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VIBRATION SERVICEABILITY OF FOOTBRIDGES UNDER HUMAN-INDUCED EXCITATION: A LITERATURE REVIEW

STANA ŽIVANOVIĆ, ALEKSANDAR PAVIC and PAUL REYNOLDS

ABSTRACT

Increasing strength of new structural materials and longer spans of new footbridges, accompanied with aesthetic requirements for greater slenderness, are resulting in more lively footbridge structures. In the past few years this issue attracted great public attention. The excessive lateral sway motion caused by crowd walking across the infamous Millennium Bridge in London is the prime example of the vibration serviceability problem of footbridges. In principle, consideration of footbridge vibration serviceability requires a characterisation of the vibration source, path and receiver. This paper is the most comprehensive review published to date of about 200 references which deal with these three key issues.

The literature survey identified humans as the most important source of vibration for footbridges. However, modelling of the crowd-induced dynamic force is not clearly defined yet, despite some serious attempts to tackle this issue in the last few years.

The vibration path is the mass, damping and stiffness of the footbridge. Of these, damping is the most uncertain but extremely important parameter as the resonant behaviour tends to govern vibration serviceability of footbridges.

A typical receiver of footbridge vibrations is a pedestrian who is quite often the source of vibrations as well. Many scales for rating the human perception of vibrations have been found in the published literature. However, few are applicable to footbridges because a receiver is not stationary but is actually moving across the vibrating structure.

During footbridge vibration, especially under crowd load, it seems that some form of human–structure interaction occurs. The problem of influence of walking people on footbridge vibration properties, such as the natural frequency and damping is not well understood, let alone quantified.

Finally, there is not a single national or international design guidance which covers all aspects of the problem comprehensively and some form of their combination with other published information is prudent when designing major footbridge structures. The overdue update of the current codes to reflect the recent research achievements is a great challenge for the next 5–10 years.

Abbreviations:

ASD—auto spectral density;

DOF—degree of freedom;

FRF—frequency response function;

MTMD—multiple tuned mass damper;

SDOF—single-degree-of-freedom;

TMD—tuned mass damper

DLF—dynamic load factor;

FE—finite element;

MDOF—multiple-degree-of-freedom;

RMS—root-mean-square;

TLD—tuned liquid damper;

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1 Introduction

In recent decades there has been a trend towards improved mechanical characteristics of materials used in footbridge construction. It has enabled engineers to design lighter, more slender and more aesthetic structures. A considerable variety of modern footbridge structural forms can be seen, for example, in recent articles by Biliszczuk et al. [1], Block and Schlaich [2], Eyre [3], Firth [4], Iso and Masubuchi [5], Mimram [6], Schlaich [7], Strasky [8], Takenouchi and Ito [9] and Wörner and Schlaich [10]. As a result of these construction trends, many footbridges have become more susceptible to vibrations when subjected to dynamic loads.

This paper is focused on human-induced dynamic loading of footbridges. This is a frequently occurring and often dominant load for footbridges as it stems from the very purpose of a footbridge—to convey pedestrians. It was noted very early that this type of dynamic excitation could cause excessive vibrations and in extreme cases even a collapse of the structure. It is known that in 1154 a timber footbridge collapsed under a crowd that wanted to greet the Archbishop William [11]. However, details related to the exact crowd behaviour are not known. Probably the oldest case of footbridge failure due to dynamic human-induced load reported in detail was the one which occurred in 1831 in Broughton, UK while 60 soldiers were marching across a bridge. It was this event that prompted the placement of the famous notices on a considerable number of bridges with a warning to troops to break step when crossing [12]. One of the notices displayed on a railway suspension bridge at Niagara Falls, USA reads as follows [13]:

> "A fine of $50 to $100 will be imposed for marching over this bridge in rank and file or to music, or by keeping regular step. Bodies of men or troops must be kept out of step when passing over this bridge. No musical band will be allowed to play while crossing except when seated in wagons or carriages."

Although there have been many reported cases of lively footbridges in the past [14], [15], [16], [17] and [18], this problem attracted considerably greater public and professional attention after the infamous swaying of the new and attractive Millennium Bridge in London during its opening day on 10 June 2000 [19]. The Millennium Bridge problem attracted more than 1000 press articles and over 150 broadcasts in the media around the world. In this and almost all other previously reported problems related to footbridge vibrations, the excessive vibrations were caused by a near resonance of one or more modes of vibration. The reason for this is that the range of footbridge natural (vertical or lateral) frequencies often coincides with the dominant frequencies of the human-induced load [20]. It is important to note that the problems have occurred on a range of different structural types, such as cable-stayed, suspension and girder bridges, as well as on footbridges made of different materials (e.g. timber, composite steel-concrete, steel, reinforced and prestressed concrete). The problem of footbridge vibrations is becoming so alarming that a major international conference, entitled Footbridge 2002, was recently held in Paris and was almost completely devoted to this issue. Among many articles given at the conference, three presented the current state-of-the-art in footbridge vibration serviceability design, especially with regard to human-induced dynamic load [7], [21] and [22].

Nowadays, it is generally accepted that vibration produced by human-induced loads is usually a serviceability rather than a safety (i.e. strength-related) problem [14], [23], [24] and [25]. This is because human beings are very sensitive to vibration levels as low as 0.001 mm [26]. This high sensitivity usually triggers the vibration serviceability problem much before the vibration levels are even remotely sufficient to cause damage of the structure itself.

The ISO 10137 guidelines [27] define the vibration source, path and receiver as three key, but separate, issues which require consideration when dealing with the vibration serviceability of any structure. Following the notion of such an analytical framework, this paper reviews these three issues for footbridges separately.

The literature review contains six parts. In the first three, the materials related to the vibration source, path and receiver are presented. The fourth part outlines the human–structure dynamic interaction phenomenon while in the fifth some important design procedures/recommendations are reviewed. In this paper, the term "design procedure" or "design recommendation" stands for

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design checking methods offered by different authors which are not formally codified, while for codified procedures the terms "design guidelines/codes (of practice)" are used. Naturally, some of the design recommendations tend to be adopted in key design codes of practice which deal with footbridge vibration serviceability. These codes are also presented. The last part reviews typical remedial measures which can be undertaken to lessen the excessive vibrations of footbridges.

2 Humans as Vibration Source for Footbridges

During walking, a pedestrian produces a dynamic time varying force which has components in all three directions: vertical, horizontal-lateral and horizontal-longitudinal [14]. This single pedestrian walking force, which is due to accelerating and decelerating of the mass of their body, has been studied for many years. In particular, the vertical component of the force has been most investigated. It is regarded as the most important of the three forces because it has the highest magnitude. Other types of human-induced forces important for footbridges are due to running and some forms of deliberate vandal loading (jumping, bouncing or horizontal body swaying). Some of these types of human-induced forces have been studied not only for a single person, but also for small groups of people. However, large groups of pedestrians have seldom been formally investigated.

2.1 Early Works

Probably the oldest report of noticeable vibrations in footbridges was made by Stevenson in 1821 [69]. In addition to this, the same author reported severe vibrations due to a marching regiment crossing over a bridge, indicating very early a need to consider human-induced dynamic loads in bridge design. It is interesting that 10 years after Stevenson's observations, as previously mentioned, a bridge collapse in Broughton was caused by marching soldiers.

Tilden [28] wrote an excellent article for that time primarily devoted to the crowd load. However, he also reported some experiments in which, although not having precise measurement devices, he tried to quantify the dynamic effect of a force generated by a single person due to different activities.

2.2 Single Person Force Measurements

One of the first measurements of pedestrian-induced forces was conducted by Harper et al. [29] and Harper [30] with the aim to investigate the friction and slipperiness of a floor surface. They measured horizontal and vertical force from a single footstep using a force plate [31]. The shape of the vertical force with two peaks and a trough of the kind shown in Fig. 1a was recorded. This general shape of the force time history was confirmed by other researchers such as Galbraith and Barton [33], Blanchard et al. [34], Ohlsson [35], Kerr [36] and many others.

A lot of research into walking forces has been done in the field of biomechanics, usually with the aim to investigate differences in the step patterns between patients who are healthy and those with abnormalities. In one of these investigations, Andriacchi et al. [32] measured, similar to Harper et al. [29], single step walking forces in all three directions by means of a force plate. Typical shapes are presented in Fig. 1. They also reported that increasing walking velocity led to increasing step length and peak force magnitude. In other words, the dynamic effect of the forces was changing with the walking speed. This demonstrates the complex nature of human-induced dynamic forces and their dependence on many parameters. For example, tests with control of only one of the parameters, such as the pacing frequency, speed or step length, each produce different relationships between the walking speed and the pacing frequency [37]. Also, with increasing walking speed, the variability in vertical and lateral forces over successive steps increases, whereas the longitudinal force has the minimum variability at a normal walking speed [38].

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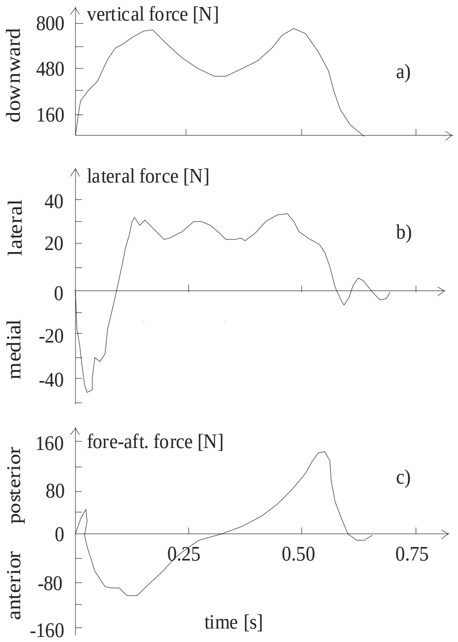

Figure 1: Typical shapes of walking force in (a) vertical, (b) lateral and (c) longitudinal direction (after Andriacchi et al. [32]).

Galbraith and Barton [33] measured a single step vertical force on an aluminium plate, ranging from slow walking to running. They reported that the shape of running force differed from the walking force in having only one peak (Fig. 2). Subject weight and step frequency were identified as important parameters which increase led to higher peak amplitudes of the force. On the other hand, the force was not dependent on the type of footwear and walking surface. By combining individual foot forces, which are assumed to be identical, a continuous walking or running force can be obtained artificially (Fig. 2). During walking there are some short time periods when both feet are on the ground which gives an overlapping between the left and right leg in the walking time history (Fig. 2—right). On the other hand, during running there are periods when both feet are off the ground leading to zero force recorded (Fig. 2—left).

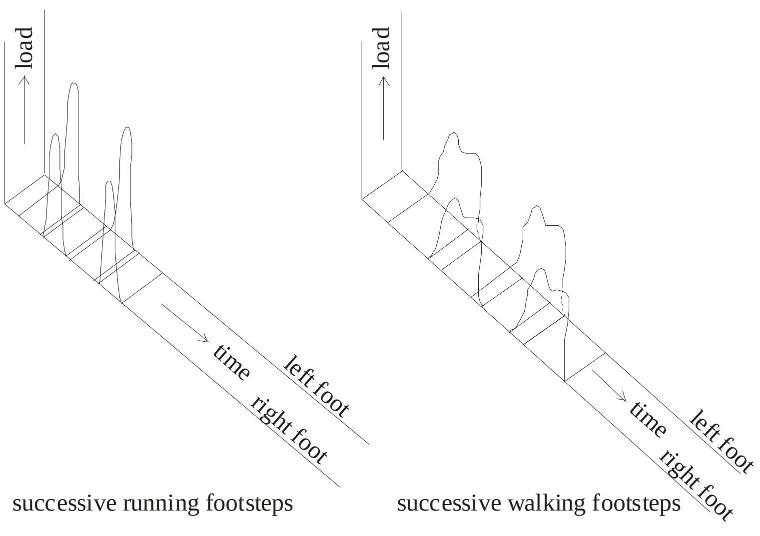

Figure 2: Typical pattern of running and walking forces (after Galbraith and Barton [33]).

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A very comprehensive research into human forces relevant to footbridge dynamic excitation was conducted by Wheeler [39] and [40] who systematised the work of other researchers related to different modes of human moving from slow walking to running (Fig. 3). He also presented dependence of many walking parameters, such as step length, moving velocity, peak force and contact time (the time while one foot is in the contact with the ground) as a function of the pacing frequency (Fig. 4). It was noted that all these parameters are different for different persons, but some general conclusions can be drawn. For example, that with increasing step frequency the peak amplitude, stride length and velocity increase while contact time decreases.

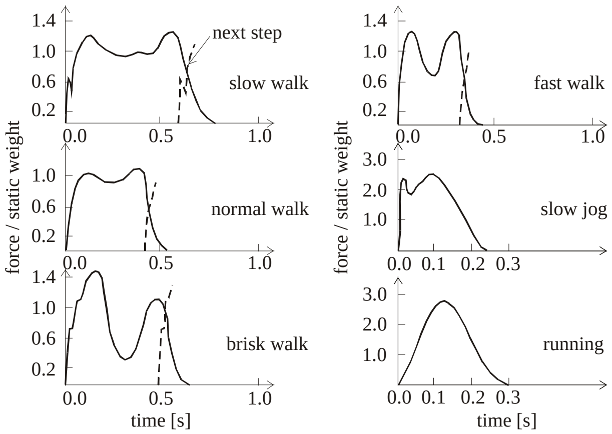

Figure 3: Typical vertical force patterns for different types of human activities (after Wheeler [40]).

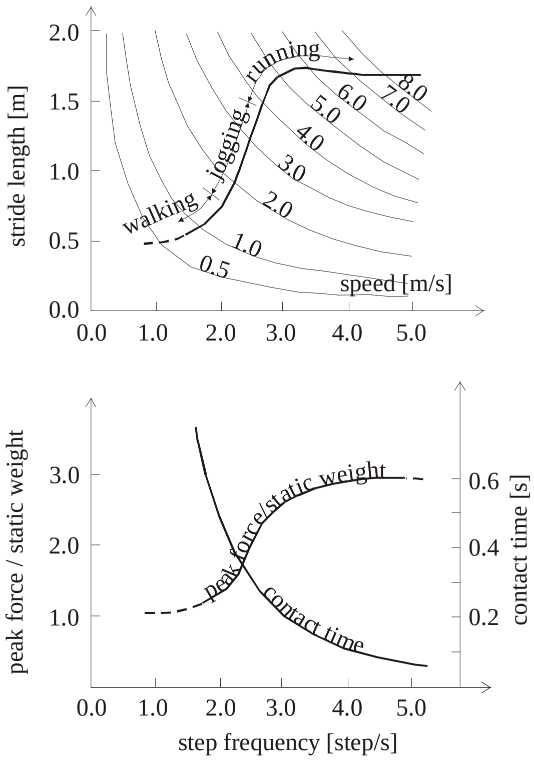

Figure 4: Dependance of stride length, velocity, peak force and contact time on different pacing rates (after Wheeler [40]).

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Measurements of individual step forces were followed by the more advanced and informative measurements of continuous walking time histories comprising several steps. For this purpose Blanchard et al. [34] used a gait machine described by Skorecki [41], Rainer et al. [42] used a floor strip, whereas Ebrahimpour et al. [43] and [44] used a platform instrumented with several force plates. The measured time histories were obviously near periodic with the (average) period equal to reciprocal value of the (average) step frequency. Unfortunately, in all these works the attention was paid only to vertical forces. However, based on measurements by Andriacchi et al. [32] and taking into account that the fundamental frequency of the lateral walking-induced force is two times lower than its counterpart relevant to the vertical and longitudinal forces [14], general shapes for continuous forces in all three directions can be constructed if their perfect periodicity is assumed (Fig. 5).

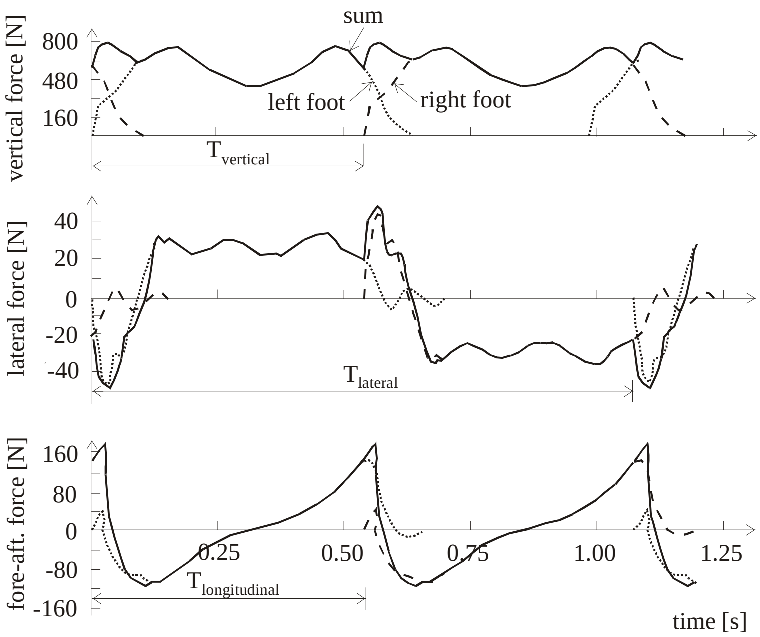

Figure 5: Periodic walking time histories in vertical, lateral and longitudinal directions.

A reliable statistical description of normal walking frequencies was first given by Matsumoto et al. [45] and [46] who investigated a sample of 505 persons. They concluded that the frequencies followed a normal distribution with a mean pacing rate of 2.0 Hz and standard deviation of 0.173 Hz (Fig. 6). Kerr and Bishop [47] obtained a mean frequency of 1.9 Hz but from an investigation of only 40 subjects. It is also interesting that Leonard in 1966 concluded that the normal walking frequency range is 1.7–2.3 Hz, which is in broad agreement with what Matsumoto et al. [46] and successive researchers have found. Similar comprehensive statistically based investigations, such as the one given by Matsumoto et al. [46] for walking, do not exist for other types of human-induced forces. However, there are some proposals as to the typical frequency ranges for different human activities (running, jumping, bouncing, etc.). For example, Bachmann et al. [48] defined typical frequency ranges of 1.6–2.4 Hz for walking, 2.0–3.5 Hz for running, 1.8–3.4 Hz for jumping, 1.5–3.0 Hz for bouncing and 0.4–0.7 Hz for horizontal body swaying while stationary.

Taking an alternative approach to the problem, Ohlsson [35] was interested more in the energy and frequency content than in the exact time history of the vertical force. He concluded that a single step force had most of its energy in the frequency range from 0 to 6 Hz. His successor Eriksson [49] investigated this issue more closely. He measured a continuous walking force indirectly, concluded that it was a narrow-band random process and, quite conveniently, presented it in terms of its auto (or power) spectral density (Fig. 7).

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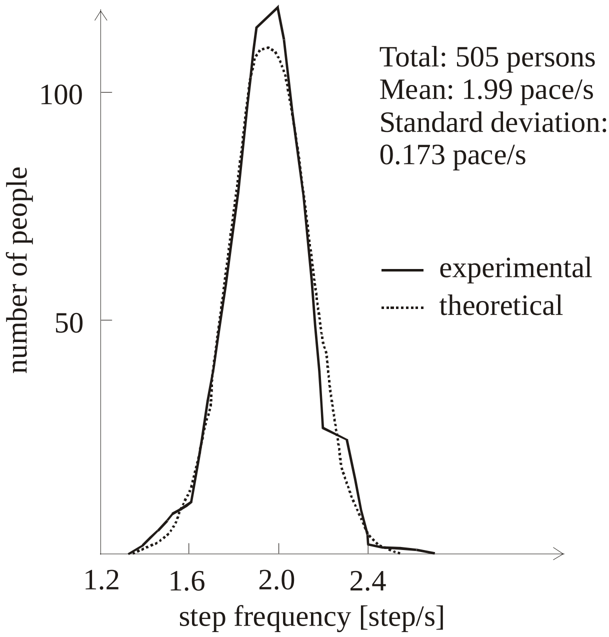

Figure 6: Normal distribution of pacing frequencies for normal walking (after Matsumoto et al. [45]).

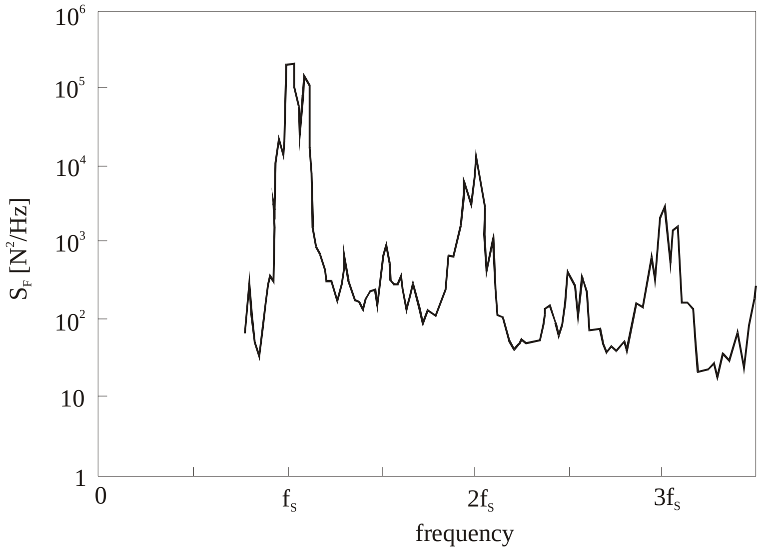

Figure 7: ASD of a walking force (after Eriksson [49]).

With regard to all this, it is prudent to stress that practically all mentioned measurements of the walking forces were conducted on various forms of rigid surface, such as a force plate on stiff ground, gait machine or high-frequency structure (for indirect measurements of the kind performed by Rainer et al. [42] and Eriksson [49]). This leaves the possibility that the reported forces could be different from the ones that actually occur on low-frequency footbridge structures that move perceptibly.

Regarding vandal loading, some researchers have presented jumping forces from individuals [48], [50], [51], [52] and [53]. During jumping, the peak forces have been found to be several times higher then the jumper's weight. Moreover, very recently gathered experimental evidence has shown that horizontal forces due to vertical jumping also exist [54], where the front-to-back force was considerably larger than its side-to-side counterpart. Also, time histories of the

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bouncing force were recently presented by Yao et al. [52] who, quite differently from other researchers, measured these forces on a flexible and perceptibly moving instrumented platform.

It should be noted that the majority of past investigations have been into forces due to activities of a single person. However, some limited measurements of forces produced by groups of people do exist and will be mentioned later.

2.3 Force Modelling

To successfully apply the measured dynamic forces in design it is necessary to model them analytically. Two types of such models can be found in the literature: time- and frequency-domain models. Although the former is much more common than the latter, in both cases mathematical modelling of human-induced dynamic forces is a complicated task. This is because:

1. there are many different types of human-induced forces and some of them change not only in time but also in space (e.g. walking and running);
2. forces are dependent on many parameters as demonstrated in the preceding text;
3. dynamic force generated by a single person is essentially a narrow-band process which is not well understood and therefore difficult to mathematically model;
4. the influence of the number of persons as well as their degree of synchronisation/correlation is difficult to generalise; and
5. there are strong indications that the forces are different in cases of perceptibly and not so perceptibly moving footbridges because of different behaviour of people in these two situations.

However, force models do exist and are used in contemporary design. They are based on some more or less justifiable assumptions which will be presented.

2.3.1 Time Domain Force Models

Generally, two types of time domain models have been found in the literature: deterministic and probabilistic. The first type intends to establish one general force model for each type of human activity, while the other takes into account the fact that some parameters which influence human force, such as the previously mentioned activity frequency, person's weight and so on, are random variables whose statistical nature should be considered in terms of their probability distribution functions.

In any case, time-domain models for walking and running are based on an assumption that both human feet produce exactly the same force and that the force is periodic. The assumption of perfect repetition is also frequently used in modelling of vandal loading generated by a single person and small groups.

