An Augmentation Scheme for Fault Tolerant Control using Integral Sliding Modes

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***Abstract*— In this paper a novel fault tolerant control allocation scheme is proposed, which has the capability to maintain closed-loop nominal performance in the case of faults/failures by effectively managing the actuator redundancy, and without reconfiguring the underlying control law. The proposed scheme relies on an *a posteri* approach, building on an existing state feedback controller designed using only the primary actuators. An integral sliding mode scheme is integrated with the existing controller to introduce fault tolerance. The proposed scheme uses the measured or estimated actuator effectiveness levels in order to redistribute the control signals to the healthy ones which allows a certain class of total actuator failures to be mitigated. The effectiveness of the proposed scheme is tested in simulation using a high fidelity nonlinear model of a transport aircraft model.**

I. Introduction

One of the key attributes of any Fault tolerant control (FTC) system is an ability to maintain some level of functionality/controllability in the face of faults and failures within the system. One way to achieve a high level of 'availability' is to ensure a suitable level of redundancy in terms of the actuators and sensors within the system. One paradigm is to subdivide and classify the different actuators into ones of primary and secondary status. Then, in the case of faults or failures in the primary actuators, secondary actuators can be exploited to retain acceptable performance [1].

One way to manage the redundancy which is created by the use of primary and secondary actuators is to deploy control allocation (CA) schemes to distribute the control effort over the effector suite. Some benefits of using CA schemes in terms of FTC are discussed in [2] and a comparison of the performance of different control allocation methods is documented in [3]. In aircraft systems the principal idea behind CA schemes is to translate the commanded moments produced by the 'virtual' controller into physical control signals which are distributed to the actuators. The combination of CA and 'conventional' sliding mode control is used in [4], [5] and [6] as a mechanism for FTC. Sliding mode control techniques [7], [8] are well known for their inherent robustness properties against matched uncertainties and so their combination with CA is an intuitively appealing method for creating FTC. However the inherent robustness properties of 'conventional' sliding mode control approaches are only guaranteed after the occurrence of a sliding mode [7]. To circumvent this, the idea of integral sliding mode (ISM) control was introduced in [7] to enforce a sliding mode throughout the entire system response thus eliminating the reaching phase. In [9], ISM control was combined with CA in order to introduce tolerance to actuator faults/failures throughout the entire system response.

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The FTC technique proposed in this paper is quite different to the techniques proposed in [4], [6] and more recently [9]. The techniques in [4], [9] are designed based on the open-loop plant with no cognizance of any existing controller. In [9], all the parameters associated with the integral sliding mode scheme are synthesized simultaneously based on a model of the open loop plant and the closed loop performance (in both fault free conditions and in the presence of faults) is completely determined by this design process. In this paper for controller design purposes the actuators are classified as primary and secondary. It is assumed a controller based *only on primary actuators has already been designed* to provide appropriate closed loop performance in a fault free scenario. The technique proposed in this paper involves creating an *a posteri* integral sliding mode design, *building on the existing state feedback controller*. The idea is to use *only* the primary actuators in the nominal fault free scenario, and to engage the secondary actuators *only* if faults/failures occur. Crucially, in the fault free case, the closed loop system behaviour is entirely dependent on the original controller, and the overall scheme behaves exactly as though the ISM scheme was not present. Only in the fault/failure case does the FTC scheme become active. In this way the proposed integral sliding mode FTC scheme can be retrofitted to almost any existing control scheme to induce fault tolerance. *This requires a totally different design philosophy compared to [9].* The scheme proposed in this paper has an advantage over [4], [9] from an industrial perspective, since the proposed scheme can be retrofitted to an existing control scheme without the need to remove or alter existing control loops. Furthermore the nominal fault free performance can be specified according to any design paradigm. The scheme proposed in this paper uses measured or estimated actuator effectiveness levels in order to distribute the control signals among the actuators. In the case of faults/failures, the controller structure does not need to be changed and the control signals are automatically redistributed to healthy actuators to maintain the closed-loop performance close to nominal.

II. System Description and Problem Formulation

An LTI system subject to actuator faults/failures and an external disturbance can be modelled as

$$\dot{x}_p(t) = A_p x_p(t) + B_p W(t) u(t) + f_p(t, x_p) \qquad (1)$$

where $A_p \in \mathbb{R}^{n\times n}$, $B_p \in \mathbb{R}^{n\times m}$ and $W(t) \in \mathbb{R}^{m\times m}$ is a diagonal semi-positive definite weighting matrix representing the effectiveness of each actuator where the elements $0 \leq w_i(t) \leq 1$ for $i = 1, ..., m$. If $w_i(t) = 1$, the corresponding i^{th} actuator has no fault, whereas if $1 > w_i(t) > 0$, an actuator fault is present. In a situation where $w_i(t) = 0$, the actuator has completely failed. This representation of actuator faults/failures, has been used by many researchers: see for example [10]. The function $f_p(t, x_p)$ represents uncertainties/nonlinearities. Suppose the input distribution can be partitioned as

