\overline{x}+3\delta-c)(c-\overline{x}+\delta)}{4\delta(\overline{x}-\underline{x})} & \text{if} \quad c \in [\overline{x}-\delta, \overline{x}+\delta]. \end{cases}$$

It is easy to check that the joint density function of X and Z_I is given by:

$$f(x, z_I) = \frac{1}{2\delta(\overline{x}-\underline{x})}.$$

The density function of X given the realized value of Z_I is:

$$f_X(x \mid z_I) = \begin{cases} \frac{1}{z_I+\delta-\underline{x}} & \text{if} \quad z_I \in [\underline{x}-\delta, \underline{x}+\delta) \\ \\ \frac{1}{2\delta} & \text{if} \quad z_I \in [\underline{x}+\delta, \overline{x}-\delta) \\ \\ \frac{1}{\delta-z_I+\overline{x}} & \text{if} \quad z_I \in [\overline{x}-\delta, \overline{x}+\delta]. \end{cases}$$

Equilibrium Bidding in SAP

Theorem 1 in Baye et al. [5] demonstrates that in any Nash equilibrium of this game, the expected payoff of both bidder's is zero, and that both bidders randomize continuously on $[0, E(X)]$. In a symmetric equilibrium, this implies that for any $b_i \in [0, E(X)]$

$$\Pi_i^{SAP}(b_i) = K(b_i)\, E(X) - b_i = 0$$

where $K(\cdot)$ is the distribution function of the symmetric equilibrium mixed strategy. Thus,

$$K(b_i) = \frac{b_i}{E(X)}.$$

Since both bidders have expected payoffs of zero, the expected revenue of this auction is $E(X)$.

Equilibrium Bidding in AAP

Appendix E provides the unique equilibrium of this game. In this equilibrium, when the informed bidder observes z_I he/she bids according to the function

$$\begin{aligned}\beta(z_I) &= F_{z_I}(z_I)\, E\left(E(X \mid Z_I) \mid Z_I \leq z_I\right) \\ &= \int_{\underline{x}-\delta}^{z_I} E(X \mid Z_I = s)\, f_{Z_I}(s)\, ds.\end{aligned}$$

When $z_I \in [\underline{x}-\delta, \underline{x}+\delta)$, this is

$$\begin{aligned}\beta(z_I) &= \int_{\underline{x}-\delta}^{z_I} \left(\frac{\underline{x}+s+\delta}{2}\right)\left(\frac{s+\delta-\underline{x}}{2\delta(\overline{x}-\underline{x})}\right) ds \\ &= \frac{(2\underline{x}+z_I+\delta)(z_I-\underline{x}+\delta)^2}{12\delta(\overline{x}-\underline{x})}.\end{aligned}$$

When $z_I \in [\underline{x}+\delta, \overline{x}-\delta)$ this is

$$\begin{aligned}\beta(z_I) &= \int_{\underline{x}-\delta}^{\underline{x}+\delta} \left(\frac{\underline{x}+s+\delta}{2}\right)\left(\frac{s+\delta-\underline{x}}{2\delta(\overline{x}-\underline{x})}\right) ds + \int_{\underline{x}+\delta}^{z_I} s\left(\frac{1}{(\overline{x}-\underline{x})}\right) ds \\ &= \frac{3z_I^2+\delta^2-3\underline{x}^2}{6(z_I-\underline{x})}.\end{aligned}$$

When $z_I \in [\overline{x} - \delta, \overline{x} + \delta]$ this is

$$\begin{aligned}
\beta(z_I) &= \int_{\underline{x}-\delta}^{\underline{x}+\delta} \left(\frac{\underline{x} + s + \delta}{2}\right)\left(\frac{s + \delta - \underline{x}}{2\delta(\overline{x} - \underline{x})}\right) ds + \\
&\quad \int_{\underline{x}+\delta}^{\overline{x}-\delta} s\left(\frac{1}{(\overline{x} - \underline{x})}\right) ds + \\
&\quad \int_{\overline{x}-\delta}^{z_I} \left(\frac{\overline{x} + s - \delta}{2}\right)\left(\frac{\overline{x} + \delta - s}{2\delta(\overline{x} - \underline{x})}\right) ds. \\
&= \frac{2\overline{x}^3 + (z_I - \delta)^3 + 6\underline{x}^2\delta - 3\overline{x}^2(z_I + \delta)}{12\delta(\overline{x} - \underline{x})}
\end{aligned}$$