2.3.1.1 Deterministic Force Models

It is well-known that each periodic force $F_p(t)$ with a period T can be represented by a Fourier series [48]:

$$F_p(t) = G + \sum_{i=1}^{n} G\alpha_i \sin\left(2\pi i f_p t - \phi_i\right) \quad (1)$$

where:

G – person's weight [N],

α_i – Fourier's coefficient of the i^{th} harmonic i.e. dynamic load factor (DLF),

f_p – activity rate [Hz],

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ϕ_i – phase shift of the i^{th} harmonic,

i – order number of the harmonic,

n – total number of contributing harmonics.

Based on Fourier decomposition, many researchers have tried to quantify DLFs which are the basis for this most common model of perfectly periodic human-induced force. Blanchard et al. [34] proposed a simple walking force model based on resonance due only to the first harmonic with the DLF equal to 0.257 and pedestrian weight $G = 700\ \mathrm{N}$. This was given for footbridges with a vertical fundamental frequency of up to 4 Hz. For fundamental frequencies between 4 and 5 Hz some reduction factors were applied to account for the lower amplitude of the second harmonic because this frequency range could not be excited by the first harmonic of walking. On the other hand, Bachmann & Ammann [14] reported the first five harmonics for vertical walking force and also harmonics for the lateral and longitudinal direction. They reported that the 1st and 3rd harmonics of the lateral and the 1st and 2nd harmonics of the longitudinal force are dominant (Fig. 8). It is interesting that in the latter case some sub-harmonics also appeared. Bachmann & Ammann [14] explained it as a consequence of "more pronounced footfall on one side". The same authors suggested DLF values for the first harmonic of the vertical force between 0.4 (at frequency 2.0 Hz) and 0.5 (at 2.4 Hz), with linear interpolation for other frequencies inside the 2.0–2.4 Hz range. For the second and third harmonic they suggested identical DLFs equal to 0.1 for step frequencies near to 2 Hz.

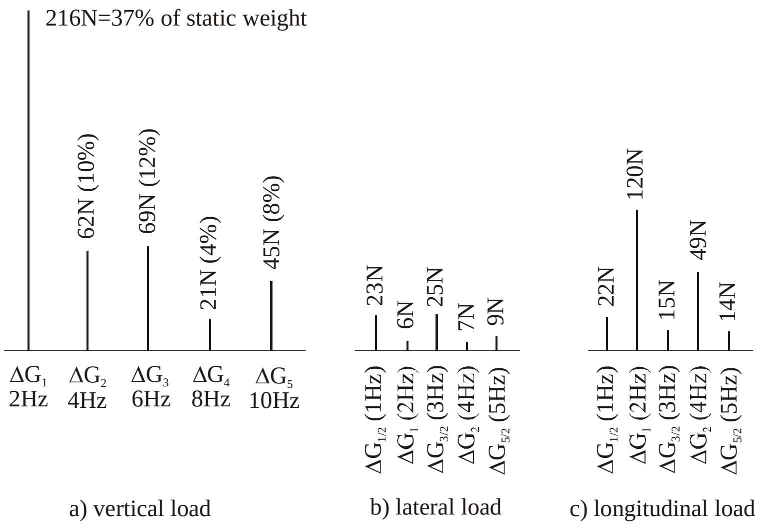

Figure 8: Harmonic components of the walking force in (a) vertical, (b) lateral and (c) longitudinal directions (after Bachmann and Ammann [14]).

In 1982, Kajikawa formulated "correction coefficients" (i.e. DLFs) for walking and running as a function of step frequency [55]. This function, together with person's velocity is given in Fig. 9. A significant boost to the field was provided in the late 1980s by an excellent work of Rainer et al. [42]. They measured continuous single-person force not only from walking but also from running and jumping. It was confirmed that DLFs strongly depended on the frequency of the activity. Values of the first four DLFs were presented (Fig. 10). The only shortcoming of this work was that measurements had been done with only three human test subjects and therefore lacked statistical reliability. Much more extensive work, but only for the walking force, was presented by Kerr [36] in his Ph.D. thesis. His 40 subjects produced about 1000 force records covering walking rates ranging from unnaturally slow 1 Hz to equally unnaturally fast 3 Hz. Kerr reported large scatter in the DLF values. However, the first harmonic had a clear trend to increase with increasing pace frequency and these results were similar to those reported by Rainer et al. [42].

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However, DLFs of the higher harmonics in Kerr's work were very scattered, so they have been characterised statistically by mean values and coefficient of variation (Fig. 11).

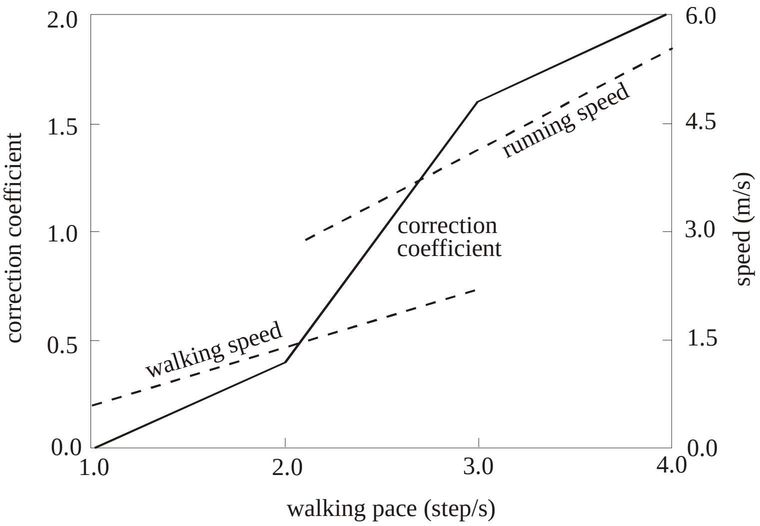

Figure 9: DLFs and pedestrian velocity as a function of step frequency (after Yoneda [55]).

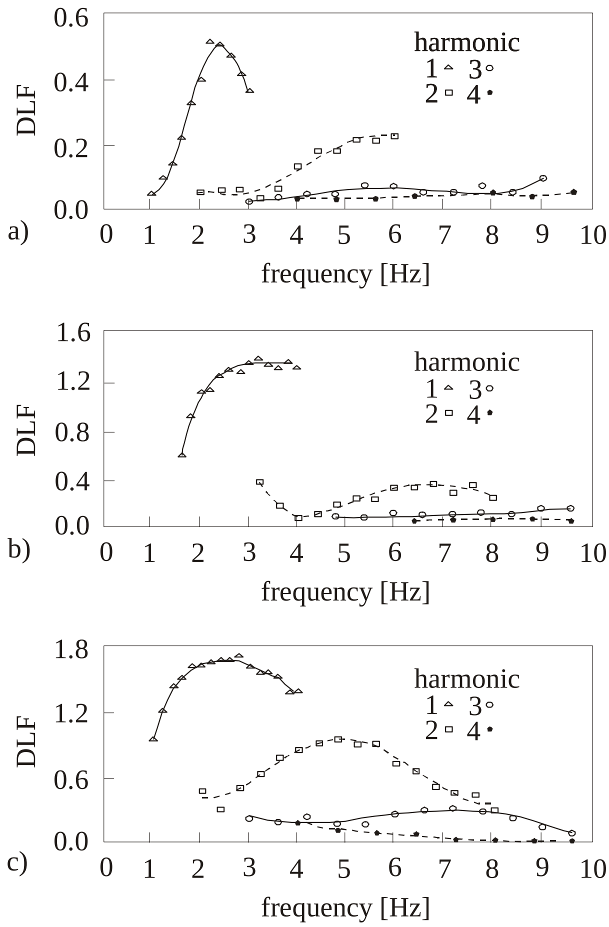

Figure 10: DLFs for the first four harmonics for (a) walking, (b) running and (c) jumping force (after Rainer et al. [42]).

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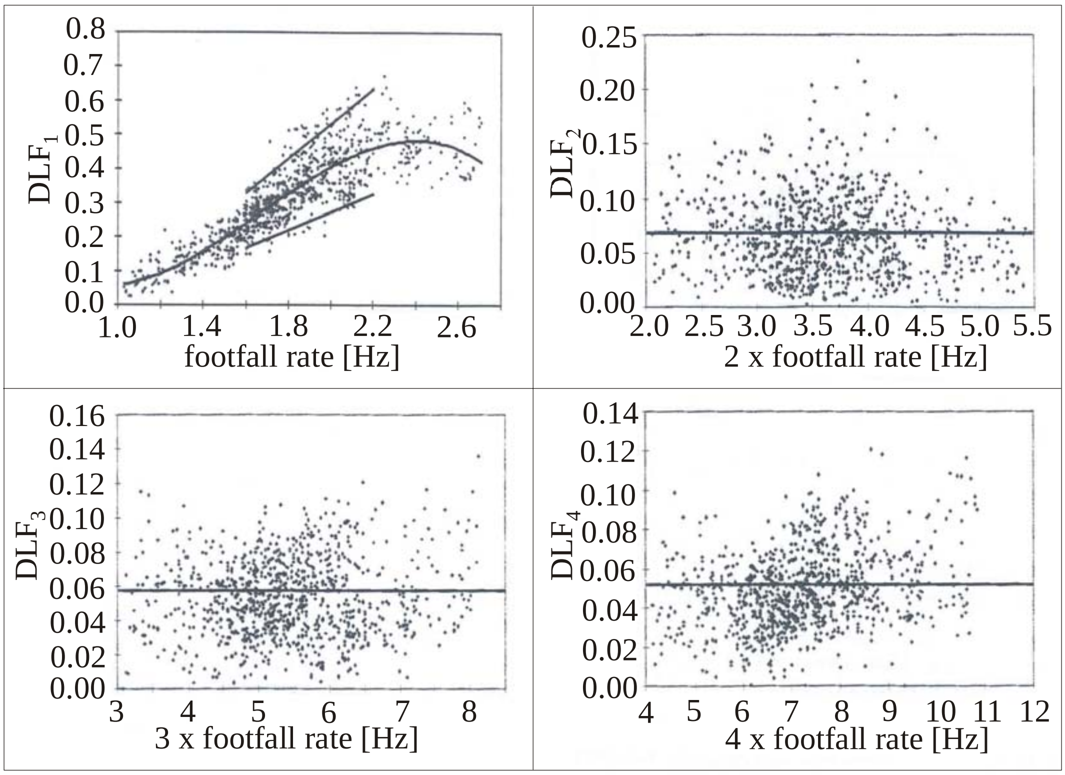

Figure 11: DLFs of walking force for the first four harmonics (after Kerr [36]).

Young [56] presented the work of Kerr and others (Fig. 12) and outlined basic principles which are used by Arup Consulting Engineers when modelling walking forces and the corresponding structural responses. He proposed DLFs for the first four harmonics as a function of the walking frequency assumed to be in the range from 1 to 2.8 Hz. The design values of DLFs presented as

$$\begin{aligned} \alpha_1 &= 0.41(f-0.95) \le 0.56;\ f = 1-2.8Hz \\ \alpha_2 &= 0.069+0.0056f;\ f = 2-5.6Hz \\ \alpha_3 &= 0.033+0.0064f;\ f = 3-8.4Hz \\ \alpha_4 &= 0.013+0.0065f;\ f = 4-11.2Hz, \end{aligned} \tag{2}$$

where f is the frequency of an appropriate harmonic, had 25% chance of being exceeded. This is the first attempt known to the authors of this review to take into account the stochastic nature of human walking in day-to-day design. Statistical mean values of DLFs defined by Young [56] are given in Table 1.

It should be stressed again that in the described investigations, the DLFs were obtained by direct or indirect force measurements on rigid surfaces. However, Pimentel [58] found that, for two full-scale footbridges investigated both analytically and experimentally, DLFs for resonant vertical harmonics (the first and second harmonics) were considerably lower than those reported in literature. It seemed that the human-induced force differed from that measured on a rigid surface probably due to an interaction which exists between humans and low-frequency structures like footbridges. Yao et al. [52] and [53] found this to be the case when jumping on a perceptibly moving structure, but similar direct measurements of the walking force are yet to be made.

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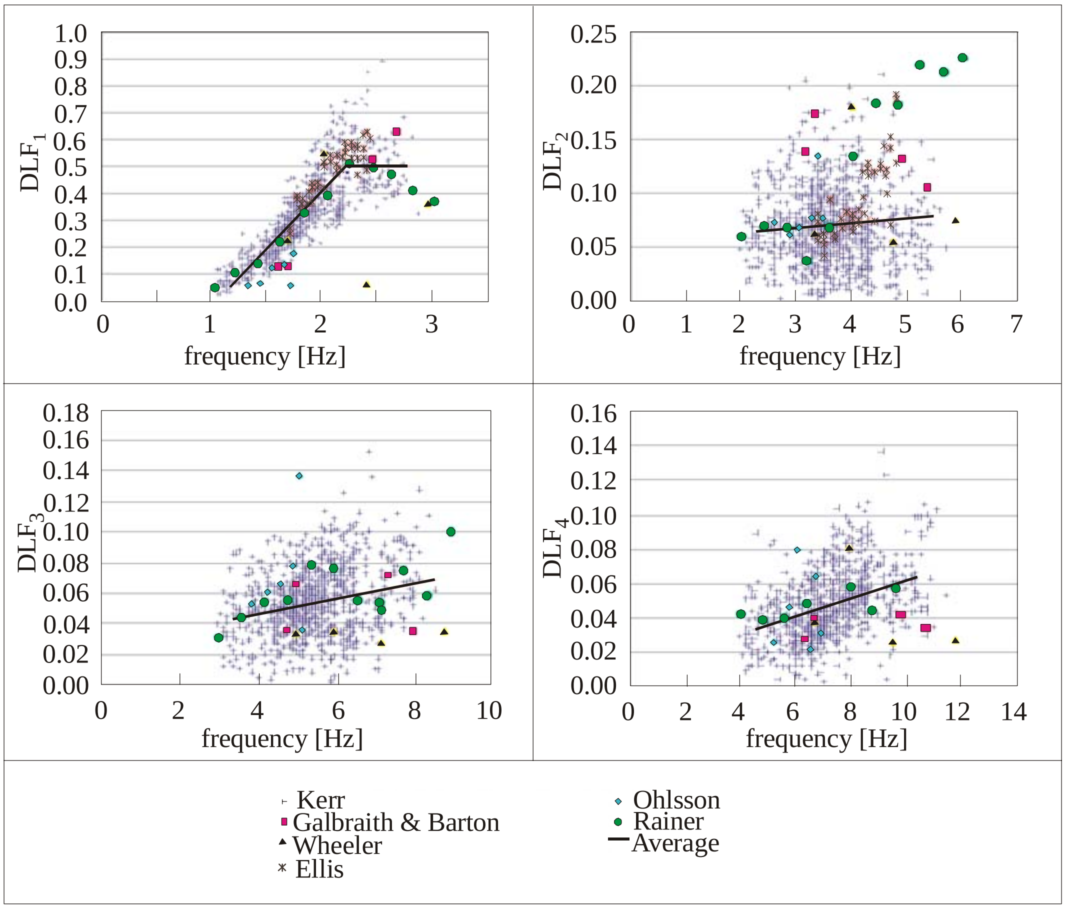

Figure 12: Review of DLFs for the first four harmonics after different authors (after Young [56]).

A jumping force can be modelled in a similar way using the Fourier series. The shape of the time history of this force is qualitatively similar to that one from running (Fig. 3) with the difference that jumping force is not moving across the structure. During one jumping cycle, a period of time, also known as the contact time, is spent in contact with the jumping surface and the rest of the jumping cycle is when the jumper is flying and not touching the surface. Bachmann and Ammann [14] described a half-sine jumping force model and presented dependence of the first four harmonics on the ratio of the contact time to the duration of the jumping cycle, which is known as the contact ratio. Earlier, Wheeler [39] and [40] suggested modelling all walking, running and jumping forces using the “half-sine” model defined by a set of parameters which vary for different activities.

Bachmann et al. [48] divided jumping into two categories: normal and high jump. For the latter case they reported the jumping DLFs for the first three harmonics as high as 1.9, 1.6 and 1.1, respectively at the jumping frequency of 2 Hz. Compared with the walking force, it can be noticed that more harmonics are needed to accurately describe the jumping force. The same authors reported DLF values for vertical bouncing with hand clapping (0.38 and 0.12 for the first two harmonics corresponding to the 2.4 Hz rate of the activity) and for horizontal in-place body swaying (0.5 for the first harmonic). However, Yao et al. [52] measured the first two DLFs of 0.7 and 0.25 during bouncing. This was done in a test when the test subject was asked to bounce freely and in a way so as to produce maximum physically possible response of a flexible and perceptibly moving structure having fundamental frequency of 2 Hz.

The overview of DLFs for single-person force reported by different authors is given in Table 1.

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Table 1. DLFs for single person force models after different authors.

Author(s)	DLFs for considered harmonics	Comment	Type of activity and its direction
Blanchard et al. [34]	$\alpha_1 = 0.257$	DLF is lessen for frequencies from 4 to 5 Hz	Walking – vertical
Bachmann & Ammann [14]	$\alpha_1 = 0.4 - 0.5$	Between 2.0 Hz and 2.4 Hz	Walking – vertical
	$\alpha_2 = \alpha_3 = 0.1$	At approximately 2.0 Hz	
Schulze (after Bacmann & Ammann, [14])	$\alpha_1 = 0.37$ $\alpha_2 = 0.10$ $\alpha_3 = 0.12$ $\alpha_4 = 0.04$ $\alpha_5 = 0.08$	At 2.0 Hz	Walking – vertical
	$\alpha_1 = 0.039$ $\alpha_2 = 0.01$ $\alpha_3 = 0.043$ $\alpha_4 = 0.012$ $\alpha_5 = 0.015$	At 2.0 Hz	Walking – lateral
	$\alpha_{1/2} = 0.037$ $\alpha_1 = 0.204$ $\alpha_{3/2} = 0.026$ $\alpha_2 = 0.083$ $\alpha_{5/2} = 0.024$	At 2.0 Hz	Walking – longitudinal
Rainer et al. [42]	$\alpha_1, \alpha_2, \alpha_3$ and α_4	DLFs are frequency dependent (Figure 10)	Walking, running, jumping – vertical
Bachmann et al. [48]	$\alpha_1 = 0.4/0.5$, $\alpha_2 = \alpha_3 = 0.1/-$	At 2.0/2.4 Hz	Walking – vertical
	$\alpha_1 = \alpha_3 = 0.1$	At 2.0 Hz	Walking – lateral
	$\alpha_{1/2} = 0.1$, $\alpha_1 = 0.2$ $\alpha_2 = 0.1$	At 2.0 Hz	Walking – longitudinal
	$\alpha_1 = 1.6$, $\alpha_2 = 0.7$, $\alpha_3 = 0.2$	At 2.0–3.0 Hz	Running – vertical
Kerr [36]	α_1 , $\alpha_2 = 0.07$ $\alpha_3 \approx 0.06$	α_1 is frequency dependent (Figure 11)	Walking – vertical
Young [56]	$\alpha_1 = 0.37(f - 0.95) \leq 0.5$ $\alpha_2 = 0.054 + 0.0044f$ $\alpha_3 = 0.026 + 0.0050f$ $\alpha_4 = 0.010 + 0.0051f$	These are mean values for DLFs.	Walking – vertical
Bachmann et al. [48]	$\alpha_1 = 1.8/1.7$, $\alpha_2 = 1.3/1.1$ $\alpha_3 = 0.7/0.5$	Normal jump at 2.0/3.0 Hz	Jumping – vertical
	$\alpha_1 = 1.9/1.8$, $\alpha_2 = 1.6/1.3$ $\alpha_3 = 1.1/0.8$	High jump at 2.0/3.0 Hz	Jumping – vertical
	$\alpha_1 = 0.17/0.38$, $\alpha_2 = 0.10/0.12$ $\alpha_3 = 0.04/0.02$	At 1.6/2.4 Hz	Bouncing – vertical
	$\alpha_1 = 0.5$	At 0.6 Hz	Body swaying while standing – lateral
Yao et al. [52]	$\alpha_1 = 0.7$ $\alpha_2 = 0.25$	Free bouncing on a flexible platform with natural frequency of 2.0 Hz	Bouncing – vertical

Some work on jumping forces from groups of people, usually at controlled frequencies, has also been carried out. For example, Rainer et al. [42] reported that individuals jumping in groups of two, four and eight people produced on average lower DLFs than when jumping alone. This holds particularly well for higher harmonics, but not for the fundamental harmonic which DLF exhibits values approximately the same as when a single person is jumping. Pernica [59] added that the average vertical DLFs per person tend to decrease with increasing number of people (in

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all walking, running and jumping activities). This clearly suggests that larger groups have reduced synchronisation between jumping people.

Investigations, specifically related to low-frequency footbridges, on forces due to activities performed by groups of people are very limited. Because of this, it is interesting to mention some work related to floors. For example, Allen [60] indirectly measured the force from 10 to 25 people jumping on a floor and proposed individual averaged DLFs of 1.5, 0.6 and 0.1 for the first three harmonics, respectively. He reported that synchronisation above 2.75 Hz was very difficult. Willford [61] confirmed that several people jumping cannot achieve perfect synchronisation. Using the half-sine model and Ebrahimpour's [50] proposal for statistical distribution of time delays between jumping people, Willford used a Monte Carlo approach and simulated the group effects. He independently confirmed Allen's proposal for reduced, in comparison with a single person, group DLFs. In another investigation related to walking across a floor, a large group of 32 people was involved in experiments related to uncontrolled and controlled walking on a high-frequency floor [62] and [63]. In both cases the response of the high-frequency floor was similar to the one due to a single person walking in such a way that one of the higher harmonics matched the floor resonance. However, it should not be forgotten that walking patterns in floors and footbridges are different, particularly because bridges are usually much narrower and longer structures having only one dominant dimension (length).

2.3.1.2 Probabilistic Force Models

A more detailed probabilistic approach to the walking force model is based on the fact that a person will never produce exactly the same force-time history during repeated experiments. In the case of two persons it is even more so [64]. For a single person force, which is still assumed to be periodic, randomness can be taken into account by probability distributions of person's weight, pacing rate and so on. For several people, the probability distribution of time delay between people who perform a particular activity can be added. The main idea of this philosophy is to get a reliable estimate of the force from a group of people by combining forces from individuals. Naturally, for a reliable statistical description of human forces, a large database of measurements with a single person should be provided. Some work on this was done by Tuan and Saul [65] who measured forces from many different activities mainly typical for grandstands, among which was jumping.

In his Ph.D. thesis, Ebrahimpour [50] continued the work of Tuan and Saul and conducted measurements of different types of forces using a specially constructed force platform. Among many types of forces typical for activities on grandstand structures, a single jump and periodic jumping with controlled frequencies at 2, 3 and 4 Hz were investigated. For a statistical description of continuous jumping force-time histories from individuals, Ebrahimpour chose the first three harmonics of the Fourier series and the force repeating period. Then, by comparing the measured force from two people simultaneously with computer simulations obtained by a combination of forces from individuals, he identified the time delay distribution between two people who were trying to perform synchronised jumping. The idea was to use this time delay distribution together with statistically described individual time histories to enable the calculation of the resulting force from any number of people. The procedure was experimentally verified for only four people. Further Monte Carlo computer simulations revealed that the force peak amplitude per person decreased with increasing the number of people, which was in line with the already mentioned findings of Pernica [59] related to DLFs. However, this model was hardly applicable in practice because of the fact that peak force amplitude is not enough to describe that force. A very good digested version of whole procedure is given by Ebrahimpour and Sack [66].

In a subsequent experimental work Ebrahimpour et al. [67] found that the previously used computer program gave good estimates of the peak jumping forces for groups of up to 40 people. Three years later, Ebrahimpour and Sack [68] tried to improve Ebrahimpour's previous design suggestion by proposing design curves for the first three harmonics of jumping load as a function of the group size, which was a much more practical proposal.

