

## R-method: A simple ranking method for multi-attribute decision-making in the industrial environment

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### CHRONICLE

#### Article history:

Received: March 22, 2021

Received in revised format:

April 2, 2021

Accepted: May 2, 2021

Available online:

May 2, 2021

#### Keywords:

Multi-attribute decision-making (MADM)

Ranking of alternatives

Ranking of attributes

Composite scores

### ABSTRACT

A simple multi-attribute decision-making method based on ranking of alternatives and attributes is proposed in this paper. The method ranks the alternatives with respect to each of the attributes based on the corresponding performance measures. Similarly, the ranks are assigned to the attributes based on their importance as perceived by the decision maker. The ranks assigned to the alternatives with respect to each of the attributes and the ranks assigned to the attributes are converted to appropriate weights and the final composite scores of the alternatives are computed using these weights. An interesting feature of the proposed method is that the qualitative attributes (i.e. the attributes expressed in linguistic terms) need not require the use of fuzzy logic. The proposed method is very simple and useful in situations of limited time availability, presence of qualitative attributes, imprecise/incomplete/partial data, and decision maker's limited attention and capability to process the information. The proposed method is proved easier and better compared to the other widely used decision-making methods. The proposed method will be tested further on more realistic problems of the industrial environment and the results will be reported soon.

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## 1. Introduction

Multi-attribute decision-making (MADM) is a part of multi-criteria decision-making (MCDM). The MADM problems deal with selection of a right alternative from amongst a large but finite number of alternatives keeping in view of the performance of the alternatives with respect to a set of attributes. The common questions faced by the decision maker in MADM problems are: how to assign the weights to the attributes, how to make use of the attributes' data corresponding to various alternatives, how to deal with the qualitative attributes, how to deal with imprecise/incomplete/partial information, and how to arrive at a final decision. A number of MADM methods are available in literature and some of these methods have become very popular such as Simple Additive Weighing (SAW), Analytical Hierarchy Process (AHP), Analytical Network Process (ANP), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), Višekriterijumsko KOmpromisno Rangiranje (VIKOR), ELimination Et Choix Traduisant la REalité (ELECTRE), Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE), Complex Proportional Assessment (COPRAS), Grey Relational Analysis (GRA), Data Envelopment Analysis (DEA), Decision-making Trial and Evaluation Laboratory (DEMATEL), Step-wise Weight Assessment Ratio Analysis (SWARA), Best-Worst Method (BWM), etc. These methods are proved successful in different decision-making situations. However, these methods have their own merits and demerits. For example, the TOPSIS method involves lengthy calculations which become more complex with the increase in the number of alternatives and the attributes. Different methods of normalizing the data in TOPSIS method may lead to different rankings of the alternatives. The VIKOR method involves more computation. Furthermore, the weight of the strategy of "the majority of attributes" can change from 0 to 1 and different ranking lists can result for the same weights of the attributes. The ELECTRE method uses the concept of outranking relationship and the computational procedure is quite complex. The method involves lengthy calculations of net concordance and net

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Table 1 shows the weights assigned to different ranks of the alternatives or attributes up to 15. However, the table can be easily extended for more than 15 by following the procedure explained in the next section. The proposed MADM method is named as “**R-method**” as it is based on ranking of the attributes and alternatives. The MATLAB code of the proposed method is given in the Appendix. The method is demonstrated by means of four examples in the next section.

### 3. Demonstration of working of the proposed R-method

#### 3.1 Example 1: Vendor selection

To demonstrate the working of the proposed R-method, a vendor selection problem is considered. The vendor is required to supply the raw material. The four pertinent attributes are identified as, raw material cost in \$/kg, raw material quality (Q), expected delivery time in months (T), and the vendor’s service performance (P). Suppose four alternative vendors are identified. The data is given in Table 2.

