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Imperfect Platform Competition: A General Framework

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Imperfect Platform Competition:
A General Framework*

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Abstract

We propose a general model of imperfect competition among multi-product firms, the consumption of whose goods yields externalities from one consumer to another. We extend the allocation approach of Weyl (2010)'s monopoly model, proposing a solution concept, *Insulated Equilibrium*, that allows for tractable analysis of competition. In such an equilibrium each firm's price on one side of the market adjusts to all firms' participation levels on the other side, so as to insulate its own allocation. This eliminates both the indeterminacy of consumer reactions once platforms have set their tariffs and the multiplicity of reaction functions that platforms can have to one another's tariffs. Our approach allows us to derive intuitive first-order conditions characterizing equilibrium without restrictive assumptions and to analyze the effects of competition, mergers and regulation.

Keywords: Two-sided Markets, Multi-sided Platforms, Quality Competition, Oligopoly, Antitrust of Network Industries

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1 Introduction

Network neutrality, anticompetitive practices by operating system makers, mergers between search-advertising providers, regulation of the credit card industry and managing the competitive effects of declining newspapers have been among the leading concerns of competition authorities and applied industrial economists in recent years (Evans et al., 2006). Consequently a burgeoning theoretical literature has developed that models "two-sided markets" and "multi-sided platforms" in which firms have market power over services to several groups of users whose consumption has externalities for the value of consumption by other users.[1] Unfortunately this literature has had a relatively limited impact on on applied and empirical analysis because, while applied analysis of "one-sided markets" has increasingly incorporated richer and more realistic structures of demand and supply, modeling of platform competition has been restricted to highly stylized models. In what follows we argue that the restricted reach of the theory of platform competition is due to a basic indeterminacy in this theory that makes solving rich models intractable. Our primary contribution is then to propose a solution concept to clear this roadblock.

Video game developers care not only about the license fees they pay but also about the number of consumers who own the console for which they are developing. Conversely, gamers' value from owning a console derives largely from the media available for it (Lee, 2010b). Therefore (Katz and Shapiro, 1985), a console producer's profits depend not only on prices charged, but also on consumer expectations about other consumers' participation. This leads to the possibility multiple equilibria (Ellison and Fudenberg, 2003) in the interaction *among consumers, holding prices fixed*. This makes it difficult to even clearly define the static game played by the platforms. Such concerns make it attractive for a modeler to allow, as in Rochet and Tirole (2003), the quite realistic possibility of firms conditioning their prices on the participation of consumers on the other side of their platform. However, as first observed by Armstrong (2006), the flexibility this allows for off-equilibrium beliefs permits, for reasons closely related to the argument of Klemperer and Meyer (1989), virtually any outcome to be consistent with equilibrium. As a result of these two challenges, *consumer coordination* and *Armstrong's paradox*, only very tightly restricted models of multi-sided platforms have been tractable, none of which come close to nesting as a special case the richness of standard static industrial organization models such as Berry et al. (1995).

Weyl (2010) shows how consumer coordination can safely be ignored when a monop-

[1] An excellent recent survey of this literature, pioneered by Caillaud and Jullien (2003) and Rochet and Tirole (2003), is given by Rysman (2009).

olist platform charges *insulating tariffs* that adjust the price on one side of the market, in response to changes in the number of participants on the other side, so as to maintain the desired level of participation. In doing so, insulating tariffs guarantee to members of each side that a given level of participation on the other side will indeed occur in equilibrium.

In Section 5 we extend his argument in two ways. First, we show how to generalize the insulating tariff to the context of oligopoly: each firm charges an insulating tariff on the residual inverse demand system for her product determined by the tariffs charged by all other firms. These together form an *insulating tariff system* under which there is always a unique equilibrium among consumers. That is, the consumer coordination problem disappears. Second, we show that because the insulating tariff uniquely ties down the shape of price functions it also obviates Armstrong's paradox, as firms' best responses are generically unique. Thus, under the refinement of *Insulated Equilibrium*, measurement of the demand system and equilibrium prices suffice to identify firms' marginal costs, returning us to the familiar standard conditions of static industrial organization.

The foundation of our approach is a natural extension, described in Section 4 of a classical result from quasi-linear general equilibrium theory to our smooth, large two-sided economy. While consumers may coordinate on a variety of outcomes, given any set of prices that prevail, any given *allocation* (vector of participation rates on the two sides of the market) is consistent with only a single vector of prices on the two sides, representing the utility of marginal consumers at that allocation. Thus, regardless of consumer beliefs, a given allocation always leads to the same payoffs for all agents. Therefore, despite the fact that our solution concept is Nash-in-prices in the sense that firms take other firms' pricing functions as given when choosing their own, it is useful to think of the firms as choosing the allocation consistent with their residual inverse demand that maximizes profits. This closely resembles Myerson (1981)'s argument that it is more convenient to solve for optimal allocations in auctions and then derive from them the implementation of this allocation than to directly compare implementations.

