STRATEGIES FOR AEROELASTIC PARAMETER IDENTIFICATION FROM BRIDGE DECK FREE VIBRATION DATA

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ABSTRACT

Several techniques for identification of aerodynamic derivatives (ADs) from free vibration test data are compared using simulated data and test data obtained from wind tunnel tests. These identification methods include system identification from one degree of freedom or two degree of freedom response to either transient excitation or to turbulent buffeting. The experimental and analytical difficulties involved in each method are highlighted and suggestions made for the best approach to determination of ADs in both model and full-scale studies.

Time domain methods using step relaxation provided the best results as long as air-flow turbulence does not cause severe signal to noise ratio problems with the free vibration decay. When, as in full-scale, the turbulence is the primary forcing function, time domain and frequency domain methods can be used to recover the full set of ADs concerning vertical and torsional response.

KEYWORDS

flutter derivatives, system identification, wind tunnel, bridge section model, free vibration

INTRODUCTION

The possible adverse effects of wind on the dynamic behaviour of a suspended span bridge were graphically illustrated by the spectacular collapse of the Tacoma Narrows Suspension Bridge[1] due to wind-induced instability. The investigations following this incident were the beginnings of a major research discipline when aerodynamic techniques and theories developed to deal with 'flutter' instability of aircraft wings were applied to civil engineering structures. One major research tool in these studies has been the wind tunnel test, in which a geometrically and aerodynamically representative scale model of a length of a bridge deck is mounted in a wind tunnel. The aerodynamic forces arising from motion of the bridge section in the air flow can then be measured either directly or indirectly. These motion-dependent forces feed back into the dynamics of the bridge as aerodynamic damping and stiffness and the effect is termed 'aeroelasticity' and described via coefficients or aerodynamic derivatives[2] (ADs) or 'flutter derivatives' which are non-dimensional functions of wind speed, geometry and frequency of oscillations.

Identification of these effects via estimates of the ADs is a vital step in performance predictions for a wind-sensitive full-scale structure. Being non-dimensional functions of the shape of the section they can be applied directly to full-scale in a piecewise manner. While computational fluid dynamics techniques are increasing in power and sophistication, at the time of writing they appear still to have a long way to go before they are capable of simulating with convincing accuracy the effect of intricate geometric details as well as structural motions and turbulence, so wind tunnel tests are likely to remain the most effective tool for years to come.

Wind tunnel testing of bridges take the form of full bridge models[3], taut-strip models[4] and section models[5]. Full models are expensive to build, require large wind tunnels and demand similarity of mass distribution, reduced frequency and mode shapes while taut-strip models, which are much simpler, are intended to model the prototype mode shapes of the deck alone. Generally both of these will have a larger scale ratio and may not be good in study of vortex shedding. In simulating turbulence in wind tunnel bridge tests, similarity of Reynolds number is impractical[6], which will usually not matter for study of bluff body aerodynamics. For study of vortex shedding of streamlined shapes and curved surfaces Reynolds number similarity will be more relevant.

For aerodynamically and geometrically representative models of sections of a deck, only the similarity of reduced frequency (Cauchy number) is required, provided vortex shedding is not an issue. They are intended to represent only the deck; by evaluating the modal integrals involving the prototype mode shapes, the behaviour can be extrapolated to full-scale. Aerodynamics of towers and cables require separate study to avoid introduction of extra variables into the experiment and can be included in the extrapolation[7] to full-scale.

SECTION MODEL TESTS TO DETERMINE AERODYNAMIC DERIVATIVES

Various methods[8] are used to extract the ADs from wind tunnel tests on geometrically and aerodynamically representative models of short sections of the deck. While it is possible to identify the forces from the difference of inertial and excitation forces on a structure forced to vibrate at a single frequency[9], or potentially from pressure taps on the section[10] it is usually experimentally simpler to obtain and analyse free vibration response records[11]. The free vibration may be in response to a transient deflection (step relaxation) or to buffeting caused by the airflow turbulence. Having less emphasis on elaborate equipment and more on the signal processing and data reduction techniques, these procedures are more applicable to full-scale data.

The ADs are usually identified through the effect they have on the free decay vibration characteristics of the model section. A typical wind tunnel test involves suspending a rigid section model from a set of springs so that it can oscillate vertically and in torsion (about a transverse axis) as flutter traditionally involves only these two degrees of freedom. The section can be considered as a rigid body, having (in still air) a pair of uncoupled rigid-body vibration modes each with corresponding natural frequency and damping ratio.

When set in motion in an airflow, changes in the frequency and damping of the two vibration modes and interaction effects between them are identified and ADs are obtained. If the model is restrained to move in pure torsion the effect of the wind will typically be to increase the damping ratio and reduce the natural frequency of the oscillation, and each effect is described by one derivative. From study of damping and stiffness effects in pure vertical or torsional motion four so-called 'direct derivatives' can be obtained for the two degrees of freedom (DOF). When motional restraints are removed the aerodynamic cross-coupling effects between the DOF inherent in the recorded response can be used to identify all the ADs including the additional set of four 'cross-derivatives' linking the DOF. A different approach is to estimate all ADs 'simultaneously' from the response data of a model moving freely in both vertical and torsional direction, due to buffeting loading or an initial excitation or deflection[12,13,14].

