Pipe Smoothing Genetic Algorithm for Least Cost Water Distribution Network Design

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ABSTRACT
This paper describes the development of a Pipe Smoothing Genetic Algorithm (PSGA) and its application to the problem of least cost water distribution network design. Genetic algorithms have been used widely for the optimisation of both theoretical and real-world non-linear optimisation problems, including water system design and maintenance problems. In this work we propose a pipe smoothing based approach to the creation and mutation of chromosomes which utilises engineering expertise with the view to increasing the performance of the algorithm compared to a standard genetic algorithm. Both PSGA and the standard genetic algorithm were tested on benchmark water distribution networks from the literature. In all cases PSGA achieves higher optimality in fewer solution evaluations than the standard genetic algorithm.

Categories and Subject Descriptors
G.1.6 [**Numerical Analysis**]: Optimization – *Constrained Optimization.*

Keywords
Water Distribution, Genetic Algorithm, Heuristic, Optimisation

1. INTRODUCTION
Evolutionary algorithms (EAs) are widely used for the optimisation of both theoretical and real-world problems. These problems tend to be highly complex and incorporate one or more constraints that limit the feasible space to be searched. One such problem is that of optimising a water distribution network where the task is to determine the optimally least-cost network design that still meets the requirements of the network (typically the provision of the required pressure at each of the points of demand). EAs have been shown to be excellent tools for optimising such networks, but many formulations fail to take into account engineering expertise. As such, the solutions they propose can be excellent from an objective function perspective, but are not able to be implemented in the real-world without considerable modification.

In this work, we propose the use of a heuristic based approach for the initialization and mutation of chromosomes based on human engineering expertise and demonstrate this method on some water distribution network design problems. Although the heuristic used is specific to the problem, the method of constraining the solutions could be applied to other network problems in the literature including sewer networks, communications networks etc. The heuristic-based 'pipe smoothing' approach is shown to perform better than a standard evolutionary algorithm on all water distribution network design problems tested, both in terms of outright performance and engineering feasibility.

1.1 Water Distribution Network Design Problem
Water distribution network (WDN) design is a complex non-linear optimisation problem, commonly involving a large number of different network components and hydraulic constraints. Due to the inherent complexity of WDN design, a simplified formulation of the problem is commonly employed when applied to optimisation techniques. This method is commonly comprised of the allocation of a diameter to each pipe in a given network layout, with the objective of minimising cost whilst satisfying pressure constraints at the nodes [1]. In this simplified version, design considerations such as water quality and network reliability are not included in the formulation of the problem. This method provides the designer with a base from which to solve the overall problem and allows the comparison of new optimisation techniques with the large amount of literature that employs this technique of problem formulation.

The optimal design of a water distribution network is presented here using the following mathematical statement. The objective function is defined as the total cost of the network with regard to pipe length and diameter

$$f(D_1, \ldots, D_n) = \sum_{i=1}^{N} c(D_i, L_i)$$

where $c(D_i, L_i)$ = cost of pipe i with diameter D_i and length L_i with N = number of pipes in the network. This function is to be minimised whilst satisfying the following constraints. For each junction (excluding the source) the following continuity constraint has to be satisfied

$$\sum Q_{in} - \sum Q_{out} = Q_e$$

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where $Q_{in} =$ inflow to the junction, $Q_{out} =$ outflow from the junction and $Q_e =$ external flow or junction demand which in this case is always positive. The pressure drop due to friction or head loss h_f for a specific pipe i is calculated using the following equation

$$h_f = \omega \frac{L_i}{C_i^a D_i^b} Q_i^a$$

where C_i = Hazen-Williams roughness coefficient, Q_i = flow and a, b, and ω are parameters of the equations.

The minimum head constraint for each junction in the network is as follows

$$H_j \geq H_j^{min}; j = 1, \ldots, M$$

where H_j = hydraulic head (water pressure) at junction j, $H_j^{min} =$ minimum head requirement at junction j and $M =$ total number of junctions present in the network.

In the case of this formulation of the WDN design problem the optimisation is exclusively concerned with the selection of pipe diameters. Each individual problem has a set of available pipe diameters which can be selected for each decision pipe in the network. These decisions are encoded as a binary bit sub-string with a length dictated by the number of pipe sizes available. The substrings are then concatenated to form the chromosome to represent the entire solution.