Armed with these technical tools, in Section 6 we develop first-order conditions characterizing the Insulated Equilibria. In particular, we show that platforms' prices under Insulated Equilibrium bear an intuitive relationship to the monopoly pricing formula of Weyl (2010). An appealing feature of this monopoly formula, which we discuss in detail in Section 6, is its identification of the two forces that push a two-sided monopolist to induce an allocation that departs from the one that is socially optimal. One of these forces is the classical *market power distortion* and the other is the *Spence distortion*, owing its name to the seminal analysis in Spence (1975) of a monopolist's choice of quality.

We show that under competition, these continue to be the two fundamental forces governing the relationship between the equilibrium allocation and the optimum. While, as a general matter, the effect that an intensification of competition has on the market power distortion is well known, the effect of such an intensification on the Spence distortion is less well understood and potentially ambiguous in a way that depends on a larger set of factors. Importantly, our analysis applies to a much broader class of economies than that of any previous literature in this area of which we are aware. In particular, we make no specific assumptions on any of the following

- Functional forms for firm costs or distribution of user preferences
- The dimensions of heterogeneity of consumer preferences
- Number and symmetry of firms
- Single versus multi-homing (i.e. consumption patterns)

While the main model we consider has exactly two sides, in Section 7, we show how our model may be easily extended to accommodate an arbitrary number of sides. We have also worked out a version of the model that includes within-side externalities and will include this in the next version of the paper, which we discuss below. We therefore believe our approach allows the analysis of models of platform competition at least as rich as those typically used to evaluate competition and regulatory policy in markets without consumption externalities according to principles which are intuitive extensions of the standard reasoning in static industrial organization models.

To illustrate this, and the advantages it offers compared to previous work on multi-sided platforms, we precede the development of our formal model, in Section 3, by illustrating, in Section 2, the predictive payoffs our approach delivers, as well as the dimensions described above along which we generalize. Following this, we derive our main technical results in Sections 4 and 5. In Section 6, we discuss our results, comparing socially optimal versus Insulated Equilibrium pricing and considering several examples. In the near future, we plan to have an updated version of this work, with numerous additional sections. In Section 8, we conclude by discussing these as well as broader directions for future research.

2 Our Contribution In Context

In this section we preview the payoffs delivered by the solution concept that we propose in the main section of the paper, and we describe the ways in which our results enrich previous literature on multi-sided platforms.

2.1 Our contribution

Our model and solution concept provide precise and intuitive, but general, first-order conditions characterizing the market equilibrium of competing multi-sided platforms. These generalize both the optimality conditions for a multi-sided monopolist, derived by Weyl (2010), to a competitive setting, and the classical conditions for Nash-in-prices equilibrium in a differentiated products industry, to a multi-sided setting.

Let j denote a particular firm, $\mathcal{I}$ denote a side of the market, P denote price, N denote the fraction of consumers participating, $C_{\mathcal{I}}$ denote marginal cost of serving side i, μ denote the inverse (partial) hazard rate of demand (the standard "market power" or Cournot distortion often denoted by $P'Q$) and $\mathbf{D}$ represent the *diversion ratio matrix* with i,jth element $\frac{\frac{\partial N^i}{\partial P^j}}{-\frac{\partial N^i}{\partial P^i}}$. The first-order condition for insulated equilibrium pricing is that for each firm j, on each side of the market $\mathcal{I}$

$$\underbrace{P^{\mathcal{I},j} = C^j_{\mathcal{I}} + \mu^{\mathcal{I},j}}_{\text{exactly as in standard market}} \quad -N^{\mathcal{J},j} \cdot \underbrace{\left(\left[\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{P}^{\mathcal{J}}}\right]^{-1}\left[\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{N}^{\mathcal{I}}}\right]\right)_{j,\cdot}}_{\approx\text{average value of marginal opposite-side user}} \cdot -\mathbf{D}^{\mathcal{I}}_{\cdot,j} \tag{1}$$

