# Bridge Structural Condition Assessment Using Systematically Validated Finite Element Model

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## Abstract

Structural condition assessment of highway bridges is largely based on visual observations described by subjective indices, and it is necessary to develop methodology for accurate and reliable condition assessment of aging and damaged structures. This paper presents a method using a systematically validated finite element model for quantitative condition assessment of a damaged reinforced concrete bridge deck structure, including damage location and extent, residual stiffness evaluation and load-carrying capacity assessment. In a trial of the method in a cracked bridge beam, the residual stiffness distribution was determined by model updating thereby locating the damage in the structure. Furthermore the damage extent was identified through a defined damage index and the residual load-carrying capacity was estimated.

**Key words**: Finite element model updating, condition assessment, bridge, damage, residual stiffness, load-carrying capacity.

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## 1. Introduction

Many existing bridges have been deemed structurally deficient (Aktan et al. 1996), but despite the extent of the problem, condition assessment of bridges is still largely based on visual observations and described by subjective indices which do not permit accurate evaluation of serviceability and safety. Subjective or inaccurate condition assessment has been identified as the most critical technical barrier to the effective management of highway bridges. Perhaps the most significant challenge to bridge condition assessment is the quantification of information on bridge condition by development of technologies for objective and accurate condition assessment and reliability evaluation.

A significant research effort has focussed on the condition assessment of existing bridges, and relevant research has accelerated in recent years (Aktan et al. 1996,1997,1998; Enright and Frangopol 1999; Wahab and Roeck 1999). However, most investigations were based on field testing and numerical analysis without attempts to quantify structural condition. In addition, most of research effort has focussed on damage detection with very little research on stiffness and load-carrying capacity assessment.

A procedure that can be applied in condition assessment is the finite element (FE) model updating method (Mottershead and Friswell 1993) that integrates finite element analysis (FEA) and experimental modal analysis (EMA). This model updating technique can produce a systematically validated FE model by correcting uncertainties from modeling, geometry, physical and analysis to improve the analytical results closing to real values based on the dynamically measured data. It has emerged in the 1990s as a subject of great importance for mechanical and aerospace structures. However the technique may be difficult to apply for civil engineering structures, because of the constraints on prototype testing and experimental data analysis resulting from the nature, size, location and usage of these structures. Recently, the civil engineering community has begun to apply this technique. For example, Cantieni (1996) investigated model updating of a concrete arch bridge while Pavic et al. (1998) and Reynolds et al. (1998) have been applying the technique to footbridges and concrete floors, while applications to cable supported bridges have been reported by Brownjohn (1997) and Brownjohn and Xia (1999; 2000).

This paper describes the application to quantitative condition assessment of a damaged reinforced concrete (RC) bridge deck model, whose FE model was pre-validated and post-validated systematically by model updating based on dynamically measured data from the undamaged and damaged structure. The damage location and quantification of the damaged structure were then identified, leading to residual stiffness and load-carrying capacity assessment.

## 2. Damaged Bridge Structure and Finite Element Modeling

The bridge deck used in the investigation was a RC slab/beam structure comprising a shallow lightly RC slab supported by two deep RC edge beams and simulating a short span pedestrian bridge, details of which are shown in Fig. 1. The edge beams were $250mm$ deep by $150mm$ wide and overall structure width was $1m$. The bridge deck was simply supported on the concrete blocks.

In order to produce damage in the RC bridge deck, the static loads were applied at the midspan location of the structure until a maximum load of $47kN$ was sustained. Being an under-reinforced beam, the failure (yield of tension steel) was accompanied by tensile cracks that developed at midspan of one edge beam component. Because of the damage in the beam components of the bridge deck, the structural condition (stiffness and strength) had been degraded, leading to a reduction of the serviceability and load-carrying capacity. If such a structure is to remain in use after damage, accurate assessment of the condition would be vital for safety.

In FE modeling the bridge deck structure, two edge beams of the test structure were modeled using 3D beam elements, the slab between the beam was modeled using shell elements and the boundary conditions were simulated using eight linear springs. However, preparation of a FE model to be a candidate for updating requires some specific considerations of additional factors not normally taken into account in conventional FE model construction. Of these, an important one is that uncertainties in a structure must be expressed quantitatively as parameters. When damage is known to exist in a localised area in a structure, one way to simulate the damage is to incorporate some 'weak' elements (Brownjohn and Xia 1999) into an FE model. This avoids the problem of damage detection that would require far more unknowns. For the purpose, four beam elements were used to represent the damage zones at the midspan in the edge beams components. The resulting FE model for model updating is shown in Fig. 2 in which numbers represent beam elements of the edge beam component ands the 'weak' beam elements are numbered 10-13. If the parameters take the real values for the damage zones, then the FE model is taken to represent the damaged bridge deck, but if the parameters match those for the rest of the beam the FE model represents the undamaged bridge. By estimating the parameters of the 'weak' beam elements through model updating based on measured data, the damage is quantified and the residual stiffness and load-carrying capacity would be determined.

