# Noise Characteristics of Sensors for Extreme Low Level Vibration Measurements

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ABSTRACT

Performing vibration assessment of structures and sites having stringent vibration requirements poses problems for vibration sensing equipment routinely used for experimental modal analysis of civil structures. When vibration levels are specified in terms of microns, microns per second or micro-g, special consideration has to be given to the whole of the instrumentation chain for data sensing and acquisition and it is vital to establish the performance limits of available sensor systems.

This means devising methods for testing and presenting the noise characteristics of equipment and providing meaningful and unambigous specifications of noise characteristics, especially when engaging a third party for vibration assessment. For tutorial benefit, experiences are presented here with using accelerometers and a seismometer for a number of low level structural vibration measurements.

## 1 INTRODUCTION

Trends in increased power and miniaturization for personal computers and other information technology products push technology for economic mass production at 'nano-scale' to greater challenges. Meanwhile scientists explore new techniques for studying the structure of matter and of our universe and these demand extreme precision in measurement.

Examples include micro-chip fabrication plants (fabs) and factories for disk drives and flat panel display screens, facilities such as synchrotrons or lasers that use precisely aligned beamlines to examine the atomic structure and behaviour of materials, and telescopes for detecting gravitational waves. In extreme cases such as stability of beamlines in a synchrotron, positional stability may be measured in terms of picometers ($10^{-12}$m).

Failure to provide stable i.e. low vibration environments for the instruments and machines used in these examples can lead either to costly production losses or failure of instrumentation with huge loss of man-years and of opportunity to the scientific community.

Hence precision vibration measurements may be required at the design stage to check the vibration levels of the site and later to check as-built structural performance of such a facility.

For either exercise, a major factor in the assessment of vibration levels is the deployment and performance of the measuring instruments. In particular knowledge of the noise characteristics and frequency response characteristics are required in order to choose an instrument of suitable capability.

## 2 FEATURE SIZES AND SIGNAL FORMS

Manufacture to very tight dimensional tolerances requires extreme stability of equipment in order to machine and then check the finished product. Feature sizes such as (disk drive) media track widths and chip features are now specified in nanometers (nm, $10^{-9}$m). As a perspective, as well as the beamline example, disc drive heads may have as little as two or three air molecules clearance over the media, i.e. a few nm.

Velocity is usually used as the measure of movement because measurement of displacement either implies or physically requires a stationary reference point (which, as shown later, may be hard to define), because it has been found that sensitivity of instrumentation is bounded by constant velocity curves in the higher frequency region[1], and because low level vibrations have historically been measured using velocity-sensing seismographs.

Assessment of vibration levels for sensitive structures relates to vibrations one or two orders of magnitude below human perception threshold. For example the most stringent of the 'VC curves' [2] is VC-E which is equivalent to a 1/3rd octave RMS limit of 3 micron/sec i.e. 3μm/sec which, when realized as a pure harmonic oscillation at 8Hz, equates to 200μm/sec$^2$ or 20μg. This value is below the noise floor of standard accelerometers used for measuring response of civil structures.

## 3 SENSOR TYPE AND SPECIFICATION

Making a choice of sensor raises two questions:
- What are the expected vibration levels to be measured?
- What are the noise characteristics of the sensor specified?
- How can sensor noise characteristics be compared to (expected) site vibration levels?

For random vibrations without any distinct harmonic component, standard piezo-electric accelerometers (such as ICP, LIVM or other proprietary forms) will often fail to resolve clearly such weak signals above their own electronic and mechanical noise levels. To achieve adequate signal to noise ratios requires the use of either high-grade accelerometers or seismometers used by Vibration Engineering Section (vibration.shef.ac.uk) are compared here:

**ENDEVCO 7754-1000 'ISOTRON' accelerometer (now obsolete)** (Fig. 1)
*Supply, cabling and signal conditioning*
This accelerometer uses an ICP-standard constant current source. Standard cabling for this type of sensor is 'microdot', which is essentially small diameter BNC with a core and braid shield. Generally this type of cable kinks easily and our experience is that kinked cables add noise to the signal. Also exposing the cable to temperature variations or moving is affects the signal, so we use large diameter signal cable with fail-safe XLR connectors.
*Mounting*
Mounting is via a threaded stud to a magnet or metal base.
*Frequency response*
Response amplitude is flat to within 5% down to 0.2Hz and up to 400Hz. This is not a DC accelerometer due to the AC coupling required to isolate the 18-24V ICP supply voltage from the accelerometer.
*Noise characteristics*
Residual noise is quoted as 10μg RMS (typical) from 0.1-100Hz or 0.5μg/√Hz.

