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# Merger waves in vertically related industries

Zhiyong Yao and Wen Zhou\*

## Abstract

This article studies merger waves in the vertically related industries where firms can engage in both vertical and horizontal mergers. All firms may remain independent. But when mergers do take place, they always come in waves which is either vertical or horizontal depending on the relative benefits of eliminating double markup and eliminating horizontal competition. Meanwhile, merger waves may happen with or without fundamental changes.

**Keywords:** merger wave, horizontal mergers, vertical mergers, stable market structure

**JEL Code:** L13, L42, D43

## 1 Introduction

It has been well documented that mergers come in waves, clustering in time and by industries. Some merger waves are horizontal, consisting mainly of mergers between competing firms in the same industry, while some other waves are vertical, consisting mainly of mergers between suppliers and customers across vertically related industries. For example, the first great merger wave in the U.S. (1893-1904) is horizontal. Consolidation between competitors generated corporate giants in the steel, telephone, oil, mining, railroad and other major manufacturing and transportation industries. The second great merger wave (1919-1929), by contrast, is vertical. Ford and General Motors emerged as the major automobile manufacturers through vertical integration, acquiring every business along the supply chain from iron and coal mines, railroads and ore boats, steel mills, body and assembly all the way to finished vehicles. More recently, in February 2011 Nokia and Microsoft announced a comprehensive plan to ally (vertically) in the Smart phone area in response to the success of Apple's iPhone. Several months later, Google acquired Motorola Mobility in a similar move.

Furthermore, the horizontal merger waves can happen parallel in the vertically related industries. And these two kinds of merger waves, horizontal and vertical, can switch sometimes. For example, a series of mergers in the US pharmaceutical industry occurred in 2007 to counter the increased power in the downstream health care industry, where

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consolidation had been taking place. Semiconductor companies used to be vertically integrated. With rising costs and changing cost structure, economies of scale became more important, and firms started to disintegrate vertically and merge horizontally.

These examples highlight the need to study horizontal and vertical mergers together, as a merger will alter the incentives of other mergers, and the impacts are likely to be different depending on whether the mergers are vertical or horizontal. Given the empirical observation that mergers occur in waves and that a wave can be either vertical or horizontal, the following questions immediately emerge: Why do mergers occur in waves? What causes a merger wave to take place? And what determines the wave to be vertical or horizontal? When will these two kinds of merger waves switch? There have been some economic studies on horizontal merger waves in the same industries and a few on vertical merger waves, but virtually none put the two together. This research takes a novel approach by studying mergers in an economy with vertically related industries and is therefore uniquely positioned in addressing last two questions. In doing so, however, the analysis has also shed new lights to answering the first two questions.

A simple model is hitherto constructed which contains two vertically related industries, each consisting of two firms that may engage in both horizontal and vertical mergers. Although merger decisions are made simultaneously, the equilibrium concept is defined such that a firm considers “responses” from other firms when contemplating a merger. As a result, the interaction between mergers becomes crucial in determining the equilibrium market structure. Merger waves will be analyzed as an equilibrium outcome with multiple mergers, which means all firms’ merger incentives must be considered collectively. Furthermore, to explain why a merger wave takes place at a particular time instead of earlier, we must be able to justify the original situation, i.e., all firms remaining independent, as an equilibrium, too. That is, a merger wave must be endogenized, representing a change from one equilibrium to another. Comparison of the conditions between the two equilibria will help answer the question of what triggers a merger wave.

The model demonstrates that indeed mergers occur in waves: In any equilibrium where a merger takes place, it must be accompanied by a second merger. And an equilibrium may consist of two mergers, but it may also consist of no mergers, i.e., all four firms remain independent even though any individual merger would have been profitable. A merger is profitable only when the other two firms’ configuration is fixed, i.e., they remain independent. If these firms also merge, however, the first pair will be hurt so much that they prefer the original situation in which no firms merge. It is this concern that prevents any firm from carrying out a merger.

Therefore, no-merger and merger wave can both be equilibria, under either different or the same conditions. A merger wave can consequently be endogenized so that the economy changes from one equilibrium in which no firm merges, to another equilibrium in which all firms merge. Such endogenization further points to two potential triggers of merger waves: Equilibrium may change because the underlying conditions have changed, or because it is a switch between multiple equilibria without any change in the fundamentals. The first trigger corresponds to an economic shock that changes the cost, demand, regulation or trading opportunities. Such a trigger is tangible. By contrast, the second trigger is intangible, corresponding to something trivial or totally unrelated to the underlying economic conditions, such as rumor, change in mood or expectation.

Whether a merger wave is vertical or horizontal depends on the relative benefits of these two mergers. Vertical mergers remove double markup but intensify downstream competition. Horizontal mergers eliminate horizontal competition but exacerbate the double markup problem. Whether a merger wave is vertical or horizontal depends on the relative benefits of eliminating double markup on the one hand and eliminating horizontal competition on the other. The two merger waves can switch to each other due to either the tangible trigger or the intangible trigger.

It is now well known that mergers tend to occur in waves at both the economy and the industry levels, which is usually explained by economic shocks at the respective levels (Andrade et al, 2001; Andrade and Stafford, 2004; Harford, 2005; Mitchell and Mulherin, 1996). In the industrial organization literature, there are only a few theoretical researches on merger waves (Fauli-Oller, 2000; Qiu and Zhou, 2007). However, they all focus on the horizontal merger waves in the same industry. Qiu and Zhou (2007) and Toxvaerd (2008) both explained horizontal merger waves by strategic complementarity between horizontal mergers, where the complementarity arises because a merger raises other firms' profits in the first case, and reduces the availability of potential targets in the second case. These two papers also attributed merger waves to economic shocks that change a merger's profitability. While sharing the same focus on multiple mergers and the attempt to endogenize merger waves, this paper is broader in its setting with both horizontal and vertical mergers, and richer in its finding of both tangible and intangible triggers of merger waves.

Bonanno and Vickers (1988) and Lin (1988) have demonstrated that vertical disintegration can be an equilibrium because it dampens downstream competition. They obtained vertical disintegration by assuming away double markup, which is driving vertical integration in our model. Furthermore, unlike their setting in which the only alternative to vertical integration is no-merger, firms in our model have additional options of horizontal mergers. Like the present paper, Colangelo (1995) studied the interaction between horizontal and vertical mergers in vertically related industries. While his main finding is that vertical mergers tend to preempt horizontal ones, we are more interested in merger waves, focusing in particular on justifying no-merger as an equilibrium and thereby endogenizing merger waves.

The plan of the paper is as follows. Section 2 presents and analyzes the main model. Section 3 generalizes the main model to five different variations. Section 4 concludes.

