An observer based controller design with disturbance feedforward framework for formation control of satellites

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Abstract

In this paper, a bespoke sliding mode nonlinear observer and a linear controller framework is proposed for achieving robust formation control of a cluster of satellites in the case of a circular reference orbit. Exploiting the structure of the satellite dynamics, a nonlinear observer is proposed based on super-twist sliding mode ideas. The observer estimates the states and any unknown bounded disturbances in *finite time*. The stability properties of the observers are demonstrated using Lyapunov techniques. A distributed controller, based on the estimated states and the relative position output information, depending on the underlying communication topology, is proposed. A polytopic representation of the collective dynamics which depends on the eigenvalues of the Laplacian matrix associated with the communication topology is used to synthesize the gains of the proposed control laws. A simulation example is used to demonstrate the efficacy of the proposed approach.

1 Introduction

In recent years, multiple, cheap satellites have been deployed into earth orbit and controlled to perform together collectively to meet a common goal - often of a scientific nature. Centralized, decentralized and distributed control techniques have been suggested/developed for this purpose. Many researchers have investigated such problems from different paradigms. From a viewpoint of the operational cost, optimal control laws are often preferred to account for deviations from a desired orbit (a J_2 invariant orbit) resulting from external sources such as atmospheric drag, solar and lunar gravity, and the effect of the Earth's oblateness (also known as the J_2 effect) [1, 2]. The literature on spacecraft formation flying control has to a large extent focused on the relative position control problem, see Refs.[3, 4] for extensive details of state-of-the-art guidance and control techniques for formation control of spacecraft; however certain exceptions exist.

In satellite formations, since the inter-satellite distances are small compared to the radii of the orbits, the dynamics governing the relative behaviour of the satellites, in the case of circular reference orbits can be represented using Hill's equations [2, 5]. This has been used as the basis of many of the control laws which have been proposed[6, 7]. Different leader follower paradigms, and control paradigms have been considered by different researchers. The key advantage of the leader-follower method is that the control problem becomes a tracking problem, which can be designed and analyzed using standard control theoretic techniques. A disadvantage is that the formation does not tolerate failures at the leader level, since the leader's trajectory is independent of the motion of the associated followers. Recently graph theoretic methods have been widely applied to formation problems: see for example [8, 9, 10, 11, 12, 13, 14] and the references therein. In [8], a swarm of satellites around a planet on a circular orbit, modelled as Clohessy-Wiltshire (CW) equations, is considered, and disturbance terms are assumed to be absent.

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Centralized, decentralized and distributed output feedback controllers are designed (aiming to minimis the relative error in the positions) and compared. In [9], relative position information alone is used to achieve a formation. Decentralized controller implementations are realized - mainly formulating them as convex optimization problems, with a possibility of interchange of various precalculated local measurement or communicated variable options for control. In [10], the coupled translational and attitude dynamics of satellites are considered, and the formation problem is realized as a special type of synchronization problem. Nonlinear tracking control laws (with exponential convergence) based on contraction principles and phase coupled oscillator laws for the synchronization problem (formation) have been studied. Consensus ideas as seen in [12, 13] have been applied to the problem of formation control of simplified satellite models in [11] (posed as a class of consensus problem). A leader following strategy (in circular orbits) has been studied in [14] using the tools from graph theory and linear matrix inequalities. Asymptotically stabilizing state feedback control laws have been proposed in [14] (also in the presence of saturation, uncertainties and disturbances) for the case, where the states are measured in an inertial and moving reference frame. In [6, 7], sliding mode control techniques have been employed to attain formation control of a leader-follower configuration. A behavioural approach is studied in [15] for an identical problem. In this approach, the applied control action for each satellite is a weighted average of the possible control actions corresponding to different desired behaviours for the satellite. This has advantages when multiple competing objectives are considered. One main drawback of the methodology is that it is difficult to apply rigorous analytical analysis to the closed loop and hence it is difficult to obtain an explicit guaranteed proof of collective stability of the system. Adaptive control has been employed in [16] to obtain a less conservative and generic adaptive scheme for the position tracking of formations of satellites. In [16] the underlying dynamics considered were not based on Hill's equations but on fully nonlinear ones as in [17], where a novel learning controller has been realized for formation control. An advantage of the proposed method in [16] is that the results hold globally, and the positions are adaptively tracked against parameter variations. However, a limitation is that the scheme requires both the relative position and velocity information. Furthermore, estimation of the entire states of the system from measurements and making use of this information for control, has been explored in [18, 19], one is based on covariance estimation principles and the other completely based on fuzzy rules.

In [20] the application of graph theoretic methods to a number of different applications including satellite formation problems [14] is discussed. Ref. [11] provides a detailed look at consensus protocols for a similar problem. According to [8, 10, 11, 12, 14], formation flying of multiple satellites can be modeled as a spatially distributed network of vehicles which collaborate to perform a unique mission requirement. Graph theoretic methods, that are discussed in detail in [20], have been exploited in [10] to address the formation flying problem from the perspective of synchronization of a network of Lagrangian systems. An attractive generalized framework, which also utilizes the basics of contraction theory, is proposed in [10]. In [9] linear controllers have been designed using the relative measurements together with an observer. In addition, the possibility of decentralization of the designed controllers, and possible switching of the underlying communication topologies without affecting the stability and performance of the collective system, have also been investigated. See Refs. [21, 22] and references therein for a discussion on distributed control and decentralized estimation architectures in formation flying. Ref.[23] reports a fuel optimal control design addressing disturbance rejection based on LQR ideas. The advantage of this method is its cost benefits when compared with Lyapunov and period-matching methods [23]. However, the stability arguments yield local results - which may be considered as a drawback of the scheme.

Sliding mode tracking control laws (decoupled in terms of in-plane and cross-track) are developed in [6] in order for the follower satellites to maintain a close formation with a leader satellite. The underlying dynamics considered for the design are Hill's equations. Nonlinear simulations are carried out in the presence of disturbances to validate the efficacy of the proposed scheme. Continuous sliding mode control schemes driven by a sliding mode disturbance observer and a formation controller using a super twisting second order sliding mode controller are proposed in [7]. In addition, an integral sliding mode controller is proposed in [7]. All the sliding mode control methods are implemented using pulse width modulation techniques for precision robust tracking. An advantage of the sliding mode approach is its inherent robustness to matched uncertainties. However, a criticism of using sliding modes in the control loop is its requirement of high bandwidth and fast sampling, even when the infamous chattering issues are conveniently addressed.

In this paper, a leader, and a cluster of multiple satellites is considered and treated as a spatially distributed network of dynamical systems. The individual satellite dynamics are studied using Hill's equations (as in many other similar studies as discussed above). A novel nonlinear observer, exploiting structures in the satellite dynamics is proposed to robustly estimate the states and the disturbances in the follower satellites from relative position information only (together with the leader information). Furthermore, a robust distributed output feedback control scheme is developed in an elegant way treating the network system from a polytopic perspective. An advantage of the proposed scheme is that fault tolerance and structural robustness of the formation is enhanced because each satellite simultaneously monitors the local relative state information as well as the disturbance/exogenous signals, from only relative position measurements. By feeding-forward the estimates of the disturbance/exogenous signals, their effect on the performance of the closed-loop system is attenuated. Using the polytopic design approach, it is possible to alleviate any strong dependence on the specific form of the dynamics and the formulation topology. Also relative position information alone has been used for creating the distributed component of the control law. The attitude determination and control system (ADCS) of most satellites includes an observer, usually an extended Kalman filter, and the proposed nonlinear observer can be regarded as part of such a system. By following this approach, the conventional criticism regarding the use of sliding modes in the control loop (such as high gain and bandwidth requirements) is avoided as the sliding mode now occurs instead in the observer loop and generates accurate disturbance estimates in *finite time*. However the benefit of recovering robustness is still achieved with a feed-forward term, and the control scheme retains a simple (traditional) state feedback structure.

