Price Competition with Consumer Confusion*

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Abstract

This paper proposes a model in which identical sellers of a homogenous product compete in both prices and price frames (i.e., ways to present price information). Frame choices affect the comparability of price offers, and may lead to consumer confusion. In the symmetric equilibrium price and price frame dispersion coexist and firms make positive profits. Moreover, the nature of equilibrium depends on whether frame differentiation or frame complexity is more confusing, and an increase in the number of firms can raise industry profits and harm consumers.

Keywords: bounded rationality, framing, oligopoly markets, frame dispersion, price dispersion

JEL classification: D03, D43, L13

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1 Introduction

Sellers use various ways to convey price information to consumers. Price promotions are often framed differently using, for instance, a direct price reduction, a percentage discount, a multi-unit discount, or a voucher.[1] Some restaurants, hotels, or online booksellers offer a single price, while others divide the price by quoting the table service, the breakfast, the internet access, or the shipping fee separately. A search for a popular textbook at a book price comparison website led to the following results:

	Price	Shipping fee
Books Express	£32.78	£4.75
Quartermelon	£32.97	£4.32
Blackwell Online	£38.95	Free

Prices of Jean Tirole's "The Theory of Industrial Organization"

www.abebooks.co.uk (November 15, 2009)

Similarly, airlines and travel agencies present differently the fees they charge for card payment. For example, Wizz charges a £4 flat fee per person, while Virgin Atlantic charges 1.3% of the total booking.[2] Retailers offer store cards with diverse terms such as "10% off first shop if opened online or 10% off for first week if opened in store", "500 bonus points on first order", or "£5 voucher after first purchase". Financial product prices are also often framed distinctively: mortgages might have the arrangement fees rolled in the interest rate or not; some loans may specify the "repayment amount", while others the "APR".

Despite the prevalence of price framing, the practice has received little attention in the economic literature. There is no explanation why different firms employ different price frames or why the same firm changes its price frame over time (as in the case of supermarket discounts). If firms use different price presentation modes to compete better for consumers, industry-specific pricing schemes whose terms allow for better comparisons should emerge. But, the persistence of much variation in price frames seems more likely to confuse consumers and harm competition. In addition, as the above examples suggest, markets with price frame dispersion often exhibit price dispersion, too.

To address this phenomenon, we consider a model in which firms supply a homogeneous product and choose both price frames and prices simultaneously. We assume that

[1] For example, to buy a 50 ml whitening toothpaste in a grocery store one can choose between Macleans sold at £2.31 with a "buy one get one free" offer and Aquafresh which "was £1.93 now is £1.28 saves 65p".

[2] See "Calls for airline charges clean-up" on BBC News on July 17, 2009 (*http://news.bbc.co.uk)*.

price framing can confuse consumers and, as a result, they fail to identify the best available deal in the market.[3] More specifically, if consumers get confused by price frames, they choose one product randomly; otherwise, they behave rationally. Consumers might be confused either by *frame differentiation* (i.e., they fail to compare prices in different frames) or by *frame complexity* (i.e., they fail to compare prices in a common complex frame). The marketing literature has provided relevant evidence that consumers have difficulties in comparing prices which are presented differently or prices which are complicated (see, for instance, Estelami, 1997, Morwitz et al., 1998, and Thomas and Morwitz, 2009). In the economic literature, Kalayci and Potters (2011), and Kalayci (2010) document experimental evidence that increasing the number of product attributes or price scheme dimensions can create confusion and lead to suboptimal consumer choices.[4]

In the market, sometimes frame differentiation is a main source of confusion. Comparing two prices both including the VAT or both excluding the VAT (given that the same percentage tax applies) is easy, but comparing a price excluding VAT with an all-inclusive one might be more difficult. Similarly, variation in grocery price promotions might make it harder for consumers to compare the actual prices, although each discount method is not particularly involved. Other times, frame complexity is also a source of consumer confusion, and it may even be the dominant one. Comparing offers which quote separately the shipping fees might be confusing if the applicable fees differ across sellers. The same is true in the mortgage market where deals with the service fee quoted separately are usually harder to compare than deals with the service fees rolled in the interest rate. Much evidence suggests that consumers do not understand well the prices when the sellers use complex price frames (which involve many elements or pieces of information) in markets such as financial services or electricity and gas.[5] This paper considers both sources of confusion.

[3] Research in psychology has long recognized framing effects in decision making (for instance, Tversky and Kahneman, 1981). People's responses to essentially the same decision problem may differ systematically if the problem is framed differently. In this paper, we focus on frames as price presentation modes and on their ability to cause confusion in price comparison.

[4] An emerging empirical literature on price framing effects investigates how shrouding a price component or making it less salient affects consumers' perception of prices and their choice behavior. See, for instance, Hossain and Morgan (2006), Chetty, Looney, and Kroft (2009), and Brown, Hossain, and Morgan (2010).

[5] An EU study of mortgage markets states that "even if consumers do have the relevant information [to make a decision] they do not necessarily understand it". (See the "White paper on the integration of EU mortgage credit markets", 2007.) Research on the gas and electricity market in UK by the consumer organization *Which?* says that "complex tariff structures made it very difficult for consumers to understand what type of deal they were on and how to reduce energy use and costs". (See "Customers confused by energy tariffs" at *http://www.which.co.uk/news* on May 7, 2009.)

We address the following questions: Can price frame dispersion and price dispersion coexist, as we observe in many markets? Do market outcomes depend on whether frame differentiation or frame complexity is more confusing? In the presence of price framing, does an increase in the number of competitors reduce market complexity and enhance consumer welfare?

In Section 2, we illustrate the coexistence of frame and price dispersion in a duopoly setting. Intuitively, if the market is relatively transparent (for instance, when both firms use a simple frame), a firm has an incentive to create more confusion and take advantage of confused consumers. While if the market is already confusing enough (for instance, when firms use different frames), a firm has an incentive to reduce confusion through its frame choice and undercut its rival. Due to this conflict, in any equilibrium firms mix on frames. Then, as the firms face both price aware buyers (who compare prices perfectly) and confused buyers (who shop at random) with positive probability, they will also mix on prices in equilibrium.[6] We also show that the nature of equilibrium depends on which source of confusion dominates. If frame complexity is more confusing than frame differentiation, then the more complex frame is always associated with higher prices. In contrast, if frame differentiation is more confusing than complexity, there is no clear monotonic relationship between the prices associated with different frames.

Section 3 studies the oligopoly model and shows that when the number of firms increases, with a limited number of frames, the ability of frame differentiation to reduce price competition decreases, and firms rely more on frame complexity. In particular, in fragmented oligopolies, firms tend to use complex frames almost surely. Consequently, industry profits are bounded away from zero regardless of the number of competitors. A decrease in concentration has the usual positive effect (pressure down on prices), but also a negative effect (higher market complexity and less competitive pressure). When the latter effect dominates, an increase in the number of firms boosts industry profits and harms the consumers. Therefore, when firms compete in both prices and frames, a competition policy which focuses exclusively on increasing the number of firms might have undesired effects. Section 4 discusses robustness issues and alternative interpretations of our model (such as product framing/differentiation, and confusion as a result of costly information processing).

Our analysis predicts that, if frame differentiation is the dominant confusion source, there is no clear ranking (on average) among prices associated with different frames.

[6] In the price dispersion literature (see the survey by Baye et al., 2006) mixed-strategy equilibria are associated with both cross-sectional and intertemporal variation. Some of our motivating examples relate to cross-sectional frame dispersion (e.g., the restaurant or online bookstore cases), while others could be linked to intertemporal frame variation (e.g., the supermarket case).

For example, there should be no significant price differences across different discounting methods. This is an empirically testable result and seems consistent with casual observations of grocery store promotions, for instance. In markets where frame complexity is the dominant source of confusion, our model predicts that the more complex frame is always associated with higher prices. Woodward (2003), and Woodward and Hall (2010) provide evidence that, in the mortgage market, the deals with the arrangement fees rolled in the interest rate are on average better than the deals with separate fees.[7] Our model also predicts that, in markets where some frames are more complex than others, an increase in the number of firms can increase prices and harm consumers. Hortaçsu and Syverson (2004) document evidence that in the S&P index fund market where multi-dimensional fee schemes prevail, a decrease in concentration between 1995-99 indeed triggered an increase in the average price.

An emerging economic literature investigates price complexity and firms' intentional attempts to degrade the quality of information to the consumers. Ellison and Ellison (2009) provide empirical evidence on retailers' use of obfuscation strategies in online markets. They show, for instance, that retailers deliberately create more confusing websites to make it harder for the consumers to figure out the total price. On the theoretical side, one stream of literature adopts the information search framework (e.g., Carlin, 2009, and Ellison and Wolitzky, 2008). If it is more costly for consumers to assess complex prices, firms may have incentives to increase price complexity and thereby reduce consumers' incentive to gather information and weaken price competition.[8] Another stream of literature regards price complexity as a device to exploit boundedly rational consumers. For example, in Spiegler (2006), consumers who face complex (multi-dimensional) prices (e.g., insurance schemes) sample just one random dimension and buy from the firm with the lowest sampled fee. As a result, firms have incentive to introduce variation across different price dimensions.[9] Our model also considers price complexity, but unlike previous

[7]Notice that frame differentiation seems to prevail in markets where the consumers purchase with relatively high frequency (e.g., supermarkets). If some frames were associated with higher prices, even consumers with high cognitive costs might be able to figure it out over time and avoid buying these products. On the other hand, price complexity seems to be mostly common in markets where the consumers participate infrequently. In that case, even if more complex frames are always associated with higher prices, consumers may not have the opportunity to learn to infer prices from presentation modes.

[8]In the standard search framework, however, each firm has no individual incentive to increase price complexity. Both Carlin (2009), and Ellison and Wolitzky (2008) make specific assumptions on the search technology to deal with this issue. See a detailed discussion in subsection 4.2.

[9]Gabaix and Laibson (2004) study product complexity using a random utility model. They assume that a more complex product makes the consumer's valuation noisier. If each firm can control the degree of complexity of its product, increasing the number of firms may induce them to supply more

studies, it combines the effects of price frame differentiation and price frame complexity in a unified framework. In particular, we regard frame differentiation as an important source of "market complexity", albeit different from frame complexity. In effect, our study disentangles the relative effects of frame differentiation and frame complexity on market outcomes.

In a closely related but independent article, Piccione and Spiegler (2009) also examine frame-price competition.[10] They focus on a duopoly model but with a more general frame structure, and mainly examine the relation between equilibrium properties and the frame structure. Our duopoly example in Section 2 could be regarded as a special case of their model, but we develop an oligopoly model which allows us to examine the impact of greater competition on firms' framing strategies and market outcomes. We discuss other differences between their work and ours in Section 4.1.

More broadly, our work contributes to the growing literature on bounded rationality and industrial organization (see Ellison, 2006, for instance). In our model, it is the inability of boundedly rational consumers to compare framed prices that leads to equilibrium frame dispersion. Finally, our study is also related to the literature on consumer search and price dispersion (see Baye et al., 2006 for a survey). However, we focus on how firms may confuse consumers by randomizing their frame choices, and in our model price dispersion is a by-product of price frame dispersion.

2 A Duopoly Example

This section introduces the model and presents some of the main insights in a two-firm example. Consider a market for a homogeneous product with two identical sellers, firms 1 and 2. The constant marginal cost of production is normalized to zero. There is a unit mass of consumers, each demanding at most one unit of the product and willing to pay at most $v = 1$. Suppose that there are two possible price presentation modes, referred to as frames A and B. Frame A is weakly simpler than frame B. The firms simultaneously and noncooperatively choose price frames and prices p_1 and p_2. Each firm can choose just one of the two frames. The timing reflects the fact that both frames and prices can be changed relatively easily.[11]

complicated products in order to soften price competition.

[10] Gaudeul and Sugden (2009) consider a similar issue of price and "standard" choices. In particular, they emphasize that, if consumers have strong preferences for firms which are using the same standard and refuse to consider all other firms which are using "individuated" standards, then a common-standard competitive equilibrium may emerge.

[11] We discuss an alternative two-stage setting where firms choose frames before engaging in price competition in subsection 4.2.

