An Evaluation of Shareholder Activism

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March 2014
JEL classifications: G30, G34, G11, G14
Key words: shareholder activism, activism, evaluation of activism, hedge funds, corporate governance, diversified portfolios

Abstract

We develop a method to evaluate shareholder activism when an activist targets firms whose shareholders are diversified portfolio holders of possibly correlated firms. Our method of evaluation takes the portfolios of all of the shareholders, including the activist, as its basis of analysis. We model the activist from the time of the acquisition of a foothold in the target firm through the moment when the activist divests the newly acquired shares. We assume that during this period, all exchanges of securities, and their corresponding prices, are achieved in Walrasian markets in which all participants, including the activist, are risk-averse price-takers. Using the derived series of price changes of all the firms in the market, as well as the derived series of changes in all the portfolio holdings over this period, we evaluate the impact of activism on the activist, on the group of other shareholders, and on the combined group. We show that when activism is beneficial to the activist, the group of other investors may not benefit; furthermore, even when the activist benefits from activism, the value of the market may decrease. When the activist benefits from activism, an increase in the value of the market is a necessary but not sufficient condition for the group of other investors to benefit also from activism. In addition, we show that the combined group, the activist plus the group of other investors, benefits if and only if the value of the market increases and, under this condition, either the activist or the group of other investors, but not necessarily both, benefits.

1 Introduction

We develop a method to evaluate shareholder activism when an activist targets firms whose shareholders are diversified portfolio holders of possibly correlated firms. Our method of evaluation takes the portfolios of all of the shareholders, including the activist, as its basis of analysis. We model the activist from the time of the acquisition of a foothold in the target firm through the moment when the activist divests the newly acquired shares. We assume that, during this period, all exchanges of securities, and their corresponding prices, are achieved in Walrasian markets in which all participants, including the activist, are risk-averse price-takers. Using the derived series of price changes of all the firms in the market, as well as the derived series of changes in all the portfolio holdings over this period, we evaluate the impact of activism on the activist, on the group of other shareholders, and on the combined group. Our evaluation provides answers to the following questions: Who benefits from activism? If the activist benefits, is it at the expense of the other investors? Do the benefits of activism, when they occur, imply an increase in the value of the market over the period of activism?[1]

Our contribution to the literature is the proposal of a method of evaluation of activism which is applicable not only to the activist but also to other market participants, and which takes into account the diversification of shareholders' portfolios.[2] Using our method, we show that when activism is beneficial to the activist, the group of other investors may not benefit; furthermore, even when the activist benefits from activism, the value of the market may decrease. When the activist benefits from

[1] Variants of these questions have been raised elsewhere, for example, in Kahan and Rock (2007), Bebchuk and Weisbach (2010), and Edmans (2013).

[2] See Hansen and Lott (1996) who emphasize that, in the presence of externalities, the appropriate objective of analysis is the portfolio, in which spillovers can be incorporated, rather than the individual stock prices and their responses to announcements.

activism, an increase in the value of the market is a necessary but not sufficient condition for the group of other investors to benefit also from activism. In addition, we show that the combined group, the activist plus the group of other investors, benefits if and only if the value of the market increases and, under this condition, either the activist or the group of other investors, but not necessarily both, benefits.

Our approach to activism differs from others not only in its dealing with diversified portfolio holders[3] and in its method of evaluation, but also in describing the process by which the activist acquires and ultimately divests of new shares in the target firm.[4] In other models, one or more of the following, which we assume, are not assumed: Owners of the target firm are diverse portfolio holders, owners of the target firm are risk-averse investors, all market participants are involved as price-takers in a Walrasian market, and the focus is on the entire period of involvement of the activist. Furthermore, other models generally do not focus on evaluating activism from the perspective of the activist as distinct from the group of other shareholders.[5]

Elsewhere when evaluation is discussed, evaluation depends on the impact on the target firm alone.[6] For example, in the empirical literature activism is judged as

[3] An exception to the general lack of consideration of diversified shareholders' portfolios is Admati et al. (1994) where diversified portfolios are considered but, unlike in our approach, the activist is given extraordinary power in first choosing the size of the foothold and only following that does the market come into play. Though obviously in this approach the activist benefits, attention in the paper is directed to equilibrium in the securities market (where small passive investors benefit in a free-rider sense), but not to an explicit evaluation of the impact of activism on the other shareholders as distinct from the activist or on the value of the market.

[4] See, for example, Edmans (2013) for a thorough review of theoretical and empirical literature on blockholders and shareholder activism.

[5] There are exceptions, as in, for example, Clifford (2008), Becht et al. (2009) and Boyson and Mooradian (2011).

[6] For example, Bebchuk et al. (2013) argue that activism does not produce long term deleterious effects on target firms. Exceptions to the focus on the evaluation of activism on a single target firm include Lee and Park (2009) and Gantchev et al. (2013) who find spillover effects from a target firm

being beneficial based on the increase in the price of shares of the target firm at the time the activist announces acquiring those shares via a Schedule 13D filing.[7] As our results show, neglecting the diversification of shareholders in the method of evaluation may lead to incorrect conclusions regarding the benefits of activism. Other issues of interpretation arise when statements concerning the benefits to shareholders do not distinguish between those pertaining to the activist, those pertaining to the group of other investors in the target firm, or those pertaining to the combined group.

In Section 2 we model the sequence of equilibria prices and holdings of diversified shareholders over the course of activism. In Section 3 we develop the conditions on which the initial decision of activism is based. We propose a method of evaluation of activism in Section 4, and use the results derived in Sections 2 and 3 to implement this proposal and investigate its ramifications. In Section 5 we raise issues for discussion and suggest possible extensions to our model.

2 The Impact of Activism on Prices and Portfolio Rebalancing

The model that we consider specifies four moments in time at which investors gather together to compete for shares in firms for their portfolios. These moments are

to others.

[7] See, for example, Brav et al. (2009) and Klein and Zur (2009). Both studies highlight the increase in average excess return around the time of Schedule 13D filing, and its persistence. Primarily on that basis, both studies posit activism benefits target firm shareholders. Boyson and Mooradian (2011) and Clifford (2008), for example, find that both activist hedge funds and shareholders benefit from activism when considering a single firm. Becht et al. (2009) in a study of a single U.K. fund, find activism benefits that fund and also its shareholders. Becht et al. (2014), studying activism in Asia, Europe and North America, find activism is associated with abnormal returns to the target firm in the three regions.

distinguished by the information sets available to investors at each of these points in time. At time $t = 0$, all participants hold the same view regarding the future values of the firms, and come together to buy shares in these firms based on that commonly held information. We refer to the set of portfolios determined in this manner as the benchmark portfolios. We assume that the benchmark portfolios remain the same until one of those investors, called the activist, comes to believe that his involvement can alter the performance of a firm. Since his belief in the future value of the firm is different from that of all other market participants at this point, the acquisition of new ownership would be difficult if this information were shared with other investors. Thus, we assume that the activist must surreptitiously acquire these new shares, keeping his belief in the future value of the target firm to himself.

Given this belief, the activist must first decide whether it would be advantageous for him to act on the basis of this belief. If not, activism obviously does not occur. Should the decision to act be taken, then the activist moves at time $t = 1$ to acquire shares to facilitate his objective. This move at time $t = 1$ precipitates a new competitive market equilibrium with asymmetric information: The activist acts on his private information while the views of all other investors concerning the future values of the set of firms remain unchanged. If the activist acquires a sufficient number of shares, then, at time $t = 2$, the activist announces this publicly by filing Schedule 13D.[8] At the time of the filing, the other investors become informed of the activist's intent to improve the performance of the firm. Note, time $t = 2$ might follow quite closely after time $t = 1$. Having gained knowledge of the activist's intent, the remaining investors enter into a new competitive equilibrium for shares. Here, the activist refrains from entering into trading since he needs the shares he has already acquired to carry out

[8] When an owner acquires 5% or more of the voting power of a registered security, and has the intent to attempt to alter the policies of the current management, SEC rules require that Schedule 13D (the so-called beneficial ownership report), be filed within 10 days.

his activist program. Subsequently, at time $t = 3$, it becomes known to all market participants whether or not the activist has been successful in his plans to improve the firm.[9] This new information acquired by all market participants induces a new competitive equilibrium with all investors participating. Should the activist's holdings fall sufficiently, he announces this by filing an amended Schedule 13D (Schedule 13D/A). The time between $t = 2$ and $t = 3$ can be lengthy. Finally, at time T, all uncertainty concerning the firms is resolved and all the firms are liquidated.

In each competitive market equilibrium we assume that there exists the same set of N risky assets and a riskless one. Each of the M risk-averse investors is a price-taker and a von Neumann-Morgenstern expected utility of end-of-period wealth maximizer. We now introduce some notation. Let $\mathbf{x}_{it}$ be the N x 1 vector of shares held by investor i, $i = 1, ..., M$, at time t, $t = 0, 1, 2, 3$, in the N firms. Let y_{it} be the amount investor i borrows (lends) at time t to facilitate purchases. Let $\widetilde{\mathbf{p}}_{it}$ be an N x 1 vector of random prices per share of the N firms that would prevail at time T as perceived by investor i at time t, and let p_0 be the price of the riskless asset. Let u_i be the utility function of investor i, w_{it} be the wealth with which the i^{th} investor comes to the market at time t and, for convenience, let $p_0 = 1$.