1.2 Previous Work on WDN Optimisation

The optimal design of water distribution networks is considered a NP-hard problem [2] and has been solved with a number of approaches, such as classic methods that include linear and dynamic programming [3][4][5][6][7] and various heuristic algorithms. Due to the discrete nature of the decision space and the advent of effective hydraulic solvers, the application of global stochastic optimisation algorithms has been proven to be a good approach to the WDN design problem. These approaches, although effective can induce a large number of hydraulic evaluations which in the case of large, real world WDNs can become extremely computationally expensive. Over the last two decades a considerable amount of research has been applied to the problem of WDN design especially in the field of EAs such as Genetic Algorithms (GAs) [8][9][10][11][12], Simulated Annealing[13], Shuffled Complex Evolution [14], Ant Colony Optimisation [15] and Harmony Search [16]. These techniques have proven to be effective on a number of benchmark WDN design problems.

1.3 Constraint Handling In EAs

In their basic form, EAs are unconstrained optimisation procedures. However, many problems have constraints imposed upon them especially in real-world optimisation problems. A common approach to dealing with constrained optimisation problems is to incorporate the constraints into the fitness function of the EA by adding a penalty function to the fitness function, where the value obtained from the penalty function represents the solution's distance from feasibility. A frequently used approach is the static penalty [17], where the penalty factors remain constant throughout the evolutionary process. Another approach is the use of a dynamic penalty where the penalty function is varied over time, commonly tightening the constraints as the EA's population develops. The notion of allowing an EA to explore the search space unimpeded before increasing the focus of the search and therefore potentially improving the scope of the search has lead some researchers to argue that dynamic penalties perform better than a static penalty approach. However, it has been found that deriving an effective dynamic penalty function is as difficult to achieve as producing good penalty factors for static functions [18].

Another approach when handling the constraints of a problem is to employ a repair algorithm. The repair algorithm has proven a popular choice for many combinatorial optimisation problems as it is often relatively easy to 'repair' an infeasible solution through the iterative modification of individual decision variables. When a solution can be transformed from infeasible to feasible at a low computational cost, repair algorithms have proven to be effective. However, it is not always possible to repair an infeasible solution at an acceptable computational cost and in some cases the algorithm can harm the evolutionary process by introducing a strong bias in the search [19].

A further method is to use an indirect representation where the genes do not code for variables in the problem directly, but via a heuristic that determines the phenotype given the genotype developed by the algorithm. These approaches have been shown to work well in timetabling problems [20] but the relationship between the genotype and phenotype is more complex leading to a more multimodal fitness landscape.

1.4 Pipe Smoothing Approach

This paper describes the development of a Pipe Smoothing Genetic Algorithm (PSGA) and its application to the least cost WDN design problem. This method actively promotes engineering feasibility by directly influencing chromosome construction and mutation. PSGA is based upon a standard genetic algorithm and incorporates a modified population initialiser and mutation operator which directly targets elements of a network with the aim to increase network smoothness (in terms of progression from one diameter to the next) using network element awareness and an elementary heuristic. Experiments are conducted to compare a standard genetic algorithm and PSGA for a number of benchmark WDN design problems. The results show that the PSGA approach exhibits improved performance over the standard GA, especially for parallel expansion WDN design problems.

2. PIPE SMOOTHING GENETIC ALGORITHM

The Pipe Smoothing Genetic Algorithm (PSGA) is based around the principle that in a gravity fed WDN the diameter of any pipe is never greater than the sum of the diameter(s) of the directly upstream pipes. Networks that adhere to this rule can be seen to 'smoothly' transition from large to small diameters from source to the extremities of the network. This rule is routinely and implicitly applied by engineers when designing such networks as it makes little sense to follow a smaller diameter pipe with a larger one in the majority of circumstances. The larger pipe will cost more to install and will not add to the hydraulic capability of the system as it will be constrained by the smaller diameter pipe upstream. One further negative aspect of this arrangement is that velocities will be lower in the larger pipe and high water age can become an issue. A standard GA of course will mutate some of these inconsistent pipe selections from the final solution as they have a corresponding improvement in the cost function and no hydraulic penalty. However extensive experimentation has shown that even well-optimised solutions after hundreds of thousands of generations of a standard EA still contain significant numbers of incorrectly sized pipes in larger networks.