The main purpose of the present paper is to present and discuss some procedures for obtaining, from free vibration section model tests, a set of ADs that can be used in the extrapolation to full-scale. A secondary aim is to identify procedures that would have greatest chance of success in extracting derivatives from full-sale response data. The methods are classified according to whether they work on motion restricted to one degree of freedom (with aerodynamic cross coupling disabled) or allow unrestricted interaction, and whether they work in time or frequency domain.

TEST ARRANGEMENT FOR FREE VIBRATION RESPONSE

Figure 1 shows a schematic arrangement of a bridge section model in a wind tunnel with horizontal incident wind having mean speed U. The deck has chord B, mass m and moment of inertia I about the geometric centreline. Accidental or deliberate mass eccentricity is described by a mass m_e at radius r_e leading to total inertia I_T and total mass m_T. The section is attached to a rigid test frame at each corner by linear springs with

stiffness k arranged at distance e upstream or downstream of the geometric centreline. The contributions of spring mass to total mass and inertia are accounted for by adding 1/3 of their mass at their point of attachment. Vertical and torsional displacements and their time-derivatives at mid-chord are respectively denoted $h, \dot{h}, \ddot{h}, \theta, \dot{\theta}, \ddot{\theta}$ and can be recovered from measurement and subsequent signal processing of acceleration records $\ddot{y}_1, \ddot{y}_2$ from the leading and trailing edges of the section. It is also possible to obtain motion records via optical displacement transducers or load cells.

In still air without aerodynamic influence and zero (resultant) mass eccentricity the natural frequencies of the deck for rigid body vibration are in theory obtained as

$$f_\theta = (1/2\pi)\sqrt{k_\theta / I_T} \quad \text{and}$$
$$f_h = (1/2\pi)\sqrt{k_h / m_T}$$

where $k_\theta = 8ke^2$ and $k_h = 8k$.

In practice the test rig and model do not present exact rigid body modes and there may be a degree of torsion or bending present in the model. These effects can be minimised by good construction but may have to be accounted for if measured motion parameters are not representative of the whole section.

In particular test rig flexibility at the connections with the model will lead to apparent spring rates different from nominal values of k. The exact effective values can be identified via the still-air natural vertical natural frequency given the known mass of the deck. Likewise the effective torsional inertia would be obtained from the torsional natural frequency once k is known.

Note that the above convention is not unique; a popular convention used in aeronautics is obtained by simple rotation about the wind axis.

CHOICE OF STRUCTURAL/GEOMETRIC PARAMETERS FOR SECTION TEST

For a wind tunnel with maximum wind speed $\hat{U}$ the values of k and e are chosen to obtain a range of non-dimensional wind speeds U / fB consistent with prototype values of f and design wind speed. For example if the prototype has vertical mode frequency 0.2Hz, a chord of 40m and a design wind speed of 60m/sec, a model with chord 0.6m used in a wind tunnel with top speed of 22.5m/sec should have a maximum vertical mode frequency f_h set via:

$$(U/f_h B)_{prototype} = 7.5 = (\hat{U}/f_h B)_{model} \quad \text{i.e. } f_h = 5\text{Hz, with similar factors applying to } f_\theta. \text{ To provide}$$

stability against torsional divergence and for other practical considerations it is better that $f_\theta > f_h$.

The model mass depends on B and the recommended range[15] of 3 to 8 for the ratio of chord to span, to minimise the effect of deck flexibility in rigid body modes. For best detailing in a limited tunnel width the lower limit may be approached, although the end effects at extremities of the section then become more significant. Appropriate materials are used to achieve geometric accuracy with adequate stiffness to prevent occurrence of the low frequency deformation modes in the model. Given the resulting model mass the spring rate k can be chosen to achieve f_h, f_θ.

Additional considerations for the test rig and suspension arrangement are that a good linear range of spring deflection should be allowed and that the ratio of f_θ to f_h should be adjustable in a wide range either side of unity to suit the requirements of different identification techniques.

The set of eight vertical springs offers no restraint against small lateral or longitudinal deflections nor against rotation about a vertical axis and mechanical arrangements are used to restrain or restrict these. For example, drag wires can be installed to resist but not entirely constrain these motions. Practical issues relating to constraints and geometric effects of the springs are well documented by Hjorth-Hansen[5]. In case it is

desirable to provide restraint against rotation, a set of four additional drag wires may be connected to a rigid vertical bar attached to the section. Likewise, vertical motion may be restrained by anchoring a pair of roller bearings (to allow free rotation about a centroidal cross-wind axis). The drag wires do not offer complete restraint but they increase the stiffness of the restrained DOF to the extent that it cannot contribute significantly to the aerodynamic effects.