**QA-700 Q-Flex servo-accelerometer (sold by Honeywell Aerospace)** (Fig. 2)
*Supply, cabling and signal conditioning*
This accelerometer generates a current to centre the proof mass, and this current (which is directly proportional to linear acceleration of the housing), is available through the accelerometer signal pin to drop across a load resistor.
*Mounting*
The QA needs to be clamped with even tightness to a metal fixture using the supplied mounting ring.
*Frequency response*
The QA is a DC accelerometer operating from 0Hz to approximately 200Hz.
*Noise characteristics*
Resolution/Threshold is quoted as <1μg max.

**GURALP CMG-3ESPD tri-axial seismometer** (Fig. 3)
*Supply, cabling and signal conditioning*
This seismometer is connected directly to a built-on 24-bit digitiser. As with many seismometers, the Guralp can operate without connection but is connected to a PC by RS232 to change acquisition setup and download waveforms.
*Mounting*
The Guralp has three adjustable pointed feet and spirit bubble for levelling. The three sensors need to be locked while the unit is being moved.
*Frequency response*
The Guralp is quoted as operating from 120 seconds (1/120Hz) to 100Hz.
*Noise characteristics*
Guralp cites a USGS report which states that the unit's self-noise remains below the Peterson 'New low Noise Model' between 40s (1/40Hz) and 16Hz. Fig. 4 reproduces the evidence in the Guralp brochure, taken from the cited USGS report. The Y-axis label reads 'Equivalent peak earth acceleration (20 Log M/sec2)' and the axis limits are -240 to -30.
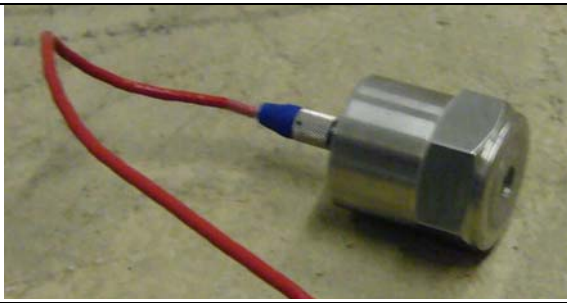
Figure 1 Endevco 7754-1000 accelerometer


Figure 2 QA-700 accelerometers


Figure 3 Guralp CMG-3ESP –no digitiser

## 4 NOISE CHARACTERISTICS

The different ways of stating sensor noise i.e. two different numerical ways for the accelerometers and the plot for the Guralp highlight the difficulties in determining whether a sensor will be capable of resolving the vibrations to be measured against their own 'self-noise'.

Another of the difficulties is the way in which the vibration to be measured is quantified and the way in which is analysed. Typically a signal is recorded for a finite time $T$ and processed by a digital signal processor using discrete Fourier transform (DFT) to generate some form of spectrum of vibration level.

If a signal $f(t)$ is precisely periodic e.g. $g(t) = a_n \sin 2n\pi t/T$ it will appear in the DFT as a line height $a_n$. The same line can also represent a narrow-band signal with


