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Pricing Systematic Ambiguity in Capital Markets†

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Abstract

Asset pricing models assume that probabilities of future outcomes are known. In reality, however, there is ambiguity with regard to these probabilities. Accounting for ambiguity in asset pricing theory results in a model with two systematic components, beta risk and beta ambiguity. The focus of this paper is to study the empirical implications of ambiguity for the cross section of equity returns. We find that systematic ambiguity is an important determinant of equity returns. We also find that the Fama-French factors contribute to the explanatory power of the two main drivers of returns; namely, systematic risk and systematic ambiguity.

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Abstract

Asset pricing models assume that probabilities of future outcomes are known. In reality, however, there is ambiguity with regard to these probabilities. Accounting for ambiguity in asset pricing theory results in a model with two systematic components, beta risk and beta ambiguity. The focus of this paper is to study the empirical implications of ambiguity for the cross section of equity returns. We find that systematic ambiguity is an important determinant of equity returns. We also find that the Fama-French factors contribute to the explanatory power of the two main drivers of returns; namely, systematic risk and systematic ambiguity.

JEL Classifications: G12, D81

Key Words: Capital asset pricing, Ambiguity, Beta Ambiguity

1. Introduction

The most basic question in financial economic—what determines asset prices—has been the subject of many theories and a multitude of empirical tests. Over the years the original Sharp-Lintner-Mossin (henceforth SLM) CAPM went through many transformations. The basic hypothesis that differences in risk should explain differences in expected return has not changed. The main task of the various theories was to determine the risk factors that affect asset prices differentially.

Almost all empirical tests of asset pricing models use the methodology developed by Fama and MacBeth (1973, henceforth FM). This too has undergone some changes due to econometric issues that were raised including measurement and estimation issues.[1] Though the very first tests (e.g. Black, Jensen and Scholes (1972, henceforth BJS) and FM (1973)) seem to confirm the basic positive relationship between risk and return, the evidence was not that strong. It was obvious that there are some missing "factors". The theory then went on a 'hunt' for these missing factors, either to be added to existing ones, or to replace them. The new models were accompanied by empirical tests, mostly using the three-stage approach. For example, a study by Amihud (2002) shows that expected market illiquidity is a factor that affects expected returns of stocks. Acharia and Pedersen (2005) introduce illiquidity betas along with the market beta and obtain illiquidity factors associated with illiquidity risks.[2] Another path of research focused on the empirical tests, identifying issues with past tests and offering better methodologies and tests, for example, Shanken (1992), Jagannathan and Wang (1996), Pasquariello (1999).

The risk factor(s) in all these models and the empirical tests that follow assume implicitly that the probabilities are known with certainty, that there is no *ambiguity* (also known as Knightian uncertainty) about the probabilities. In reality, however, investors face such *uncertainty* all the time. Should this ambiguity affect asset prices differentially? Is it a missing factor in asset pricing models? Is it systematic? A recent asset pricing model developed by

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[1] A comprehensive analysis of the methodology is given in Cochrane (2005). Several studies dealt with specific assumptions made in the FM tests (see, for example, Brenner (1976)).

[2] Some of these tests produce very high cross-sectional R^2. In a recent paper Lewellen, Nagel and Shanken (2010) argue that cross-sectional tests which try to explain average returns on size-B/M portfolios are misleading, the high R^2 actually provides weak support for the model. This critique does not apply to our tests since we do not form size-B/M portfolios.

Izhakian (2012) incorporates ambiguity as an additional factor that explains differences in expected return on financial assets.

The objective of our study is to test whether ambiguity is indeed a missing factor that plays an important role in determining asset prices. Specifically, we test the effect of systematic ambiguity, as distinct from systematic risk, on expected returns. To conduct our empirical tests we use two main data sets: (*i*) Intraday stock quotes taken from the TAQ database. (*ii*) Daily and monthly stock return taken from the CRSP database. The dependent variable, the monthly rate of return on common stocks, is adjusted for dividend as reported in the CRSP database. Our sample covers all available common stocks from March 1993 and December 2011.

The basic approach is the FM (1973) approach with the proper adjustments necessary to address the known issues with their methodology. Accordingly, we construct 25 value weighted portfolios ranked by systematic ambiguity. We account for known issues like heteroskedasticity, serial correlation and errors in variables. The main results show that systematic ambiguity is priced and plays an important role in determining asset prices.

The rest of the paper is organized as follows; Section 2 presents the model and its implications. Section 3 describes the methodology and the data. Section 4 provides the tests and an analysis of the results. Section 5 is a summary and conclusions.

2. The asset pricing model with ambiguity

The one factor CAPM introduced by SLM is an equilibrium model in the expected utility paradigm where the sole determinant of expected return is systematic risk (measured by *beta*); the sensitivity of an asset return to the one factor called market. For any risky asset, the expected return on equity in any period is conditional on the return on the market, where the implicit assumption is that investors know, or act as if they know, the probabilities of all states of nature. A basic issue with these models is that in reality the investor does not know the precise probabilities of events, i.e., there is ambiguity about the probability distribution. Investors are exposed not only to *risk* but also to *ambiguity* (*Knightian uncertainty*). Thus, potentially what is missing is the sensitivity of asset returns to ambiguity about market returns, which we call *systematic ambiguity*. In other words, tests of asset pricing have left out an important determinant, namely ambiguity, and the sensitivity to it, called *beta ambiguity*.

To test the effect of ambiguity on asset prices we use the model proposed by Izhakian (2012), where expected return encompasses a premium for ambiguity. The central concept of this model is that not only are the returns on assets random but the probabilities of these returns are themselves random. The model separates systematic ambiguity from systematic risk. The two betas can be estimated separately but can also be combined to obtain an estimate of *beta uncertainty*.

Let r be the random return on an asset and r_f be the risk free rate of return, which also serves as the reference point relative to which outcomes are classified either as loss or as gain. That is, any return lower than r_f is considered a loss and any return higher than r_f is considered a gain.

The main idea of this theory is that, just as we measure the degree of risk by the variance of outcomes, so can the degree of ambiguity be measured by the variance of the probability of loss (or gain). Formally, let P_L and P_G denote the random probabilities of loss and of gain, respectively. Their expectation, $\mathrm{E}\left[\mathrm{P}_L\right]$ and $\mathrm{E}\left[\mathrm{P}_G\right]$, taken with respect to second-order probabilities, are

$$\mathrm{E}\left[\mathrm{P}_L\right] = \int_{\mathbf{P}} \mathrm{P}\left(r \le r_f\right) d\chi\left(\mathrm{P}\right) \qquad \text{and} \qquad \mathrm{E}\left[\mathrm{P}_G\right] = \int_{\mathbf{P}} \mathrm{P}\left(r > r_f\right) d\chi\left(\mathrm{P}\right), \tag{1}$$

where $\mathbf{P}$ is the set of probability measures and the second order probability $\chi\left(\mathrm{P}\right)$ is the probability of the probability distribution $\mathrm{P} \in \mathbf{P}$. The expected return, $\mathrm{E}\left[r\right]$, and the variance of return, $\mathrm{Var}\left[r\right]$, are evaluated using the expected probabilities, i.e., a double expectation of the random probability of return and the second-order probabilities. The *measure of ambiguity*

$$\mathrm{J}^2\left[r\right] = 4\,\mathrm{Var}\left[\mathrm{P}_L\right] \tag{2}$$

is four times the variance of the probability of loss, or four times the variance of the probability of gain, which are taken with respect to the second order probability distribution χ.