At time t, $t = 0, 1, 2, 3$, the equilibrium process is defined as follows. Taking the N x 1 vector $\mathbf{p}_t$ as given, investor i determines $\mathbf{x}^*_{it}$ which satisfies $\arg\max_{\mathbf{x}_{it}} E_{it} u_i(y_{it} + \mathbf{x}'_{it}\widetilde{\mathbf{p}}_{it})$ $s.t.$ $y_{it} + \mathbf{x}'_{it}\mathbf{p}_t = w_{it}$ where E_{it} is the expectation of investor i at time t with respect to the distribution of $\widetilde{\mathbf{p}}_{it}$ and a prime denotes a transpose operation. The equilibrium price vector at time t, $\mathbf{P}_t$, yields the demands $\mathbf{x}^*_{it}$ so that all shares are sold, i.e., $\sum_{i=1}^{M} \mathbf{x}^*_{it} = \mathbf{Q}$ where $\mathbf{Q}$ is the N x 1 vector whose elements are the total number of shares in each of the N risky firms. For convenience, we normalize $\mathbf{Q}$

[9] In our model we do not allow the leakage of information as to the success of the activist between time $t = 2$ and time $t = 3$; however, we mention the additional complications such leakage might engender in Section 5 below.

and represent it by $\mathbf{1}$, an N x 1 vector whose elements are 1, so that $\mathbf{x}_{it}$ represents the vector of proportional ownership of investor i at time t in the N risky firms. We assume that each investor has an exponential utility function with Pratt-Arrow coefficient of absolute risk aversion a_i. We further assume that the random vector $\widetilde{\mathbf{p}}_{it}$ is normally distributed with mean vector $\boldsymbol{\mu}_{it}$ and positive definite covariance matrix $\boldsymbol{\Omega}_{it}$. With these assumptions, the equilibrium solution at each time t is the solution to a specific nonhomogeneous (homogeneous) portfolio problem based on the changing information. Solutions to each of these problems are derived by applying the results from Rabinovitch and Owen (1978).

Maximizing the expected utility for each of the participants at each moment of time results in the maximum expected utility over the time period $t = 0$ to $t = 3$. This follows because, since borrowing and lending are allowed, the only carryover when optimizing at time t is the resulting wealth from the optimization at time $t - 1$. However, as shown in Rabinovitch and Owen (1978), the optimum solutions, $\mathbf{x}_{it}^*$ and $\mathbf{P}_t$, at time t do not depend on this preceding wealth. Therefore, each local optimization is separate from any other. Furthermore, our choice of four trading moments is based on the assumption that trading only takes place at those times when a change of information occurs, and we assume these changes are independent of one another.

In our model, we have chosen to abstract from the usual activities of the activist, for example, from attempting to acquire representation on the board, changing dividend policy, changing CEO salary, and/or selling parts of the firm, etc. Instead, we have chosen to characterize activities into ways in which they alter the future distribution of prices. Specifically, some activities will affect the mean, others the variance and still others the covariance of the target firm with other firms. Indeed, some activities will affect these three features in various combinations. This abstraction permits us to deal with the issue of diversified ownership.

We now introduce the specifics of our model. At time $t = 0$, all investors agree on their assessments of the distribution of prices that will occur at time T. Thus, in this case, $\boldsymbol{\mu}_{i0} = \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_{i0} = \boldsymbol{\Omega}_0$. We state this well-known equilibrium solution result without proof in the next proposition.

Proposition 1. At time $t = 0$, $\boldsymbol{\mu}_{i0} = \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_{i0} = \boldsymbol{\Omega}_0, i = 1, ..., M$. Then the equilibrium solution yielding the benchmark is $\mathbf{x}^*_{i0} = \frac{d_i}{d}\mathbf{1}$, $i = 1, ..., M$, and $\mathbf{P}_0 = \boldsymbol{\mu}_0 - \frac{1}{d}\boldsymbol{\Omega}_0\mathbf{1}$ where $d_i = \frac{1}{a_i}$ and $d = \sum d_i$.

Following this market exchange, one of the investors comes to believe that, with sufficient shares in a particular firm, he can improve its performance and thereby benefit from his activism.[10] We designate this activist as investor 1, and refer to the activist as A. The single firm that is the target of A's interest is firm 1.[11] Since we have assumed that all investors can borrow, lend, as well as sell short, A must have these capabilities as well. Thus, our model necessarily excludes mutual funds as activists, but includes both hedge fund activists and other entrepreneurial activists such as individual investors and private equity funds.[12]

If A proceeds with his plan to acquire additional shares, it is done surreptitiously, and it forces a new round of trading. A comes to this round of trading with predictions as to how his involvement in the target firm would alter the future distribution of

[10] Although we do not explore the case in which the activist might benefit even if his activities are detrimental to the target firm, our model could be used to examine this situation. See comments in Section 5, below.

[11] The activist has only one target firm in our model. This assumption is made for convenience of exposition.

[12] Mutual funds are subject to the Investment Company Act of 1940 which, among other things, prevents them from selling short, borrowing, and holding concentrated positions. Hedge funds, by having a small number of high net worth investors, are not subject to this Act, and, accordingly, are not governed by the regulation of fees specified in the Act. See, for example, Brav et al. (2008, pp. 1734-1736) for a discussion of differences between mutual funds and hedge funds.

prices of all securities. In particular, we assume this involvement would change the mean and the covariance matrix of A's distribution by the amounts $\Delta\boldsymbol{\mu}$ and $\Delta\boldsymbol{\Omega}$, respectively. We note that both these changes depend on the change that would occur should A be successful with his plans, the change that would occur should A be unsuccessful with his plans, and the probability of each. For convenience, we assume that should A be unsuccessful, the parameters revert to those at time $t = 0$, i.e., $\Delta\boldsymbol{\mu} = \Delta\boldsymbol{\Omega} = \mathbf{0}$.[13] This framework leads to a heterogeneous information equilibrium whose solution is given in Proposition 2. The proof of this proposition, and all following propositions and the lemma, can be found in the Appendix.

Proposition 2. Let the distributional parameters for A be $\boldsymbol{\mu}_{11} = \boldsymbol{\mu}_0 + \Delta\boldsymbol{\mu}$ and $\boldsymbol{\Omega}_{11} = \boldsymbol{\Omega}_0 + \Delta\boldsymbol{\Omega}$ and let those for investor i, $i = 2, ..., M$, be $\boldsymbol{\mu}_{i1} = \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_{i1} = \boldsymbol{\Omega}_0$. Then, at $t = 1$, the equilibrium solution is given by

$$[d\mathbf{I} + (d - d_1)\Delta\boldsymbol{\Omega}\boldsymbol{\Omega}_0^{-1}](\mathbf{P}_1 - \mathbf{P}_0) = d_1(\boldsymbol{\Delta\mu} - \boldsymbol{\Delta\Omega}\mathbf{1}/d)$$

$$\mathbf{x}_{11}^* - \mathbf{x}_{10}^* = (d - d_1)\boldsymbol{\Omega}_0^{-1}(\mathbf{P}_1 - \mathbf{P}_0) \text{ and}$$

$$\mathbf{x}_{i1}^* - \mathbf{x}_{i0}^* = -d_i\boldsymbol{\Omega}_0^{-1}(\mathbf{P}_1 - \mathbf{P}_0) \text{ for } i = 2, ..., M.$$

Proposition 2 establishes the relationship between the changes in prices and the changes in the portfolios held by all investors due to activism. These changes are based on the changes in the mean and covariance matrices, $\boldsymbol{\Delta\mu}$ and $\boldsymbol{\Delta\Omega}$, respectively. Since $\boldsymbol{\Delta\mu}$ and $\boldsymbol{\Delta\Omega}$ are arbitrary in this proposition, we now restrict them, in keeping with our modelling of A. We assume at time $t = 1$ that A is active only in firm 1, and believes that the expected price per share of firm 1 will increase by $m > 0$ if he succeeds, and remain the same otherwise.[14] The expected values of the remaining firms are unchanged. The variance of the price of firm 1, as well as the covariances of

[13] Not making this assumption would introduce additional free parameters complicating, but not changing, our results.

[14] The issue of whether the activist could benefit if $m < 0$ is discussed later.

the price of firm 1 with the other firms, might, however, change.[15] The covariances between two prices, neither of which involves firm 1, are unchanged. Thus, we assume that the covariance matrix of prices might change in the first row and first column if the activist succeeds and would remain the same otherwise. We next make these changes explicit.

We introduce the following notation. The subscript -1 is used for a vector or matrix to denote that vector or matrix without its first element or first row, respectively, e.g., the N x 1 vector $\mathbf{v}$, with first element v_1, is written as $\mathbf{v}' = (v_1, \mathbf{v}'_{-1})$. We let $\boldsymbol{\Omega}_0^{-1} = (\boldsymbol{\omega}^1, ..., \boldsymbol{\omega}^N) = \begin{pmatrix} \omega_1^1 & \boldsymbol{\omega}_{-1}^{1\prime} \\ \boldsymbol{\omega}_{-1}^1 & \mathbf{R} \end{pmatrix}$ where $\mathbf{R}$ is a positive definite $N-1$ x $N-1$ symmetric matrix. The omission of the first row of the matrix $\boldsymbol{\Omega}_0^{-1}$ will be written as $\boldsymbol{\Omega}_{-1,0}^{-1}$. If we define the N x N matrix $\mathbf{V} = \begin{pmatrix} v_1 & \mathbf{v}'_{-1} \\ \mathbf{v}_{-1} & \mathbf{0} \end{pmatrix}$ and π as the probability that A will succeed in his plans, A approaches the market at $t = 1$ with parameters $\boldsymbol{\mu}_{11} = \boldsymbol{\mu}_0 + \pi m \mathbf{e}_1$ and $\boldsymbol{\Omega}_{11} = \boldsymbol{\Omega}_0 + \pi \mathbf{V}$ where $\mathbf{e}_1$ is an N x 1 vector with 1 in the first position and zeros elsewhere. The other investors remain with their previous information, i.e., $\boldsymbol{\mu}_{i1} = \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_{i1} = \boldsymbol{\Omega}_0$, $i = 2, ..., M$. We next present a lemma that permits us to solve explicitly for the inverse needed to determine the equilibrium price changes in Proposition 2.

In what follows, we let $(\mathbf{P}_1 - \mathbf{P}_0)' = ((\mathbf{P}_1 - \mathbf{P}_0)_1, ..., (\mathbf{P}_1 - \mathbf{P}_0)_N)$, where $(\mathbf{P}_1 - \mathbf{P}_0)_j$ is the j^{th} component of $(\mathbf{P}_1 - \mathbf{P}_0)$. Scalar components for other vectors are indicated in a similar manner.