where $\mathbf{M}_{j,\cdot}$ and $\mathbf{M}_{\cdot,j}$ represent, respectively, the jth row and column of the matrix $\mathbf{M}$. Note that the first terms come directly from Neoclassical industrial organization theory: price equals marginal cost plus the optimal differentiated Bertrand mark-up, the partial inverse hazard rate of demand. To interpret the additional "two-sided markets" term it is useful to compare it to that arising in the monopoly setting of Weyl (2010) where $\left(\left[\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{P}^{\mathcal{J}}}\right]^{-1}\left[\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{N}^{\mathcal{I}}}\right]\right)_{j,\cdot} \cdot -\mathbf{D}^{\mathcal{I}}_{\cdot,j}$ collapses to the *average* willingness of a *marginal* side $\mathcal{J}$ user to pay for the participation of a *marginal* user on side $\mathcal{I}$. This is the part of the externality created by this marginal side $\mathcal{I}$ user that the platform can extract per user on side $\mathcal{J}$.

As we discuss extensively in Subsection 6.2, the broader expression we show is valid under oligopoly is a natural oligopoly extension of this same notion. The jth diagonal entry of $\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{P}^{\mathcal{J}}}$ is the density of $\mathcal{J}$-side users just indifferent between consuming a bundle including platform j and consuming a bundling excluding it: the mass of j's marginal users. This matrix's i,jth entry for $i \neq j$ is (if platforms are never complements) the mass of users indifferent between consuming a bundle including platform i but not platform j and consuming a bundle including platform j but not platform i: the mass of "switching" users marginal between i and j. Thus $\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{P}^{\mathcal{J}}}$ is a natural multi-product extension of the "mass of marginal users". Similarly, we show that the jth diagonal entry of $\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{N}^{\mathcal{I}}}$ is the product of the density of j's marginal users and the average value these place on a marginal side i

user, while its i, jth entry for $i \neq j$ is the density of switch of i, j switching users multiplied by the average value such users would place on a marginal side $\mathcal{I}$ user joining platform j if they were to join j. Thus this matrix is a natural extension of the product of the mass of marginal users and their average marginal valuations for users on the other side.

We can therefore see $\left(\left[\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{P}^{\mathcal{J}}}\right]^{-1}\left[\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{N}^{\mathcal{I}}}\right]\right)_{j,\cdot} \cdot -\mathbf{D}^{\mathcal{I}}_{\cdot,j}$ as a straightforward generalization to the oligopoly setting of Weyl (2010)'s monopoly pricing rule, in the same way that, for example, the matrix equation for a multivariate regression generalizes the ratio of the covariance to the variance of the regressor. For example, in the case considered by Armstrong (2006) when all marginal values are constant and homogeneous across all individual-platform pairs, this quantity collapses to exactly that marginal value.

This allows us to consider the impact of intensified competition on the relationship between social and private objectives. While in standard markets it is well-known that intensified competition will reduce incentives for distortionary above-cost pricing, Weyl argues that market power introduces a second Spence (1975) distortion into pricing, as platforms have an incentive to incorporate externalities to average marginal users, rather than the average of all users. As we argue in Subsections 6.4, whether competition is likely to alleviate or exacerbate the Spence distortion depends on the nature of heterogeneity *among platforms*. If, as in Subsection 6.5, platforms differ along horizontal or vertical dimensions orthogonal to consumer valuations of externalities, then competition is likely to ameliorate the Spence distortion as it leads platforms to attend switching rather than exiting users' valuation of externalities, which are more likely to be representative of the full population of participating users. However, if platforms differentiate themselves vertically in the number of users they have on the other side of the market, users switching between the platforms are likely to have valuations for users on the other side below those of the "high quality" and above those of the "low quality" platform, while exiting marginal users may be more representative of the participating users on each platform.

2.2 Context

Why were not such simple and general results feasible in prior work? The two issues of multiplicity we discussed in the introduction, consumer coordination and Armstrong's paradox, stymied the tractability of a general model of multi-sided platforms. We now discuss these two challenges as well as the dimensions along which we generalize, and fail to generalize, the existing literature.

Caillaud and Jullien (2003), Ellison and Fudenberg (2003), Ellison et al. (2004) Hagiu (2006), Ambrus and Argenziano (2009), Lee (2010a) and Anderson et al. (2010) study the coordination of consumers given prices. Many equilibria are possible in these settings and the payoffs in the game played by platforms depend sensitively on which equilibrium consumers are assumed to coordinate on by refinements in the second stage. Our approach instead attributes the role of coordination to *platforms*, who have a large stake in the matter and significant powers over the outcome, rather than to consumers, who are multitudinous and dispersed.[2]