## 3. Validating FE Models

FE models can be validated systematically by correcting uncertainties in the structure based on the dynamically measured data. Dynamic properties of a structure are normally very sensitive to boundary conditions and these were uncertainties for the bridge deck structure. Also, the Young's modulus and to a lesser extent mass density of concrete were not well defined. Hence it was very important to determine these physical parameters for the structure, in the undamaged state, to establish a reliable initial FE model prior to condition assessment of the damaged bridge. The validation of FE model thus consisted of two steps:

(1) pre-validating the FE model for the undamaged structure before loading, based on the dynamically measured data on the undamaged structure, to determine the uncertain physical parameters of the structure and provide a reliable initial FE model for damage identification of the damaged structure; and

(2) post-validating FE model for the damaged structure, based on the dynamically measured data from the damaged structure, to identify the damage and assess the structural condition.

If FE model validating goes straight to the damaged case, then the identified damage may reflect the uncertain structural parameters not associated with damage, furthermore the updating may not converge because of too large differences between the modal properties generated by the FE model and those measured. For this reason, vibration tests were performed using modal testing technique (Ewins 1984) on the undamaged and damaged structures for updating their respective FE models. In both cases, dynamic properties i.e. frequencies and mode shapes were identified using excitations provided by instrumented hammer and electrodynamic long stroke inertial shaker. Both techniques applied a broadband excitation, the shaker via a continuous chirp or fast sine sweep (Godfrey 1993) and the hammer by impulsive load. Using two methods provided a high quality of test data. The measured frequencies $f_e$ are listed in the second and sixth column in Table 1 respectively for the undamaged and damaged structure. The measured mode shapes are shown in the first column of Fig. 3 (the dots represented the measured data) and show some important features:

(1) there were significant responses at the supports (concrete blocks) which were obviously not rigid, as commonly assumed. These were simulated by linear springs in the FE model, as shown in Fig. 2;

(2) the frequencies of the cracked structure were slightly reduced consistent with the expected reduced stiffness compared to the undamaged structure;

(3) there was slightly sharper curvature of first and third mode shapes at midspan of the damaged structure and

(4) the frequencies and mode shapes were slightly different for damaged and undamaged structures.

Based on the measured frequencies and mode shapes, a FE model can be validated and then updated. Generally, a sensitivity analysis based model updating procedure includes three aspects (Brownjohn and Xia 1999; 2000):

(1) selection of responses as reference data which are normally the measured data such as measured frequencies and mode shapes;

(2) selection of physically and geometrically uncertain parameters to which changes in the selected reference responses should be sensitive; and

(3) model tuning, an iterative process to update or modify the selected parameters based on the selected reference data.

After model updating, the validity of the FE model can be checked by correlation indicators such as difference between FE model and experimental natural frequencies and by the modal assurance criterion or $MAC$ (Allemang and Brown 1982) defined by:

$$MAC(\phi_a, \phi_e) = \frac{\left|\phi_a^T \phi_e\right|^2}{\left(\phi_a^T \phi_a\right)\left(\phi_e^T \phi_e\right)} \tag{1}$$

where, $\phi_a$ and $\phi_e$ are the analytical and experimental mode shape vectors, respectively. Given a set of experimental modes and a set of predicted modes, a matrix of *MAC* values can be computed. The mode shapes with a *MAC* value equal to 100% represent a perfect correlation, modes which are completely orthogonal have 0% *MAC*. Generally, it is found that a value in excess of 90% should be attained for correlated modes depending on how the degrees of freedom to be correlated are selected; for a small number of modes visual correlation may work better.

In the FE model pre-validating for the undamaged bridge deck, the stiffness of the boundary supports, Young's modulus and mass density of concrete were uncertain, as described previously, and selected to update. The updated frequencies $f_u$, frequency differences $\Delta_f$ and *MAC* values correlating the measured and updated data are listed in the first, third and fourth column respectively in Table 1. The low $\Delta_f$ and high *MAC* values show an excellent correlation between updated FE model and test model. The comparison of the updated and measured mode shapes is shown in the first column in Fig. 3, where. The pre-validated FE model could be taken as a reliable initial FE model with correct boundary conditions and physical parameters for further damage identification. The updated value of Young's modulus of concrete was found to be $E_c = 2.8\times10^4 N/mm^2$, a reduction on the value $E_c = 3\times10^4 N/mm^2$ estimated based on sample cube tests.