1.1 Notation

The notation in the paper is quite standard. The set of real numbers, real-valued vectors of length m, and real-valued $m \times n$ matrices are given by $\mathbb{R}$, $\mathbb{R}^m$, and $\mathbb{R}^{m\times n}$ respectively. $\mathscr{C}ol(.)$ and $\mathscr{D}iag(.)$ are used to denote column vectors and diagonal matrices. For a symmetric positive definite (s.p.d) matrix $P = P^{\mathrm{T}} > 0$, $\lambda_{min}(P)$ and $\lambda_{max}(P)$ are the minimum and maximum eigenvalues. The graph theoretic terminology employed is also quite standard [24]. For a graph $\mathscr{G}$, the adjacency matrix $\mathscr{A}(\mathscr{G}) = [a_{ij}]$, is defined by setting $a_{ij} = 1$ if i and j are adjacent nodes of the graph, and $a_{ij} = 0$ otherwise. This creates a symmetric matrix. The symbol $\Delta(\mathscr{G}) = [\alpha_{ij}]$ represents the degree matrix, and is an $N \times N$ diagonal matrix, where α_{ii} is the degree of the vertex i. The Laplacian of $\mathscr{G}$, $\mathscr{L}(\mathscr{G})$, is defined as the difference $\Delta(\mathscr{G}) - \mathscr{A}(\mathscr{G})$.

2 System Description and Problem Definition

A cluster of $N+1$ satellites, consisting of a leader satellite and N follower satellites, which are in nearby orbits, is considered. The following simplifications are assumed to hold. The leader satellite is on a circular Keplerian orbit. The follower satellites can estimate the relative distance between all the nearby satellites as well as the leader satellite. The coupling effect between the attitude and translational dynamics of the satellites is assumed to be weak and is ignored. The follower satellites have information about the control forces employed in the leader.

The simplified equations for the relative motion of satellites, with an elliptic reference orbit, known as the Tschauner-Hempel (TH) equations, containing periodic coefficients, are:

$$\begin{aligned}
\ddot{x}_i - 2\dot{\theta}\dot{y}_i - \ddot{\theta}y_i - (\dot{\theta}^2 + 2\frac{\mu}{R_0^3})x_i &= \bar{u}_{xi} + \bar{d}_{xi} \\
\ddot{y}_i + 2\dot{\theta}\dot{x}_i + \ddot{\theta}x_i - (\dot{\theta}^2 - \frac{\mu}{R_0^3})y_i &= \bar{u}_{yi} + \bar{d}_{yi} \\
\ddot{z}_i + \frac{\mu}{R_0^3}z_i &= \bar{u}_{zi} + \bar{d}_{zi}
\end{aligned} \tag{1}$$

where $R_0 = p/(1 + e\cos\theta)$. In the equations above, R_0 is the radius, θ is the true anomaly, $p = A_0(1 - e^2)$, A_0 is the semi-major axis, and e is the eccentricity. The term μ is the gravitational parameter of the Earth. If $e = 0$, the orbit is circular and $\dot{\theta} = n = (\mu/A_0^3)^{1/2}$, i.e., the orbital mean motion. Exploiting the fact that $\dot{\theta}^2 = \mu/R_0^3 = \omega_n^2$, the relative dynamics of the i^{th} follower satellite can be studied using the so-called Hill's equations or the Clohessy-Wiltshire equations [2]:

$$\ddot{x}_i - 2\omega_n\dot{y}_i - 3\omega_n^2 x_i = \bar{u}_{xi} + \bar{d}_{xi} \tag{2}$$

$$\ddot{y}_i + 2\omega_n\dot{x}_i = \bar{u}_{yi} + \bar{d}_{yi} \tag{3}$$

$$\ddot{z}_i + \omega_n^2 z_i = \bar{u}_{zi} + \bar{d}_{zi} \tag{4}$$

In (2)-(4), x_i, y_i and z_i represent displacements in the radial, tangential and out-of-plane directions respectively with respect to the leader satellite. The angular velocity of the orbit is given by ω_n. The quantities $\bar{u}_{xi}$, $\bar{u}_{yi}$ and $\bar{u}_{zi}$ represent the net control force in each axis (m/s^2), and $\bar{d}_{xi}$, $\bar{d}_{yi}$ and $\bar{d}_{zi}$ correspond to the net specific disturbances experienced in each axis by the follower satellites. This could be, for example, due to atmospheric drag, tesseral resonance and the J_2 effects.

Note that the out-of-plane dynamics are decoupled from the radial and tangential dynamics and are ignored in this paper. (The out-of-plane dynamics can be addressed separately if required.) The disturbances in the (totally decoupled) second order dynamics in (4) can be estimated using the 'classical' super-twisting scheme as discussed in [26]. Only the radial and tangential $(x - y)$ plane dynamics, which are coupled, are addressed in this paper. Following [2, 6, 7], natural time t can be scaled, and a new time $\tau := \omega_n t$ can be defined. With respect to time τ, the dynamics in the $(x - y)$ plane (normalized Hill equations) can be written as:

$$\ddot{x}_i - 2\dot{y}_i - 3x_i = u_{xi} + d_{xi} \tag{5}$$

$$\ddot{y}_i + 2\dot{x}_i = u_{yi} + d_{yi} \tag{6}$$

where now the 'dot' notation represents the differentiation w.r.t time τ. The net specific control forces relative with respect to the leader u_{xi} and u_{yi} can be written as

$$u_{xi} = u_{xi}^f - u_x^l \tag{7}$$

$$u_{yi} = u_{yi}^f - u_y^l \tag{8}$$

where the superscripts f and l indicate the follower and leader respectively, and so for example, u_{xi}^f is the control signal applied to the i^{th} follower satellite in the radial direction. The units of the specific control forces (u_{xi} and u_{yi}) and the disturbances (d_{xi} and d_{yi}) are in terms of force per unit mass per mean motion squared (ω_n^2). Hence, all the terms in (5) and (6) have a length dimension in new time τ.