That is, the equilibrium bid function for the informed bidder in AAP auctions is

$$\beta(z_I) = \begin{cases} \frac{(2\underline{x}+z_I+\delta)(z_I-\underline{x}+\delta)^2}{12\delta(\overline{x}-\underline{x})} & \text{if} \quad z_I \in [\underline{x} - \delta, \underline{x} + \delta) \\ \\ \frac{3z_I^2+\delta^2-3\underline{x}^2}{6(z_I-\underline{x})} & \text{if} \quad z_I \in [\underline{x} + \delta, \overline{x} - \delta) \\ \\ \frac{2\overline{x}^3+(z_I-\delta)^3+6\underline{x}^2\delta-3\overline{x}^2(z_I+\delta)}{12\delta(\overline{x}-\underline{x})} & \text{if} \quad z_I \in [\overline{x} - \delta, \overline{x} + \delta]\,. \end{cases}$$

In equilibrium, the uninformed bidder will mix on the interval $[0, E(X)]$ according to the following distribution function:

$$\begin{aligned}
J(b) &= \text{Prob}\left[\beta(Z_I) \le b\right] \\
&= F_{Z_I}\left(\beta^{-1}(b)\right).
\end{aligned}$$

So, the uninformed bidder will mix according using this distribution function:

$$J(b) = \begin{cases} \frac{\left(\beta^{-1}(b)-\underline{x}+\delta\right)^2}{4\delta(\overline{x}-\underline{x})} & \text{if} \quad b \in \left[\beta\left(\underline{x}-\delta\right), \beta\left(\underline{x}+\delta\right)\right) \\ \frac{\beta^{-1}(b)-\underline{x}}{(\overline{x}-\underline{x})} & \text{if} \quad b \in \left[\beta\left(\underline{x}+\delta\right), \beta\left(\overline{x}-\delta\right)\right) \\ \frac{4\delta(\overline{x}-\underline{x}-\delta)+\left(\overline{x}+3\delta-\beta^{-1}(b)\right)\left(\beta^{-1}(b)-\overline{x}+\delta\right)}{4\delta(\overline{x}-\underline{x})} & \text{if} \quad b \in \left[\beta\left(\overline{x}-\delta\right), \beta\left(\overline{x}+\delta\right)\right]. \end{cases}$$

The expected payoff of the informed bidder is given by:

$$\Pi_I^{AAP}(z_I) = \begin{cases} \frac{(z_I-\underline{x}+\delta)^3}{12\delta(\overline{x}-\underline{x})} & \text{if} \quad z_I \in \left[\underline{x}-\delta, \underline{x}+\delta\right) \\ \frac{3(\underline{x}-z_I)^2-\delta^2}{6(\overline{x}-\underline{x})} & \text{if} \quad z_I \in \left[\underline{x}+\delta, \overline{x}-\delta\right) \\ \frac{(\overline{x}-z_I+\delta)^3}{24\delta(\overline{x}-\underline{x})} + \frac{(\overline{x}+z_I-\delta)}{2} - \frac{(\overline{x}-\underline{x})}{2}. & \text{if} \quad z_I \in \left[\overline{x}-\delta, \overline{x}+\delta\right]. \end{cases}$$

The ex ante expected payoff of the informed bidder is

$$\begin{aligned} E\left(\Pi_I^{AAP}\right) &= \int_{\underline{x}-\delta}^{\overline{x}+\delta} \Pi_I^{AAP}(z_I)\, dz_I \\ &= \frac{5\left(\overline{x}-\underline{x}\right)^3 - 10\delta^2\left(\overline{x}-\underline{x}\right) + 8\delta^3}{30\left(\overline{x}-\underline{x}\right)^2}. \end{aligned}$$

For the parameters employed in our design, $E\left(\Pi_I^{AAP}\right) = 33.2301$. Recall that the uninformed bidder has an expected payoff of zero.

The ex ante expected revenue for the seller is found by subtracting the ex ante expected payoff of the informed bidder from the expected value of X. This yields

$$\begin{aligned} E\left(R^{AAP}\right) &= E\left(X\right) - E\left(\Pi_I^{AAP}\right) \\ &= \left(\frac{\overline{x}+\underline{x}}{2}\right) - \frac{5\left(\overline{x}-\underline{x}\right)^3 - 10\delta^2\left(\overline{x}-\underline{x}\right) + 8\delta^3}{30\left(\overline{x}-\underline{x}\right)^2}. \end{aligned}$$

For the parameter values used in our design $E\left(R^{AAP}\right) = 91.7699$.