For the damaged bridge deck, the cracking in the concrete was widest and deepest at midspan, whereas narrower and shallower minor cracks developed towards the supports. The cracks effectively reduced the section moment of inertia and stiffness. Hence the moment of inertia of the beam and the cross section area were taken as uncertain parameters to update for the four midspan 'weak' beam elements in the FE model around midspan. To observe the effect of beam bending stiffness on the dynamic properties of the bridge deck, the sensitivity analysis of dynamic properties to the beam cross section's moment of inertia in bending was conducted through sensitivity coefficients. These are defined as the rate of change of a particular response quantity with respect to a change in a structural parameter and the set of sensitivities of $N$ structural responses $R_i$ ($i = 1,\ldots,N$) to $M$ structural parameters $P_j$ ($j = 1,\ldots,M$) are collected in a sensitivity matrix $[S]$ defined as,

$$[S]_{ij} = \frac{\partial R_i}{\partial P_j} \tag{2}$$

Fig. 4 and Fig. 5 shows the envelop of sensitivities of the measured frequencies and $MAC$ values to the cross section's moment of inertia along the beam, respectively i.e.

values in one column of *S*. The horizontal coordinates in Fig. 4 and Fig. 5 represent the distribution of beam elements in which element 11 and 12 were at the midspan of the beam. It can be found that the most sensitive area is located at element 7 and 16, while the midspan (element 11 and 12) has low sensitivity. Hence the changes in dynamic properties of the damaged structure were not too obvious in spite of the serious damage at the midspan.

The initial estimate of midspan moment of inertia should be realistic; if the initial value is too far from the true value, then the iterative updating process may diverge. Often, it is required to carry out manual tuning by engineering judgement or relevant preliminary estimation. For the damaged bridge deck, the initial value of moment of inertia of cracked beam section at the midspan was estimated to be $I_{bcr} = 5.3\times10^7 mm^4$ which was obtained from analysis of the RC beam at the post-cracking stage (Nawy 1990).

Based on the model tuning based using the measured data from the damaged structure, the updated frequencies $f_u$ of the FE model with ‘weak’ beam elements are listed in the fifth column in Table 1. The frequency differences $\Delta_f$ and correlation *MAC* values between updated and measured are listed in the seventh and eighth column respectively showing very small $\Delta_f$ and very high *MAC* values. The other ways to correlate the updated data with the measured data are shown in Fig. 6 and Fig. 7. Fig. 6 shows pairing of frequencies between the updated and measured models emphasizing errors as departures from a diagonal line with unit slope. Fig. 7 shows the *MAC* matrix with high values were high for comparable modes and dissimilar modes off the diagonal indicated by values close to zero. The mode shapes of the updated FE model for the damaged structure are visualized in the second column in Fig. 8 and very close to the measured mode shapes. All of these correlation analyses between updated and measured data illustrate that the model updating was successful for validating the FE model with 'weak' beam elements, safeguarding the accuracy of damage identification and condition assessment.

## 4. Damage and Stiffness Assessment

After post-validating the FE model for the damaged structure, the updated value of moment of inertia $I_b$ along the beam span was obtained. The value for the damaged beam cross section at the midspan was updated to be $I_{bm} = 3.1 \times 10^7 mm^4$, smaller than the initial estimate $I_{bcr} = 5.3 \times 10^7 mm^4$. The stiffness distribution $E_c I_b$ along the beam span was also estimated and is shown in Fig. 8. The smallest stiffness was at midspan with $E_c I_{bm} = 9 \times 10^5 Nm^2$ with the stiffness distribution increasing gradually towards the supports.

It is worth pointing out that the updated minimum stiffness at midspan, $E_c I_{bm}$ was smaller than the estimated value $E_c I_{bcr} = 1.5 \times 10^6 Nm^2$ for the beam at the post-cracking stage, which meant that the damaged beam had gone beyond the post-cracking stage and entered the post-serviceability stage (Nawy 1990). At this stage it is difficult to estimate theoretically the residual stiffness due to the yielding of the tension steel in the RC beam (for an under-reinforced member). However, it is important to recognize the reserve deflection capacity determined by stiffness as a measure of ductility in structures e.g. in earthquake zones and in other applications where there can be serious overload.

In terms of the stiffness distribution of the damaged beam, the extent of damage in the beam also can be identified. The cross-section with reduction of stiffness located the damage in the beam. A damage index $D_i$ which determines the extent of damage is defined as:

$$D_i = \frac{\Delta(EI)}{(EI)_0} \times 100\% \tag{3}$$

where $\Delta(EI)$ denotes the change in stiffness between original and damaged cross-section of the beam and $(EI)_0$ denotes the original stiffness of the undamaged cross section of the beam. Fig. 9 shows the damage index $D_i$ along the beam. Obviously, the extent of damage at midspan of the damaged beam was most serious, at 71%.