PSGA applies the rule described above directly to the genotype without evaluating the effect this process has on the phenotype. The heuristic employed by PSGA is developed from the network topology of a specific problem and remains constant throughout the evolutionary process. The heuristic is applied to a solution at initialization and through the mutation operator; where the probability of the heuristic being applied is defined by a preset algorithm parameter. It is the aim of the heuristic to guide the algorithm's search to the engineering feasible solution space to locate smoother WDN designs whilst maintaining the performance of a standard genetic algorithm. The PSGA mutation operator does not perform any additional partial or full fitness evaluations, except a single hydraulic simulation at initialisation to determine flow directions. This was an important consideration when developing PSGA as additional fitness evaluations would require further hydraulic evaluations, increasing algorithm run time.

PSGA is essentially a standard GA (SGA) which incorporates some additional features; these include a pipe smoothing initialiser heuristic based mutation operator. The standard GA used was a steady-state GA with tournament selection with tournament size t and single-point crossover with probability c. A Gray-coded binary string comprising of N sub-strings was employed where each sub-string represents the diameter of each pipe in the WDN. Mutation was conducted as a random bitwise mutation with probability m.

2.1 Pipe Smoothing Initialiser

The initial population of solutions is constructed by the pipe smoothing initialiser which applies a basic rule where the diameter of any pipe is never greater than the sum of the diameters of the pipes directly upstream. The operator first sets the diameter of all pipes directly connected to a reservoir to the maximum allowable diameter for the specific problem. The algorithm then selects each remaining downstream pipe in turn to allocate a diameter. The operator achieves this by calculating the maximum allowable diameter for the given pipe and randomly selecting a pipe diameter so as not to violate this constraint. Although the diameter selection is random, a skewed roulette wheel approach is employed to prioritise the selection of larger diameters. This results in larger allowable diameters having a greater probability of being chosen from the available list. It is later shown that when this operator is used to generate all chromosomes in the initial population this has a detrimental effect on the subsequent search of the genetic algorithm due to lack of diversity in the population. Therefore a mechanism was introduced to allow the probability of the pipe smoothing initialiser chromosome application to be varied.

2.2 Pipe Smoothing Mutation Operator

The pipe smoothing mutation operator randomly selects a pipe to be mutated. The sum of all the diameters of the directly upstream pipes is set as the maximum allowable diameter the current pipe can be. Much the same as the pipe smoothing initialiser, this operator employs a skewed roulette wheel approach to the random selection of the pipe diameter. This is achieved be weighting the larger pipe diameters that fall within the maximum allowable size so that the larger the diameter, the higher the probability there is of selection. Upon selection the pipe being mutated is changed to the selected diameter.

To function correctly both the pipe smoothing initialiser and mutation operator require each pipe in the network to be 'aware' of the pipes directly up and down stream of their location. When changes are made to a WDN there is a possibility that flow direction could change in some pipes hence swapping up & down stream pipes relative to the pipe in question. The flow direction is logged at each hydraulic evaluation of the network, therefore to preserve this hydraulic data the pipe smoothing mutation operator precedes the crossover operator. This is illustrated in figure 1.

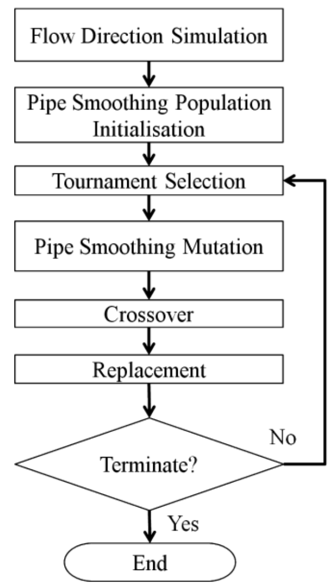

Figure 1. Flow diagram illustrating the structure of PSGA

3. COMPUTATIONAL RESULTS

PSGA was coded in C++ and run on an Intel Core i7 3.07GHz PC. The test problems used to evaluate the algorithm including a number of benchmark networks from the literature. The following test cases can be found at *emps.exeter.ac.uk/engineering/research/cws/downloads/benchmarks.* In all test cases both PSGA and SGA are run using identical common parameters.

3.1 Algorithm Development - Hanoi Problem

PSGA was applied to the Hanoi problem; a single reservoir, gravity fed water distribution network consisting of 32 junctions and 34 pipes organised in 3 loops. The Hanoi problem was used in this case to explore the effectiveness of the pipe smoothing operators when the probability of application to a standard GA was varied. The current best known solution for this specific benchmark is $6.081 million, achieved with a GA variant[21].