**Table 2**

Data of the alternatives and attributes corresponding to example 1

Alternative vendor	Raw material cost (\$/kg) (C)	Raw material quality (Q)	Delivery time (months)(T)	Vendor’s service performance (P)
V1	5	Very Low	3.5	Poor
V2	6.5	Very high	3.5	Average
V3	6.0	High	1	Very good
V4	5.5	Low	2	Excellent

Lower values of C and T and higher values of Q and P are desirable. In this example, the attributes Q and P are expressed linguistically. However, there is no need of using fuzzy logic and its rules here to convert the qualitative attribute into quantitative attribute. The proposed method simply ranks the alternatives in terms of 1, 2, 3, and 4 with respect to any qualitative attribute. Now, the ranks are assigned to the alternatives and attributes. Let the attribute C is considered very important by the decision maker and hence the rank of 1 is assigned to C. The second rank is given to Q, third rank is given to T and fourth rank is given to P. Now, from C point of view, based on the performance measures of the alternative vendors, vendor V1 is considered best and hence the rank of 1 is assigned to V1. The second rank is given to V4, third rank is given to V3, and fourth rank is given to V2. Similarly, the ranks are assigned to the alternatives with respect to the other two attributes Q and T. The vendors V1 and V2 have the same value of T and hence are assigned the average rank of 3.5 (i.e. average of 3 and 4). The ranks assigned to the alternatives and attributes by the decision maker are given in Table 3.

**Table 3**

Ranks assigned to the alternatives and attributes

Alternative vendor	Raw material cost (\$/kg) (C)	Raw material quality (Q)	Delivery time (months)(T)	Vendor’s service performance (P)
V1	1	4	3.5	4
V2	4	1	3.5	3
V3	3	2	1	2
V4	2	3	2	1
Ranks assigned to the attributes→	1	2	3	4

The weights of the ranks of the alternatives (and attributes) are calculated as given below.

$$\begin{aligned}
 &1/\text{reciprocal of rank 1:} && 1/(1/1) = 1.000000 \\
 &1/\text{reciprocals of ranks up to 2:} && 1/(1/1 + 1/2) = 0.666666 \\
 &1/\text{reciprocals of ranks up to 3:} && 1/(1/1 + 1/2 + 1/3) = 0.545454 \\
 &1/\text{reciprocals of ranks up to 4:} && 1/(1/1 + 1/2 + 1/3 + 1/4) = 0.48 \\
 &\text{Total} = 1.000000 + 0.666666 + 0.545454 + 0.48 = 2.69212
 \end{aligned}$$

∴ Weight assigned to rank 1 =  $1.000000/2.69212 = 0.37145$ ; Weight assigned to rank 2 =  $0.666666/2.69212 = 0.24763$ ; Weight assigned to rank 3 =  $0.545454/2.69212 = 0.20261$ ; and Weight assigned to rank 4 =  $0.48/2.69212 = 0.17829$ . In this problem, the number of alternatives and the attributes is the same, i.e. 4. However, if the number is different, then a similar procedure will be followed for deciding the weights. For example, if the number of attributes (or alternatives) is 3, then following the similar steps, one can obtain the weights assigned to ranks 1, 2 and 3 as 0.452, 0.301, and 0.24657 respectively (i.e.  $1.000000/2.21212$ ,  $0.666666/2.21212$ , and  $0.545454/2.21212$ ). Table 4 shows the weights assigned to the alternative vendors and the attributes. The scores and the composite ranks of the vendors are also shown in Table 4. For example, the composite score for alternative vendor V1 is calculated as,  $0.3714 \times 0.3714 + 0.2476 \times 0.1783 + 0.2026 \times 0.1904 + 0.1783 \times 0.1783 = 0.2524$ .

**Table 4**

Weights assigned to different alternative vendors and attributes, scores and the composite ranks of the vendors

Alternative vendor	C	Q	T	P	Composite scores	Composite ranks of the vendors
V1	0.3714	0.1783	0.1904	0.1783	0.2524	3
V2	0.1783	0.3714	0.1904	0.2026	0.2362	4
V3	0.2026	0.2476	0.3714	0.2476	0.2559	2
V4	0.2476	0.2026	0.2476	0.3714	0.2585	1
Weights assigned to the attributes→	0.3714	0.2476	0.2026	0.1783		

From the composite scores of the vendors, it can be noted that the vendor, designated as V4, is the first choice and V3 is the second choice. The ranking of vendors is V4-V3-V1-V2.

