Dynamic performance of high frequency floors

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ABSTRACT

Hospitals and micro-electronics fabrication facilities require ultra-low vibration environments, and to mitigate the effects of the governing vibration source, usually footfalls, the floors are typically designed to have high natural frequencies so that response takes the form of a series of transients that decay rapidly between successive foot impacts. For low frequency floors evaluation of performance is relatively simple, involving simulation of resonant response in modes up to no more than 10Hz with simplifications possible through dynamic amplification factors. For high frequency floors the problem is complicated by the transient nature of the loading time history and various difficulties with the classical normal mode analysis approach.

Two approaches are in use to assess the velocity response, one simple empirical formula for RMS velocity depending on floor frequency and stiffness, and a second more recently derived empirical formula for deriving an equivalent impulse that depends on walker and floor characteristics.

This paper studies the origin and validity of the two approaches, compares their predictions. Predictions for a high frequency sample are compared with more elaborate evaluations involving time consuming FE tine history analysis and costly prototype testing.

INTRODUCTION

Vibration serviceability is now a high-profile research topic thanks to a few public failures under human dynamic loading. These failures have been so judged according to qualitative or quantitative vibration tolerance by the same human occupants dynamically exciting the structure. There is however a class of structures for which acceptance is judged according to well specified and stringent criteria evolved largely by North American experience in designing structures for housing highly vibration-sensitive manufacturing equipment, such as wafer fabrication plants or 'fabs'. With the high capital investment, profitability requires low defect rates in the fabrication process, for which a prerequisite is an extremely low level of vibration at fabrication machine supports. Rather than specifying limiting vibration levels, for example at specific frequencies, for specific machines, the accepted approach for fab vibration control is to specific a generic vibration criteria or VC at one of five levels, VC-A through VC-E [1]. The VC, which are formally defined for vibrations down to 4Hz and specify root mean square (RMS) vibration levels which should not be exceeded in any one-third octave band, e.g. VC-C and VC-B specify maxima of 12.5 μm/sec and 25μm/sec respectively in any 1/3rd octave frequency band.

The process of vibration control for a fab then involves three stages. First, vibration sources and levels either as forces for direct excitation or base excitations for ground-borne vibration are identified and quantified. Second, response calculations are performed, e.g. via finite element modelling for the proposed structure, using the given vibration inputs. Third, the RMS levels are estimated for the given inputs and structural model and are compared with the specified VC.

Hence as usual in a vibration serviceability problem, there are three components: vibration source, vibration path and vibration receiver. While the last is well defined via the VC, the dynamic structural analysis based on a given design has inherent errors due to likely deficiencies and simplifications in modeling structural properties, including damping, the effects of boundary condition, adjacent structures and non-structural elements to name a few. Even if structural modeling is dealt according to best practice, there remains the most significant problem of estimating the vibration inputs, the most significant of which are often due to pedestrians (workers moving around).

Vibration control exercises for fabs will generally result in 'high frequency floors' designed to have high natural frequencies. The aim is to avoid both the effects of vehicle-induced ground-borne vibration, whose dominant frequencies tend to lie below approximately 12Hz, and to avoid the possibility of resonant amplification by a higher harmonic of footfall forces generated during walking. If the floor frequency is set above that of the highest 'significant' footfall force harmonic, calculations of dynamic response can avoid considerations of resonance and anything to do with 'steady state'. Instead, each footfall can be considered as an individual transient, with the assumption that footfalls are separated by so many cycles of floor vibration that resonant build up is not achieved. Hence floors with frequencies above 10Hz (depending on which guidance is followed) are high frequency floors driven by impulsive loads.

If the effects of resonance can be dismissed (with, as will be seen later, some underestimation of response) then vibration control could be reduced to considering the floor as one or more linear oscillators responding to some idealized form of impulse. In other words, the problem should reduce to studying the SDOF oscillator result:

$$V(t) = \frac{I_{eff}}{m\omega_D}\frac{d}{dt}\left(\exp^{-\zeta\omega t}\sin\omega_D t\right) \qquad 1)$$

where V is velocity, I_{eff} is the magnitude of a delta function with units of N.s representing the actual transient, m is oscillator mass and ω, ω_D are undamped and damped circular natural frequencies.