Armstrong's Paradox, whereby infinitely many allocations can be supported as equilibria, or not, among competing platforms, stems from Proposition 3 of Armstrong (2006). Armstrong argues that firms' best responses are determined by the exact degree to which competitors' prices on one side of the market respond to changes in the number of consumers on the other side, but that only the level of prices, and not the slope of such responses, is tied down by equilibrium. We discuss this issue in more detail in Subsection 5.3. While the motivation for Insulated Equilibrium is the view that it is reasonable to expect platforms to pin down consumer behavior, if platforms indeed do this, then Armstrong's Paradox is resolved.[3]

Regarding the ways in which our model generalizes with respect to existing literature,[4] a crucial aspect is its accommodation of arbitrary preference heterogeneity among consumers. Weyl (2010) shows that the comparative statics of a model of a two-sided monopolist depend crucially on whether consumers differ primarily in their valuations for *membership* or in their valuations for *interaction* with other consumers. However, with little or no empirical basis for these assumptions, in prominent theoretical models of competing platforms, such as those in Anderson and Coate (2005), Armstrong (2006), Armstrong and Wright (2007) and Peitz and Valletti (2008), and in applied works, such as those of Rysman (2004), Kaiser and Wright (2006) and Argentesi and Filistrucchi (2007), consumers are assumed to be homogenous in their interaction values. [5] A crucial implication of such a setup is that it rules out, *ex hypothesi*, the Spence distortion (discussed in the introduction

[2]Dybvig and Spatt (1983) discuss a seemingly similar but in most cases quite different notion of insurance.

[3]In a recent paper, Reisinger (2010) proposes an alternative approach to getting around Armstrong's Paradox, in a setting with the particular assumptions of Armstrong's model and abstracting from the issue of multiplicity of Consumer Equilibria. In essence, this approach points out that, under two-part tariffs, introducing heterogeneity in consumers' interaction behavior is equivalent to allowing them to price discriminate in a regime with flat pricing. This, in turn, ties down platforms' competitive responses to one another.

[4]Pioneering works in the literature on multi-side platforms include Caillaud and Jullien (2003), Rochet and Tirole (2003), Evans (2003) and Parker and Van Alstyne (2005). The best-known model of a monopoly platform is perhaps that of Rochet and Tirole (2006), which is generalized by Weyl (2010), while the best-known model of competing platforms is likely Armstrong (2006).

[5]As Armstrong (2006) says of consumer preferences that are heterogeneous in both membership and interaction values, "A full analysis of this case is technically challenging in the case of competing platforms" (p. 671).

and in Section 6). Our model provides a framework for analyzing the interaction between this distortion and variation in the competitive environment.[6]

Our approach also does not require making assumptions on functional forms of, for instance, the distribution of consumer preferences or the platforms' cost curves. In contrast, a common assumption in models of competition, following Armstrong (2006), has been that of a two-sided Hotelling (1929) setup giving rise to linear demand. Several benefits come from relaxing this assumption, including compatibility with the approach taken in the empirical industrial organization literature (e.g. Berry et al. (1995)), reduced vulnerability to the forms of criticism given in Werden et al. (2004) to using such models as bases for arguments in antitrust cases. Furthermore, Jaffe and Weyl (2010a) have recently shown that with more than two firms, it is impossible for a discrete choice model to generate linear demand.

This paper's framework does not restrict the number of firms that can compete or require them to be symmetric, making the model more realistic. In addition, not requiring symmetry among platforms protects against the possibility of making unusual-seeming findings that may be driven by this assumption (see, for instance, Amir and Lambson (2000) as well as the criticism in Berry et al. (1995) of the substitution patterns in the logit model). This is particularly true in models of competing platforms, in which equilibria can be sensitive to "tipping", as discussed in Sun and Tse (2007). Moreover, our model is amenable to merger analysis, which cannot be performed using models in the style of Armstrong (2006), due to their setup with two platforms and non-market-expanding demand.[7]

Our approach gives consumers free reign over their consumption choices, as they can select any bundle of platforms they find optimal. Existing models in which consumers "multi-home" (Caillaud and Jullien, 2003; Rochet and Tirole, 2003; Rysman, 2004; Armstrong, 2006; Doğanoglu and Wright, 2006; Armstrong and Wright, 2007), consider cases with just two platforms and/or exogenously impose single-homing on one side of the market. In the one-sided discrete choice literature, works such as Hendel (1999) and Gentzkow (2007) have moved towards incorporating such flexibility into consumers' choice set, and our approach follows in this spirit.

The restrictions that have so far been present in the theory of platform competition have made carrying out empirical studies of such industries more difficult, forcing authors

[6]In Bedre-Defolie and Calvano (2009) and White (2009) consumers are heterogenous in both dimensions, but they do not learn their interaction benefits until *after* they have selected a platform.