Assume that the return on every asset is normally distributed, but the mean, μ, and the variance, σ^2, governing the distribution are random. The random probability of loss on asset j then takes the form

$$\mathrm{P}_L = \Pr\left[r_j \le r_f\right] = \int_{-\infty}^{r_f} \frac{1}{\sqrt{2\pi\sigma_j^2}} e^{-\frac{\left(r_j-\mu_j\right)^2}{2\sigma_j^2}} dr_j = \Phi\left(r_f;\mu_j,\sigma_j\right), \tag{3}$$

where $\Phi(\cdot)$ stands for the standard normal cumulative probability distribution. The degree of ambiguity associated with asset j is then measured by

$$\mho^2\left[r_j\right] = 4\,\mathrm{Var}\left[\Phi\left(r_f;\mu_j,\sigma_j\right)\right]. \tag{4}$$

Given the measure of ambiguity $\mho^2$ and the measure of risk, Var, the mean-variance space can be extended to *mean-uncertainty* space, i.e., mean-variance-ambiguity space. In this space a combined *measure of uncertainty*, aggregating risk and ambiguity, is defined by

$$\Upsilon^2\left[r_j\right] = \frac{\mathrm{Var}\left[r_j\right]}{1-\mho^2\left[r_j\right]}. \tag{5}$$

The measure of uncertainty Υ^2 aggregates the two dimensions of uncertainty, with respect to the outcome and with respect to probabilities, to a single unified measure.

Let m denote the market portfolio. The expected return on asset j takes the form

$$\mathrm{E}\left[r_P\right]-r_f = \beta_{R,j}\left(\mathrm{E}\left[r_m\right]-r_f\right)+\beta_{A,j}\left(\mathrm{E}\left[r_m\right]-r_f\right), \tag{6}$$

where *beta risk* is defined by

$$\beta_{R,j} = \frac{\mathrm{Cov}\left[r_j,r_m\right]}{\mathrm{Var}\left[r_m\right]}, \tag{7}$$

and *beta ambiguity* is defined by

$$\beta_{A,j} = \frac{\mathrm{Cov}\left[\Phi\left(r_f;\mu_m,\sigma_m\right),\phi\left(r_f;\mu_m,\sigma_m\right)\left(\frac{\sigma_{j,m}}{\sigma_m^2}\left(\mu_m-r_f\right)-\left(\mu_j-r_f\right)\right)\right]}{1-\mho^2\left[r_m\right]}. \tag{8}$$

The random cumulative probability of loss $\Phi\left(r_f;\mu_m,\sigma_m\right)$ on the market portfolio is defined by Equation (3) and $\phi\left(r_f;\mu_m,\sigma_m\right)$ is its density at the reference point. Recall that $\sigma_{j,m}$ is also a random variable whereas Cov is computed using expected probabilities.

The risk premium $\beta_{R,j}\left(\mathrm{E}\left[r_m\right]-r_f\right)$ is the reward for systematic risk, and the ambiguity premium $\beta_{A,j}\left(\mathrm{E}\left[r_m\right]-r_f\right)$ is the reward for systematic ambiguity. The uncertainty premium is then defined as

$$\beta_{K,j}\left(\mathrm{E}\left[r_m\right]-r_f\right), \tag{9}$$

where

$$\beta_{K,j}=\beta_{R,j}+\beta_{A,j}. \tag{10}$$

We now turn to test the model empirically. We first present the data and the methodology, and then test the model.

3. Data and methodology

3.1 Data

The data used in the empirical tests is intraday trading data on stocks taken from the TAQ database[3] The data covers all common stocks in the period from March 1993 to December 2011, 226 months in total. Monthly and daily returns adjusted for dividends were obtained from the CRSP database. The initial data covers all common shares listed in the TAQ and CRSP databases. The following filters were applied to the data. Intraday data regarding a stock that doesn't have at least 10 quotes in different 15 min time intervals were omitted from the day. For a given month, a stock that doesn't have at least 10 days that satisfy the previous condition was omitted from that month. In every month stocks with a price lower than \$2 or higher than \$1000 were omitted (see, for example, Acharya and Pedersen (2005)).[4] Observations with extreme price changes (minus or plus 20 log returns) within 15 min were omitted. Stocks with beta uncertainty and beta risk that is more than 3 standard deviations away from 1.0 were winsorized. For beta ambiguity we used 3 standard deviations away from 0.0. The number of monthly observations, after filtering, was 803763 which is, on the average, 3540 stocks per month.

[3] The Trade And Quotes (TAQ) database; Wharton Research Data Services (WRDS).

[4] We have used a lower stock price, \$2 instead of \$5, since there were many large companies, especially banks, where the stock priced declined drastically during the financial crisis.

As a proxy for the market portfolio we use the exchange-traded fund SPDR (Standard & Poor's Depositary Receipts, ticker symbol: SPY).[5] The stocks in the SPDR have the same weights as in the S&P500 index and it is designed to track the index, net of expenses.[6] The expense ratio is about 7-8 basis points and the bid-ask spread is 1-2 basis points. The quarterly dividends are added to the index every 3 months. It can be sold short like any other stock and short interest is sometimes as high as 50 percent. A typical volume for the SPDR is between 200-300 million shares per day, which is the highest of any US stocks traded on any exchange. We use the SPDR as a proxy for the market portfolio and not the S&P index itself since the SPDR trades continuously, while the index contains illiquid stocks and so its values are stale.

3.2 Methodology

The basic approach in the design of our empirical tests was the FM (1973) three-stage approach. To minimize the effects of known issues in empirical asset pricing tests, as pointed out by several researchers (e.g. Shanken (1992), Chocrane (2005)), we have formed large portfolios as first suggested by BJS (1972) and employed in most asset pricing tests. In the first stage, the risk measures (betas) and factor loadings, derived from the Fama-French factors, are independent monthly estimates, based on daily observations. In the second stage, the cross sectional regressions stage, we used Weighted Least Squares accounting mainly for heteroskedasticity. To avoid spurious relationships the dependent variable, excess return, was at $t+1$ while the independent variables were computed in time t. In the third stage we tested for the significance of the coefficients (“price of risk”). The details of the empirical tests design is described next.

The first step is to compute the time series values of the variables that will be used in the tests. We first compute the degree of ambiguity, given in Equation (4), for each stock and for each month.[7] For each stock we apply the following procedure. We sample the prices of the

[5] The SPDR (Standard & Poor's Depositary Receipts) is comprised of all the stocks in the Standard & Poor's 500 Index. It began trading on the American Stock Exchange (AMEX) on January 29, 1993.

[6] Since dividends are added to the SPDR every three months, we adjust the return on the SPDR, the explanatory variable, to monthly dividend yields, using the dividend yields on the S&P-500 index, taken from the CRSP database.

[7] For consistency with other cross section studies we use one month intervals.

stock every 15 minutes starting from 9:30 until 16:00: 27 prices in total for each day.[8] In case there was no trade at a specific time interval, we took the volume-weighted average of the closest trading price. Using these prices we compute 15 minute returns, 26 returns in total for each day.[9] The choice of 15 minute intervals is dictated by the measure of ambiguity.[10] To obtain a statistically meaningful monthly measure of ambiguity we need a daily estimate of probability derived from a daily distribution of rates of return, which, in turn, requires intraday observations.

For each day we used 26 observations to compute the mean and the variance of return. Depending on the number of trading days in the month, we have, for each month and for each stock, between 440 and 572 observations. Using Equation (3) we compute for each day the probability of a loss, P_L. For each month, there are 20 to 22 different loss-probabilities. Using these loss-probabilities we compute a variance to obtain the **degree** of ambiguity, $\mho^2$, for an individual stock in a given month.

Assuming that the daily ratio $\lambda = \frac{\mu}{\sigma}$ is normally distributed with mean $\mathrm{E}[\lambda]$ and variance $\mathrm{Var}[\lambda]$, then P_L is uniformly distributed over the month.[11] This method assigns lower weights to values of λ that deviate from the monthly mean λ. In this procedure, the realized probabilities of loss serve as a proxy for the expected probability. These probabilities are extracted from daily means and variances, μ and σ, which are computed using 15 minute intervals. The variation in the probability of loss, P_L, is due to the variation in the ratio λ.