Price framing affects consumer choice in the following way. If both firms choose the relatively simple frame A, all consumers can perfectly compare the two prices and buy the cheaper product with a positive net surplus. Formally, in this case firm i's demand is

$$q_i(p_i, p_j) = \begin{cases} 1, & \text{if } p_i < p_j \text{ and } p_i \leq 1 \\ 1/2, & \text{if } p_i = p_j \leq 1 \\ 0, & \text{if } p_i > p_j \text{ or } p_i > 1 \end{cases} \quad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j. \tag{1}$$

The supposition that nobody gets confused when both firms use A is only for expositional simplicity, and our main results hold qualitatively if a small fraction of consumers will also get confused in this case.[12]

If the two firms adopt different frames, a fraction $\alpha_1 > 0$ of consumers get confused and are unable to compare the two prices.[13] In this duopoly example, for simplicity we assume that they then shop at random, i.e., half of them buy from firm 1 and the other half buy from firm 2. The remaining $1 - \alpha_1$ fraction of consumers are still able to accurately compare prices. In the oligopoly model in section 3, we allow consumers to favour the simpler frame A whenever they get confused between different frames, that is, more than half (but not all) of the confused consumers will buy from the firm using frame A. Our main insights still hold in that case.

If both firms choose the relatively complex frame B, a fraction $\alpha_2 \geq 0$ of consumers get confused and shop at random. The following table presents the fraction of confused consumers for all possible frame profiles, where z_i is the frame chosen by firm i and z_j is the frame chosen by firm j.

Table 1: Confused consumers

$z_i \setminus z_j$	A	B
A	$\alpha_0 = 0$	α_1
B	α_1	α_2

Then, firm i's profit is

$$\pi_i(p_i, p_j, z_i, z_j) = p_i \cdot [\frac{1}{2}\alpha_{z_i,z_j} + q_i(p_i, p_j)(1 - \alpha_{z_i,z_j})] \ ,$$

where α_{z_i,z_j} is presented in Table 1 and $q_i(p_i, p_j)$ is given by (1).

We make the behavioral interpretation that consumers are confused by frame differentiation or by the complexity of frame B. Alternatively, in our model confusion might

[12] More precisely, using the notation below, we only require that $\alpha_0 \leq \alpha_2$ and $\alpha_0 \neq \alpha_1$.

[13] But we still assume that even confused consumers will not pay more than $v = 1$ for the product (for instance, consumers have a budget constraint at one or they have a rough idea of how much they are paying eventually).

capture "unconscious mistakes": consumers think that they make accurate comparisons, when they actually make errors. Or, confusion might be the result of consumers' rational decision to give up costly information processing. We further discuss the latter possibility in subsection 4.2.

When consumers get confused, we assume that their choices are entirely independent of the two firms' prices. This is a tractable way to model the idea that confusion in price comparison induces consumers to make unsystematic errors and so reduces their price sensitivity. It might be more sensible to assume that confusion only leads to noisy price comparisons (rather than entirely blocking price comparisons), so that consumers' choices still depend somewhat on prices. In this case, firms still have incentives to mix on price frames, but it becomes less tractable to characterize the equilibrium. We discuss this alternative model in subsection 4.1.

There are two sources of confusion in our model: one is *frame differentiation* (measured by α_1) and the other is the *complexity of frame B* (measured by α_2). In some circumstances such as grocery price promotion, frame differentiation may be more confusing than frame complexity, i.e., $\alpha_1 > \alpha_2$. This captures the idea that when frame B is only slightly more complex than A, comparing two prices in a similar format may be easier. In particular, if $\alpha_0 = \alpha_2 = 0$, the two frames are symmetric. In this case, consumers are confused only by frame differentiation (for instance, frame A is "Price incl. VAT" and frame B is "Price excl. VAT"). In other cases, frame complexity may dominate frame differentiation in confusing consumers, i.e., $\alpha_2 > \alpha_1$. If frame B is a complex multi-dimensional price and frame A is a single price, then comparing two multi-dimensional prices may require more effort/time than comparing a simple price with a complex one, and so it may result in more confusion.

Let us now characterize the equilibrium in the duopoly case.[14] We first show that there is no pure strategy framing equilibrium. Then we prove the existence and uniqueness of a symmetric mixed-strategy equilibrium. All proofs missing from this section are relegated to Appendix A.

Lemma 1 *If $\alpha_1 \neq \alpha_2$, there is no equilibrium in which both firms choose deterministic price frames.*

Proof. (a) Suppose both firms choose frame A for sure. Then, the unique candidate equilibrium entails marginal-cost pricing and zero profit. But, if firm i unilaterally

[14] Our duopoly example can be regarded as a reduced-form model of the bi-symmetric graph case in Piccione and Spiegler (2009). All their results apply to our model, except that in our setting it is subtler to exclude the possibility of firms adopting deterministic frames. In their model, consumers are always able to perfectly compare prices in the same frame (i.e., frame differentiation is the only confusion source), so it is easy to see that firms will never adopt deterministic frames.

deviates to frame B and a positive price (no greater than one), it makes a positive profit. A contradiction.

(b) Suppose both firms choose frame B for sure. For clarity, consider two cases. (b1) If $\alpha_2 = 1$ (and so $\alpha_1 < \alpha_2$), at the unique candidate equilibrium $p_i = 1$ and $\pi_i = 1/2$ for all i. But, if firm i unilaterally deviates to frame A and price $p_i = 1 - \varepsilon$, it earns $(1-\varepsilon)\left[\alpha_1/2 + (1-\alpha_1)\right] > 1/2$ for ε small enough. (b2) If $\alpha_2 < 1$, the unique candidate equilibrium dictates mixed strategy pricing according to a cdf on $[p_0, 1]$ as in Varian (1980), and each firm's expected profit is $\alpha_2/2 = p_0(1-\alpha_2/2)$.[15] If $\alpha_1 > \alpha_2$, firm i can make a higher profit $\alpha_1/2 > \alpha_2/2$ by deviating to frame A and price $p_i = 1$. If $\alpha_1 < \alpha_2$, firm i can make a higher profit $p_0(1-\alpha_1/2) > p_0(1-\alpha_2/2)$ by deviating to frame A and price $p_i = p_0$. Both (b1) and (b2) lead to a contradiction.

(c) Suppose firm i chooses frame A and firm j chooses B. Again consider two cases. (c1) If $\alpha_1 = 1$, the unique candidate equilibrium entails $p_i = 1$ and $\pi_i = 1/2$ for all i. But, then, firm j is better off deviating to frame A and $p_j = 1 - \varepsilon$, in which case its profit is $1 - \varepsilon > 1/2$ for any $\varepsilon < 1/2$. (c2) If $\alpha_1 < 1$, then the unique candidate equilibrium is again of Varian type and dictates mixed strategy pricing according to a cdf on $[p_0, 1]$, with each firm earning $\alpha_1/2 = p_0(1-\alpha_1/2)$. But if firm j deviates to frame A and price $p_j = p_0$, it makes a higher profit p_0. Both (c1) and (c2) lead to a contradiction. This completes the proof.[16] ■

If both firms use the same simple frame (that is, A or, when $\alpha_2 = 0$, could also be B), they compete *à la* Bertrand and make zero profits. A unilateral deviation to a different frame supports positive profits as some consumers are confused by "frame differentiation" and shop at random. For $\alpha_2 > 0$, Lemma 1 also shows that at equilibrium, the firms cannot rely on only one source of confusion. Otherwise, a firm which uses frame B would have a unilateral incentive to deviate to the simpler frame A to attract some price aware consumers. However, if $\alpha_1 = \alpha_2 > 0$, there is an equilibrium in which both firms use frame B (see further details in the end of this section).

The rest of this section focuses on the general case with $\alpha_1 \neq \alpha_2$. By Lemma 1, in any candidate equilibrium at least one firm will randomize its frame choice. Therefore, there is a positive probability that firms have bases of fully aware consumers, *and* also a positive probability that they have bases of confused consumers who cannot compare prices at all. The conflict between the incentives to fully exploit confused consumers and to vigorously compete for the aware consumers leads to the inexistence of pure strategy pricing equilibria. The proof of the following result is standard and therefore omitted.

[15] See Baye et al. (1992) for the uniqueness proof in the two-firm case.

[16] Although parts (a) and (c) used the fact that consumers can compare prices perfectly when both firms use frame A, our result still holds even if $\alpha_0 > 0$ provided that $\alpha_0 \neq \alpha_1$ (the logic in (b) applies).

Lemma 2 *If $\alpha_1 \neq \alpha_2$, there is no equilibrium in which both firms charge deterministic prices.*

Lemmas 1 and 2 show that in the duopoly model there are only equilibria which exhibit dispersion in *both* price frames and prices.

In continuation, we focus on the *symmetric mixed-strategy equilibrium* (λ, F_A, F_B) in which each firm assigns probability $\lambda \in (0,1)$ to frame A and $1-\lambda$ to frame B and, when a firm uses frame $z \in \{A, B\}$, it chooses its price randomly according to a cdf F_z which is strictly increasing on its connected support $S_z = [p_0^z, p_1^z]$.[17] We first show that F_z is continuous (except when $\alpha_2 = 1$).

Lemma 3 *In the symmetric mixed-strategy equilibrium (λ, F_A, F_B), the price distribution associated with frame A (F_A) is always atomless and the one associated with frame B (F_B) is atomless whenever $\alpha_2 < 1$.*

Denote by

$$x_z(p) \equiv 1 - F_z(p)$$

the probability that a firm using frame z charges a price higher than p. Suppose firm j is employing the equilibrium strategy. Then, if firm i uses frame A and charges a price $p \in [p_0^A, p_1^A]$, its expected profit is

$$\pi(A, p) = p\{\lambda x_A(p) + (1-\lambda)[\alpha_1/2 + (1-\alpha_1) x_B(p)]\} . \tag{2}$$

With probability λ, the rival is also using A such that the firms compete *à la* Bertrand. With probability $1-\lambda$, the rival is using B, such that a fraction α_1 of the consumers are confused (by frame differentiation) and shop at random, and the firms compete *à la* Bertrand for the remaining $1-\alpha_1$ fully aware consumers.

If instead firm i uses B and charges $p \in [p_0^B, p_1^B]$, its expected profit is

$$\pi(B, p) = p\{\lambda[\alpha_1/2 + (1-\alpha_1) x_A(p)] + (1-\lambda)[\alpha_2/2 + (1-\alpha_2) x_B(p)]\} . \tag{3}$$

With probability λ, the rival is using A so that a fraction α_1 of the consumers are confused (by frame differentiation) and shop at random. With probability $1-\lambda$, the rival is also using B so that a fraction α_2 of the consumers are confused (by frame complexity) and shop at random.[18]

[17] A symmetric mixed-strategy equilibrium can also be expressed as $(F(p), \lambda(p))$ in which $F(p)$ is the price distribution and $\lambda(p)$ is the probability of adopting frame A conditional on price p.

[18] Note that the profit functions apply for any price p as $F_z(p) = 0$ for $p < p_0^z$ and $F_z(p) = 1$ for $p > p_1^z$.

The nature of the equilibrium depends on which source of confusion dominates. Intuitively, when $\alpha_1 < \alpha_2$, if a firm switches from frame A to B, more consumers get confused regardless of its rival's frame choice. Thus, each firm charges higher prices when it uses frame B than when it uses frame A. For $\alpha_1 > \alpha_2$, when a firm switches from frame A to B, more consumers get confused if its rival is using A, while fewer consumers get confused if its rival is using B. Hence, there is no obvious monotonic relationship between the prices associated with A and B. The remainder of this section analyses these two cases separately.

- *Frame differentiation dominates frame complexity:* $0 \leq \alpha_2 < \alpha_1$

The unique symmetric equilibrium in this case dictates $F_A(p) = F_B(p)$ and $S_A = S_B = [p_0, 1]$ (see Appendix A for the proof). That is, a firm's price is independent of its frame. Then let $F(p)$ be the common price distribution and $x(p) \equiv 1 - F(p)$. Substituting them into the profit functions (2) and (3) and using the frame indifference condition $\pi(A, p) = \pi(B, p)$, we obtain

$$\lambda = 1 - \frac{\alpha_1}{2\alpha_1 - \alpha_2} \ . \tag{4}$$

If the two frames are symmetric ($\alpha_2 = 0$), then firms are equally likely to adopt each frame (i.e., $\lambda = 1/2$). When frame B becomes more complex, firms adopt it more often (i.e., $1 - \lambda$ increases with α_2).

Note that (4) can be re-written as $(1 - \lambda)\alpha_1 = \lambda\alpha_1 + (1 - \lambda)\alpha_2$ and it actually requires the expected number of confused consumers to be the same when a firm uses frame A (the left-hand side) and when it uses frame B (the right-hand side). Given that in duopoly there are only two types of consumers (the confused and the fully aware), it implies that the expected market composition along the equilibrium path does not depend on a firm's frame choice. Since the pricing balances the incentives to extract surplus from the confused and to compete for the fully aware, a frame-independent market composition implies frame-independent pricing. This explains why $F_A = F_B$. (Note that this result may not hold if confused consumers have a bias toward the simple frame as we formally show in the general oligopoly model.)