GENERAL EQUATIONS OF MOTION

For identification of all eight derivatives involving only vertical and torsional motion, the equations of motion for a 2DOF section with length L and width B, in air flow with density ρ and speed U according to the conventions of Figure 1, are:

$$m_T \ddot{h} + c_h \dot{h} + k_h h + m_e r_e \ddot{\theta} = \frac{\rho U^2 BL}{2} \left[h_1 \frac{\dot{h}}{U} + h_2 \frac{B\dot{\theta}}{U} + h_4 \frac{h}{B} + h_3 \theta \right] + L_{buf} \qquad 1)$$

$$I_T \ddot{\theta} + c_\theta \dot{\theta} + k_\theta \theta + m_e r_e \ddot{h} = \frac{\rho U^2 BL}{2} \left[a_1 \frac{B\dot{h}}{U} + a_2 \frac{B^2 \dot{\theta}}{U} + a_4 h + a_3 B\theta \right] + M_{buf} \qquad 2)$$

On the left hand side the mechanical damping coefficients are c_h, c_θ for each DOF. The right hand sides are aerodynamic lift and moment forces L_{ae}, M_{ae} which evidently depend on non-dimensional coefficients or ADs. The 'direct derivatives' h_1, h_4, a_2, a_3 represent effects within a single DOF response while 'cross-derivatives' h_2, h_3, a_1, a_4 represent coupling between the DOF. Buffeting lift and moment forces are denoted L_{buf}, M_{buf} respectively.

An alternative form for aerodynamic lift and drag forces uses ADs which are frequency dependent coefficients:

$$L_{ae} = \rho U^2 BL \left[KH_1^*(K) \frac{\dot{h}}{U} + KH_2^*(K) \frac{B\dot{\theta}}{U} + K^2 H_3^*(K)\theta + K^2 H_4^*(K) \frac{h}{B} \right] \qquad 3)$$

$$M_{ae} = \rho U^2 B^2 L \left[KA_1^*(K) \frac{\dot{h}}{U} + KA_2^*(K) \frac{B\dot{\theta}}{U} + K^2 A_3^*(K)\theta + K^2 A_4^*(K) \frac{h}{B} \right]. \qquad 4)$$

Note that there are different forms of equations 3) and 4) using for example the half-chord $B/2$ as reference instead of B and using $\rho U^2 BL/2$ instead of $\rho U^2 BL$. Some alternate forms are presented by Zasso[16].

Simple algebraic relations such as $h_1 = 2KH_1^*(K)$ link the A_i^*, H_i^* in equations 3) and 4) to the a_i, h_i in equations 1) and 2) where $K = B\omega/U = 2\pi f B/U$ is the reduced frequency. In equations 1) through 4), the effect of p-derivatives i.e. those relating to lateral (drag) motion, are not considered since this type of motion is restrained. A few treatments[17,18] relating to ultra-long span suspension bridges where interactions with lateral motion are believed to be important are beginning to use the full formulation. Also at least one identification method for the full set of 18 derivatives for 3DOF has been presented[19] and the 'CBHM' method described here has also been extended to cater for 3DOF.

However, cases of classical vertical/torsional flutter are still practically covered using only the vertical and torsional DOF hence the principles are illustrated for 2DOF systems. Whereas both the A_i^*, H_i^* and a_i, h_i are functions of wind speed the form of equations 1) and 2) is used here with the convention of Figure 1, as it delays a decision about which frequency to use in K.

EQUATIONS OF MOTION FOR SDOF RESPONSE

Equations 1) and 2) are examined in single degree of freedom (SDOF) and 2DOF form for parameter identification. Considering SDOF vibration with zero resultant mass eccentricity, equations 1) and 2) simplify to

$$m_T \ddot{h} + c_h \dot{h} + k_h h = \frac{\rho U^2 BL}{2}\left[h_1 \frac{\dot{h}}{U} + h_4 \frac{h}{B}\right] + L_{buf} \qquad 5)$$

$$I_T \ddot{\theta} + c_\theta \dot{\theta} + k_\theta \theta = \frac{\rho U^2 BL}{2}\left[a_2 \frac{B^2 \dot{\theta}}{U} + a_3 B\theta\right] + M_{buf} \qquad 6)$$

having solution for free vibration (transient) decay from an initial deflections h_o, θ_o respectively

$$h(t) = h_o e^{\lambda t} \cos(\omega t + \phi) \qquad 7)$$

$$\theta(t) = \theta_o e^{\lambda t} \cos(\omega t + \phi) \ . \qquad 8)$$

For still-air vertical response, $-\lambda = \xi_h \omega_h$ and $\omega = \omega_h \sqrt{1 - \xi^2}$ where $\xi_h = c_h / 2m_T \omega_h$, $\omega_h = \sqrt{k_h / m_T}$.
For still-air torsional response, $-\lambda = \xi_\theta \omega_\theta$ and $\omega = \omega_\theta \sqrt{1 - \xi^2}$ where $\xi_\theta = c_\theta / 2I_T \omega_\theta$, $\omega_\theta = \sqrt{k_\theta / I_T}$.

For response to random excitation such as by turbulent buffeting the auto-spectrum of vertical response is

$$S_{hh}(\omega) = \frac{S_{ll}}{k_h^2\left[\left(1 - (\omega/\omega_{h'})^2\right)^2 + \left(2\xi_{h'}\,\omega/\omega_{h'}\right)^2\right]} \qquad 9)$$

where S_{ll} is the spectrum of lift forces. The factor S_{ll}/k_h^2 depends on static aerodynamic coefficients but is taken as constant around the model frequencies and $\omega_{h'}, \xi_{h'}$ are natural frequency and damping ratios assumed to be aerodynamically modified. A similar result is obtained for torsion.

