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Carlos Canon
Toulouse School of Economics

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Matching & Information Provision by One-Sided and Two-Sided Platforms[*]

Carlos Cañón[†]

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Abstract

This paper studies a "market creating" firm (platform) that offers a matching environment by charging an access fee to a population of high and low type users who wish to form a match. We focus on an environment where users *only* observe a signal of their randomly assigned partner's type and where the informativeness of the signal is controlled by the firm. We study how both tools, access fee and signal informativeness, can be used to screen particular segments of the population. We finish by characterizing the set of optimal menus. The paper proposes three results. We show that information provision has a screening role when network effects are heterogeneous because a platform cannot induce every level of participation using only the access fee. Secondly, any platform will optimally offer a menu such that only high types participate, or where every user participates. In the former the signal is perfectly informative; in the latter it is partially informative. Lastly, the profit maximizing firm will over-provide information in relation to the surplus maximizing firm, and the higher the heterogeneity in the population, the higher the chance of the optimal menu excluding low type users.

Keywords: Pricing; Market Design; Matching; Information Provision; Heterogeneous Network Effects
Jel Codes: L11, L15, D42, D83

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[†]Toulouse School of Economics Ph.D. student. Contact information: 21 Allée de Brienne, Batiment F, Bureau MF007, 31000 Toulouse - France. Webpage: http://www.carloscanon.com. E-mail: carlos.canon@tse-fr.eu.

1 Introduction

Firms that “create markets” to offer them latter to some users are very common. These firms are also named as platforms. Their financing methods are diverse, and include advertising, merchandizing, requesting donations, and charging subscription fees, among others methods. In this paper, we are interested in those firms whose subscription fees represent a large fraction of their revenues. The following one-sided platforms, Linkedin, Dropbox, Phanfare, and Animoto all have in common a requested subscription fee, and that they provide some of their services free of charge (with the exception of Phanfare).[1] These firms are able to provide some services free of charge due to the fact that in the last decade their production and distribution costs reduced significantly.[2] Two-sided platform examples are also very common. Consider for example an average dating agency (or employment agency) available on the World Wide Web. These firms also charge some membership fees for both men and woman (or workers and firms), and they usually provide some of their services free of charge.

As expected, while many platforms have been launched providing similar products, only a few of them are very successful. Without loss of generality, we focus on the one-sided platform. Linkedin is an example of success. They currently have over 120 million users world-wide. Their revenue for the second quarter of 2011 is approximately 24 US million dollars, with their market value at over 8 US billion dollars. The cases of Dropbox and Animoto are also interesting. Dropbox has over 25 million users world-wide. Animoto has only 2 million users, however it launched only three years ago and its conversion rate is around the 10 per cent. Additionally, unsuccessful or small-sized platforms who compete with those already mentioned, are plentiful.

What are the successful platforms doing right ? We suggest that they were able to add enough value added to their product that users were happy to pay the membership fee.

Existing analyses of platforms assume that the they own an environment. For example, in an employment agency workers and employers enter to meet each other and engage in transactions that otherwise would be more expensive, or that could not be carried out completely. Workers and employers who wish to use the “new market” must pay a membership fee and/or a per transaction fee. Research questions in this environment usually focus on the characterization of the equilibrium outcomes, on the optimal pricing, or on the effect of changes in the market design on the equilibrium outcomes. All of these questions are valid, but in reality “market creating” firms may simultaneously use the pricing and market design to affect the equilibrium outcomes, and the available literature has little to say about this[3].

The class of environments we are interested in are the matching environments. We focus on these environments because they have been extensively studied, because they are empirically

[1] In fact, Phafare started off by providing some of their services free of charge, but later changed this policy.

[2] Visit websites such as www.freemium.org for more information.

[3] See Roth (2002) for an interesting discussion.

relevant, and because we believe there are strong ties between this literature and the two-sided market literature that are yet to be studied. Moreover, we wish to exploit the fact that frictions in these environments are significantly related to the market design and we want to propose a model where the "market creating" firm decides upon the optimal level of friction between workers and employers, and that, in general, "users" will have to endure.

In our approach the environment has three main properties. First, participating users cannot directly observe the characteristics of their randomly assigned partner. In particular they *only observe* a noisy signal of their characteristics. The platform selects the informativeness of the signal. The friction that the "market creating" firm will control is the precision of the information that users receive on their randomly assigned partner. Second, there are no transfers between users who form a match. We argue that it is a better approximation than the opposite, where transfers exist, because usually they are not efficient in dividing the surplus generated by the match among the matching partners. Finally, signals are affiliated to the objective characteristics, in other words higher signals are regarded as good news.

The class of "market creating" firms we consider have a strategic advantage over the potential users, and they are capable of screening any population subgroup if required. Firms from this class are capable of manipulating users' willingness to participate in the platform in two ways. In the extensive margin, and because of the indirect network effects, users' participation decision depends on the mass of other users who are also willing to participate. In the two-sided market case this effect is well understood in the dating agency example. The willingness of men to participate increases with the participation of women, and vice versa[4]. In the intensive margin, and because of the within group network effects, users' participation decisions are also affected. In the dating agency example we tend to believe direct network effects are negative because the probability of any man forming a match is a decreasing function of the mass of men participating. The second crucial aspect about firms from this class is that they suffer no competition from other platforms.

Our approach analyzes "market creating" firms that satisfy the following characteristics. The platform will face a population of vertically differentiated users who are divided into two subgroups. Users' preferences are quasi-linear. This first characteristic is very natural given that we analyze matching markets. A second, and more fundamental characteristic is that high type users only wish to form a match with other high type users, moreover we assume that they dislike forming a match with low type users. This characteristic is not as strong as it might seem, and is explained by the fact that our model is static. In a dynamic model we can relax this position to the point where high type users are willing to form a match with any user, but they prefer doing so with their peers. Finally, the platform will jointly choose an access fee and the signal's precision. In the two-sided platform case the platform can discriminate amongst sides, but not within each side. Marginal costs are normalized to zero to smooth the analysis.

[4]This intuition is not robust with heterogeneous network effects since then a man do not care anymore about the probability of forming a match, but with whom they form a match.

We have two main research questions. The first is to understand how a "market creating" firm, such as those within the class we describe, can use information provision and the access fee to screen out certain groups of the population. In particular, we wish to identify conditions under which information provision is required, on top of the access fee, as a screening tool. Alternatively, we also wish to obtain a characterization of the set of optimal menus of information provision and access fee. To wit, conditional on the environment characteristics, we will establish the optimal menu for both the profit maximizing and surplus maximizing platform. As a related issue we will discuss whether the profit maximizing firm will over or under provide information in relation to the surplus maximizing platform. We conclude by conducting some comparative statics on the set of optimal menus.

Our approach is a first step towards the understanding of a decentralized profit maximizing matching environment. In addition, while addressing the question of "who matches with whom" this paper does not take as given the market design, but this design is endogenously determined. Additionally, we address the realistic assumption that the network effects are not homogeneous in a vertically differentiated world, and we show that the platform is unable to reach all levels of their demand using only the access fee. In other words, tariffs are no longer insulating as in Weyl (2010). Market design will not only have the role of increasing users' willingness to participate, but will also help the platform to reach all demand levels. Finally, externalities embedded in the class of environments we propose will allow us to explain the discrepancies between the profit maximizing and the surplus maximizing platforms.

Our results hinge on one assumption: that while low types are indifferent with whom they match, the high types dislike matching with low types. We start by showing that there exists a matching equilibrium in the acceptance stage where the participants use strategies that are increasing in types. Then we proceed to characterize the set of matching equilibria, and we show that this set is a complete lattice in which all the equilibria can be ranked according to the economic activity embedded therein. In the next step we study the participation decisions of high and low type users, and we discuss the conditions under which the information provision works as a screening tool. We argue that access fees are not always *insulating*, as in Weyl (2010)[5]. Namely, in the case of heterogeneous network effects the presence of members from a group with a positive measure will produce a negative externality on other groups of the population, then any platform might find it optimal to retain a fraction of those users causing the externality, and the access fee will fall short as a screening tool. We show that to induce participation of a *fraction* of users from a group with a positive measure, the platform must use the provision of information as a screening tool.

We study the set of optimal menus, i.e. access fee and information provision, for a "one-sided" platform, as the main driving forces are easy to explain in this environment and still remain valid for

[5] Veiga & Weyl (2011) recently studied in general the role of heterogeneous network effect taking as given the environment. They also show that access fees are not insulating, and that the platform needs to use other non-price related tools to achieve any level of their demand.

the analysis of "multi-sided" platforms. Borrowing a terminology from the literature on advertising, well explained by Johnson & Myatt (2006), any platform can use an Informative Menu either to retain only one group of the population of users or a Hype Menu where every user is willing to participate, or a *Partial* Menu to induce partial participation of one group of the population. We show that the profit maximizing platform will not use a menu that excludes high type users in the population, and the set of optimal menus is the Hype Menu and the Informative Menu for high types. On the other hand, the set of optimal menus for the surplus maximizing platform will also include a Partial Menu where all high types and a fraction of low types participate.

The profit maximizing platform will over-provide information compared to the surplus maximizing platform. More precisely, when both platforms offer the Hype Menu we show that indeed this is the case. To understand the over-provision, it is enough to acknowledge that in the absence of transfers between users who form a match, the participation of low type users induces a negative externality on high type users, and consequently the surplus maximizing platform will find it optimal to provide less information than its profit maximizing counterpart.

The key variable in the comparative statics on the set of optimal menus is the heterogeneity of users in the population. This variable is captured by the difference between high type and low type users. A high heterogeneity increases the attractiveness of high types to the platform we analyze. In particular, we show that the region in the parameter space that supports the Informative Menu for high types increases with the heterogeneity in the population. The intuition here is reasonably simple. We show that as high types become more attractive to the platform, it would design the optimal menu to induce only them to participate.

This paper is from two strands of literature: the two-sided markets literature, and the literature of matching. The literature on two-sided markets, so far, has focused on building a theory of price (e.g. Caillaud & Jullien (2001, 2003), Rochet & Tirole (2003, 2006), Armstrong (2006), Weyl (2010)), or studing aspects of market design (e.g. Hagiu & Jullien (2010a,b), Athey & Ellison (2008), Damiano & Li (2008)). In our paper the platform can choose both the price and the signal's precision (proxy of market design). We can also find papers that give microfoundations for the user valuations (e.g. Nocke et al. (2007), Hagiu (2009), White (2009)), and others that study its dynamics (e.g. Hagiu (2006), Sun & Tse (2007), Lee (2010)). In our paper microfoundations are explained by fundamentals of a matching environment without transfers. While other papers study within-group discrimination (e.g. Gomes (2009), Doğanoglu & Wright (2010)), our paper only considers between group discrimination. Only recently Veiga & Weyl (2011) studied in general the topic of heterogeneous network effect. Differences will be made clear in the next paragraph.

The novelty we propose compared to this literature is to understand the platform's behavior under the particular environment we describe. One paper that is close to ours is Damiano & Li (2007). The main difference is that users have imperfect information, and that the platform can affect both the equilibrium probability of finding a match (intensive margin) and the users' willingness to participate (extensive margin). Another paper that is close to ours is Veiga & Weyl (2011). The main difference of our approach is that we do not take as given the environment but consider the class of matching environments without transfers between users. Then, by selecting a particular menu of access fees and information provision, our platform is also determining the matching equilibrium.

However, our paper is related to the matching literature such as Atakan (2008), Burdett & Coles (1997), and Chade (2006). There is a significant part of this literature that deals with transferable utility models, with which we are not related. The novelty we propose here is to understand how matching equilibrium is affected by the platform's activities (e.g. optimal pricing and optimal information provision) given the matching environment we propose. The closest papers to ours are Eeckhout & Kircher (2008), Poeschel (2008) and Chade (2006) but non of them study the effect of a profit maximizing platform.

Our model is inspired by Chade's (2006)[6] dynamic matching equilibrium model, and differs from it in two ways. The first is that we allow users to participate, or not, in the environment the platform offers. This in itself is an important modification because we are including the direct and indirect network effects in the analysis. Another feature our model includes is the existence of a central planner or platform who designs and offers the matching environment for a given access price. Our central planner affects the environment in two ways: in the extensive margin, given that he can affect the users' willingness to participate; and in the intensive margin as the users' probability of being accepted by their randomly assigned partner depends on the composition of participating users.

The paper proceeds as follows. Next section, Section two, presents the model and discusses the solution to the multiplicity of matching equilibria. Section three discusses the participation stage, and its goal is to show how the information provision can work as a screening device. Section four discusses the pricing rule and the information provision rule. We also characterize the set of optimal menus of access fee and provision of information. The next section, Section five, extends the analysis to a fully two-sided case. Section six concludes.

2 Model

This set-up corresponds to a one-sided market and we will extend the analysis to a two-sided market. There is a group of users who use the platform to find a match in the population; they can be high or low. Several important real life examples fit within this framework, in particular a platform

[6]We acknowledge that there are other papers in the matching literature that analyze the role of signaling, but we decided to follow Chade's (2006) approach because we did not want to let participants select their noisy signal and at the same time allow them to choose to participate. This has a flavor of second degree price discrimination and our goal is to study only third degree price discrimination. Additionally, we did not want to study *directed search*, e.g. sellers moving first in the timing. See the discussion section for more details.

that offers an environment where people want to find "friends", or "peers" or "co-authors". The fact that our model does not allow for monetary transfers is not a crucial issue because most real life environments that include transfers between users do not guarantee them to efficiently divide the surplus generated by the match, so this model can be regarded as a rough approximation to (imperfect) transferable utility model environments.

The one-sided, from hereon symmetric, model analyzes one platform that designs and offers an environment to a population of vertically differentiated users. Inside the environment a user will randomly meet another user, and can decide to form a match or not. A match is formed only when both participants accept each other. The platform will obtain revenues from the access fee and will bear a marginal cost, of both running the platform and providing information, normalized to zero[7]. Participants decide to join in if the expected payoff from the match at least covers the access fee. The platform design is such that participants only observe a noisy signal from their partner, and side transfers between participants are not allowed.

Time lasts for one period only. The population of potential users, with a mass equal to one $\overline{M} = 1$, can be divided into high types and low types, e.g. $a \in \{\underline{a}, \overline{a}\}$, where $\underline{a} < \overline{a}$, and we denote $\Pr\{a = \overline{a}\} = \overline{\lambda}$ the fraction of high types in the whole population. A user's type is private information. Users only observe a signal from their partner's type. In particular, a user will observe a signal $\alpha \in \{\underline{\alpha}, \overline{\alpha}\}$, where $\underline{\alpha} < \overline{\alpha}$, and $Pr\{\overline{\alpha} \mid \overline{a}\} = Pr\{\underline{\alpha} \mid \underline{a}\} = \rho$. Users will not observe their own signal[8].

The good news assumption states that $Pr\{\overline{\alpha} \mid \overline{a}'\} > Pr\{\overline{\alpha} \mid \overline{a}\}$ for any $a' > a$, which amounts to:

Assumption 1. $\rho > 1/2$

Payoff from the matching game. The matching payoff is assumed to be the minimum of the types involved in the match. This implies that hight type agents will only obtain a payoff equal to $\overline{a}$ when they form a match with another high type agent. In any other situation, e.g. when a high type forms a match with a low type, or when two low type agents form a match, the payoff of each agent will be $\underline{a}$. Moreover, when agents form a match they must pay an opportunity cost $(c > 0)$ that is type dependent. This last feature captures the intuition that foregone opportunities for high type agents are higher than for low type agents.

Assumption 2. *When agents $a, a' \in \{\underline{a}, \overline{a}\}$ form a match, the payoff for agent a is* $\min\{a, a'\} - ac$.

Moreover, the environment offered by the platform is such that low type users never perceive a

[7] We have carried out the calculations with both costs and results do not qualitatively change, instead formulas become more intrincated. The presence of fixed and sunk costs is non-problematic in our setup because we analyze a platform in isolation, where they will gain greater importance in the analysis of several competing platforms.

[8] We do not argue that in reality users cannot observe the signal they have attached, or that it is not important. Allowing users to observe it will unnecessarily complicate the analysis of any platform because all we need is that users' strategies on the matching game are directly affected by the platform.

negative payoff, even when they form a match with low type partners. The situation for high types is different, in particular, the net payoff of forming a match with a low type partner will yield a negative payoff.

Assumption 3. $\overline{a}(1-c) > 0 > \underline{a} - \overline{a}c$

A simplification is that low type agents care only about finding a match, while high type agents care about finding a good match. Assumption (3) guarantees that the payoff function is logsupermodular[9]. In the appendix we discuss how the paper's main results still hold for this general class of functions.

The platform charges a linear price to all potential users (P), and chooses the precision of the signal ρ. Users will participate if their expected match surplus outweighs the access fee.

The strategy of a user will consist, on the one hand, of an acceptance rule conditional on his type and his information of his partner, i.e. $\sigma_{a\alpha} : \{\underline{a}, \overline{a}\} \times \{\underline{\alpha}, \overline{\alpha}\} \to [0,1]$ where 0 is reject. On the other hand, it will consist of a participation rule conditional only on his type, i.e. $\gamma_a : \{\underline{a}, \overline{a}\} \to \{0,1\}$ where 1 stands for enter. Notice that in the participation decision we do not consider mixed strategies.

The timing is as follows. At the beginning, (i) A user privately learn his type, e.g. $a \in \{\underline{a}, \overline{a}\}$. Then, (ii) the platform determines access fee and information provision, e.g. P and ρ. At the next step, (iii) users decide to enter or stay out. Among those who participate, (iv) users meet randomly, and they privately learn the signal from their partner α. Within each pair, (v) users accept or reject their partner. Finally, (vi) if both users accept they form a match and receive the corresponding payoff, otherwise they stay single and receive zero payoff.

2.1 Matching Equilibrium

We want to find the conditions for a participation/matching equilibrium to exist. Such equilibrium is described by $(\rho_a, \sigma_{a\overline{\alpha}}, \sigma_{a\underline{\alpha}})$ for $a \in \{\underline{a}, \overline{a}\}$, such that users decisions form a Nash equilibrium. We are imposing some structure over users' behavior inside the environment. We will study the situation where high type users prefer to match with high types, rather than with low types.

One additional piece of notation is necessary. The expected payoff of accepting signal α after

[9] A function $f(x,y)$ is logsupermodular if it satisfies $f(x_2,y_2)/f(x_2,y_1) > f(x_1,y_2)/f(x_1,y_1)$ for $x_2 > x_1$ and y_2, y_1. See Topkins (1998) and Smith (2006) for further technical details and for its implications on the matching literature.

participating and given all available information[10], is

$$\begin{aligned} v(\alpha,\overline{a}) &= Pr\{\overline{\alpha}\mid\overline{a}\}\left[(\overline{a}-\overline{a}c)\sigma_{\overline{a}\overline{\alpha}}Pr\{\overline{a}\mid\alpha\}+(\underline{a}-\overline{a}c)\sigma_{\underline{a}\overline{\alpha}}Pr\{\underline{a}\mid\alpha\}\right] \\ &+ Pr\{\underline{\alpha}\mid\overline{a}\}\left[(\overline{a}-\overline{a}c)\sigma_{\overline{a}\underline{\alpha}}Pr\{\overline{a}\mid\alpha\}+(\underline{a}-\overline{a}c)\sigma_{\underline{a}\underline{\alpha}}Pr\{\underline{a}\mid\alpha\}\right] \\ v(\alpha,\underline{a}) &= (\underline{a}-\underline{a}c)\left[Pr\{\overline{a}\mid\alpha\}(Pr\{\overline{\alpha}\mid\underline{a}\}\sigma_{\overline{a}\overline{\alpha}}+Pr\{\underline{\alpha}\mid\underline{a}\}\sigma_{\overline{a}\underline{\alpha}})+Pr\{\underline{a}\mid\alpha\}(Pr\{\overline{\alpha}\mid\underline{a}\}\sigma_{\underline{a}\overline{\alpha}}+Pr\{\underline{\alpha}\mid\underline{a}\}\sigma_{\underline{a}\underline{\alpha}})\right] \end{aligned}$$

The environment is rather simple because from assumptions (2) and (3) all low type users will accept their partner, e.g. $\sigma_{\underline{a}\overline{\alpha}}=\sigma_{\underline{a}\underline{\alpha}}=1$, and we have

$$v(\alpha,\overline{a}) = \overline{a}(1-c)Pr\{Accepted,\overline{a}\mid\alpha\}+(\underline{a}-\overline{a}c)Pr\{\underline{a}\mid\alpha\} \quad (1)$$

$$v(\alpha,\underline{a}) = \underline{a}(1-c)[Pr\{\underline{a}\mid\alpha\}+Pr\{Accepted,\overline{a}\mid\alpha\}] \quad (2)$$

where the probability a high type is accepted by another high type user is $Pr\{Accepted,\overline{a}\mid\alpha\} = Pr\{\overline{a}\mid\alpha\}(Pr\{\overline{\alpha}\mid\overline{a}\}\sigma_{\overline{a}\overline{\alpha}}+Pr\{\underline{\alpha}\mid\overline{a}\}\sigma_{\overline{a}\underline{\alpha}})$.

We need to make two important distinctions. Define λ as the proportion of high types on the platform, which we distinguish from the proportion of high types in the population $\overline{\lambda}$. Similarly, we should distinguish the mass of users participating M from the total mass of users in the population $\overline{M}$.

Under this simple model we can explicitly describe the joint distribution of types and signal, in particular $Pr\{\overline{a},\overline{\alpha}\}=\rho\lambda$, $Pr\{\overline{a},\underline{\alpha}\}=(1-\rho)\lambda$, $Pr\{\underline{a},\overline{\alpha}\}=(1-\rho)(1-\lambda)$, and $Pr\{\underline{a},\underline{\alpha}\}=\rho(1-\lambda)$. Using this joint distribution we will parametrize equations (1) and (2).

Before continuing further, it is important to emphasize an obvious but significant implication of the model. Assumptions (1) - (3) guarantee that high type users experience a higher than expected payoff when they accept a high signal partner than when they accept a low signal partner. The opposite will hold for low type users, that is, their expected payoff of accepting a low signal partner is higher than with the high signal partner.

Lemma 1. *$v(\overline{\alpha},\overline{a})>v(\underline{\alpha},\overline{a})$ and $v(\overline{\alpha},\underline{a})<v(\underline{\alpha},\underline{a})$.*

Proof. See appendix. □

The previous lemma is crucial to anticipate the role of the provision of information, and in general of the market design. Platforms will select the optimal menu of access fee and information provision with the objective of attracting a particular segment of the population. For example, if it believes that the low types are the most attractive group the optimal menu will increase the value added of the environment to these users, and will decrease the value added to the high type users. The access fee will extract all low type users' trade surplus. The provision of information, and in general the market design, is a tool for the platforms to provide value added. In our set-up the value added for high (low) type users increases (decreases) with the signal's precision.

[10] At the matching game users know their type, the signal of their partner, and the fraction of high type users in the environment. We acknowledge the analysis differs if they did not know λ.

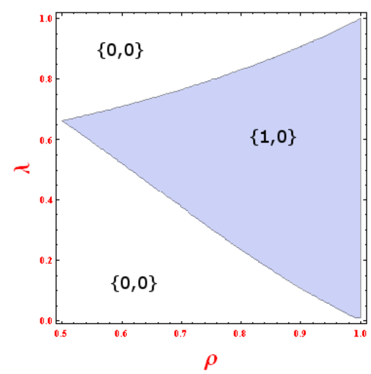

Figure 1: Case (i), $c > 0$

2.1.1 Pure Strategies

The analysis of the matching game reduces to the understanding of high type users strategies. We will start by considering the cases where these users accept or reject a randomly assigned partner with a probability equal to one. In the first case, which we call Case (i), they will only accept high signal partners $(\sigma_{\overline{a}\overline{\alpha}}, \sigma_{\overline{a}\underline{\alpha}}) = (1, 0)$. In a second case, called from hereon Case (ii), the high type users will accept every partner they encounter. Lastly, called from hereon Case (iii), they will reject any partner. We will say a few words about the region, in the space (ρ, λ), where each of these cases can be sustained.

Case (i): $(\sigma_{\overline{a}\overline{\alpha}}, \sigma_{\overline{a}\underline{\alpha}}) = (1, 0)$.