Denote by π a firm's equilibrium profit. Since all prices on $[p_0, 1]$ should result in the same profit, we obtain (e.g. from $\pi(A, 1)$ by using $x(1) = 0$)

$$\pi = \frac{\alpha_1^2}{2(2\alpha_1 - \alpha_2)} \ . \tag{5}$$

π increases with both α_1 and α_2. That is, confusion (regardless of its source) always boosts firms' payoffs and harms consumers.

Finally, the common price distribution $F(p)$ can be derived from $\pi(A, p) = \pi$, since all prices in the support of $F(p)$ should result in the same profit in a mixed-strategy equilibrium. Explicitly, $x(p) = 1 - F(p)$ solves

$$\lambda x(p) + (1 - \lambda)\left[\alpha_1/2 + (1 - \alpha_1)\, x(p)\right] = \frac{\pi}{p}\ . \tag{6}$$

Then the boundary price p_0 is defined by $x(p_0) = 1$ and one can check that $p_0 \in (0, 1)$. The price distribution for a higher α_1 (α_2) first-order stochastically dominates that for a lower α_1 (α_2). This is consistent with the observation that confusion benefits firms and harms consumers.

We summarize these findings below.

Proposition 1 *In the duopoly model with $0 \leq \alpha_2 < \alpha_1$, there is a unique symmetric mixed-strategy equilibrium in which each firm adopts frame A with probability λ and frame B with probability $1 - \lambda$, where λ is given in (4). Regardless of its frame choice, each firm chooses its price randomly according to a cdf F which is defined by (6) on $[p_0, 1]$. Each firm's equilibrium profit is π given in (5).*

Notice that the equilibrium price dispersion is driven by firms' obfuscation effort through random framing but *not* necessarily by the coexistence of price aware and confused consumers. This is best seen in the polar case with $\alpha_1 = 1$ and $\alpha_2 = 0$, where consumers are always homogeneous both ex-ante and ex-post (i.e., once a pair of frames is realized, either all consumers are confused or all of them are fully aware), but price dispersion still persists.

- *Frame complexity dominates frame differentiation:* $0 < \alpha_1 < \alpha_2$

In this case, the unique symmetric equilibrium dictates adjacent supports $S_A = [p_0^A, \hat{p}]$ and $S_B = [\hat{p}, 1]$ (see Appendix A for the proof). In particular, if $\alpha_2 = 1$, then $S_A = [p_0^A, 1)$ and $S_B = \{1\}$. That is, frame B is always associated with higher prices than frame A. This happens because when a firm shifts from frame A to frame B, regardless of the rival's frame, more consumers get confused given that $\alpha_1 < \alpha_2$.

With adjacent price supports, in the profit function $\pi(A, p)$ (in expression (2)), $x_B(p) = 1$ for any $p \in S_A$ since the B frame is always associated with higher prices. Similarly, in the profit function $\pi(B, p)$ (in expression (3)), $x_A(p) = 0$ for any $p \in S_B$. Then from the indifference condition $\pi(A, \hat{p}) = \pi(B, \hat{p})$, we can derive

$$\lambda = 1 - \frac{\alpha_1}{\alpha_2}\ . \tag{7}$$

Note that the probability of using the complex frame B $(1 - \lambda)$ *decreases* with the complexity index α_2, unlike the previous case (with $\alpha_1 > \alpha_2$). This happens because when confusion from frame complexity dominates, the prices associated with frame B are already high (so a rival who uses frame B is a softer competitor). This makes more attractive the use of frame A together with a relatively high price (but still lower than $\hat{p}$). Hence, for fixed α_1, the overall relationship between $1-\lambda$ and α_2 is non-monotonic: when $\alpha_2 < \alpha_1$, the probability of using frame B goes up with α_2 and when $\alpha_2 > \alpha_1$, it decreases with α_2.

Each firm's equilibrium profit π is given by $\pi(B, 1)$:

$$\pi = \alpha_1(1 - \frac{\alpha_1}{2\alpha_2}) \ . \tag{8}$$

As before, it can be verified that this equilibrium profit increases (and so consumer surplus decreases) with both α_1 and α_2.

Finally, $F_z(p)$ is determined by $\pi(z, p) = \pi$. Explicitly, we have

$$\lambda x_A(p) + (1-\lambda)(1-\alpha_1/2) = \frac{\pi}{p} \tag{9}$$

and

$$\lambda\alpha_1/2 + (1-\lambda)\left[\alpha_2/2 + (1-\alpha_2)x_B(p)\right] = \frac{\pi}{p} \ . \tag{10}$$

The boundary prices p_0^A and $\hat{p}$ are determined by $x_A(p_0^A) = 1$ and $x_A(\hat{p}) = 0$, respectively. One can check that both of them are well defined with $p_0^A < \hat{p}$.

We summarize these results below.

Proposition 2 *In the duopoly model,*
(i) if $\alpha_1 < \alpha_2 < 1$, there is a unique symmetric mixed-strategy equilibrium in which each firm adopts frame A with probability λ and frame B with probability $1-\lambda$, where λ is given in (7). When a firm uses frame A, it chooses its price randomly according to the cdf F_A defined on $[p_0^A, \hat{p}]$ which solves (9); when a firm uses frame B, the price cdf is F_B defined on $[\hat{p}, 1]$ which solves (10). Each firm's equilibrium profit π is given in (8).
(ii) if $\alpha_1 < \alpha_2 = 1$, the equilibrium has the same form except that F_B is a degenerate distribution on $\{1\}$ and F_A is defined on $[p_0^A, 1)$.

When $\alpha_2 \to \alpha_1$, it follows from both Propositions 1 and 2, that the firms use frame B almost surely (i.e., $\lambda \to 0$), and the price distributions associated with frame B in the two cases tend to coincide. Therefore, when $\alpha_1 = \alpha_2 > 0$, there is a unique symmetric equilibrium in which both firms use frame B.

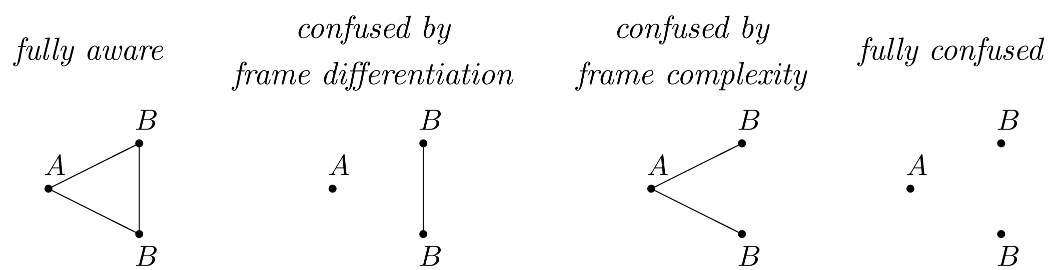
3 The Oligopoly Model

This section analyses the general oligopoly version of the model. Our main objective is to investigate the impact of an increase in the number of firms on market outcomes.

Consider a homogeneous product market with $n \geq 2$ identical sellers and, as before, two categories of frames, A and B. Let A be a simple frame so that all prices in this frame are comparable. Frame B may involve some complexity so that with probability $\alpha_2 \geq 0$ the consumers are unable to compare prices in this frame. Consumers can also be confused by frame differentiation and therefore unable to compare prices in different frames with probability $\alpha_1 > 0$. We assume that confusion from frame differentiation and confusion from frame complexity are independent.

In the duopoly case, for any realized frame profile, there is at most one confusion source, and so there are at most *two* types of consumers: fully aware (who always buy the cheaper product) and totally confused (who shop randomly). However, with more than two firms, for a realized frame profile (e.g., (A, B, B)), both confusion sources might be present. Consequently, there are up to *four* types of consumers: $(1-\alpha_1)(1-\alpha_2)$ fully aware consumers, $\alpha_1(1-\alpha_2)$ consumers confused only by frame differentiation, $(1-\alpha_1)\alpha_2$ consumers confused only by frame complexity, and $\alpha_1\alpha_2$ consumers confused by both confusion sources.[19] The following is an illustrative example.

Example 1 *Consider a case with* 3 *firms. Suppose firm 1 uses frame* A*, and firms 2 and 3 use frame* B*, respectively. The following graphs show the comparability among options for the four types of consumers. If two offers are comparable they are connected; if they are not comparable there is no link between them.*



Moreover, with more than two firms, even if there is only one confusion source, a consumer may be only partially confused as the following example shows.

Example 2 *Consider a case with* 3 *firms. Suppose firm 1 uses frame* A *and charges price* p_1*, and firms 2 and 3 use frame* B *and charge prices* p_2 *and* p_3*, respectively. If* $\alpha_1 = 1$ *and* $\alpha_2 = 0$ *(i.e., frame* B *is also simple), then only frame differentiation causes confusion. All consumers can accurately compare* p_2 *with* p_3 *since they are presented in the same frame, but cannot compare* p_1 *with either* p_2 *or* p_3*. So consumers are neither fully aware nor totally confused.*

[19] Note that in our model consumer confusion occurs at frame level. For example, across all pairs of one A and one B offer, a consumer is either able to compare all or none.

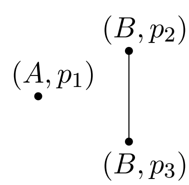
The above discussion raises the issue of how a consumer chooses from a "partially ordered" set in which some pairs of alternatives are comparable, but others are not. This is also the key difference between the duopoly model and the model with more than two firms. To deal with this consumer choice issue, following the literature of incomplete preferences, we adopt a *dominance-based* consumer choice rule. (See Section 4.1 for a discussion of alternative choice rules and the robustness of our results.) The basic idea is that consumers will only choose, according to some stochastic rule, from the "maximal" alternatives which are not dominated by any other comparable alternative. From now on, we will use "dominated" in the following sense.

Definition 1 *Firm i's offer* $(z_i, p_i) \in \{A, B\} \times [0, 1]$ *is* ***dominated*** *if there exists firm* $j \neq i$ *which offers alternative* $(z_j, p_j < p_i)$ *and the two offers are comparable.*

Notice that the set of maximal or undominated alternatives is well-defined and non-empty (for example, the firm which charges the lowest price in the market is never dominated), and it can be constructed, for example, by conducting pairwise comparisons among all alternatives.[20]

Before formalizing our consumer choice rule, let us illustrate it in the following example.

Example 3 *Consider Example 2 and let* $p_1 < p_2 < p_3$.



Offers in frame B are comparable since B is also a simple frame. Then firm 3's offer is dominated by firm 2's offer given that $p_2 < p_3$. *Offers in different frames are not comparable since* $\alpha_1 = 1$, *so both firm 1's offer and firm 2's offer survive. Then, consumers buy from firm 1 with some probability and from firm 2 with the complementary probability.*

Now we can formally state our *dominance-based* consumer choice rule as follows:

[20] In our model, the comparability of two offers is independent of their comparability with other available offers. This excludes transitivity of comparability. Consider a consumer who can compare offers in different frames, but cannot compare offers in frame B. Then the presence of an offer in frame A (which is comparable with any of the B offers) does not help the consumer compare offers in frame B directly. This might be the case, if the consumers use different procedures to compare prices in different formats and to compare prices in a complex format.

1. Consumers first eliminate all dominated offers in the market.

2. They then purchase from the undominated firms according to the following stochastic purchase rule (which is independent of prices): (i) if all these firms use the same frame, they share the market equally; (ii) if among them $n_A \geq 1$ firms use frame A and $n_B \geq 1$ firms use frame B, then each undominated A firm is chosen with probability $\phi(n_A, n_B)/n_A$ and each undominated B firm is chosen with probability $[1 - \phi(n_A, n_B)]/n_B$, where $\phi(\cdot) \in (0, 1)$ is non-decreasing in n_A and non-increasing in n_B and $\phi(n_A, n_B) \geq n_A/(n_A + n_B)$.

Note that $\phi(n_A, n_B) \geq n_A/(n_A + n_B)$ in 2(ii) allows the consumers to favor the simple frame A.[21] This generalizes the random purchase assumption in our duopoly example. $\phi(\cdot) < 1$ excludes the possibility that all consumers favor the simple frame.[22] The monotonicity assumption in 2(ii) is a natural one and it simply means that the presence of more undominated firms with one frame increases the overall probability that consumers buy from them. Note that the uniformly random purchase rule $\phi(n_A, n_B) = n_A/(n_A + n_B)$ satisfies all the conditions.

For the rest of the paper, let

$$\phi_k \equiv \phi(1, k)$$

denote the probability that a consumer buys from the A firm when there are other k undominated B firms to choose from. Then, 2(ii) implies that $\{\phi_k\}_{k=1}^{n-1}$ is a non-increasing sequence: when more B firms survive, the undominated A firm has less demand, and $\phi_k \in [1/(1+k), 1)$. Note that $\phi_k = 1/(1+k)$ represents the uniformly random purchase rule when consumers have no bias toward the simple frame.