## 2 Model

### 2.1 Setup

Consider two vertically related industries, the upstream and the downstream. The upstream industry consists of two identical firms  $A$  and  $B$ , and the downstream industry also consists of two identical firms 1 and 2. A homogeneous input is produced by the upstream firms at constant marginal cost  $c$  and sold through arm's length transaction to the downstream firms, which then transform it into a homogeneous final product at zero extra cost on a one-for-one basis. Firms compete à la Cournot in both industries. Each downstream firm regards the input price, denoted as  $t$ , as given and chooses its

output facing the demand for the final product,  $p = \alpha - Q$ , where  $Q$  is the total output produced by the downstream firms. The downstream Cournot equilibrium will give rise to  $Q$  as a function of  $t$ , which is then inverted to generate the inverse demand for the input, i.e.,  $t$  as a function of  $Q$ . Facing this derived demand, the upstream firms engage in their Cournot competition.

Firms play a two-stage merger game. In stage one, each firm chooses simultaneously a merger partner. A firm can commit to independence by choosing itself as the partner. A merger can be either horizontal (between two downstream firms or two upstream firms) or vertical (between a downstream firm and an upstream firm), and it takes place if and only if two firms choose each other as the partner. A merged entity is not allowed to participate in any further mergers. The merger decisions result in some market structure, referred to as a *configuration*, which is publicly known.

Then the game proceeds to stage two. Given the realized market configuration, all remaining firms, merged or non-merged, compete a la Cournot or choose their quantities in order to maximize their payoffs. After that, within a merged entity, the two partners share the merger surplus equally. The merger surplus is calculated assuming fixed configuration among the other firms. For example, a merger between firms  $A$  and  $1$  in  $N \equiv \{A, B; 1, 2\}$  leads to  $S \equiv \{B; A1, 2\}$ , so  $\pi_A^S = \pi_A^N + \frac{1}{2}(\pi_{A1}^S - \pi_A^N - \pi_1^N)$  and  $\pi_1^S = \pi_1^N + \frac{1}{2}(\pi_{A1}^S - \pi_A^N - \pi_1^N)$ , where  $A1$  denotes the entity resulting from a merger between firms  $A$  and  $1$ , and  $\pi_i^X$  is firm  $i$ 's payoff in configuration  $X$  with  $i \in X$ .

We look for stable market configurations. A configuration is stable if neither any individual firm nor any pair of firms have any profitable deviation. A deviation is profitable if all deviators are better off (strictly for at least one) for any possible configuration among the other firms.

Since our merger game essentially belongs to one-to-one matching games, following the matching literature (for example, Roth and Sotomayor, 1990), we check both the unilateral deviation by a single firm and the collective deviation by a pair of firms. More precisely, since a merger involves two firms, the unilateral deviation alone is not enough to address merger issues. The collective deviation must also be considered, as deviating into a new merger requires the coordination of two firms changing their strategies simultaneously. Furthermore, since the merged or non-merged firms will compete or interact with each other, our merger game is a one-to-one matching with externalities. Following Sasaki and Toda (1996), our equilibrium concept requires that a deviation must be profitable under all possible configurations of the other firms. Such a requirement is obviously sufficient to justify the deviation. Meanwhile, It turns out that the requirement is just strong enough to make the set of stable configurations non-empty and small.<sup>1</sup>

## 2.2 Analysis

The game is solved by backward induction. Suppose that at the beginning of stage two, there are  $v$  vertically integrated firms,  $u$  upstream firms, and  $d$  downstream firms in the realized market configuration. A vertically integrated firm participates only in the downstream competition, and it differs from an independent downstream firm in that its

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<sup>1</sup>For more details, please refer to Sasaki and Toda (1996), who have proved the existence of the equilibrium in one-to-one matching with externalities.

input is procured at cost  $c$  rather than the market price  $t$ .<sup>2</sup>

In the downstream competition, a vertically integrated firm chooses  $q_v$  to maximize  $\pi_v \equiv (\alpha - Q - c)q_v$ , which leads to the first-order condition  $\alpha - q_v - Q = c$ , where  $Q = vq_v + dq_d$  (using symmetry), so

$$\alpha - (v + 1)q_v - dq_d = c.$$

Likewise, the first-order condition for a downstream firm is  $\alpha - q_d - Q = t$ , or

$$\alpha - (d + 1)q_d - vq_v = t.$$

These two equations lead to

$$q_v = \frac{\alpha - (d + 1)c + dt}{d + v + 1} \quad \text{and} \quad q_d = \frac{\alpha - (v + 1)t + vc}{d + v + 1}.$$

The demand for the input for independent upstream firms is therefore

$$Q_i \equiv dq_d = \frac{d[\alpha + vc - (v + 1)t]}{d + v + 1},$$

or

$$t = \frac{\alpha + vc}{v + 1} - \frac{d + v + 1}{d(v + 1)}Q_i,$$

where  $Q_i \equiv dq_d \equiv uq_u$  is the total quantity by the independent upstream or downstream firms. Facing this derived demand, an upstream firm will choose  $q_u$  to maximize  $\pi_u \equiv \left[ \frac{\alpha + vc}{v + 1} - \frac{d + v + 1}{d(v + 1)}Q_i - c \right] q_u$ . The Cournot equilibrium is then given by

$$q_u = \frac{d(\alpha - c)}{(u + 1)(d + v + 1)}.$$

Consequently,

$$q_d = \frac{u(\alpha - c)}{(u + 1)(d + v + 1)}, \quad q_v = \frac{[d + (u + 1)(v + 1)](\alpha - c)}{(u + 1)(v + 1)(d + v + 1)},$$

and  $Q = \frac{[(u + 1)(v + 1)(d + v) - d](\alpha - c)}{(u + 1)(v + 1)(d + v + 1)}.$

It is clear that all profits are proportional to  $(\alpha - c)^2$ , so we can normalize  $\alpha - c \equiv 1$  without losing any generality. Then, the profits of the three types of the firms are:

$$\begin{aligned} \pi_v(v, u, d) &= \frac{[d + (u + 1)(v + 1)]^2}{(u + 1)^2(v + 1)^2(d + v + 1)^2}, \\ \pi_u(v, u, d) &= \frac{d}{(u + 1)^2(v + 1)(d + v + 1)}, \\ \pi_d(v, u, d) &= \frac{u^2}{(u + 1)^2(d + v + 1)^2}. \end{aligned}$$

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<sup>2</sup>As argued by Salinger (1988), a vertically integrated entity will withdraw from the upstream competition—it neither buys the input from other upstream firms nor sells it to other downstream firms.

Furthermore, given our quantity competition environment, the commitment issue is not as serious as the price competition model (Hart and Tirole, 1990).

Now move back to stage one. There are six possible market configurations as listed in Table 1. Firms' payoffs can be calculated using the formula derived above. For example, in  $S_1 \equiv \{A, B; 1, 2\}$ ,  $v = 0$ ,  $u = 2$ ,  $d = 2$ . As specified earlier, each merged entity divides the merger surplus/deficit equally between the two partners.