## 5. Load-Carrying Capacity Assessment

Load-carrying capacity of the bridge deck can not be assessed directly from the updated results because the ability to resist the bending moment induced by applied loads is due to reinforcing steel embedded in the tension zones. In the design of a RC cross section for a given moment with given material strengths, two basic quantities to be determined are geometric dimensions and steel area that will provide the ultimate moment (Spiegel and Limbrunner 1998). Therefore, the ultimate moment of the structure may in effect be determined by the tensile steel ratio or percentage for a given cross section of given material strengths, and since the stiffness also depends on the steel ratio there is a link from stiffness assessment to capacity assessment. This strategy can be applied to an existing RC flexural structure. Once the ultimate moment is determined, the load-carrying capacity can be estimated.

Normally, it is difficult to identify the effective steel ratio of a damaged RC structure due to unknown details of damage in the structure. However, as described previously, the moment of inertia of the damaged cross section can be identified by model updating. If the relationship between the moment of inertia and the steel ratio of the damaged beam cross section is developed, then the steel ratio can be estimated, leading to determination of the ultimate moment and load-carrying capacity.

Based on extensive testing verification (Nawy 1990) and the requirement of the design codes (BSI 1997; ACI 1995), it is assumed for the RC beam that: (1) concrete does not resist any tension; (2) compatibility of deformation between steel and concrete exists; (3) flexural ultimate limit state occurs as tension failure with reinforcement yielding before concrete crushes; (4) the maximum steel tensile stress equals yield stress; (5) the concrete compressive strain is less than the maximum useable concrete compressive strain at the extreme fibre; and (6) the steel compression stress is less than its yield stress. With these assumptions, two sets of equations for the edge beam shown in Fig. 1 are developed relating moment of inertia to steel area (equation (4)) and then relating steel area to moment capacity:

$$\begin{cases} I_{bcr} = \frac{1}{3} b c_c^3 + \rho n b d \left(d - c_c\right)^2 + \left(n-1\right) A_s^{'} \left(c_c - d^{'}\right)^2 \\ \frac{1}{2} b c_c^2 + \left[\rho n b d + \left(n-1\right) A_s^{'}\right] c_c - \rho n b d^2 - \left(n-1\right) A_s^{'} d^{'} = 0 \end{cases} \tag{4}$$

$$\begin{cases} M_u = \left(0.85\sigma_c^{'}\right) \beta c_u b \left(d - \frac{\beta c_u}{2}\right) + 0.003 A_s^{'} \frac{c_u - d^{'}}{c_u} E_s \left(d - d^{'}\right) \\ \left(0.85\sigma_c^{'} b \beta\right) c_u^2 + \left(0.003 E_s A_s^{'} - \rho d b \sigma_y\right) c_u - 0.003 E_s A_s^{'} d^{'} = 0 \end{cases} \tag{5}$$

where $c_c$ is the neutral axis depth at the cracking stage; $M_u$ is the ultimate moment of the edge beam; $c_u$ is the neutral axis depth at ultimate state; $\rho = A_s / bd$ is the steel ratio;

$n = E_s / E_c$ and $E_s$ is steel modulus of elasticity; $\sigma_c^{'}$ is the compressive concrete stress; β is a constant that depends on the strength of concrete (normally, $\beta = 0.80$); $\sigma_y$ is the steel yield stress.

The set of equations (4) develop the relationship between the moment of inertia $I_{bcr}$ and the steel ratio ρ and provide a means to determine steel ratio ρ for a given value of moment of inertia $I_{bcr}$. From equations (4), the cracked beam moment of inertia was calculated as $I_{bcr} = 5.3 \times 10^7 mm^4$ which was used as the starting value of damaged beam moment of inertia at the midspan for model tuning as described previously. For the damaged beam structure, the updated value of the moment of inertia of the midspan beam was $I_{bu} = 3.1 \times 10^7 mm^4$, leading to a steel ratio solved as $\rho = 0.43\%$.

The set of equations (5) determine the relationship between the ultimate moment $M_u$ and the steel ratio ρ. For the damaged beam, the ultimate moment corresponding to the percentage of steel ratio 0.43% was estimated as $M_u = 14kNm$ by which the load-carrying capacity of the damaged beam structure could be determined. For a simply supported beam subjected to midspan load $P_u = {}^{4M_u}\!/_{l}$, the ultimate load of the whole structure with two damaged beams was $22.5kN$, reduced by 52% compared with the ultimate load value $47kN$ of the undamaged deck structure according to the static load testing.