3.1.1 Probability of Pipe Smoothing Mutation

The probability that the pipe smoothing mutation operator was employed was varied throughout a number of experiments. When the pipe smoothing mutation operator was not employed, the standard bitwise mutation operator was used instead. The pipe smoothing initialisation operator was not employed for this first set of runs.

The base algorithm used in the following runs was built on a simple single objective steady state genetic algorithm with a mutation rate of 0.05 and tournament size of 0.05N where N is the population size which in this case is 100. The probability of pipe smoothing mutation was varied between 0% and 100% at 25% intervals. For each parameter set the algorithm was run a total of 20 times for 10,000 iterations (20,000 fitness evaluations) with different initial random seeds on the Hanoi problem with a penalty factor of 1,000,000 $/m head deficit. Below are the average results from these experiments.

Table 1. Pipe Smoothing Rate Results for PSGA on Hanoi

Probability of Pipe Smoothing Mutation	Mean Best Feasible Solution Cost ($)	Mean Best Feasible Solution Cost Standard Deviation ($)
0%	6,344,188	132,334
25%	6,308,339	111,084
50%	**6,297,480**	113,869
75%	6,303,149	**102,811**
100%	6,380,882	141,364

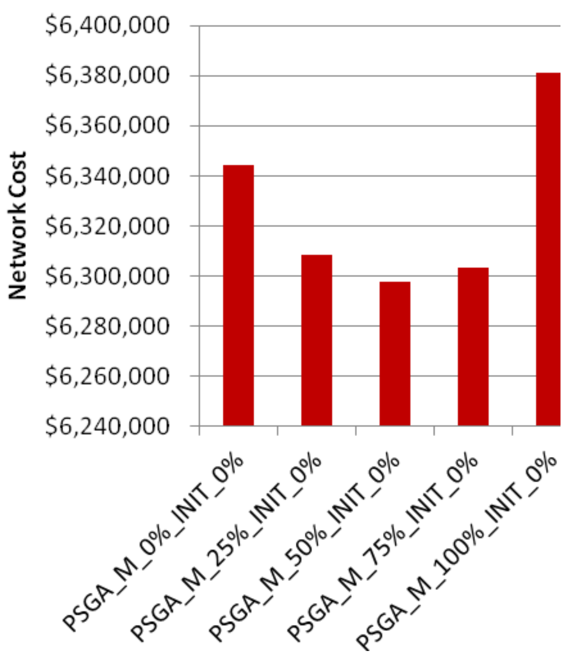

Figure 2. Mean Best Feasible Solution Cost

The preceding results show that PSGA appears to perform best when pipe smoothing mutation and standard bitwise mutation are applied in equal proportions. The results do however suggest that when the pipe smoothing mutation operator is applied continually the algorithm prematurely converges on a suboptimal solution compared to that of the standard genetic algorithm. The following figure (Figure 3) show the comparison between the standard genetic algorithm (PSGA_M_0%) and PSGA with 50% probability of pipe smoothing mutation. Note that this figure shows the average feasible solution (i.e. a solution that meets all hydraulic constraints) cost for all 20 runs.

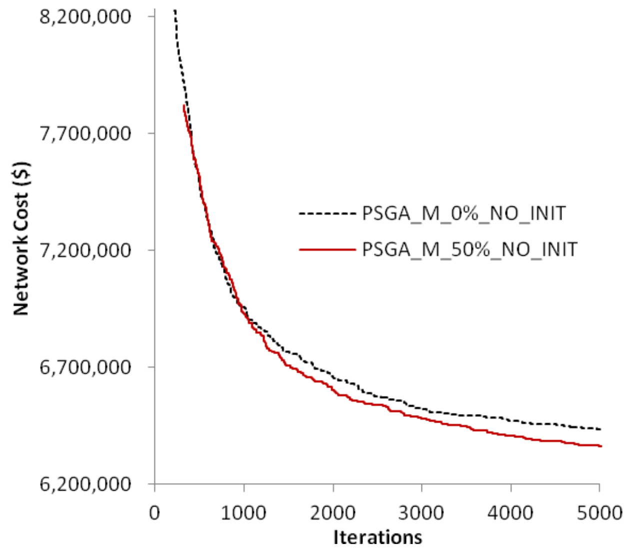

Figure 3. Mean Best Feasible Solution Cost- PSGA-Hanoi

Figure 3 shows that the standard genetic algorithm has a faster initial rate of convergence in the first 250 iterations compared to the Pipe Smoothing Genetic Algorithm (PSGA). However PSGA does surpass the standard GA beyond 1000 iterations.

