A methodology for probabilistic real-time forecasting – an urban case study

[Short title: A methodology for probabilistic real-time forecasting – An urban case study]

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ABSTRACT

The phenomenon of urban flooding due to rainfall exceeding the design capacity of drainage systems is a global problem and can have significant economic and social consequences. The complex nature of quantitative precipitation forecasts (QPFs) from numerical weather prediction (NWP) models has facilitated a need to model and manage uncertainty. This paper presents a probabilistic approach for modelling uncertainty from single-valued QPFs at different forecast lead times. The uncertainty models in the form of probability distributions of rainfall forecasts combined with a sewer model is an important advancement in real-time forecasting at the urban scale. The methodological approach utilized in this paper involves a retrospective comparison between historical forecasted rainfall from a NWP model and observed rainfall from rain gauges from which conditional probability distributions of rainfall forecasts are derived. Two different sampling methods, respectively, a direct rainfall quantile approach and the Latin hypercube sampling based method were used to determine the uncertainty in forecasted variables (water level, volume) for a test urban area, the city of Aarhus. The results show the potential for applying probabilistic rainfall forecasts and their subsequent use in urban drainage forecasting for estimation of prediction uncertainty.

Key words | forecasted rainfall, numerical weather prediction model, observed rainfall, real-time forecast, sewer model, uncertainty in rainfall forecast

INTRODUCTION

Real-time flood forecasting systems at the basin scale are operational in many parts of the world (Parker & Fordham 1996; Todini *et al*. 2005, and references therein) and most of them rely on hydrometeorological models to develop the flood forecast. These systems are established to issue warnings to flood plain residents and other authorities before a critical threshold is exceeded, thus allowing time to mitigate the consequences.

Globally there is a well-recognized need for improved techniques and approaches to flood forecasting (Krzysztofowicz 2001; Pappenberger & Beven 2006; Demeritt *et al*. 2007). Accurate predictions of flood levels resulting from precipitation and subsequent run-off have long been the goal of hydraulic engineers and hydrological modellers. More accurate predictions of water levels would not only influence demographic decision-making and urban planning in flood-prone areas but would also lead to improvements in flood warning systems.

State-of-the-art systems incorporate quantitative precipitation forecasts (QPFs) either from rainfall radar, numerical weather prediction (NWP) models, or a combination of the two, with the objective of increasing the forecast lead time (De Roo *et al*. 2003). Provision of longer lead times implies more time for mitigation action.

The shift towards QPFs from NWP models for operational forecasting systems has highlighted the need to address uncertainty in the rainfall forecast (Demeritt *et al*. 2007). Deterministic and probabilistic approaches both play an integral part of flood forecasting. In fact, they complement each other to provide additional insights into the flood risk. Making decisions under uncertainty is one of the most difficult management decisions but is the most important one. Addressing uncertainty as a reality in real-time flood forecasting shifts the question from ‘should a flood warning be issued’ to ‘with what confidence might it succeed’?

Over the last few decades operational flood forecasting systems have increasingly moved towards use of meteorological ensemble prediction systems (EPS) rather than deterministic forecasts to drive flood forecasting systems (Cloke & Pappenberger 2009). Refer to Seo *et al*. (2000) and Gouweleeuw *et al*. (2005) for examples of such systems at the river basin scale. However, using rainfall ensembles to make hydrological predictions is not as straightforward and has proven to be very challenging (Wu *et al*. 2011). According to Schaake *et al*. (2007) much remains to be done to make them reliable enough for operational hydrological predictions.

The use of single-valued QPFs for hydrological and hydrodynamic modelling at the urban scale is challenging as rainfall forecast of high temporal and spatial resolution is required. A few studies have assessed the feasibility of using QPFs as well as probabilistic forecasting schemes in combination with a sewer model in urban flood forecasting (Rico-Ramirez *et al*. 2009; Schellart *et al*. 2009; Liguori *et al*. 2012).

The approach presented here has some similarities to the method described first by Schaake *et al*. (2007), which was then further improved by Wu *et al*. (2011), in the sense that it portions the historical observed and corresponding forecasted data into four sub-regions to estimate the uncertainty of the rainfall forecast and provide a probabilistic hydrological forecast. The conditional distribution of rainfall for a given single-valued QPF is used to represent the corresponding probability distribution for the given forecast. Although the approach presented here shares similarities in the estimation methods with the method presented in Wu *et al*. (2011), the application is different as well as some of the assumptions and hence their implications are different.

The novelty in this work lies within the forecast method which is focused on pluvial flooding in urban drainage systems. Urban drainage systems differ from river systems in the way that the concentration time is much smaller (in the order of 1–3 hours) and hence the computations needs to be much faster for real-time applications. So far, operational implementations on flood forecasting for urban drainage systems have all been based on deterministic QPFs (Henonin *et al*. submitted for review). The current research develops and analyses a probabilistic pluvial urban flood forecasting method.

This article outlines a method to estimate the probability distributions of single-valued QPFs based on a retrospective comparison of archived forecasted and observed rainfall. Single-valued QPFs are used instead of meterological ensembles because single-valued QPFs are more easily available, thus making the method more attractive. The method presented here is simple and thus easy to implement in real time. It can be attached to a forecasting system to produce a probabilistic quantitative precipitation forecast (PQPF) that can be used as input to a sewer model. The approach is implemented in conjunction with a Latin hypercube sampling (LHS) approach and a direct quantile approach that produces 12-hour hourly time series of rainfall forecasts. The results are consequently ingested into a sewer model using both approaches for comparison, and the results of the forecasted water levels and volumes are compared to the resulting water levels and volumes obtained from simulation using the observed rainfall.

The paper is organized as follows. The first section describes the methodology for estimation of probabilistic rainfall forecasts from single-valued QPFs. The second section describes the case study and presents the implementation of the method for the case study as well as demonstrate the overall performance and validity of the probabilistic rainfall model. The subsequent section presents the results of application of the probabilistic rainfall forecasts for providing probabilistic forecasts of the sewer system. Finally, a discussion of the significance of the proposed approach and the conclusions are given.