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An identical procedure was applied on walking loads by Ebrahimpour et al. [44]. A vertical dynamic load by a group of pedestrians was investigated. As a result, a design proposal for only the fundamental DLF was given as a function of a number of people (Fig. 13). The reason was probably the fact that the spectrum of measured uncorrelated force for four people revealed that only the first harmonic is important. It is even more so in case of a larger number of people. However, this design proposal, although it includes up to 100 people, does not take into account the fact that people in such large crowds sometimes adjust their step according to the movement of others. The authors stressed that this effect, which is dependent on the crowd density, should be added but did not explain how. The DLFs given in Fig. 13 are lower in comparison with the results of Pernica [59].

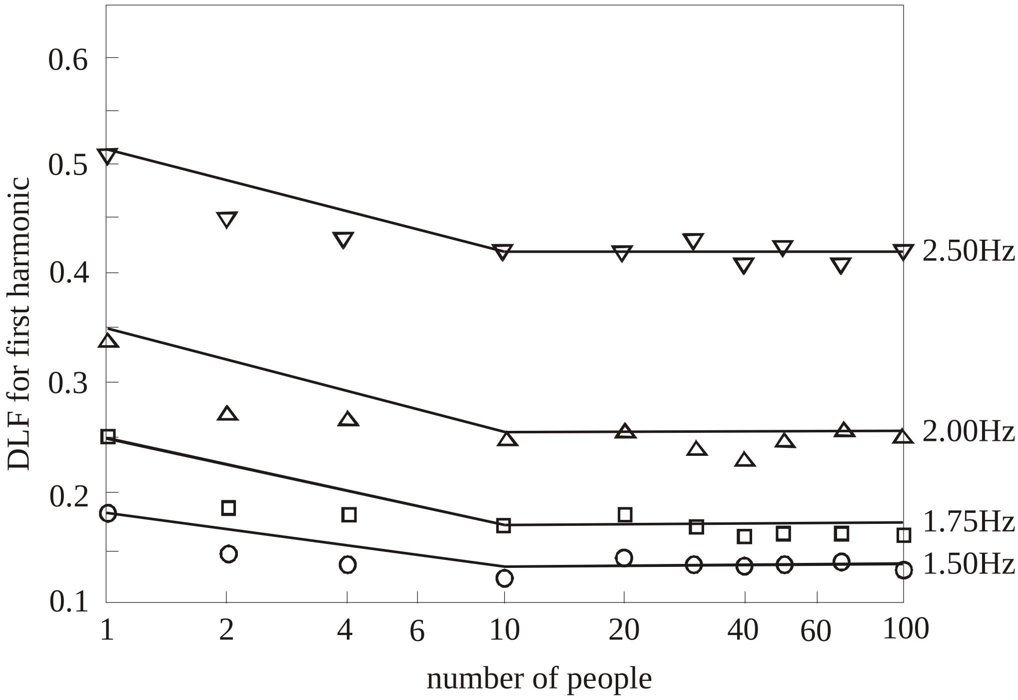

Figure 13: DLF for the first harmonic of the walking force as a function of number of people and walking frequency (after Ebrahimpour et al. [44]).

2.3.2 Frequency Domain Force Models

In his Ph.D. thesis, Ohlsson [35] introduced at that time a rather new concept for mathematical modelling of the human-induced force. Namely, as previously mentioned in Section 2.2, he measured a single-step force and then produced continuous walking force assuming artificial force periodicity. Then, he determined the auto-spectral density (ASD) of the force treating it as a transient signal where identical steps were repeated perfectly but for a limited number of times. Ohlsson studied only the high-frequency content of the ASD between 6 and 50 Hz because he was investigating behaviour of high-frequency building floors made of timber. This approach was further developed over the next 10–15 years and extended to low-frequency floors. As a result, in his Ph.D. thesis Eriksson [49], who was Ohlsson's student, focused on low-frequency floors and made use of the ASD frequency range below 6 Hz. Fig. 7 presents this part of the spectrum of a measured continuous force lasting about 100 s. Eriksson explained that the fact that each peak has some width in this spectrum means that human walking cannot be perfectly periodic and therefore cannot be accurately described by DLFs (i.e. by a time-domain model). This whole procedure is based on the assumption that human-induced force can be treated as a stationary random process. This model, and several others based on the frequency-domain approach, will be explained in more detail in Section 6, because they tend to be "packaged" with the structural modelling and/or assessment.

2.3.3 Vandal Loading

This type of load was not researched very much in the past. Although recognised as an issue in the literature [12], [14], [34], [39], [40], [69], [70], [71] and [72], vandal loading is not precisely defined in terms of which type of human activity, besides jumping, could be considered as it. Probably, deliberate horizontal body swaying can be added [14] as well as deliberate bouncing. The earliest record of vandal loading and its consequences dates from 1821 when Stevenson reported very strong vibrations on a bridge for "foot passengers and led horses" when three or four persons were "amused" by noticeable vibration on the bridge and tried to increase it deliberately. The result was that one of the supporting bridge chains broke. Although we do not know exactly how this action was performed, it demonstrated very early the potential and consequences of deliberate synchronised human action. Also, Tilden [28] reported that "jouncing" (i.e. bouncing) has a "high kinetic intensity", whatever that may mean. He did not explain more precisely this term, but it could be deduced from his paper that it was quite possible that this type of load was capable of producing a high level of response.

Vandal loading has been much more debated in terms of whether it is relevant for a particular type of a structure than in terms of how it could be modelled. For example Blanchard et al. [34] only confirmed that the data about this load type are very scarce, while Wheeler [40] rather boldly concluded that synchronisation of people had not been a real possibility. This was based on a measurement of the response to jumping in unison of two and three people which proved to be similar to the response in a single person case. As previously mentioned, Rainer et al. [42] reported a similar case when investigating correlated jumping of two, four and eight persons. However, Bachmann [21] stated that synchronisation of a small number of people seems possible at least when considering the first loading harmonic. In that case, he proposed to simply multiply single person influence by the number of persons involved, meaning perfect synchronisation. Grundmann et al. [72] suggested to link the dynamic amplitude due to rhythmical knee-banding with the displacement of the centre of gravity for a single person. However, the dependence between this amplitude and the frequency of this excitation was not stated. Also, the synchronisation factor for the case of several people was not suggested. Finally, Pimentel and Fernandes [73] claimed that there were no known cases of footbridge damage from vandals, probably not being aware of the case mentioned by Stevenson in 1821.

In summary, it may be said that vandal load does not occur often in practice but it certainly deserves greater attention. This is especially so nowadays when footbridges are very light structures which can be excited relatively easily. Also, it should be remembered [20] that this type of load should probably be related to and treated as a social problem. The BS 5400 bridge design code [74] only requires robust construction of bearings and some reserve in reinforcement for prestressed bridges as measures against vandal loading. BS 5400 does not contain any more explicit design procedure related to vandal loading and/or applicable for bridges of different materials.

3 Footbridge Structures as Vibration Path

The vibration path which transmits vibrations from the source to the receiver is the footbridge structure itself. Knowing mass, damping and stiffness properties of a footbridge, together with the previously defined force model, is necessary to calculate its dynamic response according to the well known equation of motion of a multiple-degree-of-freedom (MDOF) system [75]:

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{f}(t) \tag{3}$$

where $\mathbf{M}, \mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices respectively, each of order $n \times n$ where n is the number of degrees of freedom. In addition, $\ddot{\mathbf{x}}(t), \dot{\mathbf{x}}(t)$, $\mathbf{x}(t)$ and $\mathbf{f}(t)$ are $n \times 1$ vectors of acceleration, velocity, displacement and external force. Mass and stiffness

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matrices depend on the geometry of the footbridge and material properties. They are usually determined by the finite element (FE) concept. This implies a discretisation of the real structure having an infinite number of DOFs into an ensemble of finite elements which are interconnected at a limited number of points (nodes) and which possess a finite number of DOFs. These elements and their corresponding DOFs form the basis for further calculations. Namely, for each element type, mass and stiffness element matrices are defined and by their combination the mass and stiffness matrices for whole structure can be determined. In practice, however, the damping matrix cannot be evaluated in the same way. In footbridge vibrations, it is usually expressed via modal damping ratios ζ_n which are experimentally determined.

Assuming that the system is linear and proportionally damped, which is a fair assumption for most footbridges, the given system of n coupled equations with n unknown variables can be uncoupled into the n equations each featuring only one variable, that is, to n single-degree-of-freedom (SDOF) systems which standard form is [75]:

$$\ddot{Y}_n(t) + 2\zeta_n\omega_n\dot{Y}_n(t) + \omega_n^2 Y_n(t) = \frac{P_n(t)}{M_n}. \tag{4}$$

Here, $\ddot{Y}_n(t), \dot{Y}_n(t)$ and $Y_n(t)$ are modal (or generalised) acceleration, velocity and displacement, ζ_n and ω_n are the damping ratio and natural circular (or angular) frequency for the n^{th} mode of vibration, while $P_n(t)$ and M_n are the modal force and mass for the same mode. Then, the total displacement vector $\mathbf{x}(t)$ can be presented as a linear combination of mode shape vectors ϕ_n, where coefficients of that combination vary with time and are generalised displacements $Y_i(t)$, $i = 1,...,n$:

$$\mathbf{x}(t) = \phi_1 Y_1(t) + \phi_2 Y_2(t) + ... + \phi_n Y_n(t). \tag{5}$$

Generally, the most popular method for establishing and solving Eq. (3) is the FE method [75]. However, when one mode dominates, which often happens in footbridges, the response can be estimated sufficiently accurately using an SDOF modal equation (Eq. (4)) for the appropriate mode. This is very often implemented in practice when checking footbridge vibration serviceability (see design procedures outlined in Section 6).

Therefore, for a reliable estimate of the structural response, it is necessary to determine dynamic properties of the footbridge which feature in Eqs. (3) and (4) as accurately as possible. In the next two sections issues related to the accurate determination of the mass, stiffness and damping in footbridges will be considered. The most convenient way to present these properties is in their modal form.

3.1 Mass and Stiffness

Knowing the characteristics of structural materials and geometry, an FE model of a bridge can be developed. After an eigenvalue extraction, performed using the established mass and stiffness properties, footbridge natural frequencies and mode shapes, can be determined. However, sometimes the obtained results can contain large errors due to uncertainties in the FE modelling process. For example, Deger et al. [76] reported an error in the first natural frequency of 37% when compared with the test of a full-scale structure mainly due to inadequate modelling of footbridge boundary conditions. In circumstances when both analytical and experimental results exist, the FE model can be updated by their comparison assuming that the experimental results are correct. This approach helps future modelling of bridges with similar layouts. However, it should be noted that an FE model updating cannot be successful without good quality of experimental data [77] and [78]. The most uncertain and sensitive parameters considered in the updating of footbridges are boundary conditions, material properties and modelling of certain aspects of the key structural and nonstructural elements (such as decks,

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cables, handrails) which have the potential to influence significantly the footbridge dynamic behaviour. However, it should also be noticed that changes in the temperature can cause changes in dynamic properties. For example, increasing in temperature from 21.4 to 42.1°C was accompanied by a decrease of the fundamental frequency of a pedestrian walkway of 7.1% [79].

General procedures for FE model updating are given in the now classical textbook by Friswell and Mottershead [80]. However, this technology, widely used in the mechanical and aerospace engineering disciplines, requires special considerations when applied to civil structural engineering problems [78] and [81]. Generally, FE model updating can be done manually, by trial and error, or automatically by using software developed for that purpose, where the latter is much faster than the former [78] and [82]. However, it is recommended to conduct manual updating first to develop an FE model which features meaningful starting parameters for the automatic procedure where the choice of these parameters is very important [78] and [81]. Usually, the most uncertain parameters are the stiffness of some nonstructural elements, dynamic modulus of elasticity for concrete, stiffness of cracked concrete and stiffness of the supports. They should be parametrically investigated until a good agreement with experimental data, usually with frequencies only or frequencies and mode shapes, is obtained. The level of matching is typically checked by calculating the Modal Assurance Criterion (MAC) and/or the Coordinate Modal Assurance Criterion (COMAC) which represent degree of correlation between the analytical and experimental modes of vibration [80]. Ideally, due to orthogonality of mode shapes, MAC should be equal to 1 when the same modes are compared, and 0 in other cases [83]. However, MAC values, for the same modes, as low as 0.7 are acceptable in civil engineering applications due to imperfect measurements typically made in noisy environments. Similar considerations apply to COMAC values which should also be between 1 and 0. Prior to updating, it is very important to conduct a sensitivity analysis to determine which parameters have the biggest influence on the target values of natural frequencies, MAC and COMAC [81]. Some practical observations as to the modelling of footbridges are given in the remainder of this subsection.

For the cases when there are significant axial forces in structural elements, second-order effects should be taken into account. However, this does not apply to internally prestressed concrete elements where second-order effects do not develop. When required, the geometric stiffness should be considered together with the elastic stiffness [84], having in mind that compression force reduces and tension force increases the stiffness. This geometric nonlinearity effect is present in cable-stayed bridges. It can change the overall stiffness of the structure and, consequently, influence mode shapes and frequencies. Large axial forces are typical not only for cables but also for girders and towers in these structures [85]. In an investigation of a cable-stayed footbridge, the geometric stiffness was taken into account only for cables, while it was neglected for other bridge elements because their axial compression forces were small compared with the buckling forces [85] and [86]. The cable behaviour was very difficult to model having in mind big differences (up to 40%) in the measured and calculated cable forces and the sensitivity of results to the cable modulus of elasticity. Similarly, in the FE modelling of a suspension footbridge, Brownjohn et al. [87] modelled the whole bridge using 3D beam elements adding the geometric nonlinearity effect for tower pylons, cables and hangers. However, Pimentel [58] found that, for a fibre-reinforced cable-stayed footbridge investigated, taking into account the geometric stiffness induced only small differences in the obtained natural frequencies in comparison with the model where nonlinear effects were neglected. Khalifa et al. [88] modelled cables as truss elements and took into account geometric nonlinearities but their influence on the dynamic performance of a fibre-reinforced plastic cable-stayed footbridge was not given. For a cable-stayed footbridge, modelling the footbridge timber deck as a plate element gave much better agreement between experimentally and analytically obtained modal properties [85] and [86] than treating the deck only as a mass as in some previous case studies for the same bridge [89] and [90]. Moreover, Brownjohn et al. [87] found that prestressed, precast, concrete panels of which the deck of a suspension footbridge was made, had potential to influence strongly the horizontal lateral frequencies and should be modelled in an FE model as plate elements. However, in suspended bridges lateral and torsional deck stiffnesses have little influence on vertical modes, which means that the structure can be modelled as a 2D model with the deck presented as a beam element when modelling vertical oscillations [91].

Brownjohn et al. [87] reported differences of up to 10% in the footbridge natural frequencies obtained analytically and experimentally. The authors quoted uncertainties in the dynamic Young's modulus for concrete and the exclusion of the stiffness of railings and asphalt surfacing from the FE model as likely reasons for this discrepancy. To illustrate the variability of the dynamic modulus for concrete it should be mentioned that the values of 30.8 and 42.5 GPa were obtained by Pimentel [58] in the updating process of two tested footbridges. The same author reported nearly 300 times greater horizontal stiffness of the elastomeric bearings for a composite bridge than the manufacturer's design static value. He also found that handrails in a stressed ribbon footbridge increased the fundamental frequency by about 20%. Obata et al. [92] found that 50% of handrail stiffness was effective in investigated footbridges.

In conclusion, in all examples mentioned, the footbridge FE models were either developed using only beam elements or the deck was additionally modelled using plate elements. The FE model can be useful in detecting closely spaced modes of vibration or modes with combined lateral and torsional motion. The latter is typical when the mass and the shear centre of the footbridge section do not coincide [85] and [86]. Finally, there is sufficient evidence that footbridge handrails can increase, sometimes significantly, frequencies of vertical modes of vibration.

3.2 Damping

Damping represents energy dissipation in a vibrating structure [93]. Each structure inherently possesses some capability to dissipate energy. That capability is very beneficial because it reduces structural response to a dynamic excitation near resonance. The near-resonant condition is the governing condition when considering footbridge vibration serviceability due to human-induced load. Therefore, it is very important to model damping as accurately as possible.

In general, there are several dissipation mechanisms within a structure, the individual contributions of which are extremely difficult to assess. They can be divided into two groups: “dissipation” mechanisms which dissipate energy within the boundaries of the structure and “dispersion” or “radiation” mechanisms which propagate energy away from the structure. The overall damping in the structure which comprises both mechanisms is often called “effective damping” and it is this damping which is actually measured as modal damping in practice [94].

However, it is very hard to model mathematically these damping mechanisms. There are several damping models [95; 96] but the most often used is the viscous one. Although this model does not describe the real behaviour of the structure, it is very convenient because of its simplicity. The usual way to express viscous damping is in its modal form i.e. by using the damping ratios ζ_n defined for each mode separately. In the case of footbridges, this is very convenient both for the FE modelling and the experimental measurements.

As previously mentioned, damping is very important if the structure vibrates at or near a resonant frequency, when the stiffness and inertial forces tend to cancel each other [27]. However, it is hard to predict it. To get better idea about damping, it is necessary to conduct testing. In testing it is very important to make the right choice of excitation which will generate resonant excitation for a mode investigated [93]. Therefore, frequency content for the excitation force should be chosen carefully.

Modern construction technologies have brought a reduction of damping in structures because of a significant decrease in the amount of friction which was present in old structures. For example, Wyatt [95] stated that until 1960 there had been a widespread belief that logarithmic decrement in bridges could not be below 0.05 (i.e. viscous damping ratio 0.8%). In the mid-1940s the minimum value had even been 0.1 (1.6%), whereas nowadays, modern steel bridges regularly exhibit damping of 0.5% or less.

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In the following sections, the damping measurements using some rather old and nowadays obsolete procedures are presented. Then, some relatively new procedures gaining popularity in footbridge testing are outlined. Although in all tests natural frequencies and mode shapes were determined too, emphasis is given on damping measurements because of their relative uncertainty. It suffices to mention here that natural frequencies are usually determined from spectral plots of response frequency versus amplitude while mode shapes, due to their spatial nature, are determined by response measurements made at different locations on the structure.

3.2.1 Research Work in the 1970s

In one of the earliest attempts to measure damping, people who were jumping in union with a frequency near the structural resonant frequency were used to excite the bridge on which they were jumping [97]. In another attempt in 1966, an impulsive force in the centre of the span of a road bridge was applied by means of cables attached to the bridge and pulled from a boat [98]. Although this attempt was not successful, which is to be expected on a bridge with a central span as long as 1013 m, it is interesting to mention it considering the development of vibration measurement techniques. A short review of dynamic testing on full-scale civil engineering structures in general is given by Hudson [99] and Severn et al. [100]. Rainer [101] wrote an excellent paper full of practical advice related to vibration measurements on civil engineering structures. It deals with the planning of tests, instrumentation, way of collecting data and data analysis and interpretation of results. In absence of more data related to footbridges, some findings obtained for road bridges are presented in this subsection too.

In the 1970s, many investigations of bridge damping were conducted by the UK Transport and Road Research Laboratory. In those experiments, resonance tests were usually conducted. This was done using a single electro-hydraulic exciter, or a pedestrian whose pacing was adjusted to match resonance by means of a metronome [102]. However, tests with the electro-hydraulic exciter were regarded as more reliable to estimate experimentally not only footbridge damping, but also its natural frequencies and mode shapes because of the ability to control the excitation (and consequently response) frequencies. These were in essence stepped-sine tests in which, when a steady-state resonant response was established, the harmonic excitation was cut and damping was obtained from the free decay trace. Usually, acceleration response was measured because it was established as the best parameter for describing people's reaction to vibrations and, also, it was easy to measure it using widely available accelerometers. Test procedures used at that time not only for footbridges, but also for highway bridges are described in detail by Leonard [102].

Leonard and Eyre [103] investigated eight bridges with steel box girder and concrete deck, among them one footbridge. However, measured logarithmic decrement in the first bending mode showed considerable variability—from 0.023 (0.37%) for the footbridge up to 0.18 (2.86%) for road bridges. Therefore, it was obvious that a single value cannot be proposed for future design for this type of structure (steel box girders with concrete deck). The authors rightly concluded that supports and end conditions have great influence on the (radiation part of) damping. Also, they found that with increasing vibration amplitude damping also increases, which suggests that damping mechanism was amplitude dependent. However, Eyre [104] could not confirm this finding during testing of a road bridge made completely of steel. It was a bridge with very low first natural frequency of 0.53 Hz where maximum achieved amplitude was 13 mm which was possibly too low to activate extra damping mechanisms.

Eyre and Tilly [105] did measurements on 23 steel and composite bridges, many of which were footbridges. All structures were steel box girder or steel plate girder bridges, with different number of spans (one to six) and span length (17–57 m, plus one road bridge with the main span of 213 m). The authors reported that damping was dependent of the number of spans (single-span bridges had higher damping then multi-span ones) and vibration mode considered (higher modes generally had higher damping). Typically, logarithmic decrement for footbridges was between 0.02 and 0.03 (0.32% and 0.48%). The authors also confirmed Leonard and Eyre's [103] finding that damping is dependent on response amplitude. Furthermore, using all available data, Tilly et al. [12] concluded that it was wrong to generalise that damping increases

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in higher modes or that it is dependent on the stiffness and span length. However, it appears to be certainly dependant on type of material—steel bridges exhibit the lowest and classically reinforced concrete bridges the highest level of damping. They also suggested that, because of amplitude dependence, it is always necessary to quote measured damping together with the level of response amplitude. This sound suggestion is, unfortunately, very often omitted in the published literature.

All these results are mainly related to damping in the vertical modes, although some measurements were done in torsional modes too.

3.2.2 New Measurement Techniques

Towards the end of the 1970s, new and more reliable techniques were introduced to experimentally determine the dynamic properties of bridges. These techniques made use of improved signal analysis techniques and were based on impulsive (hammer), forced and ambient vibration excitation. There are numerous examples of these methods, such as those presented by Rainer and Van Selst [106] and Buckland et al. [107] on ambient testing and vehicle impact, Abdel-Ghaffar [108] and Brownjohn et al. [109] on ambient testing, and Rainer and Pernica [110] on ambient testing and harmonic forced vibrations.

In general, damping can be obtained using different methods. Related to civil engineering structures Rainer [101] mentioned the time-domain free decay method after the excitation (impulsive or harmonic) stops, the frequency-domain half-power bandwidth method (for the ambient and forced tests) and the time-domain-based random decrement method (for the ambient vibration surveys). All of them can produce slightly different results as a consequence of different theoretical assumptions which usually cannot be completely satisfied in practice. Also, the frequency response function (FRF) curve (mainly circle) fitting method in the frequency domain is used very often (see Table 2) and the principle of that method can be found, together with many other methods, in standard textbooks dealing with modal identification procedures [83] and [117].

Interpretation of measurement results should be conducted very carefully. For example, the SDOF half-power bandwidth method using ambient testing response data tends to produce higher damping because of the averaging during data processing and the impossibility to have ideally stationary input necessary for ambient testing [109]. Also it can neither produce good damping estimate for closely spaced modes nor give insight into the amplitude dependence phenomenon. Problems with closely spaced modes can happen in the time-domain-based free vibration decay method too [106].

The choice of the most appropriate method for each bridge is very important. In almost all articles relevant to footbridges some examples of merits and demerits of one or more methods are given. It is known that for large structures, which are difficult to be excited artificially because of low frequencies, ambient testing is the most appropriate choice. Also, it is cheaper than forced testing and does not disrupt the normal service of the bridge. This testing is often used as preliminary investigation for other methods to give a quick and rough indication of bridge natural frequencies. However, damping values from these measurements can be unreliable as previously noted, especially when closely spaced modes exist.

For short bridges featuring higher natural frequencies and modest testing budgets, hammer testing leading to FRFs may be more appropriate. It gives relatively reliable values of damping and it is very easy to conduct such a test if a bridge can be closed. However, inevitable ambient extraneous excitation can easily make analysis more complicated.

Finally, excitation by a controlled force produced by an electrodynamic or hydraulic shaker is believed to be most reliable method suitable for bridges of medium size. In principle, it requires shorter time for data acquisition than ambient tests. Also, electrodynamic and hydraulic shakers can produce different types of excitation which give flexibility in the measurement procedure. However, this method tends to be the most expensive. Also, it is hard to excite low frequencies, especially below 1.0 Hz.