[7]A recent survey, Rysman (2009), speaks of the lack of such a framework, "Naturally, if we were to analyze the merger between two platform firms, we would need to account for complex two-sided issues that arise" (p. 137).

to adapt to the circumstances of their studies in somewhat constrained ways. For instance, in Cantillon and Yin (2008), the authors lack a model to predict platforms' equilibrium prices and instead take them as exogenous, while Wilbur (2008) uses a reduced form inverse demand function to model one side of the market. Our model, we hope, can serve as a basis for applied studies and can thus help to solve such difficulties.

While our model generalizes in the dimensions listed above, it retains two important assumptions that are typical of models in the literature on multi-sided platforms. First, we employ what Economides (1996) refers to as the "macro approach" to modeling networks, taking as exogenous the interaction among the consumers on different sides, once they join platforms and assuming consumer payoffs from joining a set of platforms depends only the number of consumers participating on and the payment to the platform(s). This approach brings useful generality when the interventions one considers are unlikely to affect the microstructure of interactions. However, if one's focus is on policies aimed at microstructure, an explicit model of such is crucial, as in Nocke et al. (2007), Hagiu (2009), White (2009) and Weyl and Tirole (2010).

Second, we assume all consumers on a given side are homogenous in the externalities they cause. That is, a consumer from one group cares about how *many* consumers of another group join each platform, but not *which* consumers these are. This restrictive and unrealistic assumption has been relaxed in a few very specific contexts (Chandra and Collard-Wexler, 2009; Jeon, 2010; Gomes, 2009; Athey et al., 2010), but work in progress by Veiga and Weyl (2010) provides the first general approach to incorporating heterogeneous externalities. They show that heterogeneity of externalities matter in pricing to the extent that valuation of participation on the other side covaries (on the margin) with the value of externalities brought by a consumer.[8]

Other issues that we do not consider include dynamics and price discrimination among sides. Anderson and Coate (2005), Hagiu (2006), Sun and Tse (2007), Lee (2010a) all include consideration of the former, while Gomes (2009), Doğanoglu and Wright (2010) and Hagiu and Lee (forthcoming) deal with the latter.

3 The Model

There are $m \in \mathbb{N}$ two-sided platforms and, on each side $\mathcal{I} \in \{\mathcal{A}, \mathcal{B}\}$, a continuum of consumers of mass 1. Let $\mathscr{M}$ denote the set of all platforms, and let $\wp(\mathscr{M})$ denote the power

[8] We are hopeful that such an extension can be incorporated without great difficulty into our framework, but given the early stage of this research on heterogenous externalities, we do not include it in the current version of this paper.

set of $\mathscr{M}$. Consumers can choose to join any combination of platforms. Let us denote the particular set that consumer i on side $\mathcal{I}$ chooses to join by the subset $\mathscr{M}_i^{\mathcal{I}} \in \wp(\mathscr{M})$.

Platforms' appeal to any given consumer depends on the broader allocation of consumers to platforms. We introduce a crucial statistic, based on the assignment of consumers to platforms, that we use hereafter.

Definition 1. *A* Coarse Allocation, $\boldsymbol{N} \equiv (\boldsymbol{N}^{\mathcal{A}}, \boldsymbol{N}^{\mathcal{B}}) \in [0,1]^{2m}$, *specifies the* total *density or "number" of consumers participating on each side of each platform. We denote a generic element by $N^{\mathcal{I},j}$, where $j \in \mathscr{M}$.*

Payoffs. Consumers have quasi-linear utility, and the payoff to user i on side $\mathcal{I}$ from joining the subset of platforms $\mathscr{M}_i^{\mathcal{I}}$ can be written as

$$v^{\mathcal{I}}(\mathscr{M}_i^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}, \boldsymbol{\theta}_i^{\mathcal{I}}) - \sum_{j \in \mathscr{M}_i^{\mathcal{I}}} P^{\mathcal{I},j}$$

where $\mathcal{J} \equiv -\mathcal{I}$ and where $\boldsymbol{\theta}_i^{\mathcal{I}} \in \Theta^{\mathcal{I}}$ denotes consumer i on side $\mathcal{I}$'s "type". The set of side $\mathcal{I}$ types, $\Theta^{\mathcal{I}} = \mathbb{R}^{L^{\mathcal{I}}}$, $2^m - 1 \leq L^{\mathcal{I}} \in \mathbb{N}$, allows for complete generality of consumer heterogeneity.The function $v^{\mathcal{I}} : \wp(\mathscr{M}) \times [0,1]^m \times \Theta^{\mathcal{I}} \to \mathbb{R}$ is thus a map to a consumer's willingness to pay from each possible consumption choice, the characteristics of the available goods and the user's individual characteristics. $P^{\mathcal{I},j}$ denotes the total price a user on side $\mathcal{I}$ must pay to join platform j, the details of which we discuss below, when defining platforms' strategies. Assumption 1 further characterizes the functions $v^{\mathcal{I}}$.