Figure 4 Guralp CMG-3ESP noise specification

between frequency limits $n - 1/2T$ and $n + 1/2T$. Likewise a perfectly sinusoidal signal which is not periodic in $T$ and hence has discontinuities at times $mT$ has energy leakage over several lines (without special windowing). Hence it is unhelpful to state a resolution requirement in terms of a signal strength at specific frequency. The noise specification for the Endevco accelerometers suggests a more helpful and rational way of comparing sensor self-noise with signal strength, i.e. specification as a power spectral density.
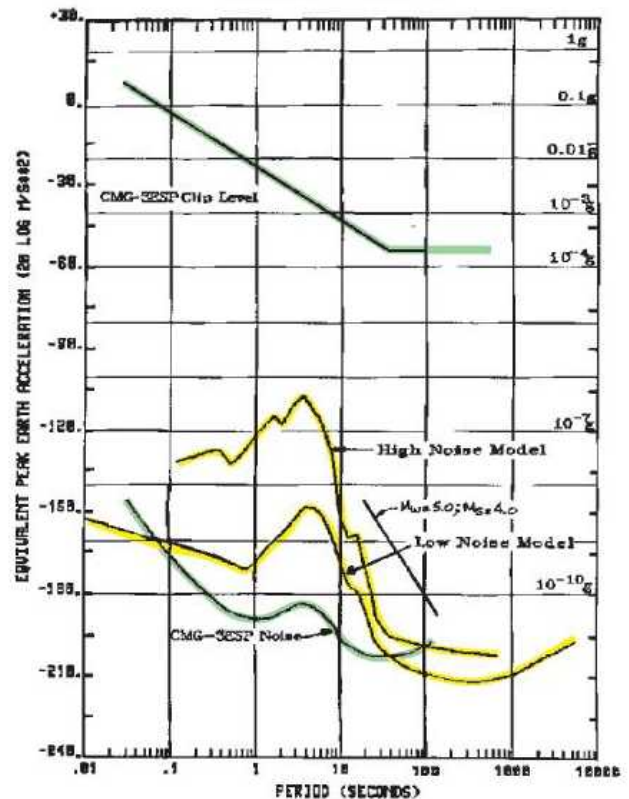
## 5 POWER SPECTRAL DENSITY REPRESENTATION OF SIGNALS

A signal $g(t_i)$ is sampled $m$ times at times $i = 0 \cdots m-1$ at sample rate $fs$ with time interval $dt$ for duration $T = m \cdot dt$ and is assumed to repeat exactly every $T$ seconds. The mean square of this signal is

$$MS = \frac{1}{T} \sum_{i=0}^{m-1} g^2(t_i) \cdot dt . \tag{1.1}$$

$g(t_i)$ has a DFT representation $F(f_i)$ sampled at $m/2$ intervals $df = 1/T$ over the range 0 to $fs/2 - df$. Strictly speaking, the DFT is 'double-sided', has values from $-m \cdot df$ to $(m-1) \cdot df$ and is also periodic, repeating at intervals $fs$ but the negative frequency side is usually ignored and its contribution accounted for in scaling of the DFT ordinates in a 'single-sided spectrum' i.e. from 0 to $fs/2 - df$.

The mean square of the signal is also computed from the single-sided DFT as

$$MS = \frac{1}{2T} \sum_{i=0}^{m/2-1} F^2(f_i) \tag{1.2}$$

Alternatively if the DFT lines are divided by $\sqrt{2df}$ i.e.

$$G^2(f_i) = F^2(f_i) / 2df \tag{1.3}$$

$$MS = \sum_{i=0}^{m/2-1} G^2(f_i) \, df \tag{1.4}$$

The quantity $G^2(f_i)$ or more simply $S(f_i)$ is termed the power spectral density (PSD) since it can be integrated via equation (1.4) to provide mean power. It has value at every frequency, including values between spectral lines (where it can be taken as some interpolation between values at DFT frequency lines).

The PSD of an acceleration signal can be converted directly to velocity and displacement PSD by dividing by $\omega^2$ and $\omega^4$ respectively where $\omega = 2\pi f$. Working in frequency domain avoids troublesome time domain integration to obtain lower time derivatives of signals which complicated by signal drift (which is sensor self-noise at very low frequencies).

## 6 NOISE PERFORMANCE OF SENSORS: ENDEVCO VS QA-700

Figure 5 shows vibration measurements in progress at night on a prepared site for a micro-chip factory, at least 100m from the nearest road. A triad of QA-700s and of Endevcos were used. QA-700s were powered by battery operated amplifier and filter, Endevcos by a battery operated four-channel Kistler 'coupler'.

This exercise provided an opportunity to compare the sensors in almost perfect extreme low-vibration environments.

