# Bailout Uncertainty in a Microfounded General Equilibrium Model of the Financial System*

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**Abstract**

This paper develops a micro-founded general equilibrium model of the financial system composed of ultimate borrowers, ultimate lenders and financial intermediaries. The model is used to investigate the impact of uncertainty about the likelihood of governmental bailouts on leverage, interest rates, the volume of defaults and the real economy. The distinction between risk and uncertainty is implemented by applying the Gilboa-Schmeidler (1989) maxmin with multiple priors framework to lenders' beliefs about the probability of bailout. Events like Lehman's collapse are conceived of as "black swan" events that led lenders to put a positive mass on bailout probabilities that were previously assigned zero mass.

Results of the analysis include: (i) An unanticipated increase in bailout uncertainty raises interest rates, the volume of defaults in both the real and financial sectors and may lead to a total drying up of credit markets. (ii) Lower exante bailout uncertainty is conducive to higher leverage - which raises moral hazard and makes the economy more vulnerable to expost increases in bailout uncertainty. (iii) Bailout uncertainty raises the likelihood of bubbles, the amplitude of booms and busts as well as the banking and the credit spreads. (iv) Bailout uncertainty is associated with higher returns' variability in diversified portfolios and systemic risks, (v) Expansionary monetary policy reinforces those effects by inducing higher aggregate leverage levels.

**Keywords and Phrases:** Risk, Uncertainty, Lehman's default, Leverage, Financial intermediaries, Bailouts, Duration mismatches.

**JEL Classification Codes:** G01, G11, G2, G18, E3, E4, E5, E6, D81, D83.

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# 1 Introduction

Financial sector bailouts in the US and more recently in Europe have revived the well known dilemma between restoration of confidence in the face of a panic and the costs of moral hazard. On one hand, when a panic engulfs financial markets, bailouts appear indispensable in order to restore confidence and prevent further collapses in the financial system. On the other, by subsidizing opportunistic behavior at the expense of taxpayers, bailouts encourage excessive risk taking on the part of financial institutions, borrowers and lenders, and plant the seeds of the next bubble.

Different experts in both policymaking circles as well as in academia often find themselves at odds regarding the ways to handle this problem. In spite of currently ongoing reforms in regulation this dilemma is, therefore, likely to be a central issue during the upcoming decade. Whether, and how exactly will bailout policies be deployed in the future is largely an open issue. However, due to the lack of consensus about the precise ways to deal with the (exante and expost) trade-offs induced by bailouts, it is extremely likely that bailout uncertainty is likely to be non negligible in the foreseeable future. The 2008 bailout zigzags in the US (Bear-Stern versus Lehman) and current uncertainties about the reaction of EMU governments to potential sovereign debt problems of a large country like Spain attest to that.

This paper develops a micro-founded general equilibrium model of the financial system and uses it in order to investigate the impact of an increase in bailout uncertainty on financial markets and the real economy. It also investigates the exante, leverage expanding, moral hazard problems created by perceived generous governmental bailout policies.

As is well known since Knight's (1921) work risk and uncertainty are distinct concepts. Modern formulations of this distinction in the context of pecuniary returns conceptualize risk as some measure of spread for a **known distribution** of the stochastic return. Uncertainty, on the other hand, is a situation in which individuals are **unsure about the probability distribution** of returns and entertain the possibility that several alternative probability distributions have positive measure. An increase in uncertainty is then viewed as a an enlargement of the set of plausible probability distributions with positive measure. Ellsberg

(1961) and others have demonstrated by means of experiments that individuals are averse to ambiguity in the sense that, other things the same, they prefer a lottery with a known probability distribution to a lottery in which several distributions are believed to be possible.

Gilboa and Schmeidler (1989) (GS in the sequel) conceptualize an investor's uncertainty by postulating that he possesses a subjective set of probability measures, or multiple priors, over outcomes. Under several axioms they show that, if the investor is averse to ambiguity his action is determined by the Gilboa-Schmeidler max-min ambiguity aversion criterion. That is, for each possible action the investor assumes that the worst (by the expected utility criterion) possible distribution will realize and chooses his action so as to attain maximum expected utility over this set of worst outcomes.

This paper utilizes the GS notion of uncertainty and the associated max-min behavioral criterion to analyze the impact of an increase in uncertainty about governmental bailout policy on financial markets, the aggregate level of credit and, through them, on the real economy. The riskiness of bailouts at the level of an individual creditor is captured by a binomial distribution in which conditional on default by a borrower there is a bailout with probability, $p$, or there is no bailout with probability $1 - p$. Bailout uncertainty then means that individuals entertain the view that several alternative binomial distributions, each characterized by a different value of $p$ possess positive mass. In this context an increase in uncertainty means that there is an enlargement in the set of possible bailout distributions.

Prior to Lehman's collapse financial market's beliefs about the probability of bailout have been relatively optimistic due to Bear-Stern's bailout in March 2008 as well as to the implicit US government guarantees of Fannie Mae and Freddie Mac's liabilities (Meltzer (2009)). In terms of the GS framework this means that the family of binomial bailout distributions with positive mass was concentrated in the relatively high range of $p$'s.

Taleb (2007) has popularized the notion of a "black swan" event. Such an event is perceived to have zero mass before it realizes for the first time. However, once it realizes, individuals assign to it (a possibly small) but positive mass. We view Lehman's collapse in mid September 2008 as such a "black swan" event. That event, deemed unthinkable, prior to this collapse had realized after all and this reduced the lowest perceived probability of bailout with positive mass.

The behavior of credit default swap (CDS) spreads during the two weeks following Lehman's collapse provides a dramatic illustration of the sensitivity of bailout expectations to public signals. In the aftermath of this collapse credit markets experienced substantial waves of deleveraging, totally drying up in some cases, and both the level and variability of CDS spreads went through the roof. Table 1[1] shows the behavior of Citibank's CDS spread index during the period just preceding Lehman's default and the final approval of the TARP bailout package at the beginning of October 2008.

| Date | Event | CDS Spread |
|---:|---|---|
| 13-14/9 | | 150 |
| 15/9 | Lehman files for chapter 11 | |
| 16-17/9 | Paulson suggests TARP to Congress | 250 |
| 18-19/9 | | 150 |
| 22-23/9 | Paulson & Bernanke address Congress | 450 |
| 24-25/9 | | 350 |
| 29/9 | Congress rejects TARP proposal | Almost 450 |
| 3/10 | Amended TARP approved by Congress | |
| 5-10/10 | Aftermath of approval | 150 |

Table 1: **Chronology of CDS spread around Lehman's collapse**

The table demonstrates the sensitivity of the CDS spread to ongoing public signals. In particular, following rejection of the proposed TARP bailout package by Congress in September 2008 the CDS spread goes up and following its approval in early October it goes down supporting the view that financial markets participants are quite sensitive to news about the likelihood of bailout.[2] Our view is that, following Lehman's collapse and the ensuing public debate among policymakers about the wisdom of governmental bailouts, the lower bound on the set of binomial distributions with perceived positive mass went down,

[1]Source: Cochrane and Zingales (2009) .

[2]Following Keynes, Akerlof and Shiller (2009) attribute changes in expectations to exogenous animal spirits. By contrast this paper takes the view that changes in expectations can be traced back to new information in noisy but relevant public signals.

say, from $\pi_0$ to $\pi_1$ (here $\pi_t$ is the lower bound of distributions with perceived positive mass in period $t$).

The analysis in the paper shows that lenders' expected utility is lower the lower is $p$. In conjunction with the GS max-min criterion this increase in bailout uncertainty implies that, once a "black swan" event like Lehman's collapse materializes, lenders become more reluctant to lend, sending shock waves through both financial and real markets. One objective of the paper is to trace some of the mechanisms through which the consequent changes in perceptions affect short term credit within the financial system, as well as credit to the real sector. Another related objective is to analyze the impact of expansionary monetary policy on leverage and risk appetite. The paper's framework makes it possible to trace out both the exante and the expost consequences of (perceived) generous bailout policies. Exante, a more generous bailout policy increases moral hazard in all segments of the financial system and induces an overall expansion of credit.[3] But expost the maintenance of a generous bailout policy may become necessary just to avoid a crisis even if government no longer desires to maintain high bailout levels.

Like Caballero and Krishnamurthy (2008) this paper attributes flight to quality episodes to Knightian uncertainty. But whereas they allow uncertainty to arise from various origins we focus on the consequences of increasing uncertainty about government bailout policy. This makes it possible to focus the analytical discussion on bailout uncertainty as a particular trigger for a flight to quality episode. A prominent example of such an episode is the increase in bailout uncertainty experienced in the immediate aftermath of Lehman's collapse.

Important features of the model include:

(i) An individual tradeoff between return seeking through higher levels of leverage and higher probability of total loss at the **individual level**.

(ii) Exante and expost relations between the worst probability of bailout and leverage at the **aggregate level**.

[3]Borio C. and M. Drehmann (2009) convincingly argue that such a credit buildup raises the likelihood of a financial crisis.

(iii) **Duration mismatches:** Borrowers need financing for two periods but get only one period loans from financial intermediaries in each period.

(iv) The model's focus is on the segment of **the shadow banking system** (like SIV and hedge funds) in which funds are secured only for short periods. Accordingly, financial intermediaries are assumed to borrow for only one period.

The rest of the paper is organized as follows. Section 2 presents a general overview of the model. Sections 3, 4 and 5 introduce a typical borrower, a typical financial intermediary and a typical lender and characterize the optimal microeconomic behavior of each type of agent. Government's bailout policy is specified in Section 5. General equilibrium of the financial system and the determination of market rates are discussed in section 6. Section 7 analyzes the impact of an exogenous decrease in perceptions about the likelihood of bailout on financial markets and utilizes it to explain some of the events observed following Lehman's collapse. Section 8 discusses the exante choice of leverage by borrowers in general equilibrium including, in particular, the impact of perceived bailout policy and the associated moral hazard problem. Section 9 reflects on the social desirability of exante commitments to a particular bailout policy. This is followed by concluding remarks in Section 10. A central result of the paper (implied by the discussion in sections 7 and 8 and elaborated in the conclusion) is that higher bailout uncertainty raises the amplitude of booms and busts. Most proofs are in the Appendix.

# 2 Framework

There is a large number of each of the following risk averse (identical within each group) 3 types of agents: Borrowers (B), Financial intermediaries (F) and Lenders (L) each possessing one unit of equity capital.[4] The initial masses of each type of agent are $M_B$, $M_L$ and $M_F$

[4]We use the following notational conventions: the subscript $j = \{B, F, L\}$ to a variable $x_{jt}$ indicates the agent type, and subscript $t = \{0, 1, 2\}$ indicates time. When the time index is omitted the variable refers to any of the time periods between 0 and 2. Random variables are identified by a tilde on top of the variable (e.g. $\widetilde{X}$).

for borrowers, financial intermediaries and lenders, respectively.[5]

There are 3 time periods labeled 0, 1 and 2. Only borrowers-investors have access to real investment decisions. All such decisions are made by them in period 0 and are long term in the sense, that once chosen, the project's size cannot be adjusted. The selected project size is $(1 + L_B)$, where 1 is the borrower's initial equity capital and $L_B$ is the leverage he selects to take. Only short term loans are available to borrowers with interest rates $r_{B1}$ and $r_{B2}$ for loans assumed in the first and in the second period, respectively. Interest rates on loans and project's yields are all specified in terms of net returns.

Each borrower can get loans only from financial intermediaries. The amount of leverage, $L_B$, demanded by a borrower is determined as a function of $r_{B1}$ and of expected $r_{B2}$, by means of individual optimization. Each financial intermediary can obtain short-term funds, $L_F$, from lenders. The intermediaries generally splits his total funds $(1 + L_F)$ between a fraction $z_F$ allocated to a partially diversified portfolio of loans to borrowers and a fraction $(1 - z_F)$ allocated to a risk free asset that pays a fixed interest rate $r_f$.[6] The return on the risk free asset, $r_f$, is determined by the monetary authority.

Financial intermediaries pay to lenders short term interest rates $r_{L1}$ and $r_{L2}$ in periods 1 and 2 (for loans taken in periods 0 and 1) respectively. A typical lender splits his initial wealth of 1 between a fraction $z_L$ of funds allocated to loans to financial intermediaries and a remaining fraction, $(1 - z_L)$, that is invested in the risk free asset. In contrast to a typical financial intermediary, whose portfolio of loans to borrowers is only partially diversified, a typical lender holds his selected portion of loans to financial intermediaries in a fully diversified portfolio of loans.

The supply of loans to borrowers by an individual financial intermediary and his demand for loans from lenders, $L_F$, are determined through the intermediary's individual optimization as a function of the interest rates $r_{B1}$, $r_{B2}$ , $r_{L1}$ and $r_{L2}$. Those interest rates are determined through general equilibrium competitive clearing in periods 0 and 1 respectively

[5] The financial markets model in the paper can be thought of as a microfounded version of general equilibrium approaches to monetary theory and policy (Brunner and Meltzer (1997) , Tobin (1969)).