Assumption 1. *The functions $v^{\mathcal{I}}$, $\mathcal{I} = \{\mathcal{A}, \mathcal{B}\}$ have the following properties:*

1. Smoothness*: $v^{\mathcal{I}}$ is C^2 in all dimensions of its second and third arguments.*
2. Normalization*: $v^{\mathcal{I}}(\emptyset, \boldsymbol{N}^{\mathcal{J}}, \boldsymbol{\theta}) = 0 \quad \forall \boldsymbol{N}^{\mathcal{J}} \in [0,1]^m, \ \forall \boldsymbol{\theta} \in \Theta^{\mathcal{I}}$*
3. Full Coverage*: $\{\boldsymbol{\theta} \in \Theta^{\mathcal{I}} : \left(v^{\mathcal{I}}(1, \boldsymbol{N}^{\mathcal{J}}, \boldsymbol{\theta}), ..., v^{\mathcal{I}}(2^m - 2, \boldsymbol{N}^{\mathcal{J}}, \boldsymbol{\theta})\right) = \boldsymbol{k}\} \neq \emptyset, \ \forall \boldsymbol{N}^{\mathcal{J}}, \forall \boldsymbol{k} \in \mathbb{R}^{2^m-2}$*
4. No Externalities to Outsiders*: if $j \notin \mathscr{X}$ then $v^{\mathcal{I}}(\mathscr{X}, \boldsymbol{N}^{\mathcal{J}}, \boldsymbol{\theta})$ is independent of $N^{\mathcal{J},j}$*

Let $f^{\mathcal{I}} : \Theta^I \to \mathbb{R}$ be the probability density function of user types on side $\mathcal{I} = \mathcal{A}, \mathcal{B}$, satisfying $\int_{\Theta^{\mathcal{I}}} f^{\mathcal{I}}(\boldsymbol{\theta}) d\boldsymbol{\theta} = 1$. We assume that each function $f^{\mathcal{I}}$ is C^1 and has full support.

Platform j's profits are given by

$$\Pi^j \equiv P^{\mathcal{A},j} N^{\mathcal{A},j} + P^{\mathcal{B},j} N^{\mathcal{B},j} - C^j(N^{\mathcal{A},j}, N^{\mathcal{B},j})$$

where $C^j(N^{\mathcal{A},j}, N^{\mathcal{B},j})$ denotes platform j's costs as a function of the number of users on each side and is assumed to be C^2.

Timing & Strategies. Platforms move first, simultaneously. Then, having observed the platforms' moves, all consumers simultaneously choose which platforms to join.

We assume that each platform can charge tariffs to consumers on side $\mathcal{I}$ that are a function, not only of the number of consumers on side $\mathcal{J}$ that join *that* platform, but rather of the *entire coarse allocation* on side $\mathcal{J}$. We believe that allowing such a broad set of feasible tariffs is important for at least two reasons. First, this is a model that collapses a complex dynamic process into a static representation, and excluding the possibility of platforms adjusting to certain market conditions potentially hinders the realism of such a representation. Second, pricing practices that are observed in network industries, such as penetration pricing, seem to correspond to such market-dependent tariff structures. Moreover, while this assumption may seem to add complication, it turns out to do the opposite, as will become apparent in Section 5, when we define Insulated Equilibrium.

Thus, a (pure) strategy for platform j, $\sigma^j \equiv \left(\sigma^{\mathcal{A},j}(\boldsymbol{N}^{\mathcal{B}}), \sigma^{\mathcal{B},j}(\boldsymbol{N}^{\mathcal{A}})\right)$, is a pair of functions each assigning a price on a given side to the coarse allocation realized on the opposite side. Formally, $\sigma^{\mathcal{I},j} : [0,1]^m \to \mathbb{R}$. We assume $\sigma^j \in \Sigma$, where Σ is the set of all pairs of C^2 price functions. To denote the profile of strategies of the entire set of platforms, we define the function $\boldsymbol{\sigma} : [0,1]^{2m} \to \mathbb{R}^{2m}$. We assume $\boldsymbol{\sigma}$ can be written $\boldsymbol{\sigma}(\boldsymbol{N}) \equiv \left(\boldsymbol{\sigma}^{\mathcal{A}}(\boldsymbol{N}^{\mathcal{B}}), \boldsymbol{\sigma}^{\mathcal{B}}(\boldsymbol{N}^{\mathcal{A}})\right)$, where $\boldsymbol{\sigma}^{\mathcal{I}}(\boldsymbol{N}^{\mathcal{J}}) : [0,1]^m \to \mathbb{R}^m$ maps to the vector of prices platforms charge on side $\mathcal{I}$.

