# "Imbalances" for the long run<sup>\*</sup>

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#### Abstract

Net exports and current account balances among developed countries, which contributed to the so called "global imbalances", are highly persistent. Despite success along many dimensions, international business cycle models have difficulty replicating these salient, low-frequency features of international capital flows. In particular, net exports and current account balances are much more persistent in the data than in standard models. We document these important empirical facts about international capital flows. Further, we show that we can account for them with a parsimonious one-good two-country model with small, persistent differences in per capita GDP growth, matching those we observe among developed countries.

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# 1 Introduction

Over recent years, the U.S. current account deficit has received considerable attention. With both the U.S. trade and current account deficits growing to higher levels, economists have tried to come up with new models, as standard international models failed to account for the persistence of large external imbalances. We do not provide a new model here, but show that a proper specification of the technology process, consistent with the data on cross-country productivity differences is both necessary and sufficient to generate external imbalances similar to what is observed in the data, in a very simple international business cycle model. Large and persistent trade balances arise as an optimal outcome of the model, that should alleviate concerns about the sustainability of the U.S. deficit.

Standard models of the current account have trouble accounting for the U.S. situation. On the one hand, small-open economy models, which traditionally supports the intertemporal approach to the current account, are ill-suited to look at the case of an economy, which represents about one third of the world's GDP. Besides the transversality condition assumed by these models which requires that net foreign asset positions converge to zero when time goes to infinity is of little help to assess the sustainability of large and persistent current account imbalances in the short to medium run. General equilibrium international business cycles models like Backus, Kehoe and Kydland (1992) on the other hand completely ignore the low frequency features of the data and of net exports and the current account in particular.

In the next section, we document some key facts about current accounts in industrialized economies that highlight the importance of low frequency movements. Not only can external imbalances be large they are also very persistent.

In order to account for this persistence, we start with a simple frictionless twocountry model with complete markets and systematically explore possible deviations from that benchmark. Neither the introduction of frictions, like adjustment costs or trade costs, nor the relaxation of the complete markets assumption have any substantial impact on the persistence of net exports.

The failure of the model to generate persistent imbalances and the observation that these imbalances arise while countries exhibit persistent productivity and output growth differences, motivate our focus on technology. We suggest a specification of the technology process that includes a transitory shock and a labor-augmenting technology "growth" shock. The transitory shock is modelled by a stationary autoregressive process as is standard in the real business cycle literature. Labor-augmenting technology follows a random walk whose drift evolves over time according to a Poisson process. This allows us to account for the persistent cross-country differences in total factor productivity growth rates we observe in the data. We estimate the parameters of this specification using the frequency domain optimization method of Levy (2003) and historical data on per capita GDP growth differences. We then show that for these parameters values the model delivers current account imbalances whose persistence matches that of the data.

The form of the technology process is crucial to obtaining these results. In the absence of persistent technology growth differences across countries, the model will only yield transitory current account imbalances. The reason is that investment is ultimately the main driver of the current account in the model. In a complete markets set-up, capital responds to differences in technology levels by flowing across countries to equalize expected marginal returns. As technology grows faster in one country than in the other, the fast growing country keeps on attracting foreign investment flows every period, which implies a current account deficit. This deficit will not reverse unless growth differences are resorbed.

This result is consistent with the conclusion of Engel and Rogers (2006) who show that expectations of higher growth in the U.S. than in other advanced economies can generate the observed U.S. current account deficit. It is similar to the result obtained by Caballero, Farhi and Gourinchas (2006) in an endowment economy. Their model however does not incorporate any consumption/saving decision. Besides it is used to emphasize cross-country differences in financial development as the main force driving current account imbalances, which we ignore here. Our explanation of current account imbalances also differs from Mendoza, Quadrini and Ríos-Rull (2006) who too focus on structural differences in financial markets characteristics, and from Fogli and Perri (2006) who consider the effect of the reduction in business cycle volatility in the United States in a model similar to ours with incomplete markets and an ad-hoc borrowing limit.

The rest of the paper is organized as follows. In Section 2, we document both the size and persistence of current accounts and net exports in developed economies. Section 3 presents the benchmark model and examines several ways to generate persistent current accounts. Section 4 provides empirical evidence for a technology growth process allowing for persistent growth differences, estimates the parameters of that process and shows how it helps to reconcile the theory with the data. Section 5 concludes.

## 2 Current accounts in developed economies

Contrary to some widespread belief, current account balances in developed economies can be both large and persistent.

## 2.1 Current account balances are large

Table 1 reports the largest trade and current account deficits and surpluses in OECD countries. In 2008, the ratio of the current account balance in absolute value to GDP for all thirty OECD countries was on average 6.6%. Half the countries had a current account balance of more than 5% of GDP in absolute value, and four of them had a balance above 10% of GDP. With a current account deficit of 4.7% of GDP, the United States ranks only tenth among OECD countries with deficits.

Net exports account for a large part of current account balances, which also include income flows and current transfers. Eight out of ten countries with the largest current account deficits are also among the ten countries with the largest trade deficits. The same is true for the countries with the largest surpluses.

The magnitude of these balances is reflected in the outstanding stocks of external assets and liabilities. At the end of 2007, net foreign asset positions in absolute value averaged 48.3% of GDP among OECD countries. In four cases, net foreign assets (Switzerland and Luxembourg) or liabilities (Iceland and Greece) were larger than 100% of GDP.

True, external balances have increased over the last ten years. Figure 1 shows that the cross-sectional dispersion of both net exports and current account balances to GDP ratios has increased since the 1960's. However, as stressed in Backus, Henriksen, Lambert and Telmer (2009), these balances are not unprecedented. Looking at historical series for a small sample of countries (Australia, Canada, France, Japan, the United Kingdom and the United States), we found that current account deficits or surpluses over 5% of GDP occurred more than 18% of the time (based on available observations prior to 1960), with balances above 10% of GDP in nearly 15% of the cases.