METHODOLOGY

Estimation of probabilistic rainfall forecasts

The stochastic approach used in this study estimates the uncertainty in rainfall forecast based on a retrospective comparison between historical forecasted rainfall and observed rainfall (herein denoted by $\hat{S}$ and S, respectively). The uncertainty model in the form of probability distributions will be estimated at different lead times by conditioning the forecast error on the rainfall forecast itself. Once the distributions are known, they can be imposed on the forecasted values to determine the probability distribution of the rainfall conditioned on the rainfall forecast, i.e.,

$$\mathrm{P}(\text{rainfall} \mid \text{rainfall forecast})$$

We estimate this probability relationship at different lead times because it is expected that the forecast error increases with lead time.

We propose to use two stochastic models, which reflect the intermittent nature of rainfall itself, i.e., non-zero (‘rain’) and zero (‘no rain’) forecasts. This study assumes that the observed rainfall is the ‘true rainfall’, i.e., no additional uncertainty is added to account for the representation error of the rain gauge measurements.

We consider conditional probability distributions corresponding to the rainfall forecast being zero or non-zero. In the case of a zero rainfall forecast $(\hat{S}=0)$, the probability distribution consists of two parts: 1) the probability of zero rainfall, and 2) the probability of non-zero rainfall. The two parts can be combined to form the probability distribution conditioned on a zero rainfall forecast:

$$P(S \le x \mid \hat{S}=0) = P(S \le x \mid S>0, \hat{S}=0) P(S>0 \mid \hat{S}=0) + P(S=0 \mid \hat{S}=0) \quad (1)$$

where the conditional probabilities are estimated from the data:

$$\hat{P}(S>0 \mid \hat{S}=0) = \frac{(No\ of\ events\{S>0 \mid \hat{S}=0\})}{(Total\ No\ of\ events\{\hat{S}=0\}\ )} \quad (2)$$

$$\hat{P}(S=0 \mid \hat{S}=0) = 1 - \hat{P}(S>0 \mid \hat{S}=0) \quad (3)$$

The marginal distribution for $P(S \le x \mid S>0, \hat{S}=0)$ may be estimated using parametric or non-parametric techniques, such as lognormal or kernel density estimates, respectively. This is done using the data points $\{S>0 \mid \hat{S}=0\}$. In the case study presented below, a parameteric distribution approach has been applied.

For the other case of a rainfall forecast larger than zero $(\hat{S}>0)$, we consider the probability distribution of S conditioned on $\hat{S}=x^*$. Again this has two components:

I. The probability of zero rainfall

$P(S=0 \mid \hat{S}=x^*)$

II. The probability of a non-zero rainfall

$P(S \le x \mid S>0, \hat{S}=x^*\ )$

Considering the first component $P(S=0 \mid \hat{S}=x^*)$, it is expected that the conditional probability of a zero rainfall will decrease with increasing rainfall forecast. To model this relationship, different approaches can be used, such as a logistic regression or any other functional relationship that would ensure a value between [0,1] for a given forecast (x^*).

For describing the second component $P(S \le x \mid S > 0, \hat{S} = x^*)$ a bivariate distribution is applied to $P(S < x, \hat{S} \le x^* \mid S > 0, \hat{S} > 0)$. This can be done by first transforming the non-zero rainfall forecast and observations to new variables that are approximately normally distributed. There are several methods for applying such transformations, including the Box Cox transformation (Box & Cox 1964) and the normal quantile transformation (NQT) (Krzysztofowicz 1997), among others. Alternatively, the bivariate distribution can be estimated using a copula approach (Nelsen 1999).

We consider here the former approach. In this case the probability distribution of the transformed rainfall s_T given a forecast $\hat{s}_T = x_T^*$ can be determined from the bivariate normal distribution. This is a normal distribution with mean and variance given by:

$$\mu_{s_T|\hat{s}_T = x_T^*} = \mu_{s_T} + \rho \sigma_{s_T} \frac{(x_T^* - \mu_{\hat{s}_T})}{\sigma_{\hat{s}_T}} \tag{4}$$

$$\sigma_{s_T|\hat{s}_T = x_T^*}^2 = \sigma_{s_T}^2 (1 - \rho^2) \tag{5}$$

where μ_{s_T} and $\sigma_{s_T}^2$ are the mean and variance of s_T, $\mu_{\hat{s}_T}$ and $\sigma_{\hat{s}_T}^2$ are the mean and variance of $\hat{s}_T$, and ρ is the linear correlation between s_T and $\hat{s}_T$ (Ersbøll & Conradsen 2007). Realizations of s_T can be obtained from the inverse transformations.

Finally, the probability conditioned on a forecasted non-zero rainfall $\hat{s} = x^*$ is obtained by combining the two components:

$$P(S \le x \mid \hat{S} = x^*) = P(S \le x \mid S > 0, \hat{S} = x^*)\ P(S > 0 \mid \hat{S} = x^*) + P(S = 0 \mid \hat{S} = x^*) \tag{6}$$

where

$$P(S > 0 \mid \hat{S} = x^*) = 1 - P(S = 0 \mid \hat{S} = x^*) \tag{7}$$

Once these relationships and probability distributions are estimated for each lead time, the probability distribution of rainfall conditioned on the rainfall forecast can be found.

Probabilistic forecasts of the sewer system

The probabilistic rainfall forecasts are used as input to a sewer model to produce probabilistic forecasts of the sewer system. This is done using an ensemble approach based on sampling of the estimated rainfall probability distribution. The ensemble is here generated using the LHS approach. For simplicity, this study is based on the assumption that there is complete temporal dependence between lead times. This means that for a given 12-hour hourly forecast, the ensemble members are generated by pairing each outcome from an interval to the same interval across lead times in the LHS, thereby creating an ensemble of 12-hour hourly time series of possible rainfall outcomes.