Consumers react to platforms' moves, which involve announcements of price functions. Thus, a pure strategy for consumer i on side $\mathcal{I}$, $\mathscr{M}_i^{\mathcal{I}}[\boldsymbol{\sigma}]$, is a functional, where $\mathscr{M}_i^{\mathcal{I}} : \Sigma^m \to \wp(\mathscr{M})$. To denote a *Side Strategy Profile*, for the set of consumers on side $\mathcal{I}$, we define the correspondence $\mathscr{M}^{\mathcal{I}}(\boldsymbol{\theta}^{\mathcal{I}}, [\boldsymbol{\sigma}])$. To avoid having to distinguish between outcomes that are equivalent from an economic perspective, we impose Assumption 2.

Assumption 2. *Strategy profiles adopted by consumers satisfy the following properties*

1. Purity: *In every subgame, each consumer takes some action with probability 1.*

2. Symmetry: *All agents sharing a common type adopt the same strategy.*

3. Convention: *When indifferent, all agents choose to join the set of platforms that comes first in the lexicographic ordering.*

It follows from the Purity and Symmetry components of Assumption 2 that $\mathscr{M}^{\mathcal{I}}$ is a functional, where $\mathscr{M}^{\mathcal{I}} : \Theta^{\mathcal{I}} \times \Sigma^m \to \wp(\mathscr{M})$ identifies all side $\mathcal{I}$ consumers' behavior in response to all $\boldsymbol{\sigma} \in \Sigma^m$. We denote the *Marketwide* consumer strategy profile by $\widetilde{\mathscr{M}}(\boldsymbol{\theta}, [\boldsymbol{\sigma}])$, where $\widetilde{\mathscr{M}} : \{\Theta^{\mathcal{A}} \times \Theta^{\mathcal{B}}\} \times \Sigma^m \to \wp(\mathscr{M})$.

A set $\{\widetilde{\mathscr{M}}, \boldsymbol{\sigma}\}$ determines a coarse allocation. Let us define the functional $\boldsymbol{N} : \{\widetilde{\mathscr{M}}\} \times \Sigma^m \to [0,1]^{2m}$, mapping from marketwide consumer strategy profile and platform strategy profile to coarse allocation. $\boldsymbol{N}[\widetilde{\mathscr{M}}, \boldsymbol{\sigma}]$ has generic elements

$$N^{\mathcal{I},j} = \int_{\{\theta^{\mathcal{I}} \in \Theta^{\mathcal{I}} : j \in \widetilde{\mathscr{M}}(\theta^{\mathcal{I}}, [\sigma])\}} f^{\mathcal{I}}(\boldsymbol{\theta}) d\boldsymbol{\theta}$$

4 Allocation $\Longrightarrow$ Price

In this section we focus on the second stage of the game in which only consumers move. Let a *Consumer Game* be a subgame that takes place once the platforms' strategy profile $\boldsymbol{\sigma}$ has been determined. Taking $\boldsymbol{\sigma}$ as a parameter, let $\widehat{P^{\mathcal{I},\mathscr{X}}}(\boldsymbol{N}^{\mathcal{J}}[\mathscr{M}^{\mathcal{J}}; \boldsymbol{\sigma}]) \equiv \sum_{j \in \mathscr{X}} \sigma^{\mathcal{I},j}(\boldsymbol{N}^{\mathcal{J}}[\mathscr{M}^{\mathcal{J}}; \boldsymbol{\sigma}])$ denote the sum of prices charged to a consumer on side $\mathcal{I}$, joining the set of platforms $\mathscr{X} \in \wp(\mathscr{M})$. Let $U_i^{\mathcal{I}}$ denote the net payoff to a consumer of type $\boldsymbol{\theta}_i^{\mathcal{I}}$, joining this bundle, where