Table 1 Payoffs in the six configurations<sup>3</sup>

Configuration	$\pi_A$	$\pi_B$	$\pi_1$	$\pi_2$	Total profit
$S_1 \equiv \{A, B; 1, 2\}$	74	74	49.4	49.4	247
$S_2 \equiv \{AB; 12\}$	62.5	62.5	31	31	188
$S_3 \equiv \{A1, B2\}$	62.5	62.5	48.6	48.6	222
$S_4 \equiv \{B; A1, 2\}$	99	42	74.5	28	243
$S_5 \equiv \{AB; 1, 2\}$	83	83	28	28	222
$S_6 \equiv \{A, B; 12\}$	55.5	55.5	55.5	55.5	222

**Proposition 1.** *Only  $S_1$  and  $S_3$  are stable.*

*Proof:*  $S_1$  is stable. There are three possible deviations, but none of them are profitable:  $A + B$  is unprofitable when 1 and 2 merge;  $1 + 2$  is unprofitable when  $A$  and  $B$  merge; and  $A + 1$  is unprofitable when  $B$  and 2 merge.<sup>4</sup>

$S_2$  is not stable because  $A + 1$  is a profitable deviation:  $A$  is indifferent and 1 is strictly better off if  $B$  and 2 merge; both  $A$  and 1 are strictly better off if  $B$  and 2 remain independent.

$S_3$  is stable. There are three possible deviations (excluding symmetric ones), but none of them are profitable: A breakup (either unilateral or collective) between  $B$  and 2 is unprofitable when  $A$  and 1 remain merged;  $A + B$  is unprofitable when 1 and 2 merge;  $1 + 2$  is unprofitable when  $A$  and  $B$  merge.

$S_4$  is not stable because  $B + 2$  is a profitable (whether  $A$  and 1 separate or remain merged).

$S_5$  is not stable because  $1 + 2$  is a profitable (whether  $A$  and  $B$  separate or remain merged).

$S_6$  is not stable because  $A + B$  is a profitable (whether 1 and 2 separate or remain merged). **Q.E.D.**

Proposition 1 says that the four firms either remain independent or carry out two vertical mergers. To understand the intuition, it is useful to summarize the payoffs in Table 1 into the following three profitability ranking:

*R1:* An exogenous merger, i.e., fixing the configuration among the remaining two firms, is always profitable. The reason is because the merger internalizes either the horizontal externality (horizontal competition) or the vertical externality (double markup), which benefits the merging partners.

*R2:* A merger always hurts other firms and consequently a breakup always benefits other firms. A vertical merger will makes the merged entity more aggressive in the

<sup>3</sup>The fractions of the payoffs are turned into numerical values by multiplying 1000 for ease of comparison.

<sup>4</sup>For convenience, we use  $i + j$  to denote a merger between firms  $i$  and  $j$ .

downstream competition by eliminating the double markup, which hurts other firms. A horizontal merger between duopolists will hurt the other industry by reducing quantities supplied or demanded.

*R3*:  $S_1$  Pareto dominates  $S_3$  which in turn dominates  $S_2$ .  $S_3$  involves only the horizontal externality,  $S_2$  involves only the vertical externality, but  $S_1$  has both. Compared to the single monopoly case (where all four firms merged), the vertical externality will reduce the quantity produced, while the horizontal externality will increase the quantity produced. In  $S_1$ , these two opposite effects will comprise each other, which leads  $S_1$  dominates both  $S_2$  and  $S_3$ . But these two opposite effects are not completely cancelled out since the vertical externality is stronger than the horizontal one here, which let  $S_3$  dominate  $S_2$ .

First, notice that it cannot be stable to have only one merger because the remaining two independent firms will be better off by merging. They gain if the originally merged firm remains merged (*R1*), and will gain even more if the merged firm breaks up (*R2*). Second, having two horizontal mergers is unstable, as an upstream firm and a downstream firm would rather merge vertically. Such a deviation is profitable if the other two firms also merge vertically (*R3*), and is even more profitable if the other two firms do not merge (*R2*).

That leaves us with two configurations: all firms remain independent ( $S_1$ ) or carry out two vertical mergers ( $S_3$ ). Both are stable. Consider first the no-merger case. An exogenous merger would have been profitable (*R1*). However, if the other two firms also merge, the first merger becomes unprofitable (*R3*). Therefore, no deviation (in the form of a merger) will be carried out. Next consider the case of two vertical mergers. It is unprofitable to either break up (if the other merged firm remains merged—*R1*) or have a horizontal merger (if the other two firms also merge horizontally—*R3*).

### 2.3 Discussion

Proposition 1 highlights four conclusions that merit discussion. First, mergers occur in waves. Whenever there is a merger, it must be accompanied by another one. A single merger can never prevail because it hurts the other two firms so much that a merger between those two becomes a dominant strategy: Given a merger, the other merger is profitable whether the first pair dissolves or remains merged.

Second, a merger wave may be vertical or horizontal depending on their relative benefits. In our 2 by 2 setting, a merger wave may consist of two vertical mergers ( $S_3$  or simply 2V) or two horizontal mergers ( $S_2$  or 2H), referred to respectively as vertical and horizontal merger waves.<sup>5</sup> As we have mentioned above, the damage or strength of double markup is greater than that of horizontal competition, and firms end up taking vertical mergers rather than horizontal ones. Notice that the relative benefits depend on the model setting. If, for example, the competition is a la Bertrand rather than Cournot, in which case the merger wave will be horizontal. In the next section, we are going to explore more about the conditions for a merger wave to be vertical or horizontal.

Third, a merger wave may take place without any fundamental change. Proposition 1

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<sup>5</sup>Here the horizontal wave consists of a horizontal merger in each of the two industries, which is slightly different from the usual meaning of multiple horizontal mergers in the same industry.



predicts multiple equilibria in this game. The firms may coordinate on  $S_1$  and all the four firms remain independent. However, they may change suddenly from  $S_1$  to  $S_3$ , carrying out two vertical mergers. Because both are equilibria, shifting from  $S_1$  to  $S_3$  does not require any change in the underlying economic conditions such as demand or technology. The trigger may be something trivial or totally unrelated: mood, expectation, rumor, etc.

Although 4I and 2V are both equilibria, they are not on an equal footing. Switching from 4I to 2V is possible or even likely because, if for whatever reason, a pair of firms expect the other pair will merge, they will surely follow suit. The reverse process of jumping from 2V to 4I, interpreted as a divestiture wave, is more difficult. If for some reason a pair breaks up, it is in the best interest of the second pair not to follow. Such an asymmetry between the two equilibria may explain why in real life, merger waves are much more common than divestiture waves.

Fourth, a merger wave may not materialize. In particular,  $S_1$  or 4I is also an equilibrium. When multiple mergers are possible, merger incentives should not be considered in isolation; the interaction between mergers becomes crucial. Qiu and Zhou (2007) have shown that in a single industry, a merger raises the profitability of another merger, so an unprofitable merger may be carried out in anticipation of subsequent mergers. In this model, the effect of a merger is the opposite: it reduces the profits of other firms. Firms refrain from mergers for fear of other mergers.