3.1.2 Pipe Smoothing Population Initialiser

Instead of generating a population of random solutions, the pipe smoothing initialisation operator uses a heuristic based approach that ensures no pipe is larger than that total diameter of pipes directly upstream whilst retaining a random element. In this set of experiments the proportion of the initial population which employs the pipe smoothing initialisation operator is varied between 0% and 100% at 25% intervals. When the pipe smoothing initialisation operator is not employed, the solution is generated randomly.

The base algorithm used in the following runs is as described in section 3.1.1. Below are the average results from these experiments.

Table 2. Pipe Smoothing Initialisation Rate Results for PSGA on Hanoi

Probability of Pipe Smoothing Initialisation	Mean Best Feasible Solution Cost ($)	Mean Best Feasible Solution Cost Standard Deviation ($)
0%	6,344,188	132,334
25%	6,321,749	103,841
50%	**6,316,813**	110,051
75%	6,335,983	117,362
100%	6,380,001	**99,466**

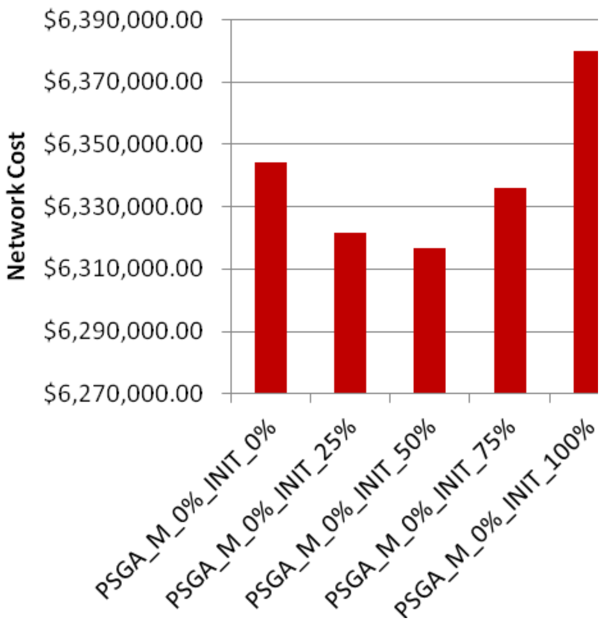

Figure 4. Pipe Smoothing Initialisation Rate Results for PSGA on Hanoi, Mean Best Feasible Solution Cost

These results suggest there is a performance increase when the pipe smoothing initialisation operator is applied to 25%, 50% and 75% of the population. However, at 100% pipe smoothing initialisation the algorithm performs worse than the standard genetic algorithm (PSGA_M_0%_INIT_0%).

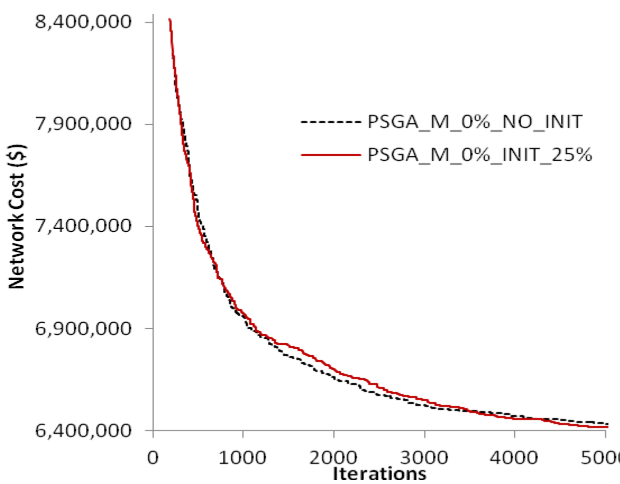

Figure 5. Mean Best Feasible Solution Cost- PSGA Initialisation – Hanoi

Figure 5 shows the comparison between the standard genetic algorithm with standard population initialisation and the standard GA with 25% pipe smoothing initialisation. The standard GA with pipe smoothing initialisation finds feasibility slightly sooner than the standard GA with standard initialisation, however there is not a notable difference in the convergence of the two algorithms.

To further explore the effect of the pipe smoothing initialisation operator, this next experiment uses the best performing version of PSGA (PS mutation at 50%) and varies the pipe smoothing initialisation rate between 0% and 100% again at 25% intervals.