The dynamics of the i^{th} satellite in the radial and tangential $(x - y)$ plane (5) - (6) can be rewritten conveniently in state space form as

$$\begin{aligned}\dot{X}_i &= AX_i + BU_i + Bd_i \qquad (9)\\ Z_{ij} &= C(X_i - X_j), \;\; j \in \mathscr{J}_i \qquad (10)\end{aligned}$$

for $i = 1,\ldots,N$ where $X_i = \mathscr{C}ol(x_{1i}, x_{2i}, x_{3i}, x_{4i}) := \mathscr{C}ol(x_i, \dot{x}_i, y_i, \dot{y}_i)$ represents the relative states of the satellite. As in [8, 9, 12], the term $Z_{ij} \in \mathbb{R}^2$ denotes the relative position output measurement between the i^{th} and the j^{th} satellites belonging to the neighbourhood set $\mathscr{J}_i$ of the i^{th} satellite. The coordinates $x_{1i} := x_i$ and $x_{3i} := y_i$ describe the position of the i^{th} follower satellite relative to the leader satellite. In (9) - (10), $U_i = \mathscr{C}ol(u_{xi}, u_{yi})$ represents the control input vector - the net specific control forces acting on the i^{th} follower satellite, and $d_i = \mathscr{C}ol(d_{xi}, d_{yi})$ represents the net specific disturbances respectively. The constant matrices in (9) - (10) are given by

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The nonempty set $\mathscr{J}_i \subset \{1,\ldots,N\}/\{i\}$ represents the indices of the other dynamical systems, for which the i^{th} dynamical system has information. Here, an assumption is made that each dynamical system has information about at least one other dynamical system. Combining all the relative information among the dynamical systems, at the i^{th} node level, equation (10) can be written as

$$Z_i = \sum_{j \in \mathscr{J}_i} C(X_i - X_j) \qquad (11)$$

This notation is consistent with that reported in [12].

Collectively, at a network level, the dynamics in (9) - (10) can be written using the Kronecker product notation as

$$\begin{aligned}\dot{X} &= (I_N \otimes A)X + (I_N \otimes B)(U + d) \qquad (12)\\ Z &= (\mathscr{L} \otimes C)X \qquad (13)\end{aligned}$$

where $X = \mathscr{C}ol(X_1,\ldots,X_N)$ represents the concatenated column vector of the N follower satellite states, $U = \mathscr{C}ol(U_1,\ldots,U_N)$ represents the concatenated control input vector, and $d = \mathscr{C}ol(d_1,\ldots,d_N)$ represents the net disturbance vector. The Laplacian of the graph $\mathscr{G}$, written as $\mathscr{L} \in \mathbb{R}^{N\times N}$, represents the relative sensing topology in (11). The Laplacian matrix $\mathscr{L}$ is defined as follows:

$$\begin{aligned}\mathscr{L}_{ii} &= |\mathscr{J}_i| \qquad (14)\\ \mathscr{L}_{ij} &= \begin{cases} -1, & j \in \mathscr{J}_i \\ 0 & j \notin \mathscr{J}_i \end{cases} \qquad (15)\end{aligned}$$

where $|\mathscr{J}_i|$ is the cardinality of the set $\mathscr{J}_i$ and represents the degree of the i^{th} node. The smallest eigenvalue of $\mathscr{L}$ is zero and the corresponding eigenvector is given by $\mathbf{1}$, (i.e. a column vector composed entirely of ones). The matrix $\mathscr{L}$ is always rank deficient, symmetric and positive semi-definite in the case of undirected graphs [24]. The Laplacian of a graph with bidirectional communication has identical properties to that of an undirected graph.

2.1 Problem Definition

The main objective of this paper is to determine the u_{xi} and u_{yi} control forces required for each follower satellite to maintain formation flight. Since it is assumed that the information from the leader satellite about the u^l_{xi} and u^l_{yi} is broadcast to all the follower satellites, from equation (7) - (8), u^f_{xi} and u^f_{yi} can be determined provided U_i is calculated by the i^{th} satellite.

A control law U_i for $i = 1,\ldots,N$ is proposed, based on a polytopic representation of the spatially distributed network, to attain a formation for the dynamics in (9)-(10). The control law U_i is assumed to be a function of estimates of X_i and measurements Z_i. Since only relative positions (x_{1i} and x_{3i}) are available, a novel nonlinear observer that has basis in a second order sliding mode control methodology [25, 26] will be employed to reconstruct estimates of X_i in *finite time*. The proposed nonlinear observer can be used to robustly reconstruct relative state information in finite time. The control law then makes use of the reconstructed (estimated) internal state measurements, plus the relative output (relative position alone) measurements.

An architecture consisting of a novel nonlinear observer and a distributed output feedback controller is proposed for obtaining the formation of the satellites, is described in sequel.

3 Design of the Nonlinear Observer

The nonlinear observer which is proposed in this paper is inspired by the second order super twisting observer proposed in [25, 26]. The design (locally) robustly estimates the states X_i and the unknown disturbances, $d_i = \mathscr{C}ol(d_{xi}, d_{yi})$, simultaneously from the measured relative position outputs (x_{1i}, x_{3i}) of each individual follower satellite.

Let the state estimate of the i^{th} follower satellite be given by $\tilde{X}_i := \mathscr{C}ol(\tilde{x}_{1i}, \tilde{x}_{2i}, \tilde{x}_{3i}, \tilde{x}_{4i})$. Consider the nonlinear observer dynamical system described by

$$\dot{\tilde{x}}_{1i} = \tilde{x}_{2i} - k_1|e_{1i}|^{\frac{1}{2}}\,\mathrm{sgn}(e_{1i}) \tag{16}$$

$$\dot{\tilde{x}}_{2i} = 3\tilde{x}_{1i} + 2\tilde{x}_{4i} - k_5 e_{1i} - k_3\,\mathrm{sgn}(e_{1i}) - k_2|e_{3i}|^{\frac{1}{2}}\,\mathrm{sgn}(e_{3i}) + u_{xi} \tag{17}$$

$$\dot{\tilde{x}}_{3i} = \tilde{x}_{4i} - k_2|e_{3i}|^{\frac{1}{2}}\,\mathrm{sgn}(e_{3i}) \tag{18}$$

$$\dot{\tilde{x}}_{4i} = -2\tilde{x}_{2i} - k_4\,\mathrm{sgn}(e_{3i}) + k_1|e_{1i}|^{\frac{1}{2}}\,\mathrm{sgn}(e_{1i}) + u_{yi} \tag{19}$$

where: $e_i = \tilde{X}_i - X_i$, such that $e_i = \mathscr{C}ol(e_{1i}, e_{2i}, e_{3i}, e_{4i})$. In (16) - (19), the $k_i \in \mathbb{R}^+$, $i = 1,\ldots,5$ represent the positive scalar design gains to be determined. This will be discussed in the sequel. The global error estimate at the network level is given by $e = \mathscr{C}ol(e_1,\ldots,e_N)$.