### 3 Generalization

The results discussed so far are derived from a highly stylized model with symmetric  $2 \times 2$  firms, constant marginal costs, homogeneous products, linear demand and Cournot competition. One may wonder whether the conclusions still hold under more general conditions. In this section we will investigate five variations of the main model by relaxing its assumptions, one at a time, so that product is differentiated, the competition is in price rather than in quantity, the marginal cost is increasing, or the demand is non-linear.<sup>6</sup> All conclusions reached in the main model are found to be robust. More importantly, since the main model is a special case of the variation models, we are able to put the conclusions in perspective and understand better their meaning, conditions and reasons. In particular, we will discuss the conditions for vertical and horizontal merger waves, and identify two types of causes that triggers a merger wave.

#### 3.1 Product differentiation

Suppose that the final products are differentiated (the inputs are still homogeneous) so that the demand for firm  $i$ 's product is  $p_i = \alpha - q_i - \beta q_j$ , where  $i, j \in \{1, 2\}$  with  $i \neq j$ , and  $\beta \in [0, 1]$  represents the degree of product differentiation. Furthermore, the competition in each industry may be in either quantity or price, which gives rise to the following three combinations: Cournot competition in both industries (Cournot-Cournot); upstream Cournot competition and downstream Bertrand competition (Cournot-Bertrand); and

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<sup>6</sup>Furthermore, a 3X3 case is analyzed in the appendix.

Bertrand competition in both industries (Bertrand-Bertrand). All other aspects of the game remain the same as in the main model. For each of the three cases, a payoff table similar to Table 1 can be constructed, where each payoff is a function of  $\beta$  only.<sup>7</sup>

**Proposition 2.**

- (1) In Cournot-Cournot,  $S_1$  is stable for  $\beta > 0.29$  and  $S_3$  is stable for any  $\beta$ .
- (2) In Cournot-Bertrand,  $S_1$  is stable for  $0.22 < \beta < 0.8$ ,  $S_2$  is stable for  $\beta > 0.66$ , and  $S_3$  is stable for  $\beta < 0.66$ .
- (3) In Bertrand-Bertrand,  $S_3$  is stable for  $\beta < 0.66$  and  $S_2$  is for any  $\beta$ .

*Proof:* See the appendix.

The equilibrium results are summarized in Table 2, which also shows the results from Proposition 3. The parameter range indicates the condition for a particular configuration to be stable, and the parenthesis below indicates the profitable deviation that makes the configuration unstable when the condition is violated. For example, in the Cournot-Cournot case,  $S_1$  is stable for  $\beta > 0.29$ . If  $\beta < 0.29$ ,  $S_1$  becomes unstable because  $A + B$  is a profitable deviation.

Table 2: Results of Propositions 2 and 3

<i>Cases</i>	$S_1$ (4I)	$S_2$ (2H)	$S_3$ (2V)
<i>Cournot-Cournot</i>	$\beta > 0.29$ (A + B)	<i>Never</i> (A + 1)	<i>Always</i>
<i>Cournot-Bertrand</i>	$0.22 < \beta < 0.80$ (A + B) (1 + 2)	$\beta > 0.66$ (A + 1)	$\beta < 0.66$ (A + B)
<i>Bertrand-Bertrand</i>	<i>Never</i> (A + B)	<i>Always</i>	$\beta < 0.66$ (A + B)
<i>Increasing mc</i>	$\gamma < 2$ (1 + 2)	$\gamma > 2$ (A + 1)	$\gamma < 2$ (1 + 2)
<i>General demand</i>	<i>Always</i>	$\sigma > 1$ (A + 1)	$\sigma \leq 1$ (A + B)

Note that the market configurations with a single merger ( $S_4$ ,  $S_5$  and  $S_6$ ) are still unstable. The reason is as what we have discussed in the main model, which will be further elaborated later. Below we explain why and under what conditions the configurations with no-merger ( $S_1$ ) and the two merger waves ( $S_2$  and  $S_3$ ) are stable.

■ Cournot-Cournot

The setting is the same as the main model except that the final products are differentiated, so the main model is a special case with  $\beta = 1$ . Greater differentiation between the final products weakens the downstream competition. If products are sufficiently differentiated ( $\beta$  is small), a downstream merger in 4I will not reduce the quantity demanded for inputs by much and therefore will do little damage to the upstream firms. Recall that upstream firms refrain from an otherwise profitable merger for fear of the negative impact of a downstream merger. If the impact is expected to be small, upstream firms will merge, and as a result 4I becomes unstable.

<sup>7</sup>Again  $\alpha - c$  can be normalized without losing any generality.

In the main model when products are homogeneous, 2V preempts 2H. Product differentiation will further strengthen the dominance of 2V over 2H because weaker downstream competition reduces 2V's disadvantage (of facing downstream competition) and 2H's advantage (of mitigating downstream competition). As a result, 2V is always stable while 2H is never stable for  $\beta \in [0, 1]$ .

■ Cournot-Bertrand

When downstream firms compete in price rather than quantity, the downstream competition is intensified. 4I is stable only if products are modestly differentiated (intermediate  $\beta$ ). As in the previous case, when products are highly differentiated, the upstream firms do not need to worry about the negative impact of a downstream merger, so they merge. When products are close substitutes, on the other hand, the downstream competition is fierce. Downstream firms will merge to avoid this competition.

As explained earlier, product differentiation favors 2V over 2H, so 2V continues to be stable when products are sufficiently differentiated (small  $\beta$ ). Conversely, when products are close substitutes (large  $\beta$ ), fierce competition in the downstream industry raises the benefits of horizontal mergers and the drawbacks of vertical mergers, so 2H becomes stable.

■ Bertrand-Bertrand

The upstream firms also compete in price now, so the upstream competition is greatly intensified. In fact it is intensified to the greatest extent as inputs are homogeneous. 4I is never stable because upstream firms earn zero profits in 4I and they will always attempt to merge.

As in the Cournot-Bertrand case, when the final products are close substitutes, 2H is stable; when products are sufficiently differentiated, 2V is stable. In the latter case when products are very differentiated, 2H is dominated by 2V, but 2H is still stable. Deviating in 2H to a vertical merger may hurt the upstream partner because its contribution to, and hence its profit share within, the vertical merger will be small if the other two firms do not merge (it will earn zero without its own merger). So 2H is stable for all degrees of product differentiation.

Colangelo (1995) has also studied the trade-off between vertical and horizontal mergers. His game 3 is similar to our Bertrand-Bertrand game except that merger decisions are made sequentially with exogenous target for the initial acquisition. Despite the difference in game rules, predictions from the two models are largely consistent: 2H for small product differentiation and 2V for large differentiation. This indicates that our game rule is able to capture the essence of merger incentives without specifying the details of the merger process.