$$B_p = \begin{bmatrix} B_1 & B_2 \end{bmatrix} \tag{2}$$

where $B_1 \in \mathbb{R}^{n \times l}$ and $B_2 \in \mathbb{R}^{n \times (m-l)}$ and $l < m$ and $l < n$. Here B_1 is the input distribution matrix associated with the primary actuators, whilst B_2 is associated with the secondary actuators which provide redundancy in the system. Partition the weighting matrix as $W(t) = diag[W_1(t), W_2(t)]$ where $W_1(t) = diag[w_1(t), ..., w_l(t)]$ and $W_2(t) = diag[w_{l+1}(t), ..., w_m(t)]$. In this paper it is assumed that the matrix $W(t)$ is estimated by some Fault Detection and Isolation (FDI) scheme, see for example [11] or by using a measurement of the actual actuator deflection compared to the demand [12]. The estimated value $\widehat{W}(t)$ will not be a perfect estimate of the real effectiveness matrix $W(t)$, and in this paper it is assumed the estimated matrix $\widehat{W}(t) = diag[\widehat{W}_1(t), \widehat{W}_2(t)]$ satisfies the relationship

$$W(t) = (I - \triangle(t))\widehat{W}(t) \tag{3}$$

where $\triangle(t) = diag[\triangle_1(t), \triangle_2(t)]$. Both $\triangle_1(t)$ and $\triangle_2(t)$ are assumed to be diagonal matrices such that the diagonal elements $\delta_i(t) \in \mathbb{R}$ satisfy $0 \leq \delta_i(t) < \triangle_{max}$ for some $\triangle_{max} > 0$ where

$$\triangle_{max} = max(\|\triangle_1(t)\|, \|\triangle_2(t)\|) \tag{4}$$

The matrices $\triangle_1(t)$ and $\triangle_2(t)$ model the level of imperfection in the fault estimation, and satisfy

$$\begin{aligned} W_1(t) &= (I_l - \triangle_1(t))\widehat{W}_1(t) \\ W_2(t) &= (I_{m-l} - \triangle_2(t))\widehat{W}_2(t) \end{aligned}$$

Assume B_1 has full column rank equal to l and therefore there exists an orthogonal matrix $T_p \in \mathbb{R}^{n \times n}$ such that

$$T_p B_1 = \begin{bmatrix} 0 \\ B_{21} \end{bmatrix} \tag{5}$$

where $B_{21} \in \mathbb{R}^{l \times l}$ (and B_{21} is nonsingular). Change coordinates to create a new state-space representation x, according to $x = T_p x_p$. In the new coordinates the input plant distribution matrix has the form

$$T_p B_p = \begin{bmatrix} 0 & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \tag{6}$$

where $B_{22} \in \mathbb{R}^{l \times (m-l)}$ and the uncertainty $f(t, x) = T_p f_p(t, T_p^{-1} x)$. Next scale the last l states to ensure that $B_{21}^T B_{21} = B_{21} B_{21}^T = I_l$ (i.e. B_{21} is orthogonal). Consequently it can be assumed without loss of generality the system in (1) can be written as

$$\dot{x}(t) = Ax(t) + BW(t)u(t) + f(t, x) \tag{7}$$

where

$$B = \left[\begin{array}{c|c} 0 & B_{12} \\ B_{21} & B_{22} \end{array} \right] =: \left[\begin{array}{c|c} B_o & B_s \end{array} \right] \tag{8}$$

Assume that the function $f(t, x)$ satisfies the matching condition [8], [7] with respect to the primary actuator channels so that

$$f(t, x) = B_o \xi(t, x) \tag{9}$$

where $\xi(t, x)$ represents uncertainty/nonlinearities in the system. Suppose $\xi(t, x)$ can be factorized as

$$\xi(t, x) = \eta(t, x)x(t) \tag{10}$$

where $\eta(t, x)$ is an unknown bounded function satisfying $\|\eta(t, x)\| < \bar{c}$ where $\bar{c} > 0$ is a known scalar. Assume that (A, B_o) is controllable and a state feedback control law

$$\nu_o(t) = Fx(t) \tag{11}$$

has been designed *for the primary control surfaces a priori* to make the system

$$\dot{x}(t) = (A + B_o F)x(t) \tag{12}$$

stable. Note that the gain F is the baseline controller designed for the primary actuators. Now a control allocation scheme will be *retrofitted* to the control law $\nu_o(t)$. The physical control law $u(t)$ applied to *all the actuators* is defined as

$$u(t) = N(t)\nu(t) \tag{13}$$

where $\nu(t) \in \mathbb{R}^l$ is the virtual control effort produced by the actuators, and will be discussed in the next section. The proposed control allocation matrix is given by

$$N(t) = \begin{bmatrix} I_l \\ N_2(t)(I_l - \widehat{W}_1(t)) \end{bmatrix} \tag{14}$$

where $\widehat{W}_1(t)$ and $\widehat{W}_2(t)$ are the estimates of the effectiveness levels and

$$N_2(t) := B_{22}^T B_{21} (B_{21}^T B_{22} \widehat{W}_2(t) B_{22}^T B_{21})^{-1} \tag{15}$$

Note: the allocation matrix in (14) is completely different to the ones proposed in [4], [9].
Define

$$\mathcal{W} = \{(\hat{w}_{l+1}, ..., \hat{w}_m) \in \mathcal{I} \ : \ \det(B_{22}\widehat{W}_2(t)B_{22}^T) \neq 0\} \tag{16}$$

where $\mathcal{I} = \begin{bmatrix} 0 & 1 \end{bmatrix} \times ... \times \begin{bmatrix} 0 & 1 \end{bmatrix}$.