[6] An intermediary's portfolio is partially diversified in the sense that only part of the idiosyncratic risk is eliminated. By contrast in a fully diversified portfolio all idiosyncratic risk is eliminated and only the systematic risk remains.

in two markets within each period: The market for loans from intermediaries to borrowers and the market for loans from lenders to financial intermediaries.[7]

As elaborated in the next section, returns on real projects are stochastic and therefore risky. They realize in period 2. A real-project yield, $\widetilde{Y}_P$, depends on two independent random variables: an aggregate economy-wide shock, $\widetilde{Y}_A$, and a specific (idiosyncratic) shock, $\widetilde{Y}_I$. The realization of the aggregate shock is not revealed prior to period 2. Although all idiosyncratic shocks realize only in period 2, the value of this future realization becomes publicly known for some borrowers already in period 1. Depending on the information available in period 1 borrowers can be classified to one of the following three groups: Lucky borrowers for whom it becomes known they will get a high $\widetilde{Y}_I$, unlucky borrowers for whom it becomes known they will get a low $\widetilde{Y}_I$, and regular borrowers for whom no advance return information is available in period 1. The availability of such information is important because it affects the borrower's ability to get refinancing in period 1. Since project yield is random and borrowers have some leverage obligations they generally may default in either of periods 1 or 2. A borrower defaults in period 1 if he does not succeed in securing credit to carry over his project on to period 2. He defaults in period 2 if the total final project return does no suffice to service the debt incurred in the previous period.

A financial intermediary can also default in period 1 or 2 if the principal and the interest rate paid to him by borrowers cannot cover his obligations to lenders. When a financial intermediary defaults lenders lose their entire investment in this intermediary including the principal and the interest rate. Government can possibly and selectively pay the debt of defaulting financial intermediaries to lenders. But governmental bailout policy is uncertain in the Knigthian sense. More precisely, individuals entertain multiple priors about the probability of bailout, or in the language of modern decision theory – government's bailout policy is ambiguous.

Figure 1 presents a bird's eye view of the model's financial system. In the figure $z_F$ and $z_L$ represent the fractions of funds Fs and Ls allocate to risky loans, and $r_B$ and $r_L$ are the rates paid by Bs and received by Ls respectively. $\widetilde{Y}_A$ and $\widetilde{Y}_I$ are aggregate and individual

[7] A loan carrying interest rate $r_t$ $(t = 1, 2)$ is contracted in period $t-1$ and settled in period $t$.

components of the total net return to a typical borrower.

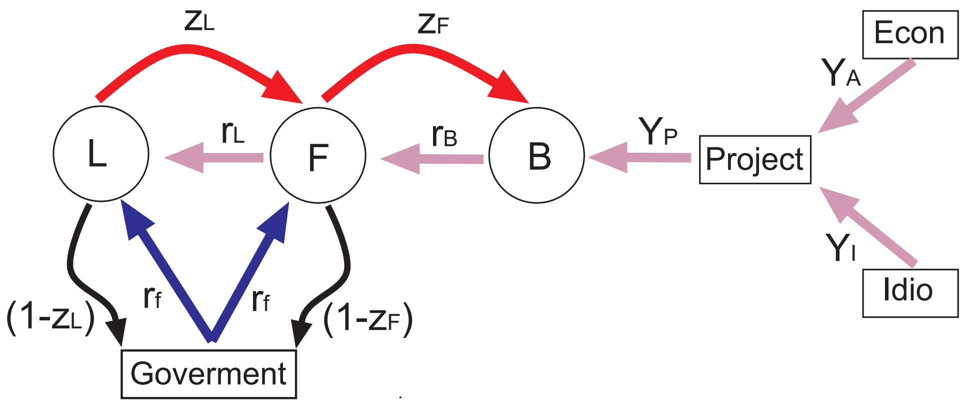


Figure 1: **The financial flows**

# 3 The Typical Investor-Borrower (B)

This section presents the borrower's problem. First it specifies B's real investments opportunities and his financial requirements in each period. It then derives conditions for his solvency and utilizes them to characterize the optimal project's size and B's optimal leverage conditional on the project's characteristics (its outcomes and their probabilities) and the cost of capital faced by him in periods 0 and period 1.

## 3.1 Real investment projects

All real investment made in period 0 are long term in the sense, that once chosen, the project's size cannot be adjusted, until returns are realized in period 2. The typical investment project yields a stochastic (net) return, $\widetilde{Y}_P$, which may be either positive or negative.[8] All real

[8]Returns, whether positive or negative, are cash flows.

projects have the same distribution of returns, and the yields of any two different projects are correlated due to presence of the aggregate common component in $\widetilde{Y}_P$.

A project's net yield is the sum of an aggregate shock $\widetilde{Y}_A$ and of an individual idiosyncratic shock $\widetilde{Y}_I$, that is

$$\widetilde{Y}_P = \widetilde{Y}_A + \widetilde{Y}_I. \tag{1}$$

We assume, for tractability, that both the aggregate and the idiosyncratic shocks, are binomially and identically distributed. That is

$$\widetilde{Y} = \widetilde{Y}_A = \widetilde{Y}_I = \begin{cases} y, & \Pr(y) = q \\ -y, & \Pr(-y) = 1-q \end{cases}, \tag{2}$$

where $0 \leq y \leq 1/2$. The random variables $\widetilde{Y}_A$ and $\widetilde{Y}_I$ are statistically independent and the idiosyncratic shock, $\widetilde{Y}_I$, is independent across projects.[9] Equations (1)-(2) imply that the distribution of $\widetilde{Y}_P$ is

$$\widetilde{Y}_P = \begin{cases} 2y, & \Pr(2y) = q^2 \\ 0, & \Pr(0) = 2q(1-q) \\ -2y, & \Pr(-2y) = (1-q)^2 \end{cases}.$$

Notice that the risk of a project is a function of $y$: Given two projects with identical $q$ a higher $y$ implies a riskier project. By equations (1)-(2) the expected return of each of the component shocks $\widetilde{Y}_i$, $i = \{A, I\}$ is

$$E\widetilde{Y}_i = \overline{Y}_i = y(2q-1), \quad i = A, I.$$

Since the project net return is the sum of $\widetilde{Y}_A$ and $\widetilde{Y}_I$, which are equal in distribution, its expected return is

$$E\widetilde{Y}_P = \overline{Y}_P = 2y(2q-1).$$

Projects must have a positive expected return to be considered; i.e. $\overline{Y}_P > 0$, which implies that $q > \frac{1}{2}$.

[9] The cases $\widetilde{Y}_A = y$ and $\widetilde{Y}_A = -y$ are referred to as expansion and contraction respectively.

**Assumption 1:** *As of period 0 the expected return on a project is higher than the expected cost of leverage needed to carry the project to completion in period 2. That is, the distribution of the return, $\widetilde{Y}_P$, on a typical project satisfies*

$$1 \leq (1 + r_{B1})(1 + r^e_{B2}) < \left(1 + \overline{Y}_P\right),$$

*where $r_{B1}$ is the interest rate paid by the borrower in period 1 and $r^e_{B2}$ is the interest rate he expects to pay in period 2 for refinancing in period 1 given the information set of period* 0. *Since the minimal project size is* 1 *and the lowest return on a project is $-2y$ we impose the constraint*

$$0 \leq y \leq \frac{1}{2}.$$

*This consraint enforces limited liability by ruling out negative realizations of wealth when there is no leverage .*

## 3.2 Borrower′s financial requirements

Projects are financed by a combination of equity and of leverage supplied by financial intermediaries to borrowers. In period 0, each borrower-entrepreneur owns one unit of equity capital. The initial financing structure (equity-1 versus leverage-$L_B$) is chosen by each B in period 0 along with the project's size, denoted by $x$. Since B's initial equity capital is 1, $x = 1 + L_B$. In each period loans by Fs to Bs are one period loans. Consequently, a B's project has to be financed by two consecutive one period loans.

In the presence of positive leverage and since projects' yields are obtained only in period 2 a B must seek refinancing in period 1. Therefore he depends on the availability and the cost of credit in period 1. If excluded from the credit market in that period he defaults and loses the entire investment project including his equity. A borrower's financial requirements in period 1 are equal to the amount needed to repay the principal, $L_B$, and period's 1 interest charges, $r_{B1}L_B$. Hence, *B's total financial requirements in period 1 are*

$$FR_{B1} = \underbrace{(1 + r_{B1}) L_B}_{Debt\ service}.$$

*When he gets credit in period 1, B's ultimate debt service in period 2 is*

$$FR_{B2} = (1 + r_{B2}) FR_{B1} = (1 + r_{B1})(1 + r_{B2}) L_B.$$

The borrower's cost of capital for the entire project's life (from period 0 till period 2) is therefore

$$r_B \equiv (1 + r_{B1})(1 + r_{B2}) - 1.$$

We assume that when the borrower cannot obtain refinancing in period 1 or cannot repay the debt in period 2, he defaults, the project is lost and neither the borrower nor the financial intermediary receives any payoff. Thus, due to limited liability, the amount needed to cover losses (if any) is 0. The next section explores the borrower's solvency condition.

## 3.3 Borrower′s solvency conditions

### 3.3.1 Period 0

A borrower is able to get a loan in period 0 only if the total expected payoff from his project is higher than the total debt service liability expected for period 2, that is

$$L_B (1 + r_B^e) \leq (1 + L_B)\left(1 + \overline{Y}_P\right), \tag{3}$$

where $r_B^e \equiv (1 + r_{B1})(1 + r_{B2}^e) - 1$ is the expected (as of period 0) cumulated interest rate factor over the lifetime of the project. Assumption 1 implies that this condition is satisfied for all non-negative leverage levels.

#### 3.3.2 Period 1

Although all borrowers are identical exante (in period 0), they split into three groups in period 1. Those groups differ in terms of the information that becomes available to markets in that period about the realizations of their idiosyncratic shocks in period 2. In particular, it becomes known in period 1 that a fraction, $\theta_{LB} < q$, of borrowers will have $\widetilde{Y}_I = y$, a fraction $\theta_{UB} < 1 - q$ will get $\widetilde{Y}_I = -y$, and no new information is revealed in period 1 about the remaining borrowers. We refer to those three types of borrowers as Lucky borrowers (LB), Unlucky borrowers (UB) and Regular Borrowers (RB) respectively.

A borrower who decides to leverage his project in period 0 is solvent in period 1 if and only if he is able to obtain the refinancing required to maintain his project alive till period 2. Financial intermediaries will offer the required credit in period 1 if and only if the expected cash flow of the project in period 2 suffices to cover period's 1 debt service. Obviously this expected cash flow differs across borrowers' types implying that borrower of type $i = \{LB, UB, RB\}$ obtains refinancing in period 1 if and only if

$$L_B (1 + r_{B1}) (1 + r_{B2}) \leq (1 + L_B) \left(1 + E\left[\widetilde{Y}_P | I_1 \cap B_i\right]\right), \ i = LB, UB, RB, \tag{4}$$

where $I_1$ is the information set of period 1. Given period's 1 information

$$E\left[\widetilde{Y}_P | I_1 \cap B_i\right] = \begin{cases} 2yq, & i = LB \\ E\left[\widetilde{Y}_P \mid I_1 \cap B_{RB}\right] = 2y\left(q + q_1 - 1\right), & i = RB \\ -2y(1-q), & i = UB \end{cases}, \tag{5}$$

and

$$q_1 \equiv \frac{q - \theta_{LB}}{1 - (\theta_{LB} + \theta_{UB})} \equiv \frac{q - \theta_{LB}}{1 - \theta} \tag{6}$$

is the probability that a regular borrower will get a good draw on the idiosyncratic shock, $\widetilde{Y}_I$, given the information available in period 1.[10]

[10]The reason this probability differs between periods 0 and 1 is that the realizations of $\widetilde{Y}_I$ become known with certainty for a fraction, $\theta$, of borrowers in period 1.

**Assumption 2:** *The expected net return of a RB on a real project conditional on the information in period 1 satisfies* $E\left[\widetilde{Y}_P \mid I_1 \cap B_{RB}\right] > r_B^e$.

Assumption 2 is basically an extension of Assumption 1 from period 0 to period 1. Together those assumptions requires that, given $r_B^e$, the expected net return perceived by a regular borrower is larger than the expected cost of leverage given the information of both periods 0 and 1. The following Lemma identifies solvency conditions in period 1 for the three types of borrowers

**Lemma 1:**

*(i) Regular borrowers are solvent in period 1 at any level of leverage,* $L_B$*, if* $r_{B2} = r_{B2}^e$.