### 3.2 Cost and demand

We now move back to the setting with homogeneous final product and Cournot competition in both industries. In the main model, the cost of input production and the demand for the final product are both linear:  $C(q) = cq$  and  $p = \alpha - Q$ . On the cost side, the model can be generalized to allow increasing marginal cost (mc):  $C(q) = \gamma q^2$

with  $\gamma \geq 0$ . Note that the main model is a special case of increasing mc with  $\gamma = 0$ .<sup>8</sup> On the demand side, it can also become more general in the form of  $p = \alpha - Q^\sigma$  with  $\alpha > 0$  and  $\sigma > 0$ . Again the main model is a special case with  $\sigma = 1$ . Note a peculiar property of this general demand function: its concavity, defined as  $\frac{p''(Q)Q}{p'(Q)}$ , is constant.<sup>9</sup>

**Proposition 3.**

(1) In the increasing marginal cost case with  $C(q) = \gamma q^2$ ,  $S_1$  and  $S_3$  are stable for  $\gamma < 2$ , and  $S_2$  is stable for  $\gamma > 2$ .

(2) In the more general demand case with  $p = \alpha - Q^\sigma$ ,  $S_1$  is stable for any  $\sigma$ ,  $S_2$  is stable for  $\sigma > 1$ , and  $S_3$  is stable when  $\sigma \leq 1$ .

*Proof:* See the appendix.

The results are summarized in Table 2. Below are the explanations for the stable configurations.

■ Increasing marginal cost

Producing the inputs at increasing marginal costs will reduce the upstream competition.<sup>10</sup> When the marginal cost increases fast with quantity ( $\gamma$  is large), 4I is unstable. This is because an upstream merger in 4I will not raise the input price by much and therefore will do little damage to downstream firms, who would then like to merge.

An increasing marginal cost favors 2H over 2V. The benefit of a vertical merger comes from output expansion due to the elimination of double markup. When the marginal cost of input increases with quantity, expansion is increasingly costly, so the benefits of vertical mergers decrease. When the marginal cost curve is flat ( $\gamma$  is small), the drop in the benefits of vertical mergers is small, so 2V continues to dominate 2H, meaning that 2V is stable while 2H is not.

■ General demand

As before, the equilibrium outcome depends on the relative strength of the vertical externality and the horizontal externality. On the one hand, when the demand is convex ( $\sigma < 1$ ), the double markup problem is more serious than the horizontal competition. Therefore, 2V dominates 2H. On the other hand, when the demand is concave ( $\sigma > 1$ ), horizontal externality is more serious so that 2H is stable while 2V is unstable.<sup>11</sup> In 4I, the horizontal externality and the vertical externality coexist and they mitigate each other, and thus 4I is always stable.

<sup>8</sup>In the main model, the exact value of  $c$  is inconsequential, as all profits are proportional to  $(\alpha - c)^2$  and so  $\alpha - c$  has been normalized without any loss of generality. We may as well think that  $c = 0$ .

<sup>9</sup>In fact,  $\frac{p''(Q)Q}{p'(Q)} = \sigma - 1$ . This function has been used by Greenhut and Ohta (1976) and other researchers in studying merger incentives and/or vertically related industries.

<sup>10</sup>In terms of oligopoly behavior, product homogeneity with increasing marginal costs is mathematically equivalent to product differentiation with constant marginal cost (Vives, 1999). In a sense, the increasing marginal cost considered here introduces differentiation to the inputs.

<sup>11</sup>According to Ziss (2005) Lemma 1, if the concavity of the demand for the final product is constant and equal to  $\sigma - 1$ , the demand for the input is also constant and equal to  $\sigma - 1$ .

### 3.3 Discussion

#### 3.3.1 Partial mergers

Out of the six possible configurations for each of the above five cases, partial merger ( $S_4$ ,  $S_5$  and  $S_6$ ) is never stable. The reason is still the two profitability ranking that we have seen in the main model: an exogenous merger between any two firms is always profitable ( $R1$ ), while a merger always hurts other firms ( $R2$ ). These two ranking are still valid in generalized models.

Consider first an upstream horizontal merger. Merging from duopoly into monopoly should always be profitable. The complication here is the presence of the downstream industry, which may affect the upstream merger's profitability. However, the demand that upstream firms face is derived from the downstream competition. As long as the downstream configuration is fixed, that demand does not change. Since the profitability of merging from duopoly into monopoly is unconditional, an upstream merger continues to be profitable when there is a downstream industry. Because the upstream merger raises the input price, it hurts downstream firms.

Next consider a downstream horizontal merger. Again it's a merger from duopoly to monopoly, and again the complication is the presence of a vertically related industry, the upstream. In particular, upstream firms may try to share the benefit of the downstream merger by raising the input price, which will reduce the profitability of the downstream merger. However, it can be verified that in the main model as well as the five models considered above, a downstream merger does not change the equilibrium input price.<sup>12</sup> As a result, a downstream merger continues to be profitable when there is an upstream industry. Because the downstream merger reduces quantity demanded for inputs without changing the input price, it hurts upstream firms.

Finally consider a vertical merger. Because the number of firms in the two industries are equal, the two merging firms must produce the same quantity without the merger. Then the merger must be profitable because it enables the merging firms to internalize the externality generated by double markup. Because the merger makes the merged firm more competitive in the downstream competition, it hurts other firms.<sup>13</sup>

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<sup>12</sup>In the main model, for example, the equilibrium input price is  $t = c + \frac{\alpha - c}{(u+1)(v+1)}$ , which does not depend on  $d$ . So if there is a downstream merger (i.e.,  $d$  decreases by 1), the equilibrium input price will not change. A downstream merger rotates the demand for inputs clockwise around the intercept. This rotation has two opposite effects on  $t$ : For any given  $t$ , the quantity demanded is smaller, so  $t$  tends to drop. At the same time, the demand becomes less elastic, so  $t$  tends to rise. In the case of linear demand for the final product, the two forces exactly cancel out each other so the equilibrium input price will not depend on the number of downstream firms. For non-linear demand, Greenhut and Ohta (1976) and Ziss (2005) have shown the same invariance as long as the demand has constant concavity, which is satisfied by the general demand we adopt here.

<sup>13</sup>A minor exception is when firms compete in prices in both industries (Bertrand-Bertrand). A vertical merger removes a competitor from the upstream industry and therefore benefits the remaining independent upstream firm (its profit increases from zero to positive). But this will not affect the equilibrium results.

### 3.3.2 No-merger

As mentioned earlier, in all cases except the Bertrand-Bertrand one, 2V eliminates double markup but retains horizontal competition, so the firms over-produce from the viewpoint of their joint profits, i.e., a horizontal externality. By contrast, 2H removes horizontal competition but has double markup problem, so the firms under-produce, which can be viewed as a vertical externality. 4I has both externalities, but since the two move in opposite directions, one mitigates the other, so the firms' joint profits tend to increase. Furthermore, If the two opposite effects are more or less balanced, 4I is stable. In the Bertrand-Bertrand case, however, only horizontal externality exists in 4I. Therefore, 4I is never stable but 2H is always stable.