## 6. Conclusions

The structural condition of a damaged reinforced concrete bridge, including the damage extent and residual stiffness and load-carrying capacity can be assessed quantitatively after the FE model of the damaged structure is validated systematically by dynamics-based model updating techniques. Load-carrying capacity assessment requires development of the relationships of moment of inertia with steel ratio and ultimate moment in the RC structure.

Prior to damage identification, it is recommended to obtain a reliable initial FE model by pre-determining other uncertainties such as boundary conditions of the structure. The model updating procedure is furthermore applied to the pre-validated initial FE model so that the damages are identified by updating the parameters which quantitatively simulate it. The finally validated FE model will represent the damaged structure.

The method is also suitable for condition assessment of an undamaged beam structure. Normally, the as-built structure differs from original design and numerical simulations also have errors due to assumptions and inaccuracies in modeling, so the estimated stiffness and/or load-carrying capacity of the as-built beam may be in error.

Finally, it is in principle also possible to apply the method in a single updating step provided the effects of damage and FE model uncertainties can be separated.

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Table 1 Correlation between updated and measurement

| Undamaged structure | | | | Damaged structure | | | |
|---|---|---|---|---|---|---|---|
| $f_u$ (Hz) (1) | $f_e$ (Hz) (2) | $\Delta_f$ (%) (3) | $MAC$ (%) (4) | $f_u$ (Hz) (5) | $f_e$ (Hz) (6) | $\Delta_f$ (%) (7) | $MAC$ (%) (8) |
| 13.3 | 12.9 | 3.20 | 98.8 | 10.0 | 10.0 | -0.40 | 99.7 |
| 31.3 | 33.0 | -5.12 | 97.6 | 32.0 | 32.2 | -0.76 | 99.0 |
| 44.5 | 42.6 | 4.40 | 97.3 | 41.1 | 40.8 | 0.69 | 97.7 |
| 74.1 | 70.6 | 4.99 | 96.6 | 70.2 | 68.8 | 1.98 | 93.2 |
| 78.2 | 81.4 | -3.95 | 98.2 | 78.6 | 78.5 | 0.13 | 97.6 |

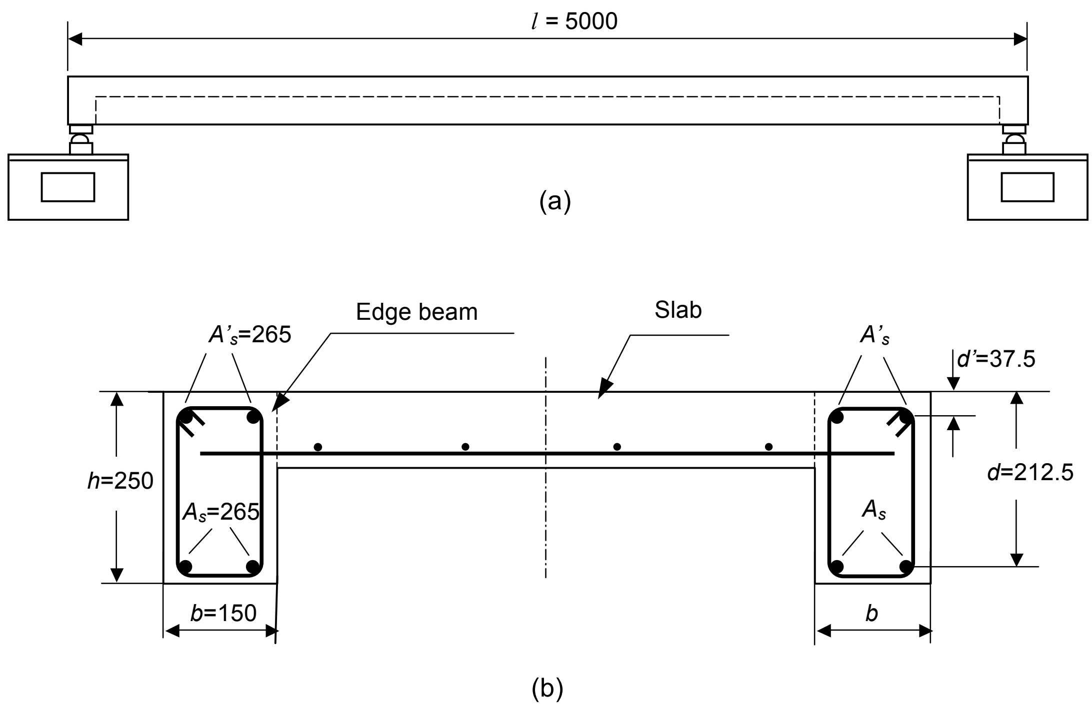


Fig. 1. Schematic of RC bridge deck structure
(a) span unit with simple supports;
(b) cross-section, unit: length(*mm*), area($mm^2$)

