A Model Predictive Approach to Control the Motion of a Virtual Player in the Mirror Game

Chao Zhai[1], Francesco Alderisio[1], Krasimira Tsaneva-Atanasova[2] and Mario di Bernardo[1]

***Abstract*— In this paper, we focus on the design of a feedback controller that drives a virtual player to follow or lead a human player in the mirror game. The movement of the end-effector of the virtual player is modeled by means of a feedback controlled Haken-Kelso-Bunz (HKB) oscillator or a damped harmonic oscillator, which is coupled with the observed motion of the human player measured in real time. A model predictive control algorithm is developed for the virtual player to generate human-like trajectories while maintaining individual motor signature and guaranteeing bounded tracking error. Experimental results based on a prototype setup show the effectiveness of our strategy and its advantages over other existing algorithms.**

I. INTRODUCTION

The study of human social interaction and coordination has attracted much research interest in the past decade [1], [2], [3], [4]. The mirror game provides a simple paradigm to study the origins and mechanisms of coordination among humans performing a joint task, as for example in improvisation theater, group dance and parade marching [5]. Human subjects can opt to play the mirror game in two different experimental conditions: the Leader-Follower (LF) condition or the Joint Improvisation (JI) condition. Specifically, in the LF mode, one participant is designated as the leader while the other has to track his/her position so that their motion becomes synchronized. In the JI mode, the two players are required to imitate each other and create interesting coordinated motion, so that leadership spontaneously emerges during the game.

In social psychology, it has been shown that people prefer to team up with others possessing similar morphological and behavioral features, and that they tend to coordinate their movement unconsciously [6], [7]. Moreover, much evidence suggests that motor processes caused by interpersonal coordination are strictly related to mental connectedness. In particular, motor coordination between two human subjects contributes to social attachment particularly when the kinematic features of their movement (or motor signatures) share similar patterns [1], [8].

Therefore, it has been suggested that manipulating the levels of similarity between the kinematic signatures of the two players can be used to enhance their coordination and social attachment. Thus, if we were able to create a Virtual Player (VP) capable of coordinating its motion with that of a human player (HP) while exhibiting desired kinematic features, we could manipulate the sense of attachment and coordination level experienced by the human player. This can be crucial, for example, to develop novel rehabilitation protocols for patients suffering from social disorders such as schizophrenia, as recently proposed in the scope of the European project AlterEgo (http://www.euromov.eu/alterego) [9], [10]. Also, it could be used in social robotics to enhance human-robot coordination.

In this paper we take the mirror game as a paradigmatic example, and model the problem of making a VP play the mirror game with a HP as a multi-objective control problem. We develop a control law able to drive the virtual player to track or lead the motion of the human being while guaranteeing certain desired kinematic properties of its own motion. The resulting strategy is a model predictive control algorithm driving the motion of the VP engaging in the mirror game. The key challenge is to design a controller so that it mimics the motion of a human being in terms of reaction times, velocity profiles and typical frequency spectra of the harmonic response.

We compare our approach to two other existing approaches in the literature: the reactive predictive controller proposed in [5] to model humans playing the mirror game, and the Human Dynamic Clamp feedback strategy presented in [11]. We show that our approach is effective in solving the problem while guaranteeing the desired kinematic properties.

II. PROBLEM STATEMENT

The experimental setup of the mirror game is shown in Fig. 1. A small orange ball is mounted onto a string so that it can be moved back and forth along the string by the HP. Meanwhile, the VP on the opposite screen moves its own ball on a parallel string. While playing the game, the two players are required to synchronize their movement. In particular, the VP can act either as a leader or a follower. A camera is installed above the ball moved by the HP, and sampled positions are made available to the cognitive architecture (control strategy) that drives the VP. The control problem is then to use these position data so as to move the end effector of the virtual player in synchrony with that of the human either by leading or following while at the same time exhibiting certain kinematic properties. Specifically, following the approach discussed in [8] we use the probability distribution function (PDF) of the velocity time series recorded in solo

[1] Chao Zhai, Francesco Alderisio and Mario di Bernardo are with Department of Engineering Mathematics, University of Bristol, Merchant Venturers' Building, Clifton, BS8 1UB Bristol, United Kingdom. chao.zhai@bristol.ac.uk, f.alderisio@bristol.ac.uk. Mario di Bernardo is also with Department of Electrical Engineering and Information Technology, University of Naples Federico II, 80125 Naples, Italy. m.dibernardo@bristol.ac.uk