*(ii) Lucky borrowers are solvent in period 1 at any level of leverage if* $2yq > E\left[\widetilde{Y}_P \mid I_1\right]$.[11]

*(iii) Unlucky borrowers are solvent in period 1 if and only if* $L_B \leq \frac{1-2y(1-q)}{2y(1-q)+r_B^e} \equiv L_B^1$

Note that since $\frac{1}{2} < q \leq 1$ the critical level of leverage, $L_B^1$, is positive. The lower this critical level the wider the range of period's 0 debt for which there is a non-zero probability that the unlucky borrower defaults in period 1. When the expected cost of capital, $r_B^e$, increases the critical level, $L_B^1$, decreases, implying that, the higher the cost of capital, the wider is the range of leverages at which a borrower might default in period 1. Increasing risk (measured in terms of returns' variance) has a similar effect since it decreases $L_B^1$.[12] On the other hand, when the probability of good returns increases (i.e $q$ increases), the critical leverage, $L_B^1$, increases widening the range of leverages for which the probability of default in period 1 is zero.

### 3.3.3 Period 2

A borrower is solvent in period 2 if the payoff from his project suffices to cover his debt obligation, that is

$$L_B (1 + r_{B1}) (1 + r_{B2}) \leq (1 + L_B) \left(1 + \widetilde{Y}_P\right). \tag{7}$$

[11] Overly strong jointly sufficient conditions for this requirement are $\theta_{LB} = \theta_{UB}$ and $q > 1 - q$..

[12] It is easily checked that increasing $y$, while keeping the probabilities of good and bad outcomes ($q$ and $(1-q)$) unchanged, increases the project's variance without changing its expected return.

Straightforward algebra shows that this is equivalent to the requirement that ultimate wealth, $W_B(.)$, is non negative

$$W_B(L_B, \widetilde{Y}_P) = 1 + \widetilde{Y}_P + \left(\widetilde{Y}_P - r_B\right) L_B \geq 0. \tag{8}$$

When the final project's payoff does no suffice to cover the principal and interest rate payments the borrower defaults and loses the entire project including his initial equity. Due to limited liability the net return on the project in this case is $\widetilde{Y}_P = -1$ and the financial intermediary who own the debt receives nothing.

Lemma 1 has shown that when period's 0 leverage is higher than some critical value the borrower is exposed to default risk in period 1. In addition positive leverage also exposes borrowers to default risk in period 2. The following Lemma identifies regular borrower's solvency conditions in period 2 for various realizations of total returns.

**Lemma 2:** *If a regular borrower's ultimate return is:*
*(i)* $\widetilde{Y}_P = -2y$ *he is solvent in period 2 if and only if* $L_B \leq \frac{1-2y}{r_B+2y} \equiv L_B^L$,
*(ii)* $\widetilde{Y}_P = 0$ *he is solvent in period 2 if and only if* $L_B \leq \frac{1}{r_B} \equiv L_B^H$,
*(iii)* $\widetilde{Y}_P = 2y$ *he is solvent in period 2 for any level of leverage.*

**Lemma 3:** Given $r_{B2} = r_{B2}^e$,
*(i) If an unlucky borrower (UB) has chosen* $L_B^1$ *in period 0 he is solvent in period 1.*
*(ii) An UB that has chosen* $L_B^1$ *is also solvent in period 2 if and only if there is an aggregae expansion*
*(iii) A lucky borrower is always solvent in period 2.*

Next, we analyze the probabilities of default in periods 1 and 2 as functions of the leverage chosen in period 0. Since payoffs are discrete those relations take the form of step functions.

**Proposition 1:** *Provided* $r_{B2} = r_{B2}^e$, *the ex-ante probabilities of default in period 1 and in period 2 (as viewed from he vantage point of period 0) are step functions of the leverage* *chosen in period 0. The precise probabilities of defaults are:*

| $L_B$ | $\Pr(D_1)$ | $\Pr(D_2)$ | $\Pr(D) = \Pr(D_1) + [1 - \Pr(D_1)]\Pr(D_2)$ |
|---|---|---|---|
| $L_B \leq L_B^L$ | $0$ | $0$ | $0$ |
| $L_B^L < L_B \leq L_B^1$ | $0$ | $(1-q)^2$ | $(1-q)^2$ |
| $L_B^1 < L_B \leq L_B^H$ | $\theta_{UB}$ | $(1-q)^2$ | $\theta_{UB} + (1-\theta)(1-q)^2$ |
| $L_B^H < L_B$ | $\theta_{UB}$ | $1-q^2$ | $\theta_{UB} + (1-\theta_{UB})(1-q^2) + ((1-\theta)2q + \theta_L)(1-q)$ |

,

*where* $\Pr(D_1)$ *and* $\Pr(D_2)$ *stand for default probabilities in period 1 and 2 respectively.*

## 3.4 Borrower's optimization

Not surprisingly the individual borrower faces a tradeoff between expected payoff and default probability. In the large, by raising leverage, he raises the expected value of terminal equity but also the chances of default. By Proposition 1 the exante probability of default is a step function of leverage. This implies that the optimal level of leverage (and by implication also the optimal project's size) must coincide with one of the four leverage levels at the jump points of the probability of default function. The reason is that, once leverage is extended beyond a given jump point the probability of default remains constant as long as leverage is not pushed beyond the next jump point. Within such an interval, raising leverage raises the expected payoff without raising the probability of default.

Hence, once leverage is raised beyond a given jump point, it is individually optimal to push it (at least) all the way till just a tiny bit before the probability function's next jump point. It follows that, from the vantage point of period 0, the optimal level of leverage is either zero or one of the following three leverage levels:

$$\begin{aligned} L_B^L &= \frac{1-2y}{r_B^e + 2y}; \\ L_B^1 &= \frac{1-2y(1-q)}{2y(1-q) + r_B^e}; \\ L_B^H &= \frac{1}{r_B^e}. \end{aligned} \tag{9}$$

The borrower's utility function is piecewise linear with a penalty in the event of default. In particular utility is linear in wealth as long as the borrower is solvent. When insolvent the borrower is subject to a penalty that increases with the magnitude of leverage he defaults on. Formally

$$u(W_B, L_B) = \begin{cases} W_B \geq 0, & \text{Solvency} \\ P_B L_B, & \text{Insolvency} \end{cases}, \tag{10}$$

where $W_B$ is his period's 2 terminal wealth after servicing all debts and $P_B$ is a fixed default penalty per unpaid leverage dollar in states of insolvency. Hence, the borrower's expected utility is

$$\begin{aligned} V(L_B) &\equiv Eu(W_B(\cdot), L_B) \\ &= [1 - \Pr(D \mid L_B)]\, E\left[W_B(\cdot) \mid W_B(\cdot) \geq 0\right] - \Pr(D \mid L_B) P_B L_B. \end{aligned} \tag{11}$$

Using Proposition 1 and the definition of $W_B(\cdot)$ in equations (8) and (9) establishes that $B's$ expected utilities at each of the five candidates for optimal leverage (four discussed above plus any level of leverage $L_B^m > L_B^H$) are given by

$$\begin{aligned} V(L_B = 0) &= q^2(1 + 2y) + 2q(1-q) + (1-q)^2(1-2y); \\ V(L_B^L) &= q^2\left[1 + 2y + (2y - r_B^e)L_B^L\right] + 2q(1-q)\left[1 - r_B^e L_B^L\right]; \\ V(L_B^1) &= q^2\left[1 + 2y + (2y - r_B^e)L_B^1\right] + 2q(1-q)\left[1 - r_B^e L_B^1\right] - (1-q)^2 P_B L_B^1; \\ V(L_B^H) &= (\theta_{LB} q + (1-\theta)q^2)\left[1 + 2y + (2y - r_B^e)L_B^H\right] - \Pr\left[D \mid L_B^H\right] P_B L_B^H; \\ V(L_B^m) &= (\theta_{LB} q + (1-\theta)q^2)\left[1 + 2y + (2y - r_B^e)L_B^m\right] - \Pr\left[D \mid L_B^m\right] P_B L_B^m, \end{aligned} \tag{12}$$

where

$$\begin{aligned} \Pr\left[D \mid L_B^H\right] &\equiv \left[\theta_{UB} + (1-\theta)(1-q^2)\right], \\ \Pr\left[D \mid L_B^m\right] &\equiv \Pr\left[D \mid L_B^H\right] + ((1-\theta)2q + \theta_{LB})(1-q). \end{aligned}$$

Let $L_B^*$ be the optimal level of leverage. The following proposition presents (overly restrictive) sufficient condition for $L_B^* = L_B^H$.

**Proposition 2:** *Provided*

$$P_B > \frac{\left(\theta_{LB} q + (1-\theta) q^2\right)\left(2y - r_B^e\right)}{\left((1-\theta) 2q + \theta_{LB}\right)(1-q)} \equiv P_B^c,$$

*there exists a dense set of values for the vector of parameters* $(q, \theta_{LB}, \theta_{UB})$, *such that* $1-q$, $q-\theta_{LB}$ *and* $\theta_{UB}$ *are all strictly positive but small, for which the borrower's optimal level of leverage is* $L_B^H$.

The broad intuition underlying this proposition can be appreciated by starting with the particular case in which the unit penalty, $P_B$, for default is zero. In this case, when the chances of good draws at the individual level are high ($q$ and $\theta_{LB}$ are large) and the likelihood that the borrower will be unlucky in period 1 is low ($\theta_{UB}$ is small), expected utility is monotonically increasing in leverage. As a matter of fact, given the full linearity of the utility function in the absence of a penalty, the borrower's optimal level of leverage is infinite in this case. However, in the presence of a sufficiently large default penalty extending leverage beyond $L_B^H$ is not individually optimal because of the increase in the risk that the penalty will be triggered once leverage crosses the $L_B^H$ threshold.

In a broad sense the conditions in the Proposition 2 are analogous to the borrower's second order condition (SOC) when the penalties from default rise continuously with leverage. In the continuous case the SOC assures that, as leverage goes up, the favorable marginal impact of higher leverage on return in good states diminishes in comparison to the unfavorable gradual increase in the default penalty. Similarly, the conditions in Proposition 2 assure that, as leverage rises, the marginal detrimental impact of the default penalty becomes more important relatively to the marginal favorable effect on likely profits.

# 4 Financial intermediaries (Fs)

For reasons that will become apparent later it is convenient to open this section with a forward look at the relation between various equilibrium rates of interest.

### 4.1 A forward look at general equilibrium

The following proposition establishes general equilibrium relations between equilibrium interest rates, $r_B$, $r_F$ and $r_L$.

**Proposition 3:** *In a general equilibrium with risk aversion on the part of borrowers, financial intermediaries and lenders, and positive levels of leverage in both the real and the financial sectors, the following inequalities hold*

$$r_f \leq r_L < r_B.$$

### 4.2 The typical financial intermediary

There is a large number of financial intermediaries (Fs) each of which possesses one unit of core funds consisting of a combination of equity and of long term (two periods) debt. A typical F can also raise **short term** (one period) funds from lenders.[13] Since the focus of this analysis is on changes in the availability of short term credit in the face of new information, the amount of short term leverage assumed by a typical F is determined endogenously while the sum of equity and of long term debt is taken to be exogenous.

Total financial resources of a typical F consist of the core funds and of short term leverage, $L_F$. The financial intermediary diversifies his total resources between the risk free asset whose rate, $r_f$, is a policy instrument, and a risky, not fully diversified, portfolio of loans to borrowers.[14] For reasons of tractability, each F lends to only two borrowers. The fraction of resources invested in the risky loan portfolio to Bs is denoted $z_F$. Let $W_F$ be the intermediary's terminal wealth after debt service in each period. F is solvent or insolvent in each period depending on whether terminal wealth is non negative or strictly negative. When solvent, F's utility is described by a CRRA utility function with a coefficient, $\delta$, of risk aversion that is close to, but not quite equal to, one. This specification implies that F is almost, but not strictly, risk neutral. When insolvent, the typical intermediary experiences

[13]For instance through various deposits including certificates of deposit (CDs).