As a result of this pairing, the LHS approach is compared to a direct quantile (DQ) approach. In order for this approach to be valid, rainfall (S) and considered quantities in the sewer system such as water level *(h)* or flow *(Q)* must have a monotonic relationship. This means that from the rainfall quantile (q_S) we can calculate the quantile of Q as $M(q_S))$, where $M(S))$ is a model that predicts Q in the urban drainage system given rainfall S as input. In that way, there is no need for multiple LHS simulations to get the distribution (or estimated uncertainty) of the variables considered in the sewer system. Mathematically, for Q, this means that the following must be true:

$$P(S < q_S) = P(Q < M(q_S)) \quad (8)$$

$$\int_0^{q_S} p_S(s)ds = \int_0^{M(q_S)} p_Q(q)dq \quad (9)$$

where p_S and p_Q are probability density functions of rainfall and flow, respectively. The quantile of Q can be derived from Equation (9) using the change of variable p_Q to p_S :

$$\int_0^{q_S} p_S(s)ds = \int_0^{M(q_S)} p_S(M^{-1}(s))\left|\frac{d}{ds}M^{-1}(s)\right|ds \quad (10)$$

In essence, Equation (10) suggests that if $M(S)$ is monotonic in S then it means that the model predicts more run-off if the rain increases, which is what is expected. However, there are situations where this may not be fulfilled, e.g., in an urban drainage system which has some intervention to control run-off as rainfall intensity increases.

The implications of using this approach means that we are only able to give an upper bound (i.e., the worst case scenario). However, recognizing this limitation under some circumstances, the probabilistic forecast information is still valuable, especially when used for risk assessment and when any other additional information is unknown. The benefit of using the direct quantile approach means that with just one hydraulic simulation we can quantify the uncertainty in the sewer system making it very attractive for real-time applications.

The assumption applied for the temporal dependence on the rainfall probability distributions between lead times also implies an upper bound on the estimated quantile (worst case scenario). This applies both for the LHS and the direct quantile approach. However, a large temporal correlation is generally expected.

CASE STUDY

The case study used in this study considers urban drainage in Aarhus (Figure 1), the second largest city in Denmark, located on the eastern coast of the Jutland peninsula. The city has a population of 250,000. This city was chosen because of the existing high quality data for the sewer system. The sewer network consists of 1,926 manholes, 65 outlets, 1,657 pipes, 196 weirs, 83 basins and 26 pumps. The sewer model has been calibrated by the municipality and is considered to be fit for purpose for this research by being able to reproduce observed flows and water levels.

The PQPF is prepared for a 12-hour period with 1-hour time steps. The 12-hour hourly product provided from the NWP model (StormGeo 2011) has a resolution of (6.2 × 11.1) km. The archived forecasted data covered the period February 2009–December 2010. Single-valued QPFs over the urban catchment were composed of two forecast grids with a total area of 138 km^2 (Figure 2). Historical observed rainfall from three tipping bucket rain gauges covering the period 2001–2011 was provided. The observed data were at a higher temporal resolution (1 minute) and volumetric resolution of 0.2 mm. The Thiessen polygon method was used for estimating areal precipitation from the rain gauge data, and the data were subsequently converted to hourly rainfall.

The areal historical meteorological forecasts were compared with areal rainfall observations for the period 2009–2010. First analyses of the data showed a number of inconsistencies. The most important issue is the frequent occurrence of a non-zero forecast and zero observed rainfall. As a result, a precipitation threshold of 0.2 mm was imposed on the rainfall forecast to define zero rainfall. See Figure 3 for comparison of observed and forecasted rainfall data. The estimated conditional probabilities of the rainfall are shown in Table 1.

Figure 3 also shows that there is a tendency of underestimation of the rainfall forecast for larger rain events and an overestimation for smaller rain events. This is supported in Table 2 by the negative values of conditional bias as the forecast gets larger and positive values of the conditional bias for smaller values of the rainfall forecast. Since there are very few data points for the larger events there are large uncertainties in the bias estimates.

From this preliminary analysis it can be concluded that there are large uncertainties in the meteorological forecasts. In the following is presented the estimation of a probabilistic forecast model for Aarhus using the approach described in the previous section.

Estimation of conditional distributions

Using the two years of observed and forecasted areal rainfall, probabilistic rainfall models are estimated for each forecast lead time, 1–12 hours.

For $\hat{S} = 0$, for each lead time the conditional probability distribution $P(S \leq x \mid S > 0, \hat{S} = 0)$ has been estimated by fitting a probability distribution to the data $\{S > 0 \mid \hat{S} = 0\}$. The lognormal distribution provided the better fit for most lead times and was therefore selected for all lead times for consistency (fit of a lognormal distribution is shown in Figure 4 for a 1-hour lead time). The probabilities $\hat{P}(S > 0 \mid \hat{S} = 0)$ and $\hat{P}(S = 0 \mid \hat{S} = 0)$ were estimated from the data and $P(S \leq x \mid \hat{S} = 0)$ is then obtained from Equation (1).

For the data $\{S > 0 \mid \hat{S} > 0\}$ for each lead time, the normality of the data was first checked by the use of simple Q-Q plots. This clearly showed that the rainfall data could not be approximated by normal distributions, and thus were transformed using the Box–Cox transformation method (Box & Cox 1964). The Box–Cox transformation is defined by

$$x_T = \begin{cases} \dfrac{x^{\lambda} - 1}{\lambda} & (\lambda \neq 0) \\ \log x & (\lambda = 0) \end{cases} \tag{11}$$

and is applied to both the observed and forecasted rainfall. There are several statistical tools available for computing an optimal λ-parameter and MATLAB was used in this case. The transformation does not necessarily guarantee normality and the transformed data were further checked for normality using probability plots. For each lead time the transformed data were found to be well described by normal distributions. Figure 5 demonstrates the estimated bivariate normal distribution of the transformed data for a lead time of 1 hour.