Lemma. The N x 1 vectors $\mathbf{x}' = (x_1, \mathbf{x}'_{-1})$ and $\mathbf{z}' = (z_1, \mathbf{z}'_{-1})$ and the matrix $\mathbf{M} = [\mathbf{I} - \alpha \binom{\mathbf{x}'}{\mathbf{v}_{-1}\mathbf{z}'}]$ satisfy $\mathbf{M}[\mathbf{I} + \alpha \mathbf{V} \boldsymbol{\Omega}_0^{-1}] = \mathbf{I}$ where

[15] See, for example, Lee and Park (2009) and Gantchev et al. (2013) who find evidence of the impact of activism in the target firm affecting other firms.

$$
\begin{aligned}
x_1 &= \frac{1}{c}[\mathbf{v}'\boldsymbol{\omega}^1 - \alpha\omega_1^1(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)] \\
\mathbf{x}_{-1} &= \frac{1}{c}[\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v} - \alpha\frac{(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})}{(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)}\boldsymbol{\omega}_{-1}^1] \\
z_1 &= \frac{\omega_1^1}{c(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)} \\
\mathbf{z}_{-1} &= \frac{1}{c(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)}[(1+\alpha\mathbf{v}'\boldsymbol{\omega}^1)\boldsymbol{\omega}_{-1}^1 - \alpha\omega_1^1\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v}] \\
c &= 1+\alpha\mathbf{v}'\boldsymbol{\omega}^1 - \alpha^2\omega_1^1(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1) \text{ and} \\
0 &< \alpha \leq 1.
\end{aligned}
$$

Since $\mathbf{M}[\mathbf{I}+\alpha\mathbf{V}\boldsymbol{\Omega}_0^{-1}] = \mathbf{I}$, it follows that $\boldsymbol{\Omega}_0^{-1}\mathbf{M}$ is the inverse of $[\boldsymbol{\Omega}_0+\alpha\mathbf{V}]$. Because this latter matrix is assumed to be positive definite, its inverse must have positive diagonal elements. It follows that the upper left diagonal element of $\boldsymbol{\Omega}_0^{-1}\mathbf{M}$ must be positive and this can only happen if $c(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1) > 0$. For the remainder of the paper we assume that the parameters satisfy $c > 0$ and $1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1 > 0$.

This lemma allows us to present the equilibrium prices at $t = 1$ explicitly. We do this in the next proposition.

Proposition 3. At time $t = 1$, $\boldsymbol{\mu}_{11} = \boldsymbol{\mu}_0 + \pi m\mathbf{e}_1$ and $\boldsymbol{\Omega}_{11} = \boldsymbol{\Omega}_0 + \pi\mathbf{V}$, and $\boldsymbol{\mu}_{i1} = \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_{i1} = \boldsymbol{\Omega}_0$, $i = 2, ..., M$. Then the equilibrium prices can be written as

$$
(\mathbf{P}_1 - \mathbf{P}_0) = \begin{bmatrix} g_1 \\ -g_2\mathbf{v}_{-1} \end{bmatrix}
$$

where

$$
\begin{aligned}
g_1 &= \frac{d_1\pi}{cd}[m - \mathbf{v}'\mathbf{1}/d + \alpha\frac{1}{d}(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)] \\
g_2 &= \frac{d_1\pi}{cd}[\frac{\alpha\omega_1^1}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1}(m - \mathbf{v}'\mathbf{1}/d) + \frac{1}{d}(1+\alpha\mathbf{v}'\boldsymbol{\omega}^1)] \text{ and} \\
\alpha &= \frac{d-d_1}{d}\pi.
\end{aligned}
$$

Propositions 2 and 3 demonstrate the result of the surreptitious acquisition of shares by A. A's predictions of the changes that his activism would produce caused him to seek to alter his portfolio holdings consistent with his predictions. Because he had to acquire shares in the market[16], and because his view of future prices was different from that of other investors, the market exchange was characterized by a heterogeneous information equilibrium. Under these conditions, Propositions 2 and 3 establish the relationship between A's predictions and their impact on prices and holdings of all market participants at time $t = 1$. In particular, Proposition 3 shows how changes in the variance or covariances affect the price change of firm 1, and all prices connected to firm 1. Furthermore, Proposition 2 extends this observation to the holdings themselves.

Should A believe that the result of his activism would have no additional effect on the covariances between firm 1 and the remaining firms, i.e., $\mathbf{v}_{-1} = \mathbf{0}$, then from Proposition 3, it follows immediately that prices other than the price of shares of the first firm would not change. However, using Proposition 2 under the condition that $\mathbf{v}_{-1} = \mathbf{0}$, we note that holdings for all investors change nevertheless. That is, a rebalancing of portfolios occurs for all investors even though only the price of the shares of the target firm changes. Since these rebalancings involve a money exchange, this demonstrates that a change in the price of the target firm, by itself, is not enough to evaluate the impact of activism on shareholders of this firm. This observation leads us to propose, in Section 4 below, a method of evaluation that avoids this criticism.

Examining the change in the price of the shares of firm 1 exhibited in Proposition 3, it is not clear, in general, that this price increases without imposing some further conditions. These conditions on g_1 will be clarified when, after discussing the remain-

[16] See, for example, Kahan and Rock (2007, p. 1069) where they state "... it is noteworthy that activist hedge funds usually accumulate stakes in portfolio companies *in order to engage in activism.*" Italics in original.

ing two equilibria, we address the preliminary decision that A would have had to have made to become an activist in the first place.[17]

Assuming A has acquired sufficient shares at time $t = 1$, then at $t = 2$, he announces this by filing Schedule 13D. With the release of information contained in his filing of Schedule 13D, all investors, except for A, institute a trading round based on this new information. A is not be involved in this trading round since we assume his acquisition of additional shares was predicated on the fact that he would continue to hold shares long enough to execute his plan.[18] Thus, the trading round at time $t = 2$ is again one of homogeneous information, but with the number of shares held by A excluded from the competition.

More precisely, at time $t = 2$, A does not trade and each of the other investors learns of the information held by A. Thus, at this time we have $M - 1$ investors sharing the same information $\boldsymbol{\mu}_{i2} = \boldsymbol{\mu}_0 + \pi m \mathbf{e}_1$ and $\boldsymbol{\Omega}_{i2} = \boldsymbol{\Omega}_0 + \pi \mathbf{V}$, $i = 2, ..., M$. The result of this competition is contained in the next proposition.

Proposition 4. At time $t = 2$, A does not trade, and $\boldsymbol{\mu}_{i2} = \boldsymbol{\mu}_0 + \pi m \mathbf{e}_1$ and $\boldsymbol{\Omega}_{i2} = \boldsymbol{\Omega}_0 + \pi \mathbf{V}$, $i = 2, ..., M$. Then the equilibrium solution yields $\mathbf{P}_2 = \boldsymbol{\mu}_0 + \pi m \mathbf{e}_1 - \frac{1}{d-d_1}(\boldsymbol{\Omega}_0 + \pi \mathbf{V})(\mathbf{1} - \mathbf{x}^*_{11})$ and $\mathbf{x}^*_{i2} = \mathbf{x}^*_{i1}$ for $i = 2, ..., M$.

Proposition 4 establishes the fact that the new information acquired by the remaining investors when A abstains from the trading round has no impact on their

[17] We need to delay the discussion for the following reason. Under the assumptions that A will have acquired additional shares, he will be able to begin his efforts to alter the direction of the firm. This, however, has come at a cost of acquiring these additional shares that can be written as $\mathbf{P}'_1(\mathbf{x}^*_{11} - \mathbf{x}^*_{01})$. In the initial decision as to whether to become an activist, A must consider this cost against the expected revenue he will subsequently receive when he has finished his activist activities and sells his extra shares on the market.

[18] See Clifford (2008) who finds that hedge funds do not seem to buy or sell additional shares when they change from a passive status to an active one, although that change in status necessitates a filing of Schedule 13D.

holdings. The intuition for this result is as follows. A would not wish to sell his recently acquired shares in firm 1 since this would undermine his purpose as an activist. Given this point, he would not wish to trade his shares in other firms either, since he already optimized his holdings in these firms in conjunction with his purchase of additional shares in firm 1 when using his private information. (In fact, he would be at a disadvantage to trade in a market in which all investors had the same information as he did.) On the other hand, the other investors, having been alerted to the activism by the Schedule 13D filing, now may want more of the shares of firm 1, and can only get those shares from among themselves. In their attempt to get more shares, the prices will change. At these changed prices, however, it becomes optimal for these other investors to end up with portfolios identical to the ones they selected at time $t = 1$.[19]

Subsequently, at time $t = 3$, there is new information since it becomes known as to whether or not A was successful. The distributional parameters held by all market participants, including A, then are either $\boldsymbol{\mu}_0 + m\mathbf{e}_1$ and $\boldsymbol{\Omega}_0 + \mathbf{V}$ if A were successful, or $\boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_0$ otherwise. Thus, all investors participate in a homogenous information equilibrium. Should this equilibrium result in the sale of sufficient shares in firm 1 by A, then at this time A files Schedule 13D/A, acknowledging the change in his ownership. The next proposition provides the results.

[19]In form, the result of Proposition 4 bears a resemblance to equation (3) in Admati et al. (1994). This resemblance is deceiving for two reasons. First, the shares acquired by the activist in Admati et al. were acquired strategically, that is, not as a price-taker, whereas our activist acquired his shares in a Walrasian market. Second, though firms are considered correlated in the Admati et al. paper, it is assumed that activism can only affect the mean of the distribution of prices whereas we assume activism can affect both the mean of the distribution and its covariance matrix. Neglecting the impact on the covariance structure obscures the necessary portfolio rebalancing and the costs associated with it.