Equilibrium Bidding in SLC

Recall that the probability that player i will obtain the good is given by:

$$p_i\left(b_i, b_j\right) = \begin{cases} \frac{b_i}{b_i+b_j} & \text{if} \quad \max\left\{b_i, b_j\right\} \neq 0 \\ \frac{1}{2} & \text{if} \qquad b_i = b_j = 0. \end{cases}$$

We assume that the marginal cost of bidding is constant and equal to one. Bidder i's seeks to maximize his expected payoff which is given by:

$$\Pi_i^{SLC} = p_i\left(b_i, b_j\right) E\left(X\right) - b_i.$$

This expenditure function is strictly concave in x_i given x_j. As discussed above, bidding zero is not an equilibrium strategy, so the best response is determined by the following first order condition:

$$\frac{b_j E\left(X\right)}{\left(b_i + b_j\right)^2} - 1 = 0.$$

Utilizing the fact that the bidders are symmetric, this yields the equilibrium bids of:

$$b_i = b_j = \frac{E\left(X\right)}{4}.$$

Using these equilibrium bids, we can easily calculate the equilibrium expected payoff of the bidders:

$$
\begin{aligned}
\Pi_i^{SLC} &= p_{i2}\left(\frac{E(X)}{4}, \frac{E(X)}{4}\right) E(X) - \frac{E(X)}{4} \\
&= \frac{E(X)}{2} - \frac{E(X)}{4} \\
&= \frac{E(X)}{4}.
\end{aligned}
$$

Revenue in this game is the expected value of the good less the expected payoffs of the bidders. Therefore, the expected revenue in this treatment, $E\left(R^{SLC}\right)$, is $\frac{E(X)}{2}$.

Equilibrium Bidding in ALC

This game is a special case of the model analyzed in the last period of Chapter II. If $a(z) = \max(\underline{x}, z - \delta)$ and $b(z) = \min(\overline{x}, z + \delta)$, then the informed bidder's problem is:

$$
\begin{aligned}
\Pi_I^{ALC}(z_I) &= \int_{a(z_I)}^{b(z_I)} \left(\frac{\zeta^{ALC}(z_I)}{\zeta^{ALC}(z_I) + b_U^{ALC}}\right) x f(x \mid z_I)\, dx - \zeta^{ALC}(z_I) \\
&= \left(\frac{\zeta^{ALC}(z_I)}{\zeta^{ALC}(z_I) + b_U^{ALC}}\right) E(X \mid z_I) - \zeta^{ALC}(z_I),
\end{aligned}
$$

where b_U^{ALC} is the bid of the uninformed ALC bidder. As in the SLC, this function is strictly concave given the bid of the uninformed bidder. The first order condition is:

$$
\frac{b_U^{ALC} E(X \mid z_I)}{\left(\zeta^{ALC}(z_I) + b_U^{ALC}\right)^2} - 1 = 0.
$$

Any $\zeta^{ALC}(z_I) \geqslant 0$ makes this condition negative if $b_U^{ALC} > E(X \mid z_I)$. Thus, the best response function of the informed bidder is:

$$\zeta^{ALC}\left(z_I\right)=\begin{cases}\sqrt{b_U^{ALC}E\left(X\mid z_I\right)}-b_U^{ALC} & \text{if}\quad z_I\geq q^{-1}\left(b_U^{ALC}\right)\\ 0 & \text{if}\quad z_I<q^{-1}\left(b_U^{ALC}\right).\end{cases}$$

where $q\left(z\right)=E\left(X\mid z_I\right)$, and $q^{-1}\left(\cdot\right)$ is the inverse of $q\left(\cdot\right)$.