[2] Krasimira Tsaneva-Atanasova is with College of Engineering, Mathematics and Physical Sciences, University of Exeter, United Kingdom. k.tsaneva-atanasova@exeter.ac.uk

Fig. 1. Mirror game setup at the University of Montpellier (France)

trials (where a HP is asked to produce interesting motion of the ball along the string without interacting with the VP) as individual motor signatures of different individuals. We use the earth mover's distance (EMD) between any two PDFs of velocity time series [12] to characterize the level of similarity or dissimilarity between different signatures. The EMD between two PDFs p_1 and p_2 can be computed as follows

$$EMD(p_1,p_2) = \int_Z |CDF_{p_1}(z) - CDF_{p_2}(z)|dz$$

where Z denotes the integration domain, and $CDF_{p_i}(z)$ denotes the cumulative distribution function of the distribution $p_i, i \in \{1,2\}$.

III. MODEL PREDICTIVE APPROACH

As model of the end effector of the VP we use a nonlinear HKB oscillator, which has been suggested in the literature as a paradigmatic example to describe human motor coordination tasks [11], [13]. Specifically, letting x and $\dot{x}$ be position and velocity of the end effector, the HKB model equation is

$$\ddot{x} + (\alpha\dot{x}^2 + \beta x^2 - \gamma)\dot{x} + \omega^2 x = u \qquad (1)$$

where the parameters α, β, γ and ω are positive constants, and u is an external control input. The problem is to design a feedback control u able to minimize the mismatch between x, $\dot{x}$ and the position and velocity of the HP, say r_p, r_v while making the distribution of the velocity $\dot{x}$ close to some desired one, encoded in the prerecorded time series $r_\sigma(t)$.

As a result, the VP produces a series of goal-directed movements influenced by both the position of the human player (goal) and the desired individual motor signature (constraint). We propose optimal control as an effective framework to allow for movement coordination and reconcile target tracking and the need to reproduce a desired individual motor signature [14]. Thus, we formulate the problem of driving the end effector motion as described by (1) on a finite time interval $[t_k, t_{k+1}]$ as the dynamic optimization problem

$$\min_{u \in R} J \qquad (2)$$

TABLE I

MODEL PREDICTIVE CONTROL ALGORITHM

set $k = 1$ and running time T_s
while (*time* $< T_s$)
    detect the position of human player $r_p(t_k)$
    estimate the position of human player $\hat{r}_p(t_{k+1})$ via (4)
    generate the control signal u by optimizing (2)
    obtain the position x and velocity $\dot{x}$ of VP by solving (1)
    $k = k+1$
end while

where

$$J = \frac{1}{2}\theta_p \underbrace{(x(t_{k+1}) - \hat{r}_p(t_{k+1}))^2}_{Temporal\ Correspondence} + \frac{1}{2}\int_{t_k}^{t_{k+1}} \theta_s \underbrace{(\dot{x}(\tau) - r_\sigma(\tau))^2}_{Signature\ Control} + \eta u(\tau)^2 d\tau \qquad (3)$$

with the constraint $\theta_p + \theta_s = 1$ and $\theta_p, \theta_s, \eta > 0$ being tunable control parameters. Here, $\hat{r}_p(t_{k+1})$ denotes the estimated position of the HP at time t_{k+1} (see (4) for further details), while r_σ refers to a prerecorded velocity time series representing the desired motor signature. The above cost function mainly consists of three terms. The first term aims at minimizing the mismatch between the position time series of the HP and the VP. The second term takes care of making the velocity profile of the motion (signature) as close as possible to the reference one. The last term guarantees boundedness of the control effort. In particular, the idea behind this cost function is that the human-like movement of the VP emerges from the integration of three different goals related to temporal correspondence, motor signature and control energy, respectively. Notice that the VP acts as a leader when θ_p is close to 0, since the term related to the position error $x - \hat{r}_p$ in the cost function is negligible and the only aim of the virtual player is to exhibit the desired motor signature. On the other hand, the VP behaves as a follower if θ_p is close to unity.