Proposition 5. At $t = 3$, if A is successful, $\boldsymbol{\mu}_{i3} = \boldsymbol{\mu}_0 + m\mathbf{e}_1$ and $\boldsymbol{\Omega}_{i3} = \boldsymbol{\Omega}_0 + \mathbf{V}$, $i = 1, ..., M$. At $t = 3$, if A is not successful, $\boldsymbol{\mu}_{i3} = \boldsymbol{\mu}_0$ and $\boldsymbol{\Omega}_{i3} = \boldsymbol{\Omega}_0$, $i = 1, ..., M$. If A is successful, the equilibrium price $\mathbf{P}_3 = \mathbf{P}_3^U = \boldsymbol{\mu}_0 + m\mathbf{e}_1 - \frac{1}{d}(\boldsymbol{\Omega}_0 + \mathbf{V})\mathbf{1}$; if A is unsuccessful, the equilibrium price is $\mathbf{P}_3 = \mathbf{P}_3^L = \mathbf{P}_0$. In either case, $\mathbf{x}_{i3}^* = \mathbf{x}_{i0}^* = \frac{d_i}{d}\mathbf{1}$.

One interesting feature of Proposition 5 is that whether successful or not at time $t = 3$, A chooses to sell the additional shares he acquired at time $t = 1$ in firm 1.[20] That is, there is no way for A, if successful, to take advantage of the improved distribution of prices once the result of his activism becomes known. In equilibrium, the combined demand of all the shareholders, including A, force this result.

The derivations of the equilibria in our model were predicated on an initial decision made by A: The decision to become an activist or not. In the next Section we discuss how this preliminary decision was made.

3 The Decision to Become an Activist

In our model, A approaches the decision to become an activist with a presumption of how the future value of the target firm, as well as the future values of other firms, would change as a result of his activism. This is summarized by the parameters of his subjective probability distribution of the future value of the target as well as other firms in the market. Under what conditions does this distribution warrant activism?

In considering this distribution, A is aware that he will have a significant impact on the equilibria that follow. A also knows that to acquire shares or to sell shares, he must involve himself in these competitive equilibria. Since A can anticipate the results of these equilibria in expectation, he can also anticipate the costs of all of the portfolio rebalancing involved as well as the portfolio he would hold when he exits

[20] See Brav et al. (2008), where it is noted that the shedding of excess shares when activism is concluded is typically via sales in the market.

the target firm. Using these results, for A to proceed, we assume that the parameters of this distribution must satisfy two conditions. First, the parameters must afford A the expectation of acquiring sufficient additional shares in the target firm at time $t = 1$ to enable his activism. Second, the parameters must afford A the expectation of avoiding a loss over the course of his activism. We assume that activism will occur only when both of these conditions are satisfied. We next show that satisfying these conditions is equivalent to placing constraints on the parameters of A's subjective probability distribution.

We denote by *CA1* the condition that A expects to acquire more shares in the target firm. Using the notation established above, we write *CA1* as $(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})_1 > 0$.[21] From Propositions 2 and 3, we have

$$\begin{aligned}(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})_1 &= (d - d_1)\boldsymbol{\omega}^{1\prime}(\mathbf{P}_1 - \mathbf{P}_0) \\ &= (d - d_1)[g_1\omega^1_1 - g_2\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}].\end{aligned}$$

Thus, the constraint *CA1* is equivalent to $g_1\omega^1_1 - g_2\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1} > 0$ and is satisfied when the parameters of A's subjective probability distribution satisfy this inequality. The expectation of acquiring additional shares does not imply that the expectation of the change in price of the shares of the target firm at time $t = 1$, g_1, is positive. That is, *CA1* can be satisfied with $g_1 < 0$, depending on whether $\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}$ is sufficiently negative.

We denote by *CA2* the condition that A expects not to suffer a loss over the course of his activism. From Proposition 5, it follows that the money exchanged in A's rebalancing resulting from the equilibrium at time $t = 3$ is $\mathbf{P}'_3(\mathbf{x}^*_{11} - \mathbf{x}^*_{13})$. Since $\mathbf{x}^*_{13} = \mathbf{x}^*_{10}$, this amount can be written as $\mathbf{P}'_3(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})$. Similarly, the money exchanged by A at time $t = 1$ due to rebalancing is $\mathbf{P}'_1(\mathbf{x}^*_{10} - \mathbf{x}^*_{11})$. Thus, the total money exchanged by A from $t = 1$ to $t = 3$ is $(\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})$. Starting with the

[21] We could have imposed the requirement $(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})_1 > \tau > 0$ but for convenience chose $\tau = 0$.

portfolio value $\mathbf{P}_0'\mathbf{x}_{10}^*$ and ending with the portfolio value $\mathbf{P}_3'\mathbf{x}_{10}^*$, A's total change in portfolio value is $(\mathbf{P}_3 - \mathbf{P}_0)'\mathbf{x}_{10}^*$. Thus, the change in value to A from his involvement in activism is given by $(\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}_{11}^* - \mathbf{x}_{10}^*) + (\mathbf{P}_3 - \mathbf{P}_0)'\mathbf{x}_{10}^*$. Since at time $t = 3$, $\mathbf{P}_3$ can take on one of two values (refer to Proposition 5), A's expected change in value from activism is $E_\pi(\mathbf{P}_3-\mathbf{P}_1)'(\mathbf{x}_{11}^*-\mathbf{x}_{10}^*)+E_\pi(\mathbf{P}_3-\mathbf{P}_0)'\mathbf{x}_{10}^*$ where E_π is the expectation taken with respect to the binary distribution of $\mathbf{P}_3$. Finally, we can write *CA2* as the constraint $E_\pi(\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}_{11}^* - \mathbf{x}_{10}^*) + E_\pi(\mathbf{P}_3 - \mathbf{P}_0)'\mathbf{x}_{10}^* > 0$. As with *CA1*, we can write the inequality of *CA2* in terms of the parameters of A's subjective distribution by using Propositions 2, 3 and 5.

Together, we call the two conditions for activism, *CA1* and *CA2, CA* and note that *CA* places constraints on the parameters that the potential activist brings to the problem. Only when *CA* is satisfied will A proceed. The implied constraints formalize the idea that among all possible targets that A might choose, only some are deemed worthy of pursuing. For the remainder of the paper we assume that the constraints in *CA* hold.

4 Methodology to Evaluate Activism

Having established the condition *CA* that permits an activist to proceed, and having presented the results of the equilibria over the course of A's involvement with the target firm, we now use these results to construct a methodology to evaluate activism. Our method of evaluating activism takes the sequence of derived equilibria as given and provides an answer to the question: How did the activist, A, and the group of investors excluding the activist, G, fare over the course of activism?

Our method of evaluation depends on the creation of a measure for A and for G, each of which involves two calculated values. The first calculated value is the sum of the money exchanged for the rebalancing of the portfolios required at each of the intervening equilibria ($t = 1, 2$ and 3) for A and G, respectively. We designate these rebalancing amounts for A and G as $R(A)$ and $R(G)$, respectively. The second calculated value is the difference between the portfolio value held at the end of activism ($t = 3$) and the portfolio value held prior to activism ($t = 0$) for A and G, respectively. We designate these differences for A and G as $D(A)$ and $D(G)$, respectively. We use these calculated values to define the measure of evaluation for A as $\Psi(A) = R(A) + D(A)$ and for the remaining investors, G, as $\Psi(G) = R(G) + D(G)$. Since the function Ψ represents the net financial gain (loss) over the course of activism, we say that activism benefits A if, at time $t = 3$, $\Psi(A) > 0$ and activism benefits G if, at time $t = 3$, $\Psi(G) > 0$. We next use the equilibria results to evaluate the Ψ functions explicitly.

We begin with A. As argued in Section 3 above, the sum of the money exchanged by A in rebalancing over the period of activism, $R(A)$, is given by $(\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10})$. (At time $t = 2$, A is not involved in the equilibrium so there is no rebalancing on his part.) Also, from Section 3, the change in A's portfolio value, $D(A)$, is given by $(\mathbf{P}_3 - \mathbf{P}_0)'\mathbf{x}^*_{10}$. Thus, the evaluation of activism for A is $\Psi(A) = (\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10}) + (\mathbf{P}_3 - \mathbf{P}_0)'\mathbf{x}^*_{10}$. Since this evaluation occurs at time $t = 3$, $\mathbf{P}_3 = \mathbf{P}^U_3$ or $\mathbf{P}^L_3$ (see Proposition 5). Note, unlike the similar calculation done by A to satisfy *CA2*, this evaluation takes place at time $t = 3$, when the value of $\mathbf{P}_3$ is known. Since, from Proposition 1, $\mathbf{x}^*_{10} = \frac{d_1}{d}\mathbf{1}$, $D(A) = \frac{d_1}{d}(\mathbf{P}_3 - \mathbf{P}_0)'\mathbf{1}$. The quantity $(\mathbf{P}_3 - \mathbf{P}_0)'\mathbf{1}$ is the actual change in the market value due to activism over its course, and we denote it by S. Thus, $\Psi(A) = R(A) + \frac{d_1}{d}S$, which depends on the change in market value caused by activism, S, and demonstrates that this change is needed in evaluating activism but in itself is not sufficient to measure the total impact of activism on A.

We now address $\Psi(G)$, the measure of gain or loss from activism for the group of other investors. We let $x^*_{Gt} = \sum_{j=2}^{M} \mathbf{x}^*_{jt}$, $t = 0, 1, 2, 3$, be the group holdings at the various equilibria. In line with the argument above, the money exchanged at time

$t = 1$ for G is $\mathbf{P}_1{}'(\mathbf{x}^*_{G0} - \mathbf{x}^*_{G1})$. At time $t = 2$, all money is exchanged among members of G itself, and therefore there is no change for the group. Using the same argument as used at time $t = 1$, and recalling that $\mathbf{x}^*_{G3} = \mathbf{x}^*_{G0}$, the money exchanged at time $t = 3$ is $\mathbf{P}_3{}'(\mathbf{x}^*_{G1} - \mathbf{x}^*_{G0})$. Thus the money exchanged due to portfolio rebalancing by G is given by $R(G) = (\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}^*_{G1} - \mathbf{x}^*_{G0})$. At time $t = 3$, G, having started with a portfolio value $\mathbf{P}'_0\mathbf{x}_{G0}$, is left with a portfolio value $\mathbf{P}'_3\mathbf{x}^*_{G0}$ at time $t = 3$. Thus, $D(G) = (\mathbf{P}_3 - \mathbf{P}_0)'\mathbf{x}^*_{G0}$ and $\Psi(G) = (\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}^*_{G1} - \mathbf{x}^*_{G0}) + (\mathbf{P}_3 - \mathbf{P}_0)'\mathbf{x}^*_{G0}$. Since $\mathbf{x}_{G0} = (1 - \frac{d_1}{d})\mathbf{1}$, $\Psi(G) = R(G) + (1 - \frac{d_1}{d})S$.