EXISTING APPROACHES TO FOOTFALL-INDUCED RESPONSE PREDICTIONS

Two approaches known to the author are the *kf* approach, originating from North American work in the 70s [2] and the Arup approach more recently developed by Young and Willford [3].

kf approach [2]

This approach is based on the partly empirical relationship

$$V = C/kf \qquad 2)$$

where V is a measure of vertical velocity response (e.g. maximum value of 1/3rd octave RMS response), C is a constant, k is the floor stiffness and f is the fundamental mode natural frequency. On the face of it this measure, supported by evidence in the original presentation and which has served long and (apparently) well, does not account for mass in any way other than via $k = m\omega^2$, so how does it work?

To begin, a footfall is idealized as shown in Figure 1a. The amplitude is set to unity and the duration represents rather slow walking but the critical part of the idealization is the initial rise, lasting t_0 seconds, which is assumed to be described by:

$$F(t) = 0.5\left(1-\cos\left(\pi t/t_0\right)\right). \qquad 3)$$

If this signal is applied to a SDOF oscillator having stiffness k, then large response is obtained when t_0 is much larger than the period of the oscillator, in the limit reaching $2x_s$ i.e. twice the static response $x_s = F/k$. As oscillator frequency f increases the ratio of peak (dynamic) to static response reduces, to values $1/2\left(ft_0\right)^2$. Results for two oscillators with unit stiffness but different frequencies (1Hz and 10Hz) are shown in Figure 1. The small ripple for the high frequency represents the dynamic response.

Hence the amplitude of the dynamic response that results from the initial rise is

$$x_{\max} = x_s/2\left(ft_0\right)^2 = F/2k\left(ft_0\right)^2 \qquad 4)$$

with acceleration amplitude

$$a_{\max} = \left(2\pi f\right)^2 x_{\max} = 2\pi^2 F/t_0^2 k \qquad 5)$$

and velocity amplitude

$$v_{\max} = \left(\pi/t_0^2\right)F/kf \qquad 6)$$

these conversions being based on an assumption of harmonic response.

Since t_0 depends on the walking, the result is apparently the dependence of velocity amplitude on *1/kf*.

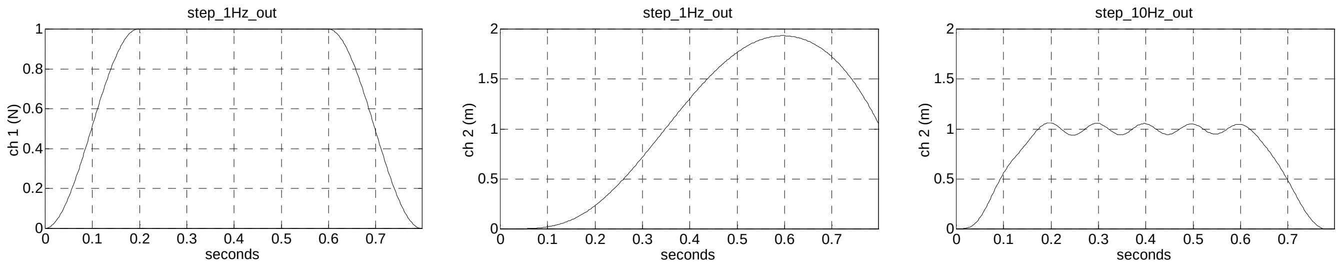


Figure 1 (a) Idealised footfall (b) 1Hz, 1N/m oscillator response (c) 10Hz oscillator response

The validity of this method is examined by direct use of the formula and response simulations using the idealized footfall as well as a real (recorded) footfall signal.

Figure 2a shows the result of using equation 6 with a rise time of 0.1 seconds and peak force 1170N for a footfall lasting 0.4 seconds. The rise time and duration are somewhat arbitrary but the 1170N amplitude gives the footfall the same impulse value (350Ns) as a real walking sample for 2.54Hz pacing rate.

Figure 2b shows the result obtained by applying the footfall time history as input to an oscillator having the same range of frequencies $5Hz \le f \le 25Hz$ and stiffness $5GN/m \le k \le 25GN/m$ as for Figure 2a.