For each stock and for the market, using the 26 intraday observations, we compute their daily mean and variance, and the covariance of the stock with the market. As a result, for each

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[8] To check for robustness, while eliminating the impact of the trading noise caused by opening and closing daily positions during the first and the last half-hour of the stock trading, we also performed our tests using only the prices from 10:00 to 15:30. The results were essentially the same.

[9] We have not included returns between closing prices and opening prices of the following day in order to eliminate the impact of overnight price changes and dividend distributions.

[10] Though Anderson, Bollerslev, Diebold and Ebens (2001) claim that 5 minute returns are sufficient to eliminate microstructure effects, we have used 15 minutes returns to minimize even further the potential microstructure effects that may be affecting the less liquid stocks.

[11] It can be shown that the density function of the random variable P_L as a function of the normally distributed random variable λ is uniform.

stock and each month we have between 20 and 26 means, variances and covariances with the market portfolio. Using Equation (8) we then compute for each month and for each stock its beta ambiguity β_A. Beta risk, β_R, is computed by Equation (7) using daily stock data.[12] Given asset's beta ambiguity $\beta_{A,j}$ and beta risk $\beta_{R,j}$, its beta uncertainty $\beta_{K,j}$ is given by their sum (see Equation (10)). Figure 1 provides a diagrammatical representation of the process for computing the ambiguity measures.

[INSERT FIGURE 1]

Figure 2, upper plot, depicts the average daily returns on the SPY over the years 1993 to 2011. The lower plot presents the monthly uncertainty, Υ, on a daily basis. During these years we observe some short periods with exceptionally large downward moves followed by large upward moves, in the market. These were associated with big changes in our uncertainty (ambiguity and risk) measure. See, for example, August-September 1998, the Russian default/LTCM debacle, and September-October 2008, the recent financial crisis. In both cases the market decline is associated with a contemporaneous increase in ambiguity, and the increase in the market is associated with a decrease in the level of ambiguity.

[INSERT FIGURE 2]

The next step was to form 25 portfolios for each month during the period March 1993 to December 2011 by ranking the stocks in each month by *Beta* ambiguity. We then compute the portfolio *betas* in the following month where the *betas* of the individual stocks are value weighted. For each portfolio we also compute the value weighted return and the other explanatory variables (size, liquidity and loadings).

The dependent variable is the monthly portfolio return, denoted r_P. The risk free rate r_f is the 1 month T-bill rate. The market return, denoted r_m, is the return on the SPY. Monthly returns are computed using the opening price on the first trading day of the month and the closing price at the last trading day of the month, and they are adjusted for monthly dividend. Table I provides summary statistics of the monthly returns. The risk-free-rate for this period is

[12] While for beta ambiguity we use intra-day data, which was dictated by the measure of ambiguity, for beta risk we use daily observations which are less noisy than intra-day ones.

about 0.26 percent monthly while the market return (cum dividends) is 0.7 percent monthly. The market's monthly standard deviation in this period was about 4.4 percent. The average excess return of the 25 portfolios is about 1.1 percent monthly while the average standard deviation is about 6.6 percent, which is in line with the market's standard deviation. To get an indication of the distributional properties of the excess return of the 25 portfolios we have tested for the significance of the average Skewness and Kurtosis. While the Skewness of the average portfolio is not significantly different from zero, the Kurtosis is significantly larger than zero which is an indication of fat tails, a known phenomenon of stock returns.

[INSERT TABLE I]

In Table II we present summary statistics of the variables that we use to explain the excess returns on the 25 portfolios. In computing the statistics in this table we use all the months for each of the 25 portfolios. The mean beta, for example, is the average of all months across the 25 portfolios. These statistics include the betas of the portfolios; β_R, with an average of 1.0 and standard deviation of 0.31, β_A with an average value of 0.14 which is positive and statistically significant at the 5 percent level. β_K is the average of the sum of β_R and β_A. The table also includes the loadings of the Fama-French factors and two other variables that may affect portfolio returns; size and liquidity. In essence, the theory that is underlying our tests contains one risk factor, uncertainty, which combines risk and ambiguity, measured separately. In the model there is no room for other factors (characteristics) like, for example, size or liquidity. However, since prior studies (e.g. Amihud (2002), Brennan, Chordia and Subrahmanyam (1998)) had shown that these variables seems to affect returns and so do the Fama-French factors, we felt that we need to include them in the tests of our model to see if the Fama-French factors are statistically significant in the presence of our ambiguity factor. The additional Fama-French factors are: *(i)* the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks (*SMB*, small minus big), *(ii)* the difference between the return on a portfolio of high book-to-market stocks and the return on a portfolio of low book-to-market stocks (*HML*, high minus low), *(iii)* the difference between the return on a portfolio of winning stocks and the return on a portfolio of losing stocks (*UMD*, up minus down).

To obtain the factor sensitivities, or loadings, λ_{HML}, λ_{SML} and λ_{UMD}, we regressed these factors on the portfolios' daily excess returns. The factor data was taken from Kenneth R. French's Website.[13,14] We also control for stock (portfolio) characteristics: size and liquidity. *SIZE* is the stock's dollar-value computed using the number of shares outstanding and stock closing price at the last day of the month. The portfolio *SIZE* is the average *SIZE* of the stocks in the portfolio. Stock liquidity, denoted *LIQ*, is the average daily Amihud (2002) measure during the month: $\text{Mean}\left[\frac{|r_j|}{vol_j \times price_j}\right]$. The liquidity of the portfolio is the value weighted monthly liquidity of the individual stocks.

[INSERT TABLE II]

Table III provides the return, the betas, the loadings, size and liquidity for each of the 25 portfolios. Recall that the portfolios were ranked by beta ambiguity in *t*-1 while the beta of the portfolio in table III is computed for period *t*, as suggested by Fama-MacBeth (1973). Thus, the portfolio betas as they appear in the table may not be monotonically descending, though the ranking is very close. Also, since the ranking was done by beta ambiguity and the correlation with beta risk is not very high, we should not expect beta risk or beta uncertainty to appear in a monotonic order.

[INSERT TABLE III]

Table IV presents the autocorrelation of all the variables used in the tests. The returns are not autocorrelated as expected while the betas exhibit first order autocorrelation which are significant but are of small magnitude, they range from 0.097 to 0.25. Most of these autocorrelations are of smaller size and non-significant.

[INSERT TABLE IV]

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[13] This data is provided via http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[14] Following Campbell, Lettau Malkiel and Xu,(2001), Xu and Malkiel (2003), Ang, Hodrick, Xing and Zhang (2006) and Bali and Cakici (2008), we use the within month daily return data.

Table V provides the cross correlation of all the variables that we are using in the regression tests. This can give us the first impression of what the test results may look like. Beta risk and beta uncertainty are highly correlated while beta risk has a low correlation with beta ambiguity. This should be helpful in distinguishing the effect of beta risk from beta ambiguity. Though the loadings of the Fama-French factors are correlated to some degree amongst themselves, they don't seem to be correlated with the betas. Thus, the potential effects of multi co-linearity are minimized. We now turn to the analysis of our main results provided by the regression tests.

[INSERT TABLE V]

4. Empirical results: the effect of ambiguity and risk on excess returns

The tests results of the theoretical model, presented in equations (6) – (8), are analyzed in this section. Table VI presents the results of the regressions, where the dependent variable is the portfolio's excess return. As a benchmark, we start by testing the standard CAPM model, using the 25 portfolios in the cross-section regression

$$r_{P,t} - r_{f,t} = \alpha_t + \Theta_{R,t}\beta_{R,P,t-1} + \varepsilon_t, \tag{11}$$

for each month *t*.