The stability of 4I is always threatened by a horizontal merger, not a vertical one. For a vertical merger to be profitable in 4I, the vertical pair will have to earn more. Because the two vertical pairs are symmetric, the four firms' total profits must therefore be higher in 2V than in 4I, which in general does not hold. By contrast, 4I may be disrupted by a horizontal merger; all it needs are benefits accruing to one horizontal pair, not both pairs. This is possible because the two industries are inherently asymmetric and so the two pairs' profits may be very different.

So it is possible for all four firms to refrain from any merger if the horizontal externality and the vertical externality are more or less balanced. When the balance is disrupted by, say, intensified or weakened competition in one industry, firms will take up a horizontal merger, and 4I will no longer be stable. Take the Cournot-Bertrand case as an example: When the final products are close substitutes, the downstream horizontal competition is strong, so the downstream firms will merge to internalize this externality. When they are very poor substitutes, but the upstream firms' competition is strong and they will merge. As a result, 4I is stable only for moderate differentiation in the final products.

### 3.3.3 Vertical and horizontal merger wave

In most cases except the Bertrand-Bertrand one, 2H and 2V are mutually exclusive: one is stable whenever the other is unstable, and vice versa. This is because the threat to the stability of a merger wave is always the other wave. As mentioned earlier, whether a merger wave is vertical or horizontal depends on the relative benefits of eliminating double markup on the one hand and eliminating horizontal competition on the other.

Intensified competition in the downstream industry reduces the benefits of vertical mergers and increases the benefits of downstream horizontal mergers, making it more likely for 2H to prevail. That is why price (rather than quantity) competition in the downstream and closer substitutability between final products favor 2H over 2V.

To summarize, a merger wave tends to be vertical when the double markup problem is more serious, and a wave tends to be horizontal when the horizontal competition is more serious.

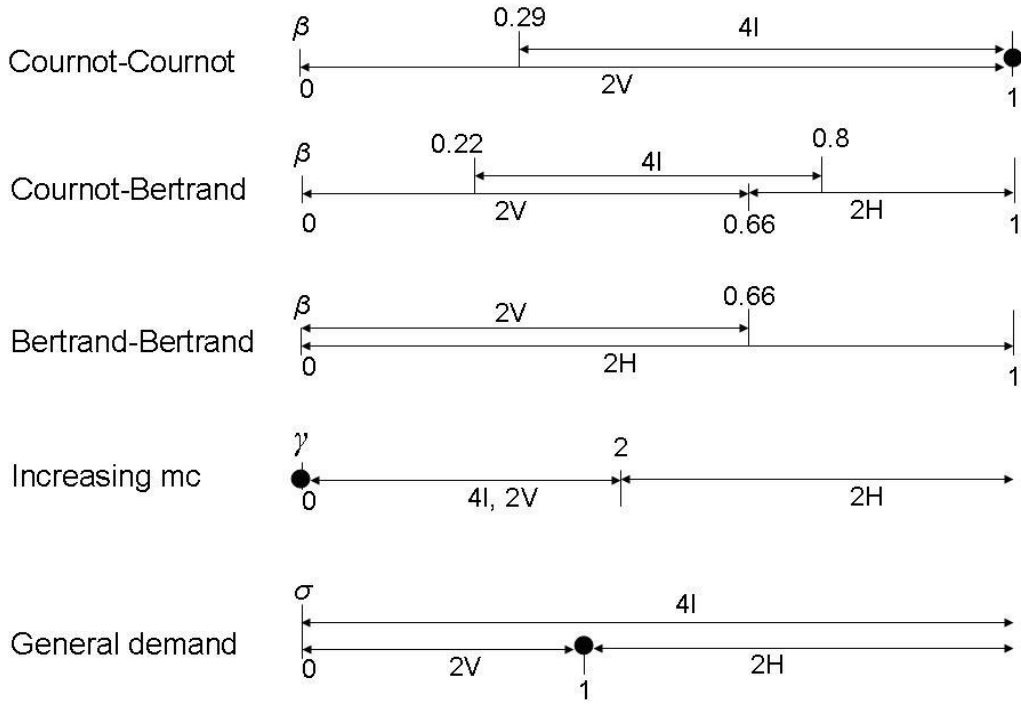


Figure 1: Stable configurations with the five cases

### 3.3.4 Causes of merger waves

So far we have been talking about the equilibrium conditions: Sometimes firms refrain from mergers; sometimes they carry out a series of mergers. The next question is, what causes firms to merge? A model of endogenous mergers must explain not only why a merger takes place, but also why it did not take place earlier. In other words, no-merger and mergers must both be justified as equilibria, possibly under different conditions. And we would like to understand what causes the equilibrium to switch from 4I to 2V or 2H. The generalized models shed some light on the answers to this question. In all these models, at any parameter value, there is at most two stable configurations and at least one. These equilibrium outcomes are demonstrated in Figure 1, where the bullet points represent the main model's result as a special case.

For some ranges of the parameters, there is a unique stable configuration, which is invariably a merger wave, sometimes 2V and sometimes 2H. If the parameter falls into that range, the merger wave should take place immediately, so the only explanation why the mergers did not take place earlier is that the parameter was outside the range, where all firms remaining independent was an equilibrium. Such a change of parameter value can be interpreted as an economic shock. For example, in Cournot-Cournot, 2V is the unique equilibrium for  $\beta < 0.29$ , and 4I is an equilibrium (though not unique) for  $\beta > 0.29$ . If something changes  $\beta$  from a value greater than 0.29 to a value below it, the market will undergo a wave of vertical mergers. Therefore, greater differentiation in

the final products (due to, say, more investment in R&D or advertising) may trigger a vertical merger wave. Likewise, a more convex demand may also trigger a vertical merger wave. Similarly, a horizontal merger wave may be triggered by smaller differentiation in the final products (Cournot-Bertrand), steeper marginal cost in input production, or greater concavity of demand.

For some other ranges of the parameters, there are exactly two equilibria and, in most cases, one is 4I and the other is either 2V or 2H. When that is the case, it is possible for a merger wave to take place without any change in the underlying economic conditions, as we have seen in the main model. For example, this may happen for a vertical merger wave when products are close substitutes (Cournot-Cournot), marginal cost rises slowly, or the demand is convex, and for a horizontal merger wave when product differentiation is moderate (Cournot-Bertrand) or the demand is concave.