[14]By contrast, as shown in the next section, the risky portfolio of suppliers of funds to Fs (lenders) is fully diversified.

a (per unit of leverage) penalty, $P_F$. Formally[15]

$$u(W_F) = \begin{cases} \frac{[W_F]^{1-\delta}}{1-\delta}, & \text{when } W_F \geq 0 \\ -P_F L_F, & \text{when } W_F < 0. \end{cases} \tag{13}$$

### 4.3 Distribution of returns, solvency and optimization

Total return to a financial intermediary depends on the performance of the two borrowers to whom he lends. Since borrowers are identical exante, the optimal risky portfolio of an F consists of a fifty-fifty split between loans to his two debtors. If both borrowers are solvent both of them pay the full face value, of the gross debt service and the payoff (from one unit) to F is $1 + r_B$. If both of them default F gets a payoff of 0 on his risky portfolio. If one borrower is solvent and the other defaults F gets the payoff $\frac{1}{2}(1 + r_B)$. Obviously, the probabilities associated with each of those three payoffs depend on the probabilities of defaults of borrowers and differ between periods 0 and 1. From the vantage point of period zero, the probability a single B defaults in period 1 , $\Pr(D_1)$, is given in Proposition 1. Since $L_B^* = L_B^H$, the probability that a B is insolvent in period 1 is $\theta_{UB}$. Since the probability of being unlucky of any borrower is statistically independent of this probability for any other borrower, the distribution of payoffs faced by a typical intermediary in period 0 is given by the following table.[16]

| State | Payoff ($\widetilde{R}_B$) | **Period 1** **Probability** | **Period 2** **Probability** |
|---|---|---|---|
| Both Bs are solvent | $1 + r_B$ | $(1-\theta_{UB})^2 \equiv \gamma_{11}$ | $q(1-q_1^2) + q_1^2 \equiv \gamma_{12}$ |
| Exactly one B is solvent | $\frac{1}{2}(1 + r_B)$ | $2(1-\theta_{UB})\theta_{UB} \equiv \gamma_{21}$ | $2(1-q)q_1(1-q_1) \equiv \gamma_{22}$ |
| Both Bs are insolvent | 0 | $\theta_{UB}^2 \equiv \gamma_{31}$ | $(1-q)(1-q_1)^2 \equiv \gamma_{32}$ |

(14)

Recall that $q$ is the probability of a positive (aggregate or idiosyncratic) shock and $q_1$,

[15] In spite of the fact that utility functions differ across the three types of agents we use the symbol $u(\cdot)$ to stand for all of them in order to economize on notation. In each case the identity of the player should be evident from the context.

[16] The notation $\gamma_{bt}$ stands for probability in time $t$ conditional on realized borrowers' types, $b$.

defined in Equation (6), is the probability of a positive idiosyncratic shock conditional on the project type that is revealed in period 1. The last column shows the probability distribution of period's 2 payoffs from loans to regular borrowers as perceived by F's in period 1.

The wealth of a typical F at the end of each period is

$$\widetilde{W}_F(\widetilde{R}_B, L_F) = (1 + L_F)\left[z_F\widetilde{R}_B + (1 - z_F)(1 + r_f)\right] - (1 + r_L)L_F, \tag{15}$$

where the distributions of $\widetilde{R}_B$ is given in Equation (14) and $r_L$ is the interest rate paid by a F on its short term obligations. A representative F chooses his leverage, $L_F$, and the fraction, $z_F$, of resources invested in the risky loan portfolio so as to maximize $Eu(\widetilde{W}_F)$ in each of periods 0 and 1. The following proposition presents a preliminary characterization of F's optimal policy.

**Proposition 4:** *Let $r_f < r_L$. Then at an optimum with positive leverage, F invests all his resources in risky loans to Bs.*

### 4.3.1 F's solvency condition

Proposition 4 and Equation (15) imply that F is solvent if and only if

$$\widetilde{W}_F(\widetilde{R}_B, L_F) = (1 + L_F)\widetilde{R}_B - (1 + r_L)L_F = \widetilde{R}_B + (\widetilde{R}_B - r_L)L_F \geq 0. \tag{16}$$

Since $r_B > r_L$, F is solvent for any level of leverage, $L_F$, when both of his borrowers are solvent, so that $\widetilde{R}_B = 1 + r_B$. In the other two cases F is solvent only if $L_F$ is sufficiently small. The precise solvency conditions are:

$$\begin{array}{lll} L_F \leq \frac{1+r_B}{1+2r_L-r_B} \equiv L_F^c & \text{when} & \widetilde{R}_B = \frac{1}{2}(1 + r_B) \\ L_F = 0 & \text{when} & \widetilde{R}_B = 0 \end{array}. \tag{17}$$

Equations (14) and (17) imply that F's probability of default is an increasing step function of F's leverage and that the precise functions for periods 1 and 2 are

$$\begin{array}{cc} \mathbf{L}_F & \Pr\left[D_t\right], \ \mathbf{t=1,2} \\ 0 & 0 \\ L_F \leq L_F^c & \gamma_{1t} \\ L_F > L_F^c & \gamma_{2t}+\gamma_{3t} \end{array} . \tag{18}$$

**Proposition 5:** *Provided:*

*(i)* $\delta$ *is sufficiently small,*

*(ii)* $\gamma_{1t}(r_{Bt}-r_{Lt})-(1-\gamma_{1t})P_F>0$,

*(iii)* $\left[\gamma_{1t}(r_{Bt}-r_{Lt})-(1-\gamma_{1t})P_F\right](L_{Ft}^*-L_F^c)>\gamma_{2t}P_FL_F$,

(iv) $(\gamma_{2t}/\gamma_{1t})$ is sufficiently small,

*F's optimal leverage is*

$$L_{Ft}^*=\left(\frac{\gamma_{1t}}{1-\gamma_{1t}}\right)^{\frac{1}{\delta}}(r_{Bt}-r_{Lt})^{\frac{1-\delta}{\delta}}\text{-}\frac{1+r_{Bt}}{r_{Bt}-r_{Lt}}. \tag{19}$$

*Here* $L_{Ft}^*$, $t=\{0,1\}$ *is the financial intermediary's optimal short term leverage in periods 0 and 1.*

The following proposition formulates the financial intermediary's solvency condition. If solvent a financial intermediary pays the full debt service to lenders. Otherwise he defaults and pays nothing.

**Proposition 6:** *Provided* $\delta$ *is sufficiently small the financial intermediary is solvent if and only if the two borrowers to whom he has lent are solvent.*

Proposition 6 implies that (since $L_{Ft}^*>L_F^c$) overly restrictive sufficient conditions for the two requirement in Proposition 5 are that $\gamma_{1t}$ is sufficiently large and/or $P_F$ sufficiently small. Next, we characterize F's optimal leverage.

**Proposition 7:** *The optimal leverage of a typical financial intermediary is higher*

*(i) the lower are the intermediary's risk aversion,* $\delta$, *and the default penalty,* $P_F$,

*(ii) the lower the cost of borrowing,* $r_{Lt}$,

*(iii) the higher the probability,* $\gamma_{1t}$, *that the intermediary remains solvent,*

*(iv) the higher the interest rate, $r_{Bt}$, paid by borrowers.*

# 5 The representative lender and government's bailout policy

Through pension or mutual funds the representative lender (L) splits his equity between a fully diversified portfolio of loans to financial intermediaries and the risk free asset.[17] Since, exante, all Fs have identical distributions of returns the optimal shares of loans to different Fs are all equal. The fraction invested in the risky loan portfolio to Fs is denoted $z_L$. The typical lender possesses mean-variance (or Constant Absolute Risk Aversion - CARA)[18] preferences

$$u\left(W_L\right) = -\frac{1}{\alpha}e^{-\alpha W_L}, \qquad \alpha \geq 0, \tag{20}$$

where $W_L$ is his terminal wealth in each period and $\alpha$ characterizes the degree of constant absolute risk aversion.

## 5.1 Perceived government's bailout policy

Government may repay the gross debt owed to lenders by defaulting Fs. The perceived probability that the debt service of a defaulting F is paid by government (a bailout) is denoted by $p$. The likelihood of bailout is independent across Fs debt. In case of bailout a lender receives the full debt service, $r_L$. In the presence of risk but no bailout uncertainty $p$ is unique. We use the Gilboa Schmeidler's (1989) multiple priors framework to formalize Knightian uncertainty.[19] Accordingly, in the presence of bailout uncertainty perceptions include the convex set of all possible binomial distributions characterized by $p$'s that the lenders believe to have positive mass (i.e. are considered as plausible). The lowest value of $p$ in the set is denoted by $\pi$. As will become clear below this is also the worst plausible prior

[17]The lender is representative in the sense that all lenders are identical.
[18]See Sargent (1987) pages 154-155.
[19]Knightian uncertainty is also known as "ambiguity" in modern decision theory.

from a typical lender's point of view.

The degree of uncertainty is determined by the "size" of the set of possible priors. That is, when circumstances become more uncertain (ambiguous) the set of possible priors expands to include priors that previously were considered implausible. Consequently, provided some of the set enlargement is toward lower $p$'s, the worst prior, $\pi$, is revised downward.

## 5.2 Representative lender's returns and optimization

### 5.2.1 Period 0

On one period loans taken in period 0 borrowers face only idiosyncratic risk because only individually unlucky borrowers default in period 1, implying that financial intermediaries who lend to them also face only idiosyncratic risk in that period. By contrast, since they fully diversify their loans across intermediaries, lenders face no risk at all in period 0. Consequently, and since lenders know from Equation (14) that only a fraction, $(1-\theta_{UB})^2$, of Fs will be solvent in period 1, equilibrium $r_{L1}$ includes a compensation for the average fraction of unpaid debt but no compensation for variability of this fraction, since this variability equals zero due to full diversification. Hence,

$$1 + r_{L1} = \frac{1 + r_{f1}}{1 - (1-\pi)\left[1 - (1-\theta_{UB})^2\right]} \tag{21}$$

where $(1-\pi)\left[1-(1-\theta_{UB})^2\right]$ is the probability that a lender loses the investment in a loan to a single intermediary. It is obtained as the product of the probabilities of the two following independent events: "the intermediary defaults" and "government does not reimburse the delinquent debt to the lender". As a consequence, in period 0, lenders are indifferent between investing in the standard risk free asset at rate $r_{f1}$ and between investing in loans to Fs.

### 5.2.2 Period 1

By contrast, in period 1, lenders face risk in loans to Fs in spite of their fully diversified portfolios. The reason is that the returns to lenders from loans to different Fs are correlated due to the common shock, $\widetilde{R}_A$, in the return to real investments of borrowers. As explained in the previous section a financial intermediary either pays his debt in full to lenders or fully defaults. When F defaults on the debt service, government may or may not step in and pay the delinquent debt service to a lender. Consequently the lender faces a binomial distribution of returns from lending to an individual F – he either gets the full debt service, $1+r_L$, (from F or from government) or 0. Although the bailout policy of government does not affect the binomial nature of the payoffs from a single F, it **does alter their distribution**. Since the risky portfolio of L's contains a large number of such binomially distributed loans the risky portfolio of Ls is normally distributed with a mean and a variance that depend on both economic ($q$) and political ($p$) uncertainties. Details appear in the following proposition.

**Proposition 8:** *For a given p period's 2 payoff to a lender on his fully diversified portfolio of loans, $\{\widetilde{R}_{L2}\}$, is normally distributed with mean*

$$E\left(\{\widetilde{R}_{L2}\}\right) = [p + \gamma_{12}(1-p)](1+r_{L2}) = \left[p + \left[q(1-q_1^2)+q_1^2\right](1-p)\right](1+r_{L2}) \qquad (22)$$

*and variance*[20]

$$Var\left(\{\widetilde{R}_{L2}\}\right) = (1-p)^2(1-q)(1-q_1)^2\left(q+2qq_1+q_1^2\right)(1+r_{L2})^2,$$

*where*

$$q_1 \equiv \frac{q-\theta_{LB}}{1-(\theta_{LB}+\theta_{UB})}, \qquad (23)$$

*and $\{\widetilde{R}_{L2}\}$ stands for the set of possible returns from loans to F's.*

A representative L chooses the fraction, $z_L$, of resources invested in the risky loan portfolio to Fs so as to maximize[21]

$$E\left(\widetilde{W}_L(z_L)\right) = -\frac{1}{\alpha}E\left(e^{-\alpha\widetilde{W}_L(z_L)}\right) \qquad (24)$$

[20]The variance is scaled by the term $(1+r_{L2})^2$ for reasons of space.

[21]The right hand side of Equation (24) is obtained by using a typical lender's utility function in equation (20).

in each period, where

$$\widetilde{W}_L(z_L) = z_L \widetilde{r}_L + (1 - z_L) r_f. \tag{25}$$

**Proposition 9:** *At an individual optimum, a lender allocates the fraction*

$$z_L^* (\pi, R_L, q, q_1) \cong \frac{E\left(\{\widetilde{R}_L\}\right) - (1 + r_f)}{\alpha Var\left(\{\widetilde{R}_L\}\right)} \tag{26}$$

*of each single \$ to the diversified risky portfolio of loans to Fs.*[22,23]

### 5.3 Partial equilibrium comparative statics

We now investigate the impact of less generous bailouts on the size of lenders' risky portfolios and the impact of ambiguity aversion in partial equilibrium. In the absence of bailout uncertainty government's bailout policy is characterized by a unique perceived probability, $p$, that government will pay the debt of delinquent F's to L's. A more generous (towards L's) bailout policy is characterized by a higher $p$ and a less generous bailout policy by a lower $p$. By changing the distribution of $\widetilde{r}_L$ the value of $p$ affects both the mean and the variance of lenders' risky portfolios.