Figures 6 and 7 compare horizontal direction acceleration signals for the two accelerometers. Top plots are time series and bottom plots are auto-spectral densities $G$ as defined in equation (1.3). The Endevco accelerometer time series show a characteristic low-frequency drift that is not evident with the QA-700s and appears as a rising trend at low frequency.
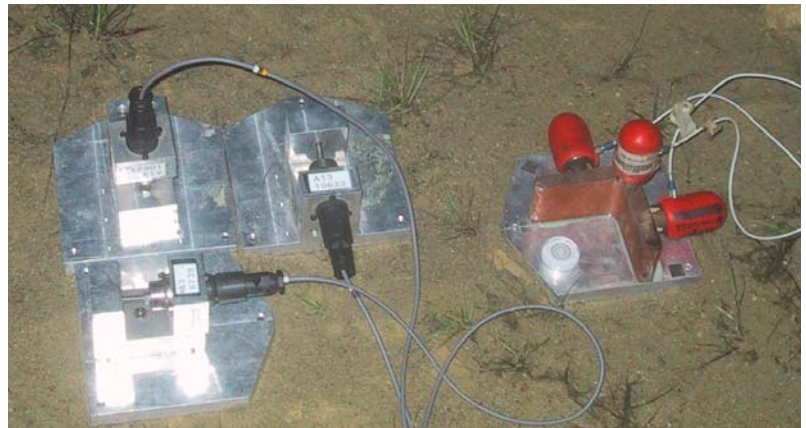

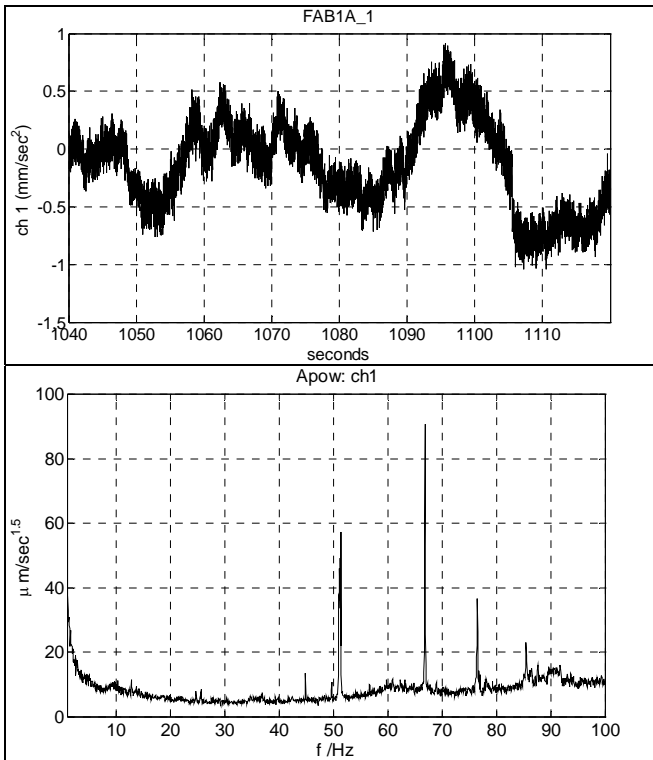
Figure 5  QA-700 and Endevco accelerometers at quiet site

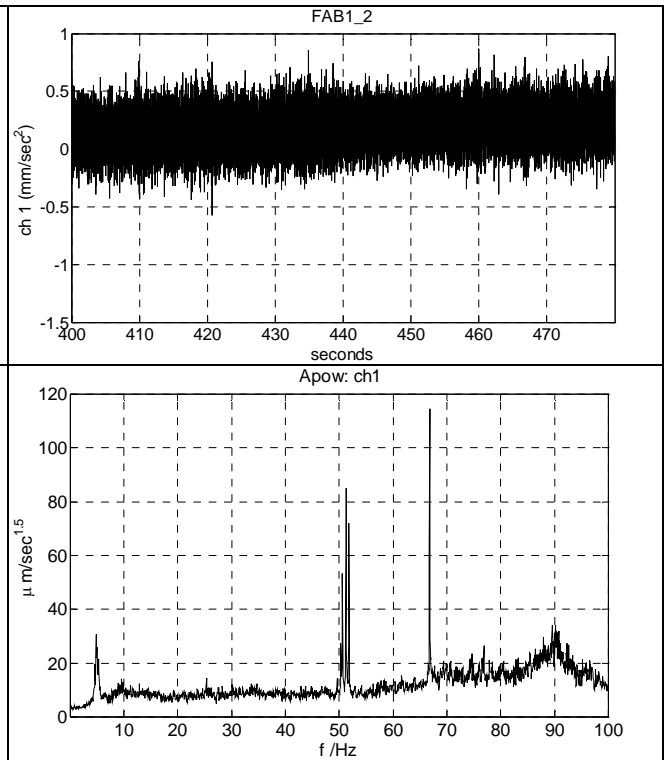| | |
|---|---|
| Figure 6 Endevo time series and ASD | Figure 7 QA-700 time series and spectra |