Remark 1 *When compared to the classical super-twisting observer proposed in [26] for estimating the states in second order systems, additional significant cross coupling terms $-k_2|e_{3i}|^{\frac{1}{2}}\,sgn(e_{3i})$ and $+k_1|e_{1i}|^{\frac{1}{2}}\,sgn(e_{1i})$ are present in (17) and (19). Furthermore in the analysis which follows the structure associated with the cross coupling terms $-2\omega_n\dot{y}_i$ and $+2\omega_n\dot{x}_i$ in Hill's equations is exploited in proposing the new nonlinear observer. The proposed nonlinear observer will be analyzed making use of the class of Lyapunov function originally proposed in [25]. A novelty of the proposed nonlinear observer above is the exploitation of the structure of the cross coupling in the satellite plant.*

Here, the scalar linear gain term $k_5 = 3$ by design, and as a consequence the error in the state estimate of the i^{th} follower satellite, e_i, is given by

$$\begin{aligned}
\dot{e}_{1i} &= -k_1|e_{1i}|^{\frac{1}{2}}\mathrm{sgn}(e_{1i}) + e_{2i} && (20)\\
\dot{e}_{2i} &= 2e_{4i} - k_3\mathrm{sgn}(e_{1i}) - k_2|e_{3i}|^{\frac{1}{2}}\mathrm{sgn}(e_{3i}) - d_{xi} && (21)\\
\dot{e}_{3i} &= -k_2|e_{3i}|^{\frac{1}{2}}\mathrm{sgn}(e_{3i}) + e_{4i} && (22)\\
\dot{e}_{4i} &= -2e_{2i} - k_4\mathrm{sgn}(e_{3i}) + k_1|e_1|^{\frac{1}{2}}\mathrm{sgn}(e_{1i}) - d_{yi} && (23)
\end{aligned}$$

where $i = 1,\ldots,N$. Note that e_{1i} and e_{3i} are available and represent the difference between the estimated and the measured relative $(x-y)$ positions of the i^{th} satellite. However, the terms d_{xi} and d_{yi} are unknown. Now the objective is to develop sufficient conditions for ensuring stability of the error dynamics in (20)-(23). The proposed design ensures the convergence of the error dynamics associated with the estimates of the states to zero *in finite time*.

Assumption 3.1 *It is assumed that the unknown disturbance terms d_{xi} and d_{yi} in the error dynamics satisfy a-priori known upper bounds. Specifically suppose $|d_{xi}| \leq \delta_1$ and $|d_{yi}| \leq \delta_2$ for known constants $\delta_1, \delta_2 \geq 0$.* This assumption is similar to the one made in [6, 7].

Remark 2 *If the disturbances are considered as unknown inputs, the problem considered in this paper is relative degree two with respect to these inputs and the measured relative positions. In the linear unknown input literature [27] as well as in 'classical' sliding mode observer approaches [28, 29], a necessary condition is that the mapping between the unknown input signal and the measured output of interest must be relative degree one and minimum phase. However, this is not the case in this paper, and hence none of the literature on linear unknown input observers is applicable. This is the main advantage of using the approach proposed in this paper.*

Consider a candidate Lyapunov function $V(e_i)$ for the error dynamics system in (20) - (23), inspired by the one in [25], given by:

$$\begin{aligned}
V(e_i) &= 2k_3|e_{1i}| + \frac{1}{2}e_{2i}^2 + \frac{1}{2}(k_1|e_{1i}|^{\frac{1}{2}}\mathrm{sgn}(e_{1i}) - e_{2i})^2\\
&+ 2k_4|e_{3i}| + \frac{1}{2}e_{4i}^2 + \frac{1}{2}(k_2|e_{3i}|^{\frac{1}{2}}\mathrm{sgn}(e_{3i}) - e_{4i})^2 && (24)
\end{aligned}$$

Note $V(e_i)$ is continuous and positive definite for all e_i, but is not differentiable at $\{e_i|e_{1i}=0, e_{3i}=0\}$. Following the arguments in Remark 1 of [25], Lyapunov methods can still be applied to those points where $V(e_i)$ is differentiable, i.e. for all $\{e_i|e_{1i}\neq 0, e_{3i}\neq 0\}$. In the sequel, it will be shown that $V(e_i)$ is indeed a Lyapunov function for system (20) - (23).

The proposed candidate Lyapunov function can be written as a quadratic form $V(\xi_i) = \xi_i^{\mathrm{T}}P\xi_i$ where $\xi_i := \mathscr{C}ol(\xi_{i1}, \xi_{i2})$ and $\xi_{i1} := \mathscr{C}ol(|e_{1i}|^{\frac{1}{2}}\mathrm{sgn}(e_{1i}),\ e_{2i})$ and $\xi_{i2} := \mathscr{C}ol(|e_{3i}|^{\frac{1}{2}}\mathrm{sgn}(e_{3i}),\ e_{4i})$. The block diagonal Lyapunov matrix defining the quadratic form is

$$P = \begin{bmatrix} P_1 & \mathbf{0}_{2\times 2} \\ \mathbf{0}_{2\times 2} & P_2 \end{bmatrix} \tag{25}$$

where

$$P_1 = \frac{1}{2}\begin{bmatrix} 4k_3 + k_1^2 & -k_1 \\ -k_1 & 2 \end{bmatrix},\ P_2 = \frac{1}{2}\begin{bmatrix} 4k_4 + k_2^2 & -k_2 \\ -k_2 & 2 \end{bmatrix}$$

Note $V(\xi_i)$ is radially unbounded if $k_3 > 0$ and $k_4 > 0$. It can be shown that the time derivative of $V(\xi_i)$ along the trajectories of the system (20) - (23) is given by

$$\dot{V}(\xi_i) = -\frac{1}{|e_{1i}|^{\frac{1}{2}}}\xi_{i1}^{\mathrm{T}}Q_1\xi_{i1} - \frac{1}{|e_{3i}|^{\frac{1}{2}}}\xi_{i2}^{\mathrm{T}}Q_2\xi_{i2} + \xi_i^{\mathrm{T}}Q_3 d_i \tag{26}$$

where

$$Q_1 = \frac{k_1}{2}\begin{bmatrix} 2k_3 + k_1^2 & -k_1 \\ -k_1 & 1 \end{bmatrix}, \quad Q_2 = \frac{k_2}{2}\begin{bmatrix} 2k_4 + k_2^2 & -k_2 \\ -k_2 & 1 \end{bmatrix}$$

and

$$Q_3 = \begin{bmatrix} -k_1 & 0 \\ 2 & 0 \\ 0 & -k_4 \\ 0 & 2 \end{bmatrix}$$

Note that significant algebraic manipulation is necessary to achieve the structure in (26) because although $V(\xi_i)$ and $\dot{V}(\xi_i)$ present a decoupled block structure as given in (24) and (26), the differential equations in (20)-(23) are coupled. In achieving (26) the structure in the coupling terms $-2\omega_n \dot{y}_i$ and $+2\omega_n \dot{x}_i$ of the satellite plant mentioned in Remark 1 have been exploited.

From assumption 3.1, the upper bounds on the unknown disturbance terms d_{xi} and d_{yi} are known. In the presence of these bounded unknown disturbance terms, it can be shown using arguments similar to those in [25] that

$$\dot{V}(\xi_i) \leq -\frac{1}{|e_{1i}|^{\frac{1}{2}}}\xi_{i1}^{\mathrm{T}}\tilde{Q}_1\xi_{i1} - \frac{1}{|e_{3i}|^{\frac{1}{2}}}\xi_{i2}^{\mathrm{T}}\tilde{Q}_2\xi_{i2} \tag{27}$$

where

$$\tilde{Q}_1 = \frac{k_1}{2}\begin{bmatrix} 2k_3 + k_1^2 - 2\delta_1 & -k_1 - 2\frac{\delta_1}{k_1} \\ -k_1 - 2\frac{\delta_1}{k_1} & 1 \end{bmatrix}$$

and

$$\tilde{Q}_2 = \frac{k_2}{2}\begin{bmatrix} 2k_4 + k_2^2 - 2\delta_2 & -k_2 - 2\frac{\delta_2}{k_1} \\ -k_2 - 2\frac{\delta_2}{k_1} & 1 \end{bmatrix}$$

and in this situation $\dot{V}(\xi_i)$ is negative definite if $\tilde{Q}_1$ and $\tilde{Q}_2$ are positive definite. Provided the scalar positive gains k_i, for $i = 1, \ldots, 4$, satisfy the following conditions

$$k_1 > 0, \quad k_3 > 3\delta_1 + 2\frac{\delta_1^2}{k_1^2} \tag{28}$$

$$k_2 > 0, \quad k_4 > 3\delta_2 + 2\frac{\delta_2^2}{k_2^2} \tag{29}$$

it can be verified that $\tilde{Q}_1$ and $\tilde{Q}_2$ are positive definite and consequently $\dot{V}(\xi_i)$ is negative definite for all $\xi_i \neq 0$ and $t > 0$.