The expected payoff of accepting a randomly assigned partner, given all available information, for a high type user can be positive or negative. Recall the participation of low type users produces a negative externality on the high users. The expected payoff of accepting a will be,

$$v(\alpha, \overline{a}) \quad = \quad \overline{a}(1 - c)Pr\{\overline{a} \mid \alpha\}\rho + (\underline{a} - \overline{a}c)Pr\{\underline{a} \mid \alpha\} \lesseqgtr 0$$

High type users' strategies in the matching game are characterized by a threshold condition, this condition is guaranteed by lemma (1). If $v(\overline{\alpha}, \overline{a}) > 0 > v(\underline{\alpha}, \overline{a})$ the high type will accept only high signal partners. Figure (1) shows the pairs (λ, ρ) such that the high type user's behavior satisfies this condition. Inside the shaded region, high type users are more selective than low type users when deciding who to accept; namely there is an equilibrium of the matching game where high types accept only high signal partners.

We can understand the high type's decision by focusing on the ratio $\frac{Pr\{\overline{a}|\alpha\}}{Pr\{\underline{a}|\alpha\}}$, for $a \in \{\underline{a}, \overline{a}\}$. This object represents the ratio of the mass of high type users versus the mass of low type users that decided to participate in the platform conditional on the observed signal. If the high type user observes a high signal the ratio becomes $\frac{\rho\lambda}{(1-\rho)(1-\lambda)}$, and this function is increasing in ρ and λ.

Then, for this situation, high type users benefit from high levels of participation from other high type users, and also from high levels information. On the other hand, if a low signal is observed the ratio is now $\frac{(1-\rho)\lambda}{\rho(1-\lambda)}$ and while still it is increasing in λ, it is decreasing in ρ. Hence high quality of the signal make this "selective" equilibrium easier to support for any λ.

Case (ii): $(\sigma_{\overline{a}\overline{\alpha}}, \sigma_{\overline{a}\underline{\alpha}}) = (1,1)$.

The expected payoff of accepting, given all available information, will now be,

$$v(\alpha,\overline{a}) \quad = \quad \overline{a}Pr\{\overline{a} \mid \alpha\} + \underline{a}Pr\{\underline{a} \mid \alpha\} - \overline{a}c \gtrless 0$$

As for Case (i) we know that lemma (1) guarantees that the high type's accept/reject decision is completely characterized by a threshold. The equilibrium condition here is that "every other" high type user that participates accepts any partner he encounters, e.g. $v(\underline{\alpha},\overline{a}) > 0$. This condition in fact determines the best reply for the high type user we analyze.

Figures (2) provides further intuition. In this figure the opportunity cost is above the minimum level[11]. We observe that this matching equilibrium can be sustained for *high enough* levels of proportion of high types λ. This fact also is robust to any level of user heterogeneity. Finally, notice that if only high type users decide to participate ($\lambda = 1$) any level of information provision is compatible with this matching equilibrium.

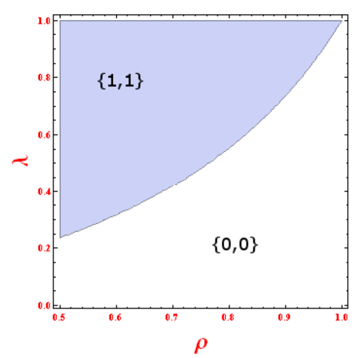

Figure 2: Case (ii), c = 0.5

Case (iii). The expected payoff of accepting, given all available information, will be negative because only low type users will accept any partner

$$v(\alpha,\overline{a}) \quad = \quad (\underline{a} - \overline{a}c)Pr\{\underline{a} \mid \alpha\} < 0$$

[11]If the opportunity cost is very close to the minimum level c_{min} we can show that any point in the parameter space (ρ, λ) supports this matching equilibrium.

The optimal decision in this case is quite trivial. High type users will reject any partner and low type users will accept any partner. Additionally, this matching case will prevail if the proportion of high type users λ is very small.

Remark 1. *Restrict the attention to Case (i) - Case (iii) when both type of users participate. In this situation the proportion of high type users is equal to* $\overline{\lambda}$ *and the mass of users is equal to one,* $M = 1$. *When* $\overline{\lambda}$ *is different from zero or one we obtain the following insights. If* $\overline{\lambda}$ *is very low the optimal reaction will be to reject any partner as long as the precision of the signal is not high enough because the high types will only accept high signal partners. On the other hand, if* $\overline{\lambda}$ *is not very low the optimal reaction will be to either accept everyone or to only accept high signal partners.*

2.1.2 Mixed Strategies.

Two additional matching equilibria in mixed strategies should be analyzed. In the first one, which we call Case (i) Mixed, high type users will reject low signal partners and will mix with high signal partners. The other case, which we call Case (ii) Mixed, high type users will accept high signal partners and will mix with low signal partners[12].

Case (ii) Mixed: $(\sigma_{\overline{a}\overline{\alpha}}, \sigma_{\overline{a}\underline{\alpha}}) = (1, \sigma^*_{\overline{a}\underline{\alpha}})$.

The expected payoff of accepting a randomly assigned partner, given all available information, is still negative. That is,

$$v(\alpha, \overline{a}) \quad = \quad \overline{a}(1-c)Pr\{\overline{a} \mid \alpha\}[\rho + (1-\rho)\sigma_{\overline{a}\underline{\alpha}}] + (\underline{a} - \overline{a}c)Pr\{\underline{a} \mid \alpha\} \gtreqless 0$$

The mixing condition for the high type users is $v(\underline{\alpha}, \overline{a}) = 0$ which yields an equilibrium probability of accepting high signal partners equal to

$$\sigma^*_{\overline{a}\underline{\alpha}} = \left(\frac{\overline{a}c - \underline{a}}{\overline{a} - \overline{a}c}\right) \frac{1}{1-\rho} \frac{Pr\{\underline{a} \mid \underline{\alpha}\}}{Pr\{\overline{a} \mid \underline{\alpha}\}} - \frac{\rho}{1-\rho}$$

Equilibrium conditions will be that $\sigma_{\overline{a}\underline{\alpha}} \in [0, 1]$. The following figure shows the region in the parameter space (ρ, λ) that sustains this matching equilibrium. The most striking feature is that this region does not include the situation where only high type users choose to participate ($\lambda = 1$), and the intuitive reason is that it makes no sense to reject a low signal partner with a positive probability if we already know only high type users are participating.

To better understand the decision of high type users notice that the expected payoff of accepting a partner with a low signal, e.g. $v(\underline{\alpha}, \overline{a})$, is determined by the ratio between the mass of high type users that participate and the mass of low type users that participate conditional on observing an

[12] We do not consider the situation where high types mix with high signal partners and accept low signal partners because by construction the expected payoff of accepting is higher with high signals than with low signals.

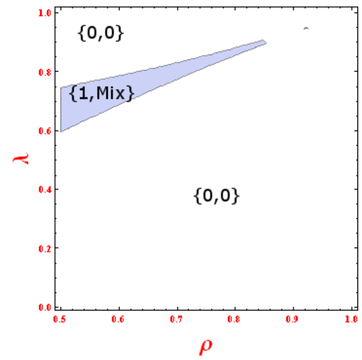

Figure 3: Case (ii) Mixed, $c > 0$

low signal, e.g. $\frac{Pr\{\overline{a}|\underline{\alpha}\}}{Pr\{\underline{a}|\underline{a}\}}$. As we discussed in Case (i), this ratio is always an increasing function on the mass of high types that participate λ, and is increasing in ρ when the high type user observes a high of signal, and decreasing when he observes a low signal.

Case (i) Mixed: $(\sigma_{\overline{a}\overline{\alpha}}, \sigma_{\overline{a}\overline{\alpha}}) = (\sigma^*_{\overline{a}\overline{\alpha}}, 0)$.

The expected payoff of accepting, given all available information, now will be,

$$v(\alpha, \overline{a}) \quad = \quad \overline{a}(1-c)Pr\{\overline{a} \mid \alpha\}\rho\sigma_{\overline{a}\overline{\alpha}} + (\underline{a} - \overline{a}c)Pr\{\underline{a} \mid \alpha\} \gtrless 0$$

The mixing condition for the high type users is $v(\overline{\alpha}, \overline{a}) = 0$ which yields an equilibrium probability of accepting high signal partners,

$$\sigma^*_{\overline{a}\overline{\alpha}} = \left(\frac{\overline{a}c - \underline{a}}{\overline{a} - \overline{a}c}\right) \frac{Pr\{\underline{a} \mid \overline{\alpha}\}}{\rho Pr\{\overline{a} \mid \overline{\alpha}\}}$$

Finally we place one bound that guarantee $\sigma_{\overline{a}\overline{\alpha}} \in (0,1)$. The upper bound condition such that the mixing probability is less than one is $0 < \overline{a}(1-c)\rho^2\lambda + (\underline{a} - \overline{a}c)(1-\rho)(1-\lambda)$. Figure (4) shows that there is an important region from the parameter space where matching equilibrium can be sustained.

2.1.3 Characterization

Two questions should be addressed, (1) the existence of a matching equilibrium[13], and (2) how to deal with the multiplicity of equilibria.

Both questions are closely related and will be jointly tackled. Using the previous section we can construct the best reply correspondence, denoted as $\Omega_a(\sigma; \lambda, \rho)$ where $\sigma = (\sigma_{\overline{a}\overline{\alpha}}, \sigma_{\overline{a}\underline{\alpha}})$ and $a \in \{\underline{a}, \overline{a}\}$, using figures (5) and (6). We observe that if costs are very low, e.g. $c = c_{min}$, only two

[13]We focus on the Nash equilibrium of the matching game between high and low type users that participate.

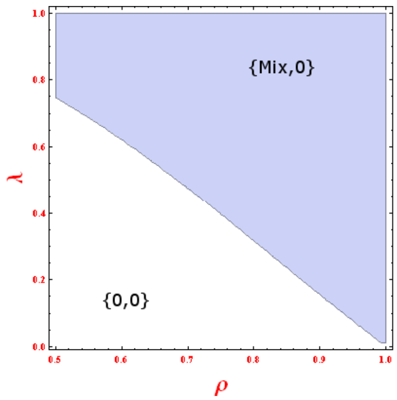

Figure 4: Case (i) Mixed, c = 0.6

matching equilibria can be supported. In particular, either high type users accept every partner they encounter, e.g. Case (ii), or they reject low signal partners and mix with high signal partners, e.g. Case (i) Mixed. Figure (5) shows this situation.

On the other hand, if the opportunity cost is above the minimum level all matching equilibrium may be supported. Using Figure (6), the highest ranked matching equilibrium is represented by the union of regions A, B and C; for the second highest ranked matching equilibrium the regions will be B and C; for the third in the ranking, the regions will be B, C and D; and for the last matching equilibrium the regions will be A, C and D. Finally, in region E the high type users will reject everyone.

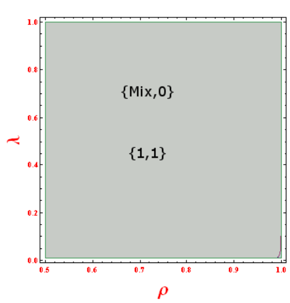

Figure 5: All Cases, $c \approx c_{min}$, Δ_a medium

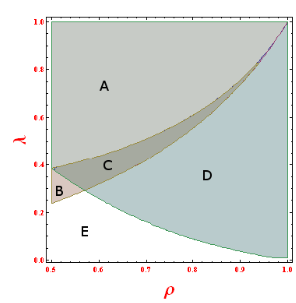

Figure 6: All Cases, $c > c_{min}$, Δ_a medium

The best reply correspondence for low type users is trivial because by construction they will accept any partner, $\Omega_{\underline{a}}(\sigma; \rho, \lambda) = \{(1, 1)\}$. High type users' case is different and depends on which region of the parameter space (ρ, λ) we are looking at. Without loss of generality consider Figure (6) where costs are above the minimum level. Denoting as (ρ^J, λ^J) all the pairs of ρ and λ that belong to region $J \in \{A, B, C, D, E\}$, we can label the best reply correspondences accordingly $\Omega_{\overline{a}}(\sigma; \rho^J, \lambda^J)$.

In region A of Figure (6) only two matching equilibrium can be supported, one where every partner is accepted (Case (ii)) and other where every partner is rejected (Case(iii)). The best reply correspondence $\Omega_{\overline{a}}(\sigma;\rho^A,\lambda^A)$ will be $\{(1,1)\}$ for the first equilibrium and $\{(0,0)\}$ for the other equilibrium. A similar argument holds for region D of Figure (6). There either the high type user only accept high signal users (Case (i)) or they do not accept any randomly assigned partner (Case (iii)). The best response correspondence $\Omega_{\overline{a}}(\sigma;\rho^D,\lambda^D)$ will be $\{(1,0)\}$ for the first equilibrium and $\{(0,0)\}$ for the other equilibrium. The last region whose best reply correspondence is very simple is region E, there only matching equilibrium Case (iii) can be supported.

Regions B and C of Figure (6) have richer best reply correspondences. In the first region three matching equilibrium can be sustained, high type users may accept any partner (Case (ii)), or they can accept high signal partners and mix with low signal partners (Case (ii) Mixed), or they can reject any partner (Case (iii)). The best reply correspondence $\Omega_{\overline{a}}(\sigma;\rho^B,\lambda^B)$ will be $\{(1,1)\}$ for the first equilibrium, $\{(1,\sigma^*_{\overline{a}\underline{\alpha}})\}$ for the second equilibrium, and will be $\{(0,0)\}$ for the last equilibrium. Finally, in region C of Figure (6) all the matching equilibria can be supported. The best reply correspondence will be

$$\Omega_{\overline{a}}(\sigma;\rho^C,\lambda^C)=\begin{cases}\{(0,0)\} & \text{if } \sigma=(0,0)\\ \{(\sigma^*_{\overline{a}\overline{\alpha}},0)\} & \text{if } \sigma=(\sigma^*_{\overline{a}\overline{\alpha}},0)\\ \{(1,0)\} & \text{if } \sigma=(1,0)\\ \{(1,\sigma^*_{\overline{a}\underline{\alpha}})\} & \text{if } \sigma=(1,\sigma^*_{\overline{a}\underline{\alpha}})\\ \{(1,1)\} & \text{if } \sigma=(1,1)\end{cases}$$

High type's best response correspondence is increasing at each region of Figure (6). Indeed, for any $\sigma',\sigma''\in[0,1]^2$, such that $\sigma'\leq\sigma''$, we can verify that for every element from the best response given σ', e.g. $b\in\Omega_{\overline{a}}(\sigma')$, we can find an element from the best response given σ'', e.g. $c\in\Omega_{\overline{a}}(\sigma'')$, such that $b\leq c$. For example in region C, taking $\sigma'=(1,0)$ and $\sigma''=(1,\sigma_{\overline{a}\underline{\alpha}})$, for $(1,0)\in\Omega_{\overline{a}}(1,0)$ we know that $(1,\sigma_{\overline{a}\underline{\alpha}})\in\Omega_{\overline{a}}(1,\sigma_{\overline{a}\underline{\alpha}})$ and on top $(1,0)\leq(1,\sigma_{\overline{a}\underline{\alpha}})$. We conclude that high type's best response correspondence is *upper and lower* increasing[14].

Proposition 1. *(i) The set of matching equilibria is nonempty, and we can determine "the greatest and the least" matching equilibria. (ii) The set of matching equilibria is a nonempty complete lattice.*

The previous result is useful at least in two ways. Though multiplicity of equilibria in this environment is unavoidable, we know enough from the structure of the set of matching equilibria to rank them according to the "economic activity". In other words, "economic activity" from the matching equilibrium where high type users accept any partner, e.g. $(\sigma_{\overline{a}\overline{\alpha}},\sigma_{\overline{a}\underline{\alpha}})=(1,1)$, is strictly higher than the one coming from the matching equilibrium where high type users accept high signal partners and mix with low signal partners[15], e.g. $(\sigma_{\overline{a}\overline{\alpha}},\sigma_{\overline{a}\underline{\alpha}})=(1,\sigma^*_{\overline{a}\underline{\alpha}})$. Using this line of reasoning

[14]We acknowledge there are different definitions of increasing correspondences, in particular in the literature is common to use Veinott's definition. We do not use this definition in Proposition (1). Our definition of "increasing correspondences" is explained in detail by Calciano (2009).

[15]Intuitively this fact comes from the linearity of $v(\alpha,\overline{a})$ on $\sigma_{\overline{a}\overline{\alpha}}$ and $\sigma_{\overline{a}\underline{\alpha}}$.

the ranking of equilibria is the following: $(0,0) \preceq (\sigma_{\overline{a}\overline{\alpha}}, 0) \preceq (1,0) \preceq (1, \sigma_{\overline{a}\underline{\alpha}}) \preceq (1,1)$.

Using proposition (1) we posit that agents will coordinate on the "greatest" matching equilibrium. Multiplicity of equilibria calls for an indifference breaking rule, and we propose one such that (high and low) users who participate will coordinate on the "greatest" matching equilibrium. We argue that this assumption is not restrictive because the expected payoff of accepting a partner, e.g. $v(\alpha, a)$, is an increasing function of $\sigma_{a\overline{\alpha}}$ and $\sigma_{a\underline{\alpha}}$. While users agree on using this matching equilibrium because their expected payoff is higher, the platform will also prefer it because the greater "economic activity" is, the greater rent he can extract using participation fee (P).

3 Participation

The objective of this section is to analyze how any platform can use both tools to induce any level of participation from high type and low type users. We will begin by showing there is a non-monotonic relationship between the mass of users participating (M) and the fraction of high type users participating (λ). We will then show that the access fee is not enough to reach every level of λ. Finally, we will comment on which matching equilibria place restrictions on the relationship between both tools.

To proceed with the analysis define two additional pieces of notation. First, let $U(a, \lambda, \rho)$, where $a \in \{\underline{a}, \overline{a}\}$, be user's expected participation payoff gross of the participation fee. We will also have four expected participation payoffs: $U^{i}(a, \lambda, \rho), U^{ii}(a, \lambda, \rho), U^{i}_{m}(a, \lambda, \rho), U^{ii}_{m}(a, \lambda, \rho)$. Second, let $\overline{A} \leq \overline{\lambda}$ be the mass of high type users willing to participate, and $\underline{A} \leq 1 - \overline{\lambda}$ be the mass of low type users willing to participate. Notice the fraction of high type users inside the environment is $\lambda = \frac{\overline{A}}{\overline{A} + \underline{A}}$.

Figure (7) shows the mass of participants (M) is a non-monotonic function of λ. There we plot at the abscissa the proportion of high type users that choose to participate, e.g. $\lambda = \frac{\overline{A}}{\overline{A} + \underline{A}}$, and the mass of participating users on the y-axis, e.g. $M = \overline{A} + \underline{A}$. Notice that in point X we find only low type users because $\lambda = 0$ and the mass of users is equal to $1 - \overline{\lambda}$, and on the other extreme, point Z, there are only high types participating because $\lambda = 1$ and the mass of users is equal to $\overline{\lambda}$. Moreover, the situation with the highest participation ($M = 1$) is at point Y, there the proportion of high type users that participate is exactly $\overline{\lambda}$. Finally, points over the segment $\overline{XY}$ represent a situation where all low type users, and a *fraction* of high type users, choose to participate; similarly, points over the segment $\overline{YZ}$ represent a situation where a *fraction* of low type users, and all high users, choose to participate.

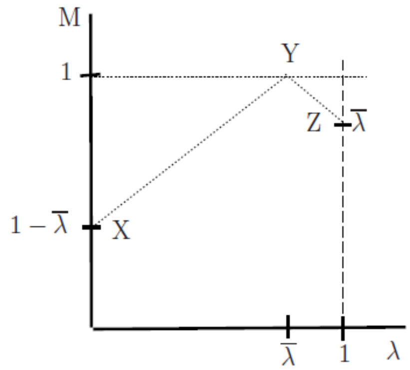

Figure 7: Non-Monotonicity Between λ and M

3.1 Example

We will narrow the analysis to the matching equilibrium where high users accept any high signal partners and reject low signal partners, Case (i). According to Proposition (1) this matching equilibrium is the third highest ranked, but we decided to use it because the explanation is easier[16]. The gross expected participation payoff for this matching equilibrium will be

$$\begin{aligned} U^i(\overline{a},\lambda,\rho) &= Pr\{\overline{\alpha}\}v(\overline{\alpha},\overline{a}) \\ &= \overline{a}(1-c)\rho^2\lambda + (\underline{a}-\overline{a}c)(1-\rho)(1-\lambda) \\ U^i(\underline{a},\lambda,\rho) &= Pr\{\overline{\alpha}\}v(\overline{\alpha},\underline{a}) + Pr\{\underline{\alpha}\}v(\underline{\alpha},\underline{a}) \\ &= \underline{a}(1-c)[(1-\rho)\lambda + (1-\lambda)] \end{aligned}$$

Begin considering a situation where the platform find optimal to only host low type users[17], $(\lambda, M) = (0, 1-\overline{\lambda})$, point X. Here the gross expected participation payoff for the high type user should be negative, $U^i(\overline{a},0,\rho) = (\underline{a}-\overline{a}c)(1-\rho) < 0$, but should be positive for the low type user, $U^i(\underline{a},0,\rho) = \underline{a}(1-c) > 0$. The platform just need to fix the access fee equal to $\underline{a}(1-c)$, the provision of information plays no particular role and its optimal level need only to satisfy the equilibrium conditions coming from Accept/Reject stage[18] and from Assumption (1).

Before continuing its important to make a remarks about the role of ρ. The information provision has a subtle screening role. The platform uses the access fee to extract all the expected

[16] If high type users play the matching equilibrium Case (ii) the platform will find useless information provision because, no matter its level, these users will accept any partner they meet. Additionally, if we assume high type users play the matching equilibrium Case (ii) Mixed, then we cannot study the situation where only high type users participate because makes no sense to mix if only high users participate.

[17] As will be shown latter, this situation may arise if the heterogeneity in the population of users is *low enough*.

[18] We need also to include restriction from the Participation stage but here they are trivially satisfied.

participation payoff from the low type users. But still we face the risk that some high type users find appealing to participate because they believe other high type users are willing to do it. The platform solves this caveat by providing a level of information where high types believe no other high type user is willing to participate. For this case this require providing a noisy signal.

Now imagine conditions related to $\Delta_a, \overline{\lambda}$, and c change and the platform wishes to induce*some* high type users to participate. This situation is equivalent to place ourself on the segment $\overline{XY}$, the proportion of high types participating will be $\lambda = \frac{\overline{A}}{\overline{A}+(1-\overline{\lambda})}$, and the mass of users will be $M = \overline{A} + (1 - \overline{\lambda})$. In this case the gross expected participation payoff of high type and low type users will be respectively $U^i(\overline{a}, \lambda, \rho)$ and $U^i(\underline{a}, \lambda, \rho)$. To induce this level of participation a platform needs to guarantee the net expected participation payoff is always zero for high type users and positive for low type users. The way to achieve this is by fixing the information provision at a level such that high type user perceive no net expected participation payoff; define as $\overline{\rho} = \overline{\rho}(\overline{A}, P)$ the provision level guaranteeing $U^i(\overline{a}, \lambda, \overline{\rho}) = P$ [19].

The provision of information has a strong role in the screening. The platform must use the provision of information not to increase the users' willingness to participate, but to guarantee high type users attain a certain level of gross expected participation payoff.

Lets move on and assume the platform wishes to get on board all users, point Y. In this situation the mass of users will be equal to one, $M = 1$, the proportion of high types will equal to $\overline{\lambda}$, and the gross expected participation payoff for high type users and low type users will be $U^{(i)}(\overline{a}, \overline{\lambda}, \rho)$ and $U^{(i)}(\underline{a}, \overline{\lambda}, \rho)$ respectively. As with point X, the access fee will be the only screening device and will be pinned down by $\min\{U^{(i)}(\overline{a}, \overline{\lambda}, \rho), U^{(i)}(\underline{a}, \overline{\lambda}, \rho)\}$. Finally, the optimal provision of information for the profit maximizing platform will differ in relation to the decision of the profit maximizing platform.

Moving on with the story, assume that conditions over $\Delta_a, \overline{\lambda}$ and c changed again and the platform wishes to keep all high type users, but only want to hold *a fraction* of low type users. This case is represented by any of the points on the segment $\overline{YZ}$, there the fraction of high type users will be $\lambda = \frac{\overline{\lambda}}{\overline{\lambda}+\underline{A}}$, and the mass of users will be $M = \overline{\lambda} + \underline{A}$. By analogy to every point in segment $\overline{XY}$, the platform must fix the provision of information to a level, say $\underline{\rho} = \underline{\rho}(\underline{A}, P)$, that guarantees low types' net expected participation payoff is equal to zero, $U^i(\underline{a}, \lambda, \overline{\rho}) = P$, and high types perceive a positive payoff, $U^i(\overline{a}, \lambda, \overline{\rho}) > 0$ [20].