## 2.2 Current account balances are persistent

Current account imbalances are not a temporary feature either. Australia for instance has been running a current account deficit since 1861 except for 29 years. More than 25% of the time (41 years out of 148), the deficit was greater than 5% of the Australian GDP, and on average over the last 30 years, it amounted to 4.4% GDP. Canada has also been running a deficit for most of the 20th century, even if the situation has reversed over the past few years.

Figure 2 provides some evidence of the persistence of both net exports and current account imbalances. The correlations coefficients for current account balances are slightly higher than for net exports. The main reason is that income flows tend to increase the persistence of current accounts. Other things being equal, countries running large trade deficits (surpluses) for several years will accumulate debt (assets) that will generate income flows in the subsequent periods even after the trade balance has reverted back to zero.

The persistence of current account balances is best seen in the frequency domain. Figure 3 plots the autocorrelation function (ACF), the periodogram and the spectrum of the ratio of annual current account balances to GDP for six countries for which we have data over more than a hundred year. See Appendix B for a description of the way these functions are computed.

For each country, the first graph represents the sample autocorrelation function (or correlogram) of current account over GDP at all lags between 1 and 50, along with the 95% confidence interval (the band between the two horizontal lines at  $\pm 2/\sqrt{n}$ , *n* being the number of observations for each country). The slow decay pattern suggests a very persistent autoregressive process, as sample autocorrelations appear significantly different from zero even for high lags.

The picture for the autocorrelation function is consistent with the two other graphs. These plot the raw periodogram and its smoothed counterpart, or spectrum estimate, for frequencies between 1/n and 1/2 (the frequencies are expressed in cycles per quarter). Both functions provide a decomposition of the variance of the series by frequency. While there appear to be some short-run (high frequency) variation, the graphs are clearly dominated by low frequency components, which can be interpreted as evidence of high persistence.

Figure 4 presents similar graphs for net exports over GDP. The picture is essentially the same as for current account balances. Income flows may actually increase the persistence of current account balances compared to net exports, as income payments depend on accumulated past trade balances.

We have shown that both trade and current account balances can be large and persistent at the same time. We now examine how these features of the data are accounted for in a standard international dynamic general equilibrium model.

# 3 The benchmark model

#### 3.1 A one-good two-country model with complete markets

Our benchmark model is the standard two-country extension of the neoclassical growth model. The countries are indexed by i = 1, 2. Each country is represented by one firm, which owns capital and makes investment decisions, and one household. Time is discrete.

#### 3.1.1 Environment

In each period, an event  $s_t$  drawn from some set S occurs that is observed by all agents in the economy. The history of events up to period t is denoted by  $s^t = [s_0, s_1, ..., s_t]$ . The unconditional probability of a particular history of events  $s^t$ is given by  $\pi(s^t)$ .

The production function in the two countries is of the Cobb-Douglas form

$$y_{i,t}(s^t) = e^{z_{i,t}(s^t)} k_{i,t}(s^{t-1})^{\alpha} n_{i,t}(s^{t-1})^{1-\alpha}, \qquad (1)$$

where  $z_{i,t}(s^t)$  is a country-specific technology shock. For simplicity, we assume that labor supply  $n_{i,t}$  is fixed and equal to one in each country.

Capital is assumed to be internationally mobile and accumulates according to

$$k_{i,t+1}(s^t) = (1-\delta)k_{i,t}(s^{t-1}) + x_{i,t}(s^t),$$
(2)

where  $x_{i,t}(s^t)$  denotes gross investment.

Preferences have the standard additive expected utility structure

$$U_i(c_i) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u_i\left(c_{i,t}(s^t)\right),\tag{3}$$

where the one-period utility function is the same for the two countries and exhibits constant relative risk aversion  $u_i(c) = c^{1-\gamma}/(1-\gamma)$ .

With complete markets, the budget constraint of country i in period t is given by

$$c_{i,t}(s^{t}) + x_{i,t}(s^{t}) + \sum_{s_{t+1}} \tilde{q}(s_{t+1}, s^{t}) a_{i,t+1}(s_{t+1}, s^{t}) \le y_{i,t}(s^{t}) + a_{i,t}(s_{t}, s^{t-1})$$
(4)

where  $a_{i,t+1}(s_{t+1}, s^t)$  denote the number of Arrow securities purchased in period tat history  $s^t$  that pay one unit of the consumption good in period t+1 if state  $s_{t+1}$ is realized and  $\tilde{q}(s_{t+1}, s^t)$  is the price of such securities, expressed in units of the consumption good in period t.

The aggregate resource constraint for the world economy is

$$\sum_{i=1,2} \left[ c_{i,t}(s^t) + x_{i,t}(s^t) \right] = \sum_{i=1,2} y_{i,t}(s^t).$$
(5)

#### 3.1.2 Equilibrium

Since the utility functions are concave, we can solve the social planner's problem for the equilibrium allocation. The Pareto problem (with equal Pareto weights on the two countries) can be written as:

Choose  $\{c_{1,t}(s^t), c_{2,t}(s^t), x_{1,t}(s^t), x_{2,t}(s^t)\}_{t=0}^{\infty}$  to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \left[ \frac{c_{1,t}(s^t)^{1-\gamma}}{1-\gamma} + \frac{c_{2,t}(s^t)^{1-\gamma}}{1-\gamma} \right]$$

subject to the aggregate budget constraint (5) and the laws of motion for capital stocks in the two countries (2) for all t, for all  $s^t$ .