Beta-risk of the portfolio is computed from daily observations during the month, and the portfolio return is the value weighted monthly return of the stocks in the portfolio. We regress the portfolio excess return, $r_{P,t}$, in month *t* on the portfolio's beta risk, $\beta_{R,P,t-1}$, computed for month *t*-1. In Table VI we report the average α_t and $\Theta_{R,t}$.[15] If the "simple" CAPM holds then α_t should be zero and $\Theta_{R,t}$ should be the market price of *beta risk*. The average coefficients were subjected to a t-test using the standard deviation of the 226 monthly values of $\Theta_{R,t}$. α_t is not significantly different from zero while the average $\Theta_{R,t}$ is positive and significant (the t-value is 2.07), which is consistent with the model. The size of $\Theta_{R,t}$, however, is larger than the market

[15] The time series of the estimated coefficient, $\Theta_{R,t}$, was subjected to an autocorrelation test and to a test for heteroskedasticity (Bartlett). Both were insignificantly different from zero.

equity premium, using the SPY as the market. The average adjusted R^2 is about 17 percent which indicates that there are some missing variables in trying to explain excess returns on stocks/portfolios.

We then turn to test the ambiguity asset pricing model, derived in Section 2 and presented in Section 3. In Equation (12) *beta ambiguity* is added to the regression.

$$r_{P,t} - r_{f,t} = \alpha_t + \Theta_{R,t}\beta_{R,P,t-1} + \Theta_{A,t}\beta_{A,P,t-1} + \varepsilon_t \,. \tag{12}$$

When we use only *beta ambiguity* the coefficient is positive and significant, though it is very large. So ambiguity itself seems to affect excess return. The model, however, suggest that the determinants are both; *beta ambiguity* and *beta risk*. It predicts that both should be positive and the combined value of the coefficients should be equal to the equity risk premium. Though both are positive, they are not significant since the two betas are highly correlated, 0.585, as can be observed in table V. Due to the co-linearity of these variables we get better information from the R^2 of the regression, which is about 0.23. In other words, *beta ambiguity* has additional explanatory power. Since *beta risk* and *beta ambiguity* are highly correlated, we have replaced them with *beta uncertainty* which is a linear combination of *beta risk* and *beta ambiguity* (Equation 10). Given the universal adaption of the Fama-French factors, we also have included these factors in our tests along with two variables that other researchers claim affect returns, namely size and liquidity.

In Equation (13) we introduce size and liquidity as additional explanatory variables

$$r_{P,t} - r_{f,t} = \alpha_t + \Theta_{K,t}\beta_{K,P,t-1} + \Theta_{SIZE,t}\gamma_{SIZE,P,t-1} + \Theta_{LIQ,t}\gamma_{LIQ,P,t-1} + \varepsilon_t \,. \tag{13}$$

Size has been shown to be associated with lower returns since larger companies tend to be more stable which may have not been captured in *beta risk.* (See, for example, Brennan, Chordia and Subrahmanyam (1998)). Liquidity was also shown to have an effect on returns (See, for example, Amihud (2002)).

The coefficient of SIZE should be negative as should the coefficient of *LIQ*. While coefficient of *beta uncertainty*, $\Theta_{K,t}$, is positive and significant, *SIZE* is not significant but *LIQ* is significant. Moreover, the adjusted R^2 has improved, it is now 33 percent. It is interesting to note the coefficient of $\beta_{K,P}$, $\Theta_{K,t}$, is about 7.7 percent on an annual basis, which is now much closer to the equity premium during these years (about 6 percent).

To test for the effect of the Fama-French (1992) three factors, we use the following regression

$$r_{P,t} - r_{f,t} = \alpha_t + \Theta_{K,t}\beta_{K,P,t-1} + \Theta_{HML,t}\lambda_{HML,P,t-1} + \Theta_{SMB,t}\lambda_{SML,P,t-1} + \Theta_{UMD,t}\lambda_{UMD,P,t-1} + \varepsilon_t , \qquad (14)$$

where λ_{HML}, λ_{SML} and λ_{UMD} are the loadings that we have obtained using their factors. When ambiguity is present we do not have a prediction as to the direction regarding the relationship between excess returns and the factor loadings, since the factor loadings λ_{HML}, λ_{SML} and λ_{UMD} are derived empirically and are not based on a theory, In Table VI we see that the introduction of the factor loadings has no added explanatory power. All three of them are not significantly different from zero and the coefficient of *beta uncertainty* has not changed. It may be argued that this is due to the multi co-linearity of the factor loadings (see Table V). In fact, the R^2 of this regression is somewhat smaller than the previous regression, where *SIZE and LIQ* are added to *beta uncertainty*. However, when we compare this test to the test with just *beta risk and beta ambiguity* we see that the R^2 is 7 percent higher, which is not that big but if combined with other variables could better explain the variability of the portfolio returns.

In regression (15), in addition to *beta* uncertainty, we include all the variables: *SIZE, LIQ* and the Fama-French factor loadings

$$\begin{aligned} r_{P,t} - r_{f,t} = \alpha_t + \Theta_{K,t}\beta_{K,P,t-1} + \Theta_{HML,t}\lambda_{HML,P,t-1} + \Theta_{SMB,t}\lambda_{SML,P,t-1} + \Theta_{UMD,t}\lambda_{UMD,P,t-1} + \\ \Theta_{SIZE,t}\gamma_{SIZE,P,t-1} + \Theta_{LIQ,t}\gamma_{LIQ,P,t-1} + \varepsilon_t \end{aligned} . \qquad (15)$$

As can be seen in Table VI, last row, *beta uncertainty* and *LIQ* have hardly changed and have the right sign as expected. None of the factor loadings is significant BUT there is an increase in the R^2, while the non adjusted R^2 is 0.55, the adjusted R^2 is about 0.40. This indicates that the factor loadings, put together, have some explanatory power. Finally, the coefficient of *beta uncertainty* is now even closer, about 6.5 percent on an annual basis, to the equity premium.

It should be noted that we subjected all the regressions to tests of heteroskedasticity and we could not reject the hypothesis of equal variances. Nevertheless all the regression tests were also conducted using Weighted OLS methodology to correct for potential heteroskedasticity. The results were virtually identical.

In conclusion, the regression tests in Table VI, show that systematic ambiguity is an important determinant of security returns. In all tests it stayed positive and significant. The other variables do have some explanatory power.

[INSERT TABLE VI]

5. Conclusions

The objective of this paper is to test the effect of ambiguity on asset prices. The uncertainty regarding the probability distribution, termed ambiguity, should command an ambiguity premium, assuming that investors are ambiguity averse. We test for this effect using an extended CAPM which focuses on the incorporation of ambiguity into asset pricing. The model (Izhakian 2012) provides an asset pricing equation with two separable *betas*: *beta risk* and *beta ambiguity*. We can now extract the pure effect of systematic ambiguity.

To minimize the effects of noise, heteroskedaticity, etc., inherent in the prices of individual securities, we formed 25 portfolios by ranking the individual stocks on the degree of systematic ambiguity. We then subjected the portfolios to regression tests to determine the importance of ambiguity in setting asset prices.

As the tests show, systematic ambiguity does have a significant effect on returns, beyond the effect of conventional risk. Adding the Fama-French factors and liquidity does improve the explanatory power of all variables combined, tough the Fama-French factors by themselves do not seem to have a significant effect on portfolio returns.

References

Acharya, V. V. and Pedersen, L. H., 2005, “Asset Pricing with Liquidity Risk,” *Journal of Financial Economics*, Vol. 77, No. 2, pp. 375–410.

Amihud, Y., 2002, “Illiquidity and Stock Returns: Cross-Section and Time Series Effects,” *Journal of Financial Markets*, 5, 31-56.