IV. IMPLEMENTATION

The cognitive architecture resulting from the use of the model predictive control (MPC) strategy presented in the previous section is shown in Fig. 2. The input of the cognitive architecture is the sampled position of the human end effector provided by the camera. A low pass filter is employed to filter out high frequency noise of the sampled positions. Then the position of the ball moved by the HP at any generic time instant t can be estimated by

$$\hat{r}_p(t) = r_p(t_k) + \hat{r}_v(t)(t - t_k), \quad t \in [t_k, t_{k+1}] \qquad (4)$$

Here, r_p refers to the measured position of the HP, while $\hat{r}_p$ and $\hat{r}_v$ represent estimated position and velocity of the reference motion over the next time interval. For the sake of simplicity, the velocity of the ball actuated by the HP is estimated as

$$\hat{r}_v(t) = \frac{r_p(t_k) - r_p(t_{k-1})}{T} \quad t \in [t_k, t_{k+1}] \qquad (5)$$

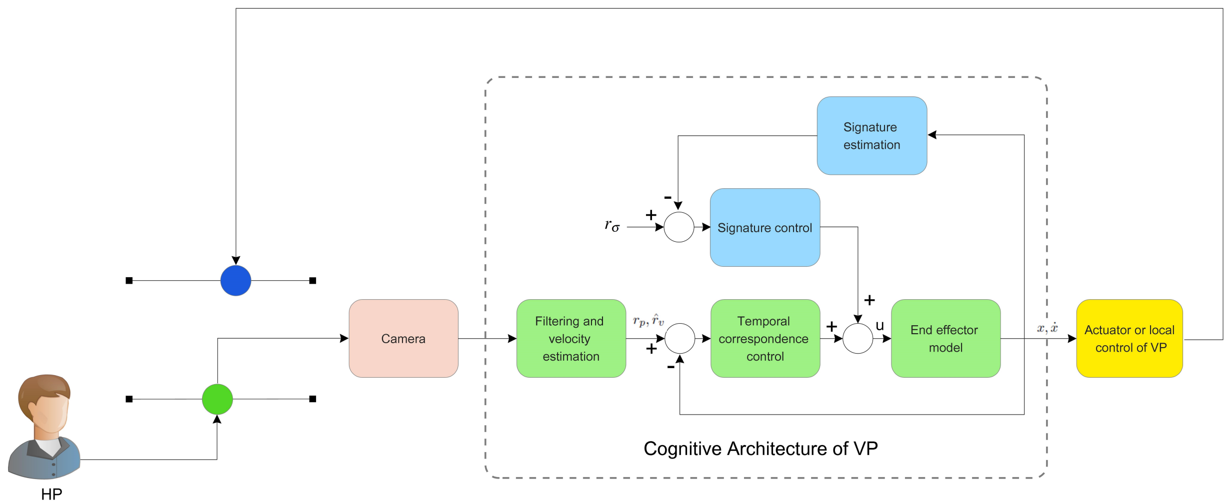


Fig. 2. Block diagram of the interactive cognitive architecture of the VP

where $k \in \mathbb{N}^*$, and $T = t_k - t_{k-1}$ denotes the sampling period of the camera. As an alternative solution, we could adopt a nonlinear observer to provide a better prediction of the player's velocity [15]. Here we find that such a complication is unnecessary to solve the problem of interest and therefore we use the simple yet effective estimation strategy discussed above. The signature estimation block provides an estimate in real time of the mismatch between the generated motion signature and the prerecorded one. The terms of the cost function in (3) correspond to the temporal control and signature control blocks, each minimizing the position mismatch and velocity profile EMDs, respectively. The resulting control algorithm is given in Table I. Finally, the output of the cognitive architecture (position and velocity of the VP) is used as the reference trajectory of the end effector of the VP.

V. CONVERGENCE ANALYSIS

Next, we show that the control algorithm presented above guarantees bounded tracking error and an acceptable performance. Since both the reference position r_p and the desired velocity r_σ are bounded, we assume $r_p \in [\underline{r}, \bar{r}]$ and $r_\sigma \in [\underline{v}, \bar{v}]$.

Theorem 5.1: The optimal control algorithm for the HKB oscillator (1) with the cost function (3) ensures bounded position error between the HP and the VP.