The first-order conditions yield the well-known results for a complete markets economy with full risk-sharing:

• Consumption is the same in the two countries:

$$c_{1,t}(s^t) = c_{2,t}(s^t) = \frac{1}{2} \sum_{i=1,2} \left[ y_{i,t}(s^t) - x_{i,t}(s^t) \right] \equiv c_t(s^t) \tag{6}$$

• Capital flows across countries to equalize expected returns on capital:

$$\beta E_t \left\{ \left( \frac{c_{t+1}(s^{t+1})}{c_t(s^t)} \right)^{-\gamma} \left( \alpha e^{z_{i,t+1}(s^{t+1})} k_{i,t+1}(s^t)^{\alpha-1} + 1 - \delta \right) \right\} = 1, \quad i = 1, 2$$
(7)

#### 3.1.3 Net exports and current account

Net exports are defined as

$$nx_{i,t}(s^t) = y_{i,t}(s^t) - x_{i,t}(s^t) - c_{i,t}(s^t).$$
(8)

The current account is the sum of net exports, net income flows and current transfers. This definition is consistent with the one used to compute balance of payments statistics (or national accounts) and does not include changes in asset prices or "valuation effects", which may be recorded in international investment position statistics. Under complete markets, current account so defined is always equal to zero, as insurance flows (recorded as current transfers) completely offset net exports. Hence we focus on net exports, the only meaningful aggregate in this set-up.

Besides "technical" considerations, there is another reason that justifies our focus on net exports. As we have seen in the previous section, the persistence as well as the magnitude of current account balances are closely linked to those of net exports. Then accounting for the latter is a necessary step toward explaining the former.

#### 3.1.4 Calibration

One period in the model corresponds to one year. We follow Backus et al. (1992) for the calibration of the model's parameters and adjust the values of the parameters to fit the model's annual frequency. The coefficient of relative risk aversion is set at  $\gamma = 2$ . The discount factor is adjusted to match a steady-state real interest rate of 4% per year. This implies  $\beta = 0.96$ .

As regards technology parameters, labor's share is  $1-\alpha = 0.64$ . The depreciation rate is set equal to the average of the ratio between consumption of fixed capital and fixed assets in the US over the last fifty years, which yields  $\delta = 0.042$ .

The technology shocks are modelled as a bivariate autoregression

$$\boldsymbol{z}_t = \Lambda \boldsymbol{z}_{t-1} + \boldsymbol{\varepsilon}_t \tag{9}$$

where  $\boldsymbol{z}_t = [z_{1,t}, z_{2,t}]'$  and  $\boldsymbol{\varepsilon}_t = [\varepsilon_{1,t}, \varepsilon_{2,t}]' \sim N(\boldsymbol{0}, \Sigma).$ 

The elements of  $\Lambda$  and  $\Sigma$  are estimated by running a VAR of order one on estimates of Solow residuals in logarithm for the United States and an aggregate of thirteen OECD countries (Australia, Belgium, Canada, Spain, France, Italy, Japan, the Netherlands, Austria, Portugal, Sweden, Switzerland and the United Kingdom). The logarithms of Solow residuals are computed from aggregate data on output, capital stock and employment, and normalized so that the mean of  $e^z$  is one. The sample period is 1970-2009 (see Appendix A for details).

The parameters in equation (9) are estimated by least squares using annual residuals for the United States and our OECD aggregate. They imply persistent technology shocks (the diagonal elements of  $\Lambda$  are close to 0.9) that are also positively correlated across countries. This calibration also allows for technology spillovers across countries. We use a "symmetrized" version of these estimates by constructing a symmetric coefficient matrix with the same eigenvalues as the estimated  $\Lambda$ . All parameter values are summarized in Table 3.

Backus et al. (1992) solve the model using a quadratic approximation of the social planner's objective function around the steady-state after substituting the nonlinear constraints. This is the standard way to proceed in the real business cycle literature. However the approximate optimal decision rules so obtained become less accurate as the economy moves far away from its steady-state. Therefore, we use a weighted residual method (McGrattan, 1999) and approximate the decision rules with Chebyshev polynomials using collocation. Though less accurate than the Galerkin method for this type of models according to Heer and Maussner (2004), the collocation procedure runs much faster, especially as the dimension of the state-space increases. We use Heer and Maussner's modified Newton-Raphson algorithm to solve for the vector of Chebyshev coefficients. The expectation on the left-hand side of the Euler equation (7) which we use as our residual function is computed with a monomial rule.

#### 3.1.5 Results

Following Watson (1993), we assess the performance of the model using spectral methods. To do so, we generate 1000 simulations of the one-good economy, each of 137 periods, and compute the average periodogram and spectrum of net exports over GDP. We then compare the implied spectrum to the ones we computed from

the data for G7 countries. The left panel of Figure 5 plots the average spectrum from simulated data (plain line) and the spectrum from US data (dotted line) as an illustration.

From this comparison, it is obvious that the model fails to capture the low frequency movements observed in the data. The flatness of the artificial spectrum contrasts with the peak at very low frequencies observed in the data.

Figure 6 plots the impulse response function of output, investment, consumption and net exports to a technology shock in one country. Right after the shock (in the same period), investment increases sharply in country 1, while it decreases in country 2 as capital flows from one country to the other to equalize expected marginal returns. The technology shock triggers an increase in output in country 1 which persists one period after the shock as capital has accumulated. The increase in consumption is much smaller than that of output in country 1, so that the trade deficit is mainly driven by changes in investment. The trade balance reverts to a surplus in the following period as investment drops, and quickly returns to zero. Hence the absence of persistent imbalances in the model.

Trade imbalances are also quite variable in this frictionless environment. The average standard deviation of the net exports to GDP ratio is 3.3% against 2.0% in the data for the United States and this result traditionally motivates the introduction of some kind of friction in the model. We therefore turn to common modifications of the model and investigate their effect on the persistence of net exports.

## **3.2** Adjustment costs

Introducing adjustment costs is standard in the literature and motivated by the need for some type of friction to dampen the volatility of investment (and therefore net exports) in response to technology shocks (see Backus et al. (1992) and Baxter and Crucini (1993)). If adjustment costs help reducing investment fluctuations, which were shown in the previous section to drive net exports, then one might expect that adjustment costs also play a role in the persistence of net exports by smoothing capital flows over time.