SYSTEM IDENTIFICATION FROM 1DOF RESPONSE

In a wind stream with velocity U and vertical response given by equation 7) the direct vertical derivatives h_1, h_4 are found from the shifts in λ, ω given by:

$$-\lambda = \xi_h \omega_h - \frac{\rho UBLh_1}{4m_T} \quad \text{and} \quad \omega^2 = \frac{k_h}{m_T} - \frac{\rho U^2 Lh_4}{2m_T} \qquad 10)$$

Similarly the direct torsional derivatives a_2, a_3 are identified from the shifts in natural frequency and damping ratio from the still-air values:

$$-\lambda = \xi_\theta \omega_\theta - \frac{\rho UB^3 La_2}{4I_T} \quad \text{and} \quad \omega^2 = \frac{k_\theta}{I_T} - \frac{\rho U^2 B^2 La_3}{2I_T} \ . \qquad 11)$$

Hence the identification of h_1, h_4, a_2, a_3 is thus relatively straightforward, almost trivial. To obtain vertical direct derivatives the torsional DOF is restrained and the model is pulled down and released in a steady wind. This method is termed 'step relaxation'. If an acceleration response data acquisition system is used it can be triggered by the sudden large acceleration. Standard curve fitting tools can be used to obtain the best fit of equation 7) to the response signal. From a practical point of view this process is very simple, and it is

possible to use the second derivative of equation 7) with acceleration data directly. Extraction of torsional direct derivatives uses an analogous process.

The free decay method is simple and accurate provided there is a clear decay signal. In the case where the wind speed is very large and the damping coefficient similarly large the useable portion of the trace may be very short and may have a poor signal to noise ratio, the noise being response to buffeting. It is also practically difficult to set a trigger threshold large enough to avoid triggering on buffeting and small enough to be mechanically achievable.

Figure 2 shows experimentally obtained free vertical vibration decay and corresponding curve fitting in still air, at low wind speed and at high wind speed, for section model test data.

For low wind speeds the slight discrepancy which can be observed between the monitored and fitted curves is a result of the mechanical damping of the model being non-linear, i.e. amplitude dependent. A linear fit is however assumed to be satisfactory, according to the linearised equations of motion 1) and 2) and errors due to mechanical non-linearity as well as amplitude-dependent aerodynamic damping can be minimised by starting from a standard amplitude. For the decay with high wind speed the fit is also not exact but for different reasons. At higher wind speeds, even for flows with very low turbulence the response is driven by the turbulence as it decays. Hence the 'better' part of the data with high 'signal' to noise ratio is rather short and also probably displays non-linear damping.

At the stage where buffeting response dominates, it is simpler to use the buffeting response data and find the values of $\omega_{h'}, \xi_{h'}$ to obtain the best fit of equation 9) to the auto power spectrum obtained from the data. Satisfactory identification of $\omega_{h'}, \xi_{h'}$ using this method is subject to a number of conditions[20] such as stationarity of input, flatness of input spectrum, adequate averaging to reduce variance errors, and using sufficient spectral resolution with respect to the width of the peak in the spectrum. Since the wind speed and turbulence spectrum are well controlled in a wind tunnel and the damping is high it is only necessary to record a few minutes of response data, which would (by scaling of frequencies from prototype to model) represent much longer full-scale time series. These data are divided into n records of length T and the minimum value of n is found to obtain a 'confident' fit and the same is repeated for torsional direct derivatives. Given good estimates of $\omega_{h'}, \xi_{h'}, \omega_{\theta'}, \xi_{\theta'}$ the direct derivatives are obtained from

$$h_1 = 4m_T \left(\omega_h \xi_h - \omega_{h'} \xi_{h'} \right) \big/ \rho UBL \qquad h_4 = 2m_T \left(\omega_h^2 - \omega_{h'}^2 \right) \big/ \rho U^2 L .$$

$$a_2 = 4I_T \left(\omega_\theta \xi_\theta - \omega_{\theta'} \xi_{\theta'} \right) \big/ \rho UB^3 L \qquad a_3 = 2I_T \left(\omega_\theta^2 - \omega_{\theta'}^2 \right) \big/ \rho U^2 B^2 L .$$

12)

Figure 3 shows buffeting response from section model test and corresponding satisfactory curve fitting at the same low and high wind speeds as Figure 2. The damping estimates are slightly different in the two figures.

As an alternative to frequency domain analysis of the random response, random decrement signature and auto-correlation function could also be used to obtain the single mode impulse response function. The auto- and cross-correlation functions are the starting point for the 2DOF time domain identification method discussed next.

EQUATIONS OF MOTION FOR 2DOF RESPONSE

The equations of motion 1) and 2) may be rewritten in matrix form:

$$\mathbf{M}\ddot{\mathbf{z}} + \mathbf{C}_{str}\dot{\mathbf{z}} + \mathbf{K}_{str}\mathbf{z} = \mathbf{C}_{ae}\dot{\mathbf{z}} + \mathbf{K}_{ae}\mathbf{z} + \mathbf{p}(t)$$