Proof: First of all, we need to demonstrate that there exists a limit cycle in the open-loop HKB oscillator

$$\begin{cases} \dot{x} = y \\ \dot{y} = -(\alpha x^2 + \beta y^2 - \gamma) y - \omega^2 x \end{cases} \tag{6}$$

To this aim, we choose the energy-like function as follows

$$V(x,y) = \frac{\omega^2 x^2 + y^2}{2}$$

The time derivative of $V(x,y)$ along the trajectory of the HKB oscillator (6) is given by

$$\begin{aligned} \dot{V}(x,y) &= \omega^2 x\dot{x} + y\dot{y} \\ &= \omega^2 xy - (\alpha x^2 + \beta y^2 - \gamma) y^2 - \omega^2 xy \\ &= -(\alpha x^2 + \beta y^2 - \gamma) y^2 \end{aligned}$$

Define

$$r_{max} := \max\left(\sqrt{\frac{\gamma}{\alpha}}, \sqrt{\frac{\gamma}{\beta}}\right), r_{min} := \min\left(\sqrt{\frac{\gamma}{\alpha}}, \sqrt{\frac{\gamma}{\beta}}\right)$$

and construct a region Ω as follows (see Fig. 3)

$$\Omega := \{(x,y) \in \mathbb{R}^2 : c_1 \leq V(x,y) \leq c_2\}$$

where the positive constants c_1 and c_2 satisfy

$$r_{min} = \max\left(\sqrt{\frac{2c_1}{\omega^2}}, \sqrt{2c_1}\right), \quad r_{max} = \min\left(\sqrt{\frac{2c_2}{\omega^2}}, \sqrt{2c_2}\right)$$

Clearly, Ω contains no stationary points of the system. Indeed, the only stationary point of the system is $(x,y) = (0,0)$, which is located outside of Ω. Moreover, $\dot{V}(x,y) \geq 0$ when $V(x,y) = c_1$ and $\dot{V}(x,y) \leq 0$ when $V(x,y) = c_2$. According to Poincare-Bendixson theorem, we can conclude that the HKB oscillator (6) has a limit cycle in Ω.

Let J^* denote the minimum value of the cost function (3) in each time interval when $u \neq 0$, and let J_0 represent the value of the same cost function when $u = 0$. Since u aims at minimizing the value of the cost function for all $k \in \mathbb{N}^*$, we can write

$$\begin{aligned} J^* \leq J_0 = &\frac{\theta_p}{2}(x(t_{k+1}) - \hat{r}_p(t_{k+1}))^2 \\ &+ \frac{\theta_s}{2}\int_{t_k}^{t_{k+1}} (\dot{x}(\tau) - r_\sigma(\tau))^2 d\tau \end{aligned}$$

Recall that $r_p(t)$ is bounded, which indicates that $\hat{r}_p(t_{k+1})$ is bounded according to (4) and (5). Moreover, also $r_\sigma(\tau)$ is bounded, and note that $x(t)$ and $\dot{x}(t)$ are bounded as well since the trajectory of the HKB oscillator converges to the

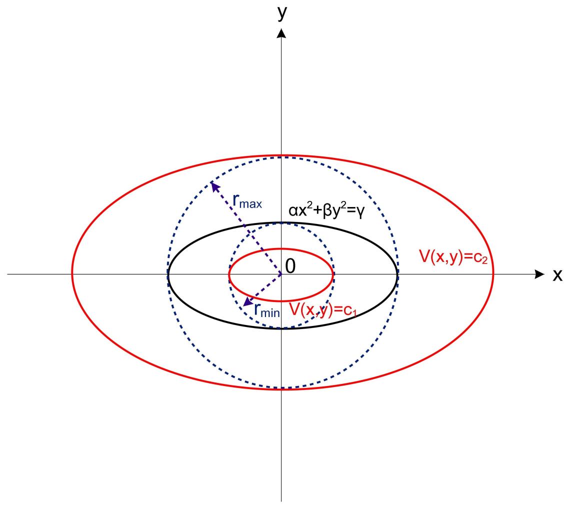


Fig. 3. Illustration on the construction of region Ω. The black ellipse is described by the equation $\alpha x^2+\beta y^2=\gamma$, and the region Ω refers to the ring-shaped area bounded by two red ellipses corresponding to $V(x,y)=c_1$ and $V(x,y)=c_2$, respectively

limit cycle in Ω. Thus, we can claim that J_0 is bounded for $k\in\mathbb{N}^*$, which implies boundedness of J^* and as a consequence of the position error between the VP and the HP. ■

If the HKB nonlinear oscillator is replaced with a simpler damped harmonic oscillator, optimality of the control algorithm can be analytically guaranteed.