We model adjustment costs in the following way:

$$\psi(k_{i,t+1}(s^t), k_{i,t}(s^{t-1})) = \varphi \frac{(k_{i,t+1}(s^t) - k_{i,t}(s^{t-1}))^2}{k_{i,t}(s^{t-1})}$$
(10)

Thus only *net* adjustments to the capital stock are costly. Note that this specification ensures that production net of investment and adjustment costs is CRS.

We tried different values for the parameter  $\varphi$  ranging from 0.1% to 2%, without any observable effect on the spectrum for net exports over GDP. Larger adjustment costs limit international capital flows without increasing their persistence.

## 3.3 Incomplete markets

Although the recent years have witnessed a huge development of international financial markets with the diffusion of new financial products, thereby providing more empirical support to the complete markets assumption, there remain some obstacles to international asset trade. If due to these frictions country-specific shocks cannot be fully insured, countries have a motive for precautionary saving. Faced with a good shock, countries accumulate foreign assets in provision of less favorable times. Hence the idea that current account imbalances should be more persistent in models with incomplete markets or limited commitment.

To test this hypothesis, we consider the most parsimonious form of market incompleteness by restricting the asset structure to a one-period risk-free bond. Country i's budget constraint becomes

$$c_{i,t}(s^{t}) + x_{i,t}(s^{t}) + \frac{b_{i,t+1}(s^{t})}{R_{t,t+1}(s^{t})} \le y_{i,t}(s^{t}) + b_{i,t}(s^{t-1}),$$
(11)

where  $b_{i,t+1}(s^t)$  denotes the number of risk-free bonds bought by country *i* in period *t* that pay one unit of consumption for sure next period and  $R_{t,t+1}(s^t)$  is the gross interest rate between periods *t* and *t* + 1, known in period *t* (so that  $1/R_{t,t+1}(s^t)$  is the price of a bond in period *t*).

While as expected bond holdings are very persistent, the spectrum for net exports over GDP remains flat. Intuition for this result is given by Baxter and Crucini (1995). As mentioned earlier the technology process specified by Backus et al. (1992) allows for spillovers across countries, so that technology shocks spread across countries over time. *In fine* most fluctuations in productivity are common across countries. Then what really matters in terms of insurance and consumption smoothing is the ability of individuals to smooth consumption across time rather than across different states, which is precisely what the risk-free bond can be used for. It is thus

not surprising that as regards international capital flows and net exports the bond economy looks similar to the complete markets one.

We did not impose any borrowing constraints (except for transversality conditions) when solving the model. However considering the results of the simulations from that model, it seems only a tight constraint would be sometimes binding. The levels of net liabilities reached by several countries in 2007 (Greece -101%, Portugal -98%, Hungary -97.5%) do not support such an assumption.

One way to have incomplete markets matter may be to introduce preference shocks, as in Stockman and Tesar (1995). What we have in mind is the case of a country which is spending a lot, e.g. to host the Olympic Games, thereby building up external debt that it has to pay back for many periods afterwards. While this may generate current account persistence thanks to persistent income flows, it is hard to reconcile with other facts that we also looked at like the persistent differences in per capita GDP across countries. Moreover it is not clear that this would generate any persistence in net exports.

## 4 Long run differences in technology growth

## 4.1 Evidence

As emphasized by several statistical agencies, there are differences in long-run average growth rates among advanced countries, both in time-series and in cross-section. Bassanini and Scarpetta report in an OECD study that a few countries have experienced an acceleration in per capita GDP growth while other major economies were lagging behind. The 2007 Economic Report of the President contains similar observations. Between 1995 and 2005, per capita GDP and its components like labor productivity have grown faster in the United States than in most other advanced industrialized country. Further, the report notes that productivity growth accelerated in the United States while it was slowing down in other major industrialized countries between 2000 and 2005.

Table 4 displays 10-year average productivity growth rates for G7 countries since 1970. These figures confirm that productivity in the United States, as measured by the Solow residuals, has been growing much faster on average than in all other G7 countries with the exception of the United Kingdom since 1990.

While we acknowledge that our measure of total factor productivity may be inaccurate, the overall picture is consistent with that obtained by looking at per capita GDP growth rates, as illustrated by Table 5.

These findings motivate us to investigate whether our benchmark model with a more carefully calibrated technology process could account for the persistence of international capital flows as well as the persistence of cross-country GDP growth rates.

## 4.2 Model specification

Following Aguiar and Gopinath (2007), we rewrite the production function as:

$$y_{i,t} = e^{z_{i,t}} k_{i,t}^{\alpha} (G_{i,t} n_{i,t})^{1-\alpha}, \quad i = 1,2$$
(12)

where  $z_{i,t}$  is a transitory shock following a stationary autoregressive process and  $G_{i,t}$  denotes the cumulative product of labor-augmenting "growth" shocks. In particular,

$$G_{i,t} = e^{g_{i,t}} G_{i,t-1}, \quad i = 1,2$$
(13)

where  $g_{i,t}$  is a growth shock generated by some autoregressive process to be specified.

Alternative modelling approaches include models allowing for both permanent and transitory innovations in each period (e.g. Quah (1990)) or a regime-switching model in the tradition started by Hamilton (1989). However, we believe that the above specification is both very intuitive and easy to deal with whereas it captures the essential features of data.

While Aguiar and Gopinath (2007) used this specification in the framework of a small-open economy model, we are looking at a two-country model. Then we need to specify the relationship between the technology processes in the two countries. In particular we restrict ourselves to processes which exhibit long-run absolute convergence, a prediction of the neoclassical growth model which has received some empirical support for OECD countries (Barro and Sala-i Martin, 1992).