The last case, which is the opposition to point X, is when the platform is only interested in high type users, point Z. In this case the proportion of high type users (λ) is equal to one, the mass of users will be equal to $\overline{\lambda}$, and the gross expected participation payoff for high and low type

[19]These conditions require finding the right level of provision $\overline{\rho}$ that solves $\frac{\overline{a}(1-c)\lambda}{(\underline{a}-\overline{a}c)(1-\lambda)}\rho^2 - \rho + 1 = P$, and guaranteeing low types' net expected participation payoff is positive, $U^i(\underline{a}, \lambda, \overline{\rho}) = \underline{a}(1-c)(\lambda(1-\overline{\rho}) + (1-\lambda)) \geq P$.

[20]These conditions require fixing the provision level $\underline{\rho}$ at $\frac{\overline{\lambda}+\underline{A}}{\overline{\lambda}}$, and guaranteeing high type users' gross expected participation payoff if positive $U^i(\overline{a}, \lambda, \underline{\rho}) = \overline{a}(1-c)\underline{\rho} + (\underline{a} - \overline{a}c)(1-\overline{\rho})\left(\frac{\underline{A}}{\overline{\lambda}+\underline{A}}\right) \geq 0$.

users will be $U^i(\overline{a}, 1, \rho) = \overline{a}(1-c)\rho^2$ and $U^i(\underline{a}, 1, \rho) = \underline{a}(1-c)(1-\rho)$, respectively. By analogy to point X, the platform will pindown the access fee with $U^i(\overline{a}, 1, \rho)$ and must pick the right level of provision.

The information provision has no role in screening. By picking an access fee that extracts all high users' expected participation payoff no low type user is willing to participate. Then, the platform do not need to use the information provision as in point X.

Remark 2. *The provision of information works as a screening device whenever the platform wishes to put on board* a fraction *of the users of one particular type, e.g. either points on the segment* $\overline{XY}$ *or on the segment* $\overline{YZ}$.

3.2 Screening & Matching Equilibria

This section discusses some restrictions different matching equilibria imposes on the screening power of the access fee and information provision. We will emphasize on one point, that the provision of information has a screening role whenever high type users accept low signal partners with a positive probability.

Within each matching equilibrium we will start explaining the three *full participation cases*: where only high type users participate ($\lambda = 1$), where every user participate ($\lambda = \overline{\lambda}$), and where only low type users participate ($\lambda = 0$). Lastly, we will proceed with the two *partial participation cases*: where all low types participate and a fraction of high types participate ($\frac{\overline{\lambda}}{\overline{\lambda}+\underline{A}}$), and vice versa ($\frac{\overline{A}}{\overline{A}+(1-\overline{\lambda})}$).

Case (ii). This is the easiest case to analyze because both high and low type users are willing to accept any partner they encounter. The direct implication we wish to stress is that the gross expected participation payoffs will not depend on the provision of information, and thus this tool will have no strategic use. The gross expected participation payoff will be,

$$\begin{aligned} U^{ii}(\overline{a}, \lambda, \rho) &= \overline{a}\lambda + \underline{a}(1-\lambda) - \overline{a}c \\ U^{ii}(\underline{a}, \lambda, \rho) &= \underline{a}(1-c) \end{aligned}$$

Using Figure (7) we will go through the conditions to be met for achieving every level of participation. If the platform wishes to put retain only high type users, point Z where $(\lambda, M) = (1, \overline{\lambda})$, provides a gross expected participation payoff for high types of $U^{ii}(\overline{a}, 1) = \overline{a}(1-c)$ and for low types of $U^{ii}(\underline{a}, 1) = \underline{a}(1-c)$. The only restriction which has to hold is that the entry fee outweighs gross expected participation payoff of low type users, e.g. $\overline{a}(1-c) \geq P > \underline{a}(1-c)$. At the point X, where $(\lambda, M) = (0, 1-\overline{\lambda})$, the gross expected participation payoff for low types is $U^{ii}(\underline{a}, 0) = \underline{a}(1-c)$ and for the high types is $U^{ii}(\overline{a}, 0) = \underline{a} - \overline{a}c$, with the usual restriction. Finally, if the platform wishes to support full participation, point Y where $(\lambda, M) = (\overline{\lambda}, 1)$, the gross expected participation payoff will be equal to $U^{ii}(\overline{a}, \overline{\lambda})$ and $U^{ii}(\underline{a}, \overline{\lambda})$, and the condition for participation is that $Min\{U^{ii}(\overline{a}, \overline{\lambda}), U^{ii}(\underline{a}, \overline{\lambda})\} \geq P$.

This matching equilibrium does not support any partial participation, e.g. points on segments $\overline{XY}$ and $\overline{YZ}$. Indeed, as the gross expected participation payoffs for both type of users are not affected by the provision of information its impossible to find a level of provision such that the net participation payoff, of high or low type users, is exactly equal to zero. To be precise, it does not exist a provision level $\overline{\rho}$ and $\underline{\rho}$ such that respectively $U^{ii}(\underline{a}, \lambda, \overline{\rho}) = P$ and $U^{ii}(\overline{a}, \lambda, \underline{\rho}) = P$.

Lets finish stressing that the provision of information do not work as a screening tool for this case. The story will be very different if the platform can affect the probability any user to be accepted, just like in the following matching equilibrium we discuss bellow.

Case (ii) Mixed. This matching equilibria is very important because the platform can influence the probability of a user to be accepted inside the environment. Indeed, here high type users will accept low signal partners with a probability less or equal to one, e.g. $\sigma_{\overline{a}\underline{\alpha}} = \sigma_{\overline{a}\underline{\alpha}}(\rho)$, then any platform can affect the probability of forming a match by manipulating ρ. The expected payoff of participation will be,

$$
\begin{aligned}
U^{ii}_m(\overline{a}, \lambda, \rho) &= (2\rho - 1)(\overline{a}c - \underline{a})\left(\frac{1-\lambda}{1-\rho}\right) \\
U^{ii}_m(\underline{a}, \lambda, \rho) &= \frac{\underline{a}}{\overline{a}}\frac{1}{(1-\rho)^2}\left[\overline{a}(1-c)(1-\rho)(1-\rho-\lambda\rho) + (\overline{a}c - \underline{a})\rho^2(1-\lambda)\right]
\end{aligned}
$$

Now discuss the conditions associated to each participation level. The platform will not retain only high type users, e.g. point Z where $(\lambda, M) = (1, \overline{\lambda})$, for this matching equilibrium because the gross expected participation payoff for high types is equal to zero. Intuitively, high type users will not follow this matching equilibrium because makes no sense rejecting a low signal partner if they know only high types are participating. Formally, if $\lambda = 1$ be obtain that $\sigma_{\overline{a}\underline{\alpha}} = -\rho/(1-\rho)$ and its clearly impossible to have this probability equal to zero because by assumption (1) we can only deal with $\rho > 1/2$.

For the other extreme case, e.g. point X where $(\lambda, M) = (0, 1-\overline{\lambda})$, the gross expected participation payoff for low types is $U^{ii}_m(\underline{a}, 0, \rho) = \frac{\underline{a}}{\overline{a}}\frac{1}{(1-\rho)^2}[\overline{a}(1-c)(1-\rho)^2 + (\overline{a}c - \underline{a})\rho^2]$ and for the high types is $U^{ii}_m(\overline{a}, 0, \rho) = (\overline{a}c - \underline{a})\frac{2\rho-1}{1-\rho}$. Additionally we should impose the condition that the access fee outweighs the gross expected participation payoff from high types, but not the one from low types. Finally, when the platform wishes to put on board every users, e.g. point Y where $(\lambda, M) = (\overline{\lambda}, 1)$, the gross expected participation payoff for the high and low type users will be, respectively, $U^{ii}_m(\overline{a}, \overline{\lambda}, \rho)$ and $U^{ii}_m(\underline{a}, \overline{\lambda}, \rho)$. The only condition to be met is that the access fee must be equal to the minimum of these two gross expected participation payoffs.

Until now the provision of information does not work as a screening device. As shown at the beginning of this section, if the platform wishes to induce a *partial participation* from either the high or low type of users, he must use the provision of information as a screening device. Starting with the situation when he wishes to keep all the high types and only *a fraction* of the low types, e.g. points on the segment $\overline{YZ}$ where $\lambda = \frac{\overline{\lambda}}{\overline{\lambda}+\underline{A}}$ and $M = \overline{\lambda} + \underline{A}$, the platform should fix the provision at a level $\underline{\rho}$ such that $U_m^{ii}(\underline{a}, \lambda, \underline{\rho}) = P$, and at the same time guarantee $U_m^{ii}(\overline{a}, \lambda, \underline{\rho}) > P$ [21].

The other *partial participation* case, e.g. points on segment $\overline{XY}$ where $\lambda = \frac{\overline{A}}{\overline{A}+(1-\overline{\lambda})}$ and $M = \overline{A} + (1 - \overline{\lambda})$, is when the platform wishes to keep all low types and only *fraction* of high type users. By analogy, the platform should fix the provision $\overline{\rho}$ at a level such that high types obtain zero net payoff from participation. Additionally, low type users' net payoff for participating should be greater or equal to zero, $U_m^{ii}(\underline{a}, \lambda, \overline{\rho}) > P$ [22].

Case (i). This case was already introduced at the beginning of this section. Let us remind that within this matching equilibrium the provision of information does serves as a screening device whenever the platform wishes to attain a participation level different from $\lambda \in \{1, 0, \overline{\lambda}\}$.

Remark 3. *Case (i) Mixed will not be used in a "one-sided" platform such as this we present. By construction we know the gross expected participation payoff for high type users will be equal to zero, then the only reason these users will consider targeting this matching equilibrium is because the access fee is non-positive. This situation is very unlikely and the results we achieve are not affected if we drop out this matching equilibrium. In the appendix we present all analytical results corresponding to this case.*

4 Pricing and Information Provision

The objective of this section is to uncover the optimal pair of access fee and information provision for all the parameter space. We will start discussing the optimal menu given different participation levels, special attention is devoted to the information provision rule and the pricing rule. We finish discussing which participation level is optimal given different combinations of $\Delta_a, \overline{\lambda}$ and c.

A platform at this point observes the environment's characteristics and decides the access fee (P) and the signal (ρ). Environments' characteristics are determined by users' heterogeneity ($\Delta_a \equiv \overline{a} - \underline{a}$), by the proportion of high types ($\overline{\lambda}$) in the population, and by the opportunity cost (c). Among these the most relevant of them, at least for our main results, is user's heterogeneity. This parameter provide's valuable information for the optimal design of the environment, for example, if the difference is significantly high a profit maximizing platform will have strong incentives to

[21] These conditions require finding the $\underline{\rho}$ that solves $\overline{a}(1-c)(1-\underline{\rho})(1-\underline{\rho}-\underline{\rho}\lambda) + (\overline{a}c - \underline{a})\rho^2(1-\lambda) = P$, and guaranteeing $U_m^{ii}(\overline{a}, \lambda, \underline{\rho}) = (\overline{a}c - \underline{a})\left(\frac{\underline{A}}{\overline{\lambda}+\underline{A}}\right)\left(\frac{2\underline{\rho}-1}{1-\underline{\rho}}\right) > P$, which is satisfied by construction.

[22] This condition is indeed satisfied. After making all the necessary replacements we obtain that $U_m^{ii}(\underline{a}, \lambda, \overline{\rho}) = \frac{\underline{a}}{\overline{a}}\left(\frac{1-\overline{\lambda}}{\overline{A}+(1-\overline{\lambda})}\right)(\overline{a} - \underline{a}) > P$.

construct a very attractive environment to high type users[23].

At this stage we need to consider additional constraints that characterize high type users. We wish that the high type users are more selective than low type users. This behavior is guaranteed as long as the difference between accepting only high signal partners (e.g. $Pr\{\overline{\alpha}\}v(\overline{\alpha}, a)$) and accepting any partner (e.g. $Pr\{\overline{\alpha}\}v(\overline{\alpha}, a)+Pr\{\underline{\alpha}\}v(\underline{\alpha}, a)$) is an increasing function of the type, e.g. $v(\underline{\alpha}, \overline{a}) - v(\underline{\alpha}, \underline{a}) < 0$. Finally we need to guarantee the high types are willing to participate, this is equivalent as saying the payoff of accepting any partner is an increasing function of the type, e.g. $Pr\{\overline{\alpha}\}[v(\overline{\alpha}, \overline{a}) - v(\overline{\alpha}, \underline{a})] + Pr\{\underline{\alpha}\}[v(\underline{\alpha}, \overline{a}) - v(\underline{\alpha}, \underline{a})] > 0$.

Remark 4. *In the characterization of the matching equilibrium we learned that if costs are very low, e.g. $c \approx c_{min}$, only two matching equilibria can be supported. Also we learned that if opportunity costs rises every marching equilibrium can be supported.*

In addition we observe that, given an opportunity cost above c_{min}, when heterogeneity is very high the matching equilibrium where high type users accept any partner cannot be supported. But as long as heterogeneity reduces, its possible to support more and more this matching equilibrium, and at the same time the regions for matching equilibrium Case (i) and Case (ii) Mixed will be reduced. At the extreme situation, where heterogeneity is the minimum, the matching equilibrium where high type users accept high signal partner and mix with low signal partner, e.g. Case (ii) Mixed, cannot be supported.

See appendix for detailed analysis.

The maximization program is constrained for each level of participation. Besides the natural constraint coming from Assumption (1), e.g. $\rho \in (0.5, 1]$, we must include the restrictions coming from the participation stage, and the accept/reject stage. Given $\lambda \in \{1, 0, \overline{\lambda}, \frac{\overline{A}}{\overline{A}+(1-\overline{\lambda})}, \frac{\overline{\lambda}}{\overline{\lambda}+\underline{A}}\}$, define the set

$$\begin{aligned}\Phi_\lambda \quad = \quad & \{\rho \times \overline{A} \times \underline{A} \in (0.5, 1] \times [0, \overline{\lambda}] \times [0, 1-\overline{\lambda}] : v(\underline{\alpha}, \overline{a}) - v(\underline{\alpha}, \underline{a}) \leq 0, \\ & Pr\{\overline{\alpha}\}[v(\overline{\alpha}, \overline{a}) - v(\overline{\alpha}, \underline{a})] + Pr\{\underline{\alpha}\}[v(\underline{\alpha}, \overline{a}) - v(\underline{\alpha}, \underline{a})] \geq 0, \\ & EquilibriumConditions\}\end{aligned}$$

where the "Equilibrium Conditions" restrictions depend on which matching equilibrium we are analyzing. For example, in the case of high type users accepting high signal partners, e.g. Case (i), the equilibrium conditions require the expected payoff for a high type user of accepting a partner with a low signal to be negative, e.g. $v(\underline{\alpha}, \overline{a}) < 0$, and also require the expected payoff of accepting a high signal partner to be positive, e.g. $v(\overline{\alpha}, \overline{a}) > 0$. Similarly, for Case (ii) we require that $v(\underline{\alpha}, \overline{a}) > 0$, for Case (i) Mixed that $v(\overline{\alpha}, \overline{a}) = 0$, and for Case (ii) Mixed that $v(\underline{\alpha}, \overline{a}) = 0$.

[23] This intuition does not necessarily apply for a surplus maximizing platform because we already know they care not for the marginal users, but for the inframarginal ones. Latter in the paper we will go into these issues in greater detail.

The second set of restrictions comes from the participation stage and will depend on which types are willing to participate,

$$\Psi_M = \{\rho \times \overline{A} \times \underline{A} \in (0.5, 1] \times [0, \overline{\lambda}] \times [0, 1 - \overline{\lambda}] : ParticipationConditions\}$$

where the mass of participating users (M) includes the cases of full group participation ($\lambda \in \{0, \overline{\lambda}, 1\}$) and the cases of partial group participation ($\lambda \in \{\overline{A} + (1 - \overline{\lambda}), \overline{\lambda} + \underline{A}\}$). For example, in Case (i) the participation constraint will be that the gross expected participation payoff of high type users must be strictly higher than the one corresponding to low type users, e.g. $\overline{a}\rho^2 > \underline{a}(1-\rho)$.

Definition 1. *Conditional on the matching equilibrium and on having* full group participation *(*partial group participation*), a information provision level (participation level) is feasible if it belongs to* $\Phi_\lambda \cap \Psi_M$.

Until now we have no restriction on the objective function. If the platform wishes to maximize its profits, the objective function is defined by revenues coming from the access fees[24]. On the other hand, if the platform cares about the social surplus, the objective function will be the sum of the gross expected participation payoffs from those willing to pay the access fee. Denoting as $F(\cdot)$ a generic function whose only argument could be either ρ, $\overline{A}$ or $\underline{A}$, the optimization program for each matching equilibrium will be $\max F(\cdot)$ such that the argument lies within $\Phi_\lambda \cap \Psi_M$.

4.1 Pricing and Information Provision Given Participation

For the remaining of the section we will begin obtaining the optimal *information provision rule* in the cases of Full Group Participation, both for profit maximizing and surplus maximizing platform. Then, we will present the optimal *pricing rule* for the case of Partial Group Participation, both for profit maximizing and surplus maximizing platform. We will conclude obtaining the optimal pair of access fee and information provision levels for every point of the parameter space.

4.1.1 *Full Group Participation*

Private Optimal Signal. The question we ask ourselves here is what is the optimal signal a profit maximizing platform will establish given a fixed participation level. The impact of higher levels of information we saw is positive for high type users, but its detrimental to low type users. Then we posit a profit maximizing platform will provide high levels of information the higher the proportion of high type users participating.

Begin analyzing the extreme participation cases. If only high type users participate the profit maximizing platform must guarantee these users face the right levels of access fee and information such that no low type user is willing to participate. As the gross expected participation payoff for

[24] Recall that we normalize all marginal costs to zero

these users is increasing in ρ the platform will have strong incentives to provide as much information as possible because this guarantee that low type users will perceive a low expected participation payoff. The platform will use the access fee to extract all the surplus from the high type users. To informally prove this state assume the platform will do the opposite, then high type users will reduce their probability of being accepted, and so they will be less willing to participate. By providing perfect information the platform will increase high type user's gross expected participation payoff, and so will his benefits because he can extract more surplus.

In the opposite situation only low type users will participate. The result here is trivial because by construction low types will accept anyone no matter the signal they have. Platform's decision in this case will be to set any feasible signal, e.g. $\rho \in \Phi_\lambda \cap \Psi_M$. The following proposition summarizes these two arguments.

Proposition 2. *(i) If only high type users participate $\lambda = 1$ the profit maximizing platform will provide perfect information, e.g. $\rho^* = 1$, and will charge an access fee equal to $\overline{a}(1-c)$. (ii) If only low type users participate $\lambda = 0$ the profit maximizing platform will provide noisy information, e.g. $\rho^* = 1/2$, and will charge an access fee equal to $\underline{a}(1-c)$.*

Proof. Here we will provide an heuristic proof, the complete version is at the appendix. The first bullet follows directly from showing the set of constraints from the accept/reject and participation stages reduce to a singleton with 1 as its element, e.g. $\Phi_1^{ii} = \Phi_{Mixed,1}^{ii} = \Phi_1^{i} = \{1\}$. The second bullet is trivial because the feasible set for ρ^* is only constrained by assumption (1), e.g. $[0.5, 1]$. The platform will reduce the precision of the signal to guarantee high type users will not participate and the correct allocation emerges. □

The surplus maximizing signal coincides with the profit maximizing signal for the extreme participation cases. This result is trivial because both platforms face the same optimization programs, indeed the objective functions and the feasible sets are the same. The following corollary will formally state this remark.

Corollary 1. *If only high (or low) type users participate the surplus maximizing platform and the profit maximizing platform will provide the same information, and charge the same access fee.*

Proof. Trivial. □

Now continue with the optimal signal when all users participate. Among all possible matching equilibria we find one case where the signal do not affect the gross expected participation payoff. The situation is the matching equilibrium when both types accept any partner they encounter disregarding the signal they observe from them, i.e. Case (ii). This equilibrium indeed is the most attractive because it embeds the highest possible *activity level*. The profit maximizing platform do not need to use the signal for this situation because it does not affect users' decision of accepting any partner. The optimal signal in this case will lie at its feasible set $\rho^* \in \Phi_{\overline{\lambda}}^{ii} \cap \Psi_1^{ii}$, see Figure (2).

The signal do play a role in the rest of matching equilibria where both types participate. Indeed, in matching equilibria Case (ii) Mixed and Case (i) the signal do affect the probability of forming a match, then the profit maximizing platform decision is nontrivial. Starting with the case high type users mix with low signal partners and accept any high signal partner, i.e. Case (ii) Mixed, the main feature is the impact of the signal on the probability a high type user accepts a low signal partner, e.g. $\sigma_{\overline{a}\underline{\alpha}}$. The optimization program for this case will be, $\max_{\rho} U_m^{ii}(\underline{a}, \overline{\lambda}, \rho)$ such that $\rho \in \Phi^{ii}_{Mixed,\overline{\lambda}} \cap \Psi^{ii}_{Mixed,1}$ [25].

For the last matching equilibrium, e.g. Case (i), still information provision does not work as a screening device. The access fee is the tool used by the platform to select which group of users will participate into the matching environment. The optimization program for this case will be $\max_{\rho} U^i(\underline{a}, \overline{\lambda}, \rho)$, such that $\rho \in \Phi^{i}_{\overline{\lambda}} \cap \Psi^{i}_{1}$ [26].

Analysis here is simple because the gross expected participation payoff for low type users is linear in the signal ρ. Then, the optimal information level must be either the highest or the lowest possible. We will show that the best decision will be to provide the least feasible information level because by doing this all low type users will participate, and by construction we know that if all low type users participate then all high type users will also participate.

The following proposition summarizes the optimal signal rule given both type of users decide to participate,

Proposition 3. *Assume all users participate $\lambda = \overline{\lambda}$. (i) For the matching equilibrium with the highest "economic activity", e.g. Case (ii), the profit maximizing platform will provide any feasible level of information, e.g $\rho^* \in \Phi^{ii}_{\overline{\lambda}} \cap \Psi^{ii}_{1}$. (ii) For the second highest ranked matching equilibrium, e.g. Case (ii) Mixed, the optimal (interior) provision satisfies the following rule:*

$$\xi_{\sigma^*_{\overline{a}\underline{\alpha}},\rho} = \frac{1-\sigma^*_{\overline{a}\underline{\alpha}}}{\sigma^*_{\overline{a}\underline{\alpha}}}$$

where $\xi_{\sigma^*_{\overline{a}\underline{\alpha}},\rho}$ is the elasticity of the $\sigma^*_{\overline{a}\underline{\alpha}}$ with respect to ρ. (iii) For the matching equilibrium Case (i) its optimal to provide the least feasible level of information.

Proof. See the appendix. □

Socially Optimal Signal. We pose two questions. First, which is the optimal information provision level for a surplus maximizing platform ? And, which are the differences vis-a-vis the profit

[25] The feasible set of signals is for this matching equilibrium will be the intersection of $\Phi^{ii}_{Mixed,\overline{\lambda}} = \{\rho \in (0.5, 1] : 0 \geq (\overline{a}c - \underline{a})(1-\overline{\lambda})(2\rho - 1) \geq \frac{\underline{a}(1-c)}{(1-\rho)\overline{a}(1-c)}[\rho^2(\overline{a}c - \underline{a})(1-\overline{\lambda}) + \overline{a}(1-c)(1-\rho)(1-\rho-\rho\overline{\lambda})], \frac{\overline{a}c-\underline{a}}{\overline{a}(1-c)}\frac{\rho(1-\overline{\lambda})}{(1-\rho)^2\overline{\lambda}} - \frac{\rho}{1-\rho} \in [0,1]\}$ and $\Psi^{ii}_{Mixed,1} = \{\rho \in (0.5, 1] : U_m^{ii}(\underline{a}, \overline{\lambda}, \rho) \geq 0\}$.