Recall that in the duopoly case the type of market equilibrium depends on whether frame differentiation or frame complexity is more confusing. The same is true in the general case. Subsections 3.1 and 3.2 analyze the corresponding symmetric equilibrium and the impact of greater competition for $\alpha_1 < \alpha_2$ and $\alpha_1 > \alpha_2$, respectively.

Before we proceed with the analysis, let us summarize two main findings. First, when $\alpha_2 > 0$ (i.e., when frame B is complex), greater competition tends to induce firms to use frame B more often. In particular, when there is a large number of firms, they use frame B almost surely. Intuitively, with more firms it becomes harder for them to differentiate from each other by frame choices, and so firms rely more on frame complexity to soften price competition. Second, when $\alpha_2 > 0$, industry profit is bounded away from zero

[21] There is evidence that people have preferences for simpler options, especially when they face many alternatives. See, for instance, Iyengar and Kamenica (2010) and the references therein.

[22] For example, some consumers might be overconfident of their ability to compare offers, so they do not favor any particular frame, but may actually make mistakes.

even when there are an infinite number of firms, and greater competition can increase industry profit and harm consumers (i.e., consumers may actually pay more in a more competitive market).[23]

3.1 Frame differentiation dominates frame complexity ($\alpha_1 > \alpha_2$)

We analyze now the case where consumers are more likely to be confused by frame differentiation than by the complexity of frame B (that is, $\alpha_1 > \alpha_2$). For simplicity, we first focus on the polar case in which prices in different frames are always incomparable (i.e., $\alpha_1 = 1$). We then discuss how the main results can be extended to the case with $\alpha_1 < 1$. All proofs missing from the text are relegated to Appendix B.1.

Lemma 4 in Appendix B.1 shows that there is no pure-strategy equilibrium whenever $\alpha_2 > 0$. If $\alpha_2 = 0$ (i.e., if both frames are simple) and $n \geq 4$, there are always asymmetric pure-strategy equilibria in which each frame is used by more than one firm and all firms charge a price equal to the marginal cost. Nevertheless, for any $n \geq 2$, there is a symmetric mixed-strategy equilibrium in which firms make positive profits.

A symmetric mixed-strategy equilibrium. Let (λ, F_A, F_B) be a symmetric mixed-strategy equilibrium, where λ is the probability of using frame A and F_z is a price cdf associated with frame $z \in \{A, B\}$. Let $[p_0^z, p_1^z]$ be the support of F_z. As in Lemma 3, it is straightforward to show that F_z is atomless everywhere (as now $\alpha_2 < 1$). For the rest of the paper,

$$P_{n-1}^k \equiv C_{n-1}^k \lambda^k (1-\lambda)^{n-k-1}$$

denotes the probability that k firms among $n-1$ ones adopt frame A at equilibrium, where C_{n-1}^k stands for combinations of $n-1$ taken k. Recall the notation $x_z(p) = 1 - F_z(p)$.

Along the equilibrium path, if firm i uses frame A and charges price p, its profit is:

$$\pi(A, p) = p\lambda^{n-1} x_A(p)^{n-1} + p \sum_{k=0}^{n-2} P_{n-1}^k x_A(p)^k \left[\alpha_2 \phi_{n-k-1} + (1-\alpha_2)\phi_1\right] . \qquad (11)$$

If k other firms also use frame A, firm i has a positive demand only if all other A firms charge prices higher than p. This happens with probability $x_A(p)^k$. Conditional on that, if there are no B firms in the market (i.e., if $k = n-1$), then firm i serves the whole market. The first term in $\pi(A, p)$ follows from this. Otherwise, firm i's demand depends

[23] An increase in the number of firms might lead to higher prices in other search and price dispersion models (see Varian, 1980). But there more fragmentation induces the firms to exploit uninformed consumers, rather than compete for shoppers. In our model, in contrast, increasing the number of firms will first influence firms' framing strategies and make the market more complex. This market complexity effect is not captured in standard search models.

on whether the consumer can compare offers from the B firms. If she is confused by frame complexity and unable to compare (which happens with probability α_2), all B firms are undominated (since no comparison between A and B is possible), and so firm i's demand is ϕ_{n-k-1}. If she is not confused by frame complexity and, therefore, able to compare prices in frame B (which happens with probability $1-\alpha_2$), only one B firm is undominated and so firm i's demand is ϕ_1.

If instead, along the equilibrium path, firm i uses B and charges price p, its profit is:

$$\begin{aligned}\pi(B,p) &= p(1-\lambda)^{n-1}[\frac{\alpha_2}{n}+(1-\alpha_2)x_B(p)^{n-1}] \\ &\quad + p\sum_{k=1}^{n-1}P_{n-1}^k\left[\alpha_2\frac{1-\phi_{n-k}}{n-k}+(1-\alpha_2)(1-\phi_1)x_B(p)^{n-k-1}\right] .\end{aligned} \tag{12}$$

The first term gives the expected profit when there are no A firms in the market: the consumers who are confused by frame complexity purchase randomly among all B firms, while those who are not confused buy from firm i only if it offers the lowest price. When $k \geq 1$ firms use frame A (note that only one of them will be undominated), if the consumer is confused by frame complexity (i.e., unable to compare prices in frame B), all B firms are undominated and have demand $1-\phi_{n-k}$ in total. Firm i shares equally this residual demand with the other B firms. If the consumer is not confused by frame complexity, to face a positive demand, firm i must charge the lowest price in group B (this happens with probability $x_B(p)^{n-k-1}$), in which case it gets the residual demand $1-\phi_1$.

Note that given $\alpha_1 = 1$ price competition can only take place among firms which use the same frame, and so $x_A(p)$ does not appear in $\pi(B,p)$ and $x_B(p)$ does not appear in $\pi(A,p)$. This also implies that both profit functions are valid even if firm i charges an off-equilibrium price. Thus, the upper bounds of the price cdf's are frame-independent: $p_1^A = p_1^B = 1$. Otherwise any price greater than p_1^z would lead to a higher profit. Then using the frame-indifference condition $\pi(A,1) = \pi(B,1)$, we can pin down a unique well-defined $\lambda \in (0,1)$. (See equation (16) in Appendix B.1). Each firm's equilibrium profit is

$$\pi = \pi(A,1) = (1-\lambda)^{n-1}[\alpha_2\phi_{n-1} + (1-\alpha_2)\phi_1] . \tag{13}$$

The price distributions F_A and F_B are implicitly determined by $\pi(z,p) = \pi$ since any price in the support of F_z should lead to the same profit in a mixed-strategy equilibrium. Both F_z are uniquely defined. The boundary prices $p_0^z < 1$ are determined by $\pi(z, p_0^z) = \pi$. Deviations to prices lower than p_0^z are not profitable since they only result in a price loss and do not increase demand. We characterize the symmetric equilibrium below.

Proposition 3 *For $n \geq 2$ and $\alpha_2 < \alpha_1 = 1$, there is a symmetric mixed-strategy equilibrium in which each firm adopts frame A with probability λ and frame B with probability $1 - \lambda$. When a firm uses frame $z \in \{A, B\}$, it chooses its price randomly according to a cdf F_z defined on $[p_0^z, 1]$, and implicitly determined by $\pi(z, p) = \pi$ with $\pi(z, p)$ given in (11) and (12) and π given in (13).*

Figure 1 below depicts the equilibrium price distributions $F_A(p)$ (the solid line) and $F_B(p)$ (the dashed line) in the case with $n = 3$, $\alpha_1 = 1$, $\alpha_2 = 0.5$, and $\phi_k = 1/(1 + k)$.

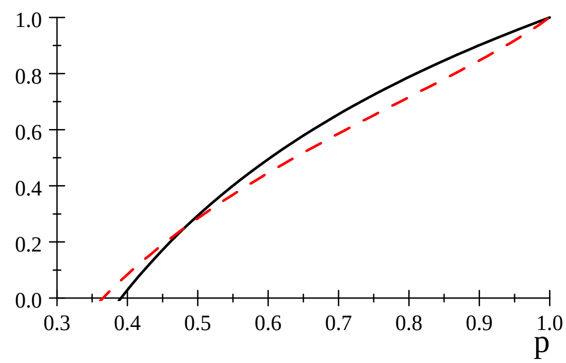


Figure 1: Price distributions with $n = 3$, $\alpha_1 = 1$ and $\alpha_2 = 0.5$

Recall that in the duopoly equilibrium in Proposition 1 pricing is frame independent (i.e., $F_A(p) = F_B(p)$) if confused consumers have no exogenous bias toward a specific frame. However, this is no longer true in the case with more than two firms, as the above example indicates. (See Appendix B.1 for a rigorous treatment of this issue.) When $F_A \neq F_B$, there is no straightforward way to analytically rank the prices associated with the two frames.

The impact of greater competition. We now study the impact of an increase in the number of firms on the equilibrium framing strategies, and on profits and consumer surplus. Our analysis is based on the equilibrium characterized in Proposition 3. We first consider a market with many sellers, which provides the key insight for our main result.

Proposition 4 *When there are a large number of firms in the market,*

$$\lim_{n\to\infty} \lambda = \begin{cases} 1/2, & \text{if } \alpha_2 = 0 \\ 0, & \text{if } \alpha_2 > 0 \end{cases} \quad ; \quad \lim_{n\to\infty} n\pi = \begin{cases} 0, & \text{if } \alpha_2 = 0 \\ > 0, & \text{if } \alpha_2 > 0 \end{cases} .$$

When frame B is also a simple frame, the only way to reduce price competition is through frame differentiation. This is why in a sufficiently competitive market λ tends to $1/2$, which maximizes frame differentiation. However, the ability of frame differentiation alone to weaken price competition is limited. When there are a large number of firms in the market, each frame is adopted by more than one firm almost surely (as long as λ is bounded away from zero and one), so price competition becomes extremely intense and the market price tends to marginal cost.

When frame B is complex, the impact of greater competition on firms' framing strategies changes completely. In a sufficiently competitive market, firms use frame B almost surely: they rely heavily on frame complexity to soften price competition. (This is true even if frame B is only slightly more complex than frame A.) The reason is that, in a large market, the effect of frame differentiation on reducing price competition becomes negligible, but the effect of frame complexity is still significant. For example, if all firms employ frame B for sure, industry profit is always α_2, regardless of the number of firms in the market. Hence, when frame B is complex, competition does not drive the market price to marginal cost. Note that these results hold even if some (but not all) confused consumers favor the simple frame.

The analysis for large n suggests that, when the number of firms increases, frame B's complexity tends to become a more important anti-competitive device. In effect, as we will show shortly, λ tends to be decreasing in the number of firms. That is, greater competition tends to induce firms to use the complex frame more frequently. Is it then possible that, in the presence of a complex frame B, greater competition raises market prices by increasing market complexity? The answer, in general, depends on the parameter values. But, we show below that, at least for sufficiently large α_2, greater competition can actually increase industry profit and harm consumers. Therefore, in the market with price framing, competition policy which focuses exclusively on an increase in the number of competitors, might have undesired effects. For tractability, we focus on the uniformly random purchase rule $\phi_k = 1/(1+k)$.

Proposition 5 *With* $0 < \alpha_2 < \alpha_1 = 1$ *and the random purchase rule* $\phi_k = 1/(1+k)$,
(i) when n *increases from* 2 *to* 3*, both* λ *and industry profit* $n\pi$ *decrease;*
(ii) for any $n \geq 3$*, there exists* $\hat{\alpha} \in (0,1)$ *such that for* $\alpha_2 > \hat{\alpha}$*,* λ *decreases but industry profit* $n\pi$ *increases from* n *to* $n+1$*.*

Beyond the limit results, numerical simulations suggest that λ tends to decrease in n, and industry profit can increase in n for a relatively large α_2.[24] The graph below

[24] For a sufficiently small α_2, increasing the number of firms will lower industry profit. This can be seen by noticing that at $\alpha_2 = 0$, we have $\lambda = 1/2$ (for any n) and industry profit is $n/2^n$, which decreases in n.

describes how industry profit varies with n when $\alpha_2 = 0.9$.