We assume that  $\ln(G_{1,t})$  and  $\ln(G_{2,t})$  are cointegrated with cointegrating vector [1, -1]. Let  $u_t \equiv \ln(G_{2,t}) - \ln(G_{1,t})$ .

$$u_t = \tau \mu_t + \rho^u u_{t-1} + \varepsilon_t^u \tag{14}$$

where  $\mu_t$  is a random variable drawn from a standard normal distribution whose value changes with probability  $\lambda > 0$  every period, and  $\varepsilon_t^u$  is the innovation to the technology trend difference ( $\varepsilon_t^u \sim N(0, \sigma_u)$ ). Different values of the variable  $\mu$  can be interpreted as corresponding to different technology epochs, in which one country is growing faster than the other. This specification thus captures the idea that countries can diverge, rather than converge, for some periods of time, while keeping the overall process consistent with absolute convergence in the very long run, as the unconditional mean of  $u_t$  is zero.

Given the assumed cointegration relationship, we can solve the model by normalizing all variables in the model by the common technology trend  $G_{1,t-1}$  to ensure stationarity. For any variable x, let  $\hat{x}$  denote its detrended counterpart:

$$\hat{x}_t \equiv \frac{x_t}{G_{1,t-1}}.\tag{15}$$

This normalization is inocuous for our purpose. What indeed matters for net exports or current account is the difference in technology  $\ln(G_{2,t}) - \ln(G_{1,t})$ . In steady-state where  $\bar{G} = 1$  in both countries, there is no current account imbalance.

With normalized variables, the Euler equations used to solve the model become (we dropped the history-dependent notation):

$$\beta e^{-\gamma g_{1,t}} E_t \left\{ \left( \frac{\hat{c}_{t+1}}{\hat{c}_t} \right)^{-\gamma} \left( \alpha e^{(z_{1,t+1}+(1-\alpha)g_{1,t+1})} \hat{k}_{1,t+1}^{\alpha-1} + 1 - \delta \right) \right\} = 1$$
(16)

$$\beta e^{-\gamma g_{1,t}} E_t \left\{ \left( \frac{\hat{c}_{t+1}}{\hat{c}_t} \right)^{-\gamma} \left( \alpha e^{(z_{2,t+1}+(1-\alpha)g_{2,t+1})} e^{(1-\alpha)u_t} \hat{k}_{2,t+1}^{\alpha-1} + 1 - \delta \right) \right\} = 1 \quad (17)$$

where  $u_t$  is defined by Equation (14).

## 4.3 Estimation

The difference in the logarithms of technology levels l between the two country is given by:

$$tfp_{2,t} - tfp_{1,t} = z_{2,t} - z_{1,t} + (1 - \alpha)u_t$$
(18)

The question is how to pin down the processes for  $u_t$  and  $z_{i,t}$ , i = 1, 2. Let  $z_{i,t} = \rho^z z_{i,t-1} + \varepsilon_{i,t}^z$ . In the model, net exports are ultimately driven by TFP differences. For simplicity we can then fix the technology level in country 1 to one,  $e^{z_{1,t}}G_{1,t}^{1-\alpha} = 1$  for all t, and let  $g_{2,t} = u_t - u_{t-1}$ . Thus we still need to estimate six parameters:  $\rho^z$  (autoregressive coefficient of the transitory shock),  $\sigma_z$  (standard deviation of the transitory shocks),  $\rho^u$  (autoregressive coefficient of the growth shock),  $\tau$  (standard deviation of the technological shifts),  $\lambda$  (probability of a technological shift) and  $\sigma_u$  (standard deviation of the innovations to the technology trend differences). Let  $\theta = (\rho^z, \sigma_z, \rho^u, \tau, \lambda, \sigma_u).$ 

The parameters are estimated using the frequency domain optimization method of Levy (2003). In particular, we select the parameter values that minimize the distance between the data and the model spectra on per capita GDP growth differences. The metric we use is given by

$$d(\theta) = \int_{-\pi}^{\pi} \xi(\omega) |f_{data}(\omega) - f_{model}(\omega)| d\omega$$
(19)

where  $\xi(\omega)$  is a frequency weighting function, and  $f_{data}(\omega)$  and  $f_{model}(\omega)$  are the spectral densities of per capita GDP growth differences between the US and an aggregate of eighteen OECD countries (the same thirteen countries for which we estimated Solow residuals plus Denmark, Finland, Germany, Norway and New Zealand) over 137 years (1870-2006). The data come from Maddison (2009). We choose a proportional weighting function by requiring that each frequency be given a weight proportional to its contribution to the series' total variance:

$$\xi(\omega) = \frac{f_{data}(\omega)}{\int_{-\pi}^{\pi} f_{data}(\omega)d\omega}$$
(20)

This ensures that the model fits the data well at those frequencies that contribute most to the fluctuations in the data.

The estimated parameter values are reported in Table 6. Other parameter values remain unchanged (see Table 3). The estimated technology process implies a simulated spectrum for per capita gdp differences very similar to the one computed from the data, as shown in Figure 7.

#### 4.4 Results

Figure 8 plots the average ACF and normalized spectrum of the ratio of net exports over GDP from a large number of simulations of 137 periods of the model given the estimated technology process. Crucial for the success of the model is the persistence of growth differences implied by our specification of the technology process. Focussing on AR(1) processes, as is done in Backus et al. (1992) and other international business cycle studies, ignores most low frequency movements in technology. Part of the reason why the various extensions of the benchmark model failed to reproduce the low frequency dynamics in net exports is because these extensions do not provide sufficiently strong propagation mechanisms for high-frequency shocks. In a one-country set-up, a similar point was emphasized by Cogley and Nason (1995). Our results show that a more careful specification of the technology process which captures the observed long-run growth differences across countries is both necessary and sufficient to get the spectrum of the trade balance right.