[26] The feasible set of signal for this matching equilibrium will be the intersection of $\Phi^{i}_{\overline{\lambda}} = \{\rho \in (0.5, 1] : \underline{a}(1-c)[(1-\rho)^2\overline{\lambda} + \rho(1-\overline{\lambda})] \geq \overline{a}(1-c)(1-\rho)\rho\overline{\lambda} + (\underline{a} - \overline{a}c)\rho(1-\overline{\lambda}), \overline{\lambda}(1-c)[\overline{a}\rho - \underline{a}(1-\rho)] - c(1-\overline{\lambda})(\overline{a} - \underline{a}) \geq 0, \overline{a}(1-c)\rho^2\overline{\lambda} + (\underline{a} - \overline{a}c)(1-\rho)(1-\overline{\lambda}) \geq 0, \overline{a}(1-c)\rho(1-\rho)\overline{\lambda} + (\underline{a} - \overline{a}c)\rho(1-\overline{\lambda}) \leq 0\}$ and $\Psi^{i}_{1} = \{\rho \in (0.5, 1] : U^i(\underline{a}, \overline{\lambda}, \rho) \geq 0\}$.

maximizing platform. The obvious difference lies on while the former cares about the inframarginal users, the latter only care for the marginal users. Platform's objective now will be to find the optimal level of information provision that maximizes the weighted sum of the gross expected participation payoff from both type of individual, where the weights are their proportion in the population, e.g.

$$\max_{\rho \in \Phi_{\overline{\lambda}} \cap \Psi_1} \overline{\lambda} U(\overline{a}, \overline{\lambda}, \rho) + (1 - \overline{\lambda}) U(\underline{a}, \overline{\lambda}, \rho)$$

We find a trivial solution for the highest ranking matching equilibrium. In this equilibrium "economic activity" is the highest because every one is willing to accept any partner they meet, naturally then the signal does not affect at all users' behavior. The surplus maximizing platform will be indifferent between any feasible signal, i.e. $\rho^* \in \Phi^{ii}_{\overline{\lambda}} \cap \Psi^{ii}_1$. The optimal signal for the other two matching equilibria is also non-trivial. With the second highest ranked matching equilibrium, e.g. Case (ii) Mixed, the optimal signal rule must include the elasticity of the probability a high type user accepts a low signal parter with respect to the signal, $\xi_{\sigma_{\overline{a}\underline{\alpha}}}$. The signal that maximizes social surplus will differ from the one from the profit maximizing platform because the latter only considers the impact of the signal on one type of users, e.g. those with the lowest gross expected participation payoff. A similar argument holds for the last matching equilibrium to considers.

The following proposition formally states the optimal signal level for a surplus maximizing platform.

Proposition 4. *Assume all users participate $\lambda = \overline{\lambda}$. (i) For the highest ranked matching equilibrium, e.g. Case (ii), the surplus maximizing platform will make the same decision as the profit maximizing platform. (ii) For the second highest ranked matching equilibrium, e.g. Case (ii) Mixed, the optimal (interior) signal satisfies the following rule:*

$$\xi_{\sigma^*_{\overline{a}\underline{\alpha}},\rho} = \omega(\rho)\left(\frac{1-\sigma^*_{\overline{a}\underline{\alpha}}}{\sigma^*_{\overline{a}\underline{\alpha}}}\right) - (1-\omega(\rho))\left[\frac{\sigma^*_{\overline{a}\underline{\alpha}} + \rho(1-\sigma^*_{\overline{a}\underline{\alpha}})}{\sigma^*_{\overline{a}\underline{\alpha}}} + \frac{\overline{a}c - \underline{a}}{\overline{a}(1-c)}\frac{1-\overline{\lambda}}{\overline{\lambda}}\frac{1}{\sigma^*_{\overline{a}\underline{\alpha}}}\right]$$

where $\xi_{\sigma^*_{\overline{a}\underline{\alpha}},\rho}$ is the elasticity of the $\sigma^*_{\overline{a}\underline{\alpha}}$ with respect to ρ, and $\omega(\rho) = \frac{\overline{a}Pr\{\overline{a}\}}{\overline{a}Pr\{\overline{a},\underline{\alpha}\} + \underline{a}Pr\{\underline{a}\}}$. (iii) For the matching equilibrium Case (i) the optimal (interior) signal will satisfy:

$$\rho = \frac{1-\overline{\lambda}}{\overline{\lambda}}\left(\frac{2\underline{a} - (\underline{a} + \overline{a})c}{2\overline{a}(1-c)}\right)$$

Proof. See the appendix. □

Propositions (3) and (4) let us identify and analyze the differences between the surplus maximizing and the profit maximizing platform. Without loss of generality consider the optimal information provision rule for the second best ranked matching equilibrium, e.g. Case (ii) Mixed. While on the left hand side of both equations we have the elasticity of $\sigma^*_{\overline{a}\underline{\alpha}}$ wrt ρ, on the right hand side we observe two differences. The first one is an additional term for the surplus maximizing platform, which we will name as *Within Group Effect* (WGE), and represents the marginal effect of the signal on the gross expected participation payoff of high type user. Moreover we observe this effect has a negative sign. The second difference between both equations are the weights attached to the marginal effects of the signal on the gross expected participation payoff.

The WGE is not a new object in economics, its sign though provides us more intuition. From standard IO text books we know the WGE is explained by the different objective functions surplus maximizing and profit maximizing platforms have, while the former care about the inframarginal user participating, the latter care about the marginal user. The fact WGE has a negative sign tells us the surplus maximizing platform will place the optimal information provision level where the elasticity of $\sigma_{\overline{a}\underline{\alpha}}$ is lower compared to the optimal profit maximizing information provision level.

Before moving forward with the analysis it is interesting to establish if there exists any over/under provision of information given a fixed participation level. From proposition (3) we learned that only in two matching equilibria the surplus maximizing and profit maximizing platform will take different decisions, e.g. Case (ii) Mixed and Case (i). The answer to this question very much depends which matching equilibrium we are looking at.

Consider Case (ii) Mixed with full participation. If the platform believes that high type users will accept with some probability low signal partners, the profit maximizing platform will be interested in increasing the demand for users receiving a low signal, and this will be done by providing as much information as possible. On the other hand, the surplus maximizing platform will not be so eager in providing as much information as before because doing so reduces low type users chances of forming a match. In other words, while the surplus maximizing platform is interested in increasing the provision of information to benefit both type of users, the profit maximizing platform wishes to increase the provision to benefit more high type users.

In the other matching equilibrium, e.g. Case (i), with full participation, the result is the opposite. Indeed, if the platform believes high type users will never accept a low signal partner the surplus maximizing platform can approximate to perfect assortative match, where high types and low types only match with their peers, by providing perfect information if possible. On the other hand, the profit maximizing platform is less concerned about perfect sorting, and thus will provide less information.

The following propositions states formally the arguments,

Proposition 5. *Assume all users participate $\lambda = \overline{\lambda}$. The profit maximizing platform will over-provide information in relation to the surplus maximizing platform, e.g. $\rho^{private} \geq \rho^{surplus}$, whenever the high type user accepts low signal partners with a positive probability. If the high type user always rejects low signal partners, now the profit maximizing platform will under-provide information.*

Proof. See appendix. □

4.1.2 *Partial Group Participation*

The provision of information plays a crucial role for the screening in these cases. Consider the example where the platform wishes that all high type users participate, but only a *fraction* of low types participate. In the previous section we showed that to achieve this the information provision should be fixed at a level such that the net expected participation payoff for low type users is exactly zero, e.g. $\underline{\rho}$ such that $U(\underline{a}, \lambda, \underline{\rho}) = P$ and $\lambda = \frac{\overline{\lambda}}{\overline{\lambda}+\underline{A}}$. Previous literature on information provision do not emphasize on this new role because they do not let users decide to participate or not into the environment.

A platform that wishes to achieve a partial group participation outcome will affect the matching equilibrium. As we shown in the previous section only two matching equilibria support partial participation, e.g. Case (i) and Case (ii) Mixed. Is not a coincidence these two matching equilibria share this feature, they are the only cases where the platform can affect the probability a user has to be accepted and form a match.

Private Optimal λ. The optimization program is very simple. If we analyze the situation where all highs participate but only "a fraction" of lows does, then by construction we know the net expected participation payoff for low types is equal to zero, and for high types is non-negative. As the expected participation payoffs satisfy $U(\overline{a}, \lambda, \rho) \geq P = U(\underline{a}, \lambda, \rho)$, the access fee will be increased up to the point where $U(\overline{a}, \lambda, \rho) = P$. The platform will pick a provision of information level $(\underline{\rho})$ such that $U(\overline{a}, \lambda, \underline{\rho}) = P = U(\underline{a}, \lambda, \underline{\rho})$.

The profit maximizing platform will determine the participation level $\underline{A} \in [0, 1 - \overline{\lambda}]$ that maximizes the revenue because costs are normalized to zero. More formally, the optimization program will be $\max_{\underline{A} \in \Phi_\lambda \cap \Psi_M} (\overline{\lambda}\underline{A}) U(\overline{a}, \lambda, \underline{\rho}(\underline{A}))$ where $\lambda = \frac{\overline{\lambda}}{\overline{\lambda}+\underline{A}}$ and $\underline{\rho} \equiv \rho(\underline{A})$.

For the other partial participation case, where "a fraction" of high types participate and all low types participate, the platform determines the participation level $\overline{A} \in [0, \overline{\lambda}]$ that maximize also the revenues. By construction this object is $(\overline{A} + (1 - \overline{\lambda}))U(\underline{a}, \lambda, \overline{\rho}(\overline{A}))$ where $\lambda = \frac{\overline{A}}{\overline{A}+(1-\overline{\lambda})}$ and $\overline{\rho}(\overline{A})$ is the level of information that guarantee $U(\overline{a}, \lambda, \overline{\rho}) = P = U(\underline{a}, \lambda, \overline{\rho})$.

The following proposition describes the interior solution for both partial group participation cases,

Proposition 6. *(i) The profit maximizing platform will extract the gross expected participation payoff from the high type and low type users that participate. (ii) If all high types participate and a fraction of low types participate, the optimal (interior) mass of lows in the platform satisfy*

$$\xi_{\underline{\rho}(\underline{A}), \underline{A}} = -\left(\frac{1 - \underline{\rho}(\underline{A})}{\underline{\rho}(\underline{A})}\right)(2\underline{\rho}(\underline{A}) - 1)$$

where $\underline{\rho}$ satisfies $U(\overline{a}, \lambda, \underline{\rho}) = U(\underline{a}, \lambda, \underline{\rho})$, where $\lambda = \frac{\overline{\lambda}}{\overline{\lambda}+\underline{A}}$, and where $\xi_{\underline{\rho}(\underline{A}), \underline{A}}$ is the elasticity of $\underline{\rho}(\underline{A})$ with respect to $\underline{A}$. (iii) If a fraction of high types participate and all low types participate, the *optimal (interior) mass of highs in the platform satisfy*

$$\xi_{\overline{\rho}(\overline{A}),\overline{A}} = \frac{1-\overline{\rho}(\overline{A})}{\overline{\rho}(\overline{A}}$$

where $\overline{\rho}$ *satisfies* $U(\overline{a},\lambda,\overline{\rho}) = U(\underline{a},\lambda,\overline{\rho})$*, where* $\lambda = \frac{1-\overline{\lambda}}{\overline{A}+1-\overline{\lambda}}$*, and where* $\xi_{\overline{\rho}(\overline{A}),\overline{A}}$ *is the elasticity of* $\overline{\rho}(\overline{A})$ *with respect to* $\overline{A}$*.*

Proof. See appendix. □

Platform's decision determine which matching equilibrium is observed, in particular, in the appendix we show each partial group participation case can be supported by only one matching equilibrium. The first case in Proposition (6) we show is supported by the matching equilibrium where high type users accept high signal partners, but mix with low signal partners, e.g. Case (ii) Mixed. The second case in Proposition (6) we show is supported by the matching equilibrium where high type users *only accept* high signal partners, e.g. Case (i).

The surplus maximizing platform decision is the same as in Proposition (6). Without loss of generality focus on the case where all high types participate but a fraction of low type users participate. Here the surplus from all high types users will be $\overline{\lambda}U(\overline{a},\lambda,\underline{\rho})$, where the fraction of highs in the platform is $\lambda = \frac{\overline{\lambda}}{\overline{\lambda}+\underline{A}}$, the mass of lows is $\underline{A} \leq 1-\overline{\lambda}$, and where the information provision is fixed at a level $\underline{\rho}$ such that $U(\overline{a},\lambda,\underline{\rho}) = U(\underline{a},\lambda,\underline{\rho})$. On the other hand, the surplus from the low types will be $\underline{A}U(\underline{a},\lambda,\underline{\rho})$, but by construction it turns out to be $\underline{A}U(\overline{a},\lambda,\underline{\rho})$. The total surplus will be $(\overline{\lambda}+\underline{A})U(\overline{a},\lambda,\underline{\rho})$ which is the same revenue a profit maximizing platform faces in this case.

Corollary 2. *The surplus maximizing platform and the profit maximizing platform take the same decisions in the partial participation cases*

4.2 Optimal Menu

The optimal menu of access fee and information provision, e.g. (P,ρ), can be established by comparing the profits/surplus obtained from all possible levels of participation, $\lambda \in \{1, 0, \overline{\lambda}, \frac{\overline{\lambda}}{\overline{\lambda}+\underline{A}}, \frac{\overline{A}}{\overline{A}+(1-\overline{\lambda})}\}$, at every point in the parameter space $\Delta_a, \overline{\lambda}$, and c. A priori we do not know if it is optimal to a platform to keep some high type or low type users, also we do not know if it is optimal to induce the partial participation of one of the groups.

From hereon our interests are twofold. First, we will analyze if it is optimal to exclude either the low or the high type users at a given point in the parameter space. Secondly, we will be do some comparative statics on the set of optimal menus.

The main results are also twofold. First, that either the profit and the surplus maximizing platforms will find optimal to propose menus where only high type users, or both type of users decide to participate. In other words, these platforms will not be interested in excluding high type users from the platform. Additionally, this is line with what others, such as Johnson & Myatt 2006, have found in models with related contexts.

The second result is about the comparative statics. Borrowing Johnson & Myatt's terminology, we will call an Informative Menu that where only high or low type users are willing to participate, and Hype Menu that were both type of users are willing to participate. We will show that the higher is the heterogeneity in the population Δ_a, the bigger will be the region, at parameter space $(\overline{\lambda}, c)$, that support the Information Menu for high type users. In short, the higher the heterogeneity the bigger the incentives for the platform to only let participate high type users.

Private Optimal Menus. Assuming every user participates, only the two highest ranked matching equilibrium may emerge. This is by no means a surprise because, the platform has strong incentives to increase the probability any user with a low signal to be accepted by his randomly assigned partner. In other words, either the platform will keep the highest ranked matching equilibrium, or he will pick the right level of information such that the probability a high type accepts a low signal partner ($\sigma_{\overline{a}\underline{\alpha}}$) approaches to one. This logic cannot be applied to matching equilibrium Case (i) and Case (i) Mixed.

Figure (28) in the appendix shows, for a given level of heterogeneity, the region where matching equilibrium Case (ii) Mixed prevail over the highest ranked equilibrium Case (ii). Additionally, in a comparative statics exercise, we observe a decrease in the population heterogeneity will reduce the shaded are to the left. For the extreme situation with a very small heterogeneity most of the parameter space $(\overline{\lambda}, c)$ will support the highest ranked matching equilibrium.

Finally, we must figure out where the Informative Menus, with either low or high type users, prevail over the Hype Menu, where every user participates and the platform provides a partly informative signal. Though a priori is unclear the comparison between the Informative Menu with the high types and the Hype Menu, we can argue the platform will not find optimal an Informative Menu with low type users.

If the fraction of high type users in the population $\overline{\lambda}$ is low, the profit maximizing platform still will prefer to include high type users. For this region an Informative Menu with low types yields higher profits than a similar Menu with hight types because the profit with the latter is $(1-\overline{\lambda})\underline{a}(1-c)$, and with the former is $\overline{\lambda}\overline{a}(1-c)$, then if $\overline{\lambda}$ is lower than $\underline{a}/(\overline{a}+\underline{a})$, the platform will keep on board only low type users. On the other hand, from Figure (28) we know that also for low values of $\overline{\lambda}$ (and for most values of c) the platform will use matching equilibrium Case (ii) Mixed to get on board every user. The profit maximizing platform must choose between only accepting low type users, which yields a profit of $(1-\overline{\lambda})\underline{a}(1-c)$, or accepting every user, which yields a profit of $U_M^{ii}(\underline{a}, \overline{\lambda}, \rho^*)$. Now is easier to posit the platform will prefer the Hype Menu over the Informative Menu with low types because with the former an extra mass of users, equivalent to $\overline{\lambda}$, are participating, and this effect outweighs any possible reduction in the access fee.

If the fraction of high type users in the population is rather high, it is also optimal to keep on board high type users. In this case the platform has two possible comparisons to make. Either he is choosing between the participation of every user, but now under the highest ranked matching equilibrium, with yields a profit of $\underline{a}(1-c)$, or the participation of only low types, which yield a profit equal to $(1-\overline{\lambda})\underline{a}(1-c)$, or he is considering to include only high type users, which yields a profit equal to $\overline{\lambda}\overline{a}(1-c)$. In both cases its clear that the profit maximizing platform will have incentives to let in high type users.

The following theorem summarizes all what has been said in the previous paragraphs.

Theorem 1. *(i) The profit maximizing platform will not offer the Informative Menu for low type users, (ii) There will be a unique optimal menu for each point in the parameter space $(\Delta_a, \overline{\lambda}, c)$, and (iii) Only Case (ii) Mixed matching equilibrium could be optimally supported at the Hype Menu.*

Proof. See the appendix. □

The following figure shows the regions, in the parameter space $(\overline{\lambda}, c)$ and given a small value of Δ_a, where either the Hype Menu or the Informative Menu with high types will appear. There we show that for high values of $\overline{\lambda}$, and not too high values of c[27], the profit maximizing platform will prefer to offer the Informative Menu for high type users (dark area). As mentioned before this menu charges an access fee equal to $\overline{a}(1-c)$, and provides full information. As for the rest of the parameter space, the platform will propose the Hype Menu, there every user in the population participates by paying an access fee equal to the gross expected participation payoff from low type users, and the platform will provide a partially informative signal

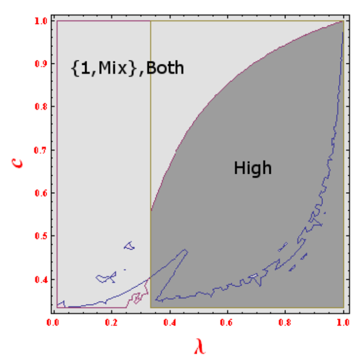

Figure 8: Δ_a Small

The previous theorem highlights the motivation behind the profit maximizing platform's decisions. In plain words, the platform wants to design an environment such that users' gross expected participation payoff is enlarged as much as possible because this will raise platform's profits. We have seen that the better the signal the higher the participation payoff for high type users, but this

[27]To be more precise, the higher is $\overline{\lambda}$ also the higher the opportunity cost could be.

is not necessarily true for low type users because they benefit from noisy signals. Then, if the profit maximizing platform prefers having only the high type users he will enlarge their participation payoff by providing more information, and latter will set the access fee to extract it all. In addition, if the platform faces other environment characteristics such that he wishes to keep on board all users, he can do so by charging a smaller access fee and providing a less informative signal. In this new situation the information is not the ideal for any type of user, high type users would have preferred a perfectly informative signal and the low types would have preferred a purely noisy signal.

Lets say a few more words about why the Informative Menu for low types is not optimal. We argued that the only region this menu could turn optimal is when the fraction of high type users in the population ($\overline{\lambda}$) is low enough, but we showed that the profit maximizing platform will prefer to include additional high type users. To be more precise, the platform faces two opposing effects associated to the inclusion of these type uses. If every user participates there is a volume effect explained by the $\overline{\lambda}$ mass of new participants, but also will have a (negative) revenue effect because the access fee will be reduced. Assumption (4) plays a key role in choosing which effect is the dominant one, in our model the volume effect will dominate the revenue effect.

Socially Optimal Menus. Platforms in this case will take different decisions because they care for the surplus of all participating users. As with the previous analysis we are interested in characterizing the set of optimal menus in the parameter space $(\overline{\lambda}, c)$. We also restrain the analysis to full group participation cases to compare the results in Theorem (1), latter we will comment on the possibility of including partial group participation.

Focusing the attention on the full group participation cases, e.g. $\lambda \in \{0, \overline{\lambda}, 1\}$, the analysis do not qualitatively differ from what we obtained with the profit maximizing platform. Indeed, in this situation still its optimal always to include high type users into the environment. To see why this is the case imagine again the fraction of high type users in the population is "low enough", e.g. $\overline{\lambda} < \underline{a}/(\overline{a}+\underline{a})$, so the platform faces the decision of keeping only low type users and reach a surplus of $(1-\overline{\lambda})\underline{a}(1-c)$, or let every user participates and reach a surplus of $\overline{\lambda}U(\overline{a}, \overline{\lambda}, \rho^*)+(1-\overline{\lambda})U(\underline{a}, \overline{\lambda}, \rho^*)$, where ρ^* satisfy the optimality condition described in the previous section. Again, the platform will contrast a positive volume effect associated to the mass $\overline{\lambda}$ of new participants, and a negative effect (previously mentioned as revenue effect) explained by the fact low type's participation payoff will be reduced. Assumption (4) is crucial here to determine that the volume effect will outweigh the negative effect.

The following theorem will formally state the result,

Theorem 2. *Restrain λ to $\{0, \overline{\lambda}, 1\}$. The surplus maximizing platform's set of optimal menus will not be qualitatively different from that in Theorem (1), that is, (i) its not optimal to offer the Informative Menu for low type users, (ii) there is a unique menu for each point in the parameter space $(\Delta_a, \overline{\lambda}, c)$, and (iii) in the Hype Menu users will use matching equilibrium Case (ii) Mixed.*

Proof. Omitted because its a repetition of Theorem (1)'s proof. □

If the information provision does not work as a screening tool we cannot make a general statement about the over/under provision of information. Despite this, we can conclude the profit maximizing platform will optimally over-provide information, vis-a-vis the benchmark, as long as both platforms offer the Hype Menu. Also we can conclude that if the profit maximizing platform offers the Hype Menu and the surplus maximizing platform offers the Informative Menu for the high type users, the former platform will now under-provide information because in the benchmark case signal is fully informative.

The following figures show the region, on the parameter space $(\overline{\lambda}, c)$ and given a medium value of Δ_a, where Hype and Informative Menus prevail. We observe that the Hype Menu (dark region) is located at the east of the regions.

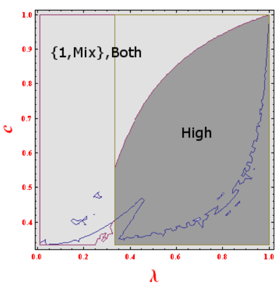

Figure 9: Profit Max., Δ_a Medium

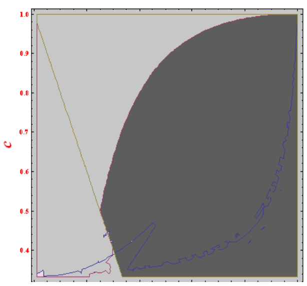

Figure 10: Surplus Max, Δ_a Medium

Its important to remark its not socially optimal to provide full information when all users participate. We tend to think the surplus maximizing platform have strong incentives to provide full information hoping high type users only match with their peers. But in this environment users cannot use transfer to efficiently divide their matching surplus. Then, the surplus maximizing platform internalize this externality and choose to provide a partially informative signal.

To finish the characterization of the set of optimal menus we proceed with the comparative statics. The most important element will the the heterogeneity of users in the population, e.g. Δ_a. In particular, we observe that the higher is the heterogeneity the bigger will be the region, in the parameter space $(\overline{\lambda}, c)$, supporting the Hype Menu. Intuitively this result is straight forward because any platform will have stronger incentives to only let high types participate, the bigger the gap between users' types. To conclude, if we take for example Figure (9), we can obtain that the higher the heterogeneity, the bigger will be the dark region.

Corollary 3. *The region, in the parameter space $(\overline{\lambda}, c)$, where the Hype Menu is optimal is positively related with the population heterogeneity, e.g.* Δ_a

This last corollary shows an unexpected relationship with the literature on information provision. Johnson & Myatt (2006) found that a key determinant about what sort of advertising a monopolist should offer is the heterogeneity of the consumers. In particular, they observe that if the heterogeneity is high the optimal advertising should target selected groups in the population (Hype), but if heterogeneity is low the optimal advertising should target the average consumer (Informative). Our model predictions follow the same spirit.

To complete the analysis we must consider the situation where any platform wishes to induce partial group participation. This situation arises when the information provision works as a screening tool. Qualitative results up to now will not change, and the question which remains to be answered is if its optimal for the profit or surplus maximizing platform to offer a menu that induces the partial participation of one of the groups. In line with the results obtained so far we posit this model should predict it is only optimal to offer a menu were all high type users participate and only a fraction of low type users participate.

5 Extension: Two-Sided Platform

Until now we focused on a "one-sided" platform which is either incapable to distinguish which group users belong to, and/or is uninterested to discriminate between users according to their group of origin. In this particular situation we learned that in the presence of heterogeneous network effects the platform must use the access fee and the information provision to attain each point of his demand. Also we learned that the optimal set of menus do not exclude high type users, and that the profit maximizing platform will over-provide information compared to the surplus maximizing platform.