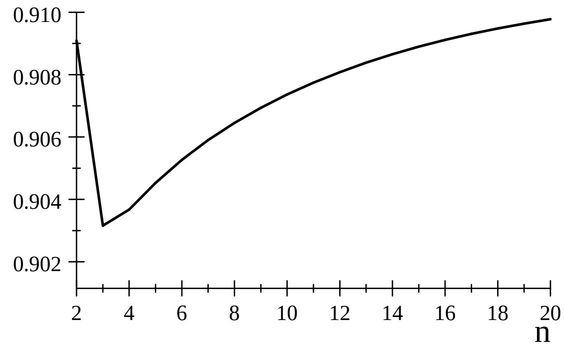


Figure 2: Industry profit and n when
$\alpha_1 = 1$ and $\alpha_2 = 0.9$

The case with $\alpha_2 < \alpha_1 < 1$. Price competition can also take place between firms using different frames. Then both $x_A(p)$ and $x_B(p)$ will appear in the profit functions $\pi(z, p)$. The related analysis becomes more involved and its details are presented in the online supplementary document.[25] There we show that if a symmetric mixed-strategy equilibrium exists, then it still satisfies $p_1^A = p_1^B = 1$. Numerical simulations suggest that greater competition can still have undesired effects (for example, when α_1 is large and α_2 is close to α_1). For example, when $\alpha_1 = 0.98$ and $\alpha_2 = 0.9$, industry profit varies with n in a way similar to Figure 2.

3.2 Frame complexity dominates frame differentiation ($\alpha_2 > \alpha_1$)

We now turn to the case where consumers are more likely to be confused by the complexity of frame B than by frame differentiation (i.e., $\alpha_2 > \alpha_1$). Again, we first consider the polar case in which prices in frame B are always incomparable (i.e., $\alpha_2 = 1$). We then discuss the robustness of our main results to the case with $\alpha_2 < 1$. Since the analysis resembles the previous one, we only report the main results here and relegate the details to Appendix B.2.

Proposition 6 *For $n \geq 2$ and $0 < \alpha_1 < \alpha_2 = 1$, there is a symmetric mixed-strategy equilibrium in which each firm adopts frame A with probability λ and frame B with probability $1 - \lambda$. When a firm uses frame A, it chooses its price randomly according to a cdf F_A defined on $[p_0^A, 1)$; when it uses frame B, it charges a deterministic price $p = 1$.*

[25] See https://sites.google.com/site/jidongzhou77/research .

Using the equilibrium in this proposition, we analyze the impact of greater competition on the market outcome. When there are many sellers in the market, the same results as in Proposition 4 for $\alpha_2 > 0$ hold. That is, $\lim_{n\to\infty} \lambda = 0$ and $\lim_{n\to\infty} n\pi > 0$. The same intuition applies: in a sufficiently competitive market, the ability of frame differentiation to soften price competition is negligible, and so firms resort to the complexity of frame B.

The following result shows that in the current case greater competition can also improve industry profit and decrease consumer surplus. In particular, this must happen when α_1 is small. The reason is that, for a small α_1, the complexity of frame B is more effective in reducing price competition, which makes the frequency of using frame B increase fast enough with the number of firms. The resulting market complexity could then dominate the usual competitive effect of larger n.

Proposition 7 *In the case with $0 < \alpha_1 < \alpha_2 = 1$, for any $n \geq 2$, there exists $\hat{\alpha} \in (0, 1)$ such that for $\alpha_1 < \hat{\alpha}$, λ decreases while industry profit $n\pi$ increases from n to $n + 1$.*

The following graph describes how industry profit varies with n when $\alpha_1 = 0.05$. [26]

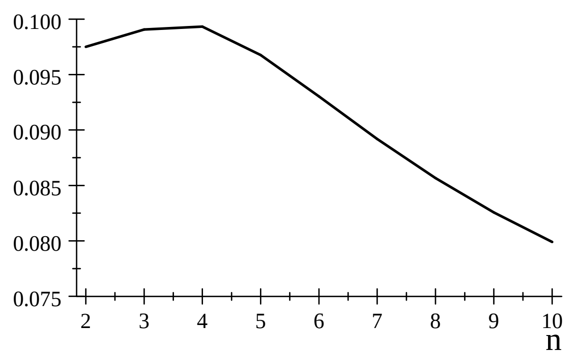


Figure 3: Industry profit and n when
$\alpha_1 = 0.05$ and $\alpha_2 = 1$

The case with $\alpha_1 < \alpha_2 < 1$. The analysis in this case is more involved, and we relegate the details to the online supplementary document. First, a symmetric separating equilibrium with $S_A = [p_0^A, \hat{p}]$ and $S_B = [\hat{p}, p_1^B]$, resembling the one in Proposition 6, still exists under some parameter restrictions (when α_1 is not too close to $\alpha_2 < 1$). Second, for fixed $\alpha_2 < 1$, if α_1 is sufficiently small, greater competition can still increase industry profit and harm consumers.

[26] For industry profit to increase at a larger n, α_1 needs to be smaller. But this is always feasible according to Proposition 7.

3.3 More frames

The oligopoly model with a general frame structure is less tractable. In the online supplementary document, we explore the relatively simple case with $m \geq 2$ completely *symmetric* frames $\{A_1, \cdots, A_m\}$ to understand how the number of frames could affect market outcomes. We show that if the set of available frames is large enough ($n < 2m$), there can only be equilibria in which firms randomize over frames. When fewer frames are available ($n \geq 2m$), there are asymmetric equilibria where each frame is used by more than one firm and all firms make zero profit. However, even in this case there also exists a mixed-strategy equilibrium in which firms randomize over frames and make positive profits. Note that when n goes up, it is more difficult for each firm to differentiate itself from rivals. As expected, numerical simulations suggest that firms compete more aggressively in prices and make lower profits. In contrast, when m increases the firms can frame differentiate and avoid price competition more easily, so that the profits go up. If each new entrant brings a new frame to the market (i.e., $m = n$ always), then simulations show that industry profit always *increases* with n (but consumer surplus is bounded away from zero). This result suggests that the frame differentiation effect is stronger than the competition effect.

Considering a general frame structure for $m \geq 3$ brings about significant technical complications. We leave this for future research, but briefly comment here on some modelling possibilities. An oligopoly model with a general frame structure could (i) assign to each frame a complexity index—the probability that the consumer gets confused among prices in this frame, and (ii) assign to each pair of frames a differentiation index—the probability that the consumer gets confused between the two frames. The dominance-based choice rule with an appropriately modified stochastic purchase rule (e.g., the uniformly random one) can apply. We conjecture that our main insights would still apply in this framework.

4 Concluding Discussions

This paper has presented a model of competition in both prices and price frames where price framing can obstruct consumers' price comparisons. We characterized the symmetric mixed-strategy equilibrium in which firms randomize over both frames and prices, and examined how the degree of competition affects firms' framing strategies, profits, and consumer welfare.

In the remainder of this section, we discuss alternative consumer choice rules and interpretations of consumer confusion.

4.1 Alternative consumer choice rules

(1) *More restrictive consideration sets.* In our dominance-based choice rule, consumers' "consideration set" includes all available options. The consumers make correct comparisons among *all* pairs of comparable alternatives, rule out the dominated alternatives, and then select from the set of undominated ones. Alternatively, consumers may restrict their consideration set at the outset (to save time and effort, for instance). The following example illustrates such choice heuristics.

Example 4 *When $n_A \geq 1$ firms use frame A and $n_B \geq 1$ firms use frame B, a consumer who cannot compare B options, restricts attention to a consideration set which consists of all A firms and $k \leq n_B$ randomly chosen B firms. She then applies the dominance-based choice rule to this restricted consideration set.*

Our benchmark choice procedure corresponds to $k = n_B$ and is the most sophisticated one in this class. When $k < n_B$, though consumers face a simpler choice problem, they may eventually choose some dominated options. For example, when a consumer can compare A to B, she would fail to eliminate the A option(s) if the B option(s) which dominate them had been excluded from her consideration set. It can be shown that (at least) for $k = 1$, our main results hold qualitatively.

(2) *A default-bias choice rule.* Our dominance-based choice rule embeds a *simultaneous* assessment of competing offers, and a consumer's choice outcome is not affected by the particular sequence of pairwise comparisons. This "simultaneous search" feature is more suitable in a market where the consumers are not influenced by their previous experiences (or are newcomers). Piccione and Spiegler (2009) consider a default-bias duopoly model in which consumers are initially randomly attached to one brand (their default option), and they switch to another brand only if that is comparable to their default and better than it. In this case, due to the sequential comparison, a consumer's final choice will depend on her default option.

In the duopoly case, the default-biased model is actually equivalent to our simultaneous assessment model (with the random purchase rule for confused consumers).[27] This is because, if the two firms' offers are comparable, in both models the better offer attracts all consumers, whereas if they are incomparable, in both models the firms share the market equally. However, when there are more than two firms, the two approaches diverge. In fact, with more than two firms, the default-bias model calls for further structure on the choice rule. To see why, consider the following example.

[27] More precisely, the equivalence requires the probability of being confused by two frames to be independent of which one is the default option.

Example 5 *There are three firms in the market. Let* $\alpha_2 = 1$ *and* $\alpha_1 = 0$ *(i.e., the only confusion source is frame complexity). Suppose that firm 1 adopts frame A and charges a price* p_1*, while firms 2 and 3 adopt frame B and charge prices* p_2 *and* p_3*, respectively, with* $p_2 < p_1 < p_3$*.*

The dominance-based rule implies that consumers will purchase only from firm 2 since firm 3 is dominated by firm 1 and firm 1 is dominated by firm 2. Now consider the default-bias model. If a consumer is initially attached to firm 2, she will not switch. If she is initially attached to firm 1, she will switch to firm 2. However, if she is initially attached to firm 3, she will switch to firm 1, but whether she will further switch to firm 2 depends on what the choice rule of the default-biased consumer dictates. Such rule should specify if the consumer will assess firm 2's offer from the perspective of her default option (i.e., firm 3) or from the perspective of her new choice (i.e., firm 2). In contrast, the dominance-based rule applies equally well regardless of the number of firms in the market.[28]

Both a more restrictive consideration set and a default bias add another dimension of bounded rationality on top of consumer confusion caused by framing. In this sense, our framework is a minimal deviation from the standard Bertrand competition model.

(3) *Noisy price comparisons.* For the sake of tractability, we have assumed in our consumer choice rule that confused consumers' choice from the set of undominated alternatives is entirely *independent* of the prices. Alternatively, confusion might only lead to noisy price comparisons, so that consumers' choice still depends somewhat on prices. For instance, in the duopoly case, when the price difference between firms 1 and 2 is $p_1 - p_2$, the consumer might misperceive it as being $p_1 - p_2 + \delta$, where δ is a frame-profile-dependent random variable. If all δ's have symmetric distributions around zero, then our result that in the symmetric equilibrium the firms randomize in both frames and prices carries over. (See a detailed argument in the online supplementary document.) However, unless we restrict attention to a duopoly case where confusion stems only from frame differentiation, it is difficult to characterize the symmetric mixed-strategy equilibrium in this setting.

[28] The fact that these two choice rules may lead to different outcomes can also be seen from the following example: consider the frame choices in example 5, but let $\alpha_2 = 0$ and $\alpha_1 = 1$, and $p_1 < p_2 < p_3$. Our approach (with the uniform purchase rule) predicts that firms 1 and 2 will share the market equally; while the default-bias rule predicts that firm 1 has demand $\frac{1}{3}$ and firm 2 has demand $\frac{2}{3}$.

4.2 Alternative interpretations

(1) *Product framing.* Although our study has been motivated by price framing, it also applies to situations where *product framing* reduces the comparability of products. For instance, the way in which nutritional information is presented might frame essentially identical food products differently. A label indicating an "improved recipe" or a "British meal" might spuriously differentiate a ready meal from its close substitutes.[29] Differences in package size or quantity premia could also make it harder to compare products. On the same shelf toothpastes come in tubes of 50, 75 or 100 ml, and refreshments, cleaning products, tea boxes occasionally come in larger containers offering, say, "extra 25% free". In addition, Betrand et al. (2010) and Choi et al. (2010) document evidence that in the personal finance market, providing some payoff irrelevant information (e.g., a female photo in the loan advertising letter or information concerning mutual fund historical returns) can significantly influence consumers' choices.[30] Our main insights also apply to the product framing case as long as framing is not a long-run decision or one observable to the rivals before price competition. This interpretation also relates our paper to the literature on endogenous product differentiation (see, for instance, Chapter 7 in Tirole, 1988). One main difference is that in our model firms make product differentiation and price choices simultaneously.

If changing the product or price frame (e.g., redesigning a contract form) is considerably costly, a two-stage game in which firms first choose (and commit to) frames and then compete in prices is more suitable. In this sequential setting, it is an equilibrium for the duopolists to choose different frames if $\alpha_1 > \alpha_2$ and choose the common (complex) frame B if $\alpha_1 < \alpha_2$. However, when $\alpha_1 > \alpha_2$, there is also a mixed-strategy equilibrium in which firms randomize their frame choices like in our simultaneous move game.