The quoted Endevco Noise level $0.5\mu g/\sqrt{Hz}$ is equal to $5\mu m/sec^{1.5}$. This is the lower bound of the signal, shown in Fig. 6, at 30Hz and is clearly optimistic. Comparison with Fig. 7 indicates that the low frequency component of the Endevco must be dominated by self-noise of the sensor and acquisition system. The QA-700 lowest noise level also appears to be $5\mu m /sec^{1.5}$, but $0.3\mu g^2/Hz$ has been achieved in other tests.

## 7 ACQUISITION SYSTEM

The acquisition system used in an exercise to check acquisition noise comprised the (home made and hence simple) amplifier and anti-alias filter for the QA-700 and the Kistler coupler for the Endevco. A 16-channel 16-bit National instruments AT16XE PCMIA card was used for recording. To check the noise levels introduced by the acquisition system, it was operated open-circuit i.e. with the accelerometers disconnected from the input. This may not be the ideal representation since, for example, cable noise is not included. Fig. 8 shows the result applying the same conversion factor as for Figs. 6 and 7 to convert acquired Volts to physical units, showing that acquisition noise seems to be significantly less than sensor noise :
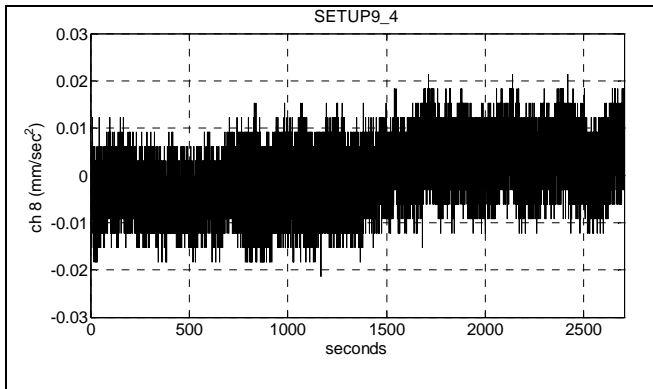


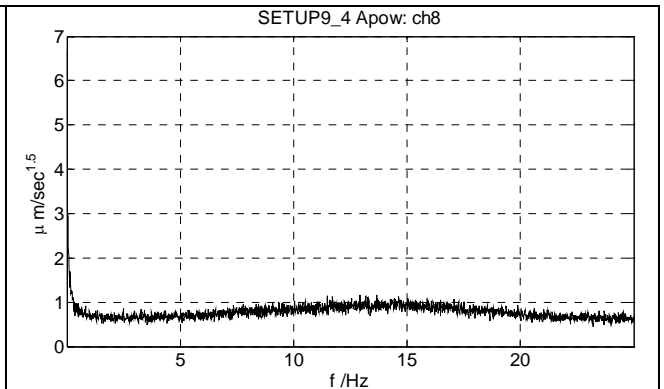| | |
|---|---|
| Figure 8a bit-level acquisition system noise | Figure 8b bit-level noise spectrum |

## 8  NOISE PERFORMANCE OF SENSORS: QA-700 VS GURALP

Because of the low-frequency noise issues associated with the Endevcos, which are made worse by the cabling, we prefer to use QA-700s. In exceptional cases of extreme low level vibrations QA performance may be inadequate and the Guralp seismometer may be necessary.

Fig. 9 shows the two sensors in use measuring vibration levels at a new high power microscope facility where vibration levels (by 1/3$^{rd}$ octave analysis) marginally exceed VC-E.

Fig. 10 compares signals for the two sensors for the same time period as velocity power spectra. The difference in signals is at least attributable to the noise levels for the accelerometer system, which are significant particularly at low frequencies. Levels are comparable for the two instruments only around (resonant) peaks of the PSD of the structural response, which is amplified around resonances.

## 9  A NOTE ABOUT UNITS

The conversion from acceleration PSD (as Figs. 7 and 8) to velocity PSD (as Fig. 10) by dividing values by $(2\pi f)^2$ has the effect of rotating the plots clockwise.

Further conversion to displacement applies another rotation.