What else can we learn in a "two-sided" platform ? One interesting aspect is that when the heterogeneity of one side (a-users) is different from that of the other side (b-users). For example, if the a-group is very heterogeneous and the b-group is very homogeneous any platform will tend to keep on board only high type a-users, and any b-type user. Another interesting aspect is that when one side of the market is bigger than the other. For example, in the labor market for skilled workers in a developing country the mass of workers looking for a job will be smaller than the mass of firms willing to hire them.

In this extension we will focus con that situation where the heterogeneity differs, e.g. $\Delta_a \neq \Delta b$, but the size of the populations is the same, e.g. $\overline{M}_a = \overline{M}_b = 1$[28]. Our goal will be to stress out which insights from the "one-sided" platform are still valid in the "two-sided" platform, and which not.

Model. The setup do not suffer any significant modification. We analyze again a one-period game, the population of users is divided in two sides (A and B), and each side is partitioned in

[28] A broader analysis of the multi-sided platform is a matter of analysis in a companion paper. Interested readers please visit the author's web page.

two subgroup of high and low users, e.g. $a \in \{\underline{a}, \overline{a}\}$ and $b \in \{\underline{b}, \overline{b}\}$. The fraction of high a-users and b-users in the population will be denoted by $\overline{\lambda}_a$ and $\overline{\lambda}_b$. Finally, as well as in the "one-sided" case, users who participate do not observe directly the objective characteristics from the randomly assigned partner, instead they observe an informative signal which can either be high or low. To wit, an a-user that participates will only observe a signal $\beta\{\underline{\beta}, \overline{\beta}\}$ from his b-user partner. Analogously, an b-user that participates will only observe a signal $\alpha\{\underline{\alpha}, \overline{\alpha}\}$ from his a-user partner.

To make a parallel with assumption (1) define first ρ_a and ρ_b as the probability a high a-users and b-user receive, respectively, a high signal $\overline{\alpha}$ and $\overline{\beta}$. The assumption we know have to make is

Assumption 4. *Let $\rho_b > 1/2$, and $\rho_a > 1/2$*

Matching payoffs are such that high type a-users and b-users dislike forming a match with a low type partner. As before, the benefits depend on who form the match, and the cost are type specific. To fix ideas pick a high type a-users. His matching payoff will be $\overline{b} - \overline{a}c$ only if his partner has a high type, otherwise his payoff is $\underline{b} - \overline{a}c$. Then we must replace assumption (2) with,

Assumption 5. *Let users $a \in \{\underline{a}, \overline{a}\}$ and $b \in \{\underline{b}, \overline{b}\}$ form a match, and let $\mathbb{1}$ be an indicator function, then the payoff for user "a" will be $\mathbb{1}_{a=\overline{a}, b=\overline{b}}\overline{b} + (1 - \mathbb{1}_{otherwise})\underline{b} - ac$, ad for user "b" will be $\mathbb{1}_{a=\overline{a}, b=\overline{b}}\overline{a} + (1 - \mathbb{1}_{otherwise})\underline{a} - bc$.*

Finally, the crucial assumption we impose in the model is that high type a-users and b-users only obtain a positive payoff when they form a match with one of their peers from the other side of the market.

Assumption 6. *Assume $\overline{a} - \overline{b}c, \overline{b} - \overline{a}c > 0 > \underline{a} - \overline{b}c, \underline{b} - \overline{a}c$*

In line with the standard two-sided market literature we will allow the platform to establish a different access fee for each group, e.g. P^a and P^b. Similarly, he will provide a different ρ for each group, e.g. ρ_a and ρ_b. Finally, the strategies for the users and the platform are adapted to the new setting. We will call $\sigma_{a\beta}$ ($\sigma_{b\alpha}$) the probability an a-user (b-user) accepts a β-signal (α-signal) partner, and γ_a (γ_b) the decision a user from side "A" ("B") enter into the matching environment. The platform's strategy specifies the access fees, e.g. P^a and P^b, and the informativeness of the signal, e.g. ρ_a and ρ_b. The timing remains unchanged[29].

Model's Main Restriction. The main restriction our approach has is over the mass of a-users and b-users in the population. Indeed we analyze the situation where both masses are equal and normalized to one, i.e. $\overline{M}_a = \overline{M}_b = 1$. The difference between both sides can be attributed to the heterogeneity in the population, i.e. $\Delta_a \neq \Delta_b$.

[29]The graph is on the appendix.

Matching Equilibrium. The expected payoff of accepting, say for an a-user, after participating and given all available information will be

$$\begin{aligned} v(\beta,\overline{a}) &= (\overline{b}-\overline{a}c)Pr\{\overline{b}\mid\beta\}\left[Pr\{\overline{\alpha}\mid\overline{a}\}\sigma_{\overline{b}\overline{\alpha}}+Pr\{\underline{\alpha}\mid\overline{a}\}\sigma_{\overline{b}\underline{\alpha}}\right]+(\underline{b}-\overline{a}c)Pr\{\underline{b}\mid\beta\} \\ v(\beta,\underline{a}) &= (\underline{b}-\underline{a}c)\left[Pr\{\overline{b}\mid\beta\}(Pr\{\overline{\alpha}\mid\underline{a}\}\sigma_{\overline{b}\overline{\alpha}}+Pr\{\underline{\alpha}\mid\underline{a}\}\sigma_{\overline{b}\underline{\alpha}})+Pr\{\underline{b}\mid\beta\}\right] \end{aligned}$$

Similarly, for an b-user will be

$$\begin{aligned} w(\alpha,\overline{b}) &= (\overline{a}-\overline{b}c)Pr\{\overline{a}\mid\alpha\}\left[Pr\{\overline{\beta}\mid\overline{b}\}\sigma_{\overline{a}\overline{\beta}}+Pr\{\underline{\beta}\mid\overline{b}\}\sigma_{\overline{a}\underline{\beta}}\right]+(\underline{a}-\overline{b}c)Pr\{\underline{a}\mid\alpha\} \\ w(\alpha,\underline{b}) &= (\underline{a}-\underline{b}c)\left[Pr\{\overline{a}\mid\alpha\}(Pr\{\overline{\beta}\mid\underline{b}\}\sigma_{\overline{a}\overline{\beta}}+Pr\{\underline{\beta}\mid\underline{b}\}\sigma_{\overline{a}\underline{\beta}})+Pr\{\underline{a}\mid\alpha\}\right] \end{aligned}$$

We can explicitly describe, within each side, the joint distribution of types and signal. For the A-side we have $Pr\{\overline{a},\overline{\alpha}\}=\rho_a\lambda_a$, $Pr\{\overline{a},\underline{\alpha}\}=(1-\rho_a)\lambda_a$, $Pr\{\underline{a},\overline{\alpha}\}=(1-\rho_a)(1-\lambda_a)$ and $Pr\{\underline{a},\underline{\alpha}\}=\rho_a(1-\lambda_a)$. For the B-side we will have $Pr\{\overline{b},\overline{\beta}\}=\rho_b\lambda_b$, $Pr\{\overline{b},\underline{\beta}\}=(1-\rho_b)\lambda_b$, $Pr\{\underline{b},\overline{\beta}\}=(1-\rho_b)(1-\lambda_b)$ and $Pr\{\underline{b},\underline{\beta}\}=\rho_b(1-\lambda_b)$.

The multiplicity of equilibria now becomes more complex. As with the "one-sided" platform we will obtain pure strategy and mixed strategy matching equilibria, but now the set of the former will be bigger. Indeed, on top of obtaining a matching equilibrium where both high type a-users and b-users accept every partner, e.g. Case (ii), and where the same users only accept high signal partners and reject low signal partners, e.g. Case (i), we will obtain another case which we will call "non-mirror" pure strategy matching equilibria (NM-Case). In this additional situation, while high type a-users accept any randomly assigned partner, high type b-users will only accept high signal partners and reject low signal partners. The opposite situation should also hold, while high type b-users accept any partner, the high type a-users will only accept high signal randomly assigned partners.

To fix ideas analyze the decision from a high type a-user. In the NM-Case where he only accepts high signal partners but high type b-users accept everyone, the expected payoff of accepting, given all available information, will be

$$v(\beta,\overline{a}) = \overline{b}Pr\{\overline{b}\mid\beta\}\underline{b}Pr\{\underline{b}\mid\beta\}-\overline{a}c \gtreqless 0$$

In the opposite situation this same expression will be,

$$v(\beta,\overline{a}) = (\overline{b}-\overline{a}c)Pr\{\overline{b}\mid\beta\}Pr\{\overline{\alpha}\mid\overline{a}\}+(\underline{b}-\overline{a}c)Pr\{\underline{b}\mid\beta\} \gtreqless 0$$

it is possible to obtain a plot in the parameter space (ρ_b,λ_b), similar to those from the "one-sided" platform, showing the region supporting this new matching equilibrium[30].

The characterization of the set of matching equilibria is not different. In the case of a "one-sided" platform we showed this set is a complete lattice and we can determine the *greatest* and

[30] All these details will be covered extensively in a companion paper.

the *least* among them. The key argument in favor of this result is that high users' best response correspondence is increasing. For the "two-sided" platform case still is true the best response correspondence satisfy this condition.

Why should we expect that the set of matching equilibria in a "two-sided" platform is not a complete lattice ? The complexity of the environment we analyze depends on the characteristics of both sides, and in particular depends on the heterogeneity of the population on each side. If both groups are not *too different* we would expect high type users from both side behaving alike, but if they are *different enough* its likely that the matching equilibrium NM-Case becomes relevant.

Participation. The message we learned from the "one-sided" case is that to reach every point in the platform's demand we require from both the access fee and the information provision. Recall that to induce those point in the demand with a participation of *a fraction* of high or low type users we require the access fee and the information provision to jointly work in a particular way. This insight is still valid for a "multi-sided" platform offering a NTU matching environment because to show it we only need to assume that network effects are heterogeneous.

Pricing & Information Provision Rules. To compare the analytical results obtained in propositions (2) - (4) with those from a "two-sided" platform we must include the non-mirror matching equilibrium (NM-Case). We will show that the only difference will arise with the Hype Menu.

The Informative Menus for high and low types are unchanged. If any platform wishes to keep on board only the high type a-users and b-users the optimal menu requires charging access fees that extract all their surplus. Additionally, the platform will provide a fully informative signal to avoid low type a-users and b-users from participating. The Informative Menu for high types will be $(P^a, P^b, \rho_a, \rho_b) = (\overline{b} - \overline{a}c, \overline{a} - \overline{b}, 1, 1)$. On the other hand, if any platform wishes to keep on board only the low type a-users and b-users he must pick the access fees that extract all user's rent, additionally he will provide a fully uninformative signal. The Informative Menu for low types will be $(P^a, P^b, \rho_a, \rho_b) = (\overline{b} - \overline{a}c, \overline{a} - \overline{b}, 1, 1)$.

The Hype Menu analysis is more interesting because of the *Spence Effect*. Like with the "one-sided" case, the platform will pick an access fee that induces the participation of all users from both sides. Additionally, he will provide a partially informative pair of signals because doing so induces the high and low type of users to participate.

The following proposition formalizes the arguments in the previous two paragraphs,

Proposition 7. *Let assumptions (4) - (6) hold. (i) The Informative Menus for high and low types are the same as with a "one-sided" platform. The profit maximizing's Hype Menu (ii.1) given matching equilibrium Case (ii) will provide any feasible level of information, (ii.2) given matching* *equilibrium Case (ii) Mixed will provide a pair of signals satisfying,*

$$\xi_{\sigma^*_{\overline{b}\underline{\alpha}}\rho_a} = \frac{1-\sigma^*_{\overline{b}\underline{\alpha}}}{\sigma^*_{\overline{b}\underline{\alpha}}} - \frac{\underline{a}-\underline{b}c}{\underline{b}-\underline{a}c}\frac{\overline{\lambda}_a\rho_b\sigma^*_{\overline{a}\underline{\beta}}}{\overline{\lambda}_b\sigma_{\overline{b}\underline{\alpha}}}\xi_{\sigma_{\overline{a}\underline{\beta}}\rho_a}$$

$$\xi_{\sigma^*_{\overline{a}\underline{\beta}}\rho_b} = \frac{1-\sigma^*_{\overline{a}\underline{\beta}}}{\sigma^*_{\overline{a}\underline{\beta}}} - \frac{\underline{b}-\underline{a}c}{\underline{a}-\underline{b}c}\frac{\overline{\lambda}_b\rho_a\sigma^*_{\overline{b}\underline{\alpha}}}{\overline{\lambda}_a\sigma_{\overline{a}\underline{\beta}}}\xi_{\sigma_{\overline{b}\underline{\alpha}}\rho_b}$$

and (ii.3) given matching equilibrium Case (i) will provide the least informative pair feasible signals.

Proof. The proof is on Proposition's (2) and (3) proof. □

The extra term we find at the right hand side of both equations is what is defined in Weyl (2010) as the *Spence Effect*. This effect will naturally appear in any other analytical result because it represents the marginal revenue obtained through the other side of the market. In order to not make a list the remaining analytical results we invite the reader to visit the appendix. In particular, the proofs of propositions (3), (4) and (6) are done not for the "one-sided" platform, but for the "two-sided" platform.

The complete analysis of the "two-sided" case requires tackling two final issues. On one hand, we need to consider the cases where the platform wishes to induce partial group participation on one side and full group participation on the other side. For example, any platform might consider inducing participation of all high a-users and partial participation of low type a-users, and inducing participation of both high and low type b-users. On the other hand, we need to characterize the set of optimal menus for all the parameter space $(\Delta_a, \Delta_b, \overline{\lambda}_a, \overline{\lambda}_b, c)$. Both of these issues are treated in a companion paper.

6 Conclusion

This paper considers a firm that designs and offers, for a fixed price, a matching market/enviroment to a population of vertically differentiated users. In addition, we acknowledge that these environments are not frictionless, and that the frictions are determined by the market design. The firm we analyzed jointly determines the market design and its pricing. Broadly speaking the research questions are twofold. First, how can the firm can jointly use the market design and pricing to screen particular segments of the population, and second, characterize the set of optimal menus of design and pricing.

Our approach restricts the matching environment and the "market creating" firm in several ways. On the one hand, the environment has two main properties. Users forming a match cannot use transfers to efficiently divide the surplus generated by the match, and users can *only* observe a signal from their randomly assigned partner's type after paying an access fee. On the other hand, the "market creating" firm will have two other main characteristics. The platform will pick the optimal level of access fee and information provision. And finally, we will assume that network effects are heterogeneous, in particular while low type users care about forming a match, high type users care about who they form a match with.

We propose two research questions. The first is to understand how a market creating firm can use the pairing of the access fee and information provision to screen particular segments of the population, and the second is to characterize the set of optimal menus of access fee and information provision.

We show that with heterogeneous network effects the access fee is not enough for screening and the platform must use the information provision to induce some levels of participation. Special attention is devoted to understand how information provision helps screening. We also show that the set of optimal menus of access fee and information provision rule out the possibility of excluding high type users. In the case it is optimal to retain every user we find the optimal signal is partially informative. Finally, we show that when both platforms decide to retain every user the profit maximizing firm will over-provide information in relation to the surplus maximizing firm. To complete the characterization we also look at some comparative statics. We observe that the region in the parameter space where it is optimal to retain only the high type users increases with the heterogeneity in the population, i.e. the difference between high and low types.

Within this framework there are several natural extensions worth analyzing. First, allow for platform competition. This extension should be a priority as it will allow us to approach less carefully the available data on platforms. We posit that the set of optimal menus will be strongly determined by the need of the platform to increase the mass of participating users. Secondly, allow for a second degree price discrimination within each group. We believe this extension is very relevant because by giving more flexibility on the pricing tool the gap between the information provided by the profit maximizing and surplus maximizing platform must shrink. Finally, we believe that regulation-related questions represent an interesting path for future research. For example, we could address again the topic of the Net Neutrality debate and study how regulations on non-pricing tools changes the status quo of the debate.

7 Appendix

7.1 Remarks & Tables

Remark (3)

The expected payoff of participation for Case (i) Mixed will be,

$$
\begin{aligned}
U_m^i(\overline{a}, \lambda, \rho) &= 0 \\
U_m^i(\underline{a}, \lambda, \rho) &= \underline{a}(1-c)(1-\lambda)\left(1-\left(\frac{\underline{a}-\overline{a}c}{\overline{a}-\overline{a}c}\right)\left(\frac{1-\rho}{\rho}\right)^2\right)
\end{aligned}
$$

Participation decision is simple for this matching equilibrium. When high type users decide to participate, $(\lambda, M) = (1, \overline{\lambda})$, gross expected participation payoff for high and low types is zero, e.g. $U_m^i(a, 1, \rho) = 0$ for $a \in \{\underline{a}, \overline{a}\}$. This situation implies high types are not willing to participate as long as $P > 0$. When low types decide to participate, $(\lambda, M) = (0, 1-\overline{\lambda})$, the gross expected participation payoff for low types is $U_m^i(\underline{a}, 0, \rho) = \underline{a}(1-c)\left(1+\left(\frac{\overline{a}c-\underline{a}}{\overline{a}(1-c)}\right)\left(\frac{1-\rho}{\rho}\right)^2\right)$, and for high types is zero; the usual access fee restriction applies. Finally, the situation where both high and low types participate, e.g. $(\lambda, M) = (\overline{\lambda}, 1)$, yield a gross expected participation payoff equal to $U_m^i(\overline{a}, \overline{\lambda}, \rho)$ and $U_m^i(\underline{a}, \overline{\lambda}, \rho)$. But as we require that $U_m^i(\underline{a}, \overline{\lambda}, \rho) \geq P$, again high types are not willing to participate as long as the access fee is nonnegative.

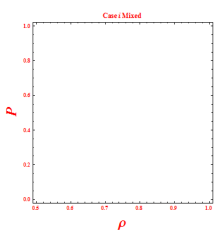

Figure 11: $(\overline{\lambda}, 1)$

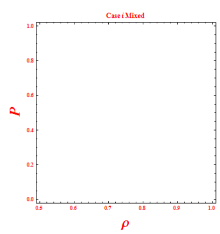

Figure 12: $(1, \overline{\lambda})$

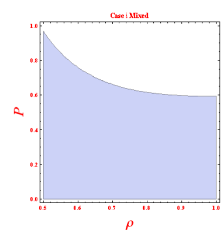

Figure 13: $(0, 1-\overline{\lambda})$

Figures (11) - (25) show only low type users are willing to participate into this matching equilibrium[31]. Indeed, only figure (25) shows a region where some users, all of them from the low type, are willing to participate.

A platform in this matching equilibrium can support only one *partial participation* situation. By construction we know the gross expected participation payoff for low type users is strictly positive, e.g. $U_m^i(\underline{a}, \lambda, \rho) > 0$, then is only possible to support a situation where all low types enter, but only a *fraction* of high types participate. The condition to be attained will be $U_m^i(\underline{a}, \lambda, \rho) = \underline{a}(1-c)(1-\lambda)[1+\frac{\overline{a}c-\underline{a}}{\overline{a}(1-c)}(\frac{1-\rho}{\rho})^2] > 0$, where $\lambda = \frac{\overline{A}}{\overline{A}+(1-\overline{\lambda})}$.

Remark (4)

Figures (14) - (16) show the cases when search costs are the minimum, e.g. $c = \frac{\underline{a}}{\overline{a}}$. The first figure shows the case where environments' heterogeneity its significant ($\overline{a} >> \underline{a}$), the second shows a situation where

[31] As with the previous cases, the figure on the left shows the case both types are willing to participate, the figure in the middle represent the case where only high types participate, and the figure on the right only low types participate.

environment's heterogeneity is not so accentuated ($\overline{a} > 2\underline{a}$), and the last a situation where environment's heterogeneity is the least ($\overline{a} \approx 2\underline{a}$). We observe from the figures that only two matching equilibria arise, one where high type users accept any partner ($(1,1)$), and another where they reject low signal partners and mix with high signal partners ($(\sigma_{\overline{a}\overline{\alpha}}, 0)$). Also we observe that the region supporting $(1,1)$ increases as search cost increases, and the reason is that the conditions such that high type users are more selective than low type users are less stringent as search cost increases. Indeed, as search cost increases is easier that the expected payoff from a high type user of accepting a low signal partner is smaller than the same expected payoff but now from a low type user.

Figures (17) - (19) show the cases where search costs are important, but not too much. As in the previous paragraph the first graph is when $\overline{a} >> \underline{a}$, the second when $\overline{a} > 2\underline{a}$, and the last when environment's heterogeneity is the minimum, e.g. $\overline{a} \approx 2\underline{a}$. The first feature to outline is that all matching equilibria may be supported if search costs are above the minimum level. In case $c_{min} = \underline{a}/\overline{a}$ we cannot sustain the matching equilibrium where high type users accept any partner they receive, e.g. $(\sigma_{\overline{a}\overline{\alpha}}, \sigma_{\overline{a}\underline{\alpha}}) = (1,1)$. Again, this result happens because the expected payoff of accepting a low signal partner is higher for high type users than low type users, which imply the former will not be more selective than low type users. The second feature to outline, see Figures (18) - (19), is that while search cost increases the regions that supports matching equilibria $(1, \sigma_{\overline{a}\underline{\alpha}})$ and $(1,1)$ decreases, and region that supports matching equilibrium $(1,1)$ increases. This feature is simply showing that as environment's heterogeneity increases, high type users becomes less more selective.

Last three figures show the situation when search costs are significant high. These cases provided no additional insights.

Timing for the Two-Sided Platform.

At the beginning, (i) the nature determines the fraction of high type a-users and b-users in the population, and every user privately learn their type. Then (ii), the platform determines P^a, P^b and ρ_a, ρ_b. At the next step, (iii) all a-users and b-users decide to enter or stay out of the environment. Among those that participate, (iv) users from one side randomly meet another user from the other side. All b-users and a-users learn α and β respectively. Now within each pair, (v) users decide to accept or not their randomly assigned partner. A match is formed when both types accept each other.

7.2 Omitted Proofs

Lemma 1 . Let assumptions (1) - (3) hold. Then, $v(\overline{\alpha}, \overline{a}) > v(\underline{\alpha}, \overline{a})$ and $v(\overline{\alpha}, \underline{a}) \leq v(\underline{\alpha}, \underline{a})$.

Proof. We will show that $v(\overline{\alpha}, \overline{a}) > v(\underline{\alpha}, \overline{a})$ (named as first condition from hereon), and $v(\overline{\alpha}, \underline{a}) < v(\underline{\alpha}, \underline{a})$ (named as second conditions from hereon) for every matching equilibrium.

In matching equilibrium Case (i)

$$v(\alpha, \overline{a}) = \overline{a}(1-c)Pr\{\overline{a} \mid \alpha\}Pr\{\overline{\alpha} \mid \overline{a}\} + (\underline{a} - \overline{a}c)Pr\{\underline{a} \mid \alpha\} \quad (3)$$

$$v(\alpha, \underline{a}) = \underline{a}(1-c)[Pr\{\overline{a} \mid \alpha\}Pr\{\overline{\alpha} \mid \underline{a}\} + Pr\{\underline{a} \mid \alpha\}] \quad (4)$$

the first condition boils down to $(2\rho - 1)(1 - \lambda) > 0$ and is satisfied by assumption (1), similarly the second condition is satisfied as it turns to $Pr\{\underline{\alpha} \mid \underline{a}\}(Pr\{\overline{a} \mid \overline{\alpha}\} - Pr\{\overline{a} \mid \underline{\alpha}\}) < 0$.