**Proposition 10:** *Holding $r_{L2}$ constant a less generous bailout policy (lower $p$)*
*(i) reduces the mean return on the portfolio of loans from lenders to financial intermediaries,*
*(ii) raises the covariance between any two loans in the (fully diversified) portfolio, and therefore, the portfolio's variance,*
*(iii) Both changes reinforce each other in inducing a "flight to safety" by lenders.*

In the presence of uncertainty about $p$, and since they are averse to ambiguity in the Gilboa-Schmeidler (1989) sense, lenders behave as if the probability of bailout is the lowest

[22] This approximation is accurate for a small risk premium, $E\left(\{\widetilde{r}_L\}\right) - r_f$.

[23] An identical allocation, $z_L^* (\pi, R_L, q, q_1)$, is obtained for constant relative risk aversion (CRRA) preferences (see Merton 1971, 1973).

within the set of $p$'s with positive mass (denoted $\pi$).[24,25] Stated differently, they choose the fraction of their portfolio invested in risky loans to Fs so as to maximize expected utility under the assumption that bailout probability is $\pi$. The operational consequence of such behavior is that $p$ should be replaced with $\pi$ in propositions 8 and 10.

Propositions 10 implies that higher bailout uncertainty has two effects: Not surprisingly it lowers the expected return from the risky loan portfolio of lenders. More surprisingly, but not less importantly, it raises the correlation between loans in the portfolio which implies in turn higher variances in lenders portfolios. This result appears surprising at first blush since intuition may lead one to conclude that an increase in bailout probability, by decreasing the likelihood of default, will increase the correlation between loans' returns in the portfolio. But this intuition is mistaken. The reason is that the correlation originates uniquely from the aggregate shock whose impact operates only through the fraction of loans in the portfolio that are no bailed out. Since the impact of this fraction on the overall correlation diminishes as more intermediaries are bailed out the variance goes down. Consequently, in the limit, when bailouts are almost certain, this variance tends to zero.

Proposition 10 implies that, when due to an increase in bailout uncertainty $\pi$ decreases, lenders reduce the share of funds supplied to financial intermediaries. This conclusion plays an important role in the following general equilibrium sections.

**Proposition 11:** *Provided* $1+2r_f \geq r_L$, *L's optimal allocation to risky loans,* $z_L^*(\pi, r_L, q, q_1)$ *is increasing in* $r_L$.

It is easy to check that the sufficient condition in the proposition is satisfied for practically all normal levels of interest rates.

[24] For simplicity we use the Gilboa-Schmeidler (1989) model to capture uncertainty, however other model of uncertainty can be incorporated into our model; Klibanoff, Marinacci and Mukerji (2005) or Izhakian and Izhakian (2009a, 2009b), for example.

[25] Lowest in the sense that under this probability distribution expected utility attains its minimal value.

# 6 General equilibrium of the financial system

Given expectations about the future, general equilibrium of the financial system is characterized by two market clearing conditions. One for credit from Ls to Fs and the other for credit from Fs to Bs. These two conditions simultaneously determine $r_B$ and $r_L$ in each period. In period 1, expectations about the future only involve realizations of period's 2 returns to borrowers. As a consequence, the formulation of this equilibrium is relatively simple. But in period 0 they also involve expectations about period's 1 market clearing values of $r_B$ and $r_L$ in period 1 ($r_{B2}^e$ and $r_{L2}^e$). Those expectations are assumed to be model consistent in the sense that, in period 0, financial market participants use the information available in that period along with their knowledge of the fact that period's 1 rates will be determined by market clearing to derive $r_{B2}^e$ and $r_{L2}^e$.

## 6.1 General equilibrium in period 0

Proposition 1 implies that a borrower is insolvent in period 1 only if he is unlucky, implying (from the discussion in Section 4) that a financial intermediary defaults in period 1 if and only if at least one of his borrowers is unlucky. Being unlucky is related to B's and Fs individual fortunes rather than to the aggregate shock. Lenders are exposed to aggregate shocks and idiosyncratic shocks; aggregate shocks are relevant only in period 2 while idiosyncratic shocks are fully diversified. Hence, since lenders are fully diversified and lend to Fs for only one period, they do not face any risk in lending to them in period 0. In particular they know for sure (from Equation (14)) that a fraction $(1-(1-\theta_{UB})^2)$ of intermediaries will default in period 1. Hence, they demand a compensation only for this known with certainty fraction of defaults. Consequently, in period 0, $r_{L1}$ is determined exogenously by the condition

$$1+r_{L1}=\frac{1+r_{f1}}{1-(1-p)\left[1-(1-\theta_{UB})^2\right]}. \tag{27}$$

Actual period's 0 equilibrium conditions in the markets for loans from Ls to Fs and from Fs to Bs are given respectively by

$$M_F L_F^*(r_{B1}, r_{L1}) = M_L z_{L1}, \tag{28}$$

$$L_B^*(r_{B1}, r_{B2}^e) = \frac{M_B}{(1+r_{B1})(1+r_{B2}^e)-1} = M_F\left(1 + L_F^*(r_{B1}, r_{L1})\right). \tag{29}$$

They determine $z_{L1}$ and $r_{B1}$ as functions of $r_{L1}$ and $r_{B2}^e$. Since $L_B^*(r_{B1}, r_{B2}^e)$ also depends on period's 0 expectation of the cost of funds to borrowers, $r_{B2}^e$, in the subsequent period a full characterization of period's 0 equilibrium requires additional conditions for the determination of $r_{B2}^e$. Those conditions are provided by the hypothesis that, in period 0, agents form their perceptions about $r_{B2}^e$ and $r_{L2}^e$ by utilizing period's 1 expected market clearing conditions given their period's 0 information. Expected period's 1 equilibrium conditions in the markets for loans from Ls to Fs and from Fs to Bs are given respectively by

$$(1-\theta_{UB})^2 M_F L_F^*(r_{B2}^e, r_{L2}^e) = (1+r_{f1}) M_L z_L^*(\pi_0, r_{L2}^e, q, q_1), \tag{30}$$

$$\frac{M_B}{(1+r_{B1})(1+r_{B2}^e)-1} = (1-\theta_{UB})\begin{bmatrix} 1 + r_{B1} + (r_{B1} - r_{L1})L_F^*(r_{B1}, r_{L1}) \\ +L_F^*(r_{B2}^e, r_{L2}^e) \end{bmatrix}. \tag{31}$$

The two period's 0 market clearing conditions and the two clearing conditions expected for period 1 jointly determine $r_{B1}$, $r_{B2}^e$, $r_{L2}^e$ and $z_{L1}$. This system is recursive since the last three equations simultaneously determine the first three variables leaving the first equation for the determination of $z_{L1}$.

## 6.2 General equilibrium in period 1

The actual period's 1 market clearing conditions in the markets for loans from Ls to Fs and from Fs to Bs are given respectively by

$$(1-\theta_{UB})^2 M_F L_F^*(r_{B2}, r_{L2}) = (1+r_{f1}) M_L z_L^*(\pi_1, r_{L2}, q, q_1), \tag{32}$$

$$\frac{M_B}{(1+r_{B1})(1+r^e_{B2})-1} = (1-\theta_{UB})\left[\begin{array}{l} 1+r_{B1}+(r_{B1}-r_{L1})L^*_F(r_{B1},r_{L1}) \\ +L^*_F(r_{B2},r_{L2}) \end{array}\right]. \qquad (33)$$

Those equilibrium conditions determine the *actual* interest rates, $r_{B2}$ and $r_{L2}$ in period 1 for predetermined, values of $r_{B1}$, $r^e_{B2}$, $r_{L1}$ and of $\pi_1$. Comparing $z^*_L(\pi_0, r^e_{L2}, q, q_1)$ from Equation (30) with $z^*_L(\pi_1, r_{L2}, q, q_1)$ from Equation (32) we note that the first two arguments of those functions differ. Not surprisingly the first, which refers to the expected equilibrium, features $r^e_{L2}$, while the second, that refers to the actual equilibrium, features $r_{L2}$. Importantly, the effective bailout probabilities, $\pi_0$ and $\pi_1$, differ across the expected and the actual period's 1 equilibria. This (at this stage) notational difference is introduced in anticipation of the discussion in the next section that introduces an unanticipated increase in bailout uncertainty.

# 7 The impact of bailout uncertainty, Lehman's collapse and the reevaluation of systemic risks

This section considers the impact of an unanticipated increase in bailout uncertainty on financial markets in period 1. Recall first that $\pi_0$ is the lowest perceived bailout probability as of period 0 for **both** periods 0 and 1.[26] The fact that, as of period 0, financial market participants do not expect this probability to change in period 1 is reflected in the formulation of the expected period's 1 general equilibrium conditions for period 0 (equations (28) through (31)).

Suppose now that, following a major indication of a shift in government's bailout policy – like not rescuing Lehman — bailout uncertainty increases. In particular, the lowest perceived bailout probability with positive mass decrease from $\pi_0$ to $\pi_1$. Proposition 10 implies that, holding $r_{L2}$ constant, this change reduces the supply of funds to Fs by Ls. Application of comparative statics methods to period's 1 equilibrium conditions shows that this change triggers a general equilibrium increases in both $r_{L2}$ and $r_{B2}$ above their expected counterparts

[26]More generally $\pi_t$ stands for the lowest belief in period $t$ about the probability of governmental bailout in all relevant future periods.

– $r^e_{L2}$ and $r^e_{B2}$. Those increases raise the fraction of defaulting borrowers in period 2 and may, under some circumstances, lead to a total drying up of credit in period 1. The string of propositions in this section provides a precise formulation of these and related results

Let

$$\frac{\partial L^*_F}{\partial r_B}, \quad \frac{\partial L^*_F}{\partial r_L} \tag{34}$$

be respectively the responses of $F's$ optimal leverage to changes in $r_{B2}$ and in $r_{L2}$ and let

$$\frac{\partial z^*_{L2}}{\partial r_L}, \quad \frac{\partial z^*_{L2}}{\partial \pi} \tag{35}$$

be the responses of $L's$ optimal share of investments in the risky portfolio to changes in $r_{L2}$ and in $\pi$.

**Proposition 12:** *Given propositions 7, 10, and 11 an increase in uncertainty about governmental bailout policy, and thus a decrease in the effective probability of bailout from* $\pi_0$ *to* $\pi_1$ *leads to:*

*(i) A flight to quality by lenders (*$z_{L2}$ *goes down)*

*(ii) An increases in* $r_{L2}$ *and* $r_{B2}$ above $r^e_{L2}$ *and* $r^e_{B2}$ respectively;

*(iii) When the increase in* $r_{B2}$ *is such that* $r_{B2} > r^e_{B2} + 2y(q + q_1 - 1)(1 + r^e_{B2})$, *period's 1 credit is denied to* ***both regular*** *and unlucky borrowers, so both types of borrowers default in period 1;*

*(iv) A sufficiently large increase in* $r_{B2}$ *beyond* $r^e_{B2} + 2y(q + q_1 - 1)(1 + r^e_{B2})$, *triggers a total "financial arrest" in period's 1 credit to Bs in the sense that credit is denied to* ***all*** *borrowers.*[27]

The comparative statics impacts in Proposition 12 accord well with the flight to quality and the general increase in the cost of debt observed following the downfall of Lehman Brothers. They are consistent with the view that much of the financial market panic, and the associated arrest of financial markets, in the aftermath of this collapse was due to an increase in uncertainty about the willingness of the US government to use public funds to compensate creditors' for losses due to defaulting financial intermediaries.

[27]This term has recently been suggested by Ricardo Caballero (2010) .

Following the outbreak of the crisis one of the expost explanations offered for the fact that so many market participants failed to see it coming was that they underestimated the systemic risks due to the correlation between the returns on differnent assets. The framework of this paper suggests that underestimation of this correlation prior to the crisis may be due to the fact that, by proposition 10 and 12, optimistic bailout expectations are associated with a low perceived correlation and therefore a low variance in lenders portofolios. On this view underestimation of the covariance between returns prior to the decision not to bailout Lehman was rational in view of the optimistic beliefs held at the time about government bailout policy. But after Lehman's collapse uncertainty about bailout policy increased. This led to a more pessimistic evaluation of the likelihood of bailout raising the covariance between the returns on loans to financial intermediaries, and with it the variance of lenders' portofolios.

## 7.1 Bailout uncertainty, the banking spread and the credit spread

The banking (or intermediation) spread.is defined as the difference between the rate, $r_B$, that financial intermediaries charge to borrowers and the rate, $r_L$, that they pay to lenders. By changing the general equilibrium level of interest rates a change in bailout uncertainty also changes the banking spread. The following proposition links the spread to bailout uncertainty.