Andersen, T. G., Bollerslev, T., Diebold, F. X., and Ebens, H., 2001, "The Distribution of Realized Stock Return Volatility," *Journal of Financial Economics*, 61, 1, 43-76.

Ang, A., Hodrick, R. J., Xing, Y., and Zhang, X., 2006, "The Cross-Section of Volatility and Expected Returns," *Journal of Finance*, 61,1, 259-299.

Bali, T. G., and Cakici, N., 2008, "Idiosyncratic Volatility and the Cross Section of Expected Returns," *Journal of Financial and Quantitative Analysis*, 43,1, 29-58.

Black, F., Jensen, M. C., and Scholes, M., 1972, “The Capital Asset Pricing Model: Some Empirical Tests,” Studies in the Theory of Capital Markets. Michael C. Jensen, ed. New York: Praeger, pp. 79-121.

Brennan, M. J., Chordia, T., and Subrahmanyam, A., 1998, "Alternative factor specifications, security characteristics, and the cross-section of expected stock returns", *Journal of Financial Economics*, vol 49, no. 3, 345 – 373.

Brenner, M., 1976, “A Note on Risk, Return and Equilibrium: Empirical Tests,” Journal of Political Economy, vol 84(2), 407-409.

Campbell, J., Lettau; Y.; M., Malkiel; B. G., and Xu, Y., 2001, “Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk.” *Journal of Finance*, 56, 1-43.

Cochrane, J. H., 1996, "A Cross-Sectional Test of a Investment-Based Asset Pricing Model," *Journal of Political Economy*, 104, 572-621.

Cochrane, J. H., 2005*, Asset Pricing*, Princeton, Princeton Univ. Press.

Fama, E. F., and MacBeth, J. D., 1973, "Risk, Return, and Equilibrium: Empirical Tests," *Journal of Political Economy*.

Fama, E. F.; French, K. R., 1993,. "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Financial Economics*, 33 (1): 3–56.

Izhakian, Y., 2012, "Capital Asset Pricing under Ambiguity," Stern School of Business, Economics Working Paper Series, ECN-12-02.

Jagannathan, R., Wang, W., 1996. "The conditional CAPM and the cross-section of stock returns," *Journal of Finance*, 51, 3-53.

Lewellen, J., Nagel, S. and Shanken, J., 2010. "A skeptical appraisal of asset pricing tests," *Journal of Financial Economics*, Vol. 96, No. 2, pages 175-194.

Pasquariell, P., 1999, “The Fama-MacBeth Approach Revisited,” unpublished manuscript dated August 1999.

Shanken, J., 1992, "On the Estimation of Beta-Pricing Models," *The Review of Financial Studies*, 5, 1-33.

Xu, Y., and Malkiel, B. G., 2003, "Investigating the Behavior of Idiosyncratic Volatility," *Journal of Business*, 76, 613-644.

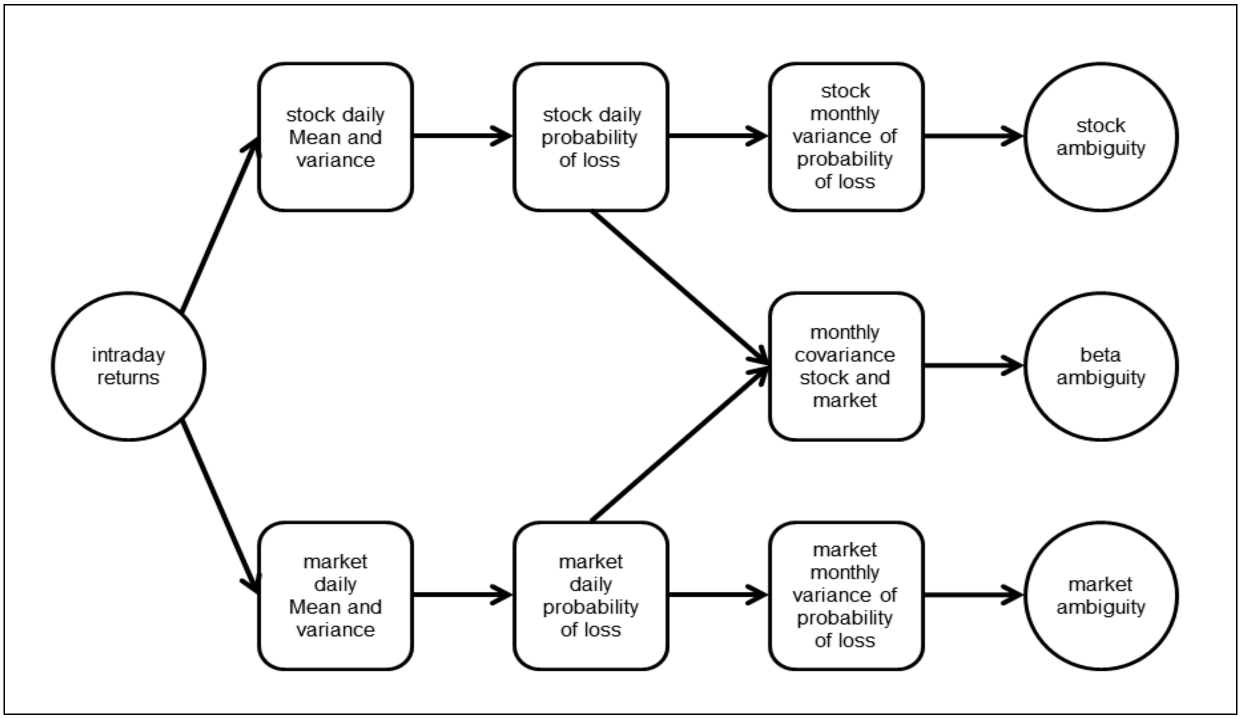

Figure 1: Computation flow

This figure provides a diagrammatical representation of the process for computing the ambiguity measures.