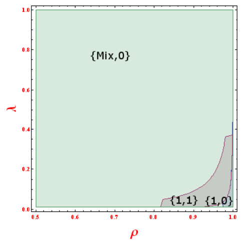

Figure 14: $c = \underline{a}/\overline{a}$

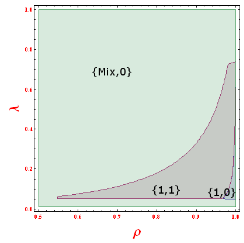

Figure 15: $c = \underline{a}/\overline{a}$

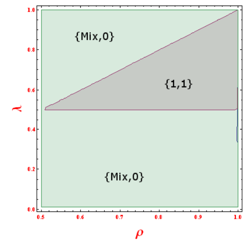

Figure 16: $c = \underline{a}/\overline{a}$

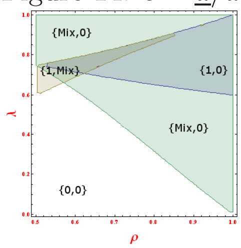

Figure 17: $c = 0.6$

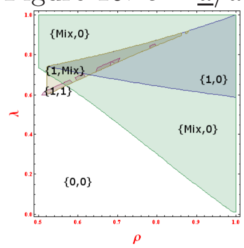

Figure 18: $c = 0.6$

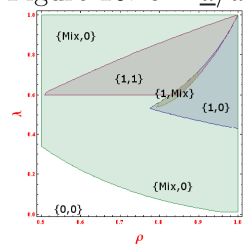

Figure 19: $c = 0.6$

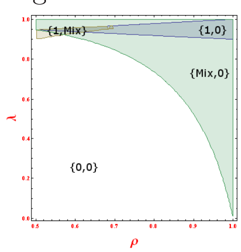

Figure 20: $c = 0.9$

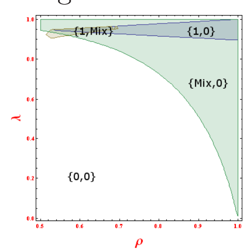

Figure 21: $c = 0.9$

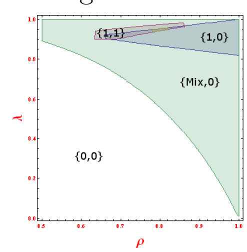

Figure 22: $c = 0.9$

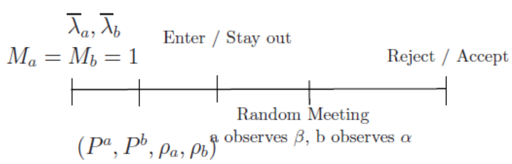

Figure 23: Two-Sided Platform Timing

For the matching equilibrium where high types access any partner, i.e. Case (ii)

$$v(\alpha, \overline{a}) = \overline{a}(1-c)Pr\{\overline{a} \mid \alpha\} + (\underline{a} - \overline{a}c)Pr\{\underline{a} \mid \alpha\} \tag{5}$$

$$v(\alpha, \underline{a}) = \underline{a}(1-c) \tag{6}$$

the first condition is $(\overline{a} - \underline{a})(2\rho - 1) > 0$ and is also satisfied by assumption (1). Second condition makes no sense in this case because the probability a low type of forming a match is equal to one.

For the matching equilibrium where high type users accept high signal partners, and mix with low signal partners, i.e. Case (ii) Mixed

$$v(\alpha,\overline{a}) = \overline{a}(1-c)Pr\{\overline{a} \mid \alpha\}[Pr\{\overline{\alpha} \mid \overline{a}\} + Pr\{\underline{\alpha} \mid \overline{a}\}\sigma_{\overline{a}\underline{\alpha}}] + (\underline{a} - \overline{a}c)Pr\{\underline{a} \mid \alpha\} \quad (7)$$

$$v(\alpha,\underline{a}) = \underline{a}(1-c)[Pr\{\overline{a} \mid \alpha\}(Pr\{\overline{\alpha} \mid \underline{a}\} + Pr\{\underline{\alpha} \mid \underline{a}\}\sigma_{\overline{a}\underline{\alpha}}) + Pr\{\underline{a} \mid \alpha\}] \quad (8)$$

the first conditions becomes $[\overline{a}(1-c)(Pr\{\overline{\alpha} \mid \overline{a}\} + Pr\{\underline{\alpha} \mid \overline{a}\}\sigma^*_{\overline{a}\underline{\alpha}}) - (\underline{a} - \overline{a}c)](Pr\{\overline{a} \mid \overline{\alpha}\} - Pr\{\overline{a} \mid \overline{\alpha}\}) > 0$, where $\sigma^*_{\overline{a}\underline{\alpha}}$ satisfies $v(\underline{\alpha},\overline{a}) = 0$. Here is immediate to show assumption (1) is enough to satisfy the inequality. The second condition boils down to guarantee that $Pr\{\overline{a} \mid \overline{\alpha}\}(Pr\{\overline{\alpha} \mid \underline{a}\} + Pr\{\underline{\alpha} \mid \underline{a}\}\sigma^*_{\overline{a}\underline{\alpha}}) + Pr\{\underline{a} \mid \overline{\alpha}\} < Pr\{\overline{a} \mid \underline{\alpha}\}(Pr\{\overline{\alpha} \mid \underline{a}\} + Pr\{\underline{\alpha} \mid \underline{a}\}\sigma^*_{\overline{a}\underline{\alpha}}) + Pr\{\underline{a} \mid \underline{\alpha}\}$ which again is satisfied by assumption (1).

Finally, for the matching equilibrium where high types reject low signal partners, and mix with high signal partners, i.e. Case (i) Mixed

$$v(\alpha,\overline{a}) = \overline{a}(1-c)Pr\{\overline{a} \mid \alpha\}Pr\{\overline{\alpha} \mid \overline{a}\}\sigma^*_{\overline{a}\overline{\alpha}} + (\underline{a} - \overline{a}c)Pr\{\underline{a} \mid \alpha\} \quad (9)$$

$$v(\alpha,\underline{a}) = \underline{a}(1-c)[Pr\{\overline{a} \mid \alpha\}Pr\{\overline{\alpha} \mid \underline{a}\}\sigma^*_{\overline{a}\overline{\alpha}} + Pr\{\underline{a} \mid \alpha\}] \quad (10)$$

where $\sigma^*_{\overline{a}\overline{\alpha}}$ guarantees $v(\overline{\alpha},\overline{a}) = 0$. The first condition turns out to be equal to $0 > \frac{\overline{a}c-\underline{a}}{Pr\{\underline{\alpha}\}Pr\{\overline{a},\overline{\alpha}\}}[Pr\{\overline{a},\underline{\alpha}\}Pr\{\underline{a},\overline{\alpha}\} - Pr\{\underline{a},\underline{\alpha}\}Pr\{\overline{a},\overline{\alpha}\}]$, and assumption (1) guarantees the term is braces is negative. The second condition is also trivially satisfied from assumption (1).

□

Proposition (1) . (i) The set of matching equilibria is nonempty, and we can determine "the greatest and the least" matching equilibria. (ii) The set of matching equilibria is a nonempty complete lattice.

Proof. Low type users' best response correspondence is trivial because they are willing to accept any partner they encounter, e.g. $\Omega_{\underline{a}}(\cdot) = \{(1,1)\}$. High type users' best response correspondence is upper and lower increasing given each region of Figure (6. Without loss of generality analyze the pairs (ρ,λ) that belong to region C from that figure. For any $\sigma' \in [0,1]^2$ there exist a least (greatest) element that belong to $\Omega_{\overline{a}}(\sigma';\rho,\lambda)$. For example, if $\sigma' = (1,\sigma_{\overline{a}\underline{\alpha}})$, we can verify that $(1,\sigma_{\overline{a}\underline{\alpha}}) \vee (0,0) = (1,\sigma_{\overline{a}\underline{\alpha}}) \in \Omega_{\overline{a}}(\sigma';\rho,\lambda)$, and similarly $(1,\sigma_{\overline{a}\underline{\alpha}}) \wedge (0,0) = (0,0) \in \Omega_{\overline{a}}(\sigma';\rho,\lambda)$. Finally, using Calciano's (2009) Theorem 2 we conclude that the set of fixed points of the matching game is nonempty, and that there exist a greatest and least fixed point.

We need to include a new piece of notation. As $[0,1]^2$ is a complete lattice, the greatest element of this set using the usual vector ordering is $(1,1)$. For a fixed $h \in [0,1]$, define the correspondence (which may be empty) $\Omega_{h,\overline{a}}(\sigma;\rho,\lambda) : [h,1]^2 \rightrightarrows [h,1]^2$ as

$$\Omega_{h,\overline{a}}(\sigma;\rho,\lambda) = \Omega_{\overline{a}}(\sigma;\rho,\lambda) \cap [h,1]^2$$

Due to the simple structure of high type users' best reply correspondence, for every $(h,h) \in \{\sigma \in [0,1]^2 : \Omega_{\overline{a}}(\sigma;\rho,\lambda) \cap [\sigma_{\overline{a}\cdot},+\infty)^2 \neq 0\}$, the correspondence $\Omega_{h,\overline{a}}$ has a least element whenever nonempty. Indeed, in our case $\{\sigma \in [0,1]^2 : \Omega_{\overline{a}}(\sigma;\rho,\lambda) \cap [\sigma_{\overline{a}\cdot},+\infty)^2 \neq 0\} = \{(1,1)\}$, so $\Omega_{1,\overline{a}} = \{(1,1)\}$ and there exist a least element. Finally, using Calciano's (2009) Theorem 3 we conclude the set of fixed points of the matching game is a complete lattice. □

Proposition (2) . (i) If only high type users participate $\lambda = 1$ the profit maximizing platform will provide perfect information, e.g. $\rho^* = 1$, and will charge an access fee equal to $\overline{a}(1-c)$. (ii) If only low type users participate $\lambda = 0$ the profit maximizing platform will provide noisy information, e.g. $\rho^* = 1/2$, and will charge an access fee equal to $\underline{a}(1-c)$.

Proof. We will proceed showing that in every matching equilibrium, when only high type users participate the optimal decision for any platform is to eliminate any noise from the signal $\rho^* = 1$, and that when only low type users participate the optimal decision for any platform is to provide a completely noisy signal, e.g. $\rho^* = 1/2$.

Lets start with the matching equilibrium where the high type users only accept high signal partners, and reject low signal partners, i.e. Case (i). If only high type users participate, $\lambda = 1$, the set of feasible information provision levels will be

$$\begin{aligned} \Phi_1^i &= \{\rho \in [0.5, 1] : \underline{a}(1-c)(1-\rho)^2 - \overline{a}(1-c)(1-\rho)\rho \geq 0, \\ &\overline{a}(1-c)\rho - \underline{a}(1-c)(1-\rho) \geq 0, \\ &\overline{a}(1-c)\rho^2 \geq 0, \overline{a}(1-c)\rho(1-\rho) \leq 0\} \end{aligned}$$

Notice the last restriction is only satisfied when $\rho = 1$, the other restrictions are satisfied at this level. Then, the feasible of ρ's reduces to a singleton. In the opposite situation, when only low type users participate, $\lambda = 0$, the set of feasible ρ's is not determined by those restriction coming from the high type users. Then, any platform will pick a ρ belonging to $[0.5, 1]$. Moreover, as neither the profit nor the surplus of those users participating depend on the signal level, because the probability of being accepted is equal to one, then platforms do not care on the signal's informativeness. Finally, under the realistic assumption information provision is costly, the optimal information provision level is the lowest possible, then $\rho = 1/2$.

Continue with the matching equilibrium where high types accept any partner they encounter, i.e. Case (ii). In case only high type users participate, $\lambda = 1$, the set of feasible signals comes to

$$\begin{aligned} \Phi_1^{ii} &= \{\rho \in [0.5, 1] : -(1-\rho)\lambda(\overline{a} - \underline{a})(1-c) \geq 0 \\ &(\overline{a} - \underline{a})(1-c) \geq 0 \\ &\overline{a}(1-c)(1-\rho) \geq 0\} \end{aligned}$$

and notice that the first restriction is satisfied only when $\rho = 1$. On the other hand, if only low type users participate the feasible set of signals remains on the original interval determined by assumption (1), because the restrictions coming from the high type users do not apply.

Continue with the matching equilibrium where high types accept high signal partners and mix with low signal partners, i.e. Case (ii) Mixed. In the case only high type users participate, i.e. $\lambda = 1$, the set of feasible signal will be

$$\Phi_{Mixed,1}^{ii} = \{\rho \in [0.5, 1] : 0 \geq \underline{a}(1-c)(1-2\rho), -\frac{\rho}{1-\rho} \in [0, 1]\}$$

The second constraint will not hold because by assumption (1) the least level of ρ is $1/2$, and consequently this set is empty. The intuition is straightforward, if only high type users participate makes no sense for them to mix. On the other hand, if only low type users participate the feasible set of ρ's is only constrained by assumption (1). The optimal decision for any platform will be to provide a pure noisy signal because neither the profit nor the surplus depends on ρ, and because information provision is usually is not costless.

Finally, lets move to the matching equilibrium where high type users reject low signal partners, and mix with high signal partners, i.e. Case (i) Mixed. If only high type users participate, $\lambda = 1$, the set of feasible signals is trivially satisfied because this set becomes $\Phi^i_{Mixed,1} = \{\rho \in [0.5, 1] : 0 \in [0, 1]\}$. This matching equilibrium is not interesting because high type users expected participation payoff is by construction equal to zero, additionally it is at the bottom of the ranking of equilibria. On the other hand, if only low type users participate, $\lambda = 0$, the arguments are the same. The set of feasible signals is only constrained by assumption (1), and any platform will optimally provide pure noisy signals, i.e. $\rho = 1/2$.

□

Proposition (3) . Assume all users participate $\lambda = \overline{\lambda}$. (i) For the matching equilibrium with the highest "economic activity", e.g. Case (ii), the profit maximizing platform will provide any feasible level of information, e.g $\rho^* \in \Phi^{ii}_{\overline{\lambda}} \cap \Psi^{ii}_1$. (ii) For the second highest ranked matching equilibrium, e.g. Case (ii) Mixed, the optimal (interior) provision satisfies the following rule:

$$\xi_{\sigma^*_{\overline{a}\underline{\alpha}},\rho} = \frac{1 - \sigma^*_{\overline{a}\underline{\alpha}}}{\sigma^*_{\overline{a}\underline{\alpha}}}$$

where $\xi_{\sigma^*_{\overline{a}\underline{\alpha}},\rho}$ is the elasticity of the $\sigma^*_{\overline{a}\underline{\alpha}}$ with respect to ρ. (iii) For the matching equilibrium Case (i) its optimal to provide the least feasible level of information.

Proof. In this case we will discuss the optimal provision of information for every matching equilibrium, given all users participate, e.g. $M = 1$. The analysis here will be performed for the more general setup where the platform is fully two-sided. The additional notation is described at Section 6.

Lets start with the highest ranked matching equilibrium, i.e. Case (ii). The set of feasible signal will be $\Phi^{ii}_{\overline{\lambda}_a\overline{\lambda}_b} \bigcap \Psi^{ii}_1$, where

$$\begin{aligned}
\Phi^{ii}_{\overline{\lambda}_a\overline{\lambda}_b} &= \{(\rho_a, \rho_b) \in [0.5, 1]^2 : -(1-\rho_b)\overline{\lambda}_b((\overline{b} - \underline{b}) - (\overline{a} - \underline{a})c) \geq 0, \\
&\quad (\overline{b} - \underline{b}) - (\overline{a} - \underline{a})c \geq 0, -(1-\rho_a)\overline{\lambda}_a((\overline{a} - \underline{a}) - (\overline{b} - \underline{b})c) \geq 0, \\
&\quad (\overline{a} - \underline{a}) - (\overline{b} - \underline{b})c \geq 0, \\
&\quad (\overline{b} - \overline{a}c)(1-\rho_b) \geq 0, (\overline{a} - \overline{b}c)(1-\rho_a) \geq 0\} \\
\Psi^{ii}_1 &= \{(\rho_a, \rho_b) \in [0.5, 1]^2 : U^{ii}(\overline{a}, \overline{\lambda}_b, \rho_a, \rho_b) \geq U^{ii}(\underline{a}, \overline{\lambda}_b, \rho_a, \rho_b), \\
&\quad U^{ii}(\overline{b}, \overline{\lambda}_a, \rho_a, \rho_b) \geq U^{ii}(\underline{b}, \overline{\lambda}_a, \rho_a, \rho_b)\} \\
&= \{(\rho_a, \rho_b) \in [0.5, 1]^2 : \overline{b}\overline{\lambda}_b + \underline{b}(1 - \overline{\lambda}_b) - \overline{a}c \geq \underline{b} - \underline{a}c, \\
&\quad \overline{a}\overline{\lambda}_a + \underline{a}(1 - \overline{\lambda}_a) - \overline{b}c \geq \underline{a} - \underline{b}c\}
\end{aligned}$$

The optimization program will be

$$\max_{(\rho_a,\rho_b)\in\Phi^{ii}_{\overline{\lambda}_a\overline{\lambda}_b} \bigcap \Psi^{ii}_1} \underline{b} - \underline{a}c + \underline{a} - \underline{b}c$$

The profit maximizing provision level is not uniquely determined because the profit function is independent on both signals. In case the provision is costly, the platform will optimally provide noisy signals.

The second highest matching equilibrium will yield an interior solution. The set of feasible signal will be $\Phi^{ii}_{Mixed,\overline{\lambda}_a\overline{\lambda}_b}\bigcap\Psi^{ii}_{Mixed,1}$, where

$$\begin{aligned}
\Phi^{ii}_{Mixed,\overline{\lambda}_a\overline{\lambda}_b} &= \{(\rho_a,\rho_b)\in[0.5,1]^2 : (\overline{a}c-\underline{b})(1-\overline{\lambda}_b)(2\rho_b-1)\geq \frac{\underline{b}-\underline{a}c}{(1-\rho_a)(\overline{b}-\overline{a}c)}[\rho_a\rho_b(\overline{a}c-\underline{b})(1-\overline{\lambda}_b) \\
&+ (\overline{b}-\overline{a}c)(1-\rho_b)(1-\rho_a-\rho_a\overline{\lambda}_a)], \\
&\quad (\overline{b}c-\underline{a})(1-\overline{\lambda}_a)(2\rho_a-1)\geq \frac{\underline{a}-\underline{b}c}{(1-\rho_b)(\overline{a}-\overline{b}c)}[\rho_b\rho_a(\overline{b}c-\underline{a})(1-\overline{\lambda}_a)+(\overline{a}-\overline{b}c)(1-\rho_a)(1-\rho_b-\rho_b\overline{\lambda}_b)], \\
&\quad \frac{\overline{a}c-\underline{b}}{\overline{b}-\overline{a}c}\frac{\rho_b(1-\overline{\lambda}_b)}{(1-\rho_b)(1-\rho_a)\overline{\lambda}_b}-\frac{\rho_a}{1-\rho_a}\in[0,1], \\
&\quad \frac{\overline{b}c-\underline{a}}{\overline{a}-\overline{b}c}\frac{\rho_a(1-\overline{\lambda}_a)}{(1-\rho_a)\overline{\lambda}_a(1-\rho_a)}-\frac{\rho_b}{1-\rho_b}\in[0,1]\}
\end{aligned}$$

The optimization program will be,

$$\begin{aligned}
\max_{(\rho_a,\rho_b)\in\Phi^{ii}_{Mixed,\overline{\lambda}_a\overline{\lambda}_b}\bigcap\Psi^{ii}_{Mixed,1}} \quad & U^{ii}_M(\underline{a},\overline{\lambda}_b,\rho_a,\rho_b)+U^{ii}_M(\underline{b},\overline{\lambda}_a,\rho_a,\rho_b) \\
&= (\underline{b}-\underline{a}c)[\overline{\lambda}_b((1-\rho_a)+\rho_a\sigma^*_{\overline{b}\underline{\alpha}}(\rho_a,\rho_b))+(1-\overline{\lambda}_b)] \\
&+ (\underline{a}-\underline{b}c)[\overline{\lambda}_a((1-\rho_b)+\rho_b\sigma^*_{\overline{a}\underline{\beta}}(\rho_a,\rho_b))+(1-\overline{\lambda}_a)]
\end{aligned}$$

where $\sigma^*_{\overline{b}\underline{\alpha}}(\cdot)$ is the probability a high type b-user accepts a low signal a-user such that $v(\underline{\alpha},\overline{b})=0$ for this particular matching equilibrium; this function has two arguments, ρ_a and ρ_b. Similarly define $\sigma^*_{\overline{a}\underline{\beta}}(\cdot)$, and this function's arguments are ρ_a and ρ_b.

The interior solution is determined by the following first order conditions,

$$\begin{aligned}
\rho_a\frac{\partial\sigma^*_{\overline{b}\underline{\alpha}}(\rho_a,\rho_b)}{\partial\rho_a} &= 1-\sigma^*_{\overline{b}\underline{\alpha}}(\rho_a,\rho_b)-\frac{(\underline{a}-\underline{b}c)\overline{\lambda}_a\rho_b}{(\underline{b}-\underline{a}c)\overline{\lambda}_b}\frac{\partial\sigma^*_{\overline{a}\underline{\beta}}(\rho_a,\rho_b)}{\partial\rho_a} \\
\rho_b\frac{\partial\sigma^*_{\overline{a}\underline{\beta}}(\rho_a,\rho_b)}{\partial\rho_b} &= 1-\sigma^*_{\overline{a}\underline{\beta}}(\rho_a,\rho_b)-\frac{(\underline{b}-\underline{a}c)\overline{\lambda}_b\rho_a}{(\underline{a}-\underline{b}c)\overline{\lambda}_a}\frac{\partial\sigma^*_{\overline{b}\underline{\alpha}}(\rho_a,\rho_b)}{\partial\rho_b}
\end{aligned}$$

Now define as $\xi_{\sigma^*_{\overline{b}\underline{\alpha}}\rho_a}$ as the elasticity of $\sigma^*_{\overline{b}\underline{\alpha}}(\rho_a,\rho_b)$ with respect to ρ_a, and analogously define $\xi_{\sigma^*_{\overline{a}\underline{\beta}}\rho_b}$. The first order conditions boils down to,

$$\begin{aligned}
\xi_{\sigma^*_{\overline{b}\underline{\alpha}}\rho_a} &= \frac{1-\sigma^*_{\overline{b}\underline{\alpha}}}{\sigma^*_{\overline{b}\underline{\alpha}}}-\frac{\underline{a}-\underline{b}c}{\underline{b}-\underline{a}c}\frac{\overline{\lambda}_a\rho_b\sigma^*_{\overline{a}\underline{\beta}}}{\overline{\lambda}_b\sigma_{\overline{b}\underline{\alpha}}}\xi_{\sigma_{\overline{a}\underline{\beta}}\rho_a} \\
\xi_{\sigma^*_{\overline{a}\underline{\beta}}\rho_b} &= \frac{1-\sigma^*_{\overline{a}\underline{\beta}}}{\sigma^*_{\overline{a}\underline{\beta}}}-\frac{\underline{b}-\underline{a}c}{\underline{a}-\underline{b}c}\frac{\overline{\lambda}_b\rho_a\sigma^*_{\overline{b}\underline{\alpha}}}{\overline{\lambda}_a\sigma_{\overline{a}\underline{\beta}}}\xi_{\sigma_{\overline{b}\underline{\alpha}}\rho_b}
\end{aligned}$$

If we restrict attention to a one-sided platform these equations further simplify. For this situation the extra term is what Weyl (2010) calls as *Spence Effect* (SE), for the first equation is

$$-\frac{\underline{a}-\underline{b}c}{\underline{b}-\underline{a}c}\frac{\overline{\lambda}_a\rho_b\sigma^*_{\overline{a}\underline{\beta}}}{\overline{\lambda}_b\sigma_{\overline{b}\underline{\alpha}}}\xi_{\sigma_{\overline{a}\underline{\beta}}\rho_a}$$

and for the second equation is

$$-\frac{\underline{b}-\underline{a}c}{\underline{a}-\underline{b}c}\frac{\overline{\lambda}_b\rho_a\sigma^*_{\overline{b}\underline{\alpha}}}{\overline{\lambda}_a\sigma_{\overline{a}\underline{\beta}}}\xi_{\sigma_{\overline{b}\underline{\alpha}}\rho_b}$$

The first order condition for the one-sided case will be,

$$\xi_{\sigma^*_{\overline{a}\underline{\alpha}}\rho}=\frac{1-\sigma^*_{\overline{a}\underline{\alpha}}}{\sigma^*_{\overline{a}\underline{\alpha}}}$$

Continue with the third highest ranked matching equilibrium where high types accept high signal users, but rejects low signal partners, i.e. Case (i). The set of feasible signal will be $\Phi^i_{\overline{\lambda}_a\overline{\lambda}_b} \bigcap \Psi^i_1$, where

$$\begin{aligned}
\Phi^i_{\overline{\lambda}_a\overline{\lambda}_b} &= \{(\rho_a,\rho_b)\in[0.5,1]^2 : (\underline{b}-\underline{a}c)[(1-\rho_b)\overline{\lambda}_b(1-\rho_a)+\rho_b(1-\overline{\lambda}_b)] \\
&- [(\overline{b}-\overline{a}c)(1-\rho_b)\overline{\lambda}_b\rho_a+(\underline{b}-\overline{a}c)\rho_b(1-\overline{\lambda}_b)]\geq 0, \\
&\quad (\underline{a}-\underline{b}c)[(1-\rho_a)\overline{\lambda}_a(1-\rho_b)+\rho_a(1-\overline{\lambda}_a)] \\
&- [(\overline{a}-\overline{b}c)(1-\rho_a)\overline{\lambda}_a\rho_b+(\underline{a}-\overline{b}c)\rho_a(1-\overline{\lambda}_a)]\geq 0, \\
&\quad \overline{\lambda}_b[(\overline{b}-\overline{a}c)\rho_a-(\underline{b}-\underline{a}c)(1-\rho_a)]-c(1-\overline{\lambda}_b)(\overline{a}-\underline{a})\geq 0, \\
&\quad \overline{\lambda}_a[(\overline{a}-\overline{b}c)\rho_b-(\underline{a}-\underline{b}c)(1-\rho_b)]-c(1-\overline{\lambda}_a)(\overline{b}-\underline{b})\geq 0, \\
&\quad (\overline{b}-\overline{a}c)\rho_b\overline{\lambda}_b\rho_a+(\underline{b}-\overline{a}c)(1-\rho_b)(1-\overline{\lambda}_b)\geq 0, \\
&\quad (\overline{b}-\overline{a}c)\rho_a(1-\rho_b)\overline{\lambda}_b+(\underline{b}-\overline{a}c)\rho_b(1-\overline{\lambda}_b)\leq 0, \\
&\quad (\overline{a}-\overline{b}c)\rho_a\overline{\lambda}_a\rho_b+(\underline{a}-\overline{b}c)(1-\rho_a)(1-\overline{\lambda}_a)\geq 0, \\
&\quad (\overline{a}-\overline{b}c)(1-\rho_a)\overline{\lambda}_a\rho_b+(\underline{a}-\overline{b}c)\rho_a(1-\overline{\lambda}_a)\geq 0\}
\end{aligned}$$

and

$$\begin{aligned}
\Psi^i_1 &= \{(\rho_a,\rho_b)\in[0.5,1]^2 : U^i(\overline{a},\overline{\lambda}_b,\rho_a,\rho_b)\geq 0, \\
&\quad U^i(\overline{b},\overline{\lambda}_a,\rho_a,\rho_b)\geq 0\} \\
&= \{(\rho_a,\rho_b)\in[0.5,1]^2 : (\underline{b}-\underline{a}c)[\overline{\lambda}_b(1-\rho_a)+(1-\overline{\lambda}_b)]\geq 0, \\
&\quad (\underline{a}-\underline{b}c)[\overline{\lambda}_a(1-\rho_b)+(1-\overline{\lambda}_a)]\geq 0\}
\end{aligned}$$

The optimization program will be,

$$\begin{aligned}
\max_{(\rho_a,\rho_b)\in\Phi^i_{\overline{\lambda}_a\overline{\lambda}_b}\bigcap\Psi^i_1} \quad & U^i(\underline{a},\overline{\lambda}_b,\rho_a,\rho_b)+U^i(\underline{b},\overline{\lambda}_a,\rho_a,\rho_b) \\
= \quad & (\underline{b}-\underline{a}c)[\overline{\lambda}_b(1-\rho_a)+(1-\overline{\lambda}_b)]+(\underline{a}-\underline{b}c)[\overline{\lambda}_a(1-\rho_b)+(1-\overline{\lambda}_a)]
\end{aligned}$$

and notice the program is linear in both arguments, then we face a bang-bang solution. The marginal profit in both arguments is negative, then the optimal decision for the platform will be to pick the least feasible level of provision. For example, the marginal profit wrt ρ_a is $-(\underline{b}-\underline{a}c)\overline{\lambda}_b<0$.