Throughout the paper it is assumed that $m \geq 2l$. This allows up to $m - 2l$ of the entries $\hat{w}_i(t)$ in the matrix $\widehat{W}_2(t)$ to be zero, and yet guarantee $\det(B_{22}\widehat{W}_2(t)B_{22}^T) \neq 0$. The set $\mathcal{W}$ will be shown to constitute the class of faults/failures for which closed-loop stability can be maintained.
Substituting (3), (9) and (13) into (7) yields

$$\dot{x}(t) = Ax(t) + \widehat{B}\nu(t) + B_o \xi(t, x) \tag{17}$$

where[1]

$$\widehat{B} = \begin{bmatrix} B_{12}(I_{m-l} - \triangle_2)\widehat{W}_2 N_2 (I_l - \widehat{W}_1) \\ B_{21}(I_l - \triangle_1)\widehat{W}_1 + B_{22}(I_{m-l} - \triangle_2)\widehat{W}_2 N_2 (I_l - \widehat{W}_1) \end{bmatrix} \tag{18}$$

[1]For notational simplicity the explicit dependence on t of the effectiveness levels $W(t)$ and the uncertainty $\triangle(t)$ has been dropped.

Since B_{21} is orthogonal by construction, $B_{21}B_{21}^T = I$, and using the definition of $N_2(t)$ in (15):

$$B_{22}\widehat{W}_2(t)N_2(t) \;=\; B_{21}B_{21}^T B_{22}\widehat{W}_2(t)N_2(t) = B_{21} \quad (19)$$

Consequently using (19), the expression in (18) simplifies to

$$\widehat{B} = \begin{bmatrix} B_{12}(I_{m-l} - \triangle_2)\widehat{W}_2 N_2(I_l - \widehat{W}_1) \\ B_{21} - B_{21}\triangle_1\widehat{W}_1 - B_{22}\triangle_2\widehat{W}_2 N_2(I_l - \widehat{W}_1) \end{bmatrix} \quad (20)$$

In the case of perfect estimation of $\widehat{W}(t)$ (i.e. $\triangle(t) = 0$) and when there is no fault in the primary and secondary actuators (i.e. $W_1(t) = I_l$ and $W_2(t) = I_{m-l}$), the system in (17) becomes

$$\dot{x}(t) = Ax(t) + B_o\nu(t) + B_o\xi(t,x) \quad (21)$$

and so only the primary control channels will be used. In a fault/failure scenario, to maintain the closed-loop performance near to nominal, the concept of integral sliding mode control is combined with the control law from (13) and (14). The nominal fault free system in (21) will be used for the design of the augmentation scheme which will be demonstrated in the sequel.

III. Integral Sliding Mode Controller Design

As in [7] the creation of an integral sliding mode controller comprises two steps. As a first design step, choose the sliding surface as $\mathcal{S} = \{x \in \mathbb{R}^n : \quad \sigma(t,x) = 0\}$ where the switching function $\sigma(t,x)$, based on the nominal system, is defined as

$$\sigma(t,x) := Gx(t) - Gx(0) - G\int_0^t (A + B_oF)x(\tau)d\tau \quad (22)$$

where $G \in \mathbb{R}^{l\times n}$ is the design freedom to be selected. The design of this switching function, as in [13], [14] and [9] aims to eliminate the reaching phase which is present in the traditional sliding mode control techniques [8]. The elimination of the reaching phase, ensures the occurrence of the sliding mode starting at $t = 0$, and guarantees the robustness of the closed-loop sliding motion throughout the entire response of the system. In this paper

$$G := B_o^T \quad (23)$$

is suggested where B_o is defined in (8). With this choice of G it follows

$$GB_o = B_{21}^T B_{21} = I_l$$

since B_{21} is orthogonal by construction, and from (20)

$$G\widehat{B} = B_{21}^T\Big(B_{21} - B_{21}\triangle_1\widehat{W}_1 - B_{22}\triangle_2\widehat{W}_2 N_2(I_l - \widehat{W}_1)\Big) \quad (24)$$

To analyze the closed-loop sliding motion associated with the integral switching function in (22) and the choice of G in (23), taking the time derivative of $\sigma(t,x)$ defined in (22) yields

$$\dot{\sigma}(t) = G\dot{x}(t) - GAx(t) - GB_oFx(t) \quad (25)$$

Substituting (17) into (25), yields the expression

$$\dot{\sigma}(t) = G\widehat{B}\nu(t) + GB_o\xi(t,x) - GB_oFx(t) \quad (26)$$

During sliding $\dot{\sigma}(t) = 0$, and by using the fact that $GB_o = I_l$, the expression for the equivalent control is given by

$$\nu_{eq}(t) = (G\widehat{B})^{-1}(Fx(t) - \xi(t,x)) \quad (27)$$

The equation of motion governing sliding is obtained by substituting (27) into (17) which yields

$$\dot{x}(t) = Ax(t) + \widehat{B}(G\widehat{B})^{-1}(Fx(t) - \xi(t,x)) + B_o\xi(t,x) \quad (28)$$

Adding and subtracting $B_oFx(t)$, equation (28) can be written as

$$\dot{x}(t) = (A + B_oF)x(t) + (\widehat{B}(G\widehat{B})^{-1} - B_o)(Fx(t) - \xi(t,x)) \quad (29)$$

which can be further simplified to

$$\dot{x}(t) = (A + B_oF)x(t) + \begin{bmatrix} \widehat{B}_1 \\ 0_l \end{bmatrix}\big(Fx(t) - \xi(t,x)\big) \quad (30)$$

where

$$\widehat{B}_1 = B_{12}(I_{m-l} - \triangle_2(t))\widehat{W}_2(t)N_2(I_l - \widehat{W}_1(t))(G\widehat{B})^{-1}$$