3.1 Finite time convergence to origin

Exploiting the very specific block diagonal structure of the Lyapunov matrix in (25), rewrite the quadratic Lyapunov function in (24) as

$$V(\xi_i) := \underbrace{\xi_{i1}^{\mathrm{T}} P_1 \xi_{i1}}_{V_1(\xi_{i1})} + \underbrace{\xi_{i2}^{\mathrm{T}} P_2 \xi_{i2}}_{V_2(\xi_{i2})} \tag{30}$$

The functions $V_1(\xi_{i1})$ and $V_2(\xi_{i2})$ are positive definite with respect to ξ_{i1} and ξ_{i2} respectively. From Rayleigh's inequality [30]

$$\gamma_{min}(P_1)\|\xi_{i1}\|_2^2 \leq V_1(\xi_{i1}) \leq \gamma_{max}(P_1)\|\xi_{i1}\|_2^2 \tag{31}$$

$$\gamma_{min}(P_2)\|\xi_{i2}\|_2^2 \leq V_2(\xi_{i2}) \leq \gamma_{max}(P_2)\|\xi_{i2}\|_2^2 \tag{32}$$

where $\gamma_{min}(.)$ and $\gamma_{max}(.)$ represent the minimum and maximum eigenvalue of the Lyapunov matrix, and $\|.\|_2$ represents the Euclidean norm. It follows that

$$|e_1|^{\frac{1}{2}} \leq \|\xi_{i1}\|_2 \leq \frac{V_1^{\frac{1}{2}}(\xi_{i1})}{\gamma_{min}^{\frac{1}{2}}(P_1)} \tag{33}$$

and

$$|e_3|^{\frac{1}{2}} \leq \|\xi_{i2}\|_2 \leq \frac{V_2^{\frac{1}{2}}(\xi_{i2})}{\gamma_{min}^{\frac{1}{2}}(P_2)} \tag{34}$$

then using identical arguments to those in [25], the inequality in (27) can be written as

$$\dot{V}(\xi_i) \leq -\frac{1}{|e_{1i}|^{\frac{1}{2}}}\gamma_{min}(\tilde{Q}_1)\|\xi_{i1}\|_2^2 - \frac{1}{|e_{3i}|^{\frac{1}{2}}}\gamma_{min}(\tilde{Q}_2)\|\xi_{i2}\|_2^2 \tag{35}$$

Using the inequalities (33) and (34), (35) can further be written as

$$\dot{V}(\xi_i) \leq -\beta_1 V_1^{\frac{1}{2}}(\xi_{i1}) - \beta_2 V_2^{\frac{1}{2}}(\xi_{i2}) \tag{36}$$

where $\beta_1 = \frac{\gamma_{min}^{\frac{1}{2}}(P_1)\gamma_{min}(\tilde{Q}_1)}{\gamma_{max}(P_1)}$ and $\beta_2 = \frac{\gamma_{min}^{\frac{1}{2}}(P_2)\gamma_{min}(\tilde{Q}_2)}{\gamma_{max}(P_2)}$ and it follows that

$$\dot{V}(\xi_i) \leq -\beta(V_1^{\frac{1}{2}}(\xi_{i1}) + V_2^{\frac{1}{2}}(\xi_{i2})) \tag{37}$$

where $\beta = \mathtt{min}(\beta_1, \beta_2)$. Since $(V_1^{\frac{1}{2}} + V_2^{\frac{1}{2}})^2 > V_1 + V_2$, because V_1 and V_2 are positive, it can be concluded that $V_1^{\frac{1}{2}} + V_2^{\frac{1}{2}} > V^{\frac{1}{2}}$. This further implies that

$$\dot{V}(\xi_i) \leq -\beta V^{\frac{1}{2}} \tag{38}$$

and hence $V(\xi_i) \equiv 0$ in *finite time*.

Remark 3 *Since the proposed nonlinear observer in (16) - (19) estimates the entire states in* finite time*, a local state feedback controller can be designed at an individual satellite level. A separation principle [30] holds, and the controller can be designed using the estimated states to stabilize the plants. For simplicity of controller synthesis, and because of the fact that the state estimates are available in* finite time*, the estimated states are assumed to be the same as the plant states (although a detailed Lyapunov analysis can be carried out to address the error in state estimate terms).*

3.2 Reconstruction of states and disturbance

As argued above, the origin $e_i = 0$ is attained in *finite time*. Consequently from (16) - (19), the estimate of the states of the i^{th} follower satellite $\tilde{X}_i := \mathscr{C}ol(\tilde{x}_{1i}, \tilde{x}_{2i}, \tilde{x}_{3i}, \tilde{x}_{4i})$ is available in *finite time*. Substituting for $e_i \equiv 0$ in (21) and (23) yields

$$\underbrace{-k_3\mathtt{sgn}(e_{1i})}_{v_{1i}} - d_{xi} = 0 \tag{39}$$

$$\underbrace{-k_4\mathtt{sgn}(e_{3i})}_{v_{3i}} - d_{yi} = 0 \tag{40}$$

Therefore $\nu_{eq,1i} := d_{xi}$ and $\nu_{eq,3i} := d_{yi}$, where $\nu_{eq,i*}$ denotes the equivalent injection signals [31] necessary to maintain sliding. Thus d_{xi} and d_{yi} can be obtained to good accuracy by low pass filtering of ν_{1i} and ν_{3i} [31]. The information about the estimates of d_{xi} and d_{yi}, defined as $\tilde{d}_{xi}$ and $\tilde{d}_{yi}$ respectively, can be employed in the control law to improve the disturbance rejection properties.