# 5 Conclusion

Three things should be taken away from this paper. First, the paper stresses the importance of the low frequency features of the data on net exports and current accounts that have been neglected by the recent international business cycle literature. Thereby we stick to the sometimes overlooked original objective of real business cycles models to provide a consistent framework to account for both longrun movements and business cycle fluctuations. Second, we show that deviations from the frictionless, complete-markets environment are not sufficient to account for the persistence of external imbalances and that carefully specifying a technology process consistent with the low frequency features of the data on productivity and per capita GDP is required to obtain such a result. We argue that long-run shifts in technology more than transitory shocks are what matter for current account dynamics, and show that once these shifts are taken into account, current account imbalances can be both large and very persistent, as is the case in the United States today. In fact, and this is our third main result, given the observed persistence of productivity and per capita GDP differences, net exports over GDP in the model are substantially larger than what is observed.

We leave many questions open. In particular, we did not address the origins of these long-run productivity growth differences. In our model with fixed labor supply, technology is a convenient concept that can be interpreted in various ways and its process may capture both "pure" technological changes as well as changes in labor supply for instance. Trying to endogenize technology has been the subject of a lot of research. Looking at this issue from an international perspective might provide new insights.

# Appendix A : Data sources

Current account, net exports and GDP data after 1960 were obtained from Datastream (national sources) and the OECD Quarterly National Accounts Database. Historical series are from Backus et al. (2009).

The expression used to compute the logarithms of Solow residuals in country i at time t is derived from the production function (1):

$$z_{i,t} = \log y_{i,t} - \alpha \log k_{i,t} - (1 - \alpha) \log n_{i,t}, \qquad (A-1)$$

where for notational concision we dropped the history-dependent notation.  $n_{i,t}$  is set equal to total employment, as comprehensive hours series are missing for many countries.  $k_{i,t}$  is total capital stock, in real terms.  $y_{i,t}$  is real GDP. This measure of Solow residuals differs from the one used in Backus et al. (1992), as it takes capital stock into account. To construct OECD countries aggregates for output and capital stock, we converted both GDP and capital stocks into US dollars using 2005 purchasing power parity data from Heston, Summers and Aten, Penn World Table Version 6.3. We used the same labor shares $(1 - \alpha)$  for all countries. Except for PPP data, all series come from the OECD Economic Outlook database.

# Appendix B : Tools for the spectral analysis of time series

Consider the covariance-stationary series  $\{x_t\}_{t=0}^{n-1}$  with mean  $\bar{x}$ . The sample autocorrelation at lag r is defined as the ratio of the autocovariance at lag r to the variance of the series:

$$\hat{acf}_r = \frac{\hat{c}_r}{\hat{c}_0} \tag{B-1}$$

where  $\hat{c}_r = \frac{1}{n} \sum_{t=|r|}^{n-1} (x_t - \bar{x})(x_{t-|r|} - \bar{x})$ . This definition which corresponds to a biased estimator of the autocovariance ensures that the sample autocorrelation lies between -1 and 1. The periodogram is the Fourier transform of the sample autocovariance sequence:

$$I(\omega_j) = \frac{1}{2\pi} \sum_{|r| < n} \hat{c}_r e^{-ir\omega_j}$$
(B-2)

where  $\omega_j = 2\pi j/n$  is the  $j^{\text{th}}$  Fourier frequency. The periodogram integrates to the sample variance:

$$\int_{-\pi}^{\pi} I(\omega) d\omega = \hat{c}_0 \tag{B-3}$$

It follows that the ordinate  $I(\omega_j)$  has a nice interpretation as the portion of the sample variance due to the harmonic component at frequency  $\omega_j$ . Note that it can be rewritten as:

$$I(\omega_{j}) = \frac{1}{2\pi} \left[ \hat{c}_{0} + 2 \sum_{r=1}^{n-1} \hat{c}_{r} (e^{ir\omega_{j}} + e^{-ir\omega_{j}}) \right]$$
$$= \frac{1}{2\pi} \left[ \hat{c}_{0} + 2 \sum_{r=1}^{n-1} \hat{c}_{r} \cos(\omega_{j}r) \right]$$
(B-4)

The spectrum is obtained by smoothing the periodogram using a q-period Bartlett window, where the choice of the bandwith parameter q results from a trade-off between reducing the variance and minimizing the bias of the estimate. Then,

$$\hat{f}(\omega_j) = \frac{1}{2\pi} \sum_{|r| < q} (1 - |r|/q) \hat{c}_r e^{-ir\omega_j}$$
$$= \frac{1}{2\pi} \left[ \hat{c}_0 + 2 \sum_{r=1}^{q-1} (1 - |r|/q) \hat{c}_r \cos(\omega_j r) \right]$$
(B-5)

# References

- Aguiar, M. and G. Gopinath (2007). "Emerging market business cycles: The cycle is the trend." *Journal of Political Economy*, **115**(1), 69–102.
- Backus, D., E. Henriksen, F. Lambert and C. Telmer (2009). "Current account fact and fiction." *NBER Working Paper*, (15525).
- Backus, D. K., P. J. Kehoe and F. E. Kydland (1992). "International real business cycles." *Journal of Political Economy*, **100**(4), 745–775.
- Barro, R. J. and X. Sala-i Martin (1992). "Convergence." Journal of Political Economy, 100(2), 223–251.
- Bassanini, A. and S. Scarpetta (2002). "The driving forces of economic growth: Panel data evidence for the OECD countries." *OECD Economic Studies*, **33**.
- Baxter, M. and M. J. Crucini (1993). "Explaining saving-investment correlations." American Economic Review, 83(3), 416–436.
- Baxter, M. and M. J. Crucini (1995). "Business cycles and the asset structure of foreign trade." *International Economic Review*, 36(4), 821–854.
- Caballero, R. J., E. Farhi and P.-O. Gourinchas (2006). "An equilibrium model of "global imbalances" and low interest rates."
- Cogley, T. and J. M. Nason (1995). "Output dynamics in real-business-cycle models." *American Economic Review*, **85**(3), 492–511.
- Engel, C. and J. H. Rogers (2006). "The U.S. current account deficit and the expected share of world output." *Journal of Monetary Economics*, 53(5), 1063– 1093.
- Fogli, A. and F. Perri (2006). "The "great moderation" and the us external imbalance." *NBER Working paper*, (12708).
- Hamilton, J. D. (1989). "A new approach to the economic analysis of nonstationary time series and the business cycle." *Econometrica*, **57**, 357–384.