Note also that for acceleration, ASD $(m/\sec^2)/\sqrt{Hz}$ is equal to $m/\sec^{1.5}$ and is the square root of the PSD unit $(m/\sec^2)^2/Hz$.

Similarly forms are used for velocity and displacement representation in spectral density form.

Occasionally PSDs (not ASDs) are given as dBs which are $20 Log_{10}(PSD/PSD_{ref})$ where $PSD_{ref}$ is likely to be given in terms of m.

## 10  FUNDAMENTAL VIBRATION MEASUREMENT LIMITS



Figure 9 Guralp vs QA-700 at quiet site
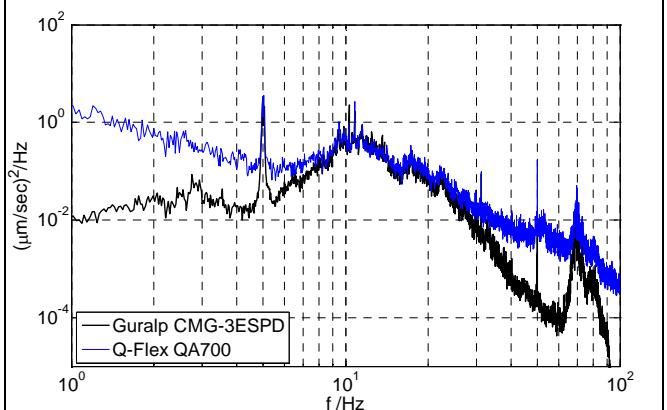


Figure 10 velocity PSDs for Guralp and QA-700

To test accelerometer noise limits it is necessary to record signals in quiet sites or somehow isolate the sensor from all vibration sources. The latter probably involves kind of suspension system that has its own natural resonances, but at very low frequencies. For broadband noise measurements the quietest site are likely to be away from urban noise, sheltered from wind and in non-seismic areas. Fig. 11 is a figure taken from reference [3] and compares vibration levels at a number of instrument sites with vibrations in a 'reference' site; a German salt mine. The plots are given with units of $\mu m^2/Hz$ which appears to be a conventional measure for scientific sites.

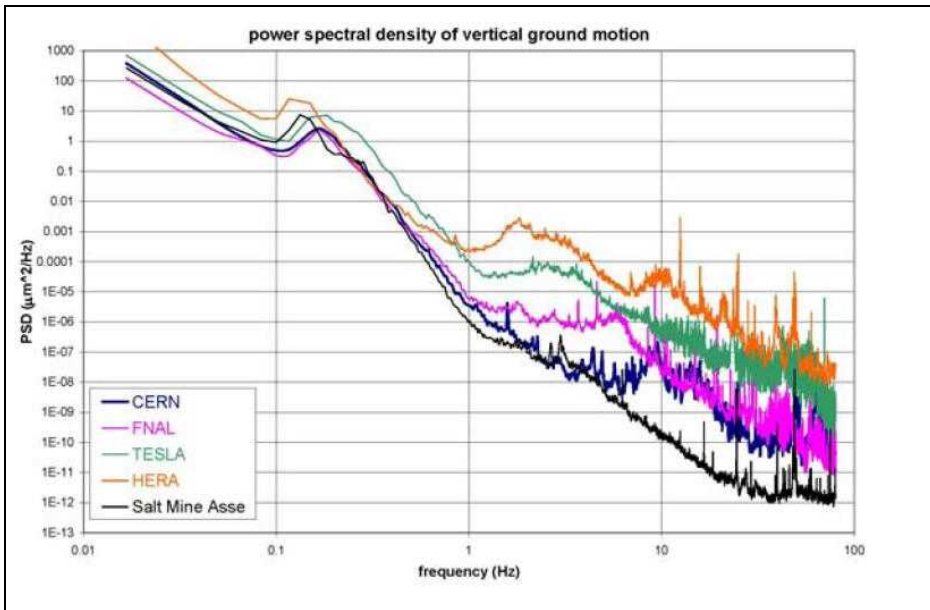Figure 11 (from Ref [3]) PSDs of ground motions at various quiet sites

Fig. 11 suggest two things:

First, that the line represented by

$$PSD = a/(2\pi f)^4$$

represents a fundamental limit to vibration levels that 'that can be measured' above 1Hz.