13)

in which

$$\mathbf{M} = \begin{bmatrix} m_T & m_e r_e \\ m_e r_e & I_T \end{bmatrix} \qquad \text{represents the mass with mechanical coupling,}$$

$$\mathbf{C}_{str} = \begin{bmatrix} 2\xi_h\omega_h m_T & 0 \\ 0 & 2\xi_\theta\omega_\theta I_T \end{bmatrix} \qquad \text{represents mechanical damping,}$$

$$\mathbf{K}_{str} = \begin{bmatrix} m_T\omega_h^2 & 0 \\ 0 & I_T\omega_\theta^2 \end{bmatrix} \qquad \text{represents mechanical stiffness,}$$

$$\mathbf{C}_{ae} = P\begin{bmatrix} h_1/U & h_2\,B/U \\ a_1\,B/U & a_2\,B^2/U \end{bmatrix} \qquad \text{represents aerodynamic damping and}$$

$$\mathbf{K}_{ae} = P\begin{bmatrix} h_4/B & h_3 \\ a_4 & a_3 B \end{bmatrix} \qquad \text{represents aerodynamic stiffness, with } P = \frac{1}{2}\rho U^2 BL.$$

Vectors of measurable response and of buffeting load are $\mathbf{z} = \begin{bmatrix} h \\ \theta \end{bmatrix}$, $\mathbf{p} = \begin{bmatrix} L_{buf} \\ M_{buf} \end{bmatrix} = \begin{bmatrix} g_h \\ g_a \end{bmatrix} u(t)$.

The buffeting input is represented by a common (wind dependent) forcing function $u(t)$ and two gain factors g_h, g_a which depend on mean wind speed, section shapes and static aerodynamic coefficients and it is implied that the lift force and moment due to buffeting are coherent.

Equation 13) can be transformed to 'state space' form:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \qquad\qquad\qquad\qquad 14a)$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}u \qquad\qquad\qquad\qquad 14b)$$

where $\mathbf{A} = \begin{bmatrix} \mathbf{M}^{-1}(\mathbf{C}_{ae} - \mathbf{C}_{str}) & \mathbf{M}^{-1}(\mathbf{K}_{ae} - \mathbf{K}_{str}) \\ \mathbf{I} & \mathbf{0} \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} \dot{\mathbf{z}} \\ \mathbf{z} \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} \mathbf{M}^{-1}\begin{bmatrix} g_h & g_a \end{bmatrix}' \\ \mathbf{0} \end{bmatrix}$.

For the case of free vibration response to a transient, $\mathbf{B}$ and $\mathbf{D}$ are null matrices, otherwise they connect with the common forcing function $u(t)$. $\mathbf{C}$ depends on which response parameter is observed. Initial conditions are given as $\mathbf{x}_o = \begin{bmatrix} \dot{h}_o & \dot{\theta}_o & h_o & \theta_o \end{bmatrix}$. For the case of response where $u(t)$ is approximately described as a Gaussian white noise process, such as excitation by turbulence in the air stream, the initial conditions are taken as zero.

SYSTEM IDENTIFICATION FROM 2DOF RESPONSE

Three methods are used for identifying the system matrix $\mathbf{A}$. The first method uses time domain free decay records, the second uses either free vibration decay or random response (from turbulent buffeting and the third, in frequency domain, uses turbulent buffeting response.

Direct curve fit to 2DOF equations of motion: PEM

For the case of free vibration due to an initial deflection, MATLAB[21] system identification routines such as 'pem'[22] are used to identify the values of $\mathbf{A}$ and $\mathbf{x}_o$ for which the time histories generated using equation (14) give the best match to the observed data. The quality of the fit is judged both visually in terms of overlays of fitted and measured data as well as by error norm values. Software 'PEM' was written around this technique based on software developed at Politecnico di Milano[23]. Figure 4 shows examples of experimental free vibration response from initial displacement, low pass filtered above 2Hz, and the best fit response generated by PEM.

Covariance Block Hankel Matrix method: CBHM

When free vibration is due to turbulence, two more methods described here are available. In the first of these, the covariance block Hankel matrix method[12], it is shown that the state matrices can be recovered from the cross-covariance estimates obtained from the two motion signals such as acceleration or displacement. The program uses MATLAB[21] elementary functions. The cross-covariance functions are known to be the same for both transient and buffeting response so the method can also be used for transient response signals.

The solution to equation 14a) is

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}_O + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}u(\tau)d\tau \qquad\qquad 15)$$

where eigenvalues of $\mathbf{A}$ are $\lambda_{\mathbf{A}} = -\xi\omega \pm i\omega\sqrt{1-\xi^2}$.

A discrete time version of equation 14) representing sampled data is

$$\mathbf{x}(i+1) = \mathbf{F}\mathbf{x}(i) + \mathbf{G}u(i) \qquad\qquad 16)$$

where $\mathbf{F} = e^{\mathbf{A}\Delta t}$ and $\mathbf{G} = \int_0^{\Delta t} e^{\mathbf{A}\tau}d\tau\mathbf{B}$.

The identification method computes, from at least N_p response data points sampled at intervals of Δt, a sequence of covariance matrices which (for no signal noise and a state variable covariance matrix $\mathbf{C_{xx}}$) depend on the system matrices as defined in equations 14) as follows:

$$\mathbf{C_{yy}}(k) = \begin{bmatrix} C_{hh}(k) & C_{h\theta}(k) \\ C_{\theta h}(k) & C_{\theta\theta}(k) \end{bmatrix} = \mathbf{C}\mathbf{F}^k\mathbf{C_{xx}}\mathbf{C}^T \quad k = 1\dots 2l-1; 2l << N_p . \qquad 17)$$

By taking the sequence $\mathbf{C_{yy}}(k)$ as blocks of a Hankel matrix with dimension 2l x 2l, the Covariance Block Hankel Matrix (CBHM) method[12] finds a suitable decomposition of the $\mathbf{C_{yy}}(k)$ to yield $\mathbf{F}$, hence $\mathbf{A}$. This technique is implemented in software CBHM developed at Norwegian Institute of Technology.