Remark 4 *The quantities $\nu_{eq,*i}$ would also contain information about actuator faults or corruption in the u^l_{xi} and u^l_{yi} data broadcast to each of the followers, thus providing a certain level of robustness. The mismatch between the actual leader control signals and whatever is received at the follower satellite level would appear as additional disturbance terms. The bounds δ_1 and δ_2 in assumption 3.1 must hold for any additional terms emanating from the corrupted signals. However, it is important to note that a total communication failure can not be handled by the proposed scheme.*

Remark 5 *If information from the leader satellite is not available to the followers, from equations* (7) *and* (8) *the quantities u^l_x and u^l_y can be absorbed into the disturbance terms d_{xi} and d_{yi} and the injection terms ν_{1i} and ν_{3i} in* (39) *and* (40) *estimate the leader forces and the disturbances.*

4 Controller Synthesis

The proposed controller consists of two parts, a local internal state feedback component, (since all the internal states X_i are available at each follower satellite in *finite time* from the proposed nonlinear observer), and a distributed component which depends on the external relative measured positions (Z_i) of the follower satellites. Consider a control law of the form

$$U_i = -KX_i - Z_i - \tilde{d}_i - \psi_i \tag{41}$$

for $i = 1, \ldots, N$, where $K \in \mathbb{R}^{2\times 4}$ is a state feedback gain, $\tilde{d}_i = \mathscr{C}ol(\tilde{d}_{xi}, \tilde{d}_{yi})$ and the term $\psi_i \in \mathbb{R}^2$ is the offset in the relative information at each node, so that each agent maintains a desired relative distance from its neighbours. Using the control law from (41) in a feedback loop, the closed loop dynamics in (9) is given by

$$\dot{X}_i = (A - BK)X_i - BZ_i + B(d_i - \tilde{d}_i) - B\psi_i \tag{42}$$

for $i = 1, \ldots, N$. Ignoring the $(d_i - \tilde{d}_i)$ terms, the closed loop dynamics of the overall network can be conveniently written as

$$\dot{X} = (I_N \otimes (A - BK))X - (\mathscr{L} \otimes BC)X - (I \otimes B)\Psi \tag{43}$$

where $X = \mathscr{C}ol(X_1, \ldots, X_N)$ and $\Psi = \mathscr{C}ol(\psi_1, \ldots, \psi_N)$ and $\mathscr{L}$ represents the graph Laplacian matrix associated with the relative sensing among the satellites. The matrix $\mathscr{L}$ is symmetric and hence a spectral decomposition is possible [32]. Consequently $\mathscr{L}$ can be written as:

$$\mathscr{L} = S\Lambda S^{\mathrm{T}} \tag{44}$$

where S is an orthogonal matrix comprising the eigenvectors of $\mathscr{L}$ and Λ is a diagonal matrix with the eigenvalues of $\mathscr{L}$ as the diagonal terms (which all are real numbers). The diagonal matrix Λ is

$$\Lambda := \mathscr{D}iag(\lambda_1, \ldots, \lambda_i, \ldots, \lambda_N) \tag{45}$$

with the property

$$0 = \lambda_1 < \lambda_2 \leq \ldots \leq \lambda_i \leq \ldots \leq \lambda_N = \lambda_{max} \tag{46}$$

Define a coordinate transformation $X \mapsto \bar{X} := TX$ where T is defined as

$$T := (S^{\mathrm{T}} \otimes I_n) \tag{47}$$

Because S is an orthogonal matrix obtained from (44), it can be shown, by making use of Kronecker product properties, that the transformation in (47) is orthogonal. Applying the coordinate transformation $T : X \mapsto \bar{X}$ in (47), the closed loop dynamics of the overall network is given by

$$\dot{\bar{X}} = ((I_N \otimes (A - BK)) - (\Lambda \otimes BC))\bar{X} \tag{48}$$

where $\bar{X} = \mathscr{C}ol(\bar{X}_1, \dots, \bar{X}_N)$. Since Λ is a diagonal matrix, a decoupling is achieved in the new coordinate system, when compared to the closed loop dynamics of the overall network in (43). As a result of the transformation, the dynamics in (48) can be written in the form

$$\dot{\bar{X}}_i = (A - BK - \lambda_i BC)\bar{X}_i \tag{49}$$

for $i = 1, \dots, N$. Since each λ_i satisfies $0 < \lambda_i < \lambda_N$ the problem of choosing K can be thus viewed as involving stabilizing a polytopic system [33, 34] where a real parameter λ_i varies in an interval $[0, \lambda_{max}]$. Consider the dynamics in (49) as a polytopic system $\mathscr{P}(\lambda)$ where λ varies in the interval $[0, \lambda_{max}]$ and thus the dynamics in (49) can be treated as a convex combination of $\mathscr{P}_{min}$ and $\mathscr{P}_{max}$, the two plants defined at the extremes of the interval. Write $\mathscr{P}(\lambda)$ as

$$\mathscr{P}(\lambda) = \rho_1 \mathscr{P}_{min} + \rho_2 \mathscr{P}_{max} \tag{50}$$

where $\rho_1 = 1 - \frac{\lambda}{\lambda_{max}}$ and $\rho_2 = \frac{\lambda}{\lambda_{max}}$ so that $\rho_1 + \rho_2 = 1$ and hence the plant $P(\lambda)$ is affine in λ. In equation (49), the plants corresponding to the extreme variations of λ are given as:

$$\mathscr{P}_{min} \;:=\; A - BK \tag{51}$$

$$\mathscr{P}_{max} \;:=\; A - BK + \lambda_{max} BC \tag{52}$$

Remark 6 *For a network with N nodes, an upper bound on λ_{max} is the maximum number of vertices in the network [20]. Hence for synthesis, this (worst-case) bound could be used.*

Hence, the problem can be posed as one of finding controller gain matrix K and a common Lyapunov s.p.d matrix $M \in \mathbb{R}^{4\times4}$ to stabilize the family of plants defined by $\mathscr{P}(\lambda) \in [\mathscr{P}_{min}, \mathscr{P}_{max}]$. This can be written as

$$(A - BK)^{\mathrm{T}} M + M(A - BK) \;<\; -2\kappa M \tag{53}$$

$$(A - \lambda_{max} BC - BK)^{\mathrm{T}} M + M(A - \lambda_{max} BC - BK) \;<\; -2\kappa M \tag{54}$$

where the scalar $\kappa > 0$ is introduced to ensure a certain level of performance. Since M and K are unknown, the above inequalities are *not* a system of LMI's in variables M and K. However, if new variables are introduced such as $\tilde{M} = M^{-1}$, $\tilde{K} = K\tilde{M}$, then the inequalities (53)-(54) are equivalent to

$$A\tilde{M} + \tilde{M}A^{\mathrm{T}} - B\tilde{K} - \tilde{K}^{\mathrm{T}} B^T \;<\; -2\kappa \tilde{M} \tag{55}$$

$$(A - \lambda_{max} BC)\tilde{M} + \tilde{M}(A - \lambda_{max} BC)^{\mathrm{T}} - B\tilde{K} - \tilde{K}^{\mathrm{T}} B^T \;<\; -2\kappa \tilde{M} \tag{56}$$

$$\tilde{M} \;>\; I \tag{57}$$

Also introduce a further inequality of the form

$$\begin{bmatrix} 0 & \tilde{K} \\ \tilde{K}^T & 0 \end{bmatrix} \;<\; \lambda I \tag{58}$$

The inequalities (55) - (58) are a system of LMI's in the decision variables $\tilde{M}$ and $\tilde{K}$. Since $\tilde{M} > I$, it follows that $\|K\| \le \|\tilde{K}\|$, and inequality (58) will constrain the solution to satisfy $\|\tilde{K}\| < \lambda$ where $\lambda > 0$ is the feasibility radius and thus avoids the controller gains becoming large. Thus the optimization problem considered here is to minimis λ subject to the inequalities (55) - (58), which is a generalized eigenvalue problem and can be solved using the standard solver `gevp` [33]). The problem then has a solution $K = \tilde{K}\tilde{M}^{-1}$, provided the system of LMIs are feasible [33].