In the particular case of a one-sided platform the solution is analogous.

Finally, the solution for the lowest ranked matching equilibrium is not analyzed because we already showed that whenever high type users participate, they will not use this equilibrium.

□

Proposition (4) . Assume all users participate $\lambda=\overline{\lambda}$. (i) For the highest ranked matching equilibrium, e.g. Case (ii), the surplus maximizing platform will make the same decision as the profit maximizing platform. (ii) For the second highest ranked matching equilibrium, e.g. Case (ii) Mixed, the optimal (interior) signal satisfies the following rule:

$$\xi_{\sigma^*_{\overline{a}\underline{a}},\rho} = \omega(\rho)\left(\frac{1-\sigma^*_{\overline{a}\underline{\alpha}}}{\sigma^*_{\overline{a}\underline{\alpha}}}\right)-(1-\omega(\rho))\left[\frac{\sigma^*_{\overline{a}\underline{\alpha}}+\rho(1-\sigma^*_{\overline{a}\underline{\alpha}})}{\sigma^*_{\overline{a}\underline{\alpha}}}+\frac{\overline{a}c-\underline{a}}{\overline{a}(1-c)}\frac{1-\overline{\lambda}}{\overline{\lambda}}\frac{1}{\sigma^*_{\overline{a}\underline{\alpha}}}\right]$$

where $\xi_{\sigma^*_{\overline{a}\underline{\alpha}},\rho}$ is the elasticity of the $\sigma^*_{\overline{a}\underline{\alpha}}$ with respect to ρ, and $\omega(\rho) = \frac{\overline{a}Pr\{\overline{a}\}}{\overline{a}Pr\{\overline{a},\underline{\alpha}\}+\underline{a}Pr\{\underline{a}\}}$. (iii) For the matching equilibrium Case (i) the optimal (interior) signal will satisfy:

$$\rho = \frac{1-\overline{\lambda}}{\overline{\lambda}}\left(\frac{2\underline{a}-(\underline{a}+\overline{a})c}{2\overline{a}(1-c)}\right)$$

Proof. Lets begin with the highest matching equilibrium where high type users accept any partner they encounter. The optimization program for the surplus maximizing platform is,

$$\begin{aligned}
\max_{(\rho_a,\rho_b)\in\Phi^{ii}_{\overline{\lambda}_a\overline{\lambda}_b}\cap\Psi^{ii}_1} \quad & \overline{\lambda}_a U^i(\overline{a},\overline{\lambda}_b,\rho_a,\rho_b) + (1-\overline{\lambda}_a)U^i(\underline{a},\overline{\lambda}_b,\rho_a,\rho_b) \\
+ \quad & \overline{\lambda}_b U^i(\overline{b},\overline{\lambda}_a,\rho_a,\rho_b) + (1-\overline{\lambda}_b)U^i(\underline{b},\overline{\lambda}_a,\rho_a,\rho_b) \\
= \quad & \overline{\lambda}_a[\overline{b}\overline{\lambda}_b + \underline{b}(1-\overline{\lambda}_b) - \overline{a}c] + (1-\overline{\lambda}_a)(\underline{b}-\underline{a}c) \\
+ \quad & \overline{\lambda}_b[\overline{a}\overline{\lambda}_a + \underline{a}(1-\overline{\lambda}_a) - \overline{b}c] + (1-\overline{\lambda}_b)(\underline{a}-\underline{b}c)
\end{aligned}$$

Its clear again the platform will optimally set any feasible level of provision because the surplus is independent of the signals. Moreover, if provision is costless the platform will provide noisy signals.

For the second highest ranked matching equilibrium again we will obtain an interior solution. The optimization program will be,

$$\begin{aligned}
\max_{(\rho_a,\rho_b)\in\Psi^{ii}_{Mixed,\overline{\lambda}_a\overline{\lambda}_b}\cap\Psi^{ii}_{Mixed,1}} \quad & \overline{\lambda}_a U^{ii}_M(\overline{a},\overline{\lambda}_b,\rho_a,\rho_b) + (1-\overline{\lambda}_a)U^{ii}_M(\underline{a},\overline{\lambda}_b,\rho_a,\rho_b) \\
+ \quad & \overline{\lambda}_b U^{ii}_M(\overline{b},\overline{\lambda}_a,\rho_a,\rho_b) + (1-\overline{\lambda}_b)U^{ii}_M(\underline{b},\overline{\lambda}_a,\rho_a,\rho_b) \\
= \quad & \overline{\lambda}_a[(\overline{b}-\overline{a}c)\rho_b\overline{\lambda}_b(\rho_a+(1-\rho_a)\sigma^*_{\overline{b}\underline{\alpha}}(\rho_a,\rho_b)) + (\underline{b}-\overline{a}c)(1-\rho_b)(1-\overline{\lambda}_b)] \\
+ \quad & (1-\overline{\lambda}_a)(\underline{b}-\underline{a}c)[\overline{\lambda}_b(1-\rho_a+\rho_a\sigma^*_{\overline{b}\underline{\alpha}}(\rho_a,\rho_b)) + (1-\overline{\lambda}_b)] \\
+ \quad & \overline{\lambda}_b[(\overline{a}-\overline{b}c)\rho_a\overline{\lambda}_a(\rho_b+(1-\rho_b)\sigma^*_{\overline{a}\underline{\beta}}(\rho_a,\rho_b)) + (\underline{a}-\overline{b}c)(1-\rho_a)(1-\overline{\lambda}_a)] \\
+ \quad & (1-\overline{\lambda}_b)(\underline{a}-\underline{b}c)[\overline{\lambda}_a(1-\rho_b+\rho_b\sigma^*_{\overline{a}\underline{\beta}}(\rho_a,\rho_b)) + (1-\overline{\lambda}_a)]
\end{aligned}$$

The first order conditions will be,

$$\begin{aligned}
\frac{\partial\sigma^*_{\overline{b}\underline{\alpha}}(\rho_a,\rho_b)}{\partial\rho_a}[(1-\rho_a)\overline{\lambda}_a(\overline{b}-\overline{a}c)\rho_b\overline{\lambda}_b + \rho_a(1-\overline{\lambda}_a)(\underline{b}-\underline{a}c)\overline{\lambda}_b] &= (1-\sigma^*_{\overline{b}\underline{\alpha}}(\rho_a,\rho_b))[(1-\overline{\lambda}_a)(\underline{b}-\underline{a}c)\overline{\lambda}_b - \overline{\lambda}_a(\overline{b}-\overline{a}c)\rho_b\overline{\lambda}_b] \\
&- \overline{\lambda}_b[(\overline{a}-\overline{b}c)\overline{\lambda}_a(\rho_b+(1-\rho_b)\sigma^*_{\overline{a}\underline{\beta}}(\rho_a,\rho_b)) - (\underline{a}-\overline{b}c)(1-\overline{\lambda}_a)] \\
&- \frac{\partial\sigma^*(\rho_a,\rho_b)}{\partial\rho_a}[\overline{\lambda}_b(\overline{a}-\overline{b}c)\rho_a\overline{\lambda}_a(1-\rho_b) + (1-\overline{\lambda}_b)(\underline{a}-\underline{b}c)\overline{\lambda}_a\rho_b] \\
\frac{\partial\sigma^*_{\overline{a}\underline{\beta}}(\rho_a,\rho_b)}{\partial\rho_b}[\overline{\lambda}_b(\overline{a}-\overline{b}c)\rho_a\overline{\lambda}_a(1-\rho_b) + (1-\overline{\lambda}_b)(\underline{a}-\underline{b}c)\overline{\lambda}_a\rho_b] &= (1-\sigma^*_{\overline{a}\underline{\beta}}(\rho_a,\rho_b))[(1-\overline{\lambda}_b)(\underline{a}-\overline{b}c)\overline{\lambda}_a - \overline{\lambda}_b\overline{\lambda}_a(\overline{a}-\overline{b}c)\rho_a] \\
&- \overline{\lambda}_a[(\overline{b}-\overline{a}c)\overline{\lambda}_b(\rho_a+(1-\rho_a)\sigma^*_{\overline{b}\underline{\alpha}}(\rho_a,\rho_b)) - (\underline{b}-\overline{a}c)(1-\overline{\lambda}_b)] \\
&- \frac{\partial\sigma^*_{\overline{b}\underline{\alpha}}(\rho_a,\rho_b)}{\partial\rho_b}[\overline{\lambda}_a(\overline{b}-\overline{a}c)\rho_b\overline{\lambda}_b(1-\rho_a) + (1-\overline{\lambda}_a)(\underline{b}-\underline{a}c)\overline{\lambda}_b\rho_a]
\end{aligned}$$

To rewrite the expressions above need to make several definitions. As with the previous proposition define $\xi_{\sigma^*_{\overline{b}\underline{\alpha}}\rho_a}$ as the elasticity of $\sigma^*_{\overline{b}\underline{\alpha}}(\rho_a,\rho_b)$ wrt to the signal assigned to a-users, ρ_a; define $\xi_{\sigma^*_{\overline{a}\underline{\beta}}\rho_b}$ as an analogous elasticity. Additionally, define the following equations

$$\begin{aligned}
\omega_a \equiv \omega_a(\rho_a,\rho_b) &= \frac{\overline{\lambda}_a(\overline{b}-\overline{a}c)\rho_b}{(1-\rho_a)\overline{\lambda}_a(\overline{b}-\overline{a}c)\rho_b + \rho_a(1-\overline{\lambda}_a)(\underline{b}-\underline{a}c)} \\
\omega_b \equiv \omega_b(\rho_a,\rho_b) &= \frac{\overline{\lambda}_b(\overline{a}-\overline{b}c)\rho_a}{(1-\rho_b)\overline{\lambda}_b(\overline{a}-\overline{b}c)\rho_a + \rho_b(1-\overline{\lambda}_b)(\underline{a}-\underline{b}c)}
\end{aligned}$$

So the first order conditions boil down to,

$$
\begin{aligned}
\xi_{\sigma^*_{\overline{b}\underline{\alpha}}\rho_a} &= \omega_a\left(\frac{1-\sigma^*_{\overline{b}\underline{\alpha}}}{\sigma^*_{\overline{b}\underline{\alpha}}}\right)-(1-\omega_a)\left[\frac{\rho_a(\overline{a}-\overline{b}c)(\rho_b+(1-\rho_b)\sigma^*_{\overline{a}\underline{\beta}})}{(\overline{b}-\overline{a}c)\rho_b\sigma^*_{\overline{b}\underline{\alpha}}}+\frac{(\overline{b}c-\underline{a})(1-\overline{\lambda}_a)\rho_a}{(\overline{b}-\overline{a}c)\overline{\lambda}_a\rho_b\sigma^*_{\overline{b}\underline{\alpha}}}\right] \\
&- \frac{\sigma^*_{\overline{a}\underline{\beta}}}{\sigma^*_{\overline{b}\underline{\alpha}}}\xi_{\sigma^*_{\overline{a}\underline{\beta}}\rho_a}\frac{\overline{\lambda}_b(\overline{a}-\overline{b}c)\rho_a\overline{\lambda}_a(1-\rho_b)+(1-\overline{\lambda}_b)(\underline{a}-\underline{b}c)\overline{\lambda}_a\rho_b}{(1-\rho_a)\overline{\lambda}_a(\overline{b}-\overline{a}c)\rho_b\overline{\lambda}_b+\rho_a(1-\overline{\lambda}_a)(\underline{b}-\underline{a}c)\overline{\lambda}_b} \\
\xi_{\sigma^*_{\overline{a}\underline{\beta}}\rho_b} &= \omega_b\left(\frac{1-\sigma^*_{\overline{a}\underline{\beta}}}{\sigma^*_{\overline{a}\underline{\beta}}}\right)-(1-\omega_b)\left[\frac{\rho_b(\overline{b}-\overline{a}c)(\rho_a+(1-\rho_a)\sigma^*_{\overline{b}\underline{\alpha}})}{(\overline{a}-\overline{b}c)\rho_a\sigma^*_{\overline{a}\underline{\beta}}}+\frac{(\overline{a}c-\underline{b})(1-\overline{\lambda}_b)\rho_b}{(\overline{a}-\overline{b}c)\overline{\lambda}_b\rho_a\sigma^*_{\overline{a}\underline{\beta}}}\right] \\
&- \frac{\sigma^*_{\overline{b}\underline{\alpha}}}{\sigma^*_{\overline{a}\underline{\beta}}}\xi_{\sigma^*_{\overline{b}\underline{\alpha}}\rho_b}\frac{\overline{\lambda}_a(\overline{b}-\overline{a}c)\rho_b\overline{\lambda}_b(1-\rho_a)+(1-\overline{\lambda}_a)(\underline{b}-\underline{a}c)\overline{\lambda}_b\rho_a}{(1-\rho_b)\overline{\lambda}_b(\overline{a}-\overline{b}c)\rho_a\overline{\lambda}_a+\rho_b(1-\overline{\lambda}_b)(\underline{a}-\underline{b}c)\overline{\lambda}_a}
\end{aligned}
$$

Finally, for the one-sided case the expression simplifies as follows

$$
\xi_{\sigma^*_{\overline{a}\underline{\alpha}}\rho} = \omega(\rho)\left(\frac{1-\sigma^*_{\overline{a}\underline{\alpha}}}{\sigma^*_{\overline{a}\underline{\alpha}}}\right)-(1-\omega(\rho))\left[\frac{\sigma^*_{\overline{a}\underline{\alpha}}+\rho(1-\sigma^*_{\overline{a}\underline{\alpha}})}{\sigma^*_{\overline{a}\underline{\alpha}}}+\frac{\overline{a}c-\underline{a}}{\overline{a}-\overline{a}c}\left(\frac{1-\overline{\lambda}}{\overline{\lambda}}\right)\frac{1}{\sigma^*_{\overline{a}\underline{\alpha}}}\right]
$$

where $\omega(\rho) \equiv \frac{\overline{a}Pr\{\overline{a}\}}{\overline{a}Pr\{\overline{a},\underline{\alpha}\}+\underline{a}Pr\{\underline{a}\}}$

Now continue with the matching equilibrium where high type users only accept high signal users. The optimization program for the surplus maximizing platform will be,

$$
\begin{aligned}
\max_{(\rho_a,\rho_b)\in\Phi^i_{\overline{\lambda}_a\overline{\lambda}_b}\cap\Psi^i_1} \quad & \overline{\lambda}_a[(\overline{b}-\overline{a}c)\rho_b\overline{\lambda}_b\rho_a+(\underline{b}-\underline{a}c)(1-\rho_b)(1-\overline{\lambda}_b)] \\
+ \quad & (1-\overline{\lambda}_a)(\underline{b}-\underline{a}c)(\overline{\lambda}_b(1-\rho_a)+(1-\overline{\lambda}_b)) \\
+ \quad & \overline{\lambda}_b[(\overline{a}-\overline{b}c)\rho_a\overline{\lambda}_a\rho_b+(\underline{a}-\overline{b}c)(1-\rho_a)(1-\overline{\lambda}_a)] \\
+ \quad & (1-\overline{\lambda}_b)(\underline{a}-\underline{b}c)(\overline{\lambda}_a(1-\rho_b)+(1-\overline{\lambda}_a))
\end{aligned}
$$

And the first order conditions immediately yield,

$$
\begin{aligned}
\rho_a &= \left(\frac{1-\overline{\lambda}_a}{\overline{\lambda}_a}\right)\left(\frac{\underline{b}+\underline{a}-(\underline{a}+\overline{b})c}{(\overline{a}+\overline{b})(1-c)}\right) \\
\rho_b &= \left(\frac{1-\overline{\lambda}_b}{\overline{\lambda}_b}\right)\left(\frac{\underline{b}+\underline{a}-(\underline{b}+\overline{a})c}{(\overline{a}+\overline{b})(1-c)}\right)
\end{aligned}
$$

Finally, for the one-sided case we have

$$
\rho = \left(\frac{1-\overline{\lambda}}{\overline{\lambda}}\right)\left(\frac{\underline{a}-\frac{\overline{a}+\underline{a}}{2}c}{\overline{a}(1-c)}\right)
$$

□

Proposition (5) . Assume all users participate $\lambda = \overline{\lambda}$. The profit maximizing platform will over-provide information in relation to the surplus maximizing platform, e.g. $\rho^{private} \geq \rho^{surplus}$, whenever the high type user accepts low signal partners with a positive probability. If the high type user always rejects low signal partners, now the profit maximizing platform will under-provide information.

Proof. Arguments for each case follow from the analysis of the optimal information provision rules. Starting with matching equilibrium Case (ii) Mixed we know the optimal level of provision for the profit maximizing platform is attained at a elasticity level of $\sigma^*_{\overline{a}\underline{\alpha}}$ wrt ρ, i.e. $\xi_{\sigma_{\overline{a}\underline{\alpha}},\rho}$, higher than the elasticity level associated to the optimal provision for the surplus maximizing platform. Finally, we only need to argue that the probability high users accept a low signal partner is an increasing a convex function of ρ, e.g. $\sigma^{*'}_{\overline{a}\underline{\alpha}}(\rho) > 0$ and $\sigma^{*''}_{\overline{a}\underline{\alpha}}(\rho) > 0$. Indeed, the first and second derivatives are

$$\frac{\partial \sigma^*_{\overline{a}\underline{\alpha}}}{\partial \rho} \gtreqless 0 \qquad iff \qquad \left(\frac{\overline{a}c - \underline{a}}{\overline{a} - \overline{a}c}\right)\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{1+\rho}{\rho}\right) \gtreqless 1$$

$$\frac{\partial^2 \sigma^*_{\overline{a}\underline{\alpha}}}{\partial \rho^2} > 0 \quad becuase \quad \frac{2}{(1-\rho)^3}\left[\left(\frac{\overline{a}c - \underline{a}}{\overline{a} - \overline{a}c}\right)\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{1+\rho}{\rho}\right) - 1\right] + \frac{2}{(1-\rho)^4}\left(\frac{\overline{a}c - \underline{a}}{\overline{a} - \overline{a}c}\right)\left(\frac{1-\lambda}{\lambda}\right) > 0$$

and while the second derivative is positive, the first one is can be positive or negative. The relevant parameters to determine the sign of the first derivative are λ and the opportunity cost c; additionally, we can show the sign is positive as long as λ is not *high enough* and simultaneously the opportunity cost is *low enough*. Finally, we should show that under these conditions the optimization program, for both platforms, is not well defined. Instead of doing so we will rely on a future result obtained in Theorem (1) which says that, for these conditions on parameters λ and c, any platform will exclude low type users from the environment. To wrap up, both the first and seconde derivative of $\sigma^*_{\overline{a}\underline{\alpha}}$ wrt ρ will be positive for the parameter range where the platform finds optimal to keep on board high and low type users.

We conclude then saying that the optimal provision level that maximizes profits will be strictly higher than the level that maximizes surplus for all the parameter space. Figures (24) and (25) show that indeed this is the case.

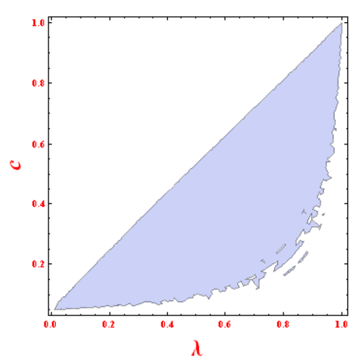

Figure 24: Case (ii) Mixed, Δ_a Medium

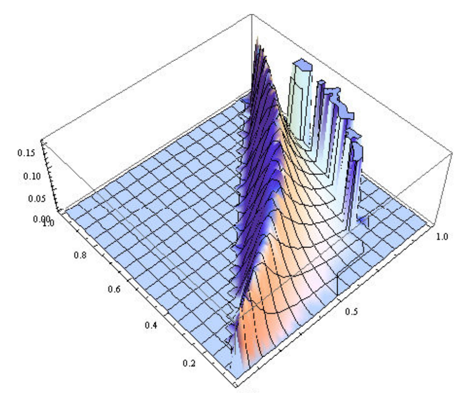

Figure 25: Case (ii) Mixed, Δ_a Medium

For the other matching equilibrium, Case (i), we observe there is no interior solution for the profit maximizing platform, moreover the optimal provision is the smallest level inside the feasible set. The situation for the surplus maximizing platform is different, there we either find an interior solution or a corner solution equivalent to the greatest level of provision in the feasible set. Figures (26) and (27) confirm that, for all parameter space, the profit maximizing platform will under-provide information compared to the surplus maximizing platform.

□

Proposition (6) . (i) The profit maximizing platform will extract the gross expected participation payoff from the high type and low type users that participate. (ii) If all high types participate and a fraction of

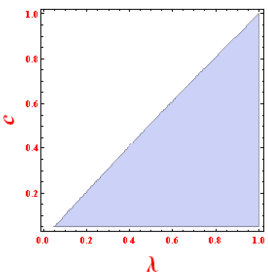

Figure 26: Case (i), Δ_a Medium

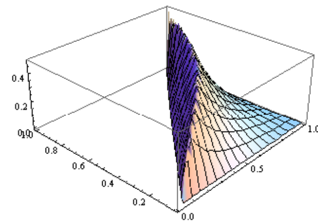

Figure 27: Case (i), Δ_a Medium

low types participate, the optimal (interior) mass of lows in the platform satisfy

$$\xi_{\underline{\rho}(\underline{A}),\underline{A}} = -\left(\frac{1-\underline{\rho}(\underline{A})}{\underline{\rho}(\underline{A})}\right)(2\underline{\rho}(\underline{A})-1)$$

where $\underline{\rho}$ satisfies $U(\overline{a},\lambda,\underline{\rho}) = U(\underline{a},\lambda,\underline{\rho})$, where $\lambda = \frac{\overline{\lambda}}{\overline{\lambda}+\underline{A}}$, and where $\xi_{\underline{\rho}(\underline{A}),\underline{A}}$ is the elasticity of $\underline{\rho}(\underline{A})$ with respect to $\underline{A}$. (iii) If a fraction of high types participate and all low types participate, the optimal (interior) mass of highs in the platform satisfy

$$\xi_{\overline{\rho}(\overline{A}),\overline{A}} = \frac{1-\overline{\rho}(\overline{A})}{\overline{\rho}(\overline{A}}$$

where $\overline{\rho}$ satisfies $U(\overline{a},\lambda,\overline{\rho}) = U(\underline{a},\lambda,\overline{\rho})$, where $\lambda = \frac{1-\overline{\lambda}}{\overline{A}+1-\overline{\lambda}}$, and where $\xi_{\overline{\rho}(\overline{A}),\overline{A}}$ is the elasticity of $\overline{\rho}(\overline{A})$ with respect to $\overline{A}$.