We note that although the equilibrium prices at time $t = 1$ play a role in our evaluation method, by themselves they are only important in so far as they contribute to $R(A)$ and $R(G)$. We next establish the relationship between $\Psi(A)$ and $\Psi(G)$.

Proposition 6.

(a) $R(A) + R(G) = 0$.

(b) $\Psi(A) + \Psi(G) = S$.

Proposition 6(a) establishes the fact that whatever financial benefit (loss) A acquires in the rebalancing of portfolios, G loses (gains). However, achieving a benefit or a loss by itself provides no information as to whether activism is beneficial, i.e., whether $\Psi > 0$. Proposition 6(b) deals with this issue. Since S is the total change in the market value due to activism, 6(b) shows that this change is split between A and G. Since neither $\Psi(A)$ nor $\Psi(G)$ need be positive, this split may not imply a benefit for both. In fact, should $S = 0$, Proposition 6 shows that the result of activism is zero-sum.

But is it reasonable to consider values of $S \leq 0$? That is, if, as a consequence of A's considerations of becoming an activist, A determines that the value of the market would fall as a result of his activism, would this imply that CA could not be satisfied? We next show that there are circumstances in which this implied decline in the value of the market would not deter the potential activist from proceeding.

Proposition 7. There are instances of A's subjective probability distributions such that despite A being aware that the impact of his activism would lower the value of the market, CA would be satisfied and A would proceed with activism. Furthermore, if successful, A would benefit but G would not.

The instance explored in the proof of Proposition 7 is where A expects that if he succeeds in his endeavors, the sole result, aside from $m > 0$, would be to increase the correlation, namely v_2, between the target firm and one other firm, firm 2. The assumption that $v_2 < dm < 2v_2$ where $0 < v_2 < \frac{1}{\alpha}$, is enough to show that CA is satisfied. It also follows that the price of the target firm increases at time $t = 1$ and at the same time the price of firm 2 decreases. This decrease causes a decrease in the value of the market at this time. However, despite this, with CA satisfied, A proceeds with his activism which, in turn, leads to a decrease in the value of the market over the entire period of activism, i.e., S falls. Finally, we show that if A succeeds, A benefits and G does not benefit. As a result of Proposition 7, in considering the benefits to those involved in activism, we must consider situations where activism could cause changes in the value of the market that are negative as well as positive.

We next examine the relationship between $\Psi(A)$ and $\Psi(G)$, making this relationship explicitly dependent on $R(A)$ and S.

Proposition 8.

(a) $\Psi(A) > 0$ and $\Psi(G) > 0$ iff $-\frac{d_1}{d}S < R(A) < (1 - \frac{d_1}{d})S$.

(b) $\Psi(A) > 0$ and $\Psi(G) < 0$ iff $R(A) > \max[-\frac{d_1}{d}S, (1 - \frac{d_1}{d})S]$.

(c) $\Psi(A) < 0$ and $\Psi(G) > 0$ iff $R(A) < \min[-\frac{d_1}{d}S, (1 - \frac{d_1}{d})S]$.

(d) $\Psi(A) < 0$ and $\Psi(G) < 0$ iff $(1 - \frac{d_1}{d})S < R(A) < -\frac{d_1}{d}S$.

The constraints in Proposition 8(a) can only be satisfied if $S > 0$. Thus, 8(a) exhibits the fact that for both parties to benefit, S must be positive and A must be constrained in the terms of the gains made in rebalancing his portfolio. Similarly, as exhibited in part (d), when neither benefit, it is necessary that S be negative and A be severely restricted in the rebalancing amounts he can make. The intervening results constrain $R(A)$ but S may or may not be positive.

Our evaluation of activism is predicated on the knowledge of the outcome of the A's activities at time $t = 3$. The evaluations will change depending on whether or not A is successful. The next proposition examines the relationship between $\Psi(A)$ and $\Psi(G)$ when this distinction is made.

Proposition 9.

(a) If A is successful, then

(1) $\Psi(A) > 0$.

(2) $\Psi(G) > 0$ iff $R(A) < (1 - \frac{d_1}{d})S$.

(3) $\Psi(G) < 0$ iff $R(A) > (1 - \frac{d_1}{d})S$.

(b) If A is not successful, then

(1) $\Psi(A) < 0$.

(2) $\Psi(G) > 0$.

Significantly, Proposition 9(a) shows that A always gains when activism is successful, while the gains or losses of G depend on the magnitude of the gains by A. As the magnitude of the A's gains increase, a point is reached where G loses. Part (b) of the proposition shows that if A is not successful, A loses while G always gains.

One can interpret Proposition 9 more generally. Given that A only proceeds having already determined that CA is satisfied, A would be assured that, if successful, he would benefit by the amount $\Psi(A)$. With this guarantee, A would proceed and, if successful, at time $t = 3$ would receive $\Psi(A)$. However, as a result of the activism, the value of the market changes over that period by the amount S. Thus, A gets

$\Psi(A)$ from the amount S, leaving the rest to G. Obviously, when $\Psi(A)$ is too large compared to S, G must make up the difference, possibly resulting in a loss for G.

Our approach has the advantage that it separates A from G in evaluating activism. To be complete, we next consider what our method of evaluation would produce if we used it to evaluate the totality of shareholders in the target firm, i.e., A and G together. The evaluation of this enlarged group is referred to as $\Psi(A+G)$. In keeping with our method of evaluation, we define $\Psi(A+G) = R(A+G) + D(A+G)$ where $R(A+G) = R(A) + R(G)$ and $D(A+G) = D(A) + D(G)$.

Proposition 10.

(a) $\Psi(A+G) > 0$ iff $S > 0$.

(b) If $\Psi(A+G) > 0$, then at least one of A and G will benefit.

(c) If $\Psi(A+G) > 0$, then activism will benefit only A or only G if $R(A)$ does not satisfy $-\frac{d_1}{d}S < R(A) < (1 - \frac{d_1}{d})S$.

Proposition 10 states that the enlarged group $A+G$ benefits over the course of activism if and only if activism leads to an increase in market value. But from Proposition 10(b) and 10(c), it follows that the benefit to the enlarged group does not necessarily translate to benefits for both A and G. Thus, in claims for the benefits of activism, it is important to make clear the particular group that is being addressed. This raises an issue with an evaluation of the impact of activism appearing in some of the literature.[22] There, the claim is made that activism benefits shareholders since the price of the target firm increases at the time of the Schedule 13D filing and persists. This claim leaves unspecified or vague whether the group that benefits in the statement in the literature is A or G or $A+G$. If the shareholders referred to in the empirical literature are either A or G, then Propositions 8 and 9 show that, with diversified portfolio holders, this conclusion cannot hold. If, as in Proposition 10, we

[22] See, for examples, Brav et al. (2009) and Klein and Zur (2009).

focus on the combined group, $A+G$, it is only under severely restrictive conditions that an increase in the target price at the time of the Schedule 13D filing yields a benefit of activism over the course of activism, even when dealing with diversified portfolio holders.

5 Discussion

Our method of evaluation of activism is applied separately to the activist and to the group of other investors, as well as to the combined group of all shareholders, when the activist and shareholders in the target firm are diversified portfolio holders of possibly correlated firms. Our method is distinguished by several features. First, it depends on two values computed from the series of price changes and portfolio changes resulting from activism, namely from the funds exchanged due to portfolio rebalancing and the change in market value over the course of activism, $R(A)$ and S, respectively. Second, the evaluation depends on all of the activities that occur from the moment that the activist decides to become an activist by acquiring additional ownership in the target firm until the moment the activist divests this additional ownership. Third, a preliminary judgement to proceed with activism, the CA condition, is described and assumed as a prerequisite to action.

We find that when the activist is successful in his endeavors, he always benefits. The fact that the activist benefits may be accompanied by a loss for the group of other investors and/or a decline in the value of the market. If the activist is not successful in his endeavors, he suffers a loss from his activities, the group of other investors gains, and the value of the market does not increase. We show, however, that the preliminary judgement to become an activist lessens the number of instances that will lead to the activist's failure to successfully complete his plans. Thus, we conclude that activism benefits the activist, possibly at the expense of the group of other shareholders. In considering the combined group of the activist and the other shareholders, we find that this combined group benefits if and only if the value of the market increases as a result of activism; furthermore, the benefits may not be shared by both the activist and the group of other investors.

Our method of evaluation enables us to draw distinctions which we think have been obscured in parts of the literature. Aside from being able to evaluate the activist separately from the group of other investors, our method exposes the costs and benefits of the two, as well as the competition between them for any benefits. For example, the funds exchanged by the activist in rebalancing his portfolio over time, $R(A)$, equals $-R(G)$. So gains made by the activist are at the expense of the group of other investors. This relationship is hidden when the focus is on the totality of all the shareholders, the activist plus the group of other investors.