Table 2. Measured damping ratios (for vertical ζ_v , horizontal ζ_h and torsional ζ_t modes) for some footbridges.

Author(s)	Bridge Type	Main Span [m]	Girders	Deck	ζ_v [%]	ζ_h [%]	ζ_t [%]	Estimation Method
Gardner-Morse & Huston [86]	cable-stayed	54.9	steel	laminated wood	0.53/0.22	–	0.46/0.36	curve fitting
Brownjohn et al. [87]	suspended	50	steel	concrete panels	2.68/0.50	1.00/0.70	0.84*/0.50	curve fitting
Brownjohn [91]	suspended	35	steel	timber	1.0/1.0	high**	2.4**/1.4**	free-decay after jumping
Cantieni & Pietrzko [111]	continuous space truss	54	wood	–	1.4/1.3	2.9/2.1***	1.4	curve fitting
Pimentel [58]	pre-cambered beam	19.9	steel	concrete	0.73 & (0.40) [0.53] /0.65	–	–	curve fitting & free-decay after (walking) [jumping]
Pimentel [58]	stressed ribbon	34	–	prestressed concrete	0.56 (0.65) /0.64 (1.02)	–	–	free-decay after walking (jumping)
Pimentel [58]	cable-stayed	63	–	glass reinforced plastic	0.84/0.94	–	–	free-decay after bouncing
Pavic & Reynolds [112]	stressed ribbon	34	–	prestressed concrete	0.53/0.65	–	0.50/0.60	curve fitting
Pavic et al. [113]	suspended	144	steel	aluminium	–	0.76/1.30	–	curve fitting
Hamm [114]	framework	68	wood and steel	–	1.2 (0.8-1.35)	–	–	half power bandwidth (free decay)
Caetano & Cunha [115]	stressed ribbon	30	–	concrete	1.7/3.6†	–	–	free-decay after skipping
Fletcher & Parker [116]	multi-cable-stayed	53	–	reinforced concrete	0.40 (0.51) / 0.21 (0.41)	0.44**	–	free decay (curve fitting)

* - half power bandwidth ** - estimated methods are not stated *** - coupled with vertical movement

† - not clear the way of identification having in mind that two closely spaced modes appeared

Note: in case of more than one damping value measured, the average value is given.

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Table 2 contains key published results related to damping measurements on footbridges. Damping ratios ζ_v, ζ_h and ζ_t for the first two vertical, horizontal and torsional modes are given whenever data were available.

Gardner-Morse and Huston [86] investigated a small cable-stayed pedestrian bridge using an impact hammer. They successfully extracted the first 14 modes. Although the deck was wooden, measured damping ratios were very low—up to only 0.75%, except for the fourth vertical mode. Brownjohn et al. [87] investigated a suspension footbridge, also using hammer testing. Besides results presented in Table 2, Brownjohn [118] obtained very different damping estimates using the half-power bandwidth method. However, it should be said that these investigations were conducted just as a preparation exercise for measurements on a long suspended road bridge. Again by using an instrumented hammer (7.25 kg), Brownjohn [91] successfully identified two closely spaced modes at frequency near 2 Hz on a suspended bridge. Cantieni and Pietrzko [111] identified the first 12 modes on a wooden bridge using a vibration generator driven by a burst random signal (0.5–25 Hz). In his Ph.D. thesis, Pimentel [58] tested two footbridges (pre-cambered beam and stressed ribbon) by means of the hammer testing. However, damping values were determined mainly using jumping and walking free-decay tests where the value obtained by jumping was higher due to presence of the test subject on the bridge. Pavic and Reynolds [112] used a electrodynamic shaker with chirp excitation (1–30 Hz) to investigate the same catenary (i.e. stressed ribbon) footbridge and found almost the same damping values as in Pimentel's [58] walking tests. Pimentel [58] also investigated a cable-stayed bridge by ambient vibrations.

Pavic et al. [119] successfully tested the London Millennium Bridge using two different shakers to excite horizontal lateral and vertical modes, respectively. The lowest lateral frequency was 0.5 Hz, which required the construction of a special hydraulic shaker which would be able to excite such a low frequency mode—an inertial shaker with moving mass of 1000 kg (Fig. 14). It is interesting to compare it with the first attempts of using mechanical exciters in dynamic investigations of bridges such as that reported by Chasteau [120] where an eccentric mass of 3.75 kg was used, or that reported by Eyre and Tilly [105] where a hydraulic actuator could not excite a vertical mode with frequency at 0.53 Hz. The damping of the Millenium Bridge was measured for different configurations while some viscous dampers and/or a tuned mass damper were in operation. These results were published by Pavic et al. [113], while in Table 2 results for the first two lateral modes of the central span without any additional damping devices are given.

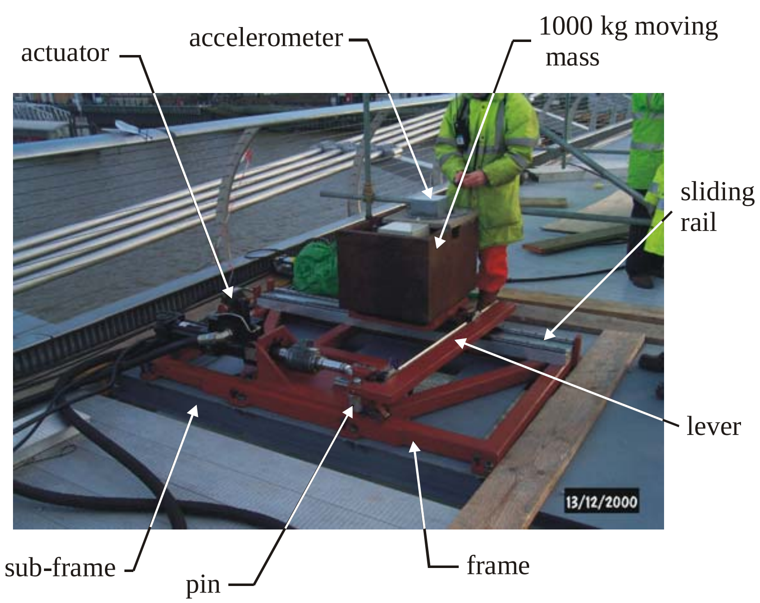

Figure 14: Horizontal shaker used for testing of the Millennium Bridge (after Pavic et al. [119]).

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A timber footbridge was investigated by Hamm [114] whose result confirmed Eurocode 5 [121] proposal for damping of this type of pedestrian bridges of 1.0% and 1.5% depending on the construction type. Unfortunately, only the first vertical mode was investigated.

Based on everything stated so far, it is obvious that it is not possible to define unique value(s) for footbridge damping. To overcome this, Bachmann et al. [57] suggested in 1995 using Table 3 as a guidance based on data collected on 43 footbridges in the 1980s [122]. Based on the new data since published, these recommendations still look very reasonable.

Table 3. Provisional values of damping ratio in footbridges (after Bachmann et al., 1995a).

	Damping Ratio ζ [%]		
Construction Type	Min.	Mean	Max.
Reinforced Concrete	0.8	1.3	2.0
Prestressed Concrete	0.5	1.0	1.7
Composite	0.3	0.6	–
Steel	0.2	0.4	–

4 Receiver of Footbridge Vibrations

The main receivers of vibrations on pedestrian bridges, who govern their vibration serviceability, are walking people. Although Walley [123] reported that “a pedestrian at rest on the bridge might ‘feel’ the passage of other pedestrians and be disturbed”, Leonard [20] claimed that it was economically unjustifiable to design footbridges where standing people would feel no vibrations.

The reaction of human beings to vibrations is a very complex issue having in mind that humans are “the greatest variables with which anyone may deal” [124]. According to Lippert [125], not only different people react differently to the same vibration conditions, but also an individual exposed to the same vibrations on different days will likely react differently. This is known as the inter- and intra-subject variability of humans and their reactions to vibrations [126]. Knowing that human sensitivity to vibrations is very high [23], it is clear that this issue is of paramount importance for footbridge vibration serviceability.

4.1 Early Works

Probably one of the first laboratory works and certainly the most often referenced in the future studies was conducted by Reiher and Meister in 1931 [127]. They investigated the effect of harmonic vibrations on ten people having different postures (laying, sitting, standing) on a test platform driven by different amplitudes, frequencies and direction of vibrations. As a result they classified the human perception into six categories and as a function of vibration amplitude and frequency (Fig. 15).

In the 1940s, some very valuable systematisations of the work until that time were published by Postlethwaite [128] and Goldman [129]. Postlethwaite [128] tried to construct perception curves by combining experimental results of different authors and by the use of “some imagination where experimental results were lacking”. The acceleration perception threshold in the low-frequency region of up to 1 Hz was 0.01 ft/s2 (0.03%g). Mallock, who investigated unpleasant vibrations at 10–15 Hz in some London houses due to traffic, found that the vibration displacement amplitude was very low (0.001 in, i.e. 0.025 mm) but the corresponding acceleration level (up to 2.3%g) caused the problem. As a result, he proposed 1%g and 5%g as

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noticeable and nuisance values, respectively [129]. This example shows the importance of the vibration descriptor in which vibration amplitude is expressed (displacements, velocities or accelerations). Goldman [129] used all known work regardless of the vibration direction, subject's posture and type of vibration to define three categories of human reaction to vibrations: perception, discomfort and maximum tolerable levels. According to this study, the minimum discomfort level was about 4.6%g while the perception value was only 0.25%g. This minimum occurred around the frequency of 5 Hz which was the main resonant frequency of the human body [130]. Dieckmann [130] also separated sensitivity to vibrations in the horizontal and vertical direction where for frequencies below about 4 Hz sensitivity was higher for horizontal vibrations.

Although these few examples of early findings are not directly related to footbridges, they present the first steps in human vibration perception research which triggered and became a basis for subsequent investigations. They also give insight into large variations of vibration threshold limits caused typically by different test conditions and illustrate the need to research this issue separately for each type of structure of interest.

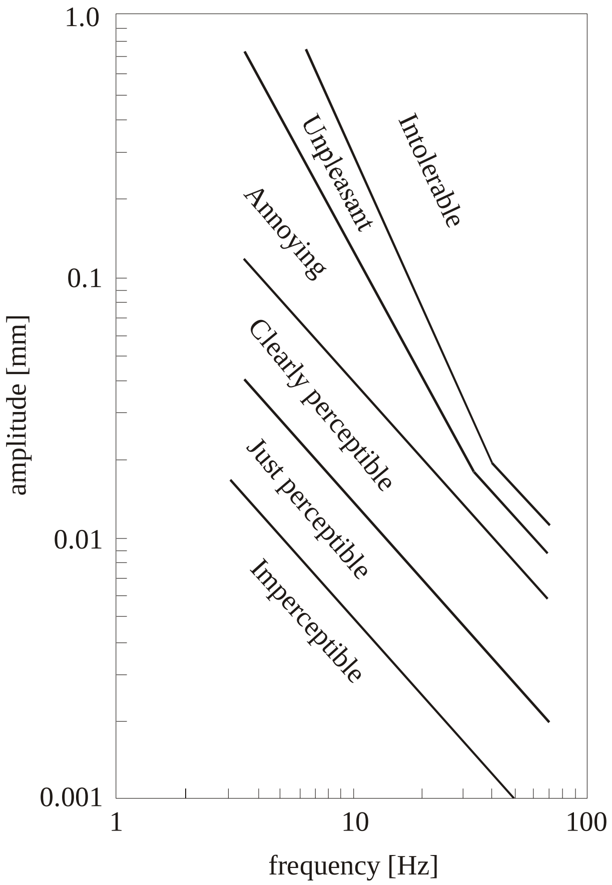

Figure 15: Reiher and Meister's scale of human perception (after Smith [139]).

4.2 Perception of Vertical Vibrations on Bridges

Trying to investigate the human perception of vibration on highway bridges, Wright and Green [127] noticed that real vibrations on bridges are much more complex than the harmonic vibrations usually used in past investigations of human perception. Also, research was conducted in laboratory conditions and therefore its applicability to footbridges is questionable. Finally, many parameters specific to bridge vibrations were not considered such as the fact that the receiver is not stationary but is moving, the transient nature of footfall excitation and the

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limited duration of exposure to vibrations. As a confirmation of the desperate situation regarding the knowledge of human perception of bridge vibrations the Committee on Deflection Limitations of Bridges [131] reported that there was no scale at that time which was appropriate for bridge applications.

In a large investigation Wright and Green [132] measured the peak oscillations on 52 highway bridges under normal traffic and found they were “unpleasant” or even in 25% cases “intolerable” according to their isosensors scale based on a refinement of Goldman's [129] work. Similar results were obtained using Reiher and Meister scale. They concluded that these, and similar, scales based on long-time vibrations might not be appropriate for bridge vibrations where peak vibrations usually lasted only for a short period of time. The duration of vibrations depends to some extent on the bridge damping which is considered as the most important factor in the human perception in Lenzen's [133] work, but related to floors.

Motivated by the lack of research related to walking and standing people under vibrations with limited duration, Leonard [20] conducted a laboratory experiment on a 10.7 m long beam driven by sinusoidal excitation at different amplitudes (up to 0.2’’ i.e. 5.08 mm) and frequencies (1–14 Hz). Forty walking and standing persons helped in these tests to define the boundary between acceptable and unacceptable vibrations in individual tests lasting up to 1 min during which vibration amplitude was held on a constant level. Results clearly indicated that a standing person is more sensitive to vibrations than a walking one (Fig. 16). Similarly to Wright and Green [132], it was shown that the Reiher and Meister scale is fairly inappropriate for application to bridges. Leonard further suggested using the curve applicable to stationary standing people for vibration perceptibility in the case of large numbers of pedestrians because of a prolonged duration of the vibration level. A similar recommendation was made regarding the perception of vibration in the horizontal direction because of the greater human sensitivity in this direction.

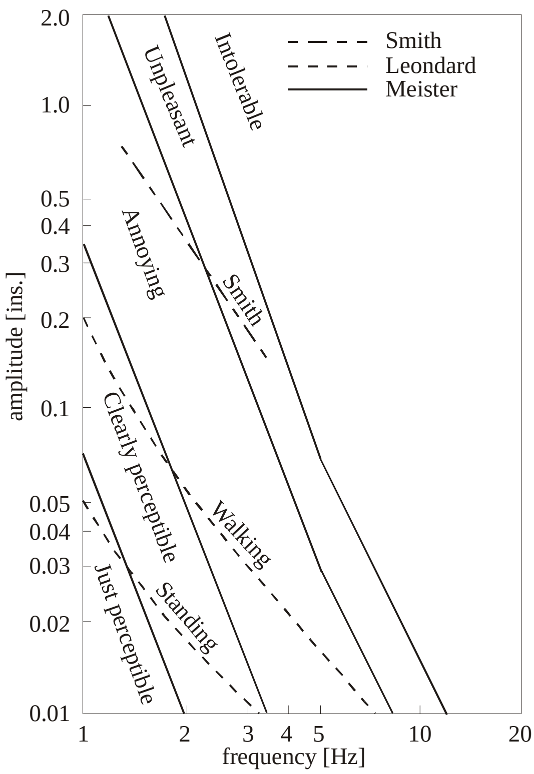

Figure 16: Leonard's and Smith's scales of human perception (after Smith [134]).

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Smith [134] conducted an experiment using a single walking pedestrian excitation on a flexible aluminium alloy plank. He pointed out that a pedestrian, even in the case of sustained sinusoidal bridge vibrations, felt the maximum amplitude only near the midspan (Fig. 17). Twenty-six subjects were asked to walk several times across the plank and to classify vibration level into three groups: acceptable, unpleasant and intolerable. During tests the frequency of the plank was adjusted to be as close as possible to the walking frequency. Because of the inter-subject variability and overlapping of the three vibration levels rated by different test subjects, Smith decided only to define regions of acceptable and unacceptable vibrations (Fig. 16). His threshold curve was much higher than the Leonard's. As a possible reason Smith mentioned possibility that Leonard chose to draw a lower limit curve rather than a mean curve. However, it could be that the length of the plank of only 4.88 m had influence too. It is interesting that in some of Smith's tests, when the resonant build up of vibration was achieved, some subjects were afraid.

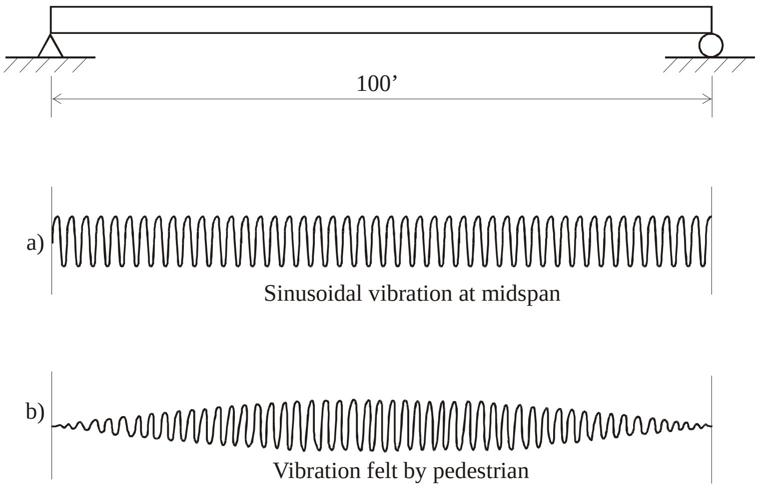

Figure 17: (a) Sinusoidal vibrations in the middle of the span. (b) Vibrations felt by a walking pedestrian (after Smith [134]).

Kobori and Kajikawa [135] conducted experiments similar to Leonard's tests, with 11 walking subjects on a vertically vibrating shaking platform driven by a sinusoidal force in the frequency range 1–10 Hz. It was found that the vibration velocity is the main parameter which influenced the human perception. The two authors formulated analytically the relationship between the vibration perception and the vibration velocity and reported a comparison between responses to: a sinusoidal vibration, vibration having two harmonic components and random vibration. They concluded that the sensitivity is the same if the “effective value of both stimuli” is the same. However, it is not quite clear what this statement precisely means and how the results are processed. The same authors investigated the possibility that a footbridge will be unserviceable under a number of pedestrians [136]. Using probability theory and assuming a Poisson distribution of pedestrian arrivals as well as a normal distribution of human response, they found the probability that serviceability of a footbridge will not be satisfied, in terms of a percentage of pedestrians who will feel an unacceptable level of vibrations. Unserviceability curves for a pedestrian bridge, as a function of arriving number of pedestrians per second and the bridge damping are given in Fig. 18. Unfortunately, none of these two articles [135] and [136] contains a list of pertinent references which could help to understand better the approach proposed. It should be noted that velocity is adopted as the parameter for evaluation of footbridge serviceability in Japan [55].

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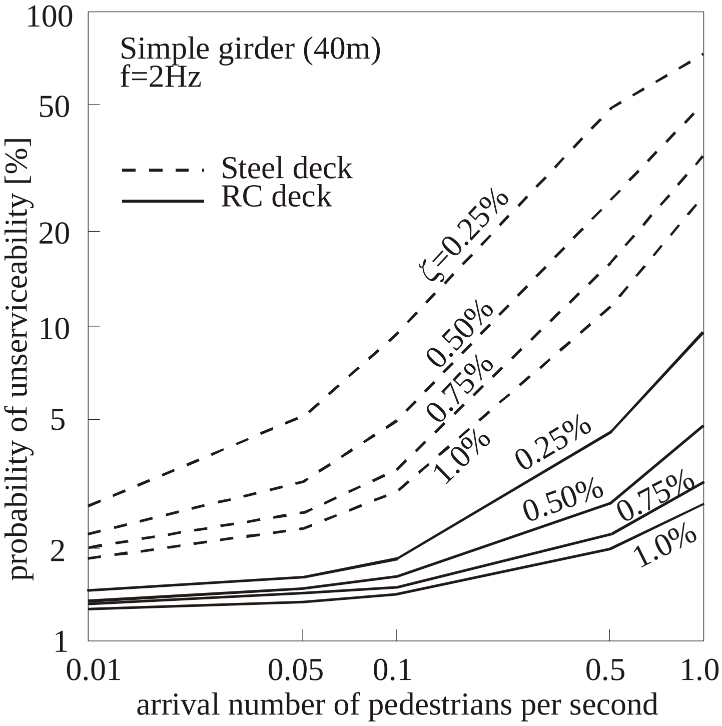

Figure 18: Probability of footbridge unserviceability (after Kajikawa and Kobori [136]).

In their design proposal for footbridges, Blanchard et al. [34] used the mean value of Leonard's and Smith's results to define a level of acceptable acceleration a_{limit}, expressed as:

$$a_{limit} = 0.5\sqrt{f}, \quad [\mathrm{m/s^2}] \tag{6}$$

where f [Hz] is the footbridge fundamental frequency. This value is adopted in the current British standard for assessing vibration serviceability of footbridges BS 5400 [137]. However, Tilly et al. [12] mentioned the possibility that the limit of $\sqrt{f}$ might be more appropriate outside the frequency range 1.7-2.2 Hz, but without detailed elaboration of this recommendation.

Irwin [138] collected data about human response to vibration from different sources based on both laboratory and tests on full-scale structures. He constructed either the perception or maximum allowable magnitude curves for different types of structures and different type of vibrations. Among them, the limits for root-mean-square (RMS) accelerations for bridges are given, separately for everyday usage and storm conditions (Fig. 19). The maximum sensitivity for everyday curve for vertical vibrations was between 1 and 2 Hz, and was 0.07 m/s^2 when expressed as an equivalent harmonic peak value. This frequency range is far lower than the 4-8 Hz range in ISO 2631-2 [140] applicable to floor vibrations. The curve for storms is obtained by multiplying the base (everyday) curve by the factor 6. However, the horizontal motion is considered only for storm conditions, while for everyday usage it is neglected as rare. This work was founded on the base curve principle, which means that curves for different purposes can be obtained from the base curve by multiplying by some factor. All perceptibility curves were expressed, contrary to a lot of previous research based on peak values, via the RMS accelerations. This quantity is the square root of the mean value of the square acceleration during time record [126]:

$$\mathrm{RMS} = \sqrt{\frac{\int_{t_1}^{t_2} \ddot{x}(t)^2 \,\mathrm{d}t}{t_2 - t_1}} \tag{7}$$

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where $\ddot{x}(t)$ is the acceleration time history, and t_1 and t_2 define the beginning and end of the time interval considered. However, the choice of RMS accelerations as the vibration perception descriptor, which became common in many guidelines related to human perception of vibrations, was based primarily on the fact that it is relatively easy to measure accelerations and the corresponding RMS values, using both analog and digital methods [126].

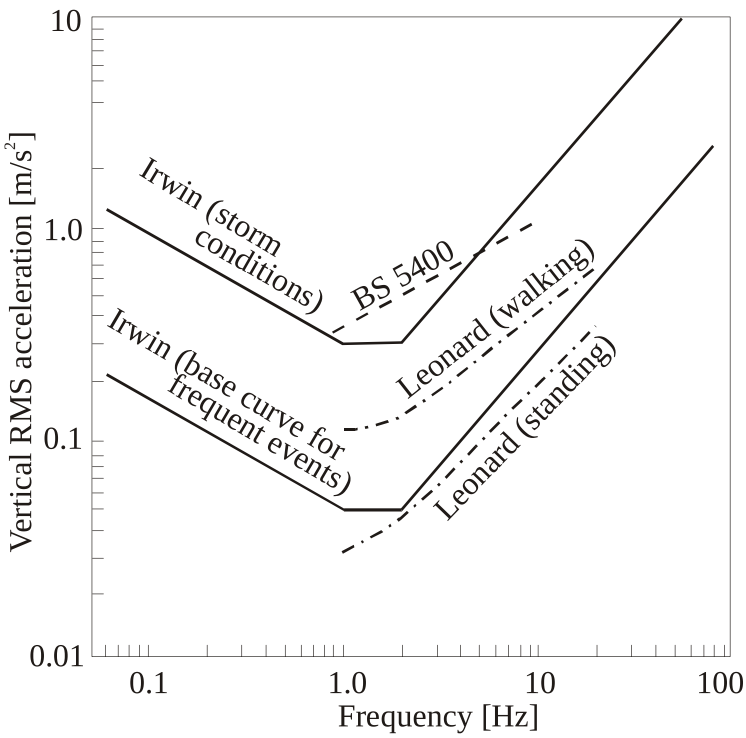

Figure 19: Acceptability of vibrations on footbridges after different scales (after Smith [139]).