For the stability analysis which follows, write (30) as

$$\dot{x}(t) = (A + B_oF)x(t) + \tilde{B}\Phi(t)\big(Fx(t) - \xi(t,x)\big) \quad (31)$$

where

$$\tilde{B} := \begin{bmatrix} B_{12} \\ 0 \end{bmatrix} \quad (32)$$

and the time varying uncertain term

$$\Phi(t) := (I_{m-l} - \triangle_2)\Psi(t)\Big(I_l - \triangle_1\widehat{W}_1 - B_{21}^T B_{22}\triangle_2\Psi(t)\Big)^{-1} \quad (33)$$

where

$$\Psi(t) := \widehat{W}_2(t)N_2(I_l - \widehat{W}_1(t)) \quad (34)$$

From (19) it is clear that $\widehat{W}_2(t)N_2(t)$ is a right pseudo inverse for $B_{21}^T B_{22}$. Then by using arguments similar to those in [4], as proved in [15], it follows $\|\widehat{W}_2(t)N_2(t)\| < \gamma_1$ for some positive scalar γ_1, provided $\det(B_{22}\widehat{W}_2(t)B_{22}^T) \neq 0$. Since

$$\|\Psi(t)\| \leq \|(I_l - \widehat{W}_1(t))\|\|\widehat{W}_2(t)N_2(t)\| < \|\widehat{W}_2(t)N_2(t)\| < \gamma_1 \quad (35)$$

it follows $\|\Psi(t)\|$ remains bounded.

A. Stability Analysis of the Closed-loop Sliding motion

In the case of perfect estimation of $\widehat{W}(t)$, (i.e. $\triangle(t) = 0$) and when there are no faults in the system (i.e. $W(t) = I$) the uncertain term $\Phi(t)$ in (31) vanishes (i.e. $\Phi(t) = 0$) and the closed-loop sliding motion in (31) simplifies to

$$\dot{x}(t) = (A + B_oF)x(t) \quad (36)$$

which is stable by the choice of the baseline controller F. From (36), it is clear that the effect of the external disturbance/uncertainty $\xi(t,x)$ is completely rejected while sliding. This concurs with (21), which demonstrates that the disturbance is matched to the primary control signals. Furthermore if $\widehat{W}_1 = I_l$ then the top matrix sub-block in the matrix $\widehat{B}$ in (20) is zero and the disturbance $\xi(t,x)$ can still be rejected provided $\|\Delta_1\| < 1$ which ensures $\det(B_{21}(I_l - \Delta_1)) \neq 0$ in the lower sub-block of $\widehat{B}$. However if $\widehat{W}_1 \neq I$ then $B_{12}(I_{m-l} -$

$\triangle_2(t))\widehat{W}_2(t)N_2(I_l - \widehat{W}_1(t))$ is generically nonzero, and from (20) the effect of $\xi(t,x)$ becomes unmatched since the range space of B_o is not contained in the range space of $\widehat{B}$. Consequently in the case of non-perfect estimation of $\widehat{W}(t)$ and in the presence of faults, the stability of (31) needs to be proven. To this end, in this most general situation, the equations governing the sliding motion in (31) are

$$\dot{x}(t) = \underbrace{(A + B_oF)}_{A_a} x(t) + \tilde{B}\Phi(t)Fx(t) - \tilde{B}\Phi(t)\xi(t,x) \quad (37)$$

Define

$$\gamma_2 = \|G_a(s)\|_\infty \quad (38)$$

where the transfer function matrix

$$G_a(s) := C_a(sI - A_a)^{-1}B_a \quad (39)$$

and

$$B_a = \begin{bmatrix} \tilde{B} & -\bar{c}\tilde{B} \end{bmatrix} \quad \text{and} \quad C_a = \begin{bmatrix} F \\ I_n \end{bmatrix} \quad (40)$$

In the definition of B_a above, $\bar{c}$ is the bound on the function $\eta(t,x)$ in (10). Also, for $\Psi(t)$ as defined in (34), define γ_1^* as the smallest number satisfying

$$\|\Psi(t)\| < \gamma_1^* \quad (41)$$

This least upper bound is guaranteed to exist since, as shown in (35), $\Psi(t)$ is bounded.

Proposition 1: Suppose that the condition

$$(1 + \gamma_3\gamma_1^*)\triangle_{max} < 1 \quad (42)$$

holds, where γ_1^* and $\triangle_{max}$ are defined in (41) and (4) and $\gamma_3 = \|B_{22}\|$, then during fault/failure conditions including failure of all primary actuators and for any $\hat{w}_{l+1}(t), ..., \hat{w}_m(t) \in \mathcal{W}$ where $\mathcal{W}$ is defined in (16), the closed loop system in (37) will be stable if:

$$\frac{\gamma_2\gamma_1^*(1 + \triangle_{max})}{1 - (1 + \gamma_3\gamma_1^*)\triangle_{max}} < 1 \quad (43)$$

where γ_2 is defined in (38).