The probability $P(S = 0 \mid \hat{S} = x^*)$ was derived using regression analysis on the entire data set as opposed to each lead time to provide a more robust estimation. In this case we found that a logistic regression based on the logistic maximum likelihood method better fitted the observed probabilities. The probability using the logistic regression is given by:

$$P(S = 0 \mid \hat{S} = x^*) = \frac{exp^{B_0 + B_1 x^*}}{1 + exp^{B_0 + B_1 x^*}} \tag{12}$$

where B_0 and B_1 are the regression coefficients. They are estimated by fitting a logit link function to the data. The data, in this case the probabilities, are approximated by splitting the data into forecast intervals and computing the probabilitiy of observing zero for this interval. The mean of the rainfall in each interval is an estimation of $P(S = 0 \mid \hat{S} = x^*)$. See Figure 6 for results of the logistic regression. The results depict what is expected, as the values of rainfall forecast increases the probability of observing zero rainfall decreases. Finally, the estimated conditional probability distribution for a non-zero rainfall forecast is obtained from Equation (6).

The estimated probability distributions are combined to create a stochastic model to generate 12-hour hourly time series of rainfall given a 12-hour hourly forecast. When the forecast is zero, the model samples from the estimated lognormal distribution. When the forecast is non-zero, the model first estimates the probability of observing zero rainfall given the non-zero forecast using the logistic regression function. Then, the rainfall value is transformed from its orginal domain to the normal domain using Equation 11 and its corresponding estimated λ-parameter for the considered lead time. The conditional mean and variance is computed using the parameters of the data in the normal domain (Equations (4) and (5), respectively) to determine the marginal distribution which is then used to generate the rainfall ensemble and rainfall quantiles. The corresponding observations for the non-zero forecast value are then back-transformed to the orginal domain using Equation (11) to create the 12-hour hourly time series.

Validity of the distribution functions – goodness of fit

The performance of the model is checked by comparing the empirical CDFs of the observed rainfall sampled for a range of rainfall forecasts to the theoretical CDFs (predicted) obtained from the estimated models. Because of very few observed larger rainfall events, CDFs are only presented up to 2.0 mm of rainfall. The theoretical CDF can be obtained but there are not enough data to construct the empirical CDF. These results for a lead time of 1 hour are presented in Figure 7.

The theoretical model gives a good description of the data by closely following the empirical CDFs thereby confirming the formulation of the stochastic model. The statistical model follows the empirical CDFs more closely for smaller rainfall forecasts (0–1 mm) than for larger rainfall forecast across lead times; however, these also have larger uncertainties due to the small number of larger rainfall events.

Implementation

For illustration of the use of the probabilistic rainfall forecasts for providing probabilistic forecasts of the sewer system two rainfall events are considered:

I. Event A where the accumulated observed rainfall is larger than the corresponding rainfall forecast (Figure 8).
II. Event B where the accumulated rainfall forecast is larger than the observed rainfall (Figure 9).

The two methods presented above (LHS sampling and the direct quantile approach) are applied for sampling the probability distributions to be used as input to the sewer model.

Latin hypercube sampling approach

For a given 12-hour rainfall forecast (Figures 8 and 9), the LHS approach was used to generate 50 ensemble members of 12-hour hourly rainfall forecasts for the two selected rain events (see Figure 10). Each hourly rainfall forecast in the 12-hour period has a corresponding probability distribution. The approach samples 50 times from each rainfall forecast of equal probability. The 50 precipitation forecasts are then used as input to the sewer model. This results in 50 simulated outputs from which the probability distribution of model outputs can be estimated.

Direct quantile approach

For the same two events (see Figures 8 and 9) a few percentiles of rainfall (see Figure 11) were extracted from the probability distributions and used as input to the sewer model. It is then assumed that the probability of the considered output corresponds to the probability of the precipitation input.

Sewer model results

For the two selected events, the resulting rainfall ensemble members from the LHS approach and the rainfall percentiles from the direct quantile approach were simulated through the sewer model. At different locations throughout the model, time series of water levels and volumes were extracted for comparison to the simulation results based on the observed rainfall. This method is used because actual observed data were not available. This, however, does not affect the assessment of the performance of the proposed approach, because the method only considers the uncertainty in the precipitation input and everything else stays the same in the model.

The plots presented in Figures 12 and 13 show the system response to the rainfall ensembles as well as rainfall percentiles. Both approaches show an overall similarity in water levels and volumes at the selected locations.

A comparison of the water levels obtained from the LHS approach and the direct quantile approach in Figures 12(a) and 12(b) for Event A show slightly larger peak values for the LHS approach. Overall, the resulting percentiles of the ensemble are almost the same compared to the system response to the direct quantile approach. Figures 12(c) and 12(d) also show similarities for both approaches except that the LHS approach again results in slightly larger forecast volume percentiles. The same is observed for the forecasted water levels and volumes presented in Figure 13 for Event B. Overall, results of the forecasted variables using both approaches show similarities, particularly for the larger percentiles. Notwithstanding the differences in peak values for the two methods, they occur at the same time for both approaches, therefore suggesting that both approaches are comparable and the difference is merely caused by the response of the sewer system controlled by pumps.

DISCUSSION

The proposed approach using probability distributions to represent the forecast uncertainty has demonstrated the feasibility of using QPF for forecasting of different variants at the urban scale. Two approaches for implementation of the probabilistic rainfall forecasts have been presented.

The main reason for the differences between the two approaches is that the results obtained from the LHS approach are a combination of the system response to each individual rainfall ensemble member simulated through the non-linear model, from which percentiles are estimated, while the direct quantile approach assumes the same probability in the outputs as in the rainfall input. The sewer model is a complex network controlled by pumps, hence making it highly non-linear. The model responds differently to each rainfall pattern. Generally, patterns with larger intensities will provide similar results because the system is running full; therefore assuming that it has a linear response. In contrast, it is easier to notice small changes in water level for smaller rainfall patterns and hence the difference in the simulated 50th percentiles.