One of the recommendations for acceptable footbridge vibrations, which is based on RMS acceleration limits, is given in the ISO 10137 guidelines for serviceability in buildings [27]. It suggests using the base curves for vibrations in both vertical and horizontal directions given in ISO 2631-2 [140] multiplied by the factor of 60 (Fig. 20 and Fig. 21). However, this recommendation is, to the best of authors' knowledge, not based on published research pertinent to footbridge vibrations.

Pimentel [58] compared vibration limits given in BS 5400 [74], Ontario Code [141], Kobori and Kajikawa [135] and ISO 10137 [27] related to footbridges. They are presented in Fig. 20, while Fig. 19 compares limits according to BS 5400, Leonard [20] and Irwin [138]. A comparison of these limits shows that, for example, BS 5400 allows the highest level of vibrations over a typical range of footbridge response frequencies. On the other hand, Bachmann et al. [57] proposed a constant acceleration acceptance level of 0.5 m/s^2. All these limits form a database of results related to footbridges. It should be noticed that the ISO [27] curve given in Fig. 20 was obtained by converting the RMS acceleration to the peak value by multiplying by the factor $\sqrt{2}$.

To account for different reactions between different people, Obata et al. [142] presented, for each of four defined perception levels (lightly perceptible, definitely perceptible, lightly unpleasant and greatly unpleasant), curves of 25%, 40%, 50%, 60% and 75% probability where the reactions to vibrations will happen. They suggested that footbridge serviceability will not be compromised for a peak velocity of 1 cm/s, while it is rare that vibrations are unpleasant up to the peak 1.4 cm/s. For a footbridge with, for example, a natural frequency of 2 Hz, converting

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these velocity peak values to corresponding peak accelerations gives 0.13 and 0.18 m/s^2, respectively, where these limits are far lower than those in Fig. 20.

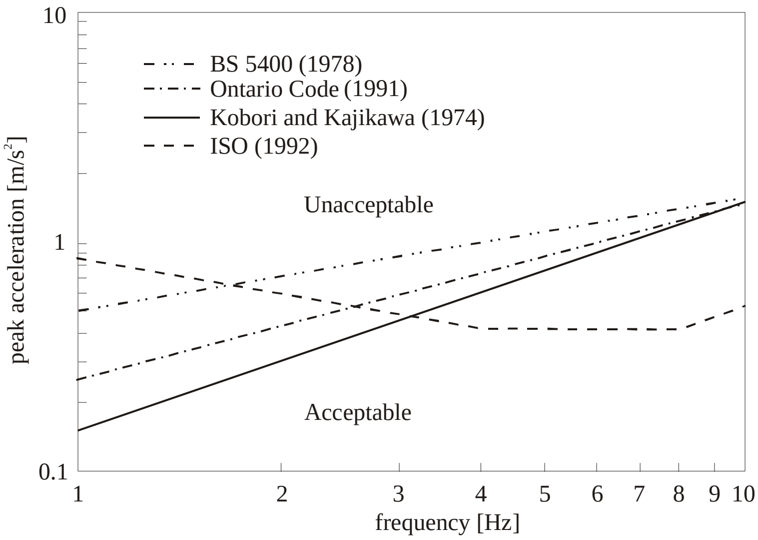

Figure 20: Acceptability of vertical vibrations in footbridges after different scales (after Pimentel [58]).

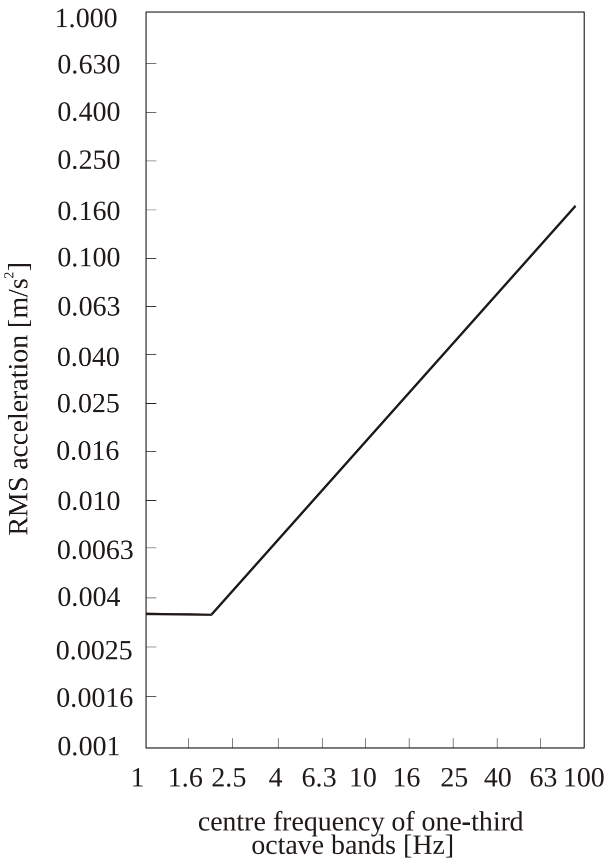

Figure 21: Acceptability of vibrations in horizontal direction. This base curve should be multiplied by the factor 60 (after ISO [27]).

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4.3 Horizontal Vibration and Its Perception within a Crowd

Data on human perception of horizontal vibration of bridges are very scarce. However, there are many works related to human perception of horizontal vibrations in buildings. For example Chen and Robertson [143] investigated the human perception threshold to horizontal sinusoidal vibrations with frequencies between 0.067 and 0.20 Hz, which are characteristic of tall building response due to wind. Although this frequency range is unlikely to be relevant for footbridges, this work is interesting because it identified the most important factors typical for this issue: the frequency of vibrations, body movement, expectancy of motion and body posture. The authors found that, also in this low-frequency region, the vibration perception threshold of walking people is higher than for a stationary person and that the perception threshold is lower when the person expects the movement. However, it should be noticed that the tolerance level (as opposed to the perception level) is higher if one expects vibrations, regardless of its direction [139]. Another interesting experiment was conducted by Nakata et al. [144]. Forty test subjects were exposed to horizontal sinusoidal vibrations at frequencies 1–6 Hz. The amplitude of vibrations was gradually increased and the perception value when the test subject felt vibration was recorded. It was concluded that the fore-aft perception threshold was higher than the side-to-side threshold in the range 1–3 Hz, while in the range 3–6 Hz the opposite was true. However, only the sitting posture was considered.

Wheeler [40] noticed that the human perception of vibration in a walking crowd on footbridges is different than for a single isolated person. An additional proof that different perception scales are necessary for circumstances involving different numbers of people was provided by Ellis and Ji [145]. They reported that during an experiment with a jumping crowd, jumpers were not concerned although the measured acceleration was 0.55 g. It is not clear if the noise, the presence of other people or something else contributed to the fact that such high accelerations were considered as tolerable.

As mentioned earlier, the only guideline which recommends a horizontal vibration limit for footbridges is ISO 10137 [27]. The perception curve is presented in Fig. 21. The highest sensitivity to this type of vibration is in the frequency region up to 2 Hz and is at about 3.1%g peak acceleration.

Probably most valuable information about the tolerance level to footbridge lateral vibrations due to crowd loading is given by Nakamura [146]. Based on pedestrian experience of vibrations on full-scale footbridges, he concluded that the amplitude of deck displacement of 45 mm (corresponding to an acceleration of 1.35 m/s^2) is a reasonable serviceability limit. At the same time he noticed that a deck displacement amplitude of 10 mm (corresponding to acceleration level of 0.3 m/s^2) was tolerable by most pedestrians, while a displacement of 70 mm (2.1 m/s^2) would make people to feel unsafe and prevent them from walking.

4.4 Concluding Remarks

In the majority of tests conducted it has been generally accepted that acceleration is the vibration parameter which should be used to describe the problem. One of the key reasons is that acceleration is convenient to be measured even though there are situations when the vibration effect can be explained better by other quantities such as velocity [126]. It is now widely accepted that the vibration tolerance for moving pedestrians on bridges is higher than for people in buildings, and that pedestrians can accept certain (initially unacceptable) level of vibrations when they accustom themselves to it [139]. Therefore, the expectation of vibrations plays a very important role in footbridge vibration serviceability. However, hard numbers which would quantify these observations are scarce.

Further work in this area for footbridges is necessary, especially with regard to the perception of vibration in the horizontal lateral direction. This should be done by simulating real conditions as much as possible and by verifying results on full-scale structures. Finally, there is some limited evidence of some probability-based approaches to vibration perception on footbridges.

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Considering the large inter- and intra-subject variability, probability based methods are likely to be the best way forward when assessing the effects of footbridge vibrations.

5 Human-Structure Dynamic Interaction in Footbridges

It is now widely accepted that during footbridge vibration some kind of human-structure interaction almost inevitably occurs. Often, this interaction can be neglected, but it is becoming more common that it cannot. In general, there are two aspects of this issue. The first considers changes in dynamic properties of the footbridge, mainly in damping and natural frequency, due to human presence. The second aspect concerns a degree of synchronisation of movement between the pedestrians themselves as well as between the pedestrians and the structure whose motion is perceived. Both phenomena are currently not well understood and research related to them has been intensified in recent years.

5.1 Dynamic Properties of Footbridges under Moving People

It is well-known that the presence of a stationary (standing or sitting) person changes the dynamic properties of a structure they occupy. The most important effect is the increase in damping in the joint human–structure dynamic system compared with the damping of the empty structure [51] and [147]. The effect is greater if more people are present [147] and [148]. Therefore, it can be concluded that the human body behaves like a damped dynamic system attached to the main structural system. Such a system can be described by biodynamics methods, structural dynamics methods or by their combination [149]. The human body is in effect a complex nonlinear MDOF system with its parts responding in different ways to structural movement [150]. In a simplified study of human body–structure interaction, the human body can be approximated by a linear SDOF system [149]. One of very few reported attempts to carry out system identification of the dynamic properties of a standing person, applicable to civil engineering, was done recently by Zheng and Brownjohn [151]. Their SDOF human body model had a damping ratio of 39% and natural frequency of 5.24 Hz. However, the simplified SDOF human body system has been shown to be frequency-dependent and cannot be always represented by the same set of mass, stiffness and damping parameters [147] and [152].

The problem is even less researched in the case of moving people, which is usual for footbridges. Ellis and Ji [51] found that a person running and jumping on the spot cannot change dynamic characteristics of the structure and, therefore, should be treated only as load. However, this investigation was conducted using a simply supported beam having a high fundamental frequency of 18.68 Hz compared with typical footbridge natural frequencies. Nevertheless a similar conclusion was reached by the same researchers regarding the effects of a moving crowd on grandstands [148].

5.2 Dynamic Forces on Flexible Footbridges

Ohlsson [35] reported that the spectrum of a force measured on a rigid surface differed from that measured on a flexible timber floor. The spectrum experienced a drop around the natural frequency of the structure where the motion was the highest. This could be a consequence of the interaction phenomenon and is in agreement with previously mentioned Pimentel's [58] findings of lower DLFs on real and moving footbridges in comparison with those measured on rigid surfaces. Ohlsson also claimed that a moving pedestrian increased the mass and the damping of the structure. However, it should be stressed again that he investigated only light timber floors where human–structure dynamic interaction is more likely due to large ratios of the mass of the humans and the empty structure. However, Willford [22] also mentioned a result of data analysis from pedestrian tests on the Millennium Bridge which indicated that walking crowd had increased the damping of the structure in the vertical direction.

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That jumping and bouncing can change dynamic properties of a flexible structure was reported by Yao et al. [52]. They found that jumping forces are lower on a more flexible structure, but it should be noted that in their investigation the subject to structure mass ratio was very high (0.41). Further, Pavic et al. [54] compared horizontal jumping forces directly measured on a force plate and indirectly measured on a concrete beam. They found that the force on the structure was about two times lower than that one on the force plate. This could also be a consequence of a human–structure interaction effect but no conclusive evidence for it was presented.

All these reported observations give only an indication that human–structure interaction really occurs without a more precise quantification of the phenomenon. Furthermore, with the exception of a paper by Pavic et al. [54], all reviewed research is related to vibrations in the vertical direction. Information on possible effects of moving people on the dynamic characteristics of footbridges in the horizontal direction is very scarce.

It is clear that research into human–structure interaction involves various human activities (e. g. waking, jumping, sitting, standing) on different types of structure. In case of footbridges, although some previous findings are quite useful, the most relevant interaction scenario appears to be a walking crowd. Considering the extremely scarce published data, this is an area that clearly requires further investigation.

5.3 Synchronisation Phenomenon of People Walking in Groups and Crowds

Ninety years ago Tilden [28] posed a question which is still unanswered:

> “Against what loads, horizontal and vertical, should an engineer design a structure which is likely to have to carry a dense crowd of human beings?”

In an attempt to consider this question, he noted that none of the following two extreme cases are real. Neither is an increase in load directly proportional to the number of people involved, in comparison with a single pedestrian force (i.e. the case of perfect synchronisation), nor should only the static weight of the crowd be taken into account (i.e. dynamic effects be neglected). Subsequent research has shown that the solution is somewhere between these two scenarios.

The first attempts to define the load induced by several pedestrians were in terms of multiplication of the load induced by a single pedestrian. One of the first proposals was given by Matsumoto et al. [46]. Assuming that pedestrians arrived on the bridge following a Poisson distribution they stochastically superimposed individual responses and found that the total response can be obtained by multiplying a single pedestrian response by the multiplication factor $\sqrt{\lambda T_0}$, where λ is the mean arrival rate expressed as the number of pedestrians per second per width of the bridge and T_0 [s] is the time needed to cross over the bridge. Therefore, $\sqrt{\lambda T_0}$ is equal to $\sqrt{n}$, where n is the number of pedestrians on the bridge at any time instant. According to random vibration theory [153], if the response due to n equal and randomly distributed inputs is $\sqrt{n}$ times higher than the response due to a single input, it means that inputs (in this case pedestrians) are absolutely uncorrelated (unsynchronised).

Similar to Matsumoto et al. [46], Wheeler [40] stochastically combined individual forces (defined deterministically using the half-sine model) assuming random arrival rate, normal distribution of step frequencies and a distribution of people's weights obtained for the Australian population. However, his simulations revealed that group loads were not a more onerous design case than a single pedestrian load, at least for footbridges with fundamental natural frequency away from approximately 2 Hz. Namely, the group load on bridges with the fundamental frequency away from the normal walking frequency range can be regarded as a nonresonant load which probably generates lower response than the one induced by a single pedestrian walking at the resonant frequency. However, the question still is if this can be applied in case of nonrandom walking of groups of pedestrians when some degree of synchronisation between people can be established.

In any case, Matsumoto et al.'s proposal was regarded as appropriate at least for footbridges with natural frequencies in the range of walking frequencies (1.8–2.2 Hz), while for bridges with natural frequencies in the ranges 1.6–1.8 Hz and 2.2–2.4 Hz a linear reduction of Matsumoto et al.'s multiplication factor $\sqrt{\lambda T_0}$ was suggested with its minimum value of 2 at the ends of these intervals in the case of more than four people present on the bridge at the same time [14]. Mouring [154] simulated a vertical force from walking groups in a way similar to Wheeler [40]. However, she described a single pedestrian force more precisely using the first 10 coefficients of the Fourier series instead of the half-sine model. As a result, she found that the effect of group loads should be considered even in case of footbridges with fundamental frequency outside the normal walking frequency range (1.8–2.2 Hz). The response obtained agreed with Matsumoto et al.'s findings. However, Pimentel [58] measured the response under three uncorrelated people on two footbridges and confirmed the inapplicability of the proposed multiplication factor for bridges with frequencies outside the normal walking frequency range, as claimed by Bachmann and Ammann [14]. It appears that group loading becomes more important precisely in the normal walking frequency range, and in that case it should be considered. Also, Matsumoto et al.'s proposal did not consider the possibility of synchronisation between people in a dense crowd, a phenomenon which has attracted a great deal of attention from researchers since the Millennium Bridge problem in London occurred in 2000.

In 1985, Eyre and Cullington [70] noticed that the vertical acceleration recorded on a footbridge in a controlled resonance test with a single pedestrian was 1.7 times lower than the one measured in normal usage which included two or more pedestrians who were not formally synchronised in any way. They explained it as a possible consequence of the occasional and by chance synchronisation between two people. Ebrahimpour and Fitts [155] reported that the optical sense plays an important role in the synchronisation of people's movement. Namely, two jumping persons who could see each other synchronised their movement better than when they were looking in opposite directions. In both cases the jumping frequencies were controlled by an audio signal. Eriksson [49] claimed that the first walking harmonic could be almost perfectly synchronised for highly correlated people within a group, while the higher harmonics should be treated as completely uncorrelated. Not surprisingly, Ebrahimpour et al. [44] therefore focussed only on the first harmonic (Fig. 13) claiming that higher harmonics cannot produce significant response for a walking crowd.

It is now widely accepted that people walking in a crowd, because of the limited space on the bridge deck and the possibility that thus can see each other, would subconsciously synchronise their steps. This becomes more likely if the crowd is dense. Bachmann & Ammann [14] reported that the maximum physically possible crowd density can be 1.6–1.8 persons per square metre of the footbridge deck. However, they concluded that a value of 1 person/m^2 is more probable. During the opening day of the Millennium Bridge in London, the maximum density was 1.3–1.5 people/m^2 [19]. The crowd density on the T-bridge in Japan (also prone to lateral movement) was between 1 and 1.5 people/m^2 [156]. In any case, crowd density influences the walking speed (Fig. 22), the degree of synchronisation between people and, consequently, the intensity of the human-induced force.

Grundmann et al. [72], proposed three models corresponding to different pedestrian configurations on a footbridge which should be considered separately. These are:

- ***Model 1***: When people walk in small groups it is probable that they will walk with the same speed v_s, and slightly different step frequencies f_s and step length l_s according to the equation:

$$v_s = f_s l_s. \tag{8}$$

In such cases, some synchronisation between these people is expected, but only when the bridge frequency is within the normal walking frequency range.

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- ***Model 2***: On bridges with a light stream of pedestrians where people can move freely and their walking frequencies are randomly distributed. The maximum density of 0.3 pedestrians/m^2 was suggested as an upper limit for unconstrained free walking. This type of walking (i.e. free walking) was considered in the previously mentioned proposal by Matsumoto et al.'s [46].

- ***Model 3***: If footbridges are exposed to pedestrian traffic of 0.6–1.0 pedestrians/m^2 then free unconstrained movement is practically impossible. In such circumstances, pedestrians are forced to adjust to some extent their step length and speed to the motion of other pedestrians. The previously mentioned swaying problems of the Millennium Bridge and the Japanese T-Bridge belong to this group, despite the fact that their pedestrian densities were higher than proposed by Grundmann et al.

As for the third model, it should be added that the case of crowd walking on a perceptibly moving bridge deck is related not only to synchronisation between people but also to synchronisation between people and the structure.

Before considering the research into the human-structure synchronisation phenomenon, two terms widely used in this article will be defined. The term 'group' of walking pedestrians is used for several people walking at the same speed as defined in Model 1 above, while the term 'crowd' is related to densely packed walking people who have to adjust their step to suit the space available, as explained in Model 3.

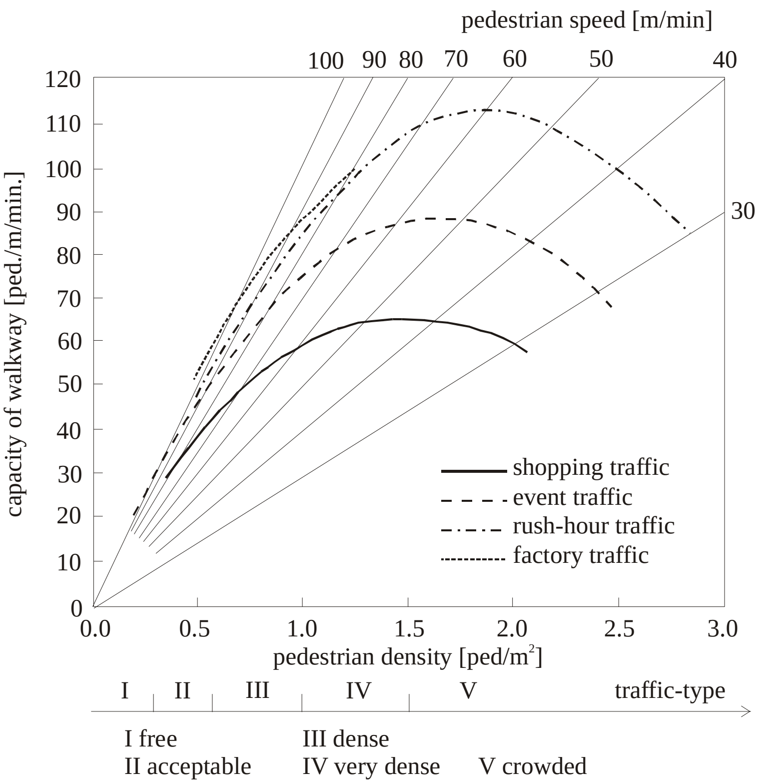

Figure 22: Relationship between the bridge capacity, pedestrian density and their velocity (after Schlaich [7]).

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5.3.1 Lateral Synchronisation

The phenomenon when people change their step to adapt it to the vibrations of the bridge, is—for the same level of vibrations—much more probable in the horizontal than in the vertical direction. This is because of the nature of human walking and desire to maintain the body balance on a laterally moving surface. When it occurs, this is known as the synchronisation phenomenon [14] or lock-in effect [19]. As a consequence of the adjusted step when people tend to walk with more spread legs, the motion of the upper torso becomes greater and the pedestrian-induced force becomes larger. This in turn increases the bridge response and, finally, results in structural dynamic instability [157]. In such circumstances, only reducing the number of people on the footbridge or disrupting/stopping their movement can solve the problem [19], [158], [159], [160] and [161]. It is interesting, however, that in a laboratory experiment [162] with a single pedestrian walking across a laterally moving platform, not every pedestrian walked in a way to boost the lateral vibrations. Some of them even managed to damp vibrations out. This fact complicates further study of pedestrian behaviour within a crowd, but also points out the need to define and investigate a factor which will describe the degree of synchronisation between people.

Typically, the excessive swaying occurs on bridges with lateral natural frequencies near 1 Hz which is the predominant frequency of the first harmonic of the pedestrian lateral force (Fig. 8). Fujino et al. [18] reported such a case on the previously mentioned T-bridge in Japan. During very crowded times, significant lateral movement occurred in the first lateral mode with frequency of 0.9 Hz. The procedure proposed by Matsumoto et al. [46] underestimated the actual bridge response. By video recording and observing the movement of people's heads in the crowd, and by measuring the lateral response, Fujino et al. concluded that 20% of the people in the crowd perfectly synchronised their walking. Fujino et al.'s assumption was also that the individual forces produced by the rest of pedestrians cancelled each other, so that their net effect was zero. Later, using image processing technique for tracking people's movement on the same bridge, Yoshida et al. [163] estimated the overall lateral force in the crowd of 1500 pedestrians at 5016 N, which gives an average of only 3.34 N per pedestrian.