**Proposition 13:**
*(i) Generally, depending on whether* $\frac{\partial L_F^*}{\partial r_B}$ *is smaller or larger than* $\mid \frac{\partial L_F^*}{\partial r_L} \mid$, *an increase in bailout uncertainty raises or reduces the banking spread,* $r_{B2} - r_{L2}$.
*(ii) For the specification of intermediaries' objective function featured in this paper an increase in bailout uncertainty raises the banking spread,* $r_{B2} - r_{L2}$.

The credit spread is generally defined as the difference between the yield on private debt instruments and government securities Within the analytical frameork of this paper the credit spread corresponds to the difference between the rate, $r_{L2}$, charged by lenders to financial intermediaries and the risk free rate, $r_f$. The following corollary summarizes the impact of a, period's 1, increase in bailout uncertainty on the credit spread.

**Corollary to proposition 12:** *An increase in uncertainty about governmental bailout policy leads to an increase in the credit spread.*

Interestingly, recent empirical proxies of credit spreads due to Gilchrist and Zakrajsek (2011) show that credit spreads rose substantially during the 2007-2009 period. In particular, the substantial increase in CDS spreads in the immediate aftermath of Lehman's collapse in September 2008 is consistent with the view that those increases were due, in large part, to an increase in bailout uncertainty.

# 8 Exante bailout perceptions and moral hazard

## 8.1 The impact of lower bailout uncertainty (higher $\pi_0$ )

Unlike period 1 in which the demand for credit by borrowers is already predetermined, period's 0 leverage depends on the borrowing rate in that period as well as on the borrowing rate expected to prevail in period 1. In general equilibrium both of those rates, as well as the rates at which financial intermediaries borrow from lenders, depend on financial markets participants' perceptions about the likelihood of bailout. Hence, by affecting equilibrium interest rates, perceptions about the likelihood of bailout in period 0 affect the volume of leverage in financial markets. This section investigates the impact of period's 0 permanent beliefs about governmental bailout policy as summarized by the parameter $\pi_0$ on the volume and the cost of period's 0 loans to Bs as well as on expected future rates. By affecting interest levels expected to prevail in period 1 the level of bailout uncertainty currently perceived for that period affects, in turn, the volume of Bs leverage in period 0. The following proposition presents the various impacts of $\pi_0$ given, overly restrictive, sufficient conditions.

**Proposition 14:** *For model consistent expectations and provided Fs risk aversion, $\delta$, is small,* $P_F < 1$ and $(1-\theta_{UB})^2 (1 + r_{B1} - r_{L1}) > 1$, *higher permanent values of $\pi_0$ (lower bailout uncertainty) are associated with*

*(i) Overall larger levels of credit by intermediaries to borrowers.and by lenders to financial*

*intermediaries in period 0.*

*(ii) Lower levels of* $r^e_{B2}$ *and of* $r^e_{L2}$.

*(iii) A higher level of* $r_{B1}$

The results of Proposition 14 arise through several interconnected channels. Proposition 8-10 imply that perceptions of a more generous bailout policy directly raises the fraction of their portfolio that lenders are expected to invest in risky loans to financial intermediaries in period 1. This effect exerts **downward** pressures on the expected future rates, $r^e_{B2}$ and $r^e_{L2}$. Since

$$L^*_B = \frac{1}{(1 + r_{B1})(1 + r^e_{B2}) - 1} \tag{36}$$

this **raises**, given $r_{B1}$, the demand for leverage by Bs. This higher demand for leverage raises $r_{B1}$ and this **reduces** the demand for leverage by Bs. However, as suggested by proposition 14 the first effect always dominates implying that, ultimately, lower exante bailout uncertainty perceived for period 1 induces a credit expansion already in period 0.

Clearly, the belief that government may repay the debt of some delinquent financial intermediaries creates a moral hazard problem. An important implication of Proposition 14 is that, by raising leverage in the economy, the perception of a more generous bailout policy aggravates this problem and increases the likelihood and the severity of a potential financial crisis in period 1.

## 8.2 The impact of a temporary expansionary monetary policy

Within the context of the model a temporary expansionary monetary in period 0, policy takes the form of a decrease in $r_f$ holding the borrowing rate expected for period 1, $r^e_{B2}$, constant. The following proposition summarizes the impact of such a policy.

**Proposition 15:** *A temporary decrease in the risk free policy rate,* $r_{f1}$*, leads to a decrease in both* $r_{B1}$ *and* $r_{L1}$*, and to an increase in leverage within both the financial and the real sectors (both* $L^*_F(r_{B1}, r_{L1})$ *and* $L^*_B$ *go up).*

When the period's 0 decrease in $r_f$ is permanent in the sense that it is expected to last also through period 1 there is a further expansionary effect on the equilibrium volume of credit in period 0. This effect operates through the same channels that an increase in $\pi_0$ does. That is, by reducing expected future rates, a lower expected future policy rate induces further increases in the current volume of credit.

**Broad interpretation of propositions 14 and 15:** The subprime crisis counterpart of period 0 in the model can be thought of as the buildup phase of the crisis. During this phase market participants believed that the set of bailout probabilities with positive mass is concentrated in a range with relatively high values of $p$. In addition, monetary policy was loose by historical standards. Propositions 14 and 15 imply that both factors contributed to the exante expansion of credit and to a real investment boom, making the system more fragile to a sudden downward revision of perceptions about the likelihood of governmental bailouts.[28]

# 9 Should government commit exante to a particular bailout policy?

This section briefly reflects on the desirability (or undesirability) of bailout uncertainty. The discussion in the previous section suggests that higher **exante** bailout uncertainty reduces the volume of leverage and with it the level of real investment activity undertaken by borrowers. The discussion in section 7 suggests that higher **expost** (after long term investment decisions have been made) bailout uncertainty leads to higher levels of defaults and, in parallel, to the destruction of existing investments by borrowers. The first result opens the door for the conclusion that some exante bailout uncertainty may be desirable since it keeps the

[28]It is noteworthy that within four to six months after the panic that followed Lehman's collapse CDS spreads returned to subsantially lower levels. In terms of the model this is consistent with the view that, after the TARP legislation was finally passed in early October 2008 and actually implemented in the following months along with a dramatic expansion of monetary policy, bailout uncertainty receded in the sense that the worst bailout probability with positive mass increased relatively to its level during the second half of September 2008. The broader implication of this observation is that government actions can reduce as well as increase bailout uncertainty.

buildup of credit in check thereby alleviating potential future burdens on taxpayers as well as potential defaults. But, in the spirit of the "lender of last resort" view, the second result provides an argument for reducing bailout uncertainty expost in order to avoid bankrupcies and the associated destruction of capital and economic activity It would therefore appear at first blush that, although higher bailout uncertainty is disruptive expost, it may be desirable exante.

This begs the question of whether the optimal level of bailout uncertainty (or ambiguity) may be internal. Although answering this important question is largely beyond the scope of this paper it is briefly touched upon at the end of this section. But the analysis in the two previous sections does implies that, whatever the optimal level of bailout uncertainty, over time changes in both directions in this level are inefficient. This statement is based on the view that welfare in the economy is higher the lower is the volume of investments destroyed or missed due to errors in evaluating bailout uncertainty. As shown in the previous two sections an increase in bailout uncertainty between periods 0 and 1 leads to period's 0 investment and leverage levels in excess of what those levels would have been in the absence of the change, and consequently to defaults and capital destruction in period 1 in excess of their levels in the absence of the change. Hence, given the initial level of bailout uncertainty, welfare is higher in the absence of the change. When bailout uncertainty goes down period's 0 investments by borrowers are lower than what they would have been, had they known in advance that period's 1 bailout uncertainty will be lower and the level of investments in the economy is suboptimally low.

We now come back to the question regarding the desirability of bailout uncertainty. One way to reduce this uncertainty is to have government commit to a particular probability of bailout, $p_c$. Such a policy, if successful, may eliminate multiple priors beliefs from the minds of individuals. But whether this encourages or discourages leverage exante depends on whether $p_c$ is higher or lower than the minimal multiple priors entertained by individuals prior to the commitment. Be that as it may one advantage of such a commitment is that it makes government's control over the public's beliefs tighter.

## 10 Concluding remarks

A major result of our analysis is that the larger the change in bailout uncertainty the stronger the pre-crisis buildup and the deeper the ensuing crisis. The detailed mechanics of this result can be appreciated by thinking of period 0 as the pre-crisis phase during which the worst scenario perceived likelihood of bailout is high and monetary policy relatively loose leading to credit expansion and to an investment boom. Taylor (2009) argues that loose monetary policy caused, prolonged and worsened the financial crisis. Period 1 can be thought of as the phase in which, due to the arrival of some major public signal — like not rescuing Lehman — financial market operators adjust their worst scenario perceptions about the likelihood of bailout downward. By Proposition 12 this adjustment induces a general increase in market interest rates, a rise in the proportion of insolvent borrowers along with the destruction of real investments and, for some realizations of real returns, a complete drying up of short term credit markets – or in Caballero's (2010) terminology – a sudden financial arrest.

By Proposition 14 the pre-crisis bubbly credit boom is larger the larger $\pi_0$ . By Proposition 12 the magnitudes of deleveraging and of insolvencies (real and financial) is larger the lower is $\pi_1$ . Since it measures the extent to which the set of possible bailout distributions widened between periods 0 and 1 the difference $\pi_0 - \pi_1$ is a natural proxy for the increase in bailout uncertainty. Combining this proxy with Propositions 14 and 12 yields the conclusion that higher changes in bailout uncertainty are associated with larger pre-crisis bubbles as well as with higher levels of insolvencies and destruction of real economic activity when the bubble bursts. The crucial variable through which those effects operate is leverage. It expands more during periods of optimism about the likelihood of bailouts but, by the same token, it shrinks more violently during periods of pessimism about the likelihood of bailouts. Given $\pi_1$ the deleveraging process during period 1 involves a larger volume of insolvencies the larger is $\pi_0$ . The reason is that a larger $\pi_0$ raises the exante leverage buildup in comparison to what market operators would have engaged in, had they known already in period 0 that the probability of bailout in period 1 will drop to $\pi_1$ . The larger this "excessive" credit buildup, the larger the expost volume of insolvencies in the real economy.

The paper shows that the perceptions of market participants about systemic risks as captured by the covariance between the returns on different loans within the fully diversified portofolio of lenders is systematically related to bailout uncertainty. In particular, even within a fully rational world, a higher level of bailout uncertainty is associated with a higher level of perceived systemic risks (propositions 10 and 12).

Somewhat analogously to Diamond and Dybvig (1983) (DD in the sequel) classic model of bank runs, a main objective of this paper was to identify circumstances that trigger a financial crisis. A main result of the DD framework is that deposit insurance eliminates runs on the banks. Although there is an analogy between the role of deposit insurance in DD and bailouts in our framework, a crucial difference between them is that, up to a given limit, deposit insurance is backed by the exante certainty of a legal act while the availability (and scope) of the generalized bailouts considered here is shrouded in uncertainty and is likely to remain in this state also in the future. Besides other obvious differences two additional difference worth emphasizing are: (i) In DD liquidity shocks are exogenous while here they are related to an increase in uncertainty due to the arrival of new information about "black swan" events. (ii) Our framework features a more detailed picture of the financial sector and is designed to make statements about the impact of monetary policy on leverage and the economy.

Reinhart and Rogoff (2009) present broad evidence supporting the view that private financial crises are often followed by substantial reductions in tax collections and defaults on sovereign debt. Motivated by this findings and some of the results in this paper we speculate in what follows on an additional channel through which higher exante leverage buildups possibly makes the economy more crisis prone when new information arrives. Higher leverage raises the probability as well as the magnitude of potential defaults, and with it the cost of potential bailouts. The more costly is a bailout to taxpayers the more reluctant is government to engage in such bailouts. As a consequence, the higher is leverage, the higher bailout uncertainty making beliefs more sensitive to news.

The punch line is that the sensitivity of expectations to various news becomes larger the larger is leverage. In terms of the Gilboa-Schmeidler (1989) uncertainty framework this means that the range of bailout probability distributions entertained by individuals becomes more sensitive to news. As a consequence, the same pessimistic new information about the likelihood of bailout is more likely to puncture a bubble the higher is leverage.