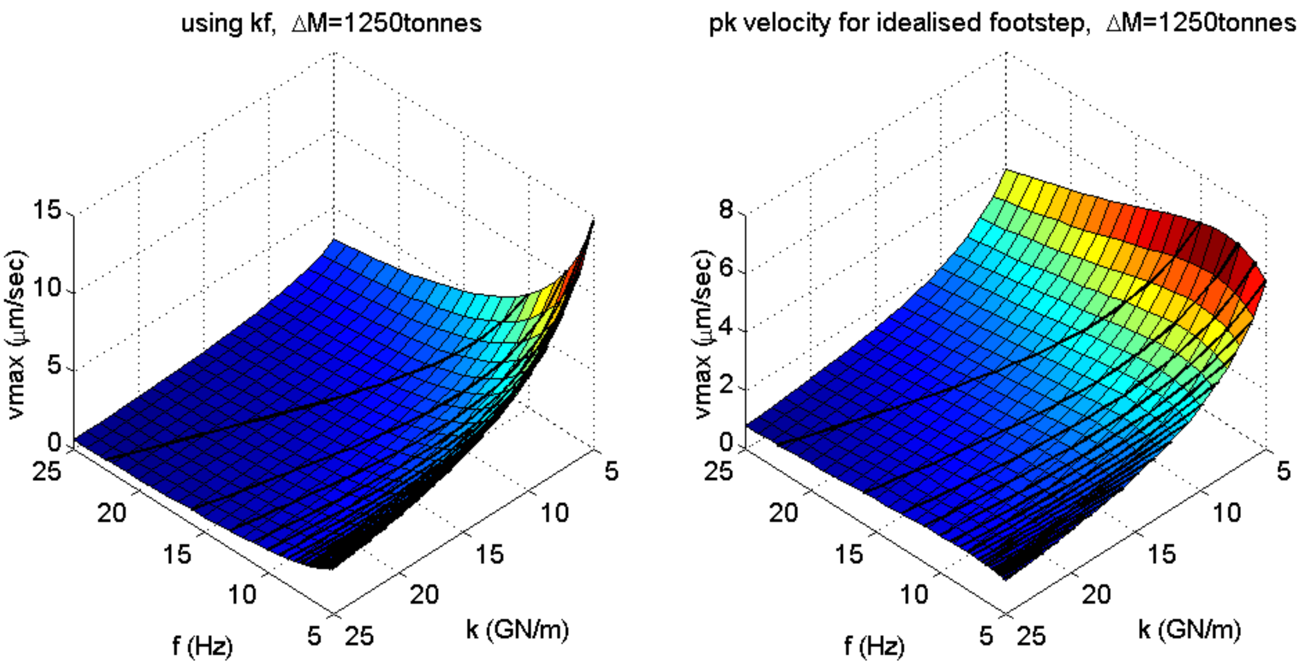


Figure 2 (a) v_{max} from *kf* formula (b) v_{max} from oscillator simulation

For *kf* the mass is apparently irrelevant while for the oscillator mass is fixed for a given *f* and *k*. The black lines on the surfaces show constant mass. The response surface for the simple formula follows the shape of the simulation so it is self-consistent.

Arup Approach [3]

The procedure followed by Arup (consulting engineers) for floors with fundamental frequencies above 10Hz uses an empirical formula,

$$I_{eff} = 42f^{1.43}/f_n^{1.3} \quad 7)$$

based on results from SDOF oscillator simulations using a large database of footfall traces, to provide values of effective impulse I_{eff} which can be used in equation 1. f is pacing rate and f_n is floor frequency.

Figure 3 shows v_{max} obtained from the formula for f=2.54Hz; damping is largely irrelevant since in the idealization the maximum velocity occurs the instant the impulse is applied and the decay is redundant. The values are in the same range as for kf but the characteristic is completely different. Obviously low k and high f means low mass and the resulting high velocity is clear in Figure 3.

Variability between subjects is considered by a coefficient of variation of 0.4 for the constant in equation 7 and the procedure can in principle be used for multi-degree of freedom systems having individual modal masses and frequencies.

SDOF Exact analysis –single footfall

Both approaches can be compared with results for real walking. First the SDOF response to a single footfall is obtained for a range of oscillator frequencies and stiffness (and hence masses) as for Figure 2b.