### 3.2 Example 2: Industrial robot selection

In this example, a jointed-arm robot is to be selected for supporting numerical control machines. The actual data was provided by the manufacturers (Braglia & Petroni, 1999; Karsak & Ahiska, 2005, 2008; Wang & Chin, 2009; Singh & Rao, 2011; Rao, 2013). The decision table containing 12 alternative robots and 5 attributes is given in Table 5. The attributes are: PC: purchasing cost in US\$, HC: handling coefficient, LC: load capacity in kg, RE: 1/repeatability in  $\text{mm}^{-1}$  and VE: velocity (m/s). Lower values of PC and higher values of the remaining four attributes are desirable. Now, following the steps of the proposed methodology, ranks are assigned to the alternatives and attributes. Out of the 5 attributes, PC is considered to be given the first importance and hence rank of 1 is assigned to it. Depending upon the importance of the remaining attributes as perceived by the decision maker, ranks 2, 3, 4 and 5 are assigned to VE, RE, HC and LC respectively. Then the 12 alternative robots are assigned ranks with respect to each of the 5 attributes as shown in Table 6. For example, from a PC point of view, a robot designated as R4 is considered best and hence the rank of 1 is assigned to R4. The second rank is given to R12, third rank is given to R5, and so on. It may be noted that the robots designated as R3, R8 and R9 have the same values of PC and hence are to be given the same rank. The ranks are considered 4, 5 and 6 but the average rank of 5 is assigned to the three robots (as the PC value is the same for all the three robots). Robots R7 and R10 have the same value of PC and hence are assigned the average rank of 10.5 (i.e. average of 10 and 11). Similar explanation is applicable to the ranks assigned to the alternatives with respect to the remaining attributes.

**Table 5**

Data table for example 2 (Braglia & Petroni, 1999; Karsak & Ahiska, 2005, 2008; Wang & Chin, 2009; Singh & Rao, 2011; Rao, 2013)

Alternative robot	PC	HC	LC	RE	VE
R1	100000	0.995	85	1.7	3.00
R2	75000	0.933	45	2.5	3.60
R3	56250	0.875	18	5.0	2.20
R4	28125	0.409	16	1.7	1.50
R5	46875	0.818	20	5.0	1.10
R6	78125	0.664	60	2.5	1.35
R7	87500	0.880	90	2.0	1.40
R8	56250	0.633	10	8.0	2.50
R9	56250	0.653	25	4.0	2.50
R10	87500	0.747	100	2.0	2.50
R11	68750	0.880	100	4.0	1.50
R12	43750	0.633	70	5.0	3.00

**Table 6**

Ranks assigned to the alternatives and attributes of example 2

Alternative robot	PC	HC	LC	RE	VE
R1	12	1	4	11.5	2.5
R2	8	2	7	7.5	1
R3	5*	5	10	3*	7
R4	1	12	11	11.5	8.5
R5	3	6	9	3*	12
R6	9	8	6	7.5	11
R7	10.5	3.5	3	9.5	10
R8	5*	10.5	12	1	5*
R9	5*	9	8	5.5	5*
R10	10.5	7	1.5	9.5	5*
R11	7	3.5	1.5	5.5	8.5
R12	2	10.5	5	3*	2.5
Ranks assigned to the attributes→	1	4	5	3	2

5\*: Average of 4, 5, and 6; 3\*: Average of 2, 3, and 4.

Now, the weightages are assigned to the ranks of the alternatives and attributes based on Table 1. Table 7 shows the weights assigned to the alternative robots and the attributes. While calculating an average weight, the average of weightages assigned to the corresponding ranks is considered. The scores and the composite ranks of the robots are also shown in Table 7. From the composite scores of the robots, it can be noted that the robot, designated as R12, is the best choice and this is same as that suggested by previous researchers (Braglia & Petroni, 1999 (using DEA method); Karsak & Ahiska, 2005, 2008 (using common weight MCDM and improved MCDM methods); Wang and Chin, 2009 (using DEA with double frontiers); Singh and Rao, 2011 (using a hybrid GTMA-AHP method). However, the weights assigned to the attributes used in the proposed method are somewhat different from the weights used by the previous researchers. Moreover, application of different methods may give different rankings to the alternatives for the considered weights of the attributes. However, so long as the first choice alternative is consistent, it does not matter. In this example, R12 has emerged as the first choice by all the decision-making methods.