Proof: The closed-loop sliding motion in (37) can be written as

$$\dot{x}(t) = A_ax(t) + B_au_a(t) \quad (44)$$
$$y_a(t) = C_ax(t) \quad (45)$$

where

$$u_a(t) = \Phi_a(t)y_a(t) \quad (46)$$

and

$$\Phi_a(t) = \text{Diag}\left(\Phi(t), \tfrac{1}{\bar{c}}\Phi(t)\eta(t,x)\right) \quad (47)$$

Note that

$$\|\Phi_a(t)\| \leq \max\left\{\|\Phi(t)\|, \|\frac{1}{\bar{c}}\Phi(t)\eta(t,x)\|\right\} \leq \|\Phi(t)\|$$

because $\|\frac{1}{\bar{c}}\Phi(t)\eta(t,x)\| \leq \|\Phi(t)\|$ since $\|\eta(t,x)\| < \bar{c}$. Using the small gain theorem, the interconnection of $G_a(s)$ with $\Phi_a(t)$ and hence equation (37) will be stable if

$$\|G_a(s)\|_\infty\|\Phi_a(t)\| < 1 \quad (48)$$

From equation (33), it is clear that

$$\|\Phi(t)\| \leq \|\left(I_l - X(t)\right)^{-1}\|\|(I_{m-l} - \triangle_2(t))\Psi(t)\| \quad (49)$$

where $X(t) = \triangle_1(t)\widehat{W}_1(t) + B_{21}^TB_{22}\triangle_2(t)\Psi(t)$. Using the fact that $\|\widehat{W}_1(t)\| \leq 1$, and $\|B_{21}^T\| = 1$ (since $B_{21}^TB_{21} = I$), from (49)

$$\begin{aligned} \|X(t)\| &\leq \|\triangle_1(t)\widehat{W}_1(t)\| + \|B_{21}^TB_{22}\triangle_2(t)\Psi(t)\| \\ &\leq \|\triangle_1(t)\| + \|B_{22}\|\|\triangle_2(t)\|\|\Psi(t)\| \\ &\leq (1 + \gamma_3\gamma_1^*)\triangle_{max} < 1 \end{aligned}$$

if the conditions of *Proposition 1* hold. Hence from (49), and using the fact that in general $\|(I - X)^{-1}\| \leq (1 - \|X\|)^{-1}$ if $\|X\| < 1$ [16], and as argued above, $\|\Phi_a(t)\| \leq \|\Phi(t)\|$,

$$\|\Phi_a(t)\| \leq \|\Phi(t)\| \leq \frac{\gamma_1^*(1 + \triangle_{max})}{1 - (1 + \gamma_3\gamma_1^*)\triangle_{max}} \quad (50)$$

From the expression in (50) and the fact that $\|G_a(s)\|_\infty = \gamma_2$, a sufficient condition to ensure the conditions of the small gain theorem in (48) hold is that

$$\frac{\gamma_2\gamma_1^*(1 + \triangle_{max})}{1 - (1 + \gamma_3\gamma_1^*)\triangle_{max}} < 1$$

This is condition (43), and the proof is complete. ∎

B. Integral Sliding Mode control laws

The second step is to design a control law which can enforce and maintain sliding on the surface in (22). Define the integral sliding mode control law as

$$\nu(t) = \nu_l(t) + \nu_n(t) \quad (51)$$

where the linear part of the control law is

$$\nu_l(t) := Fx(t) \quad (52)$$

and the nonlinear part, which induces the sliding motion, is

$$\nu_n(t) := -\rho(t,x)\frac{\sigma(t,x)}{\|\sigma(t,x)\|} \qquad \text{for } \sigma(t,x) \neq 0 \quad (53)$$

where $\rho(t,x)$ is the modulation gain whose precise value is proposed in the statement of *Proposition 2*. Now in the sequel it is demonstrated that the integral sliding mode control law in (51)-(53) satisfies the reachability condition [7], [8].

Proposition 2: Assume the conditions of *Proposition 1* hold. Then if $\rho(t,x)$ is chosen as

$$\rho(t,x) \geq \frac{(1 + \gamma_3\gamma_1^*)\triangle_{max}\|\nu_l(t)\| + \|\eta(t,x)\|\|x\| + \eta_o}{1 - (1 + \gamma_3\gamma_1^*)\triangle_{max}} \quad (54)$$

where $\eta_o > 0$ is a small positive scalar and $\eta(t,x)$ is defined in (10), the integral sliding mode control law in (51)-(53) satisfies the reachability condition and sliding on $\mathcal{S}$ in (22) is maintained.

Proof: Substitute the control law from (51)-(53) into (26). By using the fact that $GB_o = I$,

$$\dot{\sigma}(t) = (G\widehat{B})(Fx(t) - \rho\frac{\sigma(t)}{\|\sigma(t)\|}) + \xi(t,x) - Fx(t) \quad (55)$$

Since by construction $B_{21}^T B_{21} = I$, using (24) and (34) equation (55) can be written as

$$\dot{\sigma} = -\Big(\triangle_1 \widehat{W}_1 + B_{21}^T B_{22} \triangle_2 \Psi(t)\Big)(Fx(t) - \rho\frac{\sigma(t)}{\|\sigma(t)\|}) - \rho\frac{\sigma(t)}{\|\sigma(t)\|} + \xi(t,x) \quad (56)$$