The disturbances terms d_i present in (9), can be estimated in *finite time* by the proposed nonlinear observer in (16) - (23). The estimates can then be used to nullify the effect of the disturbances in the closed loop, to ensure $B(d_i - \tilde{d}_i) \approx 0$ in (42) to obtain good disturbance rejection.

Remark 7 *Additional performance can be introduced by using a scaling matrix Φ for the relative sensing information Z_i in (41) and solving the LMI's iteratively, from an initial guess for Φ.*

Following the approaches in [9, 20], weighted bias terms ψ_i (the desired spatial distances between the satellites) are introduced in the framework to provide the desired spatial separation between the satellites. These weighted bias terms can be obtained from basic vector algebra based on a steady state analysis of the network level closed loop system [35].

5 Results

A cluster of satellites consisting of a leader and four followers in a circular orbit, all represented using Hill's equations, is considered to demonstrate the proposed framework. A nearest neighbour interconnection topology as in [20] is used, yielding a Laplacian matrix

$$\mathscr{L} = \begin{bmatrix} 2 & -1 & 0 & -1 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}$$

with $\lambda_{max} = 4$. The leader satellite is assumed to be following a circular orbit. Following the arguments in [7], the reference trajectories can be obtained as $x_c(\tau) = r\sin(\tau + \theta), y_c(\tau) = 2r\cos(\tau + \theta)$ where r (the size of the ellipse) and θ (the initial angular position of the follower satellite on the path) define the desired elliptical path that the follower satellite maintains relative to the leader in a force free environment. (The initial conditions have been obtained by setting $\tau = 0$ in $x_c(\tau)$, $y_c(\tau)$ and also in their first derivatives). The angular position $\theta = -pi/2$ is used for the trajectories. The follower satellites are required to take up positions on the four corners of a square centered on the leader. To satisfy assumption 3.1, upper bounds on the unknown, matched, disturbances are assumed as $\delta_1 = 1$ and $\delta_2 = 1$. The gains k_i for $i = 1,..,4$ need to be selected satisfying the conditions (28)-(29). In the following simulations the gains of the nonlinear observer associated with each satellite are given by $k_1 = 1$, $k_2 = 1$, $k_3 = 10$ and $k_4 = 10$. These values satisfy the stability conditions in (28)-(29). Figure 1 shows the relative states of the follower satellites together with the representative random initial conditions. The simulation results are shown in the time units τ. Since, as in [7], τ is defined as the product of the mean motion and time, for an orbital period of 100.7 minutes, the actual time required to reach $\tau = 1$ in Figure 1 is 961.6 seconds. The net control forces u_{xi} and u_{yi} (the normalized values and w.r.t. time τ) necessary for the follower satellites are shown in Figure 2.

In Figure 3, the state estimation errors, in the absence of any disturbances, are shown. The underlying super-twist-like behaviour [26] of the proposed nonlinear observer is clearly seen from the error in the relative x and y positions (Figure 3). As is evident from Figure 3, since the error becomes zero, the states X_1 are *all* reconstructed perfectly in *finite time*. In all the figures, the time is in terms of τ units. Solving the LMIs (55) - (57) the controller gains are

$$K = \begin{bmatrix} 17.4254 & 5.2102 & -6.7196 & -1.8814 \\ 8.5555 & 0.9196 & 11.3258 & 4.4687 \end{bmatrix}$$

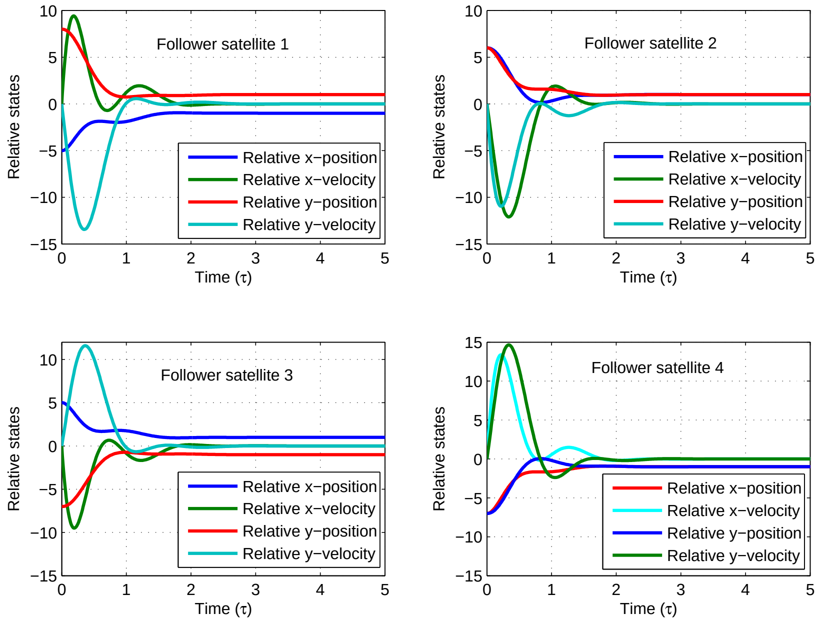

Figure 1: Evolution of relative states of follower satellites

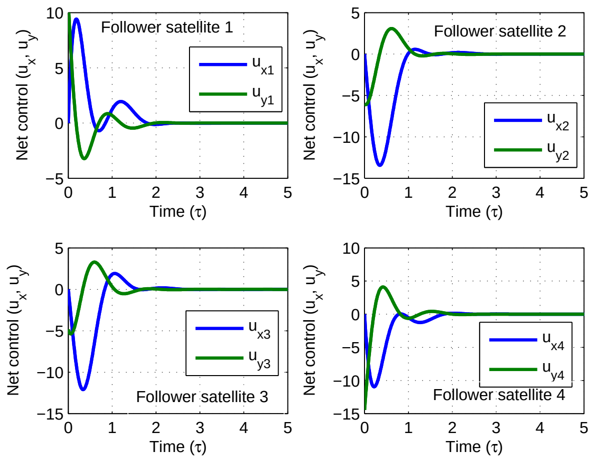

Figure 2: Evolution of control forces for follower satellites

Figure 4 shows the four follower satellites taking up the required formation (depicted in a radial-tangential plane with respect to the leader axis) centered on the leader.