The heart of the algorithm is essentially the same as the eigensystem realisation algorithm (ERA)[24] and shares its advantages of simplicity and efficiency.

Cross Power Spectrum Method; CPS

In the second method for random (turbulent) response data, the best fit between auto power and cross power spectra of acceleration responses from experiment and simulation using the parameters in the state matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ is obtained.

Taking Laplace transforms of equations 1) and 2), and including the inertia and pressure values in the coefficients by writing:

$$p_{h_1} = \left(\rho UBL/2m_T\right)h_1, \qquad\qquad p_{h_2} = \left(\rho UB^2L/2m_T\right)h_2,$$
$$p_{h_3} = \left(\rho U^2BL/2m_T\right)h_3, \qquad\qquad p_{h_4} = \left(\rho U^2L/2m_T\right)h_4,$$
$$p_{a_1} = \left(\rho UB^2L/2I_T\right)a_1, \qquad\qquad p_{a_2} = \left(\rho UB^3L/2I_T\right)a_2$$
$$p_{a_3} = \left(\rho U^2B^2L/2I_T\right)a_3 \qquad\qquad p_{a_4} = \left(\rho U^2BL/2I_T\right)a_4$$
$$p_{c_1} = m_e r_e/m_T, \qquad\qquad p_{c_2} = m_e r_e/I_T ,$$

leads to

$$\begin{bmatrix} s^2 + 2\xi_{h'}\omega_{h'}s + \omega_{h'}^2 & p_{c_1}s^2 - p_{h_2}s - p_{h_3} \\ p_{c_2}s^2 - p_{a_1}s - p_{a_4} & s^2 + 2\xi_{\theta'}\omega_{\theta'}s + \omega_{\theta'}^2 \end{bmatrix} \begin{bmatrix} H(s) \\ \Theta(s) \end{bmatrix} = \begin{bmatrix} g_h/m_T \\ g_a/I_T \end{bmatrix} U(s) \ . \tag{18}$$

Here, in-wind SDOF natural frequencies and damping ratios $\omega_{h'},\xi_{h'},\omega_{\theta'},\xi_{\theta'}$ from equation 12) are employed.

The frequency domain form of equation 18) using $s = i\omega$ provides the theoretical Fourier transforms $H(\omega)$, $\Theta(\omega)$. These are compared with measured equivalents obtained as follows.

The free vibration response vertical and torsional response to turbulence $h(t)$, $\theta(t)$ are recorded as a sequence of n records with duration T. The forward Fourier transforms $G_h(\omega)$, $G_\theta(\omega)$ for each record are obtained and averages of $G_h(\omega) \cdot G_h^*(\omega)$, $G_\theta(\omega) \cdot G_\theta^*(\omega)$, $G_h(\omega) \cdot G_\theta^*(\omega)$ and $G_\theta(\omega) \cdot G_h^*(\omega)$ computed.

The averaged measured auto-power and cross-power spectra are compared with simulated functions from equation 18) over specific narrow frequency ranges in which ADs are slowly varying and are taken as constant:

$$E\big(G_h(\omega) \cdot G_h^*(\omega)\big) = S_{hh}(\omega) \qquad \text{vs.} \qquad H(\omega) \cdot H^*(\omega) \text{ near } \omega_h \tag{19}$$

$$E\big(G_\theta(\omega) \cdot G_h^*(\omega)\big)\big/E\big(G_h(\omega) \cdot G_h^*(\omega)\big) \qquad \text{vs.} \qquad \Theta(\omega)/H(\omega) \text{ near } \omega_h \tag{20}$$

$$E\big(G_h(\omega) \cdot G_\theta^*(\omega)\big)\big/E\big(G_\theta(\omega) \cdot G_\theta^*(\omega)\big) \qquad \text{vs.} \qquad H(\omega)/\Theta(\omega) \text{ near } \omega_\theta \tag{21}$$

$$E\big(G_\theta(\omega) \cdot G_\theta^*(\omega)\big) = S_{\theta\theta}(\omega) \qquad \text{vs.} \qquad \Theta(\omega) \cdot \Theta^*(\omega) \text{ near } \omega_\theta \ . \tag{22}$$

where E() denotes averaging. The experimental transfer functions in equations 20) and 21) above are the 'H1' type[25] according to terminology used in modal analysis.