Our model relies heavily on the assumption that activism may alter the covariance structure between the target firm and other firms. In support of this assumption, we find empirical evidence that activism affects firms other than the target firm. In fact, there are instances in which the covariance structure becomes meaningful in the strategy of the activist.[23] See, for example, the description of AXA's proposed acquisition of MONY in Kahan and Rock (2007), and Lee and Park (2000) who demonstrate the impact of activist behavior on the prices of other firms. Also, Greenwood and Schor (2009) show that targets for which a merger or sale of part of the assets earn more than targets without those prospects, Becht et al. (2014) confirms a similar finding internationally, and Gantchev et al. (2013) document spillover reactions of hedge

[23] One particular recent event provides a useful example of an activity of an activist attempting to alter the covariance between firms. In this case, the hedge fund Eminence Capital owns stakes in both Men's Wearhouse and Jos. A. Bank, and has made clear that it desires the takeover bid of the latter by Men's Wearhouse to be successful. See Michael J. de la Merced, "Jos. A. Bank in Talks to Buy Eddie Bauer," *New York Times*, 2/3/14. (On 3/11/14, Men's Wearhouse agreed to buy Jos. A Bank.)

fund activism.[24]

When the activist is successful, why is it that the activist benefits while the group of other investors may not? The reason becomes clear when glancing at the sequence of equilibria over the course of activism. It is apparent at the outset at time $t = 1$ that the activist brings private information to the equilibrium (his plan to alter the future value of the target firm) giving him an advantage that carries through the rest of his involvement with the target firm. This advantage is similar to that of inside information, albeit future inside information. Furthermore, when this information leads to profitable rebalancing exchanges, $R(A) > 0$, the activist gains at the expense of the group of other investors.

Turning to our assumptions, we have assumed that there is a single activist in a single firm. This assumption can be generalized to many activist in many firms using the same approach we employed here. The further assumption that these multiple activists could form coalitions adds an additional complexity not resolvable by our approach and needs further consideration.

We have assumed the activist's activities would lead, in expectation, to an increase

[24] Other strategies that may exploit the covariance structure include those that make use of hidden ownership (that is, economic ownership held without voting rights) and empty voting (that is, voting exceeding economic ownership); see Hu and Black (2007). However, the use of derivatives, and in particular, equity swaps, to mask the accumulation of shares that would necessitiate a 13D filing, has been challenged in Federal Court in connection with a case involving The Children's Investment Fund and CSX; see, for example, Stowell (2010). According to Stowell (2010, p. 249), the 2008 ruling in the CSX vs. The Children's Investment Fund Management case "represents a strong challenge to hedge funds who attempt to conceal their true economic position through the use of derivatives." We note that more recently, the 2nd Circuit U.S. Court of Appeals, considering the same case, left unsettled exactly under what circumstances cash-settled total equity swap agreements provide beneficial ownership. In the context of the Dodd-Frank Act, since section 766(b) amends Sections 13(d) and 13(g) of the Exchange Act, this issue remains not fully settled. See Cuillerier and Hall (2011).

in the price of the target firm, $m > 0$. However, the case when $m < 0$ falls within the purview of our model since the conditions that allow activism to proceed, CA, can be satisfied in that case. In that situation, the activist's activities, in expectation, hurt the target firm while the activist gains through the induced changes in the covariance structure.

Finally, we assumed that no new public information becomes available between times $t = 2$ and $t = 3$. We could modify our model by assuming a leakage of information in that interval as to the eventual success of the activist's endeavors.

We conclude that there is a disproportionate advantage to the activist from his activism. The source of this advantage stems from the private information that the activist uses at time $t = 1$ to surreptitiously acquire additional ownership in the target firm. Policy recommendations to correct this imbalance might include ways in which the intent of activism might be exposed to the public as soon as possible. Some discussions relating to the Dodd-Frank Act along these lines are currently under consideration. Since secrecy is at the heart of the imbalance just described, we offer the following proposals that might offer more openness. First, the ten day delay before requiring the filing of Schedule 13D should be shortened. Second, no exemption from the regulation to file 13D should be permitted. Third, using derivatives etc., to obscure beneficial ownership should be precluded. Fourth, restrictions on coalitions of activists should be imposed to thwart gaming. Last, newly proposed coalitions of investors, board members, and advisers, which appear to be designed to be countervailing thrusts against activists,[25] need to be studied to see if their access to company information creates another opportunity for a form of inside information.

[25] See, David Gelles, "Unlikely Allies Seek to Check Power of Activist Hedge Funds," *New York Times*, 2/3/14.

6 Appendix

Proof of Proposition 2. In Rabinovitch and Owen (1978; hereafter RO) they proved that, under the assumptions we have made concerning utilities and distributions, the equilibrium solution for the general heterogeneous portfolio problem can be written as $\mathbf{x}_{i1}^* = d_i \mathbf{\Omega}_{i1}^{-1}[\boldsymbol{\mu}_{i1} - \mathbf{P}_1]$ where $d_i = 1/a_i$ and where $\mathbf{P}_1$ is chosen to satisfy $\sum_{i=1}^{M} \mathbf{x}_{i1}^* = \mathbf{1}$. Evaluating the RO solution under the present assumptions yields $\mathbf{\Omega}_{11}\mathbf{x}_{11}^* = d_1(\boldsymbol{\mu}_{11} - \mathbf{P}_1)$ and for $i > 1$, $\mathbf{\Omega}_0 \mathbf{x}_{i1}^* = d_i(\boldsymbol{\mu}_0 - \mathbf{P}_1)$. It follows that

$$\begin{aligned}\mathbf{x}_{11}^* - \mathbf{x}_{10}^* &= \mathbf{x}_{11}^* - \tfrac{d_1}{d}\mathbf{1} = d_1 \mathbf{\Omega}_{11}^{-1}(\boldsymbol{\mu}_{11} - \mathbf{\Omega}_{11}\mathbf{1}/d - \mathbf{P}_1) \\ &= d_1 \mathbf{\Omega}_{11}^{-1}(\boldsymbol{\mu}_0 + \Delta\boldsymbol{\mu} - (\mathbf{\Omega}_0 + \Delta\mathbf{\Omega})\mathbf{1}/d - \mathbf{P}_1) \\ &= d_1 \mathbf{\Omega}_{11}^{-1}(\mathbf{P}_0 - \mathbf{P}_1 + \Delta\boldsymbol{\mu} - \Delta\mathbf{\Omega}\mathbf{1}/d).\end{aligned}$$

Also, $\mathbf{x}_{i1}^* - \mathbf{x}_{i0}^* = \mathbf{x}_{i1}^* - \frac{d_i}{d}\mathbf{1} = d_i \mathbf{\Omega}_0^{-1}(\boldsymbol{\mu}_0 - \mathbf{\Omega}_0\mathbf{1}/d - \mathbf{P}_1)$

$$= d_i \mathbf{\Omega}_0^{-1}(\mathbf{P}_0 - \mathbf{P}_1).$$

Summing over $i = 1, ..., M$, we have

$$0 = (d - d_1)\mathbf{\Omega}_0^{-1}(\mathbf{P}_0 - \mathbf{P}_1) + d_1 \mathbf{\Omega}_{11}^{-1}(\mathbf{P}_0 - \mathbf{P}_1 + \Delta\boldsymbol{\mu} - \Delta\mathbf{\Omega}\mathbf{1}/d) \text{ or}$$

$$[(d - d_1)\mathbf{\Omega}_0^{-1} + d_1 \mathbf{\Omega}_{11}^{-1}](\mathbf{P}_1 - \mathbf{P}_0) = d_1 \mathbf{\Omega}_{11}^{-1}(\Delta\boldsymbol{\mu} - \Delta\mathbf{\Omega}\mathbf{1}/d).$$

Multiplying through by $\mathbf{\Omega}_{11}$ and substituting, we have

$$[(d\mathbf{I} + (d - d_1)\Delta\mathbf{\Omega}\mathbf{\Omega}_0^{-1}](\mathbf{P}_1 - \mathbf{P}_0) = d_1(\Delta\boldsymbol{\mu} - \Delta\mathbf{\Omega}\mathbf{1}/d).$$

This establishes the price equation.

From above, $\mathbf{x}_{11}^* - \mathbf{x}_{10}^* = d_1 \mathbf{\Omega}_{11}^{-1}(\mathbf{P}_0 - \mathbf{P}_1) + d_1 \mathbf{\Omega}_{11}^{-1}(\Delta\boldsymbol{\mu} - \Delta\mathbf{\Omega}\mathbf{1}/d)$.

Also from above, this can be written as

$$\begin{aligned}&= -d_1 \mathbf{\Omega}_{11}^{-1}(\mathbf{P}_1 - \mathbf{P}_0) + [(d - d_1)\mathbf{\Omega}_0^{-1} + d_1 \mathbf{\Omega}_{11}^{-1}](\mathbf{P}_1 - \mathbf{P}_0) \\ &= (d - d_1)\mathbf{\Omega}_0^{-1}(\mathbf{P}_1 - \mathbf{P}_0).\end{aligned}$$

Finally, since we established above that for $i > 1$, $\mathbf{x}_{i1}^* - \mathbf{x}_{i0}^* = -d_i \mathbf{\Omega}_0^{-1}(\mathbf{P}_1 - \mathbf{P}_0)$, the proposition is proved.■

Proof of Lemma. We must show that $\left[\mathbf{I} - \alpha\binom{(x_1, \mathbf{x}'_{-1})}{\mathbf{v}_{-1}(z_1, z'_{-1})}\right]\left[\mathbf{I} + \alpha\binom{\mathbf{v}'\mathbf{\Omega}_0^{-1}}{\mathbf{v}_{-1}\boldsymbol{\omega}^{1\prime}}\right] = \mathbf{I}$. We begin by solving for $\mathbf{x}$. It must satisfy

$$-\begin{bmatrix}(x_1,\mathbf{x}'_{-1})\\ \mathbf{v}_{-1}(z_1,\mathbf{z}'_{-1})\end{bmatrix}+\begin{bmatrix}\mathbf{v}'\boldsymbol{\Omega}_0^{-1}\\ \mathbf{v}_{-1}\boldsymbol{\omega}^{1'}\end{bmatrix}-\alpha\begin{bmatrix}x_1\mathbf{v}'\boldsymbol{\Omega}_0^{-1}+(\mathbf{x}'_{-1}\mathbf{v}_{-1})\boldsymbol{\omega}^{1'}\\ \mathbf{v}_{-1}(z_1\mathbf{v}'\boldsymbol{\Omega}_0^{-1}+(\mathbf{z}'_{-1}\mathbf{v}_{-1})\boldsymbol{\omega}^{1'})\end{bmatrix}=\mathbf{0}.$$