(2) *Costly information processing.* Price comparisons in the presence of framing might require costly information processing. Then, consumer confusion (and the consequent random purchase) could be the result of consumers' rational decision to opt out of information processing when its cost is too high. Therefore, our model could be interpreted as one of costly information processing with rational consumers, and not only as one of bounded rationality.

[29] The reportage "What's really in our food?" broadcast on BBC One on July 14, 2009 stressed this point. For instance, interviewed customers confessed to being misled by a ready food made with imported meat and labeled as "British meal". Also, buyers seem to have a poor understanding of what labels such as "free range" really mean.

[30] For example, Betrand et al. (2010) show that the effect of including a female photo in the loan advertising letter on increasing customers' loan take-up is as strong as a 25% reduction in the interest rate.

This idea can be easily formalized in the case where each frame is associated with the same price distribution as in Proposition 1. Let us illustrate by considering the simplest case with two firms and two symmetric *simple* frames (e.g., frame A is "Price incl. VAT" and frame B is "Price excl. VAT"). Suppose that when the two firms adopt different frames, if a consumer incurs an information processing cost s, she will be able to compare prices accurately; otherwise, she gets confused and chooses a product randomly. Consumers may have heterogeneous abilities to process information, so we assume that s is distributed in the consumer population according to a cdf $H(s)$ defined on $[0, \infty)$ which is common knowledge.

If in equilibrium a fraction α_1 of consumers give up information processing (and so get confused) when the two firms use different frames, then firms will behave as described in Proposition 1 with $\lambda = 1/2$ (each firm adopts each frame with an equal probability). For $\lambda = 1/2$ and $\alpha_2 = 0$, the price distribution is given by

$$\left(1 - \frac{\alpha_1}{2}\right)[1 - F(p)] = \frac{\alpha_1}{4}\left(\frac{1}{p} - 1\right) \tag{14}$$

with $p_0 = \alpha_1/(4 - \alpha_1)$.

Now let us determine what fraction of consumers will indeed give up information processing when the two firms use different frames and charge prices according to the price distribution in (14). Note that, in equilibrium, this fraction must be equal to α_1. When the firms adopt different frames, if no information processing effort is made, a consumer's expected payment is

$$E[p_i] = \int_{p_0}^{1} p dF(p) = p_0 + \int_{p_0}^{1} [1 - F(p)] \, dp \ ,$$

whereas if the effort is made, a consumer identifies the cheapest product and pays an expected price

$$E[\min\{p_1, p_2\}] = \int_{p_0}^{1} p d\left[1 - (1 - F(p))^2\right] = p_0 + \int_{p_0}^{1} [1 - F(p)]^2 \, dp \ ,$$

where $1 - (1 - F(p))^2$ is the cdf of $\min\{p_1, p_2\}$. Thus, the expected benefit of making the effort is

$$\chi(\alpha_1) = \int_{p_0}^{1} F(p)[1 - F(p)] \, dp = \frac{\alpha_1}{4 - 2\alpha_1}\left(\frac{2}{2 - \alpha_1} \ln \frac{4 - \alpha_1}{\alpha_1} - 2\right) \ .$$

The second step used the expression for F defined in (14). Since all consumers with a cost greater than $\chi(\alpha_1)$ will make no effort, it follows that

$$\alpha_1 = 1 - H(\chi(\alpha_1)) \ .$$

This equation must have a solution between zero and one.[31] For example, when H is a uniform distribution on $[0,1]$, we have a unique solution $\alpha_1 \approx 0.9$. In particular, $\alpha_1 = 0$ cannot form part of an equilibrium so long as $H(0) < 1$. That is, in equilibrium there will be always some consumers who make no information processing effort and get confused when facing different frames.

However, this rational interpretation does not carry over to the case with a separating equilibrium as in Proposition 2 (where the complex frame is always associated with higher prices than the simple one). This is because rational consumers should be able to infer prices from frames (*if* they can distinguish between frames) and should always choose the simple-frame product.[32] However, the separating equilibrium could still make sense if (i) consumers are yet to understand the market equilibrium or they purchase the product infrequently so that they do not have enough opportunities to learn the market equilibrium, or (ii) there is always a non-trivial mass of naive consumers who choose randomly when they get confused.

Carlin (2009) considers a setting related to our case with $\alpha_2 > \alpha_1$. In his model, if a consumer incurs a cost, she can learn *all* prices in the market, thereby purchasing the cheapest product; otherwise, she remains uninformed and shops randomly. Carlin assumes that if an individual firm increases its price complexity, then consumers regard the whole market as being more complex and information gathering as more costly, and so are more likely to remain uninformed.[33] In equilibrium, higher complexity is associated with higher prices. Carlin avoids the inference problem by exogenously assuming that consumers can only observe the aggregate market complexity, but not each firm's price complexity.

A Appendix: Proofs in the Duopoly Case

Proof of Lemma 3: Suppose that equilibrium F_z has a mass point at some price $p \in S_z$. Then, in the symmetric equilibrium, there is a positive probability that both firms use

[31] If $\alpha_1 = 1$, then the left-hand side is greater than the right-hand side; if $\alpha_1 = 0$, then one can check that $\chi(\alpha_1) = 0$ and so the left-hand side is smaller than the right-hand side.

[32] In this sense, our assumption that consumers weakly favor the simple frame (i.e., $\phi(n_A, n_B) \geq n_A/(n_A + n_B)$) partially reflects such sophistication.

[33] In a related vain, Ellison and Wolitzky (2008) considers a sequential price search model in which consumers have convex search costs (i.e., a consumer's incremental search cost increases with her cumulative search effort). Then if a firm increases its in-store search cost by presenting prices in a complicated way, it makes further search more costly and, therefore, less likely. Wilson (2010) considers an alternative two-stage model in which firms differentiate their price complexity in the first stage (e.g., one firm obfuscates and the other does not) in order to soften the price competition in the second stage.

frame z and tie at p. Given $\alpha_2 < 1$, there is always a positive measure of price aware consumers regardless of z, such that for any firm it is more profitable to offer $(z, p - \varepsilon)$ (for some small $\varepsilon > 0$) than (z, p). This leads to a contradiction.

Proof of Proposition 1: The proposed configuration is indeed an equilibrium since no deviation to $p < p_0$ is profitable. We show now that it is the unique symmetric mixed-strategy equilibrium with F_z strictly increasing on its support. Recall that, by Lemma 3, when $\alpha_2 < 1$, in any symmetric mixed-strategy equilibrium F_z is continuous on S_z. The proof entails several steps.

Step 1: $S_A \cap S_B \neq \emptyset$. Suppose $p_1^A < p_0^B$. Then if a firm uses frame A and charges p_1^A, its profit is

$$\pi(A, p_1^A) = p_1^A (1 - \lambda) [(1 - \alpha_1) + \alpha_1/2] \ .$$

The firm has positive demand only if the rival is using frame B, in which case it sells to all price aware consumers and to half of the confused ones. Clearly, this firm can do better by charging a price slightly higher than p_1^A. A contradiction. Similarly, we can rule out the possibility of $p_1^B < p_0^A$.

Step 2: $\max\{p_1^A, p_1^B\} = 1$. Suppose $p_1^z = \max\{p_1^A, p_1^B\} < 1$. Then, p_1^z is dominated by $p_1^z + \varepsilon$ (for some small $\varepsilon > 0$).

Step 3: $S_A = S_B = [p_0, 1]$. Suppose $p_1^A < p_1^B = 1$. Then, along the equilibrium path, if firm i uses frame A and charges $p \in [p_1^A, 1]$, its profit is

$$\pi(A, p) = p (1 - \lambda) [(1 - \alpha_1) x_B(p) + \alpha_1/2] \ ,$$

since it faces a positive demand only if firm j uses frame B. If firm i uses frame B and charges the same price p, its profit is

$$\pi(B, p) = p \{\lambda\alpha_1/2 + (1 - \lambda) [(1 - \alpha_2) x_B(p) + \alpha_2/2]\} \ ,$$

which should be equal to the candidate equilibrium profit. Since the supposition $p_1^A < p_1^B = 1$ and Step 1 imply that $p_1^A \in S_B$, the indifference condition requires $\pi(A, p_1^A) = \pi(B, p_1^A)$ or

$$(1 - \lambda)(\alpha_1 - \alpha_2) - \lambda\alpha_1 = 2(1 - \lambda)(\alpha_1 - \alpha_2) x_B(p_1^A) \ .$$

However, if this equation holds, $\pi(A, p) > \pi(B, p)$ for $p \in (p_1^A, 1]$ as $\alpha_1 > \alpha_2$ and x_B is strictly decreasing on S_B. This is a contradiction. Similarly, we can exclude the possibility of $p_1^B < p_1^A = 1$. Therefore, it must be that $p_1^A = p_1^B = 1$.

Then, from $\pi(A, 1) = \pi(B, 1)$, it follows that

$$\lambda\alpha_1 = (1 - \lambda)(\alpha_1 - \alpha_2) \ . \tag{15}$$

Now suppose $p_0^A < p_0^B$. Then

$$\pi(A, p_0^B) = p_0^B[\lambda x_A(p_0^B) + (1-\lambda)(1-\alpha_1/2)] \text{ and}$$
$$\pi(B, p_0^B) = p_0^B\{\lambda[(1-\alpha_1)x_A(p_0^B) + \alpha_1/2] + (1-\lambda)(1-\alpha_2/2)\} .$$

Since the supposition $p_0^A < p_0^B$ and Step 1 imply that $p_0^B \in S_A$, we need $\pi(A, p_0^B) = \pi(B, p_0^B)$, or

$$2x_A(p_0^B) = 1 + \frac{1-\lambda}{\lambda}\frac{\alpha_1 - \alpha_2}{\alpha_1} .$$

The left-hand side is strictly lower than 2 given that x_A is strictly decreasing on S_A and $p_0^A < p_0^B$. But (15) implies that the right-hand side is equal to 2. A contradiction. Similarly, we can exclude the possibility of $p_0^A < p_0^B$. Therefore, it must be that $p_0^A = p_0^B$.

Step 4: $F_A = F_B$. For any $p \in [p_0, 1]$, the indifference condition requires $\pi(A, p) = \pi(B, p)$. Using (2) and (3), we get

$$\lambda\alpha_1[x_A(p) - 1/2] = (1-\lambda)(\alpha_1 - \alpha_2)[x_B(p) - 1/2]$$

for all $p \in [p_0, 1]$. Then (15) implies $x_A = x_B$ (or $F_A = F_B$).

Proof of Proposition 2: (1) Let us first prove the result for $\alpha_2 < 1$.

(1-1) A deviation to $(A, p < p_0^A)$ is obviously not profitable. A deviation to $(A, p > \hat{p})$ generates a profit equal to

$$p(1-\lambda)[(1-\alpha_1)x_B(p) + \alpha_1/2] .$$

By using (7), one can easily check that this deviation profit is lower than $\pi(B, p)$ in (3) with $x_A(p) = 0$. The last possible deviation is $(B, p < \hat{p})$ which results in a profit equal to

$$p\{\lambda[(1-\alpha_1)x_A(p) + \alpha_1/2] + (1-\lambda)(1-\alpha_2/2)\} .$$

Again, by using (7), one can check that this deviation profit is lower than $\pi(A, p)$ in (2) with $x_B(p) = 1$.

(1-2) We now prove uniqueness. As in the proof of Proposition 1, we can show that $S_A \cap S_B \neq \emptyset$ and $\max\{p_1^A, p_1^B\} = 1$. Then the following two steps complete the proof.

Step 1: $S_A \cap S_B = \{\hat{p}\}$ for some $\hat{p}$. Suppose to the contrary that $S_A \cap S_B = [p', p'']$ with $p' < p''$. Then for any $p \in [p', p'']$, it must be that $\pi(A, p) = \pi(B, p)$, where the profit functions are given by (2) and (3). This indifference condition requires that

$$\lambda\alpha_1[x_A(p) - 1/2] = (1-\lambda)(\alpha_1 - \alpha_2)[x_B(p) - 1/2]$$

for all $p \in [p', p'']$. Since $\alpha_1 < \alpha_2$ and F_z is strictly increasing on S_z, the left-hand side is a decreasing function of p, while the right-hand side is an increasing function of p. So this condition cannot hold for all $p \in [p', p'']$. A contradiction.

Step 2: $p_1^B = 1$. Suppose $p_1^B < 1$. Then Step 1 and $\max\{p_1^A, p_1^B\} = 1$ imply that $p_1^A = 1$ and $p_1^B = p_0^A = \hat{p} < 1$. Then each firm's equilibrium profit should be equal to $\pi(A,1) = (1-\lambda)\alpha_1/2$ since the prices associated with B are lower than one. However, if a firm chooses frame B and $p = 1$, its profit is $[\lambda\alpha_1 + (1-\lambda)\alpha_2]/2$ since it sells to half of the confused consumers. This deviation profit is greater than $\pi(A,1)$ given that $\alpha_2 > \alpha_1$. A contradiction.