The uninformed bidder's problem is given by:

$$\Pi_U^{ALC}=\int_{\underline{x}-\delta}^{\overline{x}+\delta}\int_{\underline{x}}^{\overline{x}}\frac{b_U^{ALC}}{\zeta^{ALC}\left(z_I\right)+b_U^{ALC}}xf(x,z_I)dxdz_I-b_U^{ALC}$$

This yields the following first order condition:

$$\int_{\underline{x}-\delta}^{\overline{x}+\delta}\int_{\underline{x}}^{\overline{x}}\frac{\zeta^{ALC}\left(z_I\right)}{\left(\zeta^{ALC}\left(z_I\right)+b_U^{ALC}\right)^2}xf(x,z_I)dxdz_I-1=0$$

Plugging in the informed bidder's best response function and simplifying characterizes the equilibrium in this game:

$$1=\left(\frac{1}{\sqrt{b_U^{ALC}}}\right)\int_{q^{-1}\left(b_U^{ALC}\right)}^{\overline{x}+\delta}\sqrt{E\left(X\mid z_I\right)}f\left(z_I\right)dz_I-\left(1-F_{Z_I}(q^{-1}\left(b_U^{ALC}\right))\right).$$

In our experimental design $b_U^{ALC}=29.37$.

The uninformed bidder's expected payoff is given by:

$$E\left(\Pi_U^{ALC}\right)=\int_{\underline{x}-\delta}^{q^{-1}\left(b_U^{ALC}\right)}\int_{\underline{x}}^{\overline{x}}xf(x,z_I)dxdz_I+b_U^{ALC}\left(1-F_{Z_I}(q^{-1}\left(b_U^{ALC}\right))\right).$$

For the parameter values employed in our experimental design, $E\left(\Pi_U^{ALC}\right) = 29.72$.

The expected payoff of the informed bidder when he/she observes an estimate $Z_I = z_I$ is given by:

$$\Pi_I^{ALC}\left(z_I\right) = \begin{cases} 0 & \text{if} \quad z < q^{-1}\left(b_U^{ALC}\right) \\ E\left(X \mid z_I\right) - 2\sqrt{b_U^{ALC} E\left(X \mid z_I\right)} + b_U^{ALC} & \text{if} \quad z \geq q^{-1}\left(b_U^{ALC}\right). \end{cases}$$

The ex ante expected payoff of the informed bidder is given by:

$$E\left(\Pi_I^{ALC}\left(z_I\right)\right) = \int\limits_{q^{-1}\left(b_U^{ALC}\right)}^{\overline{x}+\delta} \int\limits_{\underline{x}}^{\overline{x}} x f(x, z_I) dx dz_I - b_U^{ALC}\left(3 - F_{Z_I}\left(q^{-1}\left(b_U^{ALC}\right)\right)\right).$$

For the parameter values employed in our experiment, $E\left(\Pi_I^{ALC}\left(z_I\right)\right) = 36.92$.

The ex ante expected revenue in this treatment is found by adding the expected equilibrium bid of the informed ALC bidder and the equilibrium bid of the uninformed ALC bidder. In our experimental design this is $E\left(R^{ATC}\right) = 58.74$.

APPENDIX G

This Appendix contains the description and results for all-pay auctions in which each bidder observes a private signal. Such an auction is a symmetric information all-pay auction with private signals (SAP-PRIV).

DESIGN

Participants engage in 30 all-pay auctions. In each of these all-pay auctions each bidder privately observes a signal. These signals, z_1 and z_2, are independently drawn from a uniform distribution with support $[x-8, x+8]$. In this treatment both bidders hold private information in the form of their signal. Information is symmetric in that each signal is an equally precise estimate of x. We do not have theoretical predictions for this treatment.[55] We include this treatment for comparison with the results of Chapter III. Additionally, the susceptibility of bidders to bidding above the break-even bidding strategy in such an environment is of interest.

Break-even Bidding in SAP-PRIV

A long literature experimentally studies this information structure in the context of first-price, sealed-bid auctions. It is well documented that when inexperienced bidders privately observe private signals they consistently fall victim to the winner's curse.[56] Further, Chapter III demonstrates that when inexperienced bidders in a first-price, sealed-bid auction do not observe a signal prior to bidding the winner's curse is almost completely eliminated. Including the SAP-PRIV information structure for all-pay auctions allows us to compare the results for all-pay auctions to those

[55] As noted in Athey [2], a common value all-pay auction with conditionally independent signals does not satisfy the single crossing property.

[56] The winner's curse is defined as bidding above a break-even threshold, such that when a bidder wins an auction, they have negative expected profits.

found in Chapter III. Do bidders who observe signals in all-pay auction bid above the beak-even bidding strategy when their opponent also observes a signal?