Second, a hump in the PSD spectra recurs at low frequencies, between 0.1Hz and 0.2Hz, and has been ascribed to micro-tremors induced by coastal waves.

Evidence of this is shown in Fig. 12, a recording of vibration levels at the same location as for Figs. 9 and 10, but at the dead of night in a more isolated location.
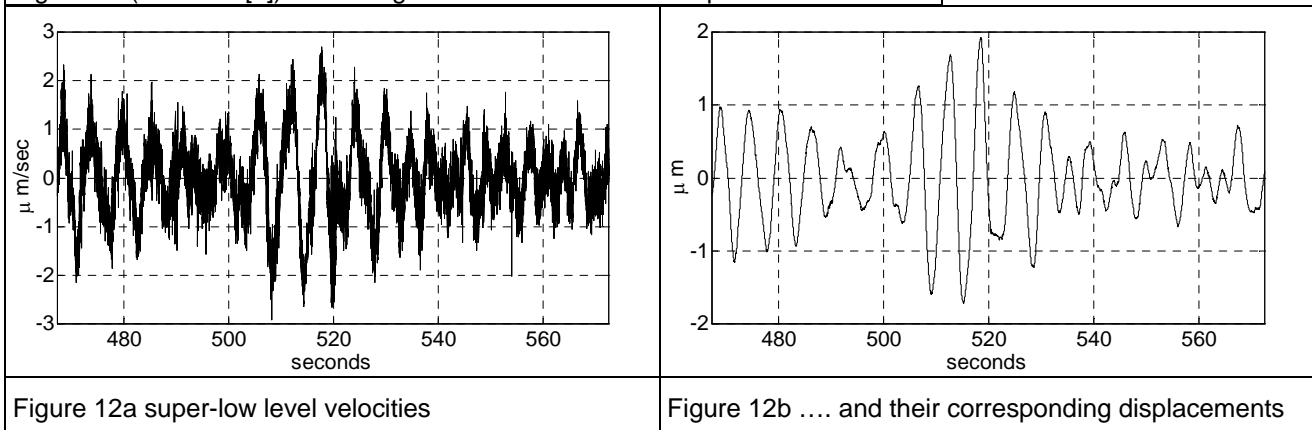


Figure 12a super-low level velocities



Figure 12b …. and their corresponding displacements

The displacement trace is obtained by numerical integration of the velocity signal.

Fig. 13 shows the PSD from this measurement; as well as a probable structural mode around 3Hz and a clear indication of mains interference at 50Hz, there is a clear low frequency motion of the structure, at 0.16Hz with amplitude of several microns. This has implications for structural design where displacements need to be controlled to levels of the order 1μm (1 micron) or less; in such cases body motion of the structure would have to be assumed with criteria relating to relative motions. This leads back to add to the reasons given earlier for quoting acceptable vibration levels for vibration-sensitive facilities.
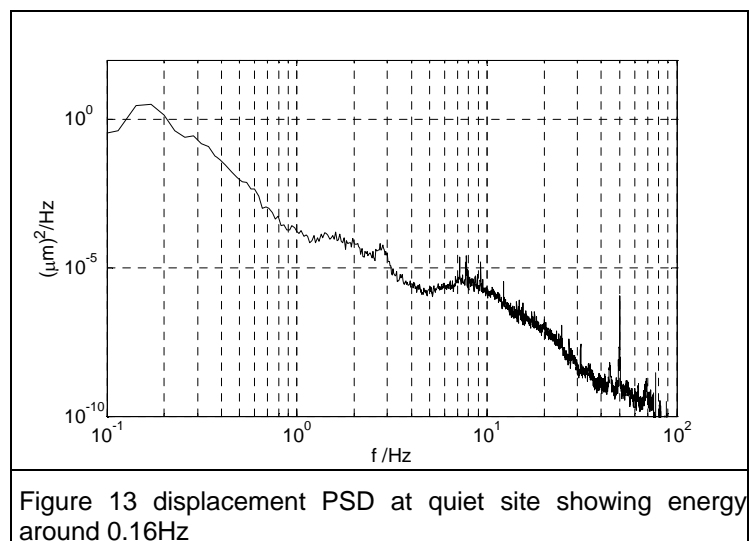