It follows that

$x_1=\mathbf{v}'\boldsymbol{\omega}^1-\alpha x_1\mathbf{v}'\boldsymbol{\omega}^1-\alpha(\mathbf{x}'_{-1}\mathbf{v}_{-1})\boldsymbol{\omega}_1^1$

$\mathbf{x}_{-1}=\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v}-\alpha x_1\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v}-\alpha(\mathbf{x}'_{-1}\mathbf{v}_{-1})\boldsymbol{\omega}_{-1}^1.$

Similarly,

$z_1=\omega_1^1-\alpha z_1\mathbf{v}'\boldsymbol{\omega}^1-\alpha(\mathbf{z}'_{-1}\mathbf{v}_{-1})\boldsymbol{\omega}_1^1$

$\mathbf{z}_{-1}=\omega_{-1}^1-\alpha z_1\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v}-\alpha(\mathbf{z}'_{-1}\mathbf{v}_{-1})\boldsymbol{\omega}_{-1}^1.$

Pre-multiplying $\mathbf{x}_{-1}$ by the vector $\mathbf{v}'_{-1}$, we have

$(\mathbf{v}'_{-1}\mathbf{x}_{-1})=(1-\alpha x_1)(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\omega_{-1}^1)$ which implies

$\mathbf{x}_{-1}=(1-\alpha x_1)\left[\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v}-\alpha\frac{\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v}}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1}\boldsymbol{\omega}_{-1}^1\right]$, and

$x_1=\mathbf{v}'\boldsymbol{\omega}^1-\alpha x_1\mathbf{v}'\boldsymbol{\omega}^1-\alpha(1-\alpha x_1)\frac{\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v}}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1}\boldsymbol{\omega}_{-1}^1.$

Solving for x_1 yields

$x_1=\frac{\mathbf{v}'\boldsymbol{\omega}^1-\alpha\omega_1^1(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)}{1+\alpha[\mathbf{v}'\boldsymbol{\omega}^1-\alpha\omega_1^1(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)]}.$

We let $c=1+\alpha[\mathbf{v}'\boldsymbol{\omega}^1-\alpha\boldsymbol{\omega}_1^1(\mathbf{v}'_{-1}\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v})/(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)]$.

The development of $\mathbf{z}$ proceeds in the same fashion yielding

$z_1=\frac{\omega_1^1}{c(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)}$ and

$\mathbf{z}_{-1}=(\frac{1}{c(1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}_{-1}^1)})[(1+\alpha\mathbf{v}'\boldsymbol{\omega}^1)\boldsymbol{\omega}_{-1}^1-\alpha\omega_1^1\boldsymbol{\Omega}_{-1,0}^{-1}\mathbf{v}].$ ■

Proof of Proposition 3. From Proposition 2, we have $[\mathbf{I}+\frac{d-d_1}{d}\Delta\boldsymbol{\Omega}\boldsymbol{\Omega}_0^{-1}](\mathbf{P}_1-\mathbf{P}_0)=\frac{d_1}{d}(\boldsymbol{\Delta\mu}-\boldsymbol{\Delta\Omega}\mathbf{1}/d)$. From the Lemma, there exists an $\mathbf{M}$ such that

$$\mathbf{P}_1-\mathbf{P}_0=\frac{d_1}{d}\mathbf{M}(\boldsymbol{\Delta\mu}-\boldsymbol{\Delta\Omega}\mathbf{1}/d)\text{ for }\alpha=\frac{d-d_1}{d}\pi.$$

Substituting the values for $\Delta\boldsymbol{\mu}$ and $\Delta\boldsymbol{\Omega}$, we have

$$\begin{aligned}
\mathbf{P}_1 - \mathbf{P}_0 &= \frac{d_1\pi}{d}\mathbf{M}[me_1 - \frac{1}{d}\begin{pmatrix}\mathbf{v}'\mathbf{1}\\ \mathbf{v}_{-1}\end{pmatrix}\\
&= \frac{d_1\pi}{d}\mathbf{M}\begin{pmatrix}m - \mathbf{v}'\mathbf{1}/d\\ -\frac{1}{d}\mathbf{v}_{-1}\end{pmatrix}\\
&= \frac{d_1\pi}{d}\{\begin{pmatrix}m - \mathbf{v}'\mathbf{1}/d\\ -\frac{1}{d}\mathbf{v}_{-1}\end{pmatrix} - \alpha\begin{pmatrix}(x_1, \mathbf{x}'_{-1})\\ \mathbf{v}_{-1}(z_1, \mathbf{z}'_{-1})\end{pmatrix}\begin{pmatrix}m - \mathbf{v}'\mathbf{1}/d\\ -\frac{1}{d}\mathbf{v}_{-1}\end{pmatrix}\}\\
&= \frac{d_1\pi}{d}[\begin{pmatrix}(1-\alpha x_1)(m - \mathbf{v}'\mathbf{1}/d) + \alpha\frac{1}{d}\mathbf{x}'_{-1}\mathbf{v}_{-1}\\ -\mathbf{v}_{-1}[\alpha z_1(m - \mathbf{v}'\mathbf{1}/d)] + \frac{1}{d}(1 - \alpha\mathbf{z}'_{-1}\mathbf{v}_{-1})\end{pmatrix}].
\end{aligned}$$

Substituting from the proof of the Lemma, we have

$$\begin{aligned}
\mathbf{P}_1 - \mathbf{P}_0 &= \frac{d_1\pi}{cd}\begin{bmatrix}m - \mathbf{v}'\mathbf{1}/d + \alpha\frac{1}{d}(\mathbf{v}'_{-1}\boldsymbol{\Omega}^{-1}_{-1,0}\mathbf{v})/(1 + \alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})\\ -\mathbf{v}_{-1}[\frac{\alpha\omega^1_1}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}(m - \mathbf{v}'\mathbf{1}/d) + \frac{1}{d}(c + \frac{\alpha^2\omega^1_1}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}(\mathbf{v}'_{-1}\boldsymbol{\Omega}^{-1}_{-1,0}\mathbf{v}))]\end{bmatrix}\\
&= \frac{d_1\pi}{cd}\begin{bmatrix}m - \mathbf{v}'\mathbf{1}/d + \alpha\frac{1}{d}(\mathbf{v}'_{-1}\boldsymbol{\Omega}^{-1}_{-1,0}\mathbf{v})/(1 + \alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1})\\ -\mathbf{v}_{-1}[\frac{\alpha\omega^1_1}{1+\alpha\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}}(m - \mathbf{v}'\mathbf{1}/d) + \frac{1}{d}(1 + \alpha\mathbf{v}'\boldsymbol{\omega}^1)]\end{bmatrix}\\
&= \begin{bmatrix}g_1\\ -\mathbf{v}_{-1}g_2\end{bmatrix}.\blacksquare
\end{aligned}$$

Proof of Proposition 4. Given the homogeneous information set, the equilibrium solution would be $\mathbf{x}^*_{i2} = \frac{d_i}{d-d_1}(\mathbf{1} - \mathbf{x}^*_{11})$ and $\mathbf{P}_2 = \boldsymbol{\mu}_0 + \pi m\mathbf{e}_1 - \frac{1}{d-d_1}(\boldsymbol{\Omega}_0 + \pi\mathbf{V})(\mathbf{1} - \mathbf{x}^*_{11})$. But from the proof of Proposition 2, for $i > 1$, $\boldsymbol{\Omega}_0\mathbf{x}^*_{i1} = d_i(\boldsymbol{\mu}_0 - \mathbf{P}_1)$ which implies that $(\mathbf{1} - \mathbf{x}^*_{11})/(d - d_1) = \boldsymbol{\Omega}_0^{-1}(\boldsymbol{\mu}_0 - \mathbf{P}_1)$. Thus, $\mathbf{x}^*_{i2} = \frac{d_i}{d-d_1}(d - d_1)\boldsymbol{\Omega}_0^{-1}(\boldsymbol{\mu}_0 - \mathbf{P}_1) = d_i\boldsymbol{\Omega}_0^{-1}(\boldsymbol{\mu}_0 - \mathbf{P}_1) = \mathbf{x}^*_{i1}$.$\blacksquare$

Proof of Proposition 5. At time $t = 3$, all participants share the same information. Thus, we have two cases of a homogeneous equilibrium, differing only in the specification of the parameters of the distribution of prices. This specification, in turn, depends on the success or failure of the activist.$\blacksquare$

Proof of Proposition 6.

Part (a):

$$\begin{aligned} R(A) + R(G) &= (\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}^*_{11} - \mathbf{x}^*_{10}) + (\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}^*_{G1} - \mathbf{x}^*_{G0}) \\ &= (\mathbf{P}_3 - \mathbf{P}_1)'[(\mathbf{x}^*_{11} + \mathbf{x}^*_{G1}) - (\mathbf{x}^*_{10} + \mathbf{x}^*_{G0})]. \end{aligned}$$

Since $(\mathbf{x}^*_{11} + \mathbf{x}^*_{G1}) = (\mathbf{x}^*_{10} + \mathbf{x}^*_{G0}) = \mathbf{1}$, the result follows.

Part (b): Since $\Psi(A) = R(A) + \frac{d_1}{d}S$ and $\Psi(G) = R(G) + (1 - \frac{d_1}{d})S$, the proof follows from part (a).■

Proof of Proposition 7. We demonstrate this proposition as follows. We first propose a subjective probability distribution that summarizes A's belief in the consequences of his activism, should he proceed. We next show that this distribution satisfies the *CA* conditions and thus A proceeds with his activism. A consequence of this decision is that the value of the market, S, falls over the period of activism. Last, we show that should A be successful, he will benefit from his activism but G will not.