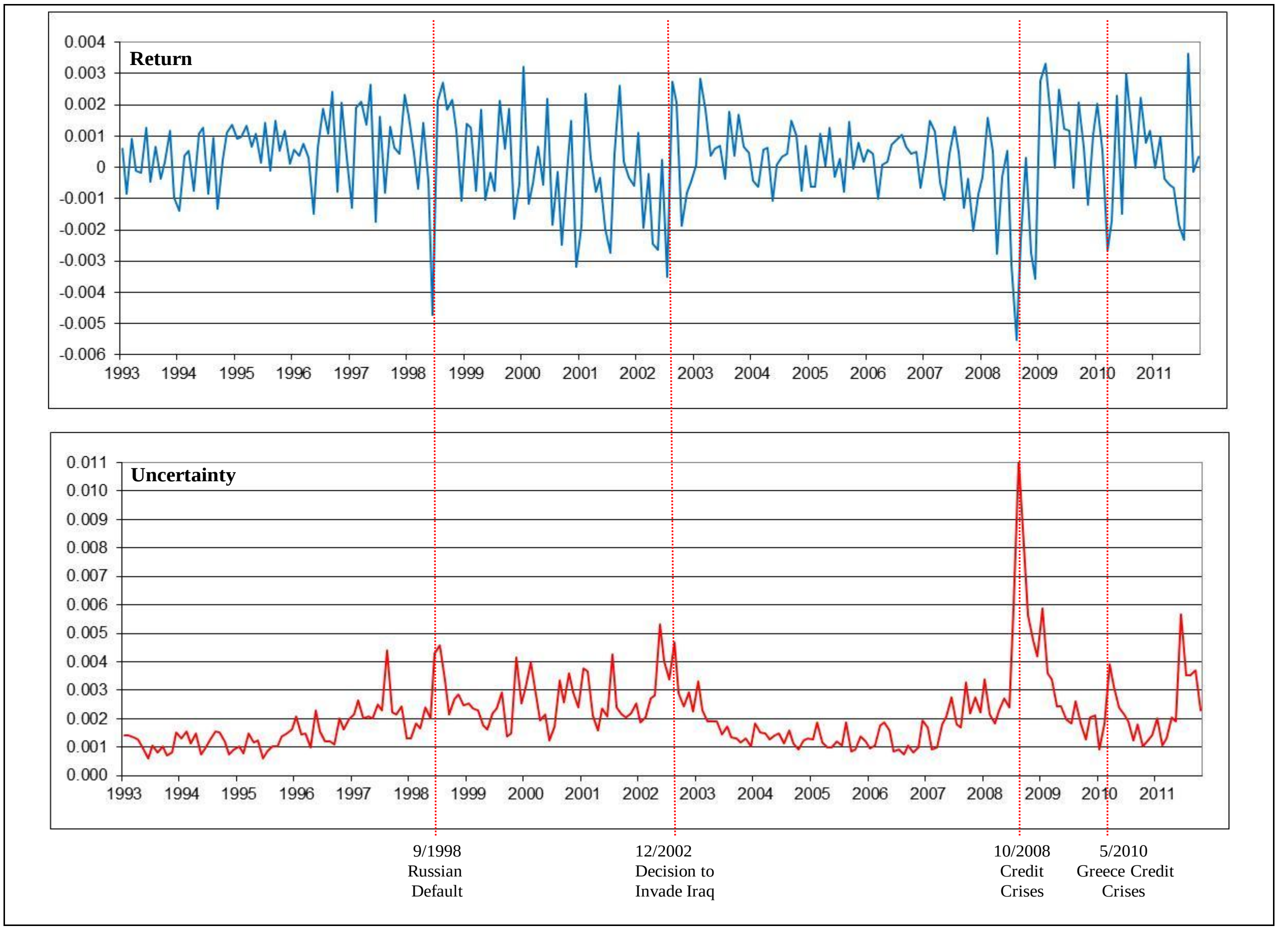

Figure 2: Market excess return and the degree of uncertainty in the market for the period 1993-2010

The upper plot provides the daily, adjusted for dividend, excess return on the SPY, which serves as a proxy for the market portfolio, between February 1993 and December 2011. The values are the average daily excess return in each month. The lower plot provides the daily degree of uncertainty, Υ^2, for each month between February 1993 and December 2011. Υ^2 is computed using 15 minutes rates of return during the month. For each day the probability of loss is computed using the mean and the variance of that day. For each month there are 20-22 probabilities of loss over which the variance is computed to provide the degree of ambiguity and the degree of uncertainty, Υ^2. The red doted vertical lines designate special events that had a significant impact on the average monthly excess returns.

Table I

Summary Statistics of the Market Portfolio (SPY) and the Risk-Free-Rate for the Period 1993-2011

This table reports summary statistics. r_m is the monthly return on the SPDR (SPY), the proxy for the market portfolio. r_f is the risk-free-rate, the 1 month t-bill rate. $r_P - r_f$ is the monthly adjusted to dividend value-weighted excess return on the portfolio of all stocks in the sample.

	Mean	**Variance**	**Skewness**	**Kurtosis**	**Min**	**Max**	**Median**	**N**
r_f	0.00259	2.83E-06	-0.23249	-1.38164	0.00000	0.00560	0.00285	226
r_m	0.00703	0.00194	-0.62558	1.00305	-0.16519	0.10915	0.01207	226
$r_P - r_f$	0.01155	0.00446	-0.11745	2.11960	-0.21330	0.26189	0.01470	226

Table II

Summary Statistics of the Cross-Sectional Regression Variables for the Period 1993-2011

This table reports summary statistics for the entire sample between February 1993 and December 2011. β_R is the systematic risk relative to the market portfolio during the month, computed using daily data. β_A is the systematic ambiguity relative to the market portfolio during the month. β_A is computed using every 15 minutes rates of return over the month and multiplied by 26 (number of 15 minutes intervals). β_K is the systematic uncertainty relative to the market portfolio, i.e., the aggregation of β_R and β_A. *SIZE* is computed using the average price in a given month multiplied by the number of shares outstanding. *LIQ* is the daily Amihud (2002) measure.

	Mean	**Variance**	**Skewness**	**Kurtosis**	**Min**	**Max**	**Median**	**N**
β_R	1.00070	0.09938	0.42501	1.25799	0.22370	2.10024	0.98861	226
β_A	0.14173	0.02906	0.55964	4.07149	-0.41524	0.79162	0.12552	226
β_K	1.14316	0.16848	0.55158	2.89680	-0.02711	2.72987	1.11821	226
λ_{SMB}	0.68459	0.25500	0.55176	1.01358	-0.46376	2.14759	0.62919	226
λ_{HML}	0.18964	0.32459	0.03028	0.69701	-1.33785	1.67069	0.18769	226
λ_{UMD}	-0.08512	0.21137	-0.01158	2.63975	-1.56495	1.30660	-0.08139	226
SIZE	3.81E+07	1.44E+15	2.41732	8.32592	2.95E+06	2.18E+08	2.75E+07	226
LIQ	5.76E-09	1.55E-14	1.46305	17.83890	-3.71E-07	4.74E-08	5.55E-09	226

Table III

Summary Statistics of Portfolios' Variables

This table reports summary statistics for the 25 portfolios. The data sample is for the period between February 1993 and December 2011. r_P is the daily adjusted to dividend return on the of the portfolio. β_R is the systematic risk relative to the market portfolio during the month, computed using daily data. β_A is the systematic ambiguity relative to the market portfolio during the month. β_A is computed using every 15 minutes rates of return over the month and multiplied by 26 (number of 15 minutes intervals). β_K is the systematic uncertainty relative to the market portfolio, i.e., the aggregation of β_R and β_A. The loadings $\lambda_{SMB}, \lambda_{HML}$ *and* λ_{UMD} are evaluated using daily returns and daily Fama-French factors. *SIZE* and *LIQ* are computed using daily data. Standard deviations are in parentheses.

	r_P	β_R	β_A	β_K	λ_{SMB}	λ_{HML}	λ_{UMD}	***SIZE***	***LIQ***
1	0.00093	1.01532	0.12233	1.13343	1.36197	0.49880	-0.16465	5400345	4.69E-08
	(0.0046)	(0.4904)	(0.4578)	(0.8041)	(1.3644)	(1.3369)	(1.1412)	(7180210)	(4.19E-08)
2	0.00074	0.97268	0.10016	1.07299	1.08925	0.31539	-0.14979	17476143	1.21E-08
	(0.0038)	(0.4026)	(0.1639)	(0.4773)	(0.8328)	(0.7756)	(0.6765)	(22143707)	(1.14E-08)
3	0.00046	0.87112	0.08420	0.95611	0.98206	0.27807	-0.10673	35345421	6.26E-09
	(0.0027)	(0.3305)	(0.1405)	(0.4023)	(0.6329)	(0.6272)	(0.4730)	(44664248)	(6.69E-09)
4	0.00054	0.85271	0.07502	0.92759	0.86469	0.27760	-0.12625	47366170	3.82E-09
	(0.0023)	(0.2647)	(0.1136)	(0.2980)	(0.5420)	(0.5310)	(0.3978)	(44798985)	(4.44E-09)
5	0.00036	0.82473	0.07658	0.90174	0.77171	0.27816	-0.07712	50820676	2.55E-09
	(0.0021)	(0.2349)	(0.1144)	(0.2733)	(0.4818)	(0.4379)	(0.3327)	(41015850)	(2.30E-09)
6	0.00040	0.83359	0.07433	0.90778	0.67609	0.27798	-0.07482	56201403	2.11E-09
	(0.0020)	(0.2110)	(0.1017)	(0.2423)	(0.4124)	(0.3815)	(0.2948)	(42896431)	(1.88E-09)
7	0.00053	0.82266	0.07743	0.89993	0.60013	0.24888	-0.07030	53471134	1.95E-09
	(0.0023)	(0.2045)	(0.1043)	(0.2353)	(0.3690)	(0.3441)	(0.2606)	(41341748)	(1.72E-09)
8	0.00046	0.82827	0.08092	0.90914	0.54421	0.24094	-0.03929	57187244	1.61E-09
	(0.0022)	(0.1928)	(0.0938)	(0.2143)	(0.3320)	(0.3167)	(0.2223)	(45203599)	(1.12E-09)
9	0.00037	0.83241	0.07827	0.91031	0.51530	0.23586	-0.05041	57642059	1.56E-09
	(0.0022)	(0.1709)	(0.0943)	(0.2025)	(0.2772)	(0.3026)	(0.2151)	(44813574)	(1.13E-09)
10	0.00028	0.84637	0.08020	0.92642	0.50447	0.23922	-0.05113	55999028	1.49E-09