**Table 7**

Weights assigned to different alternative robots and attributes, scores and the composite ranks of the robots of example 2

Alternative robot	PC	HC	LC	RE	VE	Scores	Composite ranks of robots
R1	0.0571	0.1773	0.0851	0.0579	0.1074	0.09029992	4
R2	0.0652	0.1182	0.0683	0.0668	0.1773	0.09791492	2
R3	0.0783	0.0776	0.0605	0.1	0.0683	0.07735083	9
R4	0.1773	0.0571	0.0587	0.0579	0.0639	0.09731566	3
R5	0.0967	0.0723	0.0626	0.1	0.0571	0.08032554	7
R6	0.0626	0.0652	0.0723	0.0668	0.0587	0.06425752	12
R7	0.0596	0.0909	0.0967	0.0616	0.0605	0.07013239	11
R8	0.0783	0.0596	0.0571	0.1773	0.0783	0.08971109	5
R9	0.0783	0.0626	0.0652	0.075	0.0783	0.07348433	10
R10	0.0596	0.0683	0.1477	0.0616	0.0783	0.07759921	8
R11	0.0683	0.0909	0.1477	0.075	0.0639	0.08311052	6
R12	0.1182	0.0596	0.0776	0.1	0.1074	0.09806178	1
Weights assigned to the attributes→	0.3195	0.1533	0.14	0.1742	0.213		

### 3.3 Example 3: Material selection for a given industrial product

This example is related with selection of a suitable work material for an industrial product that needs to be designed for operating in a high-temperature oxygen-rich environment (Rao, 2007; Rao, 2013). This selection problem considers 6 alternative materials and 4 attributes and the data is shown in Table 8. The attributes considered are: hardness, machinability rating, cost and the corrosion resistance. Of these four attributes, except for the cost, higher values are desirable for the remaining attributes.

**Table 8**

Data table for example 3 (Rao, 2007; Rao, 2013)

Material	Attributes			
	Hardness (HB)	Machinability rating (%)	Cost (\$)	Corrosion resistance
M1	420	25	5	Extremely high
M2	350	40	3	High
M3	390	30	3	Very high
M4	250	35	1.3	High
M5	600	30	2.2	High
M6	230	55	4	Average

The attribute ‘Corrosion resistance’ is expressed qualitatively as ‘High’, i.e. by a fuzzy linguistic term. The important feature of the proposed decision-making method is that it need not use any fuzzy conversion scales to convert a qualitative attribute to a quantitative attribute. It can easily rank the alternatives based on the qualitative attribute in terms of 1, 2, 3, etc. Now, following the steps of the proposed methodology, ranks are assigned to the alternatives and attributes. Out of the 4 attributes, Machinability rating is considered to be given the first importance and hence rank of 1 is assigned to it. Depending upon the importance of the remaining attributes as perceived by the decision maker, rank 2 is assigned to the Hardness. The attributes Cost and Corrosion resistance are considered equally important in this decision-making problem and hence a rank of 3.5 (i.e. average of 3 and 4) is assigned to both the attributes. Then the 6 alternative materials are assigned ranks with respect to each of the 4 attributes as shown in Table 9. For example, from Hardness point of view, material M5 is considered best and hence the rank of 1 is assigned to M5. The second rank is given to material M1 and the third rank is given to material M3 and so on. It may be noted that the materials designated as M2, M4 and M5 have the same linguistic expression of “High” and hence are to be given the same rank. The ranks are considered 3, 4 and 5 but the average rank of 4 is assigned to the three materials. While calculating the average, the average of weightages assigned to these ranks is considered. Materials M3 and M5 have the same value of Machinability rating and hence are assigned the average rank of 4.5 (i.e. average of 4 and 5). Similar explanation is applicable to the ranks assigned to the alternative materials with respect to the remaining attributes.