Corollary 5.1: Given the linear system

$$\ddot{x}+a\dot{x}+bx=u$$

the MPC approach described in Section IV guarantees convergence to the optimum solution over each subinterval.

Proof: According to the fundamental theorem of the calculus of variations, we need to examine the second variation of the given cost function in order to establish the optimum. From the conclusions in [16], the second variation of the cost function (3) is given by

$$\delta^2 J=\theta_p[\delta x(t_{k+1})]^2 + \int_{t_k}^{t_{k+1}} \begin{pmatrix} \delta X & \delta u \end{pmatrix} \begin{pmatrix} H_{XX} & H_{Xu} \\ H_{uX}^T & H_{uu} \end{pmatrix} \begin{pmatrix} \delta X \\ \delta u \end{pmatrix} dt$$

where H is the Hamiltonian

$$H(X,u,\lambda)=\frac{1}{2}\theta_s(\dot{x}-r_\sigma)^2+\frac{1}{2}\eta u^2+\lambda^T\begin{pmatrix} y \\ -ay-bx+u\end{pmatrix}$$

with $X=[x,\dot{x}]^T=[x,y]^T$ and $\lambda=[\lambda_1,\lambda_2]^T$. Rewriting the linear system in matrix form, we have

$$\dot{X}=AX+Bu$$

where

$$A=\begin{pmatrix} 0 & 1 \\ -b & -a\end{pmatrix},\quad B=\begin{pmatrix} 0 \\ 1\end{pmatrix}$$

Let $X=X^*+\delta X$ and $u=u^*+\delta u$, where X^* and u^* denote optimal state and optimal control, respectively. Since $\dot{X}^*=AX^*+Bu^*$, we get

$$\delta\dot{X}=A\delta X+B\delta u \tag{7}$$

where $\delta X=[\delta x,\delta\dot{x}]^T$. Thus, since $H_{Xu}=H_{uX}=[0\ \ 0]^T$, $H_{uu}=\eta>0$ and

$$H_{XX}=\begin{pmatrix} 0 & 0 \\ 0 & \theta_s\end{pmatrix}\geq 0$$

it follows that

$$\begin{aligned}\delta^2 J&=\theta_p[\delta x(t_{k+1})]^2+\int_{t_k}^{t_{k+1}}\delta X(t)^T H_{XX}\delta X(t)+\eta(\delta u(t))^2 dt\\ &=\theta_p[\delta x(t_{k+1})]^2+\int_{t_k}^{t_{k+1}}\theta_s(\delta\dot{x}(t))^2+\eta(\delta u(t))^2 dt\\ &\geq 0\end{aligned}$$

Moreover, $\delta^2 J=0$ only when $\delta x(t_{k+1})=0$, $\delta\dot{x}(t)=0$ and $\delta u(t)=0$ for all $t\in[t_k,t_{k+1}]$, which yields $\delta x(t)=\delta x(t_k)=0$ from (7). This corresponds to the optimal solution X^* and the optimal control u^*. Therefore, our control approach guarantees minimization of the cost function (3) when the plant is a linear damped oscillator. ■

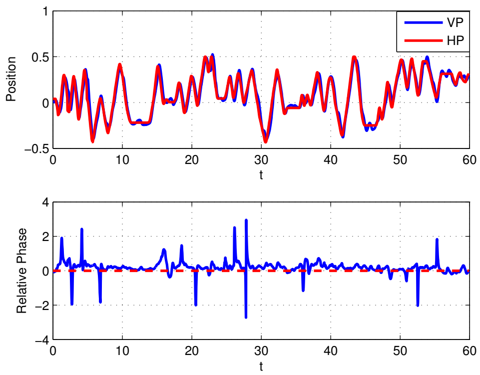
Remark 5.1: The analytical solution for the optimization problem (2) is available if a linear damped harmonic oscillator can be used as end effector model, and Pontryagin's minimum principle provides necessary and sufficient conditions to solve the minimization problem.