As such, defining the break-even bidding strategy in the context of an all-pay auction when both bidder's observe private signals is important. If bidders bid according to a monotonically increasing bid function, then the bidder who observes the highest signal will win the auction. Thus, the expected value of the good, conditional on winning the all-pay auction is the same as the expected value of the good conditional on having the highest signal. So, if bidder i bids above, $E\left(X \mid z_i > z_j\right)$, the bidder will have a negative expected payoff, conditional on winning the auction. However, if the bidder were to lose the auction, she would still have to pay her bid. As such, the break-even bidding threshold, assuming the bidders are bidding according to a monotonically increasing bid function is any bid greater than $F\left(Z_j = z_i \mid Z_i = z_i\right) E\left(X \mid z_i > z_j\right)$.

When $z_i \in [\underline{x} - \delta, \underline{x} + \delta)$,

$$
\begin{aligned}
& F\left(Z_j = z_i \mid Z_i = z_i\right) E\left(X \mid Z_i = z_i > z_j\right) \\
= & \int_{\underline{x}-\delta}^{z_i} \int_{\underline{x}}^{z_j+\delta} x f_X\left(x, z_j \mid z_i\right) dx dz_j \\
= & \int_{\underline{x}-\delta}^{z_i} \int_{\underline{x}}^{z_j+\delta} x \frac{1}{2\delta\left(z_i + \delta - \underline{x}\right)} dx dz_j \\
= & \left(\frac{z_i + \delta - \underline{x}}{4\delta}\right)\left(\frac{z_i + 2\underline{x} + \delta}{3}\right).
\end{aligned}
$$

When $z_i \in [\underline{x} + \delta, \overline{x} - \delta)$,

$$\begin{aligned}
& F(Z_j = z_i \mid Z_i = z_i) E(X \mid Z_i = z_i > z_j) \\
= & \int_{z_i - 2\delta}^{z_i} \int_{z_i - \delta}^{z_j + \delta} x f_X(x, z_j \mid z_i) \, dx dz_j \\
= & \int_{\underline{x} - \delta}^{z_i} \int_{\underline{x}}^{z_j + \delta} x \frac{1}{4\delta^2} dx dz_j \\
= & \frac{z_i}{2} - \frac{\delta}{6}.
\end{aligned}$$

When $z_i \in [\overline{x} - \delta, \overline{x} + \delta]$

$$\begin{aligned}
& F(Z_j = z_i \mid Z_i = z_i) E(X \mid Z_i = z_i > z_j) \\
= & \int_{z_i - 2\delta}^{z_i} \int_{z_i - \delta}^{z_j + \delta} x f_X(x, z_j \mid z_i) \, dx dz_j \\
= & \int_{z_i - 2\delta}^{\overline{x} - \delta} \int_{z_i - \delta}^{z_j + \delta} x \frac{1}{2\delta(\overline{x} + \delta - z_i)} dx dz_j + \int_{\overline{x} - \delta}^{z_i} \int_{z_i - \delta}^{\overline{x}} x \frac{1}{2\delta(\overline{x} + \delta - z_i)} dx dz_j \\
= & \frac{(z_i + 5\delta)(z_i - \delta) + \overline{x}(z_i + 5\delta) - 2\overline{x}^2}{12\delta}.
\end{aligned}$$

That is,

$$F(Z_j = z_i \mid Z_i = z_i) E(X \mid Z_i = z_i > z_j) = \begin{cases} \left(\frac{z_i + \delta - \underline{x}}{4\delta}\right)\left(\frac{z_i + 2\underline{x} + \delta}{3}\right) & \text{if} \quad z_i \in [\underline{x} - \delta, \underline{x} + \delta) \\ \frac{z_i}{2} - \frac{\delta}{6} & \text{if} \quad z_i \in [\underline{x} + \delta, \overline{x} - \delta) \\ \frac{(z_i + 5\delta)(z_i - \delta) + \overline{x}(z_i + 5\delta) - 2\overline{x}^2}{12\delta} & \text{if} \quad z_i \in [\overline{x} - \delta, \overline{x} + \delta]. \end{cases}$$

EXPERIMENTAL RESULTS

Table 23: Revenue in contests (including SAP-PRIV) aggregated across all rounds and sessions

Treatment	**Average observed revenue (standard deviation)**	**Average predicted revenue (standard deviation)**
SAP	119.09	125.00
	(65.77)	(0.00)
AAP	95.23	88.24
	(69.31)	(29.80)
SAP-PRIV	140.88	–
	(104.49)	
SLC	96.76	62.50
	(44.44)	(0.00)
ALC	95.97	56.13
	(56.83)	(14.65)

Revenue

Table 23 contains summary statistics regarding revenue. Notice that SAP-PRIV auctions generate more revenue than any other treatment, on average.