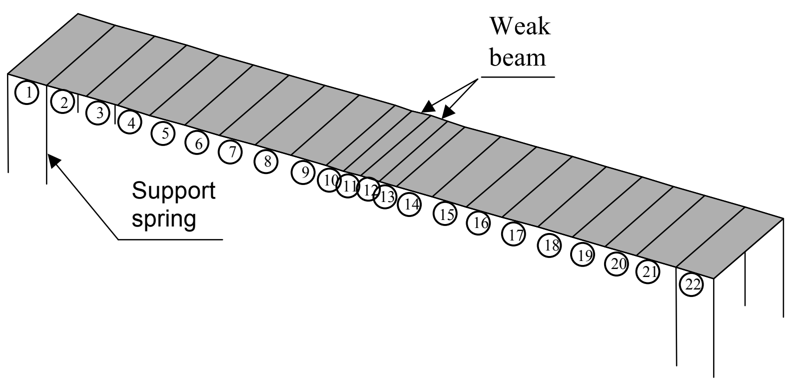


Fig. 2. A damaged bridge FE model with 'weak' beam elements

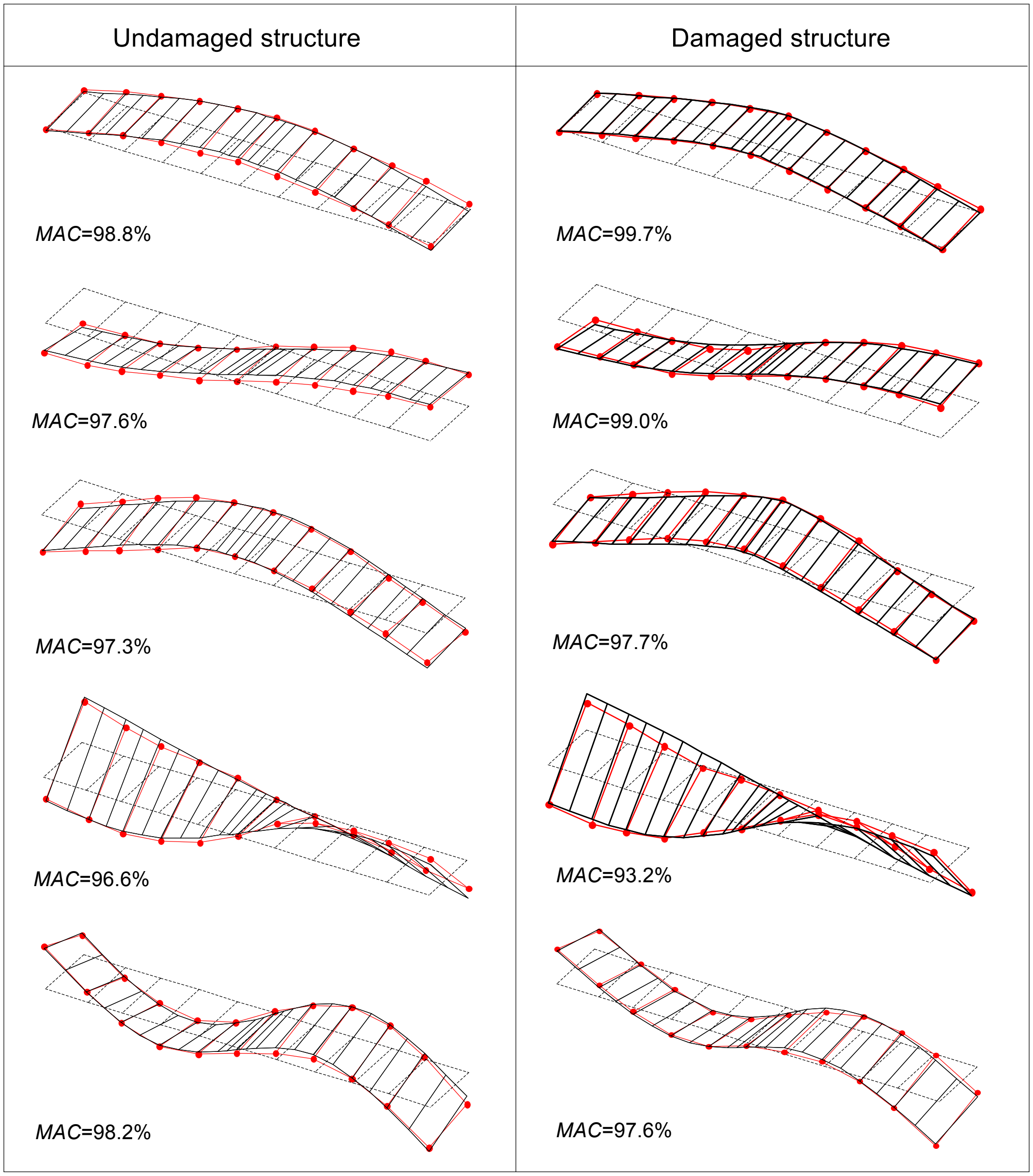


Fig. 3. Pair of mode shapes between updated and measured
----undeformed —— updated —●— measured