VI. EXPERIMENTS

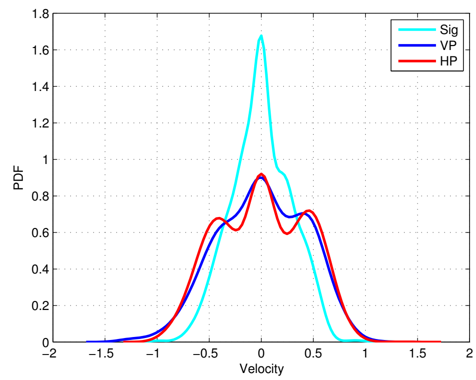
In this section we present experimental results validating our control approach on a simple, yet effective, experimental implementation of the mirror game [10]. A leap motion controller [17] is employed to detect the position of the human fingertip while playing the mirror game. The Matlab code for the interactive cognitive architecture of the VP is implemented on a laptop computer, which produces the position of the VP in order to interact with the HP in real time. In particular, the Matlab solver "bvp4c" is adopted to handle the optimization problem (2) in each time interval. More details about the experimental setup can be found in [9], [10].

A. HP-VP Interaction

The parameters of the proposed VP model are set heuristically as follows: $\alpha=1$, $\beta=1$, $\gamma=1$, $\omega=1$, $\eta=10^{-4}$ and $T=0.03$s (corresponding to the leap motion sampling time). In order for the VP to play the mirror game as a follower, we set the control parameters $\theta_p=0.9$ and $\theta_s=0.1$ so that the "tracking" term in the cost function (3) dominates onto the signature control term. As we can see from the top panel in Fig. 4(a), the VP performs well as a follower during the game; indeed, the root mean square (RMS) of the tracking error is equal to 0.057. In order to distinguish the leader from the follower in the game, we also calculate the relative phase between the HP and the VP, which is defined as $\Delta\Phi:=\Phi_{HP}-\Phi_{VP}$. Here, Φ_{HP} and Φ_{VP} are the phases of the human player and virtual player, respectively, estimated according to the method proposed in [18]. From the bottom



(a) Temporal Correspondence



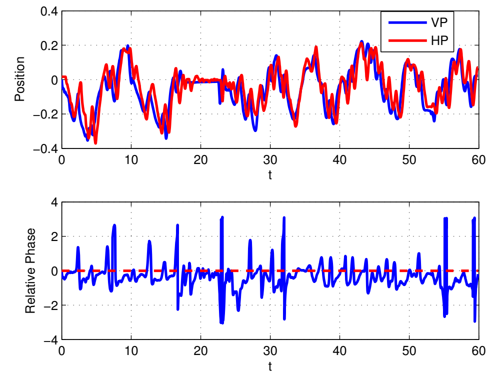
(b) Similarity

Fig. 4. Time evolution of positions and relative phase (a) and PDF of velocities (b) while the VP acts as follower in the mirror game

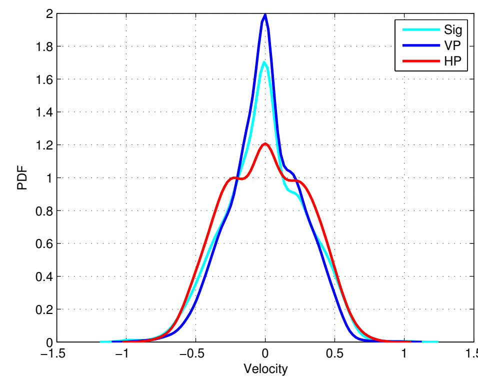
panel in Fig. 4(a) we can observe that the majority of relative phase is positive, meaning that the VP is following the HP in the game for most of the time as desired. In order to quantify the coordination level between the two players, we also compute the circular variance (CV) as follows [19]

$$CV = \left\| \frac{1}{N} \sum_{k=1}^{N} e^{i\Delta\Phi_k} \right\| \quad \in [0,1]$$