In the simulation shown above, no disturbance terms are present, i.e, $d_i = 0$ for $i = 1,..,4$. For demonstration purposes in the sequel, sinusoidal terms are added to impact on Satellite 1. A representative sinusoidal disturbance, likewise in [7], has been considered and is given by $d_{ix} = a_0 \mathtt{sin}(\omega_0 t)$ and $d_{iy} = a_1 \mathtt{sin}(\omega_1 t)$ where $a_0 = 0.75, a_1 = 0.5$ and $\omega_0 = \omega_1 = 5$. Figure 5 shows the state reconstructions from the observer associated with the Satellite 1. Clearly, as claimed, robust performance is achieved in terms of state estimation. Figure 6 shows the plots of the equivalent signals $\nu_{eq,1}$ and $\nu_{eq,3}$ satisfying (39) and (40). Clearly the injection terms track the disturbance terms d_{1x} and d_{1y} in finite time. Figure 7 shows the effect of the disturbance terms on the closed loop performance where $\tilde{d}_{1x} = \nu_{eq,1}$ and

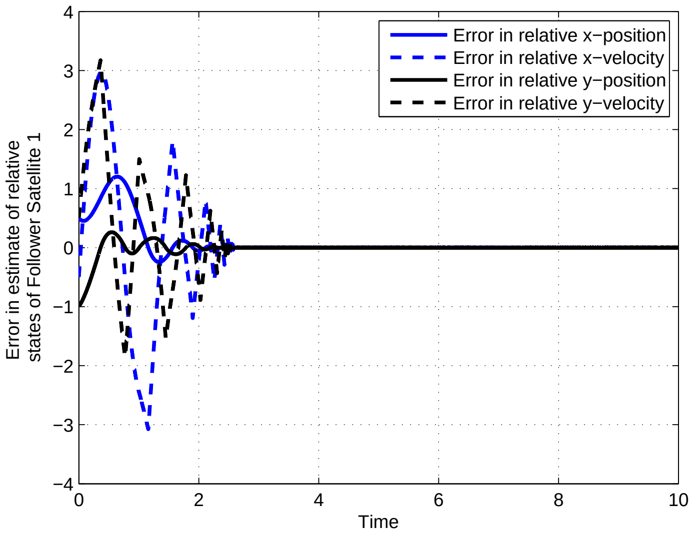

Figure 3: Error in estimates of relative states

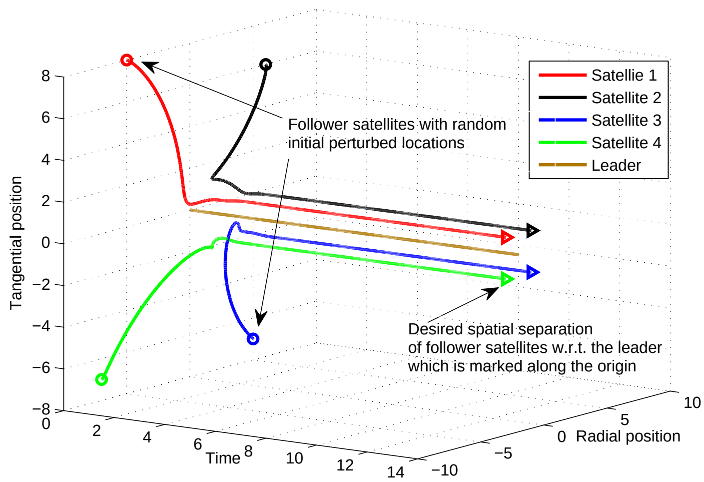

Figure 4: Formation of 4 Satellites viewed from the leader frame

$\tilde{d}_{1y} = v_{eq,3}$ are used in the controller to provide further robustness and disturbance rejection.

6 Conclusion

Formation control of a cluster of satellites has been addressed using a combination of a nonlinear observer and a distributed controller. When designing the nonlinear observer, the structure of the coupling terms in the satellite dynamics has been exploited. The proposed nonlinear observer builds on the principles of the super-twist sliding mode observer. Estimation of the entire states and unknown bounded disturbances in *finite time* is demonstrated using a global Lyapunov analysis.

A distributed controller is realized that makes use of the state estimates and the relative position output information, which depends on the underlying communication topology. The novelty in the

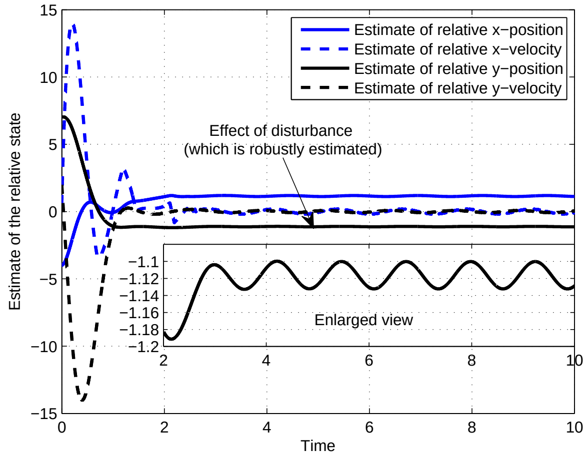

Figure 5: Estimates of relative states in presence of disturbances (but with no feedforward term)

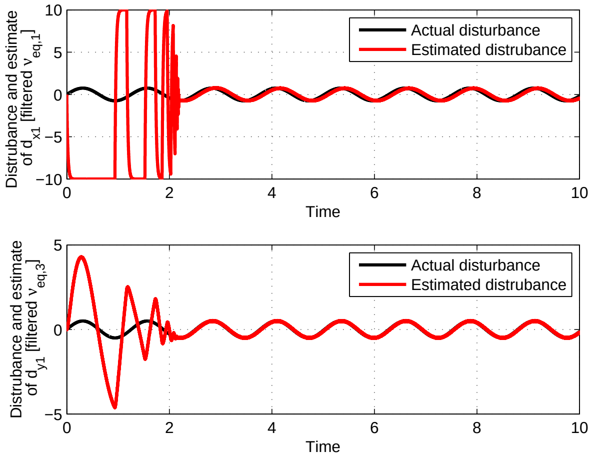

Figure 6: Estimates in disturbances

synthesis of the controller is mainly in the treatment of the underlying graph topology, the interaction among the satellites in terms of relative sensing, and the synthesis of the controller gains using a simple polytopic representation that depends on the graph Laplacian eigenvalues. The efficacy of the proposed nonlinear observer is demonstrated explicitly by simulation.

For further performance robustness evaluation, it would be interesting as future work to propagate the disturbed dynamics in an inertial frame, which pushes the propagation away from a Keplerian orbit (as assumed in both the CW and TH equations), including environment disturbances and the satellite's actuation. Here, the relative dynamics is not simulated directly. Instead the dynamics is propagated in an inertial frame, and the relative states and forces are computed online to apply the control law. In this way, the controller/observer design is based on the CW and/or TH assumptions (Keplerian propagation, eccentric or circular, and large orbital radius) but the validation is done outside these limits. This necessitates adding a further loop in the simulation, and increases the complexity and details of the simulation setup. Such a framework can also be used to test generalizations of the method for elliptical cases. However, the work presented here provides an initial verification, that substantiates a progression in the direction of generalizing the method and applying it to industrial high-fidelity formation flying

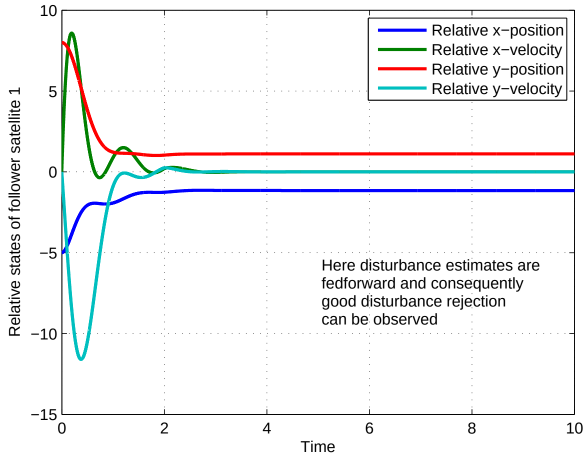

Figure 7: Relative states when estimates of disturbances are fed-forward

dynamic models.

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