Generally, the stochastic model displays a usable amount of skill for forecasting rainfall. The propagation of this forecast through the hydraulic model also displays a usable amount of skill in forecasting node water level as well as inflow volume to basins, making it possible to use in real-time application.

Notwithstanding, the overall performance depends on the quality of the data sets used. Long historical records of observed and forecasted rain data are needed for applying the developed approach. In this case, only two years of forecast data were available.

The results presented are derived from relatively small intensity rainfall events. Further investigation for large intensity rainfall is required in order to draw more concrete conclusions for such complex urban drainage systems.

The main operational difference between the two forecast methods is the computational time. Using the more comprehensive LHS approach would require a simulation for each ensemble member, while the direct quantile approach requires simulations for only a selected number of percentiles. Although the LHS approach is more theoretically sound, simply simulating a certain rainfall percentile gives a quantitative estimate of the uncertainty in the forecasted variant without costly simulations. This makes the use of the direct quantile approach for probabilistic forecasting a more attractive method for real-time forecasting at the urban scale.

In practice, the results from the proposed method are useful for decision support for urban water management and flood warning and emergency management. The method can be extended to include other sources of uncertainty in order to provide a more robust approach to decision support. Flood warnings are typically issued when a threshold is expected to be exceeded. Therefore, for selected locations in the sewer systems, certain thresholds are established, and the probabilistic forecasts give an estimate of the probability of exceeding these thresholds.

CONCLUSIONS

A method to quantify the uncertainty in rainfall forecasts from NWP models is presented. The developed method provides the opportunity to make probabilistic forecasts of different variants (water levels, volumes, discharges) in urban areas in real time by means of LHS or direct quantile simulations. The developed method does not consider other sources of uncertainty, but merely an estimate of input uncertainty (precipitation).

The method was tested by using 12-hour hourly rainfall forecasts and was applied to a 1D sewer hydraulic model. From the results it can be concluded that the direct use of rainfall quantiles to provide uncertainty estimates of water level and volume forecasts instead of LHS is promising. Moreover, the developed method is simple to apply once the data are available, and most importantly, it is computationally efficient.

However, there are some limitations to the method presented here. The direct quantile approach assumes that rainfall and sewer system reponse have a monotonic relationship, which may be violated (although not indicated in the presented case study). Both the direct quantile and the LHS approach assume complete temporal dependence across lead times. It is possible to take the temporal error structure into account using the LHS technique, although this was not considered in the study.

In order for the developed approach to be successful, it requires diligent collection of both observed and forecasted rainfall data. This is not necessarily conducted at meteorological offices, especially in developing countries. This requires a paradigm shift in the modus operandi of meteorological offices around the world, which may be the greatest limitation to the success of the proposed approach.

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Figure 1 | Setup of sewer model for Aarhus.

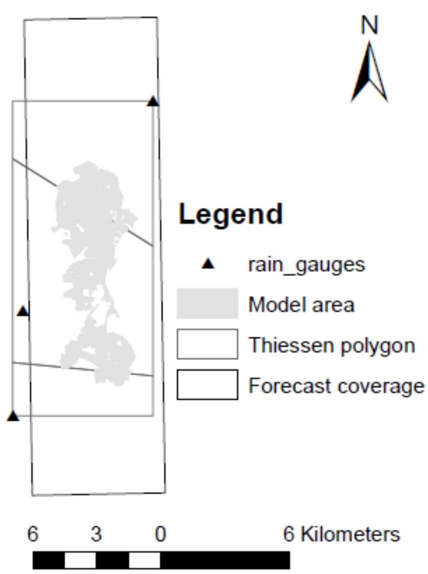

Figure 2 | Model area, total forecast coverage, Thiessen polygon and rain gauge locations.

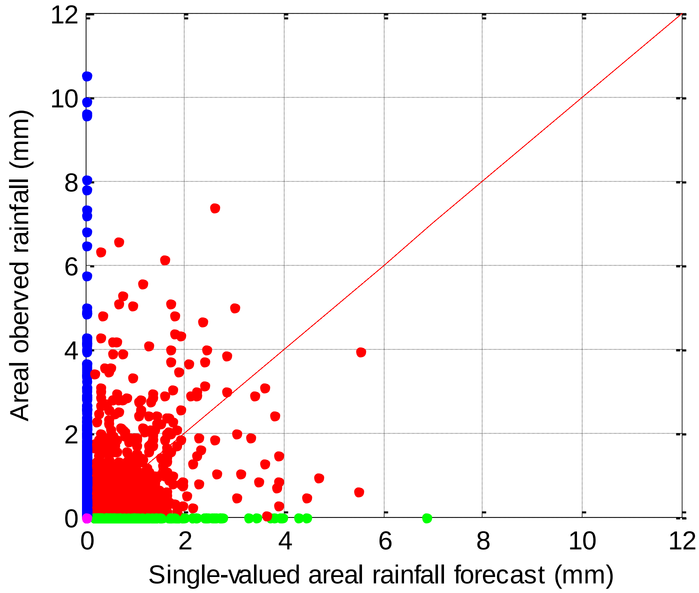

Figure 3 | Observed and forecasted rainfall for the period 2009–2010. Data points on the y-axis represent rainfall given a zero rainfall forecast, data points on the x-axis represent zero rainfall given a rainfall forecast more than zero, and the data points inside the plot represent non-zero rainfall given a non-zero rainfall forecast.