Now consider the candidate Lyapunov function$V(t) = \frac{1}{2}\sigma^T\sigma$. Taking the time derivative along the trajectories and substituting for $\dot{\sigma}(t)$ from (56) yields

$$\dot{V} = -\rho\|\sigma\| - \sigma^T\Big(\triangle_1 \widehat{W}_1 + B_{21}^T B_{22} \triangle_2 \Psi(t)\Big)Fx(t) + \rho\sigma^T\Big(\triangle_1 \widehat{W}_1 + B_{21}^T B_{22} \triangle_2 \Psi(t)\Big)\frac{\sigma}{\|\sigma\|} + \sigma^T\xi(\cdot)$$

and therefore

$$\begin{aligned}\dot{V} &\leq -\rho\|\sigma\| + \|\sigma\|(\triangle_{max} + \gamma_3\triangle_{max}\gamma_1^*)\|\nu_l\| \\ &+ \rho\|\sigma\|(\triangle_{max} + \gamma_3\triangle_{max}\gamma_1^*) + \|\sigma\|\|\xi(\cdot)\| \\ &\leq -\rho(1 - (\triangle_{max} + \gamma_3\triangle_{max}\gamma_1^*))\|\sigma\| \\ &+ \|\sigma\|(\triangle_{max} + \gamma_3\triangle_{max}\gamma_1^*)\|\nu_l\| + \|\sigma\|\|\eta(\cdot)\|\|x\| \quad (57)\end{aligned}$$

where $\triangle_{max}$ is defined in (4). By choosing the value of $\rho(t)$ as proposed in (54), the expression in (57) becomes $\dot{V} \leq -\eta_o\|\sigma\|$ which is the standard reachability condition [7], [8], and is sufficient to guarantee that sliding on $\mathcal{S}$ is maintained. ■

Finally in order to obtain the overall physical control law which is used to create the actual control signals sent to all the control surfaces, substituting (51)-(53) into (13) yields

$$u(t) = \begin{bmatrix} I_l \\ N_2(t)(I_l - \widehat{W}_1(t)) \end{bmatrix}\Big(Fx(t) - \rho(t,x)\frac{\sigma(t)}{\|\sigma(t)\|}\Big) \quad (58)$$

where $N_2(t)$ is defined in (15).

IV. Simulation: yaw damping

The proposed integral sliding mode FTC scheme employs an a-posteri approach building on an existing state feedback controller designed using only the primary actuators. In the physical control law proposed in (58), the baseline control law F is assumed to exist a-priori. The technique implemented in the proposed FTC scheme is to use the baseline controller in the nominal fault free scenario, and activates the fault tolerant features only in the case when faults or failures occur.

The lateral dynamics of a large transport aircraft are used to evaluate the proposed scheme. For design purposes, a linearization of the benchmark model from [1] is obtained about an operating condition of 7000 m altitude and 220 m/sec forward speed (Mach 0.8). By augmenting a washout filter state, the state space representation of the model is given as

$$A_p = \begin{bmatrix} -0.3330 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.0341 & 1 \\ 0 & 0.0445 & -0.0976 & -0.9952 & 0.0343 \\ 0 & -0.0018 & 1.0047 & -0.1483 & -0.0215 \\ 0 & 0.0001 & -2.9504 & 0.3054 & -0.7787 \end{bmatrix} \quad (59)$$

$$B_p = \left[\begin{array}{cc|cccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0128 & 0 & 0.0013 & -0.0013 & 0.0001 & -0.0001 \\ -0.6430 & -0.0294 & -0.0241 & 0.0241 & 0.0493 & -0.0493 \\ 0.2060 & -0.5010 & -0.3616 & 0.3616 & 0.0084 & -0.0084 \end{array}\right] \quad (60)$$

The states are $(x_{wo}, \phi, \beta, r, p)^T$, where x_{wo} is the washout filter state (rad), ϕ is the roll angle (rad), β is the side slip (rad), r is the yaw rate (rad/sec) and p is the roll rate (rad/sec). The control surfaces which are considered for the design are $\delta_{lat} = \{\delta_r, \delta_a, \delta_{sp1-5}, \delta_{sp8-12}, Tn_l, Tn_r\}^T$ where δ_r is the rudder deflection (rad), δ_a is the aileron deflection (rad), δ_{sp1-5} are grouped together as one control input the 'left spoilers' (rad), δ_{sp8-12} are grouped together as one control input the 'right spoilers' (rad) and Tn_l and Tn_r are aggregated engine thrusts (N) (scaled by 10^5) on the left and right wing. It is assumed that the left aileron moves in an antisymmetrical fashion to the right aileron. In (60) the input distribution matrix B_p is divided into primary $(\delta_r, \delta_a)^T$ and secondary $(\delta_{sp1-5}, \delta_{sp8-12}, Tn_l, Tn_r)^T$ actuators. A further transformation is required in order to have the structure in (8) and to ensure that $B_{21}B_{21}^T = I_2$. Using the set of eigenvalues in the appendix and eigenvectors suggested in [17], the ideal baseline control law F for yaw damping (considering only the primary actuators $(\delta_r, \delta_a)^T$), based on eigenstructure assignment is

$$F = \begin{bmatrix} 0.2302 & -0.6191 & -1.0912 & 1.2677 & -0.5206 \\ -5.7286 & -0.1520 & 0.3575 & 0.7138 & 0.4986 \end{bmatrix} \quad (61)$$

Details are given in the appendix.

A. *Fault Tolerant Control Law for Yaw Damping*

Here the fault tolerant control law proposed in (58) will be employed to retain performance close to the nominal. In the nominal case, the aileron is the primary control surface for ϕ tracking, and the spoilers are the redundancy; whereas the rudder is the primary control surface for β tracking (i.e. yaw damping), and differential engine thrust is the redundancy. After an appropriate change of coordinates it can be shown that $\gamma_3 = 1.0254$. For a value of $\bar{c} = 0.70$, the value of γ_2 for the *a priori* F using equation (38) is $\gamma_2 = 0.0237$. Using (41) it can be verified using a numerical search that $\gamma_1^* = 12.3477$. Hence to satisfy the stability conditions of *Proposition 1* in (42) and (43), the maximum value of the error in estimation of the actuator effectiveness levels which can be handled by the physical control law in (58), is $\triangle_{max} = 5\%$.