Figure 4 (top) shows the footfall trace; it is for a 85kg male walking at 2.54Hz pacing rate. Figure 4 (bottom) shows the dependence of v_{max} on frequency and stiffness. The same trend is observed as seen as for the Arup formula i.e. response generally increases with decreasing mass (black contours, having lower mass values for low k, high f). In this case the Arup formula underestimates w.r.t the simulation for a single example footfall time history.

Different footfalls have different shapes (but a similar trend) and this variability has effectively been accounted for using the Arup formula, for which a design value (with some safety factor) uses a constant of 54 instead of 42.

SDOF Exact analysis –multiple footfalls

How well does the single footfall (equivalent impulse) method work? For the relatively large collection of footfall time series (collected by Kerr [4]) the Arup formula should be representative. The actual response depends how the impulse is applied e.g. shared between modes, and how the response is measured e.g. as peak velocity, RMS or some other measure.

RMS is more widely used for example in fabs via the VCs which are narrow band RMS. Since all the energy will be in one of the bands, which will then govern, this is the same as simple RMS.

One problem with RMS calculations is in defining the averaging time: for a single impulse, longer durations dilute the strength, but for walking an obvious value would be the footfall interval.

Figure 5 shows how footfall sequences generate responses for two types of floor, and how different measures of response work.

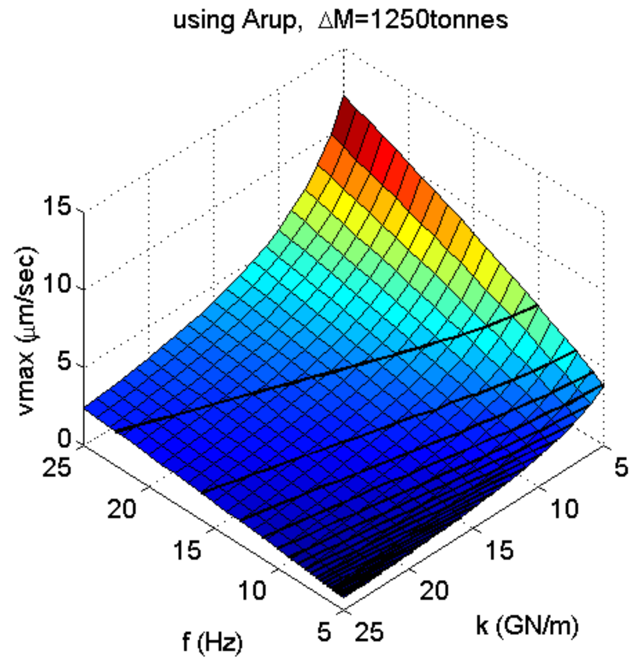
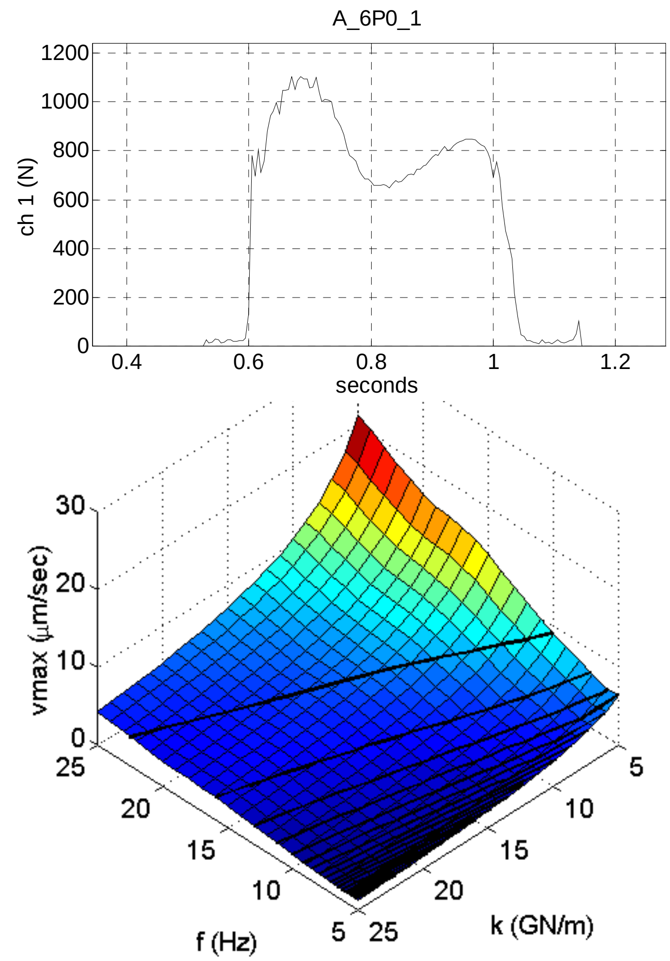