It is possible to draw some conclusions about the effects of one pair of cross-derivatives in the case of the remaining pair (either cross a-derivatives or h-derivatives) being negligible if equation 18) is written as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} H(s) \\ \Theta(s) \end{bmatrix} = \begin{bmatrix} g_h/m_T \\ g_a/I_T \end{bmatrix} U(s) \text{ from which } \begin{bmatrix} H(s) \\ \Theta(s) \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} g_h/m_T \\ g_a/I_T \end{bmatrix} U(s) \Big/ [DA - BC].$$

Assume zero mechanical coupling and zero influence of h in θ i.e. $p_{c_1} = p_{c_2} = p_{a_1} = p_{a_4} = 0$ and also that $\omega_{\theta'} \gg \omega_{h'}$ so that $A \to -\omega^2$ for 'high' frequencies around $\omega_{\theta'}$ In this case $C=0$ and we can write

$$H(\omega)/\Theta(\omega) \approx \Big\{1 - (\omega_{\theta'}/\omega)^2 - 2i\xi_{\theta'}\,\omega_{\theta'}/\omega\Big\}(g_h I_T/g_a m_T) \quad -ip_{h_2}\big/\omega - p_{h_3}\big/\omega^2 \ . \tag{23}$$

Hence the influence of θ in h can be seen in that h_2 shows up as an adjustment to the imaginary part of the uncoupled transfer function while h_3 shows up at $\omega_{\theta'}$ as an offset (from zero) of the real part. Although both responses are in principle due to the same input, the transfer function representation used here is a ratio between the responses. In practice one response (e.g. θ) can be considered to drive the other (e.g. h).

Figure 5 shows the form of the simulated functions of equations 19-22) while varying p_{h_2}, p_{h_3} the parameters proportional to H_2^* and H_3^* by multiples of 1 and 10 respectively, taking other derivatives as zero and total damping of approximately 1% for each mode. The lower curves show the effect of θ in h.

The effects of varying p_{h_2}, p_{h_3} are seen to be as described above, mainly as the effect of torsion on vertical motion at the torsional frequency. Hence in this case with the modes lightly damped, the derivatives could be estimated by studying only cross-power at one (torsional) frequency.

The coupling terms naturally affect the amplitude of vertical response (top left curve) and they also appear in the imaginary part of $\text{Im}(\theta/H)$ around f_h.

In principle single values of real and imaginary parts of equations 19) and 20) at specific frequencies together with the recovered values of $\omega_{h'}, \xi_{h'}, \omega_{\theta'}, \xi_{\theta'}$ would be enough to identify h_2, h_3 or a_1, a_4 respectively.

Software 'CPS' for iterative frequency domain curve fitting was developed which can make use of direct derivatives to start the iteration and judging convergence by graphical and numerical indicators.

Strictly speaking since the vertical and torsional response both derive from the same signal they should always be coherent and even zero coupling could be identified, but in practice the coherence of buffeting forces implied in equation (14) is not realised in the test data. If coherence values are low at one or both the modes, the relevant cross-derivatives are fixed at zero.

Note that this method is related to an earlier procedure [26] in which the phase and amplitude ratios of responses in different degrees of freedom are recovered from correlation functions obtained from free decay. Study of the correlation function is analogous to study of the CPS transfer function matrix at the modal frequencies and provides direct information on the aerodynamic coupling.

EVALUATION OF 2DOF IDENTIFICATION TECHNIQUES: DATA AND RESULTS

The various 2DOF identification techniques were tested with four data sets derived by simulations and from wind tunnel testing.

Identification from simulated data

Time histories were generated for a fictitious section having plausible derivatives defined by polynomials in *U/fB*. The section has vertical fundamental frequency of 3.6Hz, torsional mode at 5.2Hz and structural damping at 0.4% in each DOF. Acceleration and displacement data were generated using the SIMULINK[27] software for wind speeds up to 30m/sec for the following scenarios:

a) Simulated free decay from impulsive load: 512 samples at 0.01second interval.
b) Simulated response to Gaussian random excitation: 32768 samples at 0.02second interval.

Figure 6a shows results from simulated impulsive decay using time domain software PEM and CBHM. The two cross-derivatives H_2^* and A_4^* are set to zero in the simulations. The identified derivatives are plotted against non-dimensional wind speed *U/fB*.

In the plots vertical frequencies $\omega = \omega_h$ and $f=f_h$ are used in $K = \omega B/U$ and *U/fB* respectively for the ADs describing the effects of vertical motion ($H_1^*, A_2^*, A_3^*, H_4^*$) while torsional frequencies are used for ADs describing effects of torsion ($A_1^*, H_2^*, H_3^*, A_4^*$). It is also possible to use vertical frequencies for all the H_i^* and to use torsional frequencies for all the A_i^*, which may be less physically meaningful.

Clearly both techniques work perfectly with such data. The scatter (about the true value, 0) in values of A_4^* is in fact very small, but is smallest for PEM.

More challenging is the application to simulated random loading. Figure 6b shows results from CBHM (circles); the departure from the exact values are small and the method works well. CPS (squares) works well until A_3^* and H_4^* bring about 'frequency coalescence', i.e. merging of vertical and torsional frequencies. In

this case the cross-derivatives were obtained by fixing the direct derivatives at known values (e.g. from SDOF tests) in the identification. Both methods show scatter in values which should be zero.

Identification from experimental data

Response time histories were obtained from wind tunnel tests on two different section models, shown schematically in Figure 7 for the following scenarios.

a) Experimental free decay from step relaxation for section type 1: 2048 samples at 0.006second intervals.
b) Experimental response to turbulent buffeting for section type 2: 8192 samples at 0.01second intervals.

Figure 8 shows results obtained by application of the methods to experimental data. Figure 8a shows identified ADs using CBHM and PEM for experimental free decay from step relaxation of section 1 in wind with turbulence of less than 3%.

There is little to choose between the two techniques, based on scatter, and it is interesting to note the different trend for H_4^*, which had always been a difficult parameter to identify even in 1DOF tests, and until a decade ago was not normally considered. Like A_4^* it apparently depends on vertical position or acceleration, and has no physical basis according to quasi-static considerations.