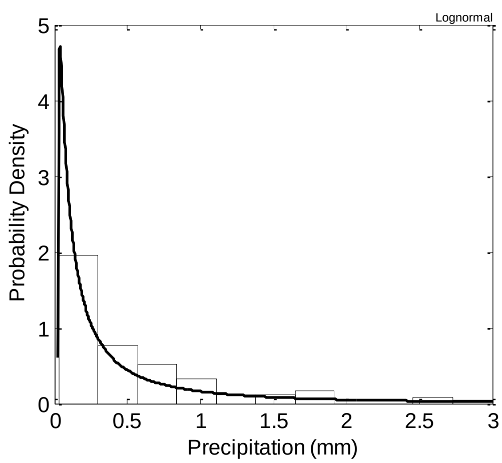

Figure 4 | Estimated log-normal distribution for a lead time of 1 hour.

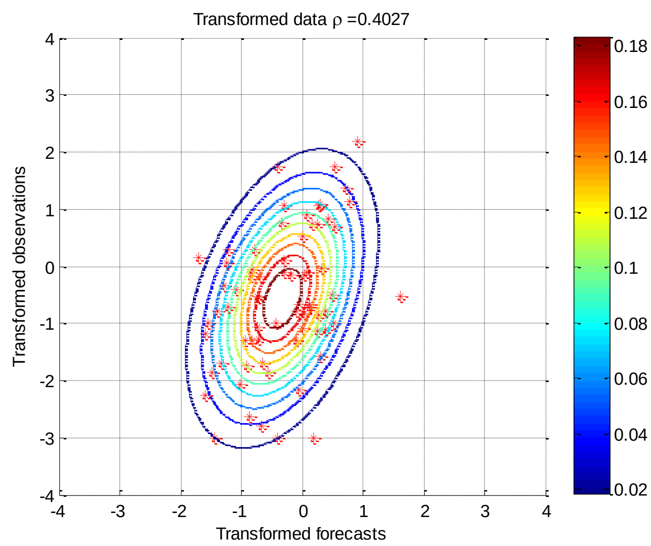

Figure 5 | Contours of the joint distribution of transformed observed and forecasted data for a lead time of 1 hour.

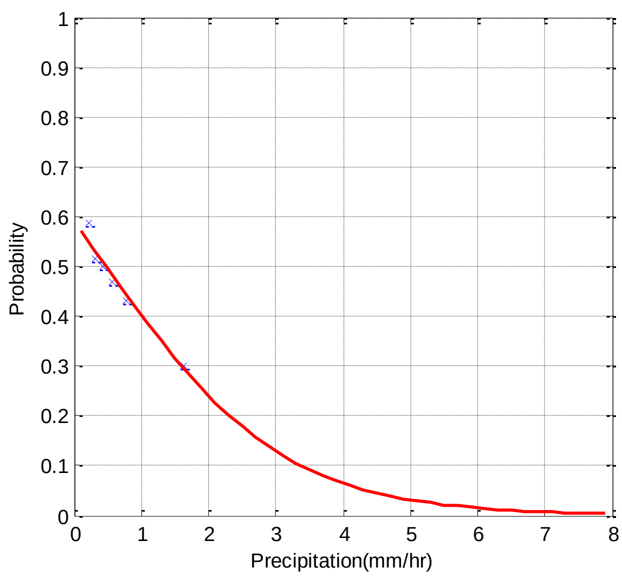

Figure 6 | Estimated logistic regression to represent the $P(S = 0|\hat{S} = x^{*})$

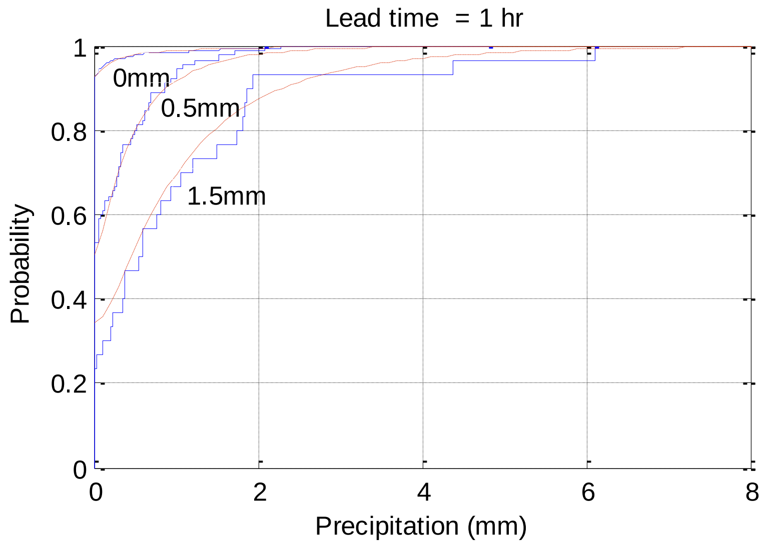

Figure 7 | Estimated versus empirical CDFs for different values of rainfall forecast – lead time 1 hour. The smooth curves represent the prediction from the developed models and the step-like curves represent the observations from the data.

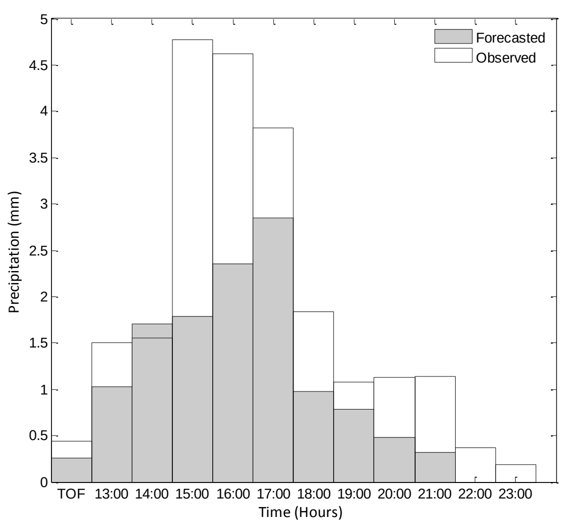

Figure 8 | Forecasted and observed rainfall (Event A).