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# A Appendix

**Proof of Lemma 1**

For regular borrowers, and given $r_{B2} = r^e_{B2}$, the condition in Equation (4) is identical to the condition in Equation (3). But, by assumption 1a, the last condition is satisfied for all $L_B \geq 0$. Hence, regular borrowers are solvent at any level of leverage. Since $2yq > 2y\,(2q-1)$ this is afortiori true for lucky borrowers. The proof for unlucky borrowers follows by using Equation (5) in solvency condition (4) and by rearranging. QED

**Proof of Lemma 2**

The proof of parts (i) and (ii) is obtained by substituting $\widetilde{Y}_P = -2y$ and $\widetilde{Y}_P = 0$ respectively into Equation (8) and by rearranging. The proof of part (iii) follows by inserting $\widetilde{Y}_P = 2y$ into Equation (8) and by utilizing Assumption 1. QED

**Proof of Lemma 3**

Part (i) is a direct consequence of Lemma 1. To prove part (ii) note that, since $\frac{1}{2} < q < 1$, $L^L_B < L^1_B < L^H_B$. The ultimate payoff of an UB is either 0 in case of expansion or $-2y$ in case of contraction. By Lemma 2, and since $L^1_B < L^H_B$, this borrower is solvent in the first case. By Lemma 2, and since $L^L_B < L^1_B$, this borrower is insolvent in the second case. Part (iii) is a direct consequence of Assumption 1 in conjunction with condition (7). QED

**Proof of Proposition 1**

The first two default probabilities follow directly from Lemmas 1 through 3. The third probability is derived by noting that, when leverage is larger than $L^1_B$ default may occur in period 1 if the borrower turns out to be unlucky (probability $\theta_{UB}$) and may also occur in period 2 if he turns out to be a regular borrower and $\widetilde{Y}_P = -2y$ (probability $(1-\theta)(1-q)^2$). The last probability follows by noting that, in addition to the states in which he defaults in the previous case, the borrower defaults also in the following two cases: (i) If he is a LB and there is a contraction, (ii) If he is a regular borrower in and $\widetilde{Y}_P = 0$. QED

**Proof of Proposition 2**

To show that $L_B^H$ is the optimal level of leverage it suffices to establish that

$$\begin{aligned} V(L_B^H) &> V(L_B^1) > V(L_B^L) > V(L_B = 0), \\ V(L_B^H) &> V(L_B^m). \end{aligned}$$

The proof is implemented by using Equation (12) to form explicit expressions for the differences $V(L_B^L) - V(L_B = 0)$, $V(L_B^1) - V(L_B^L)$, $V(L_B^H) - V(L_B^1)$, $V(L_B^m) - V(L_B^H)$ and by showing that they are all positive.

**(i)** $V(L_B^L) - V(L_B = 0) = q^2(2y - r_B^e)L_B^L - 2q(1-q)r_B^e L_B^L$.

Assumption 1 and $q$ sufficiently close to one imply that this difference is positive.

**(ii)** $V(L_B^1) - V(L_B^L) = \{q^2(2y - r_B^e) - 2q(1-q)r_B^e\}(L_B^1 - L_B^L) - (1-q)^2 P_B L_B^1$.

Since $(L_B^1 - L_B^L) > 0$ this expression is positive for $q$ sufficiently close to one.

**(iii)** Letting $\theta_{LB}$ approach $q$ from below and $\theta_{UB}$ approach zero from above

$V(L_B^H) - V(L_B^1) = q^2(2y - r_B^e)(L_B^H - L_B^1) - 2q(1-q)(1 - r_B^e L_B^1) - (1-q)^2 P_B \left[(1-q)L_B^H - L_B^1\right]$.

Since $(L_B^H - L_B^1) > 0$ this expression is positive for $q$ sufficiently close to one.

**(iv)** The condition $V(L_B^m) - V(L_B^H) > 0$ is equivalent to

$[[(1-\theta)2q(1-q) + \theta_{LB}(1-q)]P_B - [\theta_{LB}q + (1-\theta)q^2](2y - r_B^e)]\left(L_B^m - L_B^H\right) > 0$.

Since $(L_B^H - L_B^1) > 0$ this expression is positive if and only if $P_B > P_B^c$. QED

**Proof of Proposition 3**

The proof is a direct consequence of the fact that all three types of agents are risk averse and that leverage levels are positive. Consequently, financial intermediaries require a markup over their leverage costs as compensation for investing in risky loans to investors. As a consequence $r_B > r_L$. Similarly, lenders demand a risk premium when they invest in risky loans to financial intermediaries rather than in risk free asset. Hence, $r_L \geq r_f$.[29] . QED

**Proof of Proposition 4**

By construction, since $r_f < r_L$, an F with positive short term leverage and some fraction of the portfolio invested in risk free assets can increase his profits by reducing both short term

[29] As elaborated later in the general equilibrium section, the equality sign is allowed to accommodate a special case in which, due to full diversification, lenders face no risk in period 0.

leverage and, in parallel, the investment in risk free assets. Consequently, a configuration with both positive leverage and some investment in risk free assets cannot be a financial intermediary's optimum. Hence $z_f = 0$. QED

**Proof of Proposition 5**

Equation (19) is obtained by maximizing the expected utility of F with respect to $L_F$ for $z_f = 0$ under the assumption that F's optimal leverage is above $L_F^c$. Conditions (ii) and (iii) are needed to rule out the possibility that optimal leverage is at $L_F^c$ or at zero. To derive those conditions let $L_F^m$ be any leverage level above $L_F^c$ and let.

$$EV_F\left[L_F\right] \equiv Eu\left[W_F(L_F)\right]. \tag{37}$$

Then necessary conditions for the optimal level of leverage to be above $L_F^c$ are

$$\begin{aligned} EV_F\left[L_F^m\right] &> EV_F\left[L_F = 0\right] \\ EV_F\left[L_F^m\right] &> EV_F\left[L_F^c\right] \end{aligned} \tag{38}$$

Conditions (i) and (ii) are obtained by using equations (13) and (16) to express F's utility in terms of the appropriate levels of leverage, substituting the resulting expression into Equation (38) and by rearranging. To complete the proof it remains to show that, when $(\gamma_{2t}/\gamma_{1t})$ is sufficiently small, $EV_F\left[L_F^c\right] > EV_F\left[L_F^s\right]$ for any $0 < L_F^s < L_F^c$ establishing (since $EV_F\left[L_F^m\right] > EV_F\left[L_F^c\right]$) that $EV_F\left[L_F^m\right]$ is also larger than $EV_F\left[L_F^s\right]$. When $(\gamma_{2t}/\gamma_{1t})$ is small the only two terms that could potentially make $EV_F\left[L_F^s\right]$ larger than $EV_F\left[L_F^c\right]$ involve $\gamma_{2t}$, while the terms that operate to reverse this inequality involve $\gamma_{1t}$. In the extreme case, $(\gamma_{2t}/\gamma_{1t}) = 0$, it is unambiguously the case that $EV_F\left[L_F^c\right] > EV_F\left[L_F^s\right]$. By continuity this is also the case for $(\gamma_{2t}/\gamma_{1t})$ positive but sufficiently small. QED

**Proof of Proposition 6**

The condition in Equation (17) implies that, when $L_{Ft}^* > L_F^c$, F is solvent if and only if both of his debtors are solvent. Recalling that $r_{Bt} - r_{Lt} > 0$ and inspecting Equation (19) reveals that $L_{Ft}^*$ is a monotonically increasing function of $\delta$ and that it tends to infinity when $\delta$ tends to zero. It follows that, for sufficiently small but positive values of $\delta$, $L^*_{Ft} > L^c_F$ implying that F is solvent if and only if both of his borrowers are solvent. QED

**Proof of Proposition 7**

The first three parts follow directly from inspection of Equation (19). Part (iv) is established by differentiating this equation with respect to $r_{Bt}$. QED

**Proof of Proposition 8**

Calculation of the expected value is relatively straightforward. Derivation of the variance utilizes the fact that the variance of a fully diversified risky portfolio composed of (equally weighted) infinitely many identically distributed assets is equal to the covariance between any two assets within the portfolio. Calculation of this covariance simplifies the derivation of an explicit expression for $Var\left(\{\widetilde{R}_{L2}\}\right)$ but still involves some messy intermediate algebra.

Details to be completed.

The expression for L's portfolio variance in Proposition 8 is obtained by using the joint distribution of $(\widetilde{r}_{Li}, \widetilde{r}_{Lj})$ in the definition of the covariance between $\widetilde{r}_{Li}$ and $\widetilde{r}_{Lj}$ and by simplifying using the Mathematica software. QED

**Proof of Proposition 9**

Recall that, to keep notation simple, we use the symbol $\widetilde{R}_L$ to denote the gross (one plus) return on a portfolio that consist of an infinite number of loans, $\{\widetilde{r}_L\}$, and $\widetilde{R}_f = 1 + r_f$ is the gross risk free rate. A typical lender's maximization problem is given by

$$\max_{z_L} E\left[u\left(z_L(\widetilde{R}_L - R_f) + R_f\right)\right],$$

where $u(\cdot)$ stands for the utility function. The first order condition implies

$$E\left[u'\left(z^*_L(\widetilde{R}_L - R_f) + R_f\right)(\widetilde{R}_L - R_f)\right] = 0.$$

Taking a Taylor approximation of the marginal utility with respect to $\widetilde{R}_L$ around $R_f$ yields

$$E\left[u'(R_f)\left(\widetilde{R}_L - R_f\right)\right] + E\left[u''(R_f)\left(\widetilde{R}_L - R_f\right)\left(\widetilde{R}_L - R_f\right)z_L^*\right] \cong 0.$$

For a sufficiently small risk premium

$$z_L^* \cong -\frac{u'(R_f)\left(E\widetilde{R}_L - R_f\right)}{u''(R_f)\,E\left[\left(\widetilde{R}_L - R_f\right)^2\right]} = \frac{\left(E\widetilde{R}_L - R_f\right)}{-\frac{u''\left(R_f\right)}{u'\left(R_f\right)}Var\left(\widetilde{R}_L\right)},$$

but for constant absolute risk aversion, $u(x) = -e^{-\alpha x}$, the coefficient of absolute risk aversion is $-\frac{u''\left(R_f\right)}{u'\left(R_f\right)} = \alpha$, and thus $z_L^* \cong \frac{E(\{1+\widetilde{r}_L\})-\left(1+r_f\right)}{\alpha Var(\{\widetilde{r}_L\})}$. QED

**Proof of Proposition 10**

Part (i) follows immediately form Equation (22).

Part (ii) The first derivative of $Var\left(\{\widetilde{R}_{L2}\}\right)$ with respect to $p$ is

$$\frac{\partial Var\left(\{\widetilde{R}_{L2}\}\right)}{\partial q} = -2(1-p)(1-q)(1-q_1)^2\left(q + 2qq_1 + q_1^2\right),$$

which is negative since $p, q \in [0,1]$ are probabilities.

Part (iii) follows immediately form Equation (22). QED

**Proof of Proposition 11**

Writing Equation (26) as

$$z_L^*(\pi, r_L, q, q_1) \cong \frac{E\left(\{\widetilde{R}_L\}\right) - (1+r_f)}{\alpha Var\left(\{\widetilde{R}_L\}\right)} = \frac{\lambda_1(1+\widetilde{r}_L) - (1+r_f)}{\alpha\lambda_2(1+\widetilde{r}_L)^2} = \frac{1}{\alpha\lambda_2}y,$$

where $\lambda_1$ and $\lambda_2$ are determined by equations (22) and (**??**), respectively. Differentiating $y$ with respect to $\widetilde{r}_L$ yields

$$\frac{\partial y}{\partial \widetilde{r}_L} = \frac{-\lambda_1(1+\widetilde{r}_L) + 2(1+r_f)}{(1+\widetilde{r}_L)^3}.$$

Since it is a probability $\lambda_1 \in [0,,1]$. Hence, the derivative is positive for $\frac{2(1+r_f)-\lambda_1}{\lambda_1} \geq 1+2r_f \geq \widetilde{r}_L$. QED

**Proof of Proposition 12**

Part (i) is an immediate consequence of proposition 10.

Part (ii): Differentiating equations (32) and (33) totally with respect to $\pi$ yields

$$0 = (1-\theta_{UB})M_F \left[\frac{\partial L_F^*}{\partial r_B}\frac{dr_{B2}}{d\pi} + \frac{\partial L_F^*}{\partial r_L}\frac{dr_{L2}}{d\pi}\right], \tag{39}$$

$$(1-\theta_{UB})^2 M_F \left[\frac{\partial L_{F2}^*}{\partial r_B}\frac{dr_{B2}}{d\pi} + \frac{\partial L_{F2}^*}{\partial r_L}\frac{dr_{L2}}{d\pi}\right] = (1+r_{L1})M_L \left[\frac{\partial z_{L2}^*}{\partial r_L}\frac{dr_{B2}}{d\pi} + \frac{\partial z_{L2}^*}{\partial \pi}\right]. \tag{40}$$

Solving this two equations system for $\frac{dr_{B2}}{d\pi}$ and $\frac{dr_{L2}}{d\pi}$

$$\frac{dr_{B2}}{d\pi} = -\frac{\frac{\partial L_{F2}^*}{\partial r_L}\frac{\partial z_{L2}^*}{\partial \pi}}{\frac{\partial L_{F2}^*}{\partial r_B}\frac{\partial z_{L2}^*}{\partial r_L}}, \tag{41}$$

$$\frac{dr_{L2}}{d\pi} = -\frac{\frac{\partial z_{L2}^*}{\partial \pi}}{\frac{\partial z_{L2}^*}{\partial r_L}}. \tag{42}$$

By Proposition 7, $\frac{\partial L_{L2}^*}{\partial r_B} > 0$ and $\frac{\partial L_{L2}^*}{\partial r_L} < 0$. By Proposition 10 and 11, $\frac{\partial z_{L2}^*}{\partial \pi}$ and $\frac{\partial z_{L2}^*}{\partial r_L}$ are both positive. Utilization of those sign restrictions in equations (41) and (42) implies that the general equilibrium effects of a surprise decrease in $\pi$ is to raise both $r_{B2}$ and $r_{L2}$ above what those rates had been expected to be in period 0 ($r_{B2}^e$ and $r_{L2}^e$).