Table 3. Pipe Smoothing Initialisation Rate Results for PSGA on Hanoi

Probability of Pipe Smoothing Initialisation	Mean Best Feasible Solution Cost ($)	Mean Best Feasible Solution Cost Standard Deviation ($)
0%	6,297,480	113,869
25%	6,290,517	119,694
50%	**6,279,120**	**111,130**
75%	6,298,693	120,991
100%	6,353,617	142,785

The results shown in table 3 and figure 6 indicate that PSGA with a pipe smoothing mutation rate of 50% benefits from the pipe smoothing initiation operator up to 50% population application.

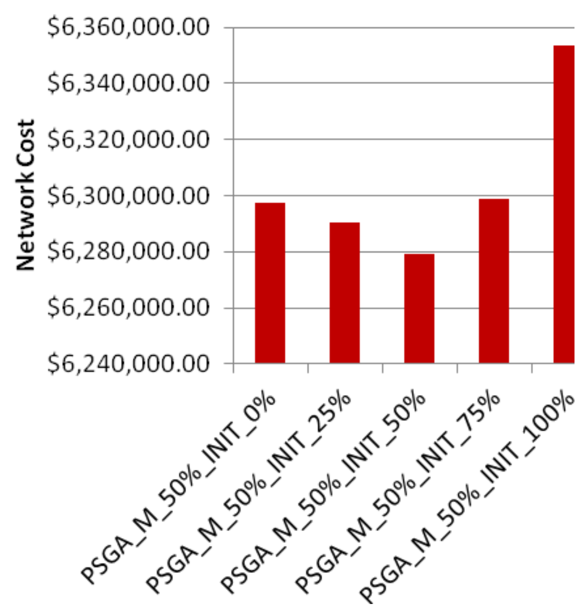

Figure 6. Pipe Smoothing Initialisation Rate Results for PSGA M 50% on Hanoi, Mean Best Feasible Solution Cost

As with the previous set of results, when pipe smoothing initialisation is applied to the whole starting population, the algorithm suffers premature convergence, achieving a suboptimal solution compared to the other algorithms.

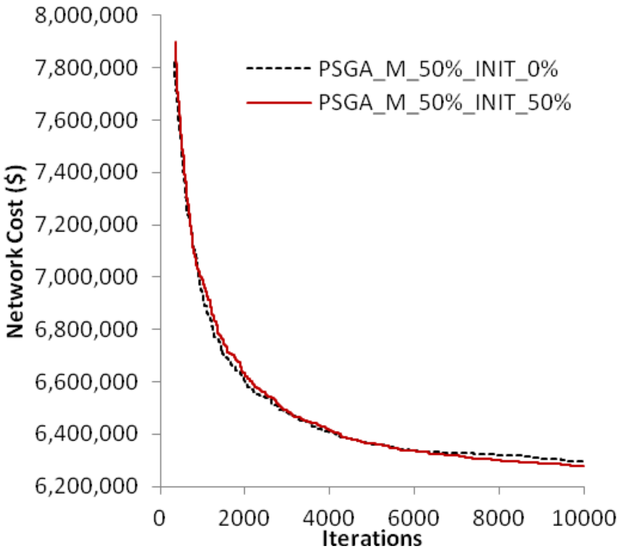

Figure 7. Mean Best Feasible Solution Cost- PSGA M50% Initialisation – Hanoi

Figure 7 shows the comparison between PSGA M50% with no pipe smoothing initialisation and the same algorithm with a pipe smoothing rate of 50%. The plots show that PSGA without pipe smoothing initialisation displays faster initial convergence, however it is outperformed in the later stages of the search by PSGA with pipe smoothing initialisation.

These initial experiments suggest that the pipe smoothing genetic algorithm benefits from an equal mix of the pipe smoothing heuristic mutation and standard bitwise mutation. Also the pipe smoothing initialisation operator adds additional performance when used to generate a moderate proportion of the initial population.

The following results show a direct comparison between the standard genetic algorithm and the best performing version of the pipe smoothing genetic algorithm.