During the opening day of the Millennium Bridge in London, lateral acceleration of 0.20–0.25 g was recorded. This corresponded with lateral displacement amplitudes of up to 7 cm. Dallard et al. [19] and [159] tried to define the problem analytically on the basis of observations made during tests with a gradually increasing number of people on the bridge (up to 275 people). Assuming that everybody contributed equally, they identified the amplitude of the modal lateral force per person (Fig. 23a) and the dependence of the lateral force on the footbridge velocity (Fig. 23b). This force was considerably higher than the one reported by Yoshida et al. [163]. Based on results in Fig. 23b, Dallard et al. concluded that people, after synchronising their movement with the movement of the structure, produced a dynamic force *F(t)* which was proportional to the deck lateral velocity *v(t)*:

$$F(t) = kv(t). \tag{9}$$

This means that moving pedestrians act as negative dampers (i.e. amplifiers) increasing the response of the structure until walking becomes so difficult, due to body balancing problems, that they have to stop. This clearly indicates the need to model differently the human-induced load before and after the synchronisation occurs. Also, it seems more relevant to investigate bridge behaviour before (and not after) the lock-in occurs, in order to predict and prevent the problem in the future. Bearing in mind several other known examples of excessive lateral vibrations of crowded bridges, Dallard et al. ([19], [158] and [159]) further concluded that the same problem can happen on every bridge with a lateral frequency below 1.3 Hz and with sufficient number of people crossing the bridge. That triggering (critical) number of people N_L was defined as:

$$N_L = \frac{8\pi cfM}{k} \tag{10}$$

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where c is the modal damping ratio, f is the lateral frequency of the bridge, M is the corresponding modal mass and k [Ns/m] is the lateral walking force coefficient introduced in Eq. (9). For the case of the Millennium Bridge it was found by back analysis that $k = 300\ \mathrm{Ns/m}$ in the lateral frequency range 0.5–1.0 Hz. However, it would be interesting to find this factor for other bridges with the lateral swaying problem to compare with this value. Also, the shape of the force time history in Fig. 23a revealed that the lock-in started at about 900 s. However, it seems that the lock-in was unsuccessfully triggered two times between 600 s and 800 s. The factors which prevented these two lock-ins are still not identified and it would be extremely beneficial to know what they are. Also, it should be emphasized that, although the predominant lateral load frequency is about 1 Hz, during the bridge opening day the first lateral mode at about 0.5 Hz was also excited. This can be caused by the reduced frequency of the lateral walking force in a crowd (down to 0.6 Hz) and by some 'meandering' patterns in human walking on moving bridge deck surfaces, as observed by Dallard et al. [159].

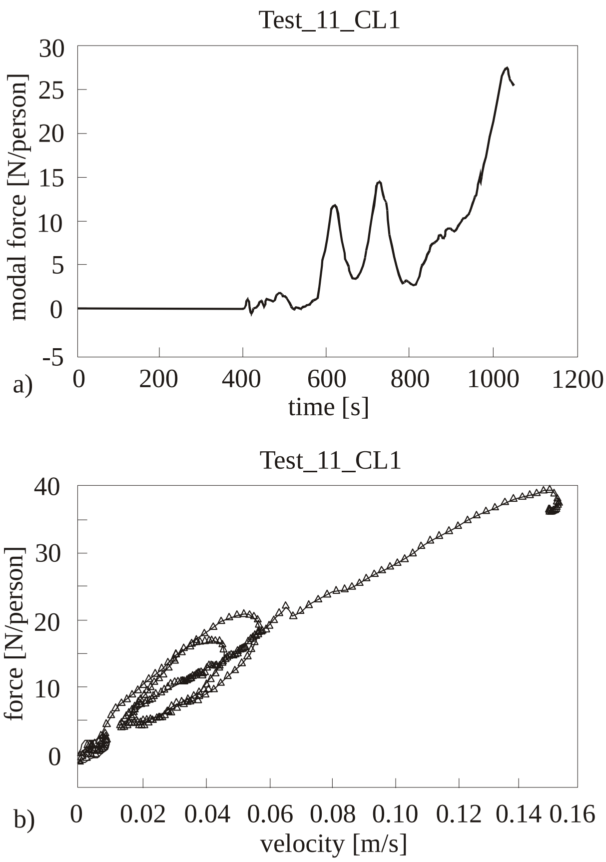

Figure 23: (a) Peak amplitude of the lateral modal force per person per vibration cycle. (b) Lateral force per person per vibration cycle vs deck velocity (after Dallard et al. [19]).

Research described in three papers by Dallard et al. [19], [158] and [159] stressed the need to investigate the dependence between the probability of synchronisation between people and the amount of bridge movement in the lateral direction. In that sense, Willford [22] reported tests with a single walking person on a platform moving laterally. The results showed that the lateral pedestrian force was increasing when the lateral movement increased. Also, he found that in the case of structural movement at 1 Hz with an amplitude of 5 mm, the probability of people adapting their step to the bridge movement is 40%. These relationships are nonlinear and dependent on frequencies of the bridge movement, even for a single person (Fig. 24). These

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observations were made for individuals and their applicability to people walking in a crowd is still unknown.

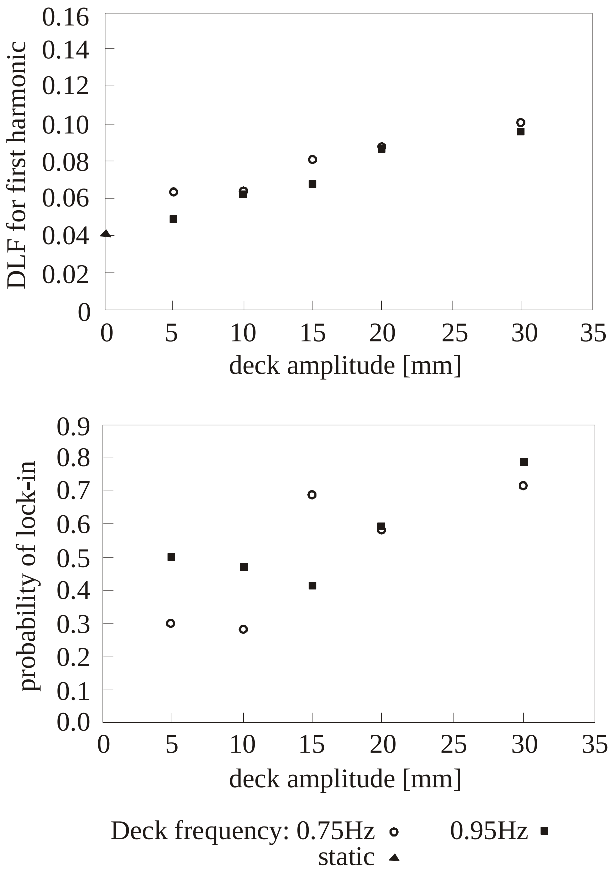

Figure 24: (a) DLF and (b) probability of “lock-in” for a single person as a function of the moving platform amplitude and frequency (after Dallard et al. [19]).

An interesting study on a lively footbridge (M-bridge) in Japan revealed that a pedestrian, walking within a crowd on a perceptibly moving deck, synchronised their movement with the bridge vibrations [146]. A phase from 120° to 160° between girder and pedestrian motion was identified. This synchronisation was only spoiled at maximum measured deck amplitude of 45 mm, when it became much harder to walk. It is interesting that excessive lateral vibrations on this footbridge occurred at two different response frequencies (0.88 Hz and 1.02 Hz) depending on the crowd density. These two frequencies corresponded to two modes as high as the sixth and seventh lateral mode of vibrations. A very low damping ratio of 0.5% and also very low bridge mass of 400 kg/m^2 certainly contributed to developing of such large vibrations. Nakamura [146] also reported that the bridge mass was lower than the mass on other two well-known lively (in lateral direction) footbridges: Millennium Bridge (about 500 kg/m^2) and T-bridge (800 kg/m^2).

Nowadays, increasing efforts are made to quantify the vibrations due to crowds using the basis of wind engineering theory. In one such attempt, Stoyanoff et al. [164] suggested a correlation factor $c_R\left(N\right)$ in a moderate crowd of N people when the density is below 1 pedestrian/m^2 similar to one from vortex-shedding theory:

$$c_R\left(N\right) = e^{-\gamma N} \tag{11}$$

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where the factor γ could be obtained from a condition that $c_R = 0.2$ (20%) for the maximum congested footbridge as it was in the work by Fujino et al. [18]. However, Fujino et al.'s methodology could not predict the structural response during the Millennium Bridge tests [159]. On the other hand, Yoneda [55] stressed that several factors influenced the synchronisation factor: the lateral natural frequency, damping, length between node points in the resonant mode, walking speed and bridge length on which synchronisation occurs. This observation was not experimentally verified on full scale structures but it deserves attention because of its generality.

Interestingly enough, an entirely different theory to the one considered so far in this section which is based mainly on observations made on the Millennium Bridge, was given by Barker [165]. He claimed that the response to crowd movement may increase without any synchronisation between people. Further, Dinmore [166] suggested treating the human-induced force as a wave which propagates through the structure. As a way to control bridge response and avoid synchronisation, he recommended to vary the dynamic stiffness through the structure using different materials which will provide energy loss due to wave reflection and refraction on their contact.

5.3.2 Vertical Synchronisation

An attempt to quantify the probability of synchronisation in the vertical direction was made by Grundmann et al. [72]. They defined the probability of synchronisation $P_S\left(a_g\right)$ as a function of the acceleration amplitude of the structure a_g (Fig. 25). They proposed that the response to N people on a structure should be calculated from the following formula:

$$a_g = P_S\left(a_g\right) N_r a_{1rz} \tag{12}$$

where a_{1rz} is the response to a single pedestrian and $N_r = NK$ is the number of people reduced by the factor $K < 1$ which takes into account that the load changes position along the structure. For a single span $K = 0.6$ was proposed. For a bridge with fundamental frequency of 2 Hz the probability of synchronisation was suggested as 0.225. Therefore, for these parameters the multiplication factor $P_S\left(a_g\right) N_r$ for the single pedestrian response a_{1rz} becomes:

$$P_S\left(a_g\right) N_r = 0.225 \cdot 0.6 \cdot N = 0.135N. \tag{13}$$

This is lower then the value $\sqrt{N}$ given by Matsumoto et al. [46] for N up to 55 people, despite the fact that Grundmann et al. took into account the synchronisation possibility, and that $\sqrt{N}$ implies N completely uncorrelated people. Grundmann et al. [72] finally suggested that for groups of up to 10 people, the multiplication factor can be taken as presented in Fig. 26, with maximum value of 3 for vertical natural frequencies between 1.5 and 2.5 Hz. The same factor was proposed for the lateral direction but corresponding to two times lower natural frequencies. It should be said that synchronisation with bridge movement in the vertical direction is much less likely, although Bachmann & Ammann [14] reported that it could happen when the vertical amplitude becomes at least 10 mm.

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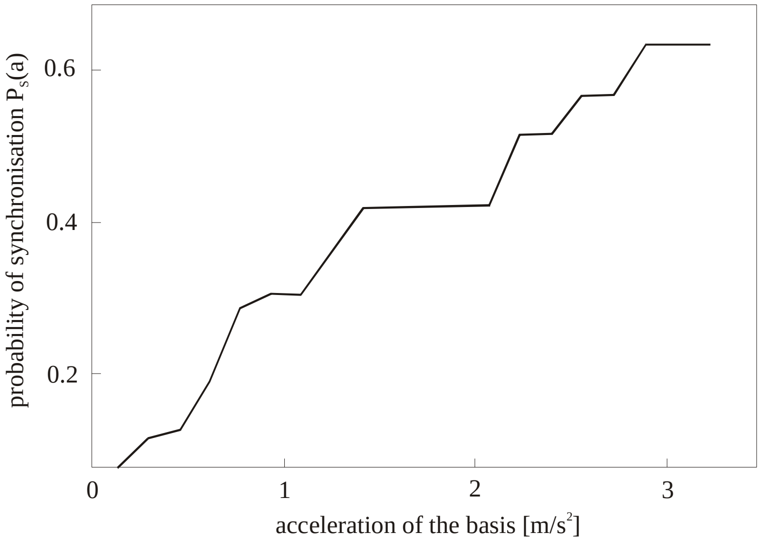

Figure 25: Probability of synchronisation as a function of the acceleration of the bridge (after Grundmann et al. [72]).

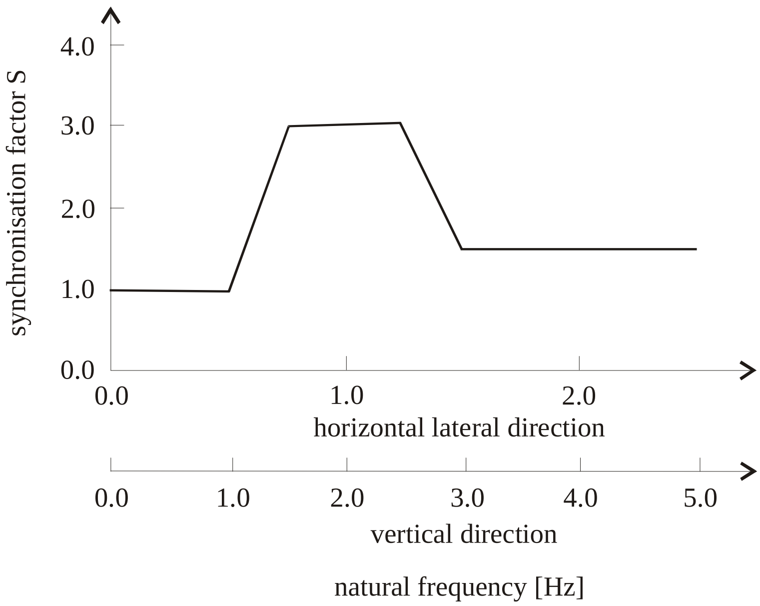

Figure 26: Multiplication factor for groups of up to 10 pedestrains (after Grundmann et al. [72]).

Dallard et al. [19] suggested using random vibration theory to predict the bridge vertical response due to crowd. The mean square acceleration response $E(a^2)$ due to N pedestrians with normally distributed pacing rates was given as:

$$E\left(a^2\right) \approx \frac{\pi N}{16c} \frac{\omega_n}{\sigma} p\left(\frac{\omega_n - \mu}{\sigma}\right)\left(\frac{F_{\omega_n}}{M}\right)^2 \tag{14}$$

where c, ω_n and M are the modal damping ratio, natural frequency and modal mass, F_{ω_n} is the amplitude of the harmonic human force while p is the probability density function for normally distributed pacing frequencies with mean value μ and standard deviation σ. However, this formula was conservative even in the Millennium Bridge case. Its assumption that

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people were uniformly distributed across the structure and that the mode shape was a sinusoid could induce errors and should be corrected according the real conditions on the bridge considered [167]. Also, the distribution of step frequencies within a crowd is unknown. Finally, Mouring [154] and Brownjohn et al. [168] identified that a quantification of the degree of correlation between people in a crowd is a primarily task for future research. Brownjohn et al. [168] went further and suggested a mathematical model for calculation of the bridge response under crowd of pedestrians based on theory of a turbulent wind on linear structures [169]. They proposed that the auto spectral density (ASD) of the response in a single mode $S_z(f)$ in a degree of freedom (DOF) specified by the coordinate z should be calculated as:

$$S_z(f) = \psi_z^2 \left|H(f)\right|^2 S_{P,1}(f) \int_0^L \int_0^L \psi_{z_1} \psi_{z_2} \mathrm{coh}(f, z_1, z_2) \mathrm{d}z_1 \mathrm{d}z_2 \tag{15}$$

where ψ_z is the mode shape ordinate in the same DOF, $H(f)$ is the frequency response function (FRF) for acceleration response, $S_{P,1}(f)$ is the ASD of the pedestrian loads per unit length while ψ_{z_1} and ψ_{z_2} are mode shape ordinates related to the location of each two pedestrians on the bridge described by coordinates z_1 and z_2. Moreover, $\mathrm{coh}(f, z_1, z_2)$ is the correlation factor, between 0 and 1, which should be further researched, as mentioned earlier. This method gave a good estimate of the response for the footbridge investigated, but needs wider verification.

An interesting suggestion for the assessment of liveliness of footbridges in the vertical direction under large crowd load, also based on the wind engineering theory [169], was given by McRobie & Morgenthal [170] and McRobie et al. [162]. It was proposed that the acceptability of vertical vibrations can be assessed by comparing the pedestrian Scruton number $vPSN$ which is achieved with the one required for a particular footbridge. This number is defined as:

$$vPSN = k_1 k_2 m \tag{16}$$

where factors $k_1 = \frac{\zeta}{0.005}$ and $k_2 = \frac{0.6}{n}$ take into account the damping ratio of the empty footbridge ζ (relative to the typical damping ratio of 0.5%) and the possibility that crowd density n could be different from an typical value of 0.6 persons/m^2, respectively. In Eq. (16) m represents the mass per unit deck area for an equivalent simply supported beam having constant cross section. To have structure which will meet vibration serviceability requirements, a larger pedestrian Scruton number (i.e. larger damping and mass and lower pedestrian densities) is preferred. Data about acceptable Scruton numbers as a function of footbridge frequency should be provided by collecting data from existing footbridges known to be lively in the vertical direction. However, this task is hampered by the fact that not many footbridges have experienced large vertical vibrations under crowd load.

In conclusion, it can be said that although the two considered types of synchronisation (among people, and between the people and the structure) are different in their nature, they usually happen simultaneously and lead to the same result – an increase in the response of the structure. In order to understand better the interaction between the moving crowd and the structure it is necessary to identify [159]:

1. the relationships between the crowd density, walking speed, walking frequency and probability of synchronisation, and
2. the probability of lock-in and effective force per person in a crowd as a function of the amplitude and frequency of the bridge motion.

6 Design Procedures and Guidelines

Having a simple and accurate model of the human-induced force, knowing the footbridge dynamic properties and having defined the tolerance level of human perception of vibrations are

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all required for checking the vibration serviceability of a footbridge. However, this is easier said than done. Based on previous sections, it is clear that researchers and practitioners have been working for many years on the formulation of footbridge vibration serviceability design procedures. As a result, some design guidelines have been adopted [74] and [141]. In general, it is important that these design procedures satisfy two somewhat contradictory requirements: to be simple and to be as accurate as possible. In this part of the review, key design procedures reported in scientific literature and design guidelines which are parts of formal national and international codes of practice are outlined.

6.1 Design Procedures Reported in Literature

The aim of most of the design procedures, defined either in the time or the frequency domain, is to determine the peak or the root-mean-square (RMS) response of a footbridge in order to assess its vibration serviceability.

6.1.1 Time Domain Design Procedures

Chronologically the first and largest group of design procedures is based on an assumption that human-induced forces are perfectly periodic and can be therefore decomposed into harmonics by means of Fourier decomposition as given in Equation (1). Then, only a single force harmonic which can, theoretically, excite footbridge resonance related to the fundamental mode shape, should be considered. This means that the structure can be regarded as a SDOF system in modal space as explained in Section 3. Usually, the first three or four excitation harmonics are considered as potentially resonant. All models presented in this sub-section are applicable to vertical forces only, if not stated otherwise. In general, the biggest problem in the modelling process is to simulate a pedestrian moving across a footbridge and the corresponding time limitation of such an excitation.

Blanchard et al. [34] proposed that serviceability should be checked in footbridges with fundamental natural frequencies f up to 5 Hz. As a serviceability criterion, they proposed that the acceleration response [m/s^2] due to one pedestrian should not exceed a limit of $0.5\sqrt{f}$, where f is expressed in Hz. Blanchard et al. proposed a walking force model which was a resonant sinusoid moving across the bridge with velocity v of $0.9f$ (Figure 27a). Modal force per modal mass from the righthand side of Equation (4) for the fundamental vibration mode was given as:

$$\frac{P_1(t)}{M_1} = \frac{P}{M_1}\phi(0.9\,ft)\sin(2\pi ft) \tag{17}$$

where P and M_1 are the force amplitude of 180 N (which corresponds to the DLF of 0.257 given in Section 2.3.1.1) and the generalised mass for the fundamental mode, respectively. $\phi(0.9\,ft)$ is the location-dependent ordinate of the first mode shape which is dependent on the position $x = 0.9\,ft$ of the pedestrian at time t after the beginning of walking. However, for simple bridge configurations (one, two or three spans), the procedure was simplified to a direct calculation of maximum acceleration response a using the formula:

$$a = \omega_1^2 y_s K \psi \tag{18}$$

where $\omega_1 = 2\pi f$ is the fundamental circular frequency of the bridge, y_s is the static deflection at the midspan due to the weight of one pedestrian, K is a configuration factor which depends on the number of spans and ψ is a dynamic response factor which takes into account the span length and footbridge damping. The last two parameters were obtained by numerical simulations on footbridges having up to three spans due to the general pedestrian load

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presented in Figure 27b. This work was probably the first attempt to define a design procedure for checking human-induced vibrations of footbridges and as such it is very valuable. However, the DLF equal to 0.257 used in Equation (17) is not representative of the whole frequency range of up to 5 Hz. In the 1970s, when Blanchard et al. [34] published this groundbreaking paper, the concept of higher harmonics of human-induced dynamic loading was still not developed, so it is not surprising that the main criticism of their approach stems from this fact. Namely, for bridge frequencies in the range 1.6–2.4 Hz the influence of the first harmonic depends strongly on the walking frequency (Figure 10) and should not be represented by one value which is constant for all frequencies. At higher frequencies the response could be overestimated because the first harmonic is not relevant there. Also, this design procedure is only concentrated on the fundamental mode of vibration. However, if this mode had low natural frequency (up to 1.4 Hz), it hardly can be relevant for bridge response under human-induced force. In such a situation, a mode having frequency in the normal walking frequency range becomes more important.

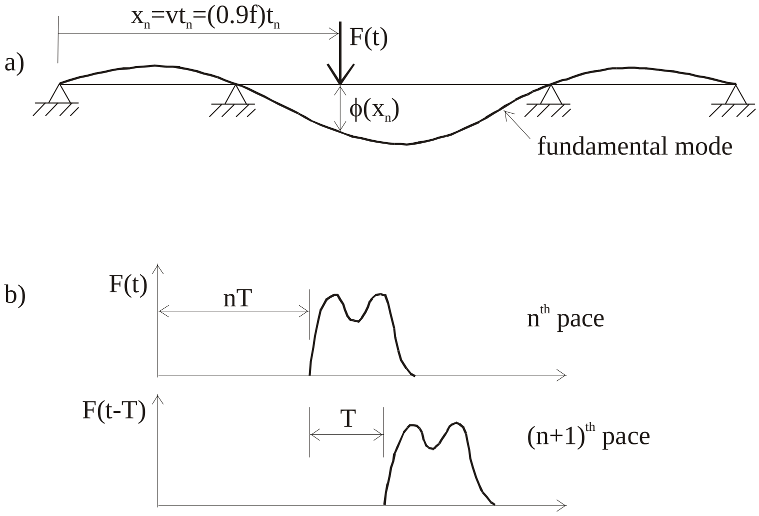

Figure 27: (a) Moving force model and (b) forcing function for a pedestrian load (after Blanchard et al. [34]).

Implementing the half-sine force model, Wheeler [39] and [40] simulated response due to a single pedestrian walking at different frequencies on 21 footbridges. He used a computer program written for that purpose. He compared maximum displacements at each frequency with those obtained experimentally on each bridge. The differences in the results are explained as a consequence of many uncertainties encountered during the dynamic modelling process. Some of them, quoted in Wheeler's article from 1980, were nonlinear cable behaviour in cable-stayed bridges, column supports modelled as pin joints, difficulties in predicting structural damping, neglecting of the mass of non-structural elements such as handrails, surfacing and so on.

A considerable improvement of Blanchard et al.'s procedure was achieved with a model which took into account that the walking (or running) force had DLFs dependent on step frequency (Figure 10), the force was moving and its duration was time limited by the length of the bridge [42]. The representative SDOF model is presented in Figure 28, where m, c and k refer to the modal mass, damping and stiffness respectively. A dynamic amplification factor Φ was introduced to account for the force moving and its limited duration, which was dependent on the structural damping and number of force cycles needed to cross the bridge. The modal peak acceleration response was then given as:

$$a = \frac{\alpha P}{m}\Phi \tag{19}$$

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where m is the modal mass, P is the pedestrian weight and α is the appropriate DLF according to Figure 10. This procedure demonstrated good agreement with an experimental study. Unfortunately, this design proposal was related only to single span footbridges. In the same article, it was proposed that the response to a jumping force could be calculated using an appropriate DLF for jumping (Figure 10) and the well-known formula for steady state response knowing that this force does not move along the bridge. Therefore, theoretically it could produce steady-state response although the practical duration of such an excitation is in question.