Figure 3 ↑ Response using Arup formula

Figure 4 ↓ Single (real) footfall (top)
Simulated response to real footfall



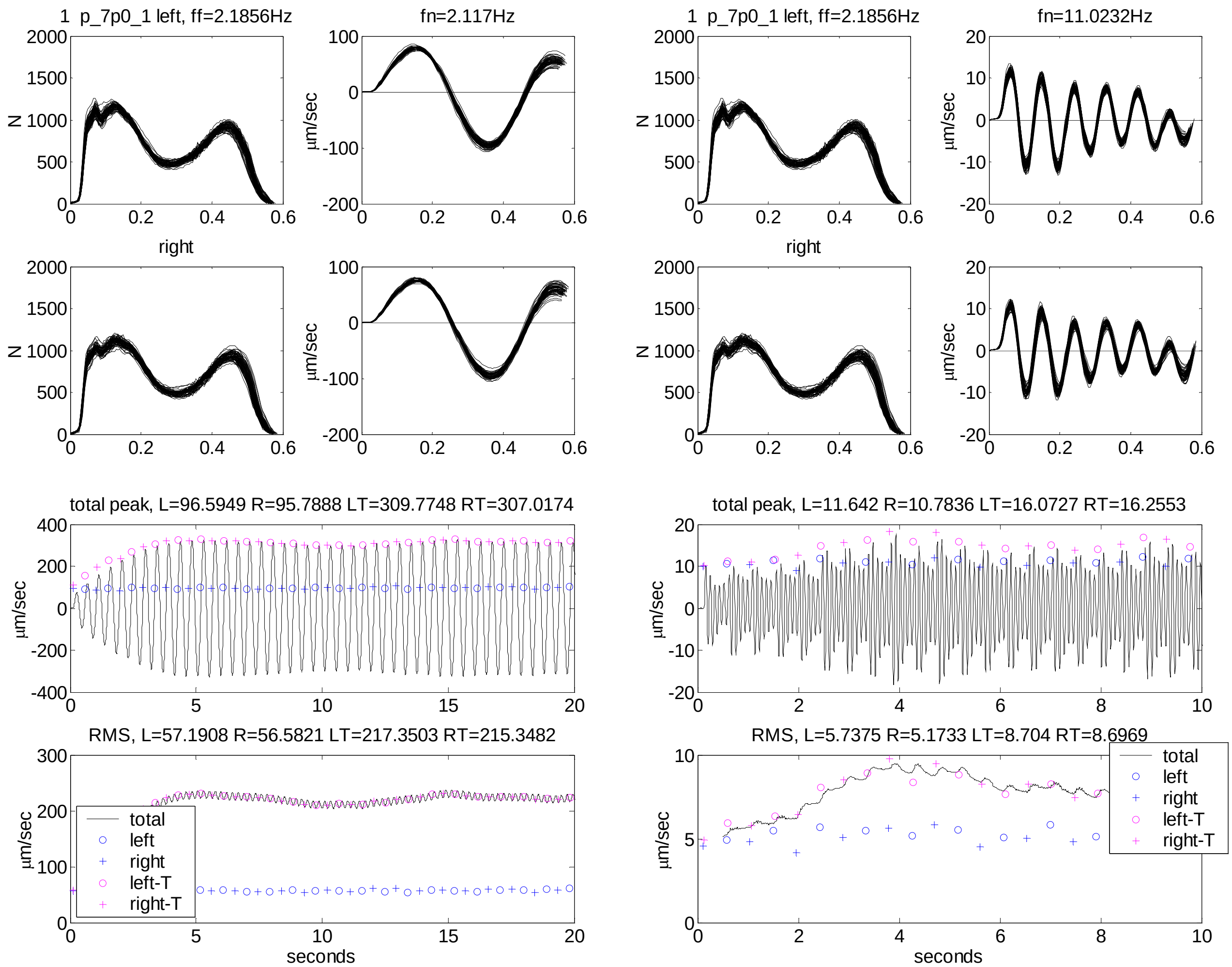


Figure 5 (clockwise from top left): (a) Footfalls and transient responses for low frequency oscillator, (b) for high frequency oscillator, (c) time series and RMS values for low frequency oscillator and (d) high frequency oscillator.

Figures 5a and 5b show (in the two left panels) the same sets of left and right footfalls for an individual pacing at 7km/h and (in the two right panels) the impulsive-like response for a 10^6 kg SDOF oscillator with 3% damping. Figure 5a represents a low-frequency (2.1Hz) floor, Figure 5b represents a high-frequency (11Hz) floor. Figures 5c and 5d show the total response of the two oscillators to the total walking force time history i.e. all footfalls added with correct time shifting. The top panels show the time series of response, the lower panels show the moving-average RMS values for the time histories.

The magenta markers track the statistics for the combined responses (that are shown) as either peak or RMS values, determined for duration of each individual footfall.

The blue markers track peak and RMS values for the separate responses of each footfall i.e. the signals in the right panels of Figures 5a and 5b.