where $\Delta\Phi_k$ represents the relative phase between two players at the k-th sampling step, N refers to the total number of time steps and $\|\cdot\|$ denotes the 2-norm. The CV between the HP and the VP is 0.95, which indicates a high coordination level. As for the distribution of the velocity, we can see in Fig. 4(b) that the VP signature (blue line) is closer to that of the HP (red line) than the desired motor signature denoted as *Sig* (cyan line). This is due to the choice of the control parameters in J that were selected to minimize the tracking error more than the signature mismatch. The measured EMDs at the end of the trial can be computed as follows: $\text{EMD}(Sig,VP) = 0.017$ and $\text{EMD}(VP,HP) = 0.005$. The VP can be enabled to play the game as a leader by changing the control parameters setting $\theta_p = 0.1$ and $\theta_s = 0.9$. Experimental results are shown in Fig. 5. The RMS of the tracking error is now 0.08, and the CV between the two players is 0.81. As depicted in the bottom panel of Fig. 5(a), the majority of the relative phase time series is negative, meaning that now the VP is leading the HP during the game for most of the time. In contrast to the previous case, the velocity distributions shown in Fig. 5(b) confirm that the VP is now matching well the desired signature (velocity profile). In this case the trade off is slightly larger with $\text{EMD}(Sig,VP) = 0.004$ and $\text{EMD}(VP,HP) = 0.008$.



(a) Temporal Correspondence



(b) Similarity

Fig. 5. Time evolution of positions and relative phase (a) and PDF of velocities (b) while the VP acts as leader in the mirror game

B. Comparison with Other Existing Approaches

In order to compare the performance of our approach with other existing methods we used a prerecorded time series from a human player as leader's motion. We then asked another HP to follow that trajectory providing a benchmark behavior to test the various methodologies against each other. We use the following models to drive the virtual follower: model predictive control (MPC), Haken-Kelso-Bunz (HKB) model [13], reactive-predictive control (RPC) [5] and Jirsa-Kelso excitator (JKE) [11]. The RMS position error and the CV between leader (pre-recorded trajectory) and follower evaluated when each of these methods is used to drive the VP are shown in Table II together with those produced by our approach. For the sake of comparison we also include the RMS and the CV recorded when a human player is asked

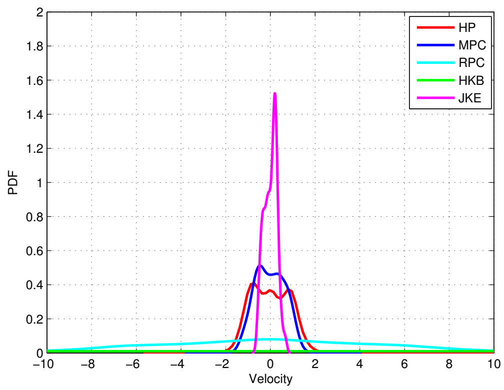


Fig. 6. Velocity distributions for the HP and VPs with different models

to follow the same leading trajectory.

TABLE II

INDEXES OF TEMPORAL CORRESPONDENCE

	HP	MPC	HKB	RPC	JKE
RMS	0.16	0.10	4.20	0.98	0.46
CV	0.70	0.90	0.06	0.24	0.51

Figure 6 shows the resulting velocity distributions recorded during the games. It is visible that the blue line (MPC algorithm) is the closest to the red line (human benchmark). Moreover, the EMDs between the velocity profile of the HP and those produced by the various strategies can be computed as follows: $\mathrm{EMD}(HP,MPC) = 0.0184$, $\mathrm{EMD}(HP,RPC) = 0.1587$, $\mathrm{EMD}(HP,HKB) = 0.4143$ and $\mathrm{EMD}(HP,JKE) = 0.0546$. Again it is possible to notice that the MPC remains closer to the benchmark signature.

VII. CONCLUSIONS

In the context of the mirror game, we succeeded in designing a control architecture able to solve the problem of driving a VP so that it coordinates its motion with that of a human player while exhibiting certain kinematic properties. By tuning the control parameters in the model, the VP is able to play as a leader or a follower with a desired degree of similarity to some prerecorded motor signature. A model predictive control algorithm has been developed to allow the VP to interact with the HP in real time. Bounded position error is also analytically guaranteed. Experiments confirmed the effectiveness of our strategy and its advantages when compared to other existing models such as the reactive predictive controller in [5] and the Human Dynamic Clamp architecture presented in [11]. Potential directions for future work may include the design of a signature generator and adaptation mechanisms of the control parameters.

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