**Table 9**

Ranks assigned to the alternatives and attributes of example 3

Material	Attributes			
	Hardness (HB)	Machinability rating (%)	Cost (\$)	Corrosion resistance
M1	2	6	6	1
M2	4	2	3.5	4*
M3	3	4.5	3.5	2
M4	5	3	1	4*
M5	1	4.5	2	4*
M6	6	1	5	6
Ranks assigned to the attributes→	2	1	3.5	3.5

4\*: Average of 3, 4, and 5. While calculating the average, the averages of weightages assigned to these ranks is to be considered.

Now, the weightages are assigned to the ranks of the alternatives and attributes based on Table 1. Table 10 shows the weights assigned to the alternative materials and the attributes. The composite scores and the composite ranks of the materials are also shown in Table 10.

**Table 10**

Weights assigned to different alternative materials and attributes, scores and the composite ranks of the materials of example 3

Material	Attributes				Composite scores	Composite ranks of the materials
	Hardness (HB)	Machinability rating (%)	Cost (\$)	Corrosion resistance		
M1	0.1884	0.1153	0.1153	0.2826	0.16523042	4
M2	0.1356	0.1884	0.1448	0.1378	0.15735336	5
M3	0.1541	0.1296	0.1448	0.1884	0.14972988	6
M4	0.1237	0.1541	0.2826	0.1378	0.16790502	3
M5	0.2826	0.1296	0.1884	0.1378	0.18021368	1
M6	0.1153	0.2826	0.1237	0.1153	0.17901152	2
Weights assigned to the attributes→	0.2476	0.3714	0.1904	0.1904		

From the composite scores of the materials, it can be noted that the material, designated as M5, is the best choice and this is the same as that suggested by SAW, WPM and TOPSIS methods (Rao, 2007) and COPRAS method (Rao, 2013). However, the weights assigned to the attributes used in the proposed method are somewhat different from the weights used by the previous researchers. Moreover, application of different methods may give different rankings to the alternatives for the considered weights of the attributes. However, so long as the first choice alternative is consistent, it does not matter. In this example, M5 has emerged as the first choice by all the above-mentioned decision-making methods.

### 3.4 Example 4: Flexible manufacturing system selection

This example is related with selection of a suitable flexible manufacturing system (FMS) by a company. Kulak and Kahraman (2005) presented the case study of a company to select the most appropriate FMS among the different alternatives. The attributes considered were: annual depreciation and maintenance costs (ADM), quality of results (Q), ease of use (E), competitiveness (C), adaptability (A), and expandability (X). Table 8 shows the linguistic expressions of the attributes. Table 8. The attributes considered in the present work are the same as those considered by Kulak and Kahraman (2005). Except for the attribute ADM, higher values are desired for the remaining attributes. Table 11 shows the data of the 4 FMS alternatives and 6 attributes.

**Table 11**

Data table for example 4 (Kulak & Kahraman, 2005; Rao, 2007)

FMS	ADM	Q	E	C	A	X
I	High	Excellent	Very good	Excellent	Very good	Very good
II	Very low	Very good	Good	Very good	Very good	Very good
III	Medium	Good	Good	Very good	Excellent	Good
IV	Low	Fair	Good	Very good	Very good	Good

Now, following the steps of the proposed methodology, Table 12 is prepared which shows the ranks assigned to the alternatives and attributes.

**Table 12**

Ranks assigned to the alternatives and attributes of example 4

FMS	ADM	Q	E	C	A	X
I	4	1	1	1	3*	1.5
II	1	2	3*	3*	3*	1.5
III	3	3	3*	3*	1	3.5
IV	2	4	3*	3*	3*	3.5
Ranks assigned to the attributes→	1	2.5	5*	2.5	5*	5*

Now, the weightages are assigned to the ranks of the alternatives and attributes based on Table 1. Table 13 shows the weights assigned to the alternative FMSs and the attributes. The scores and the composite ranks of the FMSs are also shown in Table 13.