	(0.0021)	(0.1922)	(0.0956)	(0.2227)	(0.2630)	(0.2752)	(0.1854)	(41790321)	(9.56E-10)
11	0.00033	0.87095	0.09567	0.96678	0.48858	0.22639	-0.03787	57719201	1.59E-09
	(0.0021)	(0.1757)	(0.0956)	(0.1967)	(0.2343)	(0.2586)	(0.2070)	(48697228)	(1.13E-09)
12	0.00043	0.88039	0.10190	0.98242	0.48970	0.22377	-0.05465	52497236	1.63E-09
	(0.0023)	(0.1862)	(0.0938)	(0.2164)	(0.2147)	(0.2495)	(0.2042)	(42981983)	(1.02E-09)
13	0.00048	0.89586	0.10117	0.99722	0.48542	0.21907	-0.05579	53327202	1.63E-09
	(0.0023)	(0.2071)	(0.0980)	(0.2441)	(0.2161)	(0.2552)	(0.1828)	(46520497)	(1.12E-09)
14	0.00038	0.94337	0.11479	1.05815	0.50138	0.23830	-0.05204	50962420	1.67E-09
	(0.0023)	(0.2134)	(0.0915)	(0.2295)	(0.2014)	(0.2828)	(0.1954)	(47357548)	(1.04E-09)
15	0.00039	0.95774	0.12146	1.07937	0.51527	0.20444	-0.05040	50334160	1.78E-09
	(0.0025)	(0.2231)	(0.1028)	(0.2615)	(0.1994)	(0.2618)	(0.2206)	(48255626)	(1.23E-09)
16	0.00029	0.99618	0.11994	1.11470	0.52479	0.21351	-0.05981	41711022	1.93E-09
	(0.0024)	(0.2489)	(0.1046)	(0.2915)	(0.2056)	(0.2761)	(0.2007)	(40098408)	(1.28E-09)
17	0.00037	1.03441	0.14610	1.18049	0.55665	0.18195	-0.05567	43115379	2.11E-09
	(0.0026)	(0.2712)	(0.1236)	(0.3160)	(0.2219)	(0.2512)	(0.2263)	(43131920)	(1.37E-09)
18	0.00035	1.07586	0.15338	1.22910	0.59829	0.18485	-0.07332	39757304	2.42E-09
	(0.0027)	(0.2732)	(0.1137)	(0.3284)	(0.2542)	(0.2996)	(0.2535)	(40345126)	(1.69E-09)
19	0.00036	1.11758	0.16712	1.28521	0.61270	0.13717	-0.07879	33615685	2.75E-09
	(0.0029)	(0.3069)	(0.1327)	(0.3806)	(0.2478)	(0.3366)	(0.2671)	(33190308)	(1.94E-09)
20	0.00038	1.12400	0.18955	1.31185	0.67811	0.13911	-0.08908	29761365	3.57E-09
	(0.0029)	(0.3018)	(0.1383)	(0.3765)	(0.2852)	(0.3618)	(0.2907)	(35826658)	(3.89E-09)
21	0.00037	1.18311	0.19148	1.37584	0.71212	0.13421	-0.04122	23340207	4.49E-09
	(0.0032)	(0.3683)	(0.1510)	(0.4485)	(0.3041)	(0.4366)	(0.3580)	(32023182)	(3.21E-09)
22	0.00047	1.23708	0.22865	1.46654	0.76203	0.07097	-0.08536	16598926	6.62E-09
	(0.0040)	(0.4190)	(0.1678)	(0.5198)	(0.3546)	(0.5301)	(0.4481)	(20742921)	(6.22E-09)
23	0.00063	1.30188	0.26794	1.57001	0.78905	-0.02420	-0.11735	11314978	-3.12E-08
	(0.0042)	(0.4606)	(0.1926)	(0.5807)	(0.4818)	(0.6526)	(0.4904)	(15397060)	(6.19E-07)
24	0.00065	1.35902	0.31325	1.67236	0.82124	-0.11690	-0.15333	7587408	1.66E-08
	(0.0044)	(0.4796)	(0.2154)	(0.6250)	(0.5577)	(0.8024)	(0.6344)	(12330637)	(1.40E-08)
25	0.00087	1.34023	0.38138	1.74354	0.66960	-0.18261	-0.21290	3825227	4.62E-08
	(0.0053)	(0.5525)	(0.3735)	(0.8468)	(1.0210)	(1.4035)	(1.1218)	(7335123)	(4.14E-08)

Table IV

Autocorrelation of Regression Variables for the Period 1993-2011

This table reports the autocorrelations of the variables. The data sample is for the period between February 1993 and December 2011. r_P is the daily adjusted to dividend return on the of the portfolio. β_R is the systematic risk relative to the market portfolio during the month, computed using daily data. β_A is the systematic ambiguity relative to the market portfolio during the month. β_A is computed using every 15 minutes rates of return over the month and multiplied by 26 (number of 15 minutes intervals). β_K is the systematic uncertainty relative to the market portfolio, i.e., the aggregation of β_R and β_A. The loadings $\lambda_{SMB}, \lambda_{HML}$ *and* λ_{UMD} are evaluated using daily returns and daily Fama-French factors. *SIZE* and *LIQ* are computed using daily data. t-values are in parentheses.

	t-1	***t-2***	***t-3***	***t-4***	***t-5***	***t-6***
r_P	0.08125	-0.07100	0.04449	0.02451	-0.04861	-0.03557
	(1.18526)	-(1.03573)	(0.63465)	(0.35738)	(-0.70851)	(-0.52058)
β_R	0.24942	0.07657	0.10426	0.08263	0.10390	0.05226
	(3.64375)	(1.09788)	(1.53580)	(1.20329)	(1.50696)	(0.77076)
β_A	0.16879	0.05373	0.12795	0.11512	0.08815	0.04976
	(2.48016)	(0.79239)	(1.89796)	(1.69572)	(1.29213)	(0.74658)
β_K	0.17981	0.01994	0.06107	0.05551	0.07485	0.03398
	(2.62763)	(0.29063)	(0.88325)	(0.79951)	(1.07882)	(0.50056)
λ_{SMB}	0.13189	0.08291	0.12711	0.14673	0.11949	0.08047
	(1.94526)	(1.22156)	(1.90919)	(2.20466)	(1.78085)	(1.20051)
λ_{HML}	0.09842	0.11899	0.08734	0.08574	0.06403	0.05602
	(1.44578)	(1.74472)	(1.27907)	(1.25625)	(0.93877)	(0.82401)
λ_{UMD}	0.09686	0.06024	0.07951	0.09692	0.03220	0.01006
	(1.41298)	(0.88083)	(1.17006)	(1.42069)	(0.46931)	(0.14871)
SIZE	0.12435	0.08447	0.10839	0.08882	0.07691	0.07758
	(1.82100)	(1.23617)	(1.58948)	(1.31143)	(1.13260)	(1.15115)
LIQ	0.32227	0.13784	0.16484	0.02196	0.07058	0.03092
	(4.78196)	(1.95817)	(2.34219)	(0.36570)	(1.05644)	(0.51497)