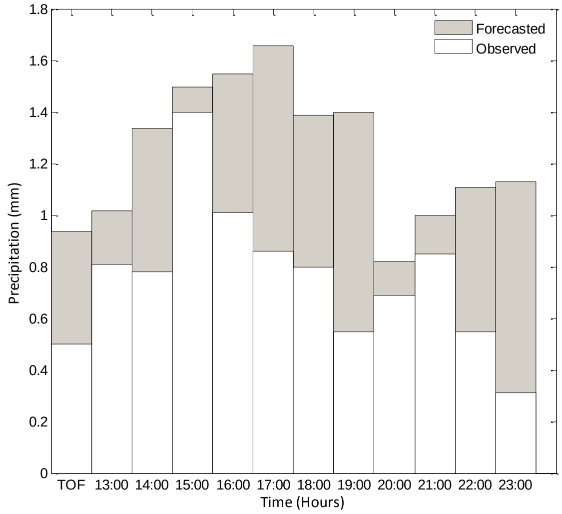

Figure 9 | Forecasted and observed rainfall (Event B).

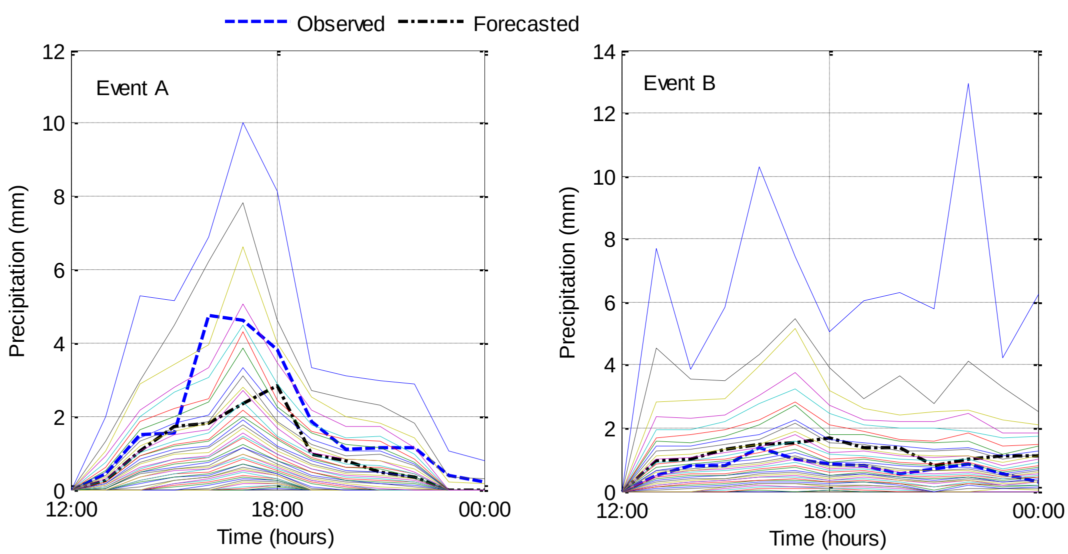

Figure 10 | Ensemble members for Events A and B, respectively. The broken lines represent the observed and forecasted rainfall, respectively, and the solid lines represent possible outcomes of rainfall from the stochastic model for the given rainfall forecast.

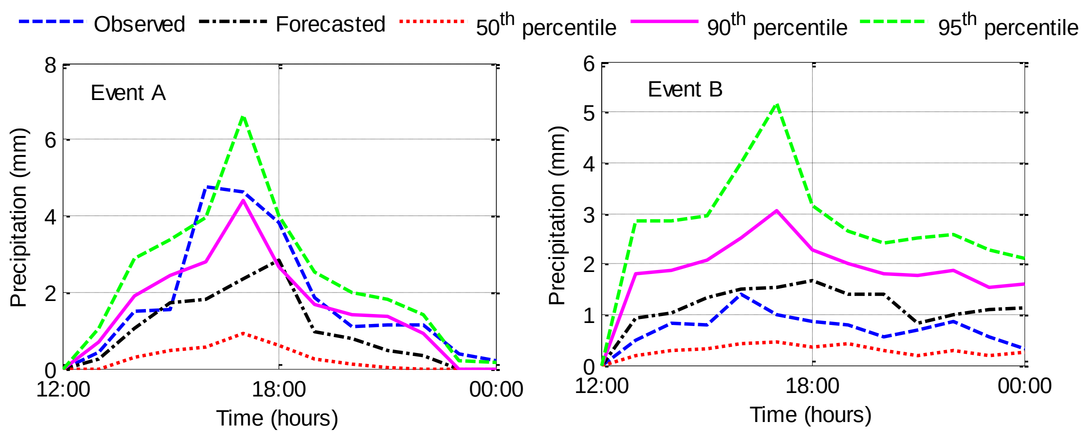

Figure 11 | Quantiles for Events A and B, respectively. The broken lines represent the observed and forecasted rainfall, respectively. The other three lines represent three selected quantiles extracted directly from the probability distributions developed for each lead time.