Figure 8b shows identified ADs using CBHM for experimental data from section 2 obtained for flow with along-wind and vertical turbulent intensities of approximately 8%. Data for the lowest wind speeds are omitted since the response is very low and the signal to (instrumentation) noise ratios are poor. In these data the in-wind modified frequencies are used in $K = \omega B/U$ and U/fB rather than the still-air values as used in other figures.

Figure 8c shows ADs estimated from the same data using CPS, and also using in-wind reduced frequencies. The data are much less satisfactory, being more scattered, and only A_3^* and H_1^* values are consistent with CBHM data. The data cover only 80 seconds, which do not allow for sufficient averaging and resolution of the response Fourier spectra. For the section studied one mode has high damping and one mode low damping so it is not possible to trade off resolution for averaging within the available data. Parameters from limited (but longer duration) records of section 1 show reasonable agreement with values from PEM.

EVALUATION OF SDOF AND 2DOF IDENTIFICATION TECHNIQUES: PRACTICAL CONSIDERATIONS

Time domain free decay identification methods are generally more satisfactory as long as the signal to (turbulence-induced) noise is adequate. For higher wind speeds where the initial free decay is drowned by the turbulent buffeting, CPS or CBHM would be preferable.

Only CPS can make use of direct derivatives to help the identification of the cross derivatives. The other methods (PEM or CBHM) do not and cannot make use of known direct derivatives, they only identify all together, which simplifies proceedings.

In order to recover a full set of derivatives the comparisons seem to show that analysis of free decay from an impulsive load or step relaxation would produce the best (at least clearest) results using either time domain method (PEM or CBHM) provided the turbulence is small. Given the luxury of being able to conduct such tests in the right conditions of smooth flow with minimal turbulence either method would be viable.

To check which of PEM and CBHM perform best in turbulent flow, Fig. 9 compares the performance of CBHM and PEM. ADs were identified from identical response data generated with the same simulated parameters as Fig. 6a, but with significant added simulated turbulent buffeting applied to both DOF, having magnitude increasing with the square of wind speed. Fig 10 shows the signal used for a mid-range wind speed of U=15m/sec; only the first 0.5 seconds of each record were used in the identification. For highest wind speeds and with the exception of H_1^*, CBHM still managed to recover the trend while PEM could not converge.

When turbulent forcing becomes significant in free decay, only the strongest part of the decay can be used. At this point ambient response methods CBHM or CPS would be more appropriate, requiring a longer time series. The scatter for free decay methods is greatest at high wind speeds (Fig 9) while for ambient response methods it is greatest at low wind speeds, but should improve for models with lower structural damping i.e. stronger response. Judgement is needed in each case as (for CBHM) the ADs are more sensitive to the length of covariance function used while in CPS the frequency functions are very 'bumpy' unless very long averaging or some kind of frequency domain smoothing (e.g. wavelets) is used.

Between the two methods (CBHM and CPS), CBHM generally works better and requires relatively few data points. CPS suffers from the usual frequency domain problems of requiring very long time series to get low variance (hence smooth curves) and good resolution to identify direct damping derivatives when damping is very low and peaks are sharp. In either case single degree of freedom tests would be available to confirm the direct derivatives.

For applications to full-scale data, free-decay and single degree of freedom methods cannot be used, since for a long span bridge only ambient multi-degree of freedom response data would available. It would be a challenge to evaluate full-scale ambient data using these methods. In reality, at full-scale the response is usually a result of turbulence at a higher level (may be 10% or higher) so methods found to work best in wind tunnels with turbulent flow, e.g. CBHM, may be appropriate.

Note that the influence of turbulence on ADs is still debatable and will depend on the section; for section 1 testing showed little effect, and Sarkar[28] found little difference for a streamlined section, while tests on a rectangular box girder bridge [29] showed galloping in smooth flow. If tests on a freely suspended section show similar performance with respect to development of large amplitude motions in smooth and turbulent flow, this would indicate small effect on the ADs. The study of the effect of turbulence places great demands on the identification methods, as for full-scale identification and the ambient response methods would be more appropriate.

One of the issues relating to the identification is the 'feel' of the procedure and the means of judging the quality of the result. Certainly as successive data points are generated in the AD plots it become clear what the trend is (if any) and whether the method is working. On the other hand when extracting a single data point (i.e. a set of derivatives at one wind speed) the success can be judged directly if the matching of time history or frequency response curves with the those simulated from the identified system can be presented. One way is to use a visual overlay of the two curves, the other is to use correlation coefficients or normalised error indicators[30].

CONCLUSIONS

Several methods for identification of aerodynamic derivatives from single degree of freedom (SDOF) or two degree of freedom (2DOF) response of wind tunnel models have been outlined and results from application of three 2DOF methods have been presented. The comparison show that free decay in smooth flow leads to the best (i.e. least scattered) results while the covariance block Hankel matrix method can use a quite small amount of ambient response data to produce acceptable results and is very robust. Frequency domain techniques can work and provide insight to the coupling effects but require lengthy time series. The methods complement each other if the model responses in smooth and turbulent flow are similar regarding critical wind speeds and large amplitude response.

ACKNOWLEDGEMENTS

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