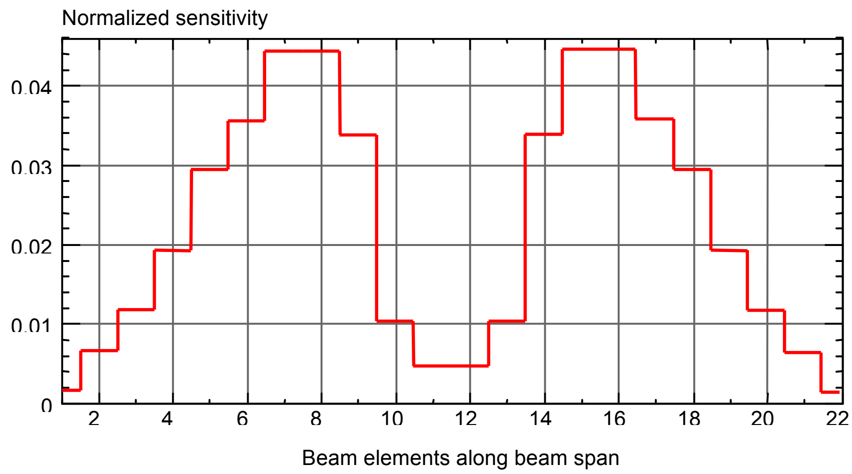


Fig. 4. Sensitivity envelop of frequencies to moment of inertia of beam

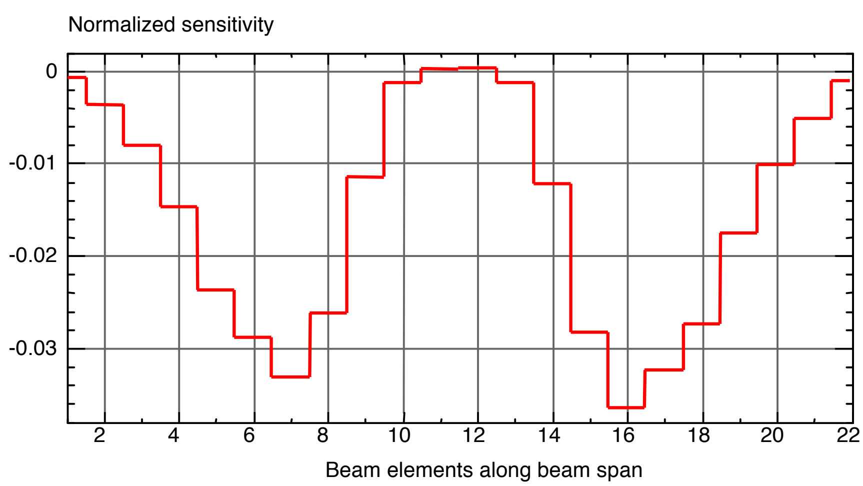


Fig. 5. Sensitivity envelop of *MAC* to moment of inertia of beam

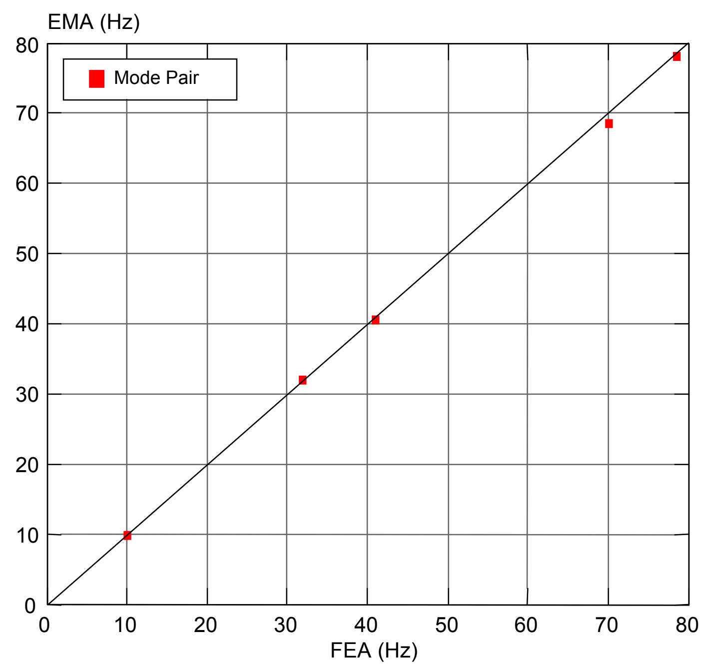


Fig. 6. Frequency pair between updated and measured data