Table 4. Standard GA vs. PSGA on Hanoi

Algorithm	Mean Best Cost ($)	Standard Deviation of Best Cost ($)
Standard GA	6,344,188	132,334
PSGA_M_50%_INIT_50%	**6,279,120**	**111,130**

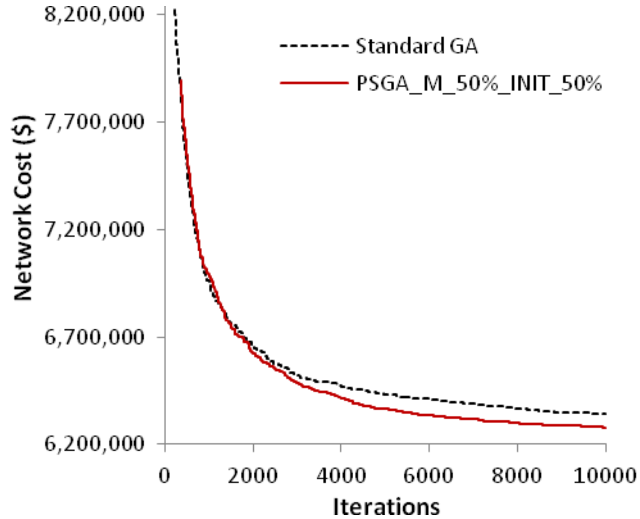

Figure 8. Mean Best Feasible Solution Cost- Standard GA vs. PSGA – Hanoi

Although the standard algorithm displays faster initial convergence over PSGA in the first 300 iterations (600 fitness evaluations), PSGA starts to outperform the standard GA after 1200 iterations (2400 fitness evaluations). PSGA achieves a better mean best feasible solution cost and with a lower standard deviation over the results of the standard GA although neither algorithm achieves the best known feasible solution ($6.081). This is not unexpected due to the number of solution evaluations allowed.

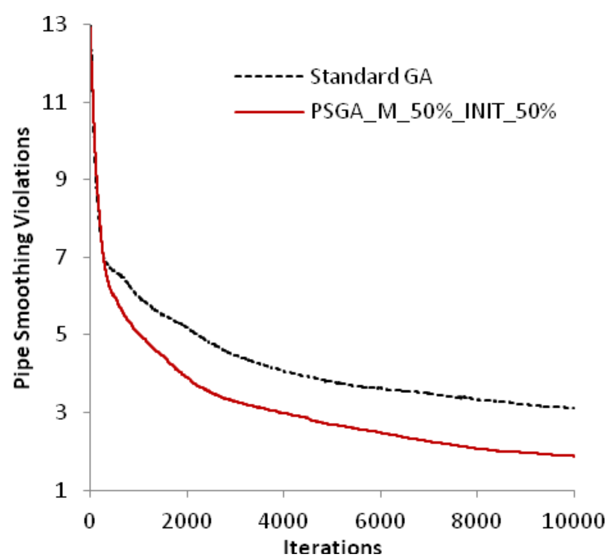

Figure 9. Average Population Pipe Smooth Violations - Standard GA vs. PSGA – Hanoi

Figure 9 shows the average population pipe smoothing violations. A violation occurs when a pipe's diameter is larger than the sum of diameters of the directly upstream pipes. From these results it is apparent that PSGA out performs the standard genetic algorithm with regard to pipe smoothing violations resulting in a population with smoother solutions and better performance. A network with fewer smoothing violations is more likely to be accepted by the engineer for implementation. The results in Figure 9 show that the PSGA solutions have approximately half the violations of those from the standard GA.

3.2 New York Tunnels Problem

The New York Tunnels Problem [3] is a parallel expansion problem consisting of 21 existing pipes and 20 junctions fed by a fixed head reservoir. The objective is to find the least cost configuration of pipes that could be installed parallel to the existing pipes to meet the head constraints of the problem. There are 16 available pipe diameters ranging from 0in to 804.0in therefore no encoding redundancy is required.

PSGA was then applied to the New York Tunnels problem and compared again to the standard GA. The base algorithm used in the following runs is built on a simple single objective steady state genetic algorithm with a mutation rate of 0.05 and tournament size of 0.05N where N is the population size which in this case is 100. For each parameter set the algorithm was run a total of 20 times for 10,000 iterations (20,000 fitness evaluations) with different initial random seeds. A penalty factor of 7,000,000 $/m head deficit was used. Below are the average results from these experiments.