Figure 5d show what is already understood i.e. that resonance because the footfalls timing is such that responses to successive footfalls sum. Hence comparing magenta with blue markers show how the effect of resonant reinforcement. Figure 5c shows a similar effect for a supposed high-frequency floor, albeit at a much reduced level. The point is that for high frequency floors resonance, or at least some enhancement due to variable timing of footfalls can occur.

Where the measure of response is a peak-hold RMS, as would be applied using third-octave derived VCs, missing out the resonant contribution can lead to significant underestimation. This worked for the trace used in Figure 5, but may be a fluke. Hence simulations were run for a set of 96 recorded walking traces for nine individuals (obtained using a treadmill [5] for different walking speeds).

Figure 6a shows the results of this simulation, as maximum RMS values for the combined footfall trace. Each ridge or trough parallel to the f0 axis represents a single walking trace fed into different oscillators.

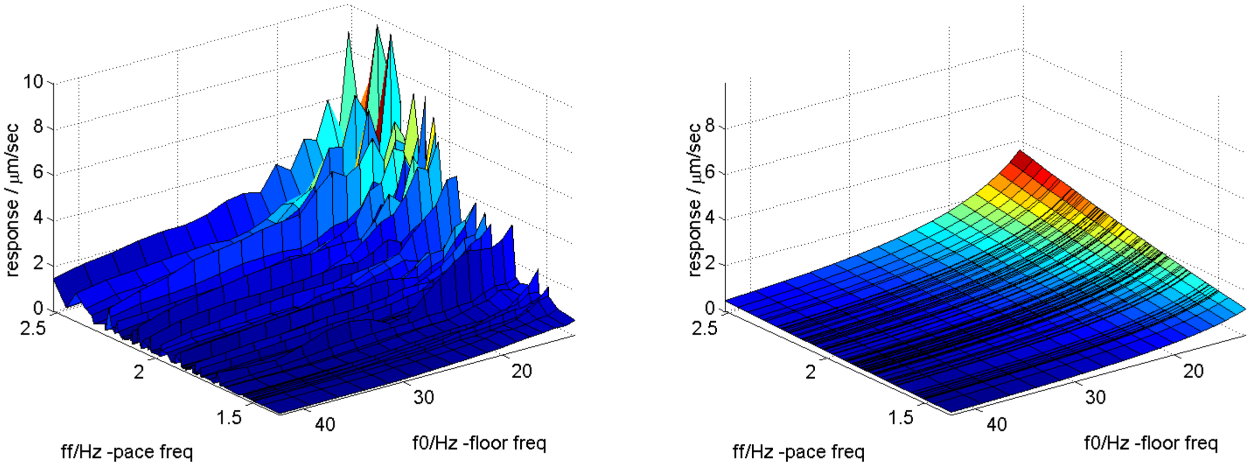


Figure 6 (a) peak hold RMS for real walking, 10^6kg oscillator (b) RMS derived via Arup formula

There is very significant variability between the traces in Figure 6a, even if weight normalization is applied. This emphasises the difficulty of using simple deterministic empirical formulae. Further, the rippling in the f0 (oscillator frequency) direction that occurs even up to 20Hz shows how much difference resonance can make.