V. Nonlinear Simulation Results for Yaw Damper

All the simulations which follow have been based on the FTLAB 747 $V6.5/7.1/2006b$ software environment which was used as the basis for the GARTEUR AG16 benchmark [1]. This high-fidelity nonlinear model contains 77 states including realistic actuators, sensors and engine dynamics, and represents a 'real world' model of a B747-100/200 aircraft.

Practically the integral sliding mode control law proposed in (53) cannot be directly used in this case, and the discontinuities in the unit vector have been smoothed using sigmoidal approximation $\frac{\sigma}{\|\sigma\|+\delta}$ [8], where value of the positive scalar $\delta = 0.05$. In all the simulations $\rho(t,x) = 1$ which is a simple, but in this case, effective choice. The objective of the simulations is to damp the lateral dynamics of the aircraft when the initial sideslip $\beta(0)$ is perturbed by $5°$. All the simulations are conducted in the presence of wind and gusts and sensor noise. The wind and gust models are embedded in the FTLAB 747 $V6.5/7.1/2006b$ [1]. The wind model generates the wind velocities ($u_{wind} = -11, v_{wind} = -12, w_{wind} = 0$) along the positive axis of the earth reference frame and the gust model uses the Dryden spectra. The sensor noise is based on a Gaussian distribution of zero mean and variance ($3e^{-8}, 3e^{-6}, 3e^{-8}, 3e^{-8}$), which appears in the measured states $(\phi, \beta, r, p)^T$. Three scenarios are investigated:

In the case when the estimation of the effectiveness level matrix $W(t)$ is perfect, $\triangle(t) = 0$ and $\triangle_{max} = 0$. Figures 1-2 demonstrate the nominal fault free performance. In Figure 1 it can be seen that the roll and yaw modes are decoupled. During the nominal fault free scenario the secondary actuators are not active (Figure 2).

A second scenario is considered here to demonstrate the efficacy of the scheme even when the estimation of the $W(t)$ matrix is not perfect. Figure 3 shows that due to imprecise information provided by the FDI, the estimate $\widehat{W}(t) \neq I$, (indicating the presence of faults) although in reality there is no fault in the system. In response to this the control allocation scheme engages the secondary actuators as shown in Figure 4 to maintain the closed-loop stability of the system and to retain nominal performance.

The third scenario demonstrates the scheme with imperfect estimates $\widehat{W}(t)$ in the case of a jam in the primary actuators (at some offset position). Theoretically the maximum percentage error $\triangle_{max}$ the proposed scheme can handle and yet ensure the stability conditions of *Proposition 1*, is 5%. Figure 6, shows that the primary actuators (rudder and ailerons) have jammed at 6 sec, and due to imprecise information provided by the FDI scheme, the effectiveness of the primary actuators is estimated at 5%, instead of 0% (Figure 5). The right wing spoilers are actively engaged by the scheme together with left and right wing engines thrust to cope with this situation, and to maintain near to nominal performance (Figure 6).

VI. Conclusion

In this paper a novel fault tolerant control allocation scheme is proposed by incorporating integral sliding modes. The controller structure does not need to be changed and the same controller is used in both nominal as well as in fault/failure scenarios. The proposed scheme employs an *a posteri* approach building on an existing state feedback controller designed using only the primary actuators. To distribute the control signals to the functional actuators, the scheme uses the estimated effectiveness levels of the actuators provided by an FDI scheme. Furthermore the proposed FTC scheme can handle a level of error in terms of estimation of the actuator effectiveness. A rigorous stability analysis for imperfect actuator effectiveness estimate has been developed. The efficacy of the proposed scheme is tested in simulation using a nonlinear benchmark model of a large transport aircraft.

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VII. Appendix

The ideal closed-loop eigenvalues for the nominal state feedback controller F associated with the primary actuators for yaw damping (which is a stability augmentation system for the lateral dynamics of an aircraft) are

$$\{-0.350, -0.468, -0.600 \pm 0.628j, -1.106\} \tag{62}$$

The motions corresponding to the stable real poles are referred to as the spiral mode (-0.350), the washout filter (-0.468) and the roll mode (-1.106). The motion corresponding to the complex poles is referred to as the Dutch roll mode. The best possible eigenvectors to ensure decoupling between these modes is described in [17]. This choice has been used to synthesize the state feedback control gain in (61).