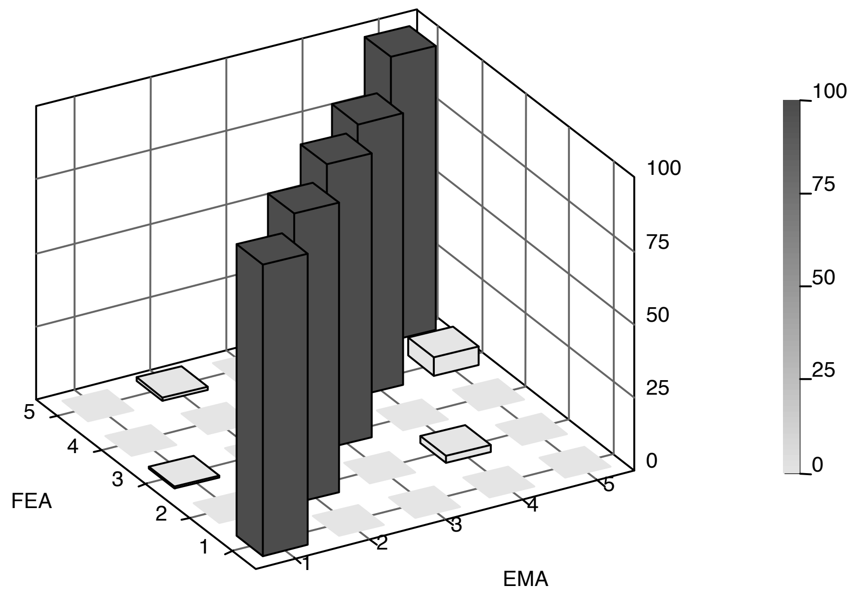


Fig. 7. *MAC* Matrix correlating updated and measured modes

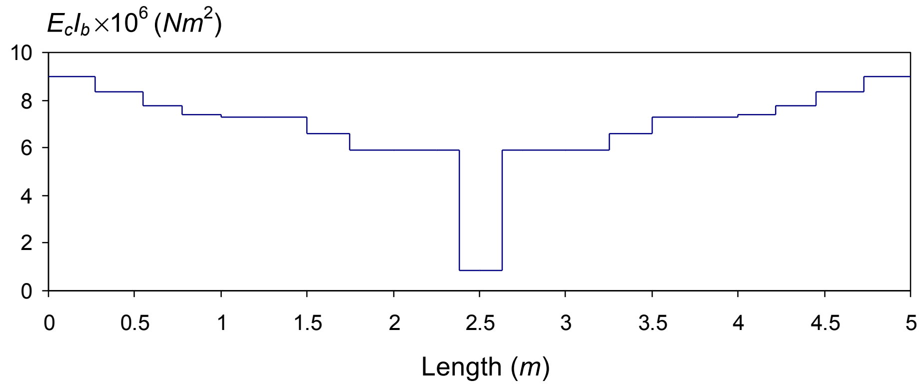


Fig. 8. Stiffness distribution along damaged edge beam

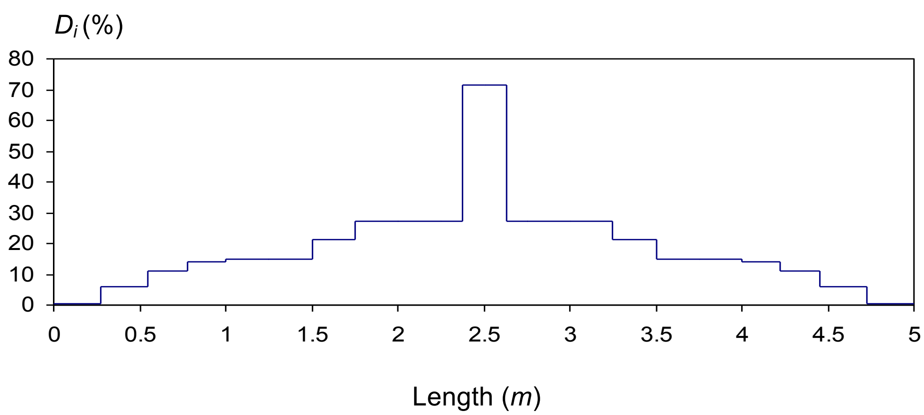


Fig. 9. Distribution of damage index along bridge deck