By comparison, Figure 6b shows (to the same scale) the equivalent surface for the Arup formula. The formula gives effective impulse and (directly) the peak velocity, and this is converted to RMS based on duration and decay rate of the ideal impulse for the duration of the footfall. Generally this appears to under-estimate, the difference is strongest in the 10-20Hz range.

CASE STUDY

Figure 7 shows the underside of a typical fab floor. A similar example was studied for serviceability assessment due to walking; Figure 8 shows part of the FE model which provided estimates of modal masses of 75×10^6 kg and 224×10^4 kg for modes at 20Hz and 22Hz that would involve the bay studied.

Using the *kf* approach was not so simple due to lack of information about the value of the constant *C*, which is derived empirically from measured results. A value of 7.5×10^4 s/m was found in some proprietary publications.

The *kf* formula uses static stiffness, which would be found using the FEM simulation shown in Figure 8.

Figure 7 High frequency floor

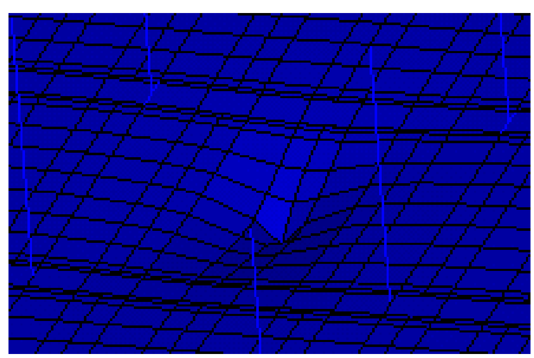


Figure 8 FEM with point loading

The mid-bay point stiffness, is 2.2×10^8 N/m resulting in a 1/3rd octave RMS velocity of 17μm/s.

Using the Arup formula, for a pacing rate of 2.03Hz (walking at 7km/h) peak velocity in each mode is 40μm/sec and 11.9μm/sec respectively. The question then arises: can these be added directly, and if so what about adding all the other modes that contribute? It is also necessary to convert these peak values to RMS values.

The final method, a simulation of walking (on the spot) is conducted using the FEM and the walking time history in a transient response simulation. Figure 9 shows the result.