- Heer, B. and A. Maussner (2004). "Projection methods and the curse of dimensionality."
- Levy, D. (2003). "Solving and simulating nonlinear stochastic equilibrium models in frequency domain."
- Maddison, A. (2009). "Statistics on world population, GDP and per capita GDP, 1-2006 AD."
- McGrattan, E. R. (1999). "Application of weighted residual methods to dynamic economic models." In *Computational Methods for the Study of Dynamic Economies*. Ramon Marimon and Andrew Scott.
- Mendoza, E., V. Quadrini and J.-V. Ríos-Rull (2006). "Financial integration, financial deepness and global imbalances."
- Quah, D. (1990). "Permanent and transitory movements in labor income: An explanation for 'excess smoothness' in consumption." *Journal of Political Economy*, 98, 449–475.
- Stockman, A. C. and L. L. Tesar (1995). "Tastes and technology in a two-country model of the business cycle: Explaining international comovements." *American Economic Review*, 85(1), 168–185.
- Watson, M. W. (1993). "Measures of fit for calibrated models." Journal of Political Economy, 101(6), 1011–1041.

T	rade balances in	% of GDP (2008)			
Largest deficits		Largest surplusses			
Portugal	-9.5%	Luxembourg	33.0%		
Greece	-8.8%	Norway	19.2%		
Spain	-5.9%	Switzerland	11.2%		
United States	-4.9%	Ireland	10.4%		
Turkey	-4.4%	Netherlands	8.3%		
Poland	-4.0%	Sweden	7.4%		
Iceland	-2.8%	Austria	6.7%		
United Kingdom	-2.6%	Germany	6.2%		
France	-2.5%	Czech Republic	4.5%		
Slovakia	-2.4%	Finland	3.9%		
Curren	t account balanc	tes in $\%$ of GDP (2008)			
Largest defic	Largest deficits		Largest surplusses		
Iceland	-34.6%	Norway	18.2%		
Greece	-14.4%	Switzerland	9.2%		
Portugal	-12.1%	Sweden	8.3%		
Spain	-9.5%	Netherlands	7.5%		
New Zealand	-8.8%	Germany	6.6%		
Hungary	-8.2%	Luxembourg	5.5%		
Slovakia	-6.5%	Austria	3.8%		
Turkey	-5.5%	Japan	3.2%		
Poland	-5.5%	Denmark	2.0%		
United States	-4.7%	Finland	1.7%		

## Table 1: Trade and current account balances in OECD countries

Sources: Datastream/OECD.

	Mean	Autocorrelation at lag:		t lag:	
Variable	(of absolute values)	4	8	12	20
net exports/GDP	3.0%	0.82	0.70	0.66	0.62
current account/GDP $$	3.5%	0.83	0.72	0.67	0.63

#### Table 2: Summary statistics (based on quarterly data)

Sources: Datastream/National sources and OECD.

Computations based on data for 18 OECD countries. The sample period covers 1957:01-2009:02.

Preferences	$\beta = .96,  \gamma = 2$
Technology	$\alpha = .36,  \delta = .042$
Productivity process	$\Lambda = \left[ \begin{array}{cc} .869 & .087 \\ .087 & .869 \end{array} \right]$
	$\Sigma = .013^2 \times \left[ \begin{array}{cc} 1 & 0.578\\ 0.578 & 1 \end{array} \right]$

 Table 3: Benchmark parameter values

Table 4: Average annual t.f.p. growth rates across G7 countries (in %)

	1970-80	1980-90	1990-2005
United States	0.09	0.49	0.47
Canada	0.12	-0.14	0.09
United Kingdom	0.35	1.00	0.49
France	0.62	0.96	0.33
Germany	-	-	0.53
Italy	1.03	0.95	-0.10
Japan	0.92	1.09	0.34

Source: OECD, authors' calculations.

	1970-80	1980-90	1990-2005
United States	2.11	2.29	1.74
Canada	2.61	1.56	1.73
United Kingdom	1.82	2.55	2.19
France	2.71	1.86	1.28
Germany	-	-	1.32
Italy	3.12	2.36	0.96
Japan	3.20	3.38	1.18

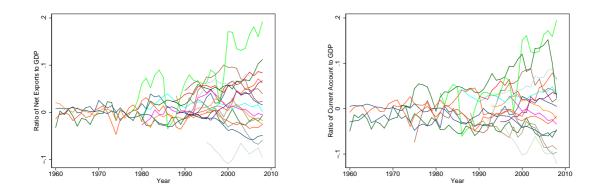
Table 5: Average annual per capita GDP growth rates across G7 countries (in %)

Source: OECD.

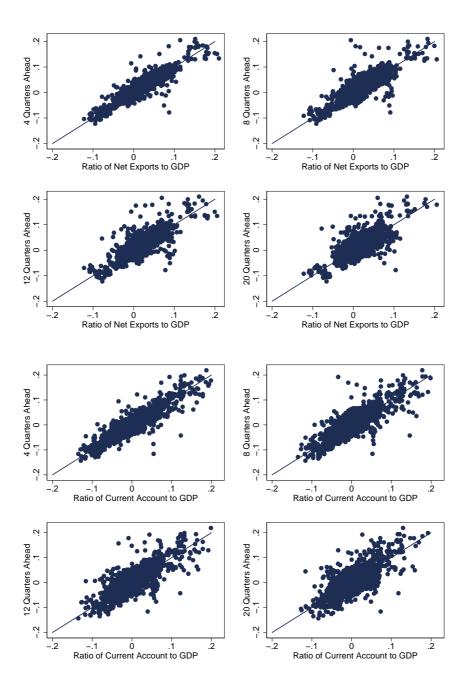
 Table 6: Estimated parameter values for the technology process

Autoregressive coefficient for the transitory shocks	$\rho_z$	0.839
Standard deviation of the transitory shocks	$\sigma_z$	0.035
Autoregressive coefficient for the growth shocks	$ ho_u$	0.920
Standard deviation of technological shifts	au	0.046
Probability of a technological shift	$\lambda$	0.099
Standard deviation of the growth shocks	$\sigma_u$	0.019

Figure 1: External deficits since 1960



The data cover 18 OECD countries: Australia, Austria, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and the United States.