Therefore, in equilibrium, it must be that $S_A = [p_0^A, \hat{p}]$ and $S_B = [\hat{p}, 1]$.

(2) The equilibrium when $\alpha_2 = 1$ is just the limit of the equilibrium in (1) as $\alpha_2 \to 1$. However, now $S_A = [p_0^A, 1)$ and $S_B = \{1\}$.

B Appendix: Proofs and Omitted Details in the Oligopoly Model

B.1 The case with $0 < \alpha_2 < \alpha_1 = 1$

Lemma 4 *In the oligopoly model with* $0 < \alpha_2 < \alpha_1 = 1$*, there is no equilibrium in which all firms adopt deterministic frames.*

Proof. We prove this lemma in three steps:

(a) In any possible equilibrium in which firms use deterministic frames, at most one firm uses frame A. Suppose to the contrary that at least two firms use frame A. Then they must all earn zero profit at any putative equilibrium. But then any of them has a unilateral incentive to deviate to frame B and a positive price, which results in a positive profit as $\alpha_2 > 0$. A contradiction.

(b) In any possible equilibrium in which firms use deterministic frames, at least one firm uses frame A. Suppose to the contrary that all firms use frame B. Then with probability α_2 consumers shop randomly, and with probability $1-\alpha_2$ they buy from the cheapest firm. This is a version of Varian (1980), and each firm earns α_2/n.[34] But then any firm can earn more by deviating to frame A and price $p = 1$, which generates a profit of at least $\phi_{n-1} \geq 1/n$. This is because at most $n-1$ B firms can survive and the deviator would never be dominated as $\alpha_1 = 1$.

(c) Consider a candidate equilibrium in which one firm uses A and all other firms use B.[35] First, the A firm must charge price $p = 1$ given that $\alpha_1 = 1$ and make a profit at

[34] For $n \geq 3$, there are both symmetric and asymmetric mixed-strategy equilibria in the Varian model, but all of them are outcome equivalent (Baye et al., 1992).

[35] This part of the proof is different from that in the duopoly case since it is hard to directly characterize the pricing equilibrium when one firm uses frame A and other $n-1 \geq 2$ firms use frame B.

least equal to ϕ_{n-1}. Second, each B firm must also earn at least ϕ_{n-1}. Otherwise, any B firm which earns $\pi_B < \phi_{n-1}$ can improve its profit by deviating to frame A and a price $1-\varepsilon$ for small ε. (The deviator would make a profit at least equal to $(1-\varepsilon)\,\phi_{n-2}$ which is greater than π_B for a sufficiently small ε given that $\phi_{n-2} \geq \phi_{n-1}$.) Then, if $\phi_{n-1} > 1/n$, the sum of all firms' profits exceeds one, and we reached a contradiction since industry profit is bounded by one. The only remaining possibility is that $\phi_{n-1} = 1/n$ and each firm earns exactly $1/n$. However, this means that all firms charge the monopoly price $p = 1$.[36] But then any B firm has an incentive to deviate to a price slightly below one given that $\alpha_2 < 1$. A contradiction. ■

Equilibrium condition for λ when $0 < \alpha_2 < \alpha_1 = 1$:

Since the price distributions for frames A and B share the same upper bound $p = 1$, letting $p = 1$ in (11)–(12) yields a frame indifference condition $\pi(A,1) = \pi(B,1)$. By dividing each side by $(1-\lambda)^{n-1}$ and rearranging the equation, we obtain

$$\alpha_2\left(\phi_{n-1} - \frac{1}{n}\right) + (1-\alpha_2)\,\phi_1 = \alpha_2 \sum_{k=1}^{n-2} \frac{C_{n-1}^{k}\left(1-\phi_{n-k}\right)}{n-k}\left(\frac{\lambda}{1-\lambda}\right)^{k} + (1-\phi_1)\left(\frac{\lambda}{1-\lambda}\right)^{n-1}. \tag{16}$$

The right-hand side of (16) increases in $\lambda \in [0,1]$ from zero to infinity, and the left-hand side is positive for any $\alpha_2 \in [0,1)$ as $\phi_{n-1} \geq 1/n$. Hence, (16) has a unique solution λ in $(0,1)$ as we claimed in the main text.

The (im)possibility of price-frame independence. Recall that in the duopoly equilibrium in Proposition 1 pricing is frame independent (i.e., $F_A(p) = F_B(p)$). This feature of the duopoly equilibrium does not carry over to the oligopoly case. With duopoly, there are only two types of consumers. When $\alpha_2 < \alpha_1$, the equilibrium λ ensures that regardless of its frame choice a firm faces the same market composition if consumers have no exogenous bias toward a particular frame. This underlies price-frame independence. With more than two firms, there are in general more than two types of consumers. Although equilibrium λ ensures that the expected number of consumers who are totally insensitive to a firm's price is the same regardless of this firm's frame choice (i.e., $\pi(A,1) = \pi(B,1)$), this no longer guarantees that this firm also faces the same expected number of other types of consumers. In general, it is impossible for a firm to face the same market composition when it shifts from one frame to the other, and so its pricing needs to adjust to different environments. The following result gives the

[36] If some firm charged a price lower than one with a positive probability, then at that price its demand would be positive (otherwise its equilibrium profit would be zero, which contradicts the fact that each firm earns $1/n$). But then consumer surplus would be positive. A contradiction.

conditions for price-frame independence. It shows that the independence result holds only in special cases.

Proposition 8 *In the oligopoly model with $\alpha_2 < \alpha_1 = 1$,*
(i) for $n = 2$, the symmetric equilibrium in Proposition 3 dictates $F_A = F_B$ only if $\phi_1 = 1/2$;
(ii) for $n \geq 3$, the symmetric equilibrium in Proposition 3 dictates $F_A = F_B$ only if $\phi_1 = 1/2$ and $\alpha_2 = 0$, or for a particular non-uniformly random purchase rule $\{\phi_k\}_{k=1}^{n-1}$.

Proof. At equilibrium, each firm's demand can be decomposed in two parts: the consumers who are insensitive to its price, and the consumers who are price-sensitive. Explicitly, we have

$$\pi(A,p)/p = \pi(A,1) + \{\lambda^{n-1} x_A(p)^{n-1} + \sum_{k=1}^{n-2} P_{n-1}^k x_A(p)^k \left[\alpha_2 \phi_{n-k-1} + (1-\alpha_2)\phi_1\right]\} ,$$

and

$$\pi(B,p)/p = \pi(B,1) + \{(1-\alpha_2)(1-\lambda)^{n-1} x_B(p)^{n-1} + (1-\alpha_2)(1-\phi_1)\sum_{k=1}^{n-2} P_{n-1}^k x_B(p)^{n-k-1}\} .$$

Suppose now $x_A(p) = x_B(p) = x(p)$, and the common support is $[p_0, 1]$. At equilibrium, $\pi(A,p) = \pi(B,p)$ must hold for any $p \in [p_0, 1]$.
(i) For $n = 2$, the last term in each demand function disappears. To have $\pi(A,p) = \pi(B,p)$ for any $p \in [p_0, 1]$, we need $\pi(A,1) = \pi(B,1)$, or equivalently

$$\frac{\lambda}{1-\lambda} = \frac{\phi_1 - \alpha_2/2}{1-\phi_1} ,$$

and $\lambda = (1-\alpha_2)(1-\lambda)$, or equivalently

$$\frac{\lambda}{1-\lambda} = 1 - \alpha_2 .$$

It follows that these two conditions hold simultaneously if and only if $\phi_1 = 1/2$.
(ii) With $n \geq 3$, to have $\pi(A,p) = \pi(B,p)$ for any $p \in [p_0, 1]$, we need $\pi(A,1) = \pi(B,1)$ (see (16)), and

$$\lambda^{n-1} + \sum_{k=1}^{n-2} P_{n-1}^{n-k-1} x(p)^{-k} \left[\alpha_2 \phi_k + (1-\alpha_2)\phi_1\right]$$
$$= (1-\alpha_2)(1-\lambda)^{n-1} + (1-\alpha_2)(1-\phi_1)\sum_{k=1}^{n-2} P_{n-1}^k x(p)^{-k} .$$

(To derive the latter, we divided each side by $px(p)^{n-1}$ and relabelled k in $\pi(A,p)$ by $n-k-1$.) Then

$$\sum_{k=1}^{n-2} b_k x(p)^{-k} = (1-\alpha_2)(1-\lambda)^{n-1} - \lambda^{n-1} \tag{17}$$

where

$$b_k \equiv P_{n-1}^{n-k-1}\left[\alpha_2\phi_k + (1-\alpha_2)\phi_1\right] - P_{n-1}^{k}(1-\alpha_2)(1-\phi_1) \ .$$

Since the left-hand side of (17) is a polynomial of $1/x(p)$ and $x(p)$ is a decreasing function, (17) holds for all $p \in [p_0, 1]$ only if $b_k = 0$ for $k = 1, \cdots, n-2$ and the right-hand side is also zero. That is,

$$\left(\frac{\lambda}{1-\lambda}\right)^{n-1} = 1-\alpha_2 \tag{18}$$

and

$$\left(\frac{\lambda}{1-\lambda}\right)^{n-2k-1} = \frac{(1-\alpha_2)(1-\phi_1)}{\alpha_2\phi_k + (1-\alpha_2)\phi_1} \quad \text{for } k = 1, \cdots, n-2 \ . \tag{19}$$

If $\alpha_2 = 0$, both of them and (16) hold for $\phi_1 = 1/2$ (in which case, $\lambda = 1/2$). Beyond this special case, (19) pins down a decreasing sequence $\{\phi_k\}_{k=1}^{n-2}$ uniquely. Substituting (18) and (19) into (16), we can solve for ϕ_{n-1}. This means that, if $n \geq 3$ and $\alpha_2 > 0$, price-frame independence can hold only for a particular sequence of ϕ_k.[37] It is easy to verify that $\phi_k = 1/(1+k)$ does not satisfy these conditions. ∎

Proof of Proposition 4: When frame B is also a simple frame (i.e., when $\alpha_2 = 0$), the equilibrium condition (16) for λ becomes

$$\frac{\lambda}{1-\lambda} = \left(\frac{\phi_1}{1-\phi_1}\right)^{1/(n-1)} \ .$$

It follows that λ tends to $1/2$ as $n \to \infty$.[38] Then industry profit $n\pi = n\phi_1(1-\lambda)^{n-1}$ must converge to zero.[39]

[37] Note that, although $\{\phi_k\}_{k=1}^{n-2}$ solved from (19) is a decreasing sequence, still ϕ_{n-1}, which is solved from (16), may not be lower than ϕ_{n-2}. For example, when $n=3$, one can check that

$$\phi_1 = \frac{1-\alpha_2}{2-\alpha_2} < \phi_2 = \frac{\phi_1 + 1/3 + \sqrt{1-\alpha_2}}{1+\sqrt{1-\alpha_2}},$$

which violates the requirement that ϕ_k is non-increasing in k.

[38] How λ varies with n also depends on the value of ϕ_1. If $\phi_1 > 1/2$, λ decreases to $1/2$ with n; if $\phi_1 = 1/2$, λ is a constant (equal to $1/2$); and if $\phi_1 < 1/2$, λ increases to $1/2$ with n.

[39] However, industry profit $n\pi$ can rise with n when n is small and ϕ_1 takes relatively extreme values. For example, when $\phi_1 = 0.95$ or 0.05, from $n=2$ to 3, industry profit $n\pi$ increases from 0.095 to about 0.099.

Now consider $\alpha_2 > 0$. Since the left-hand side of (16) is bounded, it must be that $\lim_{n\to\infty} \lambda \leq 1/2$ (otherwise the right-hand side would tend to infinity). Since $\{\phi_k\}_{k=1}^{n-1}$ is a non-increasing sequence, the right-hand side of (16) is greater than

$$\frac{\alpha_2 (1-\phi_1)}{n} \sum_{k=1}^{n-2} C_{n-1}^k \left(\frac{\lambda}{1-\lambda}\right)^k = \frac{\alpha_2 (1-\phi_1)}{n} \left[\frac{1-\lambda^{n-1}}{(1-\lambda)^{n-1}} - 1\right] .$$

So it must be that $\lim_{n\to\infty} n (1-\lambda)^{n-1} > 0$, otherwise the right-hand side of (16) would tend to infinity (given that $\lim_{n\to\infty} \lambda \leq 1/2$ and so $\lim_{n\to\infty}(1-\lambda^{n-1}) = 1$). This result implies that λ must converge to zero and industry profit

$$n\pi = n (1-\lambda)^{n-1} \left[\alpha_2 \phi_{n-1} + (1-\alpha_2) \phi_1\right]$$

must be bounded away from zero as $n \to \infty$.