After this proposal, several attempts to simplify or extend it were made. For instance, Allen and Murray [171] simplified the procedure by replacing the walking force with a stationary sinusoid acting at the centre of the span which amplitude and frequency depended on the relevant harmonic. In such conditions, the steady-state response was obtained and then a unique reduction factor R was introduced to take into account the exact nature of the force, that is the fact that it is moving and is of limited duration. The non-dimensional ratio between the harmonic peak response and gravity acceleration was then given by:

$$\frac{a}{g} = \frac{R\alpha_i P}{\beta W}\cos\left(2\pi i f t\right) \tag{20}$$

where α_i is a DLF, P is the pedestrian weight, β is the damping ratio, W is the bridge total weight while R is a reduction factor which is adopted as 0.7 for footbridges. However, this constant factor could not involve all possible situations produced by different span lengths and therefore different time needed to cross the bridge. Also, constant values for DLFs for each harmonic were adopted here as the maximum values given in Figure 10 ($\alpha_1 = 0.5$, $\alpha_2 = 0.2$, $\alpha_3 = 0.1$ and $\alpha_4 = 0.05$) which can give overconservative results. After some manipulations and taking into account the acceleration limits based on ISO [27], Equation (20) was converted into a condition for minimum natural frequency of the bridge:

$$f_0 \geq 2.86 \ln \frac{K}{\beta W} \tag{21}$$

where K is a constant equal to 8 kN. The recommended damping ratio was $\beta = 0.01$.

Further, Kerr [36] converted Rainer et al.'s [42] procedure into an analytical form to avoid using the graphs for DLFs and the factor Φ. Also, he used his own expression for DLF as a function of step frequency (Figure 11) instead of the one proposed by Rainer et al.

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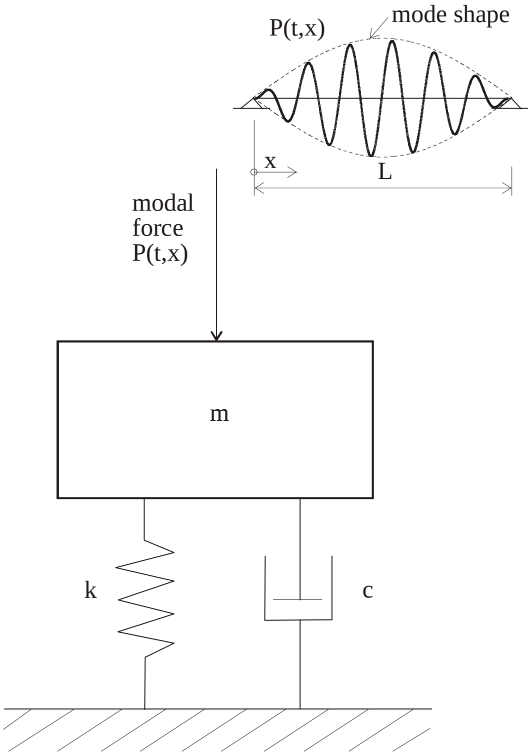

Figure 28: Pedestrian modal force model based on the design procedure by Rainer et al. [42].

Finally, Pimentel & Fernandes [73] extended the procedure proposed by Rainer et al. to footbridges with two spans introducing a factor k_a. The peak modal acceleration was given in the form:

$$a_{\max} = \omega_0^2 y_s \alpha_i \Omega_d k_a \tag{22}$$

where ω_0 is the fundamental circular bridge frequency, y_s is the static deflection due to the weight of a pedestrian, α_i is the DLF of the resonant i^{th} harmonic, Ω_d is the dynamic amplification factor and k_a is a numerically obtained factor which takes into account bridges with two spans in addition to single span structures.

Grundman et al. [72] used full theoretical expression (with transient and steady-state parts and assuming zero initial conditions) to calculate the resonant response due to a single pedestrian harmonic force. Acceleration response was given as:

$$a_{1rz} = 0.6\frac{0.4G}{M}\frac{\pi}{\delta}\left(1-e^{-n\delta}\right) \tag{23}$$

where the factor 0.4 is adopted as DLF for the first harmonic, G is the weight of a pedestrian, M is the modal mass of the structure, δ is the logarithmic decrement and n is the number of steps needed to cross the bridge. Introducing n means that a real duration of the moving force (i.e. the possibility that steady-state response is not achieved) has been taken into account. The factor 0.6 was included to involve moving of the pedestrian force i.e. the variation of the mode shape amplitude along the walking path. However, it is difficult to devise a formula representative of all footbridge span lengths and all simplifications of this kind will necessarily increase the errors in the response estimates. Grundmann et al. also proposed a similar approach for response calculation in the horizontal lateral direction but with DLF equals to one fourth of the DLF for the vertical harmonic.

Probably the biggest shortcoming of all these procedures is their limitation to girder footbridges with only a few spans, usually one. Nowadays, when new footbridges have unusual structural form, it is necessary to formulate a more general design approach based on first principles.

Young [56] made an attempt to develop a design procedure independent of footbridge type. His procedure was based on the full theoretical expression for a steady-state acceleration response a_n in a single vibration mode n (with the modal mass M, damping ratio ζ_n and natural frequency f_n) to a harmonic force with amplitude P and frequency f:

$$a_n = \mu_i \mu_j \left(\frac{f}{f_n}\right)^2 \frac{P}{M} DMF, \tag{24}$$

where μ_i and μ_j are mode shape ordinates at the walking point and response point, respectively, while DMF is the dynamic magnification factor:

$$DMF = \frac{1}{1-\left(\frac{f}{f_n}\right)^2 + i\left(2\zeta_n \frac{f}{f_n}\right)}. \tag{25}$$

at resonance, only the imaginary part of DMF remains. Young proposed a reduction factor r for this part to account for a limited duration of the pedestrian force and its moving across the structure:

$$r = 1 - e^{-2\pi\zeta N} \tag{26}$$

where N depends on the number of cycles needed for the relevant n^{th} harmonic to cross the bridge:

$$N \approx 0.55 n \frac{L}{l}. \tag{27}$$

L and l represent the span and step lengths, respectively.

In addition to controlling the vibration serviceability under single person excitation, the response of the footbridge under a stream of pedestrians, groups and a crowd of people has been also considered by some authors, as previously described in Section 5.2.

6.1.2 Frequency Domain Design Procedures

Although design procedures presented in this section are not necessarily related to footbridges, they can be used as the basis for further investigations of these structures. The idea to assess the vibration serviceability of structures by using the theory of stationary random processes appeared in the early eighties [35]. It is known that for such a process, the auto spectral density (ASD) of the response $S_y(\omega)$ can be calculated by the following relation [153]:

$$S_y(\omega) = |H(\omega)|^2 S_x(\omega) \tag{28}$$

where $H(\omega)$ is the frequency response function of the structure and $S_x(\omega)$ is the ASD of the force. The mean square value of the response $E\left[y^2\right]$ can then be calculated as an area under the spectral density curve of the response [153]:

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$$E\left[y^2\right]=\int_{-\infty}^{+\infty} S_y(\omega)d\omega \tag{29}$$

and, further, the value of the root-mean-square (RMS) or peak acceleration, on which human perception criteria are often based, can be obtained. To obtain peak accelerations crest factors are usually used.

As mentioned in Section 2.3.2, Eriksson [49] paid great attention to force modelling in the frequency range up to 6–7 Hz, which was the typical range for fundamental frequency of the low-frequency floors he investigated. He assumed that a floor structure responded predominantly in one ("weakest") mode and therefore could be modelled in the modal space as a SDOF system. Based on measurements of acceleration responses due to a single person (walking, running and jumping) and groups of 7 and 11 walking people and using the relationship between spectral densities of the force and the response of the kind given in Equation (28), Eriksson proposed a force mean square design model with constant mean square in the range of the first harmonic frequencies and frequency dependent for higher harmonics (Table 4) where F_{rms} presents the root-mean-square function of the walking force while $K_{np}\left(n_p\right)$ is a multiplication factor for a single person RMS force as a function of the number of people in the group n_p. He therefore found that group of people moving in step can be considered as almost perfectly correlated in considering its first harmonic. For that case it was proposed to multiply single force mean square by the factor of $n_p^{0.9}$ while for higher harmonics that factor was $n_p^{0.5}$ which is typical for uncorrelated people, where n_p is the number of people.

Mouring [154] and Mouring & Ellingwood [172] modelled the auto spectral density of a modal force due to crowd dynamic loading as a product of number of people and the auto spectral density of an individual force. This is equivalent to the condition that time-domain acceleration response due to a group of n uncorrelated people is $\sqrt{n}$ times higher than the acceleration due to a single person, as obtained by Matsumoto et al. [46].

Hansen & Sørensen [173] defined jumping crowd load in terms of its harmonics, each of which was based on the appropriate harmonic due to a single person. Crowd effect and lack of synchronisation between people were taken into account by a crowd reduction factor. This factor was obtained for each harmonic separately as the ratio between the magnitude of deflection spectrum for a group to the magnitude of a single person spectrum. It seems that the calibration of the crowd reduction factor was based on measurements of the jumping response only at 2 Hz.

Table 4. Design force model after Eriksson (1994).

Activity	Frequency Interval [Hz]	F_{rms} [N]	$K_{np}\left(n_p\right)$
Walking	0–2.5	220	$n_p^{0.9}$ 1)
	2.5–10.0	$180\left(f_0/f\right)$	$n_p^{0.5}$
Running	0–3.0	690	$n_p^{0.9}$ 1) 2)
	3.0–10.0	$4300\left(f_0/f\right)^2$	$n_p^{0.5}$ 2)
Jumping	0–3.5	1000	$n_p^{0.9}$
	3.5–10.0	$13000\left(f_0/f\right)^2$	$n_p^{0.5}$ 2)

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[1)] Proposed factor is applicable for well correlated group. In case of the normal traffic, the factor $n_p^{0.5}$ should be applied. [2)] Factors given inter alia in absence of sufficient data. (f_0 = 1 Hz)

Brownjohn et al. [168] paid attention to imperfections in individual human walking patterns which spread excitation energy into adjacent spectral lines in comparison with the perfectly periodic force where the whole energy is concentrated at a single harmonic frequency. The spread of energy effect was shown to be more emphasized for higher harmonics. Based on direct measurements of the vertical force time histories for three test subjects walking on a treadmill, they proposed a model which described a forcing function for the first six harmonics. Namely, the ratio between Fourier amplitudes for real and periodic forces $G_n^{'}$ was given as a function of frequency f :

$$G_n^{'}\left(\frac{f}{\overline{f}}\right) = A + Be^{-\frac{\left|\frac{f}{n\overline{f}}-1\right|^C}{D}} \tag{30}$$

where $\overline{f}$ is the fundamental frequency of the appropriate perfect periodic force, n is the order number of harmonic while A, B, C and D are constants dependent on the harmonic considered. Knowing the ASD of the force harmonic $S_{F,n}(f)$:

$$S_{F,n}(f) = \left[WG_n\left(\overline{f}_i\right)\right]^2 S_{G_n^{'}}\left(\frac{f}{\overline{f}}\right) \tag{31}$$

where W , G_n and $S_{G_n^{'}}\left(\frac{f}{\overline{f}}\right)$ are the pedestrian weight, the DLF for the appropriate harmonic of the perfectly periodic walking force and the ASD of the previously defined function $G_n^{'}$, respectively, the ASD of the response due to a single pedestrian can be obtained by applying Equation (28). On the other hand, for groups of people walking, it is suggested to evaluate the ASD of the response in terms of turbulent wind theory, as explained in Section 5.2.2. In principle, Brownjohn et al.'s model can take into account all relevant modes of the structure, including closely spaced modes.

6.1.3 Other suggestions

Attempts to use the genetic algorithm optimisation procedure in modelling the serviceability problems are given by Obata et al. [92] and Miyamori et al. [174]. Optimisation was carried out after the force and "human model" parameters, respectively. These parameters were used in calculation of the response with the aim to match it with the experimental response due to a pedestrian. However, it seems that optimisation parameters were different for different bridges which makes it difficult to generalise the model.

6.2 Design Guidelines

An early formal attempt to cope with the problem of vibrations perceptible by pedestrians on highway bridges was codified by the American Association of State Highway Officials [131]. For many years they limited the deflection due to live load to span-length ratio and the depth to span length ratio. However, Leonard [20] reported that day-to-day design practice showed that such an approach had not lead to bridges with acceptable level of vibrations. A different approach, related to composite highway bridges, was suggested by Mason & Duncan [175].

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They proposed to limit the minimum bridge natural frequency to 4 Hz and the maximum level of vertical acceleration to 0.15 m/s^2 (1.5%g).

Nowadays, there are two concepts which are present in design guidelines for footbridge vibration serviceability. The first requires a calculation of the actual dynamic response of the bridge and checking if it is within the acceptable limits for the bridge users. The second approach is based on the request to avoid footbridge natural frequencies which are in ranges coinciding with frequencies typical for human-induced dynamic excitation.

A key example of the first approach is BS 5400: Part 2 [74]. Historically, this is the first design code which dealt explicitly with the footbridge vibration serviceability issue. In its Appendix C a procedure for checking vertical vibrations due to a single pedestrian was defined for footbridges having the natural frequency of the fundamental vertical mode of vibration of up to 5 Hz. This was based on the previously described work by Blanchard et al. [34]. Many years later, based on experience with lateral vibrations of the London Millennium Bridge, an updated version of BS 5400: BD 37/01 [137] started requiring checking the vibration serviceability also in the lateral direction. For all footbridges with fundamental lateral frequencies lower than 1.5 Hz a detailed dynamic analysis is now required. However, the procedure for that is not given. Also, no improvement of the design procedure for vertical forces has been made in this updated provision. The vibration checking procedure is still based on Blanchard et al.'s [34] work despite the fact that many shortcomings of that work have been identified in the last 25 years; some of them were explained in Section 6.1 of this paper. Also, although it is understandable that the natural frequency of a footbridge with up to three spans is estimated by a simplified calculation in 1977 when Blanchard et al. published their work, it is not justifiable nowadays. Modern trends in current footbridge design practice rely on FE modelling. This and the fact that new footbridges usually have more complicated and unusual structural forms should be taken into account by proposing a methodology which is based on first principles and does not necessarily rely on simple formulae, which are very much discredited.

The Ontario Highway Bridge Design Code [141] requires a calculation of the dynamic response of a bridge due to a footfall force simulated, similar to BS 5400, by a moving sinusoidal force with amplitude 180 N and frequency equal to the fundamental frequency or 4 Hz, whichever is lower. Alternatively, if this full dynamic analysis is not done, then the simplified procedure based on Blanchard et al.'s [34] paper should be conducted. The resulting peak acceleration response should be less than an acceleration limit defined graphically. This limit acceleration is lower than in BS [74] and [137] given in Eq. (6). Therefore, this code, like BS 5400, is based on a consideration of a single pedestrian force model. To avoid the problem of coupling between horizontal and vertical modes under wind loading, the Code also requires that lateral and longitudinal frequencies of the superstructures should not be less than the smaller of 4 Hz and $1.5f$ where f is the fundamental natural frequency for vertical modes. For a footbridge with the natural vertical frequency of, say, 2 Hz this means that the lateral frequency should not be below 3 Hz. In principle, it would enable avoiding the first 2-3 harmonics of the lateral pedestrian force. However, having in mind Figure 29, this criterion can be prohibitively restrictive for footbridges with long spans. The same provisions for footbridges are given in Canadian Highway Bridge Design Code [176].

A lot of research has been conducted to check applicability of the previous two guidelines. For example, Grundamnn et al. [72] pointed to some problems with the applicability of BS 5400 to footbridges with natural frequencies around 2 Hz which are exposed to groups of pedestrians. Also, some work criticised the DLF given in the BS [74] and [137] and OHBDC [141] codes which is not representative of forces in the normal walking frequency range [58], [177], [178] and [179]. Also, these design codes were designed only for footbridges with very simple, usually beam-like configurations [73].

Eurocode 5 [121] contains some interesting information relevant to design of timber bridges. It requires the calculation of the acceleration response of a bridge due to small groups and streams of pedestrians in both the vertical and lateral directions, with the proposed frequency-independent acceleration limits of 0.7 and 0.2 m/s2 in these two directions, respectively. These limits should be checked for bridges with natural frequencies lower than 5 Hz for the vertical modes and below 2.5 Hz for the horizontal modes.

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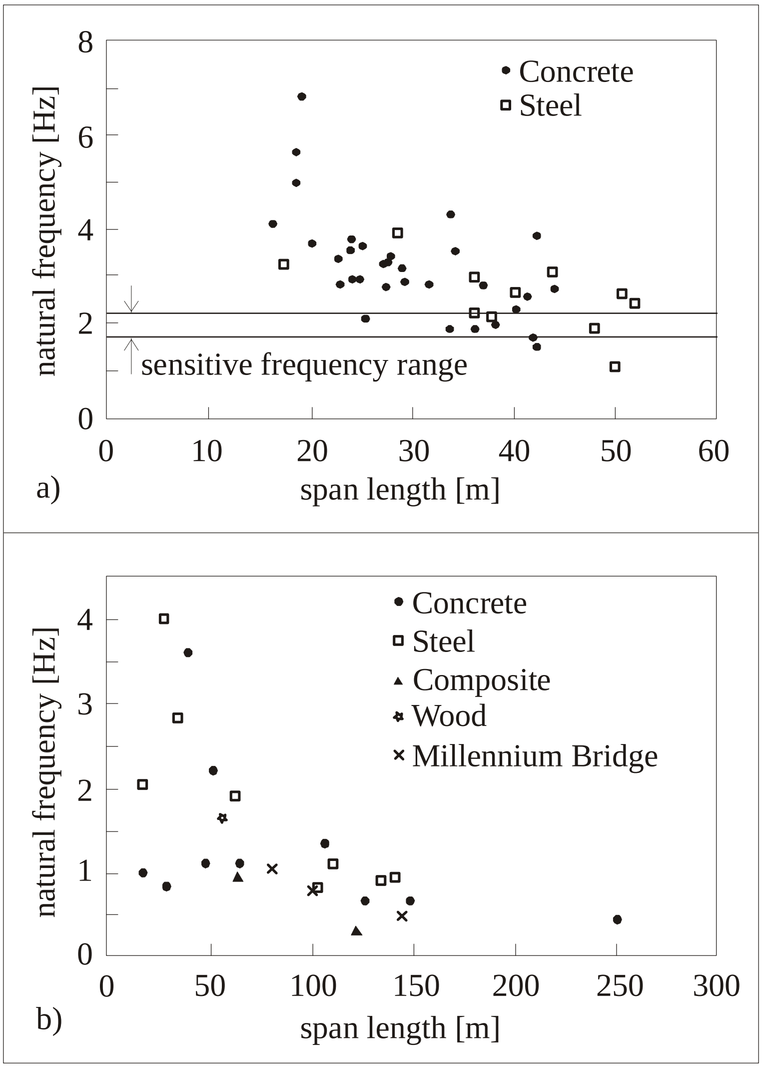

Figure 29: Dependance of the fundamental (a) vertical and (b) lateral frequency of the bridge span (after (a) Tilly et al. [12]; (b) Dallard et al. [159]).

The procedure for the calculation of the vertical acceleration response for bridges with one, two or three spans is based on obtaining $a_{vert,1}$:

$$a_{vert,1} = 165 k_a \frac{1 - e^{-2\pi n \zeta}}{M \zeta}, \tag{32}$$

which can be regarded as a single mode and a single pedestrian response. Here, k_a is a configuration factor based on Blanchard et al. [34], while n, ζ and M are the number of cycles necessary to cross the bridge with step length of 0.9 m, damping ratio and the total mass of the bridge, respectively. This formula originates from a full theoretical expression for resonant response due to a harmonic force, where the force amplitude is 165 N. This amplitude includes the DLF as well as the fact that the force moves across the bridge [114]. For the case of a footbridge which has a more general structural configuration, the acceleration response should be calculated for the force $F_{vert}(t)$ which is moving across the bridge with velocity of $0.9 f_{vert}$:

$$F_{vert}(t) = 0.28 \sin(2\pi f_{vert} t) \quad [\text{kN}], \tag{33}$$

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where f_{vert} [Hz] is the fundamental natural frequency in the vertical direction. This force amplitude of 0.28 kN is higher than in BS [74] and [137] and the Ontario design code [141], and corresponds to a DLF of 0.4 for an average pedestrian weighing 700 N. More detailed background of this formula and some possible modifications can be found in Hamm [114].

The response due to small group of pedestrians can be obtained by multiplying the calculated $a_{vert,1}$ by a factor $k_{vert,f}$ which is dependent on the footbridge fundamental natural frequency. Its maximum value of 3 is predicted for the range of common step frequencies 1.5–2.5 Hz, as given by Grundmann et al. [72] for groups of up to 10 people (Figure 26). However, if the bridge has a deck area greater than 37 m^2, then this group response should be increased further by a factor which is dependent on the area, probably to take into account the possibility of synchronisation of a crowd of pedestrians in the vertical direction. The same procedure should be followed when calculating the lateral acceleration response by replacing the quantities related to the vertical direction with the lateral direction ones. There are also differences in the force amplitudes in Equations (32) and (33) which are 40 N and 0.07 kN instead of 165 N and 0.28 kN, respectively and in the expression for the pedestrian velocity which becomes $1.8 f_{hor}$ [m/s] where f_{hor} [Hz] is the fundamental natural frequency in the lateral direction. The factor $k_{hor,f}$ has, as $k_{vert,f}$, the maximum value of 3, but in the frequency range typical for lateral force induced by pedestrians: 0.75–1.25 Hz. However, deriving this lateral response based on the procedure for the vertical one and without any experimental data could be problematic and erronerous as noted by Dallard et al. [159] and Briseghella et al. [180]. Therefore, as in previous guidelines, this proposal is also based on assumptions of the resonance condition and a single DLF value over a range of walking frequencies. However, the proposal considers a group of pedestrians which is a rather new and advanced approach in design codes.

There are few data about the applicability of this guideline. On some bridges it has successfully predicted liveliness of structures, although more detailed explanation as to the level of agreement between the calculated and measured response was not given [181]. On the other hand, an attempt to apply this procedure to the Millennium Bridge for the case of high density of people produced significantly lower response in the lateral direction than the measured one [159].

The Hong Kong Structures Design Manual for Highways and Railways [182] requires controlling the acceleration response due to a pedestrian in accordance with BS 5400. Also, it limits the lateral acceleration to 0.15 m/s^2. Moreover, it requires additionally a check of the acceleration response due to a stream of pedestrians. The assumption is that the stream is a continuous load moving with velocity 3 m/s over a simple beam. However, sufficient detail as to how to perform the load simulation was not given in the Hong Kong manual.

Swiss standard SIA 160 [183] belongs to the second group of guidelines which utilises the frequency checking and tuning approach. It requires avoiding footbridge natural frequencies in the range of the first (1.6–2.4 Hz) and the second walking harmonics (3.5–4.5 Hz) with the addition of frequencies 2.4–3.5 Hz if joggers/runners can appear on the structure. If these requirements are not fulfilled then the vibration response of the structure should be checked. Identical provisions as for the frequency ranges which should be avoided are given in the Design Code issued by Comité Euro-International du Béton [184].

The American Guide Specification [185] also proposed to avoid fundamental footbridge frequencies in the vertical direction, but below 3 Hz. However, in the case of low stiffness, damping and mass, and when running and jumping on the footbridge are possible, all frequencies below 5 Hz should be avoided to neutralise the influence of the second harmonic. However, it is not stated what the lower limits for these low dynamic properties are. If the frequency conditions are not satisfied, then the minimum natural frequency given by Allen and Murray [171] in Eq. (21) should be a target for the structure to satisfy serviceability requirements. As a possible measure to improve poor dynamic performance of bridges, installation of vibration absorbers and dampers is suggested. Finally, Yoshida et al. [163] reported that Japanese design code for footbridges requires avoiding frequencies between 1.5 and 2.3 Hz for the vertical modes.