We find dramatic results regarding revenue in all-pay auctions when both bidders observe a private signal. In particular, we find that revenue is greater than in any other treatment. Revenue in all-pay auctions where both bidders observe a private signal is greater than when neither bidder observes a signal (robust rank-order test, $\acute{U} = 3.086$, $p < 0.028$). Typically, auction theory predicts that bidders who hold private information earn a positive information rent, and reduce revenue relative to the case in which their information is unobserved or made public. Our data suggests that providing bidders with private information can increase revenue.

We also find that revenue in all-pay auctions when both bidders observe a private signal is greater than in asymmetric information all-pay auctions (robust rank-order test, $\acute{U} = n.d.$, $p = 0.004$). This is also true in lottery contests with symmetric

Table 24: Bidder payoffs in contests (including SAP-PRIV) aggregated over all rounds and sessions

Bidders	**Average observed payoffs (standard deviation)**	**Average predicted payoffs (standard deviation)**
SAP	-1.72	0
	(62.77)	(0)
AAP-Informed	26.38	27.29
	(59.50)	(27.70)
AAP-Uninformed	-6.08	0
	(44.06)	(0)
SAP-PRIV	-12.67	–
	(62.34)	
SLC	9.39	31.25
	(68.58)	(0)
ALC-Informed	22.72	31.20
	(60.96)	(26.85)
ALC-Uninformed	-3.16	29.72
	(54.68)	(0)

(robust rank-order test, $\acute{U} = n.d.$, $p = 0.004$) and asymmetric (robust rank-order test, $\acute{U} = n.d.$, $p = 0.004$).information.

Bidder Payoffs

Table 24 contains summary statistics regarding bidder payoffs. Notice that SAP-PRIV bidders have the lowest payoffs, on average.

We find that informed AAP bidders earn more than SAP-PRIV bidders (robust rank-order test, $\acute{U} = n.d.$, $p = 0.004$). SAP bidders, who hold no private information, have payoffs significantly greater than SAP-PRIV bidders, who do hold private information (robust rank-order test, $\acute{U} = 2.564$, $p < 0.048$). This surprising result is consistent with the findings in Chapter III in which bidders in common-value, first price auctions earn higher payoffs when bidders do not observe private signals

Table 25: Bidding above the break-even bidding strategy in contests (including SAP-PRIV) aggregated across all rounds and sessions

Bidders	Frequency bid exceeds break-even bid: All bidders	Frequency bid exceeds break-even bid: Winning bidders	Frequency the high (or only) signal holder wins
SAP	6.2%	12.1%	NA
	(93/1490)	(90/745)	NA
AAP-Informed	32.7%	30.4%	69.2%
	(245/750)	(158/519)	(519/750)
AAP-Uninformed	4%	11.3%	NA
	(30/750)	(26/205)	NA
SAP-PRIV	62.87%	83.73%	58.67%
	(943/1500)	(628/750)	(440/750)
SLC	8.1%	12.1%	NA
	(122/1500)	(91/750)	NA
ALC-Informed	34.3%	32.8%	50.7%
	(257/750)	(168/512)	(380/750)
ALC-Uninformed	8.3%	16%	NA
	(62/750)	(38/238)	NA

NA = not applicable.
The decimal numbers in parentheses are standard deviations.
The fractions in parentheses are relative frequencies.

than when all bidders observe private signals. We also find that uninformed AAP bidders earn significantly higher payoffs than SAP-PRIV bidders (robust rank-order test, $\acute{U} = n.d.$, $p = 0.004$).

Break-even Bidding

Table 25 contains summary statistics regarding break-even bidding.

Figure 14 illustrates how the observed signal of SAP-PRIV bidders relates to signal.