Table V

Cross correlation of Regression Variables for the Period 1993-2011

This table reports the cross correlations among the variables. The data sample is for the period between February 1993 and December 2011. r_p is the daily adjusted to dividend return of the portfolio. β_R is the systematic risk relative to the market portfolio during the month, computed using daily data. β_A is the systematic ambiguity relative to the market portfolio during the month. β_A is computed using every 15 minutes rates of return over the month and multiplied by 26 (number of 15 minutes intervals). β_K is the systematic uncertainty relative to the market portfolio, i.e., the aggregation of β_R and β_A. The loadings $\lambda_{SMB}, \lambda_{HML}$ *and* λ_{UMD} are evaluated using daily returns and daily Fama-French factors. *SIZE* and *LIQ* are computed using daily data. The significance of the cross-correlation is given by the probabilities reported in parentheses.

	r_f	r_m	r_p	β_R	β_A	β_K	λ_{SMB}	λ_{HML}	λ_{UMD}	***SIZE***	***LIQ***
r_f	1.00000	0.07221	0.04288	-0.29876	0.25151	-0.14940	0.34046	0.10936	-0.05312	-0.02139	0.13313
	(<.0001)	(0.2808)	(0.5399)	(0.0018)	(0.0275)	(0.0627)	(0.0455)	(0.0367)	(0.4238)	(0.1673)	(0.1132)
r_m	0.07221	1.00000	0.82873	-0.06624	-0.01276	-0.05787	0.07518	0.01194	-0.00423	-0.07025	0.06389
	(0.2808)	(<.0001)	(<.0001)	(0.3817)	(0.6379)	(0.4216)	(0.3189)	(0.6432)	(0.5025)	(0.3533)	(0.3375)
r_p	0.04288	0.82873	1.00000	-0.04428	-0.01439	-0.04005	0.07419	0.03567	-0.02383	-0.06295	0.09221
	(0.5399)	(<.0001)	(<.0001)	(0.4571)	(0.5968)	(0.5213)	(0.3471)	(0.5108)	(0.5813)	(0.4051)	(0.2278)
β_R	-0.29876	-0.06624	-0.04428	1.00000	0.21233	0.90577	-0.15625	-0.08559	0.04922	0.13354	-0.16223
	(0.0018)	(0.3817)	(0.4571)	(<.0001)	(0.1455)	(<.0001)	(0.0376)	(0.3697)	(0.4189)	(0.2047)	(0.0781)
β_A	0.25151	-0.01276	-0.01439	0.21233	1.00000	0.58511	0.23759	0.07482	-0.01361	-0.21040	0.30843
	(0.0275)	(0.6379)	(0.5968)	(0.1455)	(<.0001)	(<.0001)	(0.1016)	(0.2766)	(0.5316)	(0.0389)	(0.0362)
β_K	-0.14940	-0.05787	-0.04005	0.90577	0.58511	1.00000	-0.04223	-0.04222	0.03781	0.03081	-0.01608
	(0.0627)	(0.4216)	(0.5213)	(<.0001)	(<.0001)	(<.0001)	(0.1033)	(0.4070)	(0.5107)	(0.4592)	(0.1680)
λ_{SMB}	0.34046	0.07518	0.07419	-0.15625	0.23759	-0.04223	1.00000	0.38515	-0.08929	-0.10823	0.30377
	(0.0455)	(0.3189)	(0.3471)	(0.0376)	(0.1016)	(0.1033)	(<.0001)	(<.0001)	(0.2429)	(0.1447)	(0.0901)
λ_{HML}	0.10936	0.01194	0.03567	-0.08559	0.07482	-0.04222	0.38515	1.00000	0.12814	0.04272	0.07909
	(0.0367)	(0.6432)	(0.5108)	(0.3697)	(0.2766)	(0.4070)	(<.0001)	(<.0001)	(0.1472)	(0.4328)	(0.3461)
λ_{UMD}	0.42382	0.50252	0.58131	0.41895	0.53164	0.51070	0.24294	0.14723	0.00000	0.48570	0.13790
	(<.0001)	(<.0001)	(<.0001)	(0.0492)	(<.0001)	(0.0378)	(<.0001)	(0.1281)	(1.0000)	(<.0001)	(<.0001)
SIZE	-0.02139	-0.07025	-0.06295	0.13354	-0.21040	0.03081	-0.10823	0.04272	-0.01004	1.00000	-0.39255

	(0.1673)	(0.3533)	(0.4051)	(0.2047)	(0.0389)	(0.4592)	(0.1447)	(0.4328)	(0.4857)	(<.0001)	(0.0368)
LIQ	0.13313	0.06389	0.09221	-0.16223	0.30843	-0.01608	0.30377	0.07909	-0.14261	-0.39255	1.00000
	(0.1132)	(0.3375)	(0.2278)	(0.0781)	(0.0362)	(0.1680)	(0.0901)	(0.3461)	(0.1379)	(0.0368)	(<.0001)

Table VI

Regression Result

This table reports the regression results. The data sample is for the period between February 1993 and December 2011. The coefficients of the regression are as follows: Θ_R is the coefficient of beta-risk, Θ_A is the coefficient of beta-ambiguity, Θ_K is the coefficient of beta-uncertainty, Θ_{HML} is the coefficient of *HML*, Θ_{SML} is the coefficient of *SMB*, Θ_{UMD} is the coefficient of *UMD*, Θ_{SIZE} is the coefficient of size, and Θ_{LIQ} is the coefficient of liquidity. All coefficients are the average of the time series coefficients obtained from the 226 monthly cross-sectional regressions. The t-values, in parenthesis, are obtained by dividing the average value of the coefficient by its standard deviation over the 226 observations. The R^2 and the $AdjR^2$ are averages computed over the entire 226 monthly regressions.

$$r_{P,t} - r_{f,t} = \alpha_t + \Theta_{R,t}\beta_{R,j,t-1} + \Theta_{A,t}\beta_{A,j,t-1} + \varepsilon_t$$

$$r_{P,t} - r_{f,t} = \alpha_t + \Theta_{K,t}\beta_{K,j,t-1} + \Theta_{HML,t}\lambda_{HML,t-1} + \Theta_{SML,t}\lambda_{SML,t-1} + \Theta_{UMD,t}\lambda_{UMD,t-1} + \Theta_{SIZE,t}SIZE_{t-1} + \Theta_{LIQ,t}LIQ_{t-1} + \varepsilon_t$$

#	α	Θ_R	Θ_A	Θ_K	Θ_{SIZE}	Θ_{LIQ}	Θ_{SMB}	Θ_{HML}	Θ_{UMD}	R^2	$AdjR^2$
1	0.00003	0.00036								0.20387	0.16926
	(0.1998)	(2.0682)									
2	0.00034		0.00061							0.17105	0.13501
	(3.2515)		(2.0695)								
3	0.00007	0.00031	0.00014							0.29161	0.22721
	(0.5239)	(1.6760)	(0.4684)								
4	0.00008			0.00028						0.20653	0.17203
	(0.7774)			(2.3464)							
5	0.00018			0.00021	9793.31327	0.00000				0.41408	0.33037
	(1.5870)			(1.9870)	(1.1172)	(-2.2130)					
6	-0.00008			0.00021			0.00012	0.00020	-0.00029	0.41912	0.30295
	(-0.6875)			(1.9919)			(1.0442)	(1.6821)	(-1.8324)		
7	0.00013			0.00018	3711.71584	0.00000	0.00006	0.00007	-0.00019	0.54700	0.39600
	(1.1105)			(1.9563)	(0.4591)	(-2.2064)	(0.6221)	(0.6969)	(-1.3375)		