Part (iii): Although Assumption 2 requires that $E\left[\widetilde{Y}_P \mid I_1 \cap RB\right] = 2y([q+q_1-1] > r_B^e \equiv (1+r_{B1})(1+r_{B2}^e)-1$ the condition in part (ii) of this proposition implies that it is violated when $r_B^e$ is replaced with $r_B$, or in explicit notation

$$E\left[\widetilde{Y}_P \mid I_1 \cap RB\right] = 2y([q+q_1-1] < r_B \equiv (1+r_{B1})(1+r_{B2})-1. \tag{43}$$

By equation (4) adapted to **actual** period's 1 information a RB is solvent in period 1 if and only if

$$L_B(1+r_B^e) \le (1+L_B)\left[1+E\left[\widetilde{Y}_P \mid I_1 \cap RB\right]\right],$$

which is equivalent to

$$L_B \le \frac{1+2y([q+q_1-1]}{r_B - 2y([q+q_1-1]} \equiv L_B^1(r_{B1}), \tag{44}$$

where the denominator is positive by condition (43). It follows that regular borrowers do not get refinancing in period 1 if

$$L_B^* = L_B^H = \frac{1}{r_B^e} > \frac{1+2y([q+q_1-1]}{r_B - 2y([q+q_1-1]} \equiv L_B^1(r_{B1}). \tag{45}$$

Rearrangement of this inequality reveals that it is equivalent to the condition in part (ii) of the proposition establishing that RB default. Given that RB default under $r_{B2}$ UB default afortiori.

Part (iv) When $r_{B2}$ increases sufficiently beyond the bound in part (ii) even LB are denied access to credit inducing a total drying up of refinancing to borrowers. QED

**Proof of proposition 13**

Part (i): The banking spread in period 2 is

$$S_2 \equiv r_{B2} - r_{L2}. \tag{46}$$

Differentiating the spread totally with respect to $\pi_1$

$$\frac{dS_2}{d\pi_1} = \frac{dr_{B2}}{d\pi_1} - \frac{dr_{L2}}{d\pi_1} = \left(1 - \frac{\mid \frac{\partial L_{F2}^*}{\partial r_L} \mid}{\frac{\partial L_{F2}^*}{\partial r_B}}\right) \mid \frac{dr_{L2}}{d\pi_1} \mid \tag{47}$$

where the second equality follows by using equations (41) and (42). Since $\frac{\partial L_{F2}^*}{\partial r_B} > 0$, $\frac{dS_2}{d\pi_1}$ is negative or positive depending on whether $\mid \frac{\partial L_{F2}^*}{\partial r_L} \mid$ is larger or smaller than $\frac{\partial L_{F2}^*}{\partial r_B}$. Hence an increase in bailout uncertainty (a decrease in $\pi_1$) raises the spread in the first case and reduces it in the second.

Part (ii): Examination of the expression for $L_{F2}^*$ in proposition 5 reveals that $\mid \frac{\partial L_{F2}^*}{\partial r_L} \mid$and

$\frac{\partial L^*_{F2}}{\partial r_B}$ may differ only because of the second term on the right hand side of equation (19). Using this fact along with that equation yields after some algebra

$$| \frac{\partial L^*_{F2}}{\partial r_L} | - \frac{\partial L^*_{F2}}{\partial r_B} = \frac{r_{B2} - r_{L2}}{S_2^2}.$$

Since $r_{B2} - r_{L2} > 0$, $| \frac{\partial L^*_{F2}}{\partial r_L} | > \frac{\partial L^*_{F2}}{\partial r_B}$ implying, by part (i), that an increase in bailout uncertainty raises the banking spread. QED

**Proof of proposition 14**

The 3 equations system in (29), (30) and (31) determines $r^e_{B2}$, $r^e_{L2}$ and $r_{B1}$ as functions of $\pi_0$ and other exogenous variables. Differentiating this system totally with respect to $\pi_0$ and using the total differential of the first of those equations to express $\frac{dr_{B1}}{d\pi_0}$ in terms of $\frac{dr_{B2}}{d\pi_0}$ yields

$$\frac{dr_{B1}}{d\pi_0} = \frac{dr_{B1}}{dr^e_{B2}} \frac{dr^e_{B2}}{d\pi_0} \tag{48}$$

where

$$\frac{dr_{B1}}{dr^e_{B2}} = -\frac{M_B \frac{\partial L^*_B}{\partial r^e_B}}{M_B \frac{\partial L^*_B}{\partial r_B} - M_F \frac{\partial L^*_{F1}}{\partial r_B}}. \tag{49}$$

Equation (36) implies $\frac{\partial L^*_B}{\partial r^e_B} < 0$, $\frac{\partial L^*_B}{\partial r_B} < 0$ and, from proposition 7 $\frac{\partial L^*_{F1}}{\partial r_B} > 0$. Hence $\frac{dr_{B1}}{dr^e_{B2}}$ is negative implying that $\frac{dr_{B1}}{d\pi_0}$ and $\frac{dr_{B2}}{d\pi_0}$ have opposite signs. Differentiating $L^*_B$ totally with respect to $\pi_0$ and using equation (49)

$$\frac{dL^*_B}{d\pi_0} = \left\{ -\frac{M_F \frac{\partial L^*_{F1}}{\partial r_B} \frac{\partial L^*_B}{\partial r^e_B}}{M_B \frac{\partial L^*_B}{\partial r_B} - M_F \frac{\partial L^*_{F1}}{\partial r_B}} \right\} \frac{dr^e_{B2}}{d\pi_0}. \tag{50}$$

Inspection reveals that the term in curly brackets on the right hand side of this equation is negative implying that

$$\mathrm{Si}\,gn \left( \frac{dL^*_B}{d\pi_0} \right) = -\,\mathrm{Si}\,gn \left( \frac{dr^e_{B2}}{d\pi_0} \right) \tag{51}$$

Substituting equation (49) into the total differentials of equations (30) and (31) and rearranging yields the following two equations system for the determination of $\frac{dr^e_{B2}}{d\pi_0}$ and of

$\frac{dr^e_{L2}}{d\pi_0}$.

$$\begin{aligned} M_F(1-\theta_{UB})^2\frac{\partial L^e_{F2}}{\partial r^e_B}\frac{dr^e_{B2}}{d\pi_0}+\left(M_F(1-\theta_{UB})^2\frac{\partial L^e_{F2}}{\partial r^e_L}-M_L(1+r_{f1})\frac{\partial z^e_{L2}}{\partial r^e_L}\right)\frac{dr^e_{L2}}{d\pi_0} &= M_L(1+r_{f1})\frac{\partial z^e_{L2}}{\partial \pi_0} \\ \left\{\begin{array}{c} M_F(1-\theta_{UB})\left(W_{r_{B1}}\frac{dr_{B1}}{dr^e_{B2}}+\frac{\partial L^e_{F2}}{\partial r^e_B}\right) \\ -M_B\left(\frac{\partial L^*_B}{\partial r_B}\frac{dr_{B1}}{dr^e_{B2}}+\frac{\partial L^*_B}{\partial r^e_B}\right)\end{array}\right\}\frac{dr^e_{B2}}{d\pi_0}+M_F(1-\theta_{UB})\frac{\partial L^e_{F2}}{\partial r^e_L}\frac{dr^e_{L2}}{d\pi_0} &= 0 \qquad (52) \end{aligned}$$

where $L^e_{F2}$ and $z^e_{L2}$ are the model consistent expectations of those variables given the information set of period 1 and $W_{r_{B1}}$ is the derivative of $\{1+r_{B1}+(r_{B1}-r_{L1})L^*_F(r_{B1},r_{L1})\}$ with respect to $r_{B1}$. To evaluate the signs of $\frac{dr^e_{B2}}{d\pi_0}$ and of $\frac{dr^e_{L2}}{d\pi_0}$ it is convenient to utilize the following claim that is proved later.

**Claim**: Provided $\delta$ is positive but close to zero, $P_F<1$ and $(1-\theta_{UB})^2(1+r_{B1}-r_{L1})>1$ the impact of a general equilibrium change in $r^e_{B2}$ on $r_{B1}$ (i.e. $\frac{dr_{B1}}{dr^e_{B2}}$) is relatively small.

Using the claim in equations (52) and solving for $\frac{dr^e_{B2}}{d\pi_0}$ and $\frac{dr^e_{L2}}{d\pi_0}$

$$\begin{aligned} \frac{dr^e_{B2}}{d\pi_0} &\cong \frac{\frac{\partial L^e_{F2}}{\partial r^e_L}\frac{\partial z^e_{L2}}{\partial \pi_0}}{\frac{\partial z^e_{L2}}{\partial r^e_L}\frac{\partial L^e_{F2}}{\partial r^e_B}}<0 \\ \frac{dr^e_{L2}}{d\pi_0} &\cong -\frac{\left(M_F(1-\theta_{UB})\frac{\partial L^e_{F2}}{\partial r^e_B}-M_B\frac{\partial L^*_B}{\partial r^e_B}\right)\frac{\partial z^e_{L2}}{\partial \pi_0}}{M_F(1-\theta_{UB})\frac{\partial z^e_{L2}}{\partial r^e_L}\frac{\partial L^e_{F2}}{\partial r^e_B}}<0. \qquad (53) \end{aligned}$$

The negative signs of those expressions follow by noting that, from proposition 7, $\frac{\partial L^e_{F2}}{\partial r^e_B}>0$, $\frac{\partial L^e_{F2}}{\partial r^e_L}<0$; from propositions 10 and 11 $\frac{\partial z^e_{L2}}{\partial \pi_0}$ and $\frac{\partial z^e_{L2}}{\partial r^e_L}$ are both positive; and from equation (36) $\frac{\partial L^*_B}{\partial r^e_B}<0$. This establishes part (ii). The fact that $L^*_B$ is higher follows from equation (51) in conjunction with the fact that $\frac{dr^e_{B2}}{d\pi_0}<0$. This implies via equations (30) and (29) that $L^*_F(r_{B1},r_{L1})$ and $z_{L1}$ are also higher when $\pi_0$ is higher establishing part (i). Part (iii) follows from part (ii) and equation (48) by noting that $\frac{dr_{B1}}{dr^e_{B2}}$ is negative. This completes the proof of the proposition provided the claim is true.

**Proof of claim:** Noting that $\gamma_{11}=(1-\theta_{UB})^2$ and differentiating equation (19) with respect to $r_{B1}$

$$\frac{\partial L^*_{F1}}{\partial r_B} = \frac{1-\delta}{\delta}\left\{\frac{1}{P_F}\frac{(1-\theta_{UB})^2}{1-(1-\theta_{UB})^2}(r_{B1}-r_{L1})^{1-2\delta}\right\}^{\frac{1}{\delta}} + \frac{1+r_{L1}}{(r_{B1}-r_{L1})^2}.$$

Under the conditions of the proposition the term in curly parenthesis on the right hand side of this equation is larger than one. Consequently, when $\delta$ tends to zero from above $\frac{\partial L^*_{F1}}{\partial r_B}$ tends to infinity implying, by equation (49), that $\frac{dr_{B1}}{dr^e_{B2}}$ becomes very small. This completes the proof of the claim and of the proposition.

QED

**Proof of Proposition 15**

Translated into the model's timing framework a temporary decrease in the risk free rate means that $r_{f1}$ goes down without any change in $r^e_{f2}$. The decrease in $r_{f1}$ triggers, via equation (27), a decrease in $r_{L1}$, and by part (ii) of proposition 7, an increase in $L^*_F(r_{B1}, r_{L1})$. This increase translates through equilibrium condition (28) into an increase in $z_{L1}$. Furthermore, the increase in $L^*_F(r_{B1}, r_{L1})$ induces, via equilibrium condition (29), a decrease in $r_{B1}$ and an increase in $L^*_B$. QED