Quarterly data for 18 OECD countries: Australia, Austria, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and the United States.

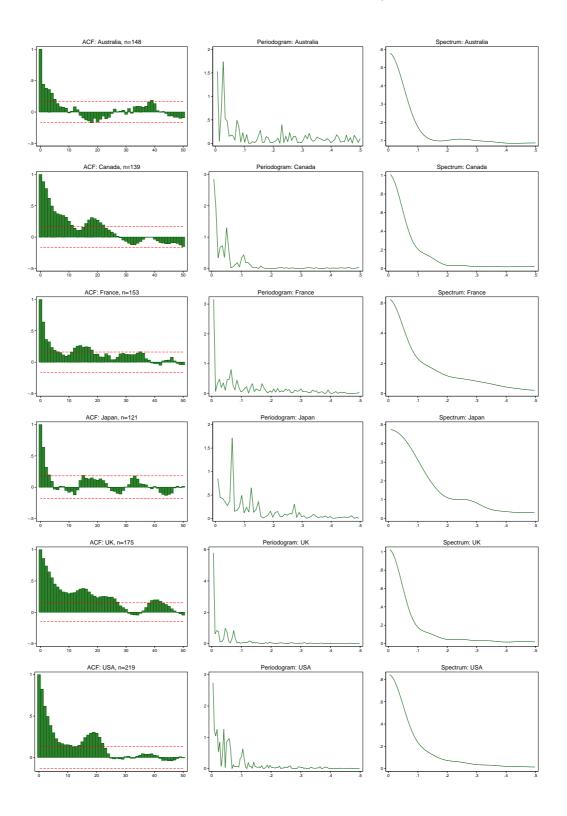


Figure 3: Current account balance/GDP

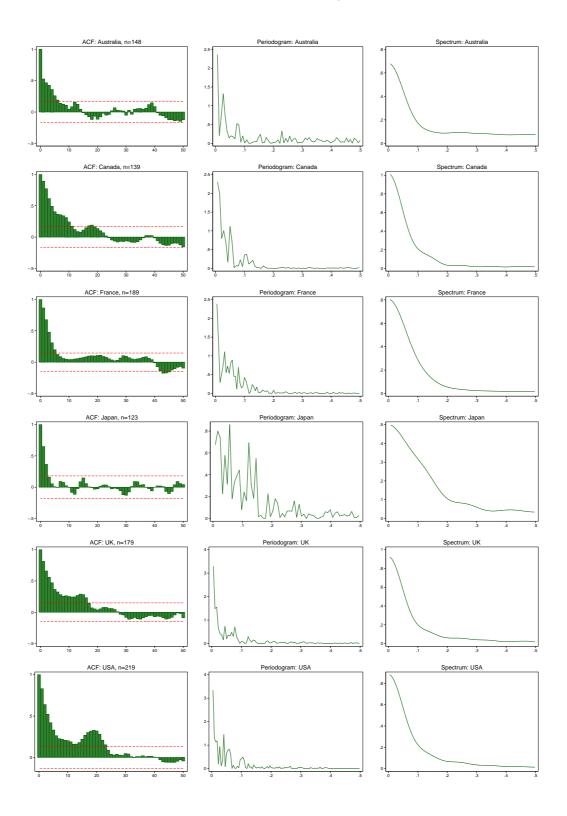
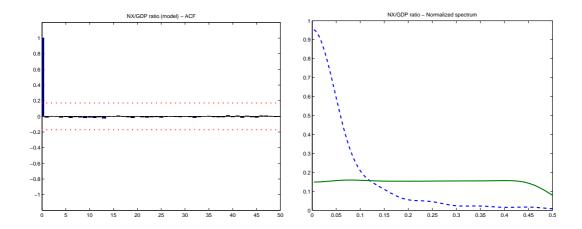


Figure 4: Net exports/GDP

Figure 5: Normalized spectrum of net exports/GDP implied by the benchmark model



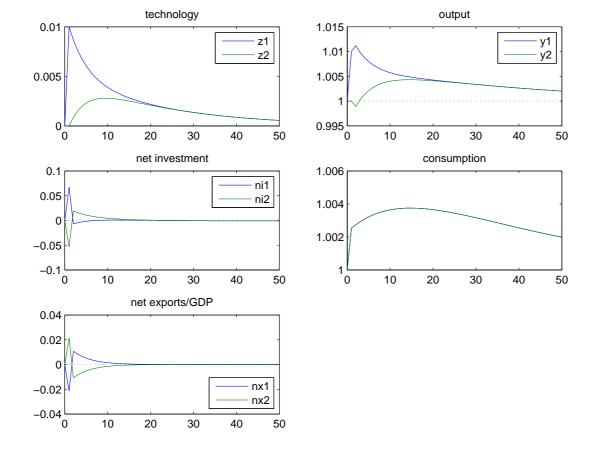


Figure 6: Impulse response functions to a 1% productivity shock in country 1 (benchmark model)



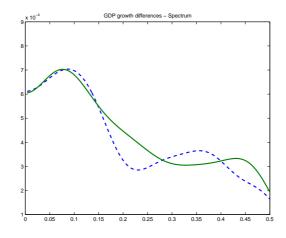


Figure 8: Normalized spectrum of net exports/GDP implied by the model with persitent growth differences