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Regarding the second group of guidelines, Pimentel et al. [179] found that the frequency tuning approach can be restrictive because there are footbridges which are serviceable although they have frequencies in the range recommended for avoidance.

A list of existing codes and their division between those which specify acceleration limits and those with frequency limits was given by Schlaich [7]. Unfortunately, the author omitted their exact references which leaves uncertainties as to the exact code titles and editions.

As has been shown in this section, some codes recommend avoiding the resonant frequency range typical for the first or second force harmonic, the others give a more or less complex design procedure to calculate the response of the bridge and check if it is acceptable. None of the codes consider all aspects of vibrations induced by humans, i.e. vibrations in both horizontal lateral and vertical direction and vibrations not only due to a single person but also due to groups and crowds. Therefore, there is a clear need to revise and update design guidelines featuring obsolete or missing information.

7 Measures against Excessive Vibrations of Footbridges

With the occurrence of the first problems related to the liveliness of footbridges, some early design recommendations, such as the one by Walley [123], proposed that the fundamental vertical natural frequency of a structure below 2.7 Hz should be avoided. It is interesting to note that this corresponds to the upper limit of the range of the first walking harmonic, although at that time little was known about the actual nature of the walking force as no widely reported measurements of it existed. Leonard [20], on the other hand, claimed that there was no need to avoid any frequency range if an appropriate damping and stiffness had been provided. For example, some footbridges are serviceable although their natural frequencies are inside the problematic ranges [179] or the damping ratio is as low as 0.4% [186]. However, with modern trends towards slenderness in footbridge design, it happens that footbridges more and more frequently do not perform well in service as far as their vibration behaviour is concerned. A list of examples of such problematic footbridges was compiled by Pimentel [58].

There are several measures which can be used to predict, prevent and resolve the problems of liveliness in footbridges [14] and [16]:

1. ***Frequency Tuning.*** As previously mentioned, this measure means avoiding the critical frequency ranges for the fundamental modes. For vertical mode these are the frequencies of the first (1.6–2.4 Hz) and, for bridges with low damping, the second walking harmonic (3.5–4.5 Hz). Although Bachmann & Amman [14] proposed the same provision for the lateral modes (namely, 0.8–1.2 Hz for the first and possibly 1.6–2.4 Hz for the second harmonic), it should be added that lower frequencies could be excited too, according to observations made on the Millennium Bridge, London where the frequency of the lowest mode excited was only 0.5 Hz [19]. For the longitudinal direction, the first sub-harmonic and the first harmonic, with frequencies 0.8–1.2 Hz and 1.6–2.4 Hz, respectively, should be avoided. Excessive vibrations in this direction are very rare, but one case was reported by Bachmann [21]. It should be stressed that the designer can influence frequencies of the footbridge by choosing an appropriate layout of the structure [58] and by studying different options for distributing its stiffness and mass. Figure 29 gives a rough guidance of the possible fundamental frequencies as a function of the bridge span for vertical [12] and lateral modes [159].

   Structural frequency can, for example, be changed by stiffening the structure (installing stiffer handrails or adding tie-down cables); Tilly et al. [12] found that footbridges with stiffness in the middle of the main span which is lower than 8 kN/mm are likely to be prone to vibrations in the vertical direction.

2. ***Detailed Vibration Response Assessment.*** This is a measure which is the basis of many contemporary design procedures (Section 6). However, it is underpinned by many uncertain modelling assumptions and its reliability is often questionable.

3. ***Measures to Reduce Vibration Response.*** These measures are:

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- Restricting the use of the bridge (for example, ban marching over the bridge);
- Increasing the damping (e.g. by adding extra damping devices such as viscous dampers or tuned mass dampers).

It can be added here that warning and/or educating people to expect vibrations can help them to tolerate higher vibration levels than they would without an explanation that their safety is not in question. This is not surprising as safety is the main concern of the bridge users in case of excessive vibrations [20].

The remainder of this section will consider the use of damping devices that are often used in practice.

7.1 Tuned Mass Dampers: Theory

Tuned mass dampers (TMDs) are spring-mass or spring-mass-damper systems which can be added to a structure to reduce its vibration response. Contrary to active vibration suppression systems, which directly monitor the structural response and accordingly adjust their dynamic behaviour to reduce response over a wide frequency range, tuned mass dampers are passive devices which are effective only in a narrow frequency range [187].

Ormondroyd and Den Hartog [188] theoretically formulated principles of TMDs in 1928. They found that adding a spring–mass system to an undamped SDOF structure, which was excited by a resonant sinusoidal force, would form a new 2DOF system in which the structural response would be completely eliminated in the case when the natural frequency of the absorber was the same as the one of the primary system. Adding damping to the absorber's spring–mass system made it efficient not only at a single frequency but also over a frequency range. The absorber damping was more effective in reducing the response of the main SDOF system than the damping already present in the main system. This was the reason to neglect structural damping in many numerical simulations of TMDs reported in the literature. Although Ormondroyd and Den Hartog [188] concluded that there was an absorber damping value (the optimum damping) which would give the maximum attenuation of the structural response, they could not find it analytically. This paper was and still is an excellent base for further research in this area.

In his textbook, which had five editions between 1934 and 1985, Den Hartog found the absorber frequency and damping which will minimise the steady-state displacement response of a structure under sinusoidal force, both as functions of a chosen ratio μ of the absorber and SDOF system masses. These optimum (tuning) parameters are [189]:

$$f = \frac{1}{1+\mu} \tag{34}$$

$$\left(\frac{c}{c_{cr}}\right)^2 = \frac{3\mu}{8(1+\mu)^3} \tag{35}$$

where f is the ratio of the absorber and structural natural frequencies, while $\frac{c}{c_{cr}}$ is the absorber damping ratio.

Footbridges with well-separated modes which have vibration serviceability problem respond mainly in one mode of vibration which are lightly damped. This means that, by using appropriate modal mass and stiffness, the excited mode can be represented as a SDOF system, and the optimum TMD parameters can be calculated using Equations (34) and (35). In that case the parameter μ becomes ratio of the absorber mass and modal (generalised) mass of the SDOF system. For a simple beam structure, the assumption that the relevant pedestrian harmonic does not move produces only small differences in the tuning parameters in comparison with a

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moving force. The effectiveness of the absorber is nevertheless lesser for the moving force case [190].

Generally, an optimisation of absorber parameters (f and $\frac{c}{c_{cr}}$) could be done for different types of excitation and considering different response parameters. A lot of work has been devoted to this issue. For example, Warburton [191] analysed an undamped SDOF system under harmonic excitation but optimised response against displacement, velocity and acceleration of the main mass, and also against the force transmitted to the base. He also did optimisation analysis for white noise excitation and harmonic base acceleration. Rana & Soong [192] analysed numerically the optimisation for a damped system due to harmonic main mass excitation and harmonic base excitation. They also pointed out the possibility to control the response in more than one structural mode by installation one TMD for each mode considered. Several TMDs can also be used for controlling SDOF system response due to wide-band random excitation [192] and [193].

In the case of footbridges, a single TMD for a dominant mode is usually considered. It is most effective to put the TMD at the point with maximum structural response, that is at the antinode [190].

7.2 Tuned Mass Dampers: Practice

Matsumoto et al. [45] reported one of the first cases of installation of a spring–mass absorber to suppress excessive vibration of a footbridge. An explanation of that exercise, as well as of an installation of two additional absorbers in another pedestrian bridge was given by Matsumoto et al. [46] in 1978. Also, Brown [194] reported briefly on another installation of a TMD on a bridge.

Chasteau [120] described the successful installation of two TMDs on a three span footbridge susceptible to wind dynamic excitation. The two TMDs were a new technology at that time which was probably the reason for clients to ask for these devices to be maintenance-free. Because of that, the author decided to use air instead of fluid damping, although it was hard to fabricate that sort of a solution. Finally, the absorbers increased the bridge damping by about five times, but their mass ratios μ were 0.043 and 0.065 which is quite high. As reported by Eyre and Cullington [70], this state of affairs probably discouraged engineers at that time to use TMDs more frequently to solve the liveliness problem in footbridges.

Jones and Pretlove [190] investigated effectiveness of a TMD on a 30 m long beam. It was demonstrated that a TMD of 70 kg having the mass ratio of only 0.006, can be quite effective. However, this was expected, having in mind very low damping ratio in the structure of only 0.13%. Jones and Pretlove also showed that Den Hartog's formulae for optimum TMD design given in Eqs. (34) and (35) can be used if the damping of the structure is below 1% and that the effectiveness of the TMD can be reduced because of the internal friction in the absorber. Fig. 30 shows the difference between theoretical and measured displacement amplitude of the beam for a range of harmonic excitation frequencies.

Bachmann and Weber [195] wrote an excellent and comprehensive article about the design and effectiveness of vibration absorbers. They showed that Eqs. (34) and (35) can be used for all lightly damped structures, especially if the damping is below 2% (Fig. 31). It was also demonstrated that the effectiveness of the absorber was much more sensitive to the error in the tuning of the TMD frequency than in the tuning of its damping. The procedure for the absorber design was outlined with a particular emphasis on the choice of the appropriate mass ratio. Namely, an absorber with, for example, mass ratio of 0.02 seems to be a solution which both successfully attenuates the structural response and also keeps the absorber mass movement within reasonable limits (Fig. 32).

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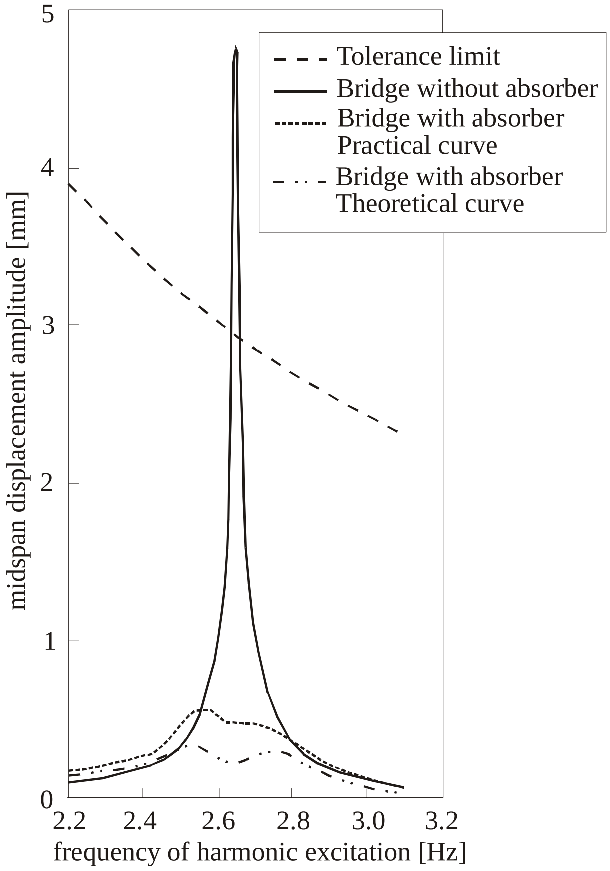

Figure 30: Influence of the absorber friction on its effectiveness (after Jones et al. [24]).

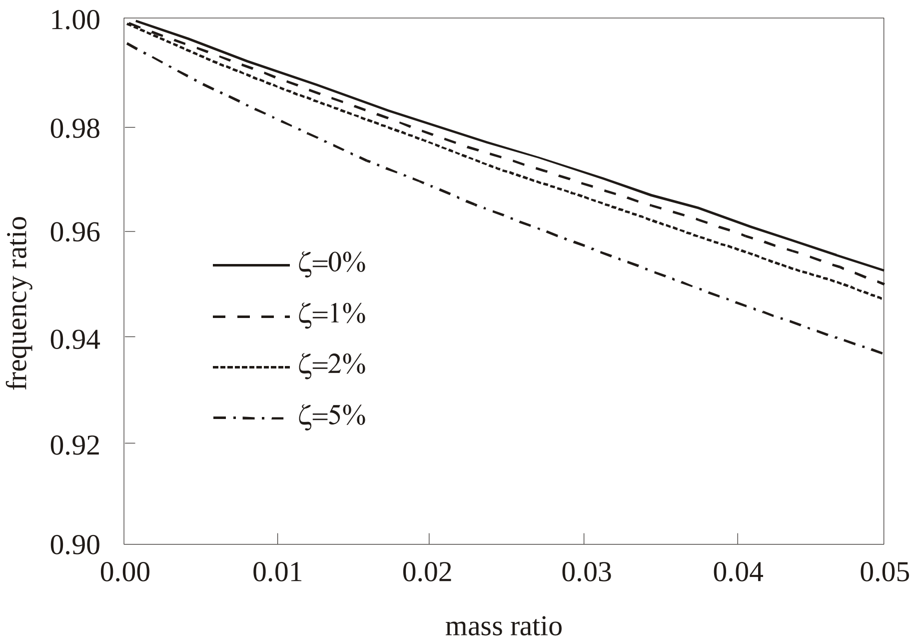

Figure 31: Frequency tuning as a function of the mass ratio and the bridge damping (after Bachmann and Weber [193]).

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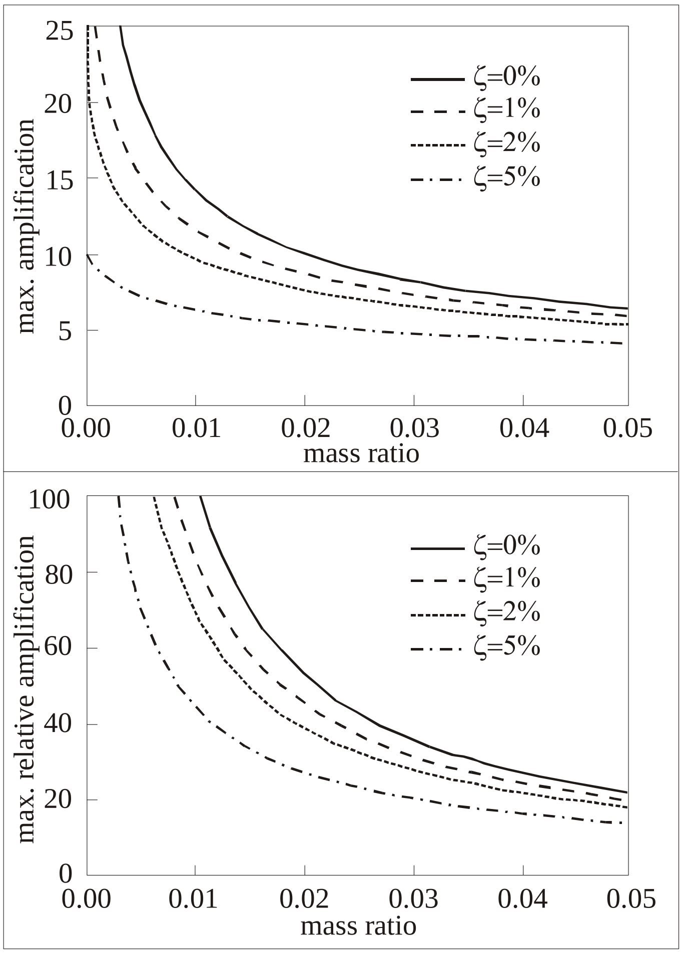

Figure 32: Dynamic amplification of the structural and relative structure-TMD response as a function of the mass ratio (after Bachmann and Weber [193]).

With regard to the usage of TMDs, it is now common practice to design them upfront, but actually manufacture and install them after the footbridge is constructed if problems of excessive vibrations are noted. Also, it is interesting that 26 pairs of TMDs were installed on the Millennium Bridge, London to prevent possible excessive movement in the vertical direction which was not noticed during the conducted tests [19]. Some examples of TMDs installed in footbridges can also be found in articles presented at Footbridge 2002 conference in Paris [196], [197] and [198].

TMDs can suppress either vertical or horizontal movement. TMDs working in the vertical direction usually consist of a mass, helical springs and viscous-fluid damper while the horizontal ones are typically constructed as pendulums [96]. Vertical TMDs are more common and they are usually attached to main girders, beneath the deck or above the deck in the plane of the handrails. They are usually low-cost and easy to maintain. However, with time the effectiveness of absorbers can be reduced because of disappearance of viscous oil [70], changing of dynamic properties of the structure [195] and changed nature of the live pedestrian load for which the footbridge was originally designed. In all these cases de-tuning of the TMDs occurs. The last two reasons initiated a quest in the research community for a better solution for vibration suppression. One of the options is replacing the passive with a semi-active damper in the TMDs. Without getting into details of this relatively new technology here, reference will be made to analyses conducted by Occhiuzzi et al. [199] and Seiler et al. [200]. They described semi active devices which, owing to their rheological properties, can adjust their stiffness and damping according to changes in the main structure. However, these devices require additional

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equipment for measurements and control and it may be some time before they become economically viable.

Also, because of the variability of the human pacing rate, the multiple tuned mass dampers (MTMD) can be a more effective solution than a single TMD. Poovarodom et al. [201], [202] and [203] reported the installation of six TMDs to reduce the vertical vibration of a footbridge. Although their weight was only 1% of the structural weight, they decreased acceleration by about three times.

7.3 Other Damping Devices

Although TMDs are most popular, other devices for suppressing bridge vibration response can also be used. For example, a very simple friction device composed of two springs was installed in the handrails of a footbridge [192]. Furthermore, 37 fluid viscous dampers were installed on the Millennium Bridge in London mostly to suppress excessive vibrations in the lateral direction. As a result, the damping ratio increased from 0.5% to 20% and near-resonant accelerations were reduced by about 40 times [13] and [19]. Also, tuned liquid sloshing dampers can be used for the same purpose. In this system liquid is contained in a shallow tank which is placed on the structure. The required height of the liquid is established by nonlinear shallow-water wave theory. The motion and viscosity of the liquid generate the required damping. This tuned liquid damper (TLD) is cost-effective, easy to install and maintain and requires a very low vibration level to which it will respond, which is sometimes a problem with standard mechanical TMDs [204]. Nakamura and Fujino [156] reported that 600 plastic tanks with 34 mm of water were employed to suppress lateral vibrations of a cable-stayed footbridge. The TLDs were placed inside the box girder. The mass ratio was only 0.007. It was shown that these TLDs were very effective at the time of installation. However, it was also reported that after 10 years without any maintenance, their effectiveness reduced considerably, mainly because the water evaporated. On the same bridge, secondary wires were used to connect stay cables and decrease their in-plane vertical oscillation which in fact is an example of stiffening rather than dampening of the structure. As a way forward, multiple TLDs with different level of liquids can be used to suppress motion over a range of frequencies [205].

8 Summary

This paper has reviewed about 200 references dealing with different aspects of vibration serviceability of footbridges under human-induced load. It is found that the whole issue is very complex and under-researched. However, rationalisation of the problem into its three key aspects: the vibration source, path and receiver [27] is adopted nowadays when dealing with vibration performance of footbridges.

Among different types of human-induced loads on footbridges, walking force due to a single pedestrian was established in the past as the most important load type because of its most frequent occurrence. Also, almost all existing force models for this type of load (defined either in the time or frequency domain) are developed from the assumption of perfect periodicity of the force and are based on force measurements conducted on rigid (i.e. high frequency) surfaces. However, footbridges which exhibit vibration serviceability problems are low-frequency flexible structures with natural frequencies within the normal walking frequency range. In such a situation, walking at a near resonant frequency is expected to generate the highest level of response as considered in the published literature. However, the walking force is not perfectly periodic [49], [64] and [168] and it could be attenuated due to interaction between the pedestrian and the structure [35], [52] and [53]. These two facts deserve more attention in future force modelling.

Apart from a single person walking, a group of pedestrians walking at the same speed to maintain the group consistency are a very frequent load type on footbridges in urban areas. This type of dynamic load was not researched much in the past, especially in relation to pedestrian bridges. Wheeler [40] and Grundmann et al. [72] were among a handful of

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researchers who investigated this issue. They found that, under this type of load, footbridges with a natural frequency of around 2 Hz are prone to experience vibrations at a higher level than those induced by a single pedestrian because of synchronisation of walking steps between people in the group. However, there is no group force model which is generally accepted. The fact that Eurocode 5 [121] recently tried to include this type of load as a compulsory consideration demonstrates a need to codify it more widely.

As this literature survey found, the problem of excessive lateral swaying of the Millennium Bridge in London in June 2000 triggered a lot of urgently needed research into crowd loads on footbridges. Attention was paid to forces induced not only in the lateral but also in the vertical direction. It was found that some degree of synchronisation between people within the crowd exists as a result of not only a limited space available when walking in a crowd but also of pacing adjusted to the bridge movement. Qualitative observations revealed that the degree of synchronisation is dependent on several factors: the natural frequency of the bridge excited by crowd walking, amplitude of the footbridge response, number of people involved, density and velocity of people and so on. However, more research is needed to quantify the influence of all these parameters on the level of synchronisation.

This review also found that forces induced by joggers, runners and vandals have not been researched much in the past. However, there is an increasing awareness that the application of vandal loading in particular to very slender light structures with low damping can generate a significant response of the structure and should be treated adequately. It is found that jumping, bouncing and horizontal body swaying are usually considered as possible vandal loads. However, contradictory proposals about the modelling of this type of loads exist. These range from the one that vandals in small groups can be perfectly synchronised [21] to the one that vandals can produce only slightly higher response than a single person performing the same activity [40] and [42]. Therefore, a clarification of the exact definition of the vandal loading, regarding its duration, type of load and number of people involved, as well as its force modelling is a task for further investigation.

For a reliable estimate of vibration serviceability performance of footbridges, an appropriate modelling of its dynamic properties (mass, stiffness and damping) is very important. This review showed that, using finite element packages, mass and stiffness can be modelled most successfully using previous experiences when modelling similar structures. In that way, good estimates of natural frequencies and mode shapes can be obtained. However, the only reliable way to determine structural damping is to conduct the testing of the full-scale structure after it is built.

As for evaluation of human-induced vibrations on footbridges, i.e. their acceptability to human receiver, it is accepted that in the case of normal footbridges, the vibration level should be evaluated for a walking and not standing person. Issues such as the transient nature of footfall excitation, limited time of exposure to vibrations and the fact that the receiver is not stationary but is moving were identified as important ones. Leonard [20] and Smith [134] investigated the acceptability of vertical vibrations to walking test subjects, having in mind the mentioned issues. As a result, they constructed scales of acceptable vibrations as function of their dominant frequency. The average value of these results has been adopted as a design rule in BS 5400 [74] and it is widely used in design practice. There is no similar widely accepted scale related to acceptability of vibrations in the lateral direction, although some recommendations related to perception within a crowd have recently been published. Also, research into differences in human acceptance of the vibrations when walking alone, in small groups or a large crowd is very scarce.

Finally, it can be said that the most advanced design guidelines, such as BD 37/01 [137] and Canadian Highway Bridge Design Code [176], which served as the basis for most of other guidelines, are founded on research data collected in the 1970s. As a consequence, they still imply some parameters which are nowadays proven as inadequate (such as a constant DLF regardless of the pacing frequency and force harmonic considered). Also, although some formal national guidelines require consideration of lateral forces induced by pedestrians, exact procedures as to how to consider them are usually not given or are proven to be inadequate. Based on these facts, the existing guidelines should be used carefully, with plenty of lateral

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thinking and along with some recently published research which could be relevant for a design case considered.

9 Acknowledgements

The authors would like to thank Drs Regina Sachse, Roberto Pimentel and Michael Schlaich as well as Douglas Taylor for providing some information used in this review. Also, the authors are grateful for the financial support which came from the Universities UK Overseas Research Student scheme, the Royal Academy of Engineering and the Engineering and Physical Sciences Research Council.

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This paper has been published under the following reference:
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This paper has been published under the following reference:
Živanović, S., Pavić, A. and Reynolds, P. (2005) Vibration serviceability of footbridges under human-induced excitation: a literature review. *Journal of Sound and Vibration*, Vol. 279, No. 1-2, pp. 1-74. **(doi:10.1016/j.jsv.2004.01.019)**

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