**Table 13**

Ranks assigned to the alternatives and attributes of example 4

FMS	ADM	Q	E	C	A	X	Composite scores	Composite ranks of the FMSs
I	0.1783	0.3714	0.3714	0.3714	0.2095	0.3095	0.2887	1
II	0.3714	0.2476	0.2095	0.2095	0.2095	0.3095	0.2742	2
III	0.2026	0.2026	0.2095	0.2095	0.3714	0.1904	0.2241	3
IV	0.2476	0.1783	0.2095	0.2095	0.2095	0.1904	0.2124	4
Weights assigned to the attributes→	0.2826	0.1712	0.1249	0.1712	0.1249	0.1249		

From the composite scores of the FMSs, it can be noted that FMS-I is the first choice for the considered weights of importance of the attributes. Kulak and Kahraman (2005) suggested FMS-II as the first choice by using weights of importance of the attributes that were different from the ones used in the present work. However, a close look at the linguistic descriptions of the 6 attributes clearly reveals that FMS-I is superior to FMS-II in the case of three attributes (Q, E, and C), equal in the case of two attributes (A and X) and inferior only in the case of ADM. Thus, considering FMS-I as the first choice using the proposed method is more genuine. The proposed method has proved its applicability by means of the four examples described in this section. The method can also be used to find the composite scores of the alternative solutions in the multi-objective optimization problems such as those presented in Rao and Keesari (2019, 2020).

#### 4. Conclusions

The proposed MADM method, named as R-method, requires only the ranking of attributes and ranking of alternatives with respect to each of the attributes for a given decision-making problem. Rank ordering the importance of attributes (and the alternatives corresponding to each attribute) is easier and convenient. The proposed method is demonstrated by means of four examples. The weights suggested by the proposed method are better and accurate than the other ranking methods. The method has given satisfactory performance and is believed to have potential to solve not only the MADM problems of the industrial environment but also the decision-making problems faced in other fields. The proposed method is very simple and useful in all situations in general, and particularly in the situations of limited time availability, presence of qualitative attributes, imprecise/incomplete/partial data, and decision maker's limited attention and capability to process the information. There is no need of using fuzzy logic and its rules for converting the qualitative attributes to quantitative attributes. An interesting feature of the proposed R-method is that even if the decision maker is interested to assign particular weights of importance to the attributes (instead of the weights obtained by using the proposed methodology), he/she can do it and the remaining procedure same as that suggested by the proposed methodology can be followed to get the composite ranks of the alternatives. The methodology developed in this paper can simultaneously consider any number of quantitative and qualitative attributes and helps to obtain the composite scores which evaluates and ranks the alternatives for a given decision-making problem. The proposed methodology offers a general procedure applicable to diverse selection problems encountered in an industrial environment that incorporate vagueness and a number of attributes and alternatives.

The results of the proposed method presented in this paper are based on the preliminary investigations. Detailed investigations are planned to be carried out in the near future. These investigations will include testing the performance of the proposed method on various complex real-life decision-making problems involving a large number of attributes and alternatives. The results of real-life applications will be compared with the results of other existing well established MADM and the statistical tests will also be conducted. Depending upon the tests the method may be improved or modified. The researchers working in the field of MADM are requested to make improvements to the proposed R-method so that the method will become much more powerful. If the proposed R-method is found having certain limitations then the researchers may suggest ways to overcome the limitations, instead of making destructive criticism, to further strengthen the proposed method.

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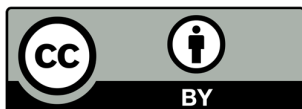
### Appendix: MATLAB code for computing the composite scores of the alternatives

%% In this code, column 2 of d matrix corresponds to n=2, column 3 of d matrix corresponds to n=3,..., and column 15 of d matrix corresponds to n=15.

```
format long
format loose
clc
clear all
disp('please enter integer number')
n=input('NUMBER =');
N=n;
n=1;
k=1;
while (n<=N)

for j=1:n
    a(j,k)=1/j;
end
b(1,k)=a(1,k);
i=2;
while(i<=n)
b(i,k)= b((i-1),k)+a(i,k);
i=i+1;
end
for i=1:n
    c(i,k) =1/(b(i,k));
end
s(1,k)=0;
for i=1:n

s(1,k)=s(1,k)+(c(i,k));
end
for i=1:n
    d(i,k)=c(i,k)/s(1,k);
end
n=n+1;
k=k+1;
end
disp(d);
*****
```



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