So there are two types of causes that may trigger a merger wave, one is tangible and the other is not. As explained above, endogenous mergers take place when the economy switches from one equilibrium in which all firms remain independent to another equilibrium in which all firms merge.<sup>14</sup> This change of equilibrium can be brought in two ways. The first is a change in the underlying economic conditions, which is interpreted as an economic shock. This is a *tangible trigger* for merger waves. The second is a shift between multiple equilibria without any change in the underlying economic conditions. This is an *intangible trigger*, corresponding to rumors, or a sudden change in expectation or mood.

Qiu and Zhou (2007, 2010) have found that horizontal merger waves can be caused by demand or cost shocks. The present work goes beyond that. The setting is broader to encompass both horizontal and vertical mergers. The finding is richer: a merger wave can be triggered by not only tangible reasons such as economic shocks, but also intangible reasons such as mood or expectation. In real life, both types of causes can be observed: Sometimes a clear trigger can be identified in the form of a sudden or gradual change in the underlying economic conditions such as demand, technology, regulation, or opening to international trade, but sometimes a series of mergers may take place without any obvious change in the fundamentals.

## 4 Conclusion

We have studied endogenous market structure in vertically related industries where firms can engage in both vertical and horizontal mergers. Because of concerns for subsequent mergers, all firms may remain independent even though any individual merger would have been profitable. When mergers do take place, they always occur together, forming a merger wave that is either vertical or horizontal depending on the relative damage of double markup and horizontal competition. By endogenizing merger waves as a change from one equilibrium in which no firm merges, to another equilibrium in which all firms merge, we are able to identify two triggers for a merger wave, a tangible one corresponding to a change in the underlying economic conditions, and an intangible one that shifts the

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<sup>14</sup>Note that in Bertrand-Bertrand, 2V and 2H can switch to each other, which can explain the semiconductor industry case mentioned in the first paragraph of this article



economy between multiple equilibria without any change in the fundamentals.

The research demonstrates the importance of interactions between mergers and the connection, or the lack of it, between merger waves and the underlying economic conditions. Mergers are carried out for competition purposes: In the absence of merger-specific cost or benefit and the antitrust authority, firms merge in order to reduce competition and/or gain competitive advantage over competitors or upstream/downstream firms. More broadly, a merger may have both a competition effect and a merger-specific effect such as cost savings or synergies. By focusing on the first effect, this research helps us understand as a first step how interaction between mergers shapes the equilibrium market structure, and how the structure may change with or without any change in the underlying economic conditions. Such understanding and analytical framework are useful in understanding intuitions where the second effect is also present. For example, cost or demand changes that are peculiar to a merger may precipitate it, which will then trigger a dramatic change in the overall market structure involving several related industries. The anticipation of subsequent changes in market structure will in turn alter the incentive of the first merger. That is, the second effect can be added to the analytical framework, and the interaction between the two effects must be analyzed simultaneously.

The model adopts the simplest possible setting with two upstream and two downstream firms, or  $2 \times 2$ . In addition to simplicity, it provides a natural setting where an exogenous merger always benefits the merging firms and hurts non-merging firms. These effects conform to common understanding of what a merger means and are robust regardless of where firms compete in price or quantity. So firms do not suffer from “merger paradox” in which a merger always hurts merging firms and benefits non-merging firms in the same industry (Salant et al, 1983). Needless to say, this simplistic setting has its drawbacks. When a horizontal merger takes place, there is no possibility for a vertical merger. So horizontal and vertical merger waves are mutually exclusive, and there is always a tradeoff: a benefit is accompanied by a drawback. Also, merging from duopoly into monopoly is special. Strange as it may appear, the merger paradox captures the important competition effect of mergers within an industry (in the absence of merger-specific cost savings) and therefore cannot be ignored. So it is important to go beyond the  $2 \times 2$  setting.

In the appendix, we analyzed the  $3 \times 3$  setting and showed that there are two equilibria, one in which all firms remain independent and the other in which three vertical mergers take place. Similar patterns have been found in settings of  $3 \times 4$ ,  $4 \times 3$  and  $4 \times 4$ , so Proposition 1 is indeed robust to the number of firms in either industry. A technical difficulty when there are more than two firms in an industry is that the equilibrium set tends to be large that contains some unreasonable market structures, so the equilibrium concept has to be modified slightly. More work needs to be done along this direction.

We have defined a special equilibrium concept in this model because existing concepts such as core or stable equilibrium are ill suited to study the questions at hand. Our equilibrium concept has two key components: simultaneous move (in terms of merger decisions) and pessimistic view (deviators worry about the worst configuration among the remaining firms). A sequential game has the advantage of generating strategic incentive for mergers and may therefore account as another explanation for merger waves (Qiu and Zhou, 2007). But the disadvantage is that the equilibrium will be sensitive

to the order of move and other details of the game setting, which a researcher has no compelling reason or sufficient information to specify exogenously. For example, Colanagelo's (1995) sequential game leads to very different predictions of the equilibrium market structure depending on which firm is the first acquisition target, which is exogenously given. Pessimistic view has been adopted in the coalition formation literature as one of several possible refinements for deviations (the  $\alpha$  game versus the  $\gamma$  game). The essence is that firms are allowed to "respond" to a deviation without specifying why or how they do so. The responses are important because a merger's profitability depends on the configuration among non-merging firms, and in real life when merger decisions are made sequentially, merging firms will look ahead and anticipate the likely configuration among the remaining firms. Therefore, the pessimistic view is particularly useful when adopted in combination with simultaneous move. The equilibrium concept thus defined is able to capture the essential interaction between mergers without specifying how the interaction is carried out. That is, the pessimistic view mitigates the disadvantage of simultaneous games.

Finally, people may have concerns about the sharing rule and the assumption that the two industries do business by arm's length transaction. We have assumed that merging firms split the merger surplus equally fixing the configuration among remaining firms, which seems to contradict our equilibrium concept where the configuration is variable when firms contemplate deviations. However, if configurations are allowed to vary in calculating profit sharing, the profits will be ill defined because there are multiple configurations. Arm's length transaction may seem problematic when an industry has only one firm. But what is really needed is double markup and that fact that input price should be affected (in principle) by the market structure. Such a setting is relevant in at least some situations. An alternative to arm's length transaction may be two-part tariff. But our Bertrand-Bertrand game removes double markup and may therefore capture what will happen when two-part tariff is assumed.

## 5 Appendix

### Procedures of proving Propositions 2 and 3

The procedures are the following. In each case, we construct the payoff table similar to Table 1 (the tables are omitted and are available up request). Each payoff will be a function containing relevant parameters. It turns out that  $\alpha$  and  $c$  can be normalized without any loss of generality. Then each payoff will depend on only one parameter.

Given the payoffs, we check the stability of each configuration. For a configuration to be stable, we have to check every possible deviation, but for any given deviation, it only needs to be unprofitable for one firm (does not have to be for both firms). For a configuration to be unstable, we only need to find one profitable deviation; if it is a collective deviation, the deviation has to be profitable for both firms.