Table 5. Standard GA vs. PSGA on New York Tunnels

Algorithm	Mean Best Cost ($)	Standard Deviation of Best Cost ($)
Standard GA	41,465,945	922,076
PSGA_M_50%_INIT_50%	**38,935,400**	**530,145**

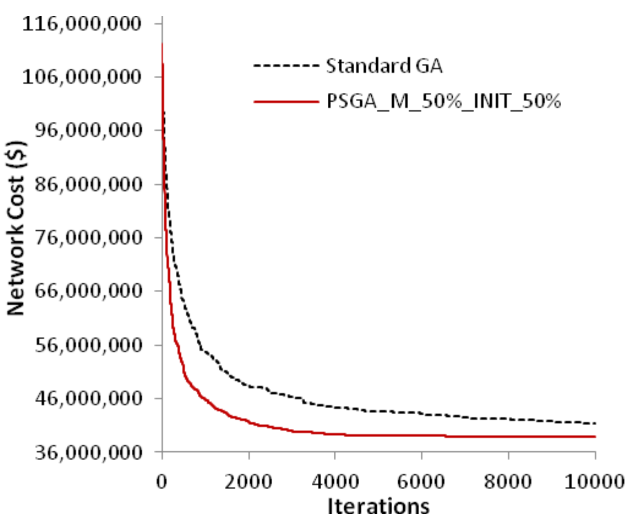

Figure 10. Mean Best Feasible Solution Cost- Standard GA vs. PSGA – New York Tunnels

In this set of runs, PSGA drastically outperforms the standard GA both in convergence rate and solution quality. It suggests that the heuristics used in PSGA directly complement the nature of the New York Tunnels problem as a parallel expansion problem. Figure 10 shows the PSGA reducing the feasible cost very close to the current best-known minimum [15] of $38.64 million in approximately 12000 fitness evaluations.

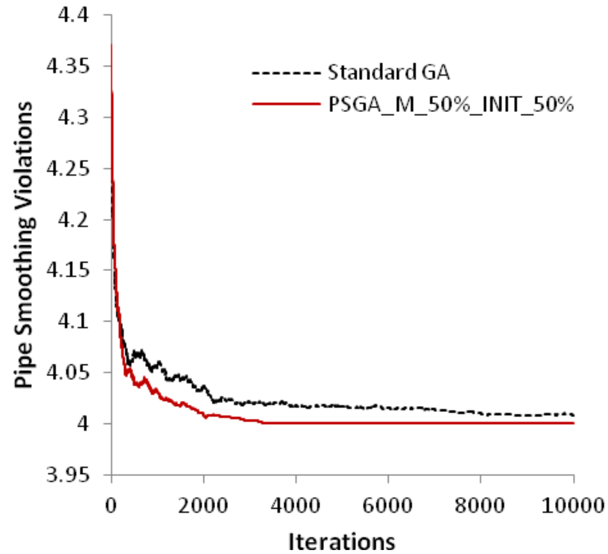

Figure 11. Average Population Pipe Smooth Violations - Standard GA vs. PSGA – New York Tunnels

Figure 11. shows the average pipe smoothing violation present in the population of both PSGA and the Standard GA. Although there is little difference between the two algorithms, PSGA does tend to outperform the Standard GA in terms of network smoothness after around 200 iterations (400 evaluations).

4. CONCLUSIONS

A 'pipe-smoothing' genetic algorithm has been created and tested on well-known benchmarks from the literature. Utilising a heuristic, PSGA encodes engineering expertise into a standard genetic algorithm with the view to improving the performance of the algorithm. The influence of the pipe smoothing initialisation and mutation operators of PSGA has proven to outperform a standard genetic algorithm on all benchmark problems tested in this paper without incurring additional fitness evaluations and hence computational complexity. For both problems tested in this paper, PSGA achieved lower cost solutions in less fitness evaluations than the standard genetic algorithm.

These experiments show PSGA will outperform a standard genetic algorithm for benchmark problems as used in this study, although further, more extensive experiments should be performed to verify the effectiveness of the new algorithm on larger real-world networks. It is also apparent that PSGA achieves a smoother solution than that of the standard genetic algorithm, making the PSGA solutions more engineering feasible for real-world application. However, the smoothness of a solution is not taken into account when assessing the network due to the nature of the fitness function which only computes the infrastructure cost and the head deficit information, thus it is most likely that smooth solutions are disregarded upon replacement. Also if we observe the behaviour of both the standard GA and the PSGA it is apparent that there is a potential correlation between the fitness of a solution and its smoothness. This leads to a question: Can search performance be improved by integrating a pipe smoothing violation component into the fitness function or separate objective in a multi-objective algorithm? Although this question falls outside the scope of this paper, this would be the natural direction to take this research in future work.

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