Figure 13 displacement PSD at quiet site showing energy around 0.16Hz

## 11 ONE-THIRD OCTAVE REPRESENTATION

Figures 14-16 show 1/3$^{rd}$ octave velocity plots for Figs. 6,7 and 13. In these plots the RMS velocities are presented in 1/3$^{rd}$ octave bands. These bands [4] are centred on frequencies $2^{\frac{n}{3}}$ Hz with bands extending from $2^{\frac{2n-1}{6}}$ Hz to $2^{\frac{2n+1}{6}}$ Hz.
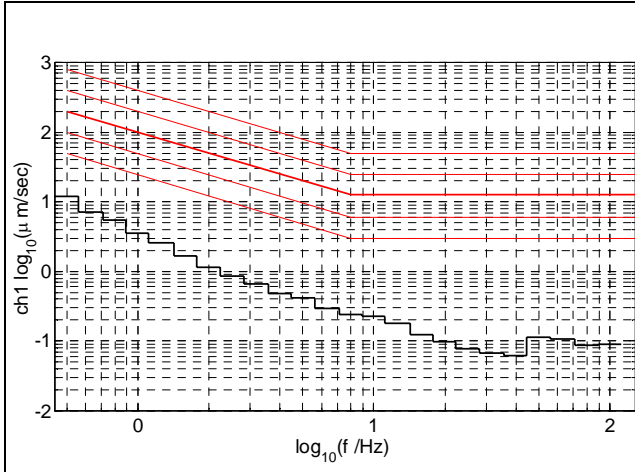


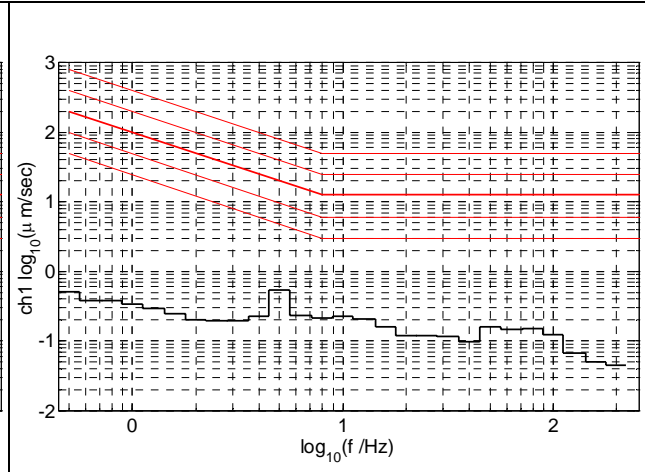Figure 14 1/3$^{rd}$ octave spectrum for Fig. 6, Endevco

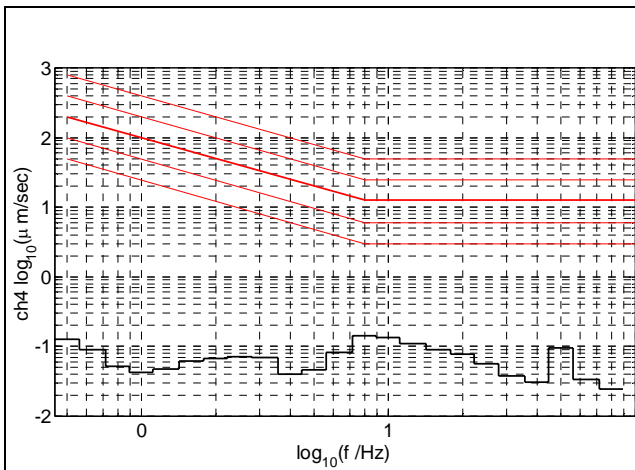Figure 15 1/3$^{rd}$ octave spectrum for Fig. 7, QA700



Figure 16 1/3$^{rd}$ octave velocity spectrum for Guralp

This type of plot is useful since it can be converted to approximate RMS values of acceleration or velocity and used to judge the peak level of harmonic (or narrow band) signal that could be recognised above the noise.

For example for the QA700 signal shown in Figs 7 and 15 (which is not just noise), 1/3$^{rd}$ octave velocity is approximately 0.1μm/sec in bands up to 100Hz, so given statistical peak factors of random signals of about 3, harmonic signals would need to have amplitudes of at least 0.3 μm/sec to be recognised above these levels, e.g. 1.8μg at 10Hz.

## 12 REFERENCES

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4. http://zone.ni.com/devzone/cda/tut/p/id/101