Proof of Proposition 5: Note that with the random purchase rule $\phi_k = 1/(1+k)$, (16) becomes

$$1 - \alpha_2 = 2\alpha_2 \sum_{k=1}^{n-2} \frac{C_{n-1}^k}{n-k+1} \left(\frac{\lambda}{1-\lambda}\right)^k + \left(\frac{\lambda}{1-\lambda}\right)^{n-1} , \tag{20}$$

and industry profit is

$$n\pi = n (1-\lambda)^{n-1} \left(\frac{\alpha_2}{n} + \frac{1-\alpha_2}{2}\right) . \tag{21}$$

(i) For $n = 2$, we have $\lambda = \frac{1-\alpha_2}{1+(1-\alpha_2)}$; and for $n = 3$, we have $\lambda = \frac{x}{1+x}$ with $x = \sqrt{4\alpha_2^2/9 + 1 - \alpha_2} - 2\alpha_2/3$. The latter is smaller if $x < 1 - \alpha_2$, which can be easily verified given that $\alpha_2 < 1$. The industry profit result follows from straightforward algebra calculation by using (21).

(ii) We consider the limit case of $\alpha_2 \to 1$. The equilibrium condition (20) implies that λ should then tend to zero. As $\lambda \approx 0$, we have $\lambda/(1-\lambda) \approx \lambda + \lambda^2$. For $n \geq 4$, the right-hand side of (20) can be approximated as

$$2\alpha_2 [\frac{n-1}{n}(\lambda + \lambda^2) + \frac{n-2}{2}(\lambda+\lambda^2)^2] \tag{22}$$

by discarding all higher-order terms. (For $n = 3$, the right-hand side of (20) is approximated by $\frac{4}{3}\alpha_2(\lambda+\lambda^2) + (\lambda+\lambda^2)^2$. One can check that the approximation result below still applies.)

Let $\alpha_2 = 1 - \varepsilon$ with $\varepsilon \approx 0$, and use the second-order (linear) approximation $\lambda \approx k_1\varepsilon + k_2\varepsilon^2$. Substituting them into (22) and discarding all terms of order higher than ε^2, we obtain

$$\frac{2(n-1)}{n} k_1 \varepsilon + \left(\frac{2(n-1)}{n}(k_2 - k_1) + \frac{n^2-2}{n} k_1^2\right) \varepsilon^2 .$$

Since the left-hand side of (20) is ε, we can solve

$$k_1 = \frac{n}{2(n-1)}; \quad k_2 = k_1 - \frac{n^2-2}{2(n-1)}k_1^2 .$$

As k_1 decreases with n, λ must decrease with n.

As $\varepsilon \approx 0$ (so that $\lambda \approx 0$), industry profit (for $n \geq 3$) can be approximated as

$$\begin{aligned} n\pi &= (1-\lambda)^{n-1}[1+(\frac{n}{2}-1)\varepsilon] \\ &\approx [1-(n-1)\lambda + C_{n-1}^2\lambda^2][1+(\frac{n}{2}-1)\varepsilon] \\ &\approx 1-\varepsilon+\frac{(n-2)n^2}{8(n-1)^2}\varepsilon^2 . \end{aligned}$$

The second step follows from discarding all terms of order higher than λ^2, and the third step comes from substituting $\lambda \approx k_1\varepsilon + k_2\varepsilon^2$ and discarding all terms of order higher than ε^2. It is ready to see that the approximated industry profit increases with n. (Notice that the first-order approximation of λ is not sufficient to tell how $n\pi$ varies with n.)

B.2 The case with $0 < \alpha_1 < \alpha_2 = 1$

We first show that there is no pure strategy equilibrium in this case either, and then prove Proposition 6.

Lemma 5 *In the oligopoly model with* $0 < \alpha_1 < \alpha_2 = 1$*, there is no equilibrium in which all firms use deterministic frames.*

Proof. We prove this lemma in three steps:

(a) In any pure strategy framing equilibrium, at most one firm uses frame A. Suppose to the contrary that at least two firms use frame A. Then, they must all earn zero profit at any putative equilibrium. But then any of them has a unilateral incentive to deviate to frame B and a positive price. A contradiction.

(b) In any pure strategy framing equilibrium, at least one firm uses frame A. Suppose to the contrary that all firms use frame B. The only candidate equilibrium entails monopoly pricing $p = 1$ and each firm earns $1/n$. But then if one firm deviates to frame A and price $1-\varepsilon$, it will earn $(1-\varepsilon)\left(\alpha_1\phi_{n-1} + 1 - \alpha_1\right)$. The reason is that, if the consumer is unable to compare prices in different frames (which happens with probability α_1), the deviator's demand is ϕ_{n-1}; if the consumer is able to compare prices in different frames (which happens with probability $1-\alpha_1$), the deviator serves the whole market (because all other firms charge $p = 1$ and so are dominated options). As $\phi_{n-1} \geq 1/n$, the deviation profit is greater than $1/n$ for a sufficiently small ε and any $\alpha_1 \in (0,1)$.

(c) The final possibility is that one firm uses A and all other firms use B. Suppose such an equilibrium exists. Let π_A be A firm's profit and π_B^j be the profit of a B firm indexed by j. (Notice that the B firms may use different pricing strategies and make different profits). Let p_A be the lowest price on which the A firm puts positive probability (it might be a deterministic price). (i) Suppose that, at equilibrium, $\pi_A > \min\{\pi_B^j\}$. Then, if the B firm which earns the least deviates to frame A and a price $p_A - \varepsilon$, it will replace the original A firm and have a demand at least equal to the original A firm's demand since it now charges a lower price *and* faces fewer competitors.[40] So, this deviation is profitable at least when ε is close to zero. A contradiction. (ii) Suppose now that, at equilibrium, $\pi_A \leq \min\{\pi_B^j\}$. Notice that $\pi_A \geq 1/n$, otherwise the A firm would deviate to frame B and a price $p = 1$, and make profit $1/n$. As industry profit cannot exceed one, all firms must earn $1/n$ at the candidate equilibrium and consumer surplus is zero. This also implies that all firms must be charging the monopoly price. But then any B firm has an incentive to deviate to frame A and price $1-\varepsilon$, in which case it makes profit $(1-\varepsilon)\left(\alpha_1\phi_{n-2} + 1 - \alpha_1\right) > 1/n$ for a sufficiently small ε. A contradiction. ■

We then characterize a symmetric mixed-strategy equilibrium (λ, F_A, F_B) in which λ is the probability of using frame A, F_A is defined on $S_A = [p_0^A, 1)$ and is atomless, and F_B is degenerate on $S_B = \{1\}$.

Along the equilibrium path, if firm i uses frame A and charges $p \in [p_0^A, 1)$, its profit is given by

$$\pi(A,p) = p \sum_{k=0}^{n-1} P_{n-1}^k x_A(p)^k \left(\alpha_1 \phi_{n-k-1} + 1 - \alpha_1\right) . \tag{23}$$

This expression follows from the fact that, when k other firms also use frame A, firm i has a positive demand only if all other A firms charge prices higher than p. Conditional on that, with probability α_1, the consumer is confused by frame differentiation and buys from firm i with probability ϕ_{n-k-1} (since all $n-k-1$ firms which use B are undominated); with probability $1-\alpha_1$, the consumer can compare A and B and, because all B firms charge price $p_B = 1 > p$ and consequently are dominated, she only buys from firm i.

A firm's equilibrium profit is equal to

$$\pi = \lim_{p \to 1} \pi(A,p) = (1-\lambda)^{n-1} \left(\alpha_1 \phi_{n-1} + 1 - \alpha_1\right) . \tag{24}$$

[40] When the consumer is unable to compare prices in different frames, the deviator's demand is ϕ_{n-2} which is (weakly) greater than ϕ_{n-1}, the original A firm's demand in this case. When the consumer is able to compare prices in different frames, the deviator is more likely to dominate the remaining B firms (and so to have a higher expected demand) than the original A firm.

Then the expression for $F_A(p)$ follows from $\pi(A,p) = \pi$, and p_0^A satisfies $\pi(A, p_0^A) = \pi$. Both of them are well defined.

If firm i uses B and charges $p = 1$, then its profit is

$$\pi(B,1) = \frac{(1-\lambda)^{n-1}}{n} + \alpha_1 \sum_{k=1}^{n-1} P_{n-1}^k \frac{1-\phi_{n-k}}{n-k} \ . \tag{25}$$

Notice that firm i has a positive demand only if all other firms also use frame B, or there are A firms but the consumer is unable to compare prices in different frames.

The equilibrium condition $\pi(B,1) = \lim_{p\to 1} \pi(A,p)$ pins down a well-defined λ:

$$\frac{1-1/n}{\alpha_1} + \phi_{n-1} - 1 = \sum_{k=1}^{n-1} \frac{C_{n-1}^k \left(1-\phi_{n-k}\right)}{n-k} \left(\frac{\lambda}{1-\lambda}\right)^k \ . \tag{26}$$

The left-hand side of (26) is positive given that $\phi_{n-1} \geq 1/n$, and the right-hand side is increasing in λ from zero to infinity. Hence, for any given $n \geq 2$ and $\alpha_1 \in (0,1)$, equation (26) has a unique solution λ in $(0,1)$.

To complete the proof of Proposition 6, we only need to rule out profitable deviations from the proposed equilibrium. First, consider two possible deviations with frame A: (i) a deviation to $\left(A, p < p_0^A\right)$ is not profitable as the firm does not gain market share, but loses on prices; (ii) a deviation $(A, p = 1)$ is not profitable either, since the deviator's profit is $(1-\lambda)^{n-1} \phi_{n-1} < \pi$.

Let us now consider a deviation to $\left(B, p \in [p_0^A, 1)\right)$. Deviator's profit is

$$\hat{\pi}(B,p) = p\pi(B,1) + p(1-\alpha_1) \sum_{k=1}^{n-1} P_{n-1}^k x_A(p)^k \ .$$

This expression captures the fact that when $n-1$ other firms also use B, or when $k \geq 1$ firms use A and the consumer is confused between A and B, firm i's demand does not depend on its price so that it is equal to $\pi(B,1)$. When $k \geq 1$ firms use A and the consumer is not confused between A and B, all other B firms (which charge price $p = 1$) are dominated by the cheapest A firm, and the consumer buys from firm i only if the cheapest A firm charges a price greater than p. Notice that, from $\pi(A,p) = \pi$ for $p \in [p_0^A, 1)$, the second term in $\hat{\pi}(B,p)$ is equal to

$$\pi - p\pi - p\alpha_1 \sum_{k=1}^{n-1} P_{n-1}^k x_A(p)^k \phi_{n-k-1} \ .$$

Then, $\hat{\pi}(B,p) < p\pi + \pi - p\pi = \pi$. The deviation to $\left(B, p < p_0^A\right)$ will result in a lower profit. This completes the proof.

Proof of Proposition 7:

From (26), it follows that $\lambda \to 1$ as $\alpha_1 \to 0$. Let $\alpha_1 = \varepsilon$ with $\varepsilon \approx 0$, and $\lambda = 1 - \delta$ with $\delta \approx 0$. Then the right-hand side of (26) can be approximated as

$$(1 - \phi_1) \left(\frac{1 - \delta}{\delta} \right)^{n-1} \approx \frac{1 - \phi_1}{\delta^{n-1}} \, ,$$

since only the term with $k = n - 1$ matters when $\delta \approx 0$. Hence, from (26), we can solve

$$\delta \approx \left(\frac{1 - \phi_1}{\frac{1}{\varepsilon}(1 - \frac{1}{n}) + \phi_{n-1} - 1} \right)^{1/(n-1)} \approx \left(\frac{n(1 - \phi_1)\varepsilon}{n - 1} \right)^{1/(n-1)} .$$

The second step follows from the fact that $\phi_{n-1} - 1$ is negligible compared to $\frac{1}{\varepsilon}(1 - \frac{1}{n})$. Given that $\varepsilon \approx 0$, it is not difficult to see that δ increases with n (e.g., one can show that $\ln \delta$ increases with n). Hence, λ decreases with n. As $\varepsilon \approx 0$, industry profit is

$$n\pi = n\delta^{n-1}[1 + (\phi_{n-1} - 1)\varepsilon] \approx \frac{n^2(1 - \phi_1)\varepsilon}{n - 1}$$

by discarding the term of ε^2. Clearly, $n\pi$ increases with n.

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