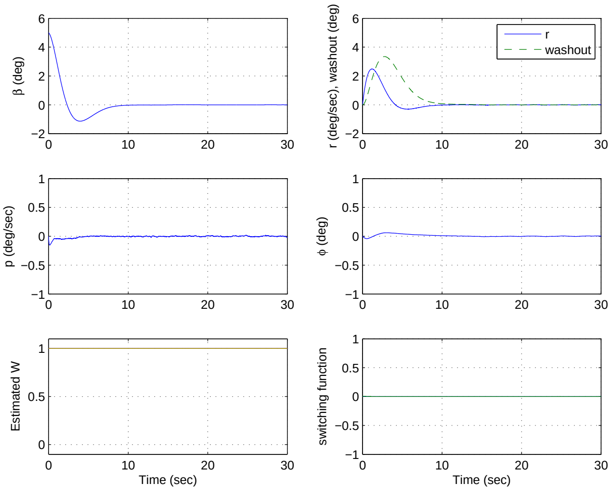

Fig. 1. No fault (perfect estimation of W): plant states

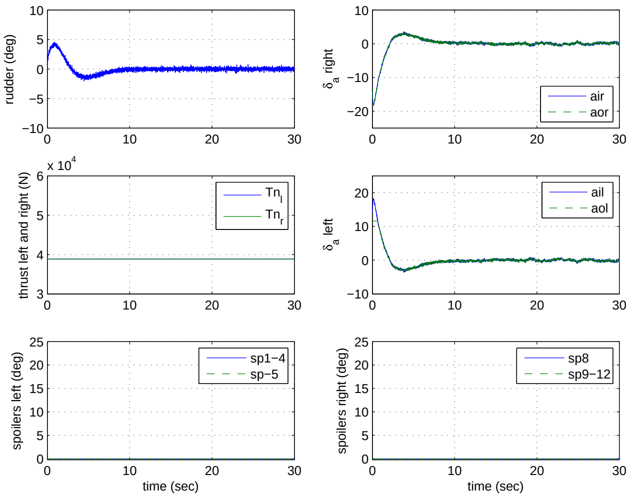

Fig. 2. No fault (perfect estimation of W): actuators

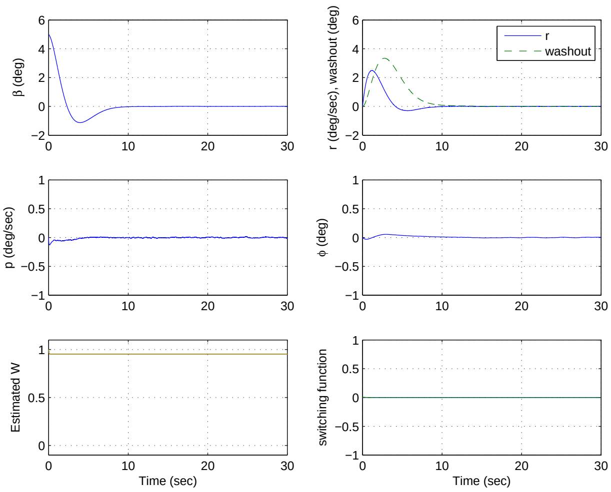

Fig. 3. No fault (imperfect estimation of W): plant states

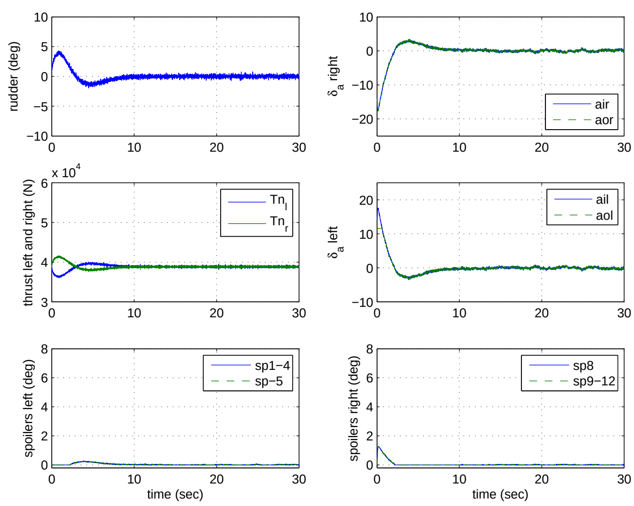

Fig. 4. No fault (imperfect estimation of W): actuators

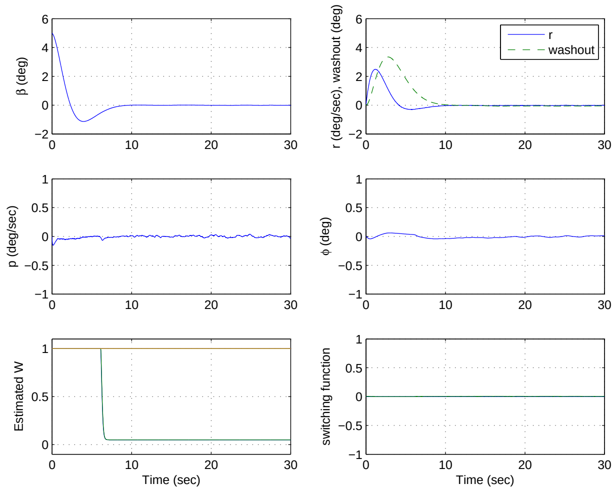

Fig. 5. Primary failure (imperfect estimation of W): plant states

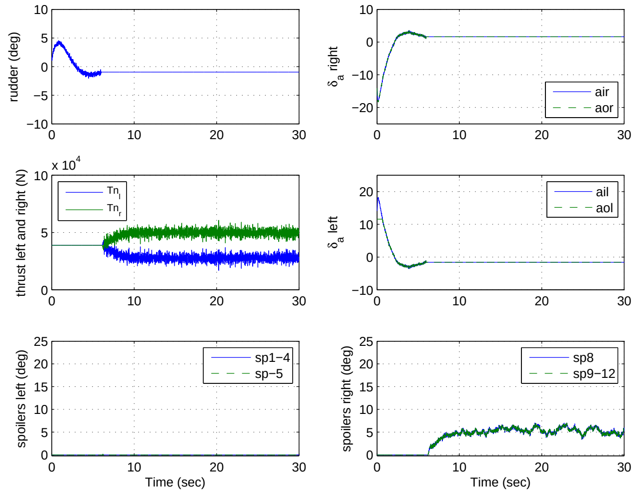

Fig. 6. Primary failure (imperfect estimation of W): actuators