For the probability distribution, we propose the following parametric values. Let $\mathbf{\Omega}_0 = \begin{pmatrix} \mathbf{I}_{2x2} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{pmatrix}$ where $\mathbf{I}_{2x2}$ is the 2 x 2 identity matrix and $\mathbf{D}$ is a positive definite matrix. Let $\mathbf{v}$ be the vector with v_2 in its second position and zeros elsewhere, with $0 < v_2 < \frac{1}{\alpha}$. Let $v_2 < dm < 2v_2$.

It follows from this specification that $\mathbf{\Omega}_0 + \alpha\mathbf{V}$ is positive definite for any $0 < \alpha < 1$. Also, since $\mathbf{\Omega}_0^{-1} = \begin{pmatrix} \mathbf{I}_{2x2} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}^{-1} \end{pmatrix}$, we have $\boldsymbol{\omega}^1 = \mathbf{e}_1$ and $\boldsymbol{\omega}^2 = \mathbf{e}_2$ where $\mathbf{e}_2$ has a 1 in its second position and zeros elsewhere. Using these values to evaluate the parameters of Proposition 3, we have $c > 0$, $g_1 = \frac{d_1\pi}{cd}[m - \frac{1}{d}v_2 + \frac{\alpha}{d}v_2^2]$ and $g_2 = \frac{d_1\pi}{cd}[\alpha(m - \frac{1}{d}v_2) + \frac{1}{d}]$ and thus $g_1 > 0$ and $g_2 > 0$. We next address the *CA* conditions.

The condition *CA1* requires that the sign of $g_1\omega^1_1 - g_2\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1}$ be positive. But $g_1\omega^1_1 - g_2\mathbf{v}'_{-1}\boldsymbol{\omega}^1_{-1} = g_1 > 0$ thus satisfying *CA1*. The condition *CA2* requires that

$E_\pi(\mathbf{P}_3-\mathbf{P}_1)'(\mathbf{x}^*_{11}-\mathbf{x}^*_{10})+E_\pi(\mathbf{P}_3-\mathbf{P}_0)'\mathbf{x}^*_{10}$ be positive. This expression can be written as $\pi(\mathbf{P}^U_3-\mathbf{P}_1)'(\mathbf{x}^*_{11}-\mathbf{x}^*_{10})-(\mathbf{P}_1-\mathbf{P}_0)'(\mathbf{x}^*_{11}-\mathbf{x}^*_{10})+\frac{d_1\pi}{d}(\mathbf{P}^U_3-\mathbf{P}_0)'\mathbf{1}$. The vector $\mathbf{P}^U_3-\mathbf{P}_0 = m\mathbf{e}_1-\frac{1}{d}\mathbf{V1}$ which by Proposition 2 equals $\frac{1}{d_1\pi}[d\mathbf{I}+(d-d_1)\pi\mathbf{V}\boldsymbol{\Omega}_0^{-1}](\mathbf{P}_1-\mathbf{P}_0)$. It follows that $\pi(\mathbf{P}^U_3-\mathbf{P}_0)'(\mathbf{x}^*_{11}-\mathbf{x}^*_{10}) = \frac{d-d_1}{d_1}(\mathbf{P}_1-\mathbf{P}_0)'\boldsymbol{\Omega}_0^{-1}[d\mathbf{I}+\pi(d-d_1)\mathbf{V}\boldsymbol{\Omega}_0^{-1}](\mathbf{P}_1-\mathbf{P}_0) = \frac{d(d-d_1)}{d_1}(\mathbf{P}_1-\mathbf{P}_0)'\boldsymbol{\Omega}_0^{-1}(\mathbf{P}_1-\mathbf{P}_0)+\frac{\pi(d-d_1)^2}{d_1}(\mathbf{P}_1-\mathbf{P}_0)'\boldsymbol{\Omega}_0^{-1}\mathbf{V}\boldsymbol{\Omega}_0^{-1}(\mathbf{P}_1-\mathbf{P}_0)$. Subtracting $(\mathbf{P}_1-\mathbf{P}_0)'(\mathbf{x}^*_{11}-\mathbf{x}^*_{10})$ from this result and substituting the presumed values of the parameters yields $E_\pi(\mathbf{P}_3-\mathbf{P}_1)'(\mathbf{x}^*_{11}-\mathbf{x}^*_{10}) = \frac{(d-d_1)^2}{d_1}[g_1^2+g_2^2v_2^2-2\pi g_1g_2v_2] = \frac{(d-d_1)^2}{d_1}[(g_1-g_2v_2)^2+2(1-\pi)g_1g_2v_2]$. Finally, *CA2* will be satisfied if $\frac{(d-d_1)^2}{d_1}2(1-\pi)g_1g_2v_2 > -\frac{d_1\pi}{d}(\mathbf{P}^U_3-\mathbf{P}_0)'\mathbf{1} = -\frac{d_1\pi}{d}(m-\frac{2}{d}v_2) = \frac{d_1\pi}{d}(\frac{2v_2}{d}-m)$. As m increases to $\frac{2v_2}{d}$ the RHS of this inequality goes to zero while the LHS remains positive. Thus, there will be an $m<\frac{2v_2}{d}$ that satisfies *CA2* and for this value of m, $S=(\mathbf{P}^U_3-\mathbf{P}_0)'\mathbf{1}=m-\frac{2v_2}{d}<0$.

Finally, assume that at time $t=3$, A is successful in his endeavors. Since *CA2* is satisfied, $E_\pi(\mathbf{P}_3-\mathbf{P}_1)'(\mathbf{x}^*_{11}-\mathbf{x}^*_{10}) > -E_\pi(\mathbf{P}_3-\mathbf{P}_0)'\mathbf{x}^*_{10}$. But $E_\pi(\mathbf{P}_3-\mathbf{P}_1)'(\mathbf{x}^*_{11}-\mathbf{x}^*_{10}) = \pi(\mathbf{P}^U_3-\mathbf{P}_1)'(\mathbf{x}^*_{11}-\mathbf{x}^*_{10})-(1-\pi)(\mathbf{P}_1-\mathbf{P}_0)'(\mathbf{x}^*_{11}-\mathbf{x}^*_{10})$. Substituting $(d-d_1)\boldsymbol{\Omega}_0^{-1}(\mathbf{P}_1-\mathbf{P}_0)$ for $(\mathbf{x}^*_{11}-\mathbf{x}^*_{10})$ using Proposition 2, we have that $E_\pi(\mathbf{P}_3-\mathbf{P}_1)'(\mathbf{x}^*_{11}-\mathbf{x}^*_{10}) < \pi(\mathbf{P}^U_3-\mathbf{P}_1)'(\mathbf{x}^*_{11}-\mathbf{x}^*_{10})$ and therefore $\pi(\mathbf{P}_3-\mathbf{P}_1)'(\mathbf{x}^*_{11}-\mathbf{x}^*_{10}) > -\frac{d_1\pi}{d}(\mathbf{P}^U_3-\mathbf{P}_0)'\mathbf{1}$. Dividing through by π yields $R(A) > -\frac{d_1}{d}S > 0$ or that $\Psi(A)>0$ if A succeeds. Furthermore, by Proposition 6, $R(G)=-R(A)$. Since $\Psi(G)=-R(A)+\frac{d-d_1}{d}S$ and $S<0$ it follows that $\Psi(G)<0$. So A benefits and G does not.■

Proof of Proposition 8.

Part (a): $\Psi(A)>0$ and $\Psi(G)>0$ iff $R(A)>-\frac{d_1}{d}S$ and $R(G)>-(1-\frac{d_1}{d})S$, respectively. But since $R(G)=-R(A)$, $\Psi(G)>0$ iff $R(A)<(1-\frac{d_1}{d})S$. Thus, $\Psi(A)>0$ and $\Psi(G)>0$ iff $-\frac{d_1}{d}S<R(A)<(1-\frac{d_1}{d})S$.

Parts (b), (c) and (d): The results of these sections follow by applying the same argument used in part (a) to these three cases.■

Proof of Proposition 9.

Part (a1): We assumed that for activism to begin, *CA* was satisfied. In particular, we assumed *CA2* that $E_\pi(\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}_{11}^* - \mathbf{x}_{10}^*) + E_\pi(\mathbf{P}_3 - \mathbf{P}_0)'\mathbf{x}_{10}^* > 0$. But this inequality is just $E_\pi\Psi(A) > 0$, i.e, the expectation taken before it becomes known whether or not the activist will be successful. For this expectation to be positive, it follows that $\Psi(A) > 0$ when A is successful. Also, when $\Psi(A) > 0$, then $R(A) > -\frac{d_1}{d}S$. Using Proposition 7, parts (a2) and (a3) follow.

Parts (b1 and b2): If A is unsuccessful, then $\mathbf{P}_3 = \mathbf{P}_0$. Thus, $S = 0$ and $\Psi(A) = -\Psi(G)$. Also, when $S = 0$, $\Psi(A) = (\mathbf{P}_3 - \mathbf{P}_1)'(\mathbf{x}_{11}^* - \mathbf{x}_{10}^*) = -(\mathbf{P}_1 - \mathbf{P}_0)'(\mathbf{x}_{11}^* - \mathbf{x}_{10}^*) = -(d - d_1)(\mathbf{P}_1 - \mathbf{P}_0)'\mathbf{\Omega}_0^{-1}(\mathbf{P}_1 - \mathbf{P}_0) < 0$ since $\mathbf{\Omega}_0$ is positive definite and the results follow.■

Proof of Proposition 10.

Part (a): By assumption, $\Psi(A + G) = R(A) + R(D) + R(G) + D(G) = \Psi(A) + \Psi(G)$. But $\Psi(A) + \Psi(G) = S$ by Proposition 6, so part (a) follows.

Part (b): If $\Psi(A + G) > 0$, then $S > 0$ by part (a). When $S > 0$, part (d) of Proposition 7 cannot hold and thus part (b) follows.

Part (c): From part (b), the first three parts of Proposition 7 can hold. However, the exclusion of the case $-\frac{d_1}{d}S < R(A) < (1 - \frac{d_1}{d})S$ disallows part (a) of Proposition 7 and the result follows.

7 References

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