For  $S_1 = \{A, B; 1, 2\}$  to be stable: (i)  $A + B$  is unprofitable:  $\pi_A^{S_1} > \min\{\pi_A^{S_2}, \pi_A^{S_5}\}$ . (ii)  $1 + 2$  is unprofitable:  $\pi_1^{S_1} > \min\{\pi_1^{S_2}, \pi_1^{S_6}\}$ . (iii)  $A + 1$  is unprofitable:  $\pi_i^{S_1} > \min\{\pi_i^{S_3}, \pi_i^{S_4}\}$  for  $i = A$  or  $1$ .

For  $S_2 = \{AB; 12\}$  to be stable: (i) Breaking  $AB$  is unprofitable:  $\pi_A^{S_2} > \min\{\pi_A^{S_1}, \pi_A^{S_6}\}$ .

(ii) Breaking 12 is unprofitable:  $\pi_1^{S_2} > \min\{\pi_1^{S_1}, \pi_1^{S_5}\}$ . (iii)  $A + 1$  is unprofitable:  $\pi_i^{S_2} > \min\{\pi_i^{S_3}, \pi_i^{S_4}\}$  for  $i = A$  or 1.

For  $S_3 = \{A1, B2\}$  to be stable: (i) Breaking  $B2$  is unprofitable:  $\pi_i^{S_3} > \min\{\pi_i^{S_1}, \pi_i^{S_4}\}$  for  $i = B$  and 2. (ii)  $A+B$  is unprofitable:  $\pi_A^{S_3} > \min\{\pi_A^{S_2}, \pi_A^{S_5}\}$ . (iii)  $1+2$  is unprofitable:  $\pi_1^{S_3} > \min\{\pi_1^{S_2}, \pi_1^{S_6}\}$ .

For  $S_4 = \{B; A1, 2\}$  to be unstable, we only need one of following cases holds: (i)  $B + 2$  is profitable:  $\pi_i^{S_4} < \min\{\pi_i^{S_3}, \pi_j^{S_4}\}$  for  $i = B, 2$  and  $j = A, 1$ . (ii)  $A + B$  is profitable:  $\pi_A^{S_4} < \min\{\pi_A^{S_2}, \pi_A^{S_5}\}$ . (iii)  $1 + 2$  is profitable:  $\pi_1^{S_4} < \min\{\pi_1^{S_2}, \pi_1^{S_6}\}$ . (iv) Breaking  $A1$  is profitable:  $\pi_i^{S_4} < \min\{\pi_i^{S_1}, \pi_j^{S_4}\}$  for  $i = A, 1$  and  $j = B, 2$ . (v)  $A + 2$  is profitable:  $\pi_i^{S_4} < \min\{\pi_i^{S_3}, \pi_j^{S_4}\}$  for  $i = A, 2$  and  $j = A, 1$ . (vi)  $B + 1$  is profitable:  $\pi_i^{S_4} < \min\{\pi_i^{S_3}, \pi_j^{S_4}\}$  for  $i = B, 1$  and  $j = A, 1$ .

For  $S_5 = \{AB; 1, 2\}$  to be unstable:  $1 + 2$  is profitable:  $\pi_1^{S_5} < \min\{\pi_1^{S_2}, \pi_1^{S_6}\}$ .

For  $S_6 = \{A, B; 12\}$  to be unstable:  $A+B$  is profitable:  $\pi_A^{S_6} < \min\{\pi_A^{S_2}, \pi_A^{S_5}\}$ . **Q.E.D.**

### The 3×3 case

Suppose there are three identical firms, namely  $A, B, C$ , in the upstream industry, and three identical firms, namely 1, 2, 3, in the down stream industry. Other things remain unchanged as the main model.

There are 10 possible market configurations as shown in the following table.

Configurations	$\pi_A$	$\pi_B$	$\pi_C$	$\pi_1$	$\pi_2$	$\pi_3$	Total profit
$S_1 = \{A, B, C; 1, 2, 3\}$	47	47	47	35	35	35	246
$S_2 = \{A1, B2, C3\}$	34	34	34	28.5	28.5	28.5	187.5
$S_3 = \{C; A1, B2, 3\}$	42.5	42.5	21	42.5	42.5	15.5	206.5
$S_4 = \{B, C; A1, 2, 3\}$	61.5	28	28	49.5	28	28	222
$S_5 = \{BC; A1, 23\}$	99	21	21	74.5	14	14	243
$S_6 = \{BC; A1, 2, 3\}$	98	31	31	42.5	15.5	15.5	234.5
$S_7 = \{B, C; A1, 23\}$	65	18.5	18.5	86	24.5	24.5	237.5
$S_8 = \{A, BC; 1, 23\}$	74	37	37	49.5	25	25	247
$S_9 = \{A, BC; 1, 2, 3\}$	83.5	41.5	41.5	28	28	28	250
$S_{10} = \{A, B, C; 1, 23\}$	41.5	41.5	41.5	62.5	31	31	250

**Proposition 4.** *In the three by three case,  $S_1$  and  $S_2$  are stable.*

Proposition 4 says that all firms will either remain independent or carry out three vertical mergers, which is consistent with the result of the main model.

*Proof:*  $S_1$  is stable. First, there is no any profitable unilateral deviation since all of the firms remain independent. Second, there is no any profitable collective deviation. To see this, there are three possible collective deviations:  $B + C, 2 + 3$ , and  $A + 1$  (due to symmetry, we don't consider other identical collective deviations).  $B + C$  is not a profitable deviation since both firms  $B$  and  $C$  are worse off in  $S_5$ .  $2 + 3$  is not a profitable deviation since both firms are worse off in  $S_5$ .  $A + 1$  is not a profitable deviation as well because both  $A$  and 1 are worse off under  $S_2$ .

$S_2$  is stable. First, there is no any profitable unilateral deviation since no any firm wants to break up with its merger partner. To see this, suppose  $C3$  breaks up. But both

$C$  and 3 are worse off in  $S_3$  compared to  $S_2$ . Second, there is no any profitable collective deviation. To see this, there are two possible collective deviations:  $B + C$  and  $2 + 3$ .  $B + C$  is not profitable since both of them are worse off in  $S_6$  compared to  $S_3$ .  $2 + 3$  is not profitable either since both of them are worse off in  $S_7$  compared to  $S_3$ .

Note that  $S_3$  is not stable since  $C + 3$  is a profitable deviation.  $S_4, S_5, S_6$  and  $S_7$  are not stable since  $B + 2$  is a profitable deviation for all of them.  $S_8, S_9$  and  $S_{10}$  are stable using our criterion for stability, but they are not reasonable equilibria: in  $S_8$ ,  $BC$  prefers to break up; in  $S_9$ ,  $BC$  also prefers to break up; in  $S_{10}$ , 23 prefers to break up. Those firms won't choose to merge horizontally at the first place due to the merger paradox argument (Salant et al., 1983). **Q.E.D.**

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