Bidding

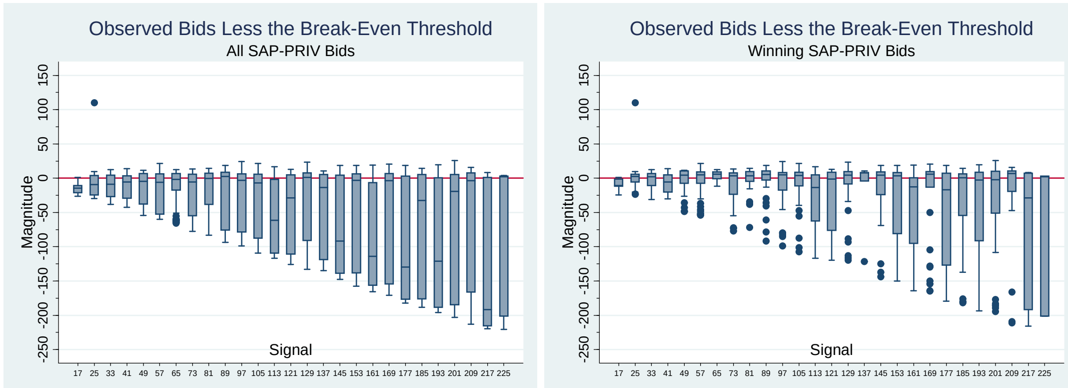


Figure 14: The difference between observed bids and break-even bids for SAP-PRIV depending on the signal

We find that SAP-PRIV bidders bid more than SAP bidders (robust rank-order test, $\acute{U} = 2.361$, $p < 0.048$). We can not reject the hypothesis that SAP-PRIV bidders bid the same amount as informed AAP bidders (robust rank-order test, $\acute{U} = 0.853$, $n.s.$). SAP-PRIV bidders also bid more than uninformed AAP bidders (robust rank-order test, $\acute{U} = n.d.$, $p = 0.004$).

APPENDIX H

What follows is a sample set of instructions from Chapter IV. Instructions for the remaining treatments are available upon request.

Introduction

Welcome. This experiment is about decision making in markets. The following instructions describe the markets you will be in and the rules that you will face. The decisions you make during this experiment will determine how much money you earn. If you make good decisions, you can earn a substantial amount of money. You will be paid in cash privately at the end of our experiment.

It is important that you remain silent and do not look at other people's work. If you have any questions, or need assistance of any kind, please raise your hand and an experimenter will come to you. If you talk, laugh, exclaim out loud, etc., you will be asked to leave and you will not be paid. We expect and appreciate your cooperation.

We will go over these instructions with you. After we have read the instructions, there will be time to ask clarifying questions. When we are done going through the instructions, each of you will have to answer a few brief questions to ensure everyone understands.

Overview

Our experiment will consist of 30 rounds. In each of these rounds, you will be randomly paired with another participant in today's experiment. Both of you will be buyers in a market. In each market, there will be a single unit of an indivisible good for sale. As a buyer, your task is to submit a bid for the purchase of the good.

You will receive earnings based on the outcome of the market. This process will be repeated until all 30 rounds have been completed.

Determination of Your Earnings

Each participant will receive a showup fee of \$5. In addition, each participant in this experiment will start with a balance of $\$3,200$ "experimental dollars" (EDs). EDs will be traded in for cash at the end of the experiment at a rate of $\$160ED = \$1$. Your starting balance can increase or decrease depending on your payoffs in each round. That is, if you have a negative payoff in a round, this loss will be deducted from your balance. If you earn a positive payoff, this is added to your balance. You are permitted to bid more than your remaining balance. However, if after a round is completed your balance is less than or equal to zero, you will not be able to participate in any future rounds.

In each round, you and the other buyer in the market will submit a bid. Both of those bids will have to be paid, but only one of the buyers will receive the good. Each of the buyers has the following probability of receiving the good:

$$\frac{\text{(Own Bid)}}{\text{(Own Bid)+(Other's Bid)}}$$

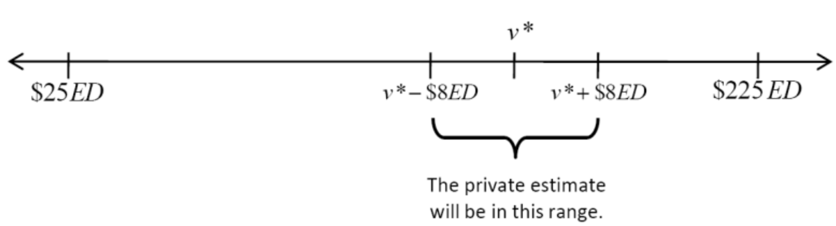
Notice that if one a buyer submits a bid of zero, there is no chance of that buyer receiving the good; the other buyer will receive the good with certainty. If both buyers submit the same bid, then each of the buyers has a 50% chance of receiving the good.