Proof. To show the profit maximizing platform extracts high types and low types gross expected participation payoff we will focus to the case where all high type users participate and a fraction of low types participate. The required condition here is that $U(\overline{a},\lambda,\rho) \geq P$ and $U(\underline{a},\lambda,\rho) = P$. The platform will fix the information provision at a level such that low type users receive zero net expected participation payoff. Additionally, the access fee will be pinned down with the gross expected participation payoff from the high type users. We conclude that the platform will extract all the gross expected participation payoff from all high types and from the fraction of low types that participate.

Only those matching equilibrium where the probability of being accepted by any randomly assigned partner is less than one, i.e. Case (i) and Case (ii) Mixed, can support a partial groups participation outcome. In other words, the matching equilibrium where high type users accept any partner cannot support partial group participation because the expected participation payoff do not depend on signals (ρ_a, ρ_b), and consequently the access fee is the unique screening tool any platform can use.

We will begin with the matching equilibrium where the high type users accept high signal partners, and mix with low signal partners, i.e. Case (ii) Mixed. Focusing on the partial group participation case where all high types participate, but only a fraction of low types participate, the fraction of high type b-users and a-users inside the environment is

$$\lambda_b = \frac{\overline{\lambda}_b}{\overline{\lambda}_b + \underline{B}}$$
$$\lambda_a = \frac{\overline{\lambda}_a}{\overline{\lambda}_a + \underline{A}}$$

where $\underline{A} \leq (1-\overline{\lambda}_a)$ and $\underline{B} \leq (1-\overline{\lambda}_b)$. The mass of a-users and b-users participating will respectively be $\overline{M}_a = \overline{\lambda}_a + \underline{A}$, and $\overline{M}_b = \overline{\lambda}_b + \underline{B}$.

The profit maximizing platform will fix the information provision at a level such that low type a-users and b-users expected participation payoff is equal to zero. To wit, this platform will pick some levels of $\underline{\rho}_a$ and $\underline{\rho}_b$ such that $U_M^{ii}(\underline{a}, \lambda_b, \underline{\rho}_a, \underline{\rho}_b) = U_M^{ii}(\overline{a}, \lambda_b, \underline{\rho}_a, \underline{\rho}_b)$ and $U_M^{ii}(\underline{b}, \lambda_a, \underline{\rho}_a, \underline{\rho}_b) = U_M^{ii}(\overline{b}, \lambda_a, \underline{\rho}_a, \underline{\rho}_b)$. Notice that $\underline{\rho}_a$ and $\underline{\rho}_b$ are functions whose arguments are $(\underline{A}, \underline{B})$.

The set of feasible participation levels will be $\Phi^{ii}_{Mixed,\lambda_a\lambda_b} \bigcap \Psi^{ii}_{Mixed,\overline{M}_a\overline{M}_b}$, where

$$
\begin{aligned}
\Phi^{ii}_{Mixed,\overline{\lambda}_a\overline{\lambda}_b} \quad = \quad & \{(\underline{A}, \underline{B}) \in [0, 1-\overline{\lambda}_a] \times [0, 1-\overline{\lambda}_b] : \\
& (\overline{a}c - \underline{b})(1-\lambda_b)(2\underline{\rho}_b(\underline{A}, \underline{B}) - 1) \geq \frac{\underline{b} - \underline{a}c}{(1-\underline{\rho}_a(\underline{A}, \underline{B}))(\overline{b} - \overline{a}c)}[\underline{\rho}_a(\underline{A}, \underline{B})\underline{\rho}_b(\underline{A}, \underline{B})(\overline{a}c - \underline{b})(1-\lambda_b) \\
+ \quad & (\overline{b} - \overline{a}c)(1-\underline{\rho}_b(\underline{A}, \underline{B}))(1-\underline{\rho}_a(\underline{A}, \underline{B}) - \underline{\rho}_a(\underline{A}, \underline{B})\lambda_a)], \\
& (\overline{b}c - \underline{a})(1-\lambda_a)(2\underline{\rho}_a(\underline{A}, \underline{B}) - 1) \geq \frac{\underline{a} - \underline{b}c}{(1-\underline{\rho}_b(\underline{A}, \underline{B}))(\overline{a} - \overline{b}c)}[\underline{\rho}_b(\underline{A}, \underline{B})\underline{\rho}_a(\underline{A}, \underline{B})(\overline{b}c - \underline{a})(1-\lambda_a) \\
+ \quad & (\overline{a} - \overline{b}c)(1-\underline{\rho}_a(\underline{A}, \underline{B}))(1-\underline{\rho}_b(\underline{A}, \underline{B}) - \underline{\rho}_b(\underline{A}, \underline{B})\lambda_b)], \\
& \frac{\overline{a}c - \underline{b}}{\overline{b} - \overline{a}c} \frac{\underline{\rho}_b(\underline{A}, \underline{B})(1-\lambda_b)}{(1-\underline{\rho}_b(\underline{A}, \underline{B}))(1-\underline{\rho}_a(\underline{A}, \underline{B}))\lambda_b} - \frac{\underline{\rho}_a(\underline{A}, \underline{B})}{1-\underline{\rho}_a(\underline{A}, \underline{B})} \in [0,1], \\
& \frac{\overline{b}c - \underline{a}}{\overline{a} - \overline{b}c} \frac{\underline{\rho}_a(\underline{A}, \underline{B})(1-\lambda_a)}{(1-\underline{\rho}_a(\underline{A}, \underline{B}))\lambda_a(1-\underline{\rho}_a(\underline{A}, \underline{B}))} - \frac{\underline{\rho}_b(\underline{A}, \underline{B})}{1-\underline{\rho}_b(\underline{A}, \underline{B})} \in [0,1]\}
\end{aligned}
$$

and additionally,

$$
\begin{aligned}
\Psi^{ii}_{Mixed,\overline{\lambda}_a\overline{\lambda}_b} \quad = \quad & \{(\underline{A}, \underline{B}) \in [0, 1-\overline{\lambda}_a] \times [0, 1-\overline{\lambda}_b] : \\
& U_M^{ii}(\underline{a}, \lambda_b, \underline{\rho}_a, \underline{\rho}_b) \geq 0 \\
& U_M^{ii}(\underline{b}, \lambda_a, \underline{\rho}_b, \underline{\rho}_a) \geq 0\}
\end{aligned}
$$

which is satisfied by construction.

The optimization program will be ,

$$
\begin{aligned}
\max_{(\underline{A}, \underline{B}) \in \Phi^{ii}_{Mixed,\lambda_a\lambda_b} \bigcap \Psi^{ii}_{Mixed,\overline{M}_a\overline{M}_b}} \quad & \overline{\lambda}_a U_M^{ii}(\overline{a}, \lambda_b, \underline{\rho}_a, \underline{\rho}_b) + \underline{A} U_M^{ii}(\underline{a}, \lambda_b, \underline{\rho}_a, \underline{\rho}_b) \\
+ \quad & \overline{\lambda}_b U_M^{ii}(\overline{b}, \lambda_a, \underline{\rho}_a, \underline{\rho}_b) + \underline{B} U_M^{ii}(\underline{b}, \lambda_a, \underline{\rho}_a, \underline{\rho}_b) \\
= \quad & (\overline{\lambda}_a + \underline{A}) \frac{\overline{a}c - \underline{b}}{1-\underline{\rho}_b(\underline{A}, \underline{B})} \frac{\underline{B}}{\overline{\lambda}_b + \underline{B}} (2\underline{\rho}_b(\underline{A}, \underline{B}) - 1) \\
+ \quad & (\overline{\lambda}_b + \underline{B}) \frac{\overline{b}c - \underline{a}}{1-\underline{\rho}_a(\underline{A}, \underline{B})} \frac{\underline{A}}{\overline{\lambda}_a + \underline{A}} (2\underline{\rho}_a(\underline{A}, \underline{B}) - 1)
\end{aligned}
$$

The first order conditions turns out to be,

$$
\begin{aligned}
0 &= \frac{(\overline{a}c-\underline{b})\underline{B}}{\overline{\lambda}_b+\underline{B}}\left[\frac{(1-\underline{\rho}_b(\underline{A},\underline{B}))[2(\overline{\lambda}_a+\underline{A})\frac{\partial\underline{\rho}_b}{\partial\underline{A}}+(2\underline{\rho}_b(\underline{A},\underline{B})-1)]+(\overline{\lambda}_a+\underline{A})(\underline{\rho}_b(\underline{A},\underline{B})-1)\frac{\partial\underline{\rho}_b}{\partial\underline{A}}}{(1-\underline{\rho}_b(\underline{A},\underline{B}))^2}\right] \\
&+ (\overline{\lambda}_b+\underline{B})(\overline{b}c-\underline{a})\left[\frac{(1-\underline{\rho}_a(\underline{A},\underline{B}))(\overline{\lambda}_a+\underline{A})[2\underline{\rho}_a(\underline{A},\underline{B})-1+2\underline{A}\frac{\partial\underline{\rho}_a}{\partial\underline{A}}]-\underline{A}(2\underline{\rho}_a-1)[1-\underline{\rho}_a(\underline{A},\underline{B})-(\overline{\lambda}_a+\underline{A})\frac{\partial\underline{\rho}_a}{\partial\underline{A}}]}{(1-\underline{\rho}_a(\underline{A},\underline{B}))^2(\overline{\lambda}_a+\underline{A})^2}\right] \\
0 &= \frac{(\overline{b}c-\underline{a})\underline{A}}{\overline{\lambda}_a+\underline{A}}\left[\frac{(1-\underline{\rho}_a(\underline{A},\underline{B}))[2(\overline{\lambda}_b+\underline{B})\frac{\partial\underline{\rho}_a}{\partial\underline{B}}+(2\underline{\rho}_a(\underline{A},\underline{B})-1)]+(\overline{\lambda}_b+\underline{B})(\underline{\rho}_a(\underline{A},\underline{B})-1)\frac{\partial\underline{\rho}_a}{\partial\underline{B}}}{(1-\underline{\rho}_a(\underline{A},\underline{B}))^2}\right] \\
&+ (\overline{\lambda}_a+\underline{A})(\overline{a}c-\underline{b})\left[\frac{(1-\underline{\rho}_b(\underline{A},\underline{B}))(\overline{\lambda}_b+\underline{B})[2\underline{\rho}_b(\underline{A},\underline{B})-1+2\underline{B}\frac{\partial\underline{\rho}_b}{\partial\underline{B}}]-\underline{B}(2\underline{\rho}_b-1)[1-\underline{\rho}_b(\underline{A},\underline{B})-(\overline{\lambda}_b+\underline{B})\frac{\partial\underline{\rho}_b}{\partial\underline{B}}]}{(1-\underline{\rho}_b(\underline{A},\underline{B}))^2(\overline{\lambda}_b+\underline{B})^2}\right]
\end{aligned}
$$

For the one-sided case the first order condition will be,

$$
\xi_{\underline{\rho}\underline{A}} = -\left(\frac{1-\underline{\rho}(\underline{A})}{\underline{\rho}(\underline{A})}\right)(2\underline{\rho}(\underline{A})-1)
$$

where $\xi_{\underline{\rho}\underline{A}}$ is the elasticity of $\underline{\rho}$ with respect to $\underline{A}$.

Continue now with the other matching equilibrium, i.e. Case (i), where high types users accept only high signal partners and reject low signal partners. Focus on the partial group participation where *a fraction* of high type a-users and b-users participate. The fraction of high types inside the environment will be,

$$
\begin{aligned}
\lambda_a &= \frac{\overline{A}}{\overline{A}+(1-\overline{\lambda}_a)} \\
\lambda_b &= \frac{\overline{B}}{\overline{B}+(1-\overline{\lambda}_b)}
\end{aligned}
$$

where $\overline{A}\leq\overline{\lambda}_a$ and $\overline{B}\leq\overline{\lambda}_b$, and the mass of a-users and b-users participating will be $\overline{M}_a=\overline{A}+(1-\overline{\lambda}_a)$ and $\overline{M}_b=\overline{B}+(1-\overline{\lambda}_b)$.

The platform must fixed the provision of information to levels $\overline{\rho}_a$ and $\overline{\rho}_b$ such that high types net expected participation payoff is equal to zero. To wit, define functions $\overline{\rho}_a(\overline{A},\overline{B})$ and $\overline{\rho}_b(\overline{A},\overline{B})$ that guarantee $U^i(\overline{a},\lambda_b,\overline{\rho}_a,\overline{\rho}_b)=U^i(\underline{a},\lambda_b,\overline{\rho}_a,\overline{\rho}_b)$ and $U^i(\overline{b},\lambda_a,\overline{\rho}_a,\overline{\rho}_b)=U^i(\underline{b},\lambda_a,\overline{\rho}_a,\overline{\rho}_b)$.

The optimization program will be

$$
\begin{aligned}
\max_{(\overline{A},\overline{B})\in\Phi^i_{\lambda_a\lambda_b}\bigcap\Psi^i_{\overline{M}_a\overline{M}_b}} \quad & \overline{A}U^i(\overline{a},\lambda_b,\overline{\rho}_a,\overline{\rho}_b)+(1-\overline{\lambda}_a)U^i(\underline{a},\lambda_b,\overline{\rho}_a,\overline{\rho}_b) \\
+ \quad & \overline{B}U^i(\overline{b},\lambda_a,\overline{\rho}_a,\overline{\rho}_b)+(1-\overline{\lambda}_b)U^i(\underline{b},\lambda_a,\overline{\rho}_a,\overline{\rho}_b) \\
= \quad & (\overline{A}+(1-\overline{\lambda}_a))(\underline{b}-\underline{a}c)\left[\frac{\overline{B}}{\overline{B}+1-\overline{\lambda}_b}(1-\overline{\rho}_a(\overline{A},\overline{B}))+\frac{1-\overline{\lambda}_b}{\overline{B}+(1-\overline{\lambda}_b)}\right] \\
+ \quad & (\overline{B}+(1-\overline{\lambda}_b))(\underline{a}-\underline{b}c)\left[\frac{\overline{A}}{\overline{A}+1-\overline{\lambda}_a}(1-\overline{\rho}_b(\overline{A},\overline{B}))+\frac{1-\overline{\lambda}_a}{\overline{A}+(1-\overline{\lambda}_a)}\right]
\end{aligned}
$$

The first order conditions will be,

$$
\begin{aligned}
0 &= (\underline{b}-\underline{a}c)\left[-(\overline{A}+1-\overline{\lambda}_a)\frac{\partial\overline{\rho}_a}{\partial\overline{A}}+\frac{\overline{B}}{\overline{B}+1-\overline{\lambda}_b}(1-\overline{\rho}_a(\overline{A},\overline{B}))+\frac{1-\overline{\lambda}_b}{\overline{B}+1-\overline{\lambda}_b}\right] \\
&+ (\underline{a}-\underline{b}c)(\overline{B}+1-\overline{\lambda}_b)\left[-\frac{\overline{A}(1-\overline{\rho}_b(\overline{A},\overline{B}))+1-\overline{\lambda}_a}{(\overline{A}+1-\overline{\lambda}_a)^2}+\frac{1-\overline{\rho}_b(\overline{A},\overline{B})-\overline{A}\frac{\partial\overline{\rho}_b}{\partial\overline{A}}}{\overline{A}+1-\overline{\lambda}_a}\right]
\end{aligned}
$$

Finally, focusing on the one-sided case, the first order condition will be

$$\xi_{\overline{\rho}\overline{A}} = \frac{1 - \overline{\rho}(\overline{A})}{\overline{\rho}(\overline{A})}$$

where $\xi_{\overline{\rho}\overline{A}}$ is the elasticity of $\overline{\rho}(\overline{A})$ with respect to $\overline{A}$.

□

Theorem (1) . (i) The profit maximizing platform will not offer the Informative Menu for low type users, (ii) There will be a unique optimal menu for each point in the parameter space $(\Delta_a, \overline{\lambda}, c)$, and (iii) Only Case (ii) Mixed matching equilibrium could be optimally supported at the Hype Menu.

Proof. The objective is to characterize the set of optimal menus, e.g. access fee and information provision, for the profit maximizing platform. The analysis for the one-sided platform is simpler than for the surplus maximizing platform because we only need to compare menus that induce full group participation, i.e. $\lambda \in \{0, \overline{\lambda}, 1\}$.

Informative Menus for high or low types where described in Proposition (2). We showed that if the platform offers a menu $(P, \rho) = (\overline{a}(1-c), 1)$ only high type users will participate, and they will suffer full rent extraction. The profits will be $\overline{\lambda}\overline{a}(1-c)$. On the other hand, if the platform offers a menu $(P, \rho) = (\underline{a}(1-c), 1/2)$ only low type users will participate, and they also will suffer full rent extraction. The profits for this case will be $(1-\overline{\lambda})\underline{a}(1-c)$.

Hype Menu, described in Proposition (3), induce all users to participate. In particular, the access fee will be equal to the low type's expected participation payoff, $U(\underline{a}, \overline{\lambda}, \rho^*)$, and the signal will neither be fully informative nor fully uninformative. The rule for optimal information provision is also described at Proposition (3). Finally low type users will suffer full rent extraction.

Platform's profits with the Hype Menu will be equal to $U(\underline{a}, \overline{\lambda}, \rho^*)$, but its value will depend on which matching equilibrium will prevail. With the highest ranked matching equilibrium profits will be $\underline{a}(1-c)$, and with the second highest ranked matching equilibrium profits will be

$$\frac{\underline{a}}{\overline{a}(1-\rho^*)^2}\left[\overline{a}(1-c)(1-\rho^*)(1-\rho^*-\rho^*\overline{\lambda}) + (\overline{a}c - \underline{a})(\rho^*)^2(1-\rho^*\overline{\lambda})\right]$$

We can also calculate the profits with the remaining matching equilibria, but its possible to show that profits under them are strictly lower than those obtained with the two highest ranked matching equilibria. Instead of doing the comparison using the analytic expression we will provide the intuition. The objective of the platform will be induce users to use the highest ranked equilibrium, but if this is not possible he will use the information provision such that the profits of the second highest ranked equilibrium match or exceeds the profits from the highest ranked equilibrium. This "strategy" is impossible with the lowest ranked equilibria because there high type users will always reject low signal partners.

Which matching equilibrium will the platform use? The parameter space is determined by $\Delta_a, \overline{\lambda}$, and c. Without loss of generality fix Δ_a. We can show that the higher is λ, the more likely is that the highest ranked matching equilibrium will outweigh the second highest ranked equilibrium. Take the extreme situation where $\overline{\lambda} = 1$. As the profits with the highest ranked equilibrium is $\underline{a}(1-c)$, and the profits with the second highest ranked is $\underline{a}(1-c)\frac{1-2\rho}{1-\rho} < 0$, the profit maximizing platform will use the former matching equilibrium. But in the other extreme, $\overline{\lambda} = 0$, while the profit with the former equilibrium remains unchanged, the profit with the second highest ranked equilibrium is $\underline{a}(1-c)+\frac{\underline{a}}{\overline{a}}(\overline{a}c-\underline{a})(\frac{\rho}{1-\rho})^2$. In this situation the profit maximizing platform will force users to play mixed strategies. Profit functions are continuous, then we posit regions with low $\overline{\lambda}$ will use matching equilibrium Case (ii) Mixed, and regions with high $\overline{\lambda}$ and low c, will use matching equilibrium Case (ii).

Figures (28) and (29) confirm this. Using Mathematica we compare at each point in the parameter space $(\Delta_a, \overline{\lambda}, c)$ the value function of the profits for matching equilibrium Case (ii) and Case (ii) Mixed. These figures use a fixed value of Δ_a but results are robust.

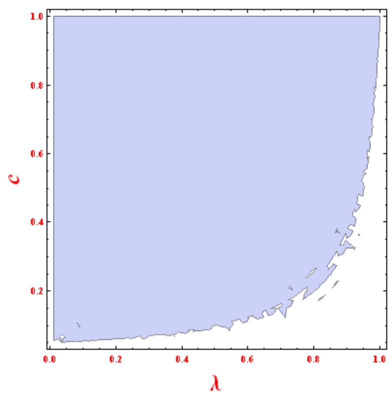

Figure 28: $\lambda = \overline{\lambda}$, Case (ii) Mixed $\geq$ Case (ii)

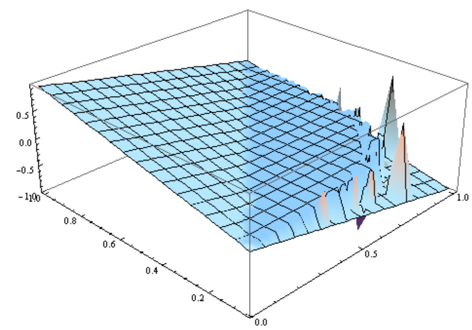

Figure 29: $\lambda = \overline{\lambda}$, Case (ii) Mixed $\geq$ Case (ii)

To determine which menus are optimal we must compare the profits coming from the Informative Menus for high and low type, and Hype Menu at each point on the parameter space $(\Delta_a, \overline{\lambda}, c)$. Instead of doing the comparison using analytic expression we opted to use numerical (constrained) optimization techniques and let Mathematica do the comparisons at each point in the parameter space. We have two reasons for this decision. First, although the model is very simple, the value function of the profits is very non linear making the comparison cumbersome. And second, using Mathematica we can obtain nice graphical representations of the regions where a particular menu becomes optimal.

We obtain two conclusions. The profit maximizing platform do not find optimal to offer an Informative Menu for the low types, this implies he always wants to keep on board the high type users. This result can be heuristically shown if we fix Δ_a, and analyze a situation with a low $\overline{\lambda}$, and another with a high $\overline{\lambda}$. In the former situation, when the platform uses a Hype Menu the profits will be close to

$$\underline{a}(1-c)+\frac{\underline{a}}{\overline{a}}(\overline{a}c-\underline{a})(\frac{\rho}{1-\rho})^2$$

, and the profits with an Informative Menu for lows yield a profit equal to $(1-\overline{\lambda})\underline{a}(1-c)$, so its clear he will prefer the first menu. In the opposite situation, with a high $\overline{\lambda}$, the profit from an Informative Menu for highs yields a profit equal to $\overline{\lambda}\overline{a}(1-c)$, and the profits from an Informative Menu for lows yields profits equal to $(1-\overline{\lambda})\underline{a}(1-c)$.

The second conclusion is about where the Informative Menu for high types is optimal. We obtain that for high values of $\overline{\lambda}$ and "low enough" values for the opportunity cost c, the profit maximizing platform will only keep on board the high type users.

Figures (30) and (31) show which regions, in the parameter space $(\overline{\lambda}, c)$, support either the Hype Menu or the Informative Menu for high types.

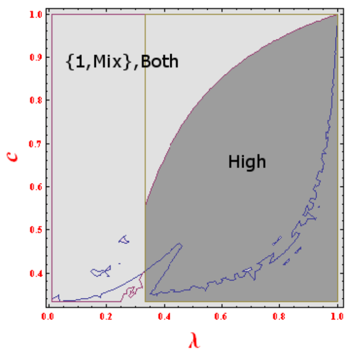

Figure 30: Profit Max., Δ_a Small

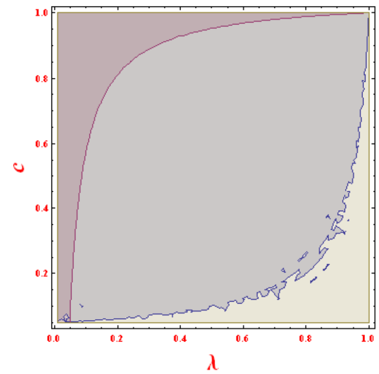

Figure 31: Profit Max., Δ_a Medium

□

Corollary (3) . The region, in the parameter space $(\overline{\lambda}, c)$, where the Hype Menu is optimal is positively related with the population heterogeneity, e.g. Δ_a.

Proof. See figures (30) and (31). Its evident that the higher it is the heterogeneity of users in the population, the bigger will be the region in the parameter space $(\overline{\lambda}, c)$ supporting the Informative Menu for high types.

□

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