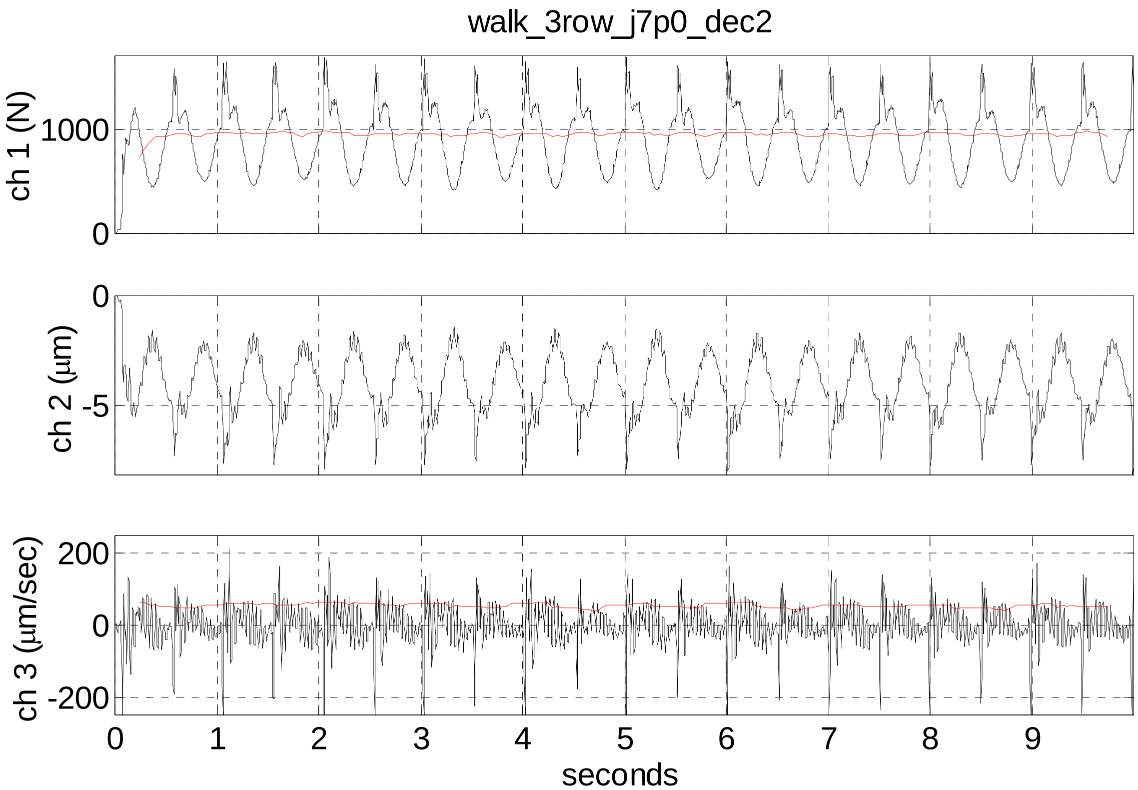


Figure 9 Simulation of walking on the spot on high frequency floor: footfall trace (top), displacement (middle), velocity (bottom). Red lines are RMS trends.

The drawback of this method is the complexity of the simulation, which is still limited to simulating walking on the spot. To move the walker along a trajectory is a practical impossibility by a full transient analysis in standard FE codes, whereas given mode shapes and other parameters, the modal superposition method could be applied.

As a final check on all the estimation procedures, the prototype floor was tested by walking at the same speed used to generate the treadmill traces. Walking was not on the spot but along the middle of a row of bays. Figure 10 shows the result: the red line again shows RMS (velocity) and the values are in line with the predictions from all methods although the form of the response is hard to compare with Figure 9.

The trace results from very heavy walking, far in excess of anything that would be possible in real operational working conditions.

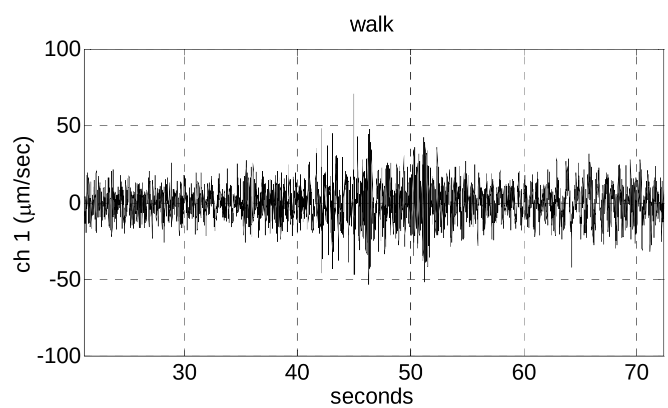


Figure 10 Prototype performance

CONCLUSIONS

Assessing the vibration performance of a high frequency floor with dynamic loading from a single person walking is a challenge for a range of reasons. Even if it is decided to use a single walker, choosing a representative characteristic is by no means simple. Even if this is agreed, there is still considerable choice with prediction methods. Exact time history methods are expensive and require known footfall traces, which are not widely available even for single footsteps while prototype testing cannot be used for prior evaluation.

Hence the simpler procedures have to be used. Of these, the *kf* approach is simple but the validity of using 1/kf seems hard to establish, and the constant *C* is an empirical result with values apparently known only to the industry. On the other hand, the values are calibrated from real life experience of performance of a number of structures, adding significant credibility.

The Arup approach is backed up by visible research focused on the characteristics of the walking, leaving the application to the user. At this point the difficulty arises in terms of how to define a mass, how to add the responses for different modes and how to convert peak to RMS values.

The full-model analysis method is costly and somewhat dependent on a specific walking trace but potentially it is the most flexible, particularly if the modal decomposition approach with modulation by mode shapes along pedestrian trajectories is applied.

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