Notice that a buyer who receives the good can end up with a negative payoff, if he/she bids more than the good is worth. The buyer who does not receive the good will always have a negative payoff if their bid was greater than zero. No buyer is permitted to submit a bid that is lower than zero.

In each round, the value of the good, which we will denote as v^*, will not be known to the buyers. The value of this good will be between \$25ED and \$225ED. Any value between \$25ED and \$225ED is equally likely to be chosen as v^*. The value of the good in any given round is independent of the value in any other round. That is, the value of the good in one round will not have *any* effect on the value of the good in a different round.

Private Information

In each market, one of the two buyers will be randomly chosen to receive some private information about the value of the good (you can think of this as flipping a coin to determine which of the buyers will receive this information, where the probability of the coin landing on each side is 50%). The person who receives the private information will be given an estimate of the value of the good. The estimate will be a randomly chosen number that is within \$8ED above or below the real value of v^* (see the illustration below). Any number between $v^* - \$8ED$, and $v^* + \$8ED$ is equally likely to be chosen as the private estimate. For example, if you receive a private estimate of \$125ED, then you know that v^* is between \$117ED and \$133ED, inclusive. It is possible for the estimate to be a value below \$25ED or above \$225ED, but the real value of v^* will always be between \$25ED and \$225ED.



Rounds

As mentioned before, there will be 30 rounds in this experiment. In each round there will be several markets going on simultaneously, with two buyers in each market. After each round you will be randomly paired with another participant in today's experiment. This random assignment is done *every round* so that two buyers will probably not be in the same market together for two consecutive rounds. Further, this pairing is anonymous. That is, if you are a buyer in a given market, you do not know which of the other participants in the experiment is the other buyer in that market. Remember that these different markets and rounds are independent from all others, and from one another. The bids and the value of the good and the private estimate in one market or round do not have any effect on other markets or rounds. Markets and rounds operate independently.

Summary

1. Each participant has a starting balance of $\$3,200ED$.
2. In every round, each participant will be a bidder in one market. Two participants are randomly assigned to a market in each round.
3. In each market each buyer gets $v^* -$ (Own bid) with probability $\left(\frac{\text{(Own Bid)}}{\text{(Own Bid)+(Other's Bid)}}\right)$, and gets $0 -$ (Own bid) with the remaining probability $\left(1 - \frac{\text{(Own Bid)}}{\text{(Own Bid)+(Other's Bid)}}\right)$. This payoff is added to the balance of each bidder (a bidder's balance will go down if the value is negative, and up if this value is positive).
4. The value of the good, v^*, is unknown. It is known that it is somewhere between $\$25ED$ and $\$225ED$. Every value between $\$25ED$ and $\$225ED$ is equally likely to be v^*.

5. One of the two bidders in each market is randomly chosen to receive a private estimate of v^*. This estimate is not observed by the other bidder in the market. This estimate is randomly drawn from the interval between $v^* - \$8ED$ and $v^* + \$8ED$, inclusive. Any number from this interval is equally likely to be chosen as the private estimate.

6. Every participant will receive the show-up fee of $5. Additionally, each participant will receive his/her balance at the end of all 30 rounds, based on the $\$3,200ED$ beginning balance and earnings in each market.

7. If a participant's balance should become negative at any point during this experiment, he/she will not be permitted to participate in future rounds.

If you have any questions, raise your hand and one of us will come help you. Please do not ask any questions out loud.

Questions

Before we begin the experiment, we would like you to answer a few questions that are meant to review the rules of today's experiment. Please raise your hand once you are done, and an experimenter will attend to you.

1. How many buyers are in each market? __________

2. Who pays their bid in each market, the bidder who gets the good, the bidder who doesn't get the good, or both? __________

3. The private estimate must be within what range of v^*? __________

4. Are you allowed to bid more than your current balance? __________

5. For each market, how many buyers get to see the estimate of v^*? __________

6. If the bid of a buyer who receives the good in a market is \$152.10ED, and the value of the good is revealed to be \$200.90ED, what is the winner's payoff for that market? ____________

7. What would the earnings from question six have been if the value of the good had been \$25.90ED? ____________

8. If Buyer 1 bids \$150.00ED, and Buyer 2 bids \$200.00ED, and the value of the good is revealed to be \$220.75ED, what are the payoffs for Buyer 1 and Buyer 2 if Buyer 2 receives the good? ____________

9. What would the earnings from question eight have been if Buyer 1 received the good? ____________

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