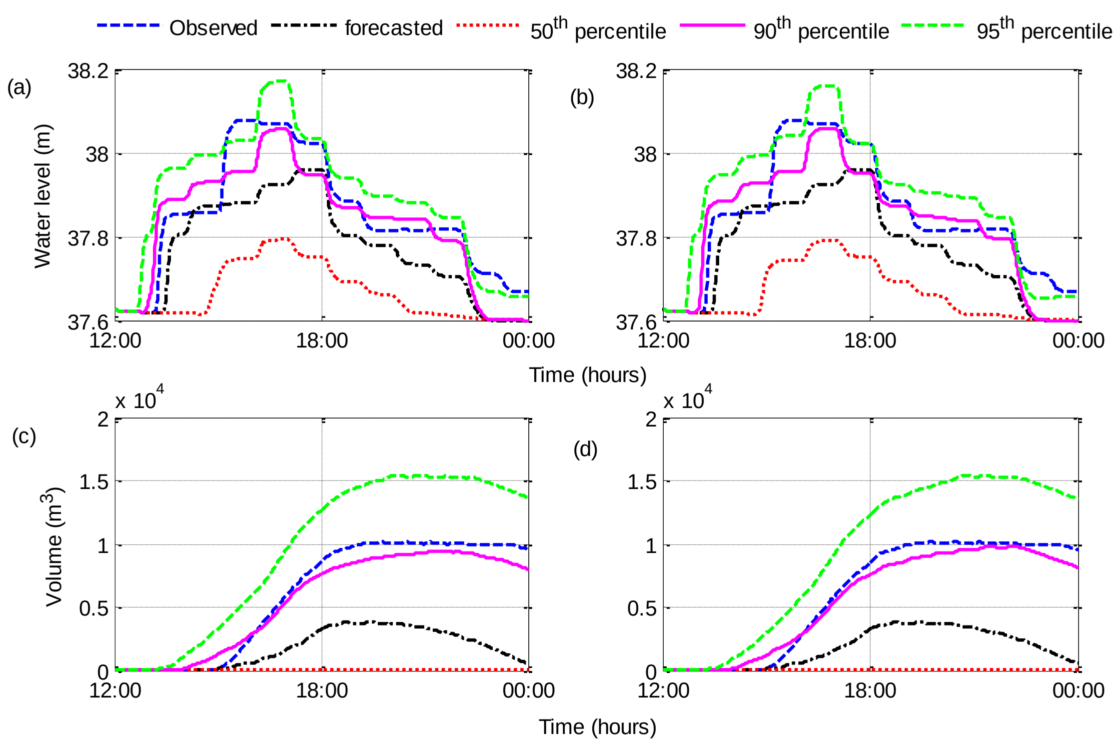

Figure 12 | Comparison of three percentiles of water level and volume at selected locations in the model of the LHS and direct quantile approach for event A. Plots (a) and (c) represent the results obtained for the LHS approach and plots (b) and (d) demonstrate the results obtained from the direct quantile approach.

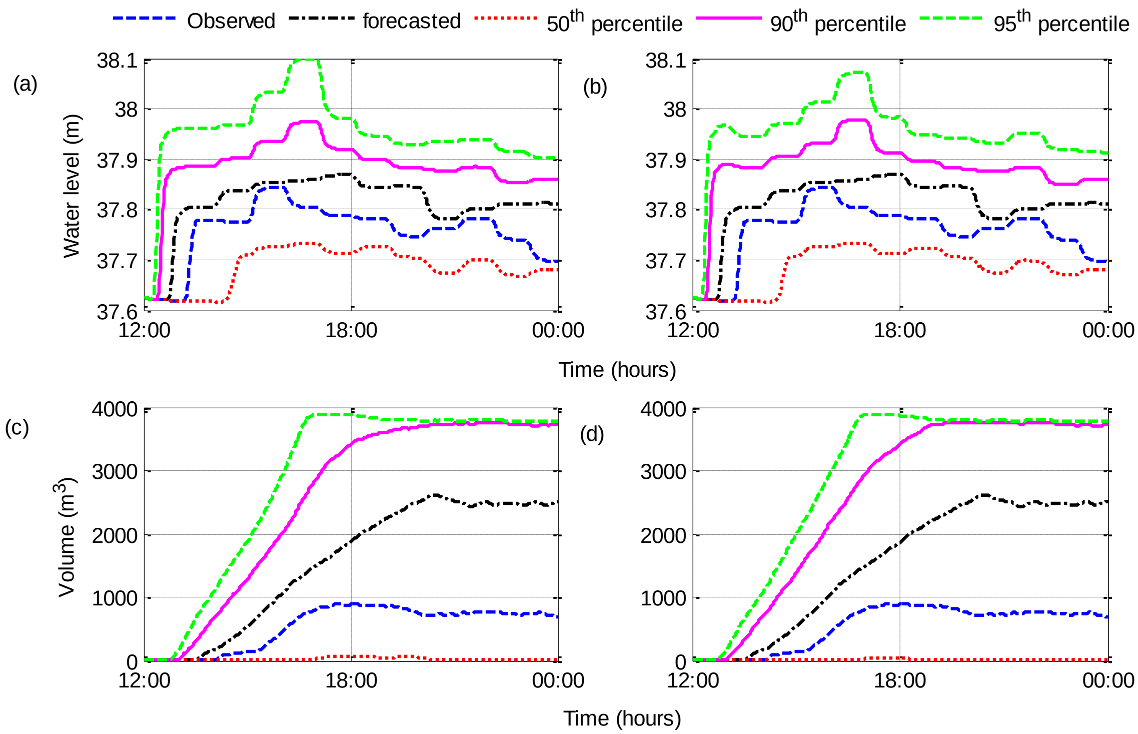

Figure 13 | Comparison of three percentiles of water level and volume at selected locations in the model of the LHS and direct quantile approach for Event B. Plots (a) and (c) represent the results obtained for the LHS approach and plots (b) and (d) demonstrate the results obtained from the direct quantile approach.

Table 1 | Conditional probabilities of rainfall given rainfall forecasts for entire data set

	Forecast = 0	**Forecast > 0**
Rainfall = 0	93.2	45.0
Rainfall > 0	6.8	55.0

Table 2 | Conditional bias and standard deviation of the rainfall forecasts for the data set used

Interval	Number of points in interval	Bias (mm)	Standard deviation (mm)
0	11954	0.026	0.259
0.01–1.0	4361	0.031	0.616
1.01–2.0	255	−0.519	1.086
2.01–3.0	35	−0.777	1.895
3.01–4.0	20	−2.636	1.020
4.01–7.0	7	−4.271	1.411