$$U_i^{\mathcal{I}}(\mathscr{X}, \boldsymbol{N}^{\mathcal{J}}; \boldsymbol{\sigma}) \equiv v^{\mathcal{I}}(\mathscr{X}, \boldsymbol{N}^{\mathcal{J}}[\mathscr{M}^{\mathcal{J}}; \boldsymbol{\sigma}], \boldsymbol{\theta}_i^{\mathcal{I}}) - \widehat{P^{\mathcal{I},\mathscr{X}}}(\boldsymbol{N}^{\mathcal{J}}[\mathscr{M}^{\mathcal{J}}; \boldsymbol{\sigma}])$$

and let $\boldsymbol{U}_i^{\mathcal{I}} \in \mathbb{R}^{2^m - 2}$ denote the vector of such payoffs, with an element for each non-empty bundle of platforms. Finally, let $\mathscr{M}^{\mathcal{I}*} : \Theta^{\mathcal{I}} \times [0,1]^m \times \Sigma^m \to \wp(\mathscr{M})$ denote the *Best Response Correspondence* for side $\mathcal{I}$, where $\mathscr{M}^{\mathcal{I}*}(\boldsymbol{\theta}_i^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}; \boldsymbol{\sigma}) \in \arg\max_{\mathscr{X} \in \wp(\mathscr{M})} U_i^{\mathcal{I}}(\mathscr{X}, \boldsymbol{N}^{\mathcal{J}}; \boldsymbol{\sigma})$, $\forall \boldsymbol{\theta}_i^{\mathcal{I}} \in \Theta^{\mathcal{I}}$. Note that by the Convention component of Assumption 2, $\mathscr{M}^{\mathcal{I}*}$ is a function. We can now state our solution concept for a Consumer Game.

Definition 2. *In Consumer Game, $\boldsymbol{\sigma}$, a marketwide consumer strategy profile, $\widetilde{\mathscr{M}}$, forms a* Consumer Nash Equilibrium (CNE) *if the associated side strategy profiles, $\{\mathscr{M}^{\mathcal{I}}\}_{\mathcal{I}=\mathcal{A},\mathcal{B}}$, satisfy $\mathscr{M}^{\mathcal{I}}(\boldsymbol{\theta}^{\mathcal{I}}; [\sigma]) = \mathscr{M}^{\mathcal{I}*}(\boldsymbol{\theta}^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}[\mathscr{M}^{\mathcal{J}}, \boldsymbol{\sigma}]; [\sigma])$, $\forall \theta^{\mathcal{I}} \in \Theta^{\mathcal{I}}$.*

Below, we establish a central result upon which we build the subsequent analysis. This is that a CNE coarse allocation implies a unique vector of prices. In other words, regardless of the particular *price functions* announced by platforms, $\boldsymbol{\sigma}(\cdot)$, if we know the *coarse allocation*, $\boldsymbol{N}^*$, corresponding to a CNE, then we can infer the *total price*, $\boldsymbol{\sigma}(\boldsymbol{N}^*)$, that each platform charges for consumers on each side to join in this CNE. We prove this in Theorem 1 below.

In order to state this theorem, we first define *Gross Consumer Surplus* on side $\mathcal{I}$, as a function of side $\mathcal{I}$ consumers' best response strategy profile and the coarse allocation on side $\mathcal{J}$. We denote this by $V^{\mathcal{I}} : \{\mathscr{M}^{\mathcal{I}}\} \times [0,1]^m \to \mathbb{R}$, where

$$V^{\mathcal{I}}\left([\mathscr{M}^{\mathcal{I}*}], \boldsymbol{N}^{\mathcal{J}}\right) \equiv \sum_{\mathscr{X} \in \wp(\mathscr{M})} \int_{\boldsymbol{\theta}^{\mathcal{I}} : \mathscr{M}^{\mathcal{I}*}(\boldsymbol{\theta}^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}; [\sigma]) = \mathscr{X}} v^{\mathcal{I}}(\mathscr{X}, \boldsymbol{N}^{\mathcal{J}}, \boldsymbol{\theta}) f(\boldsymbol{\theta}) d\boldsymbol{\theta} \qquad (2)$$

The right-hand side of expression (2) is the sum over bundles of platforms of side $\mathcal{I}$ consumers' gross payoffs, given that they are best-responding both to platform prices and to the allocation of consumers on side $\mathcal{J}$.

Since, in a CNE, consumers maximize their utility given the prevailing prices, it must be the case that allocation of bundles to consumers is Pareto optimal, given the quantity constraints imposed by the induced side $\mathcal{I}$ coarse allocation. Since consumers have quasi-linear utility, a Pareto optimal allocation also maximizes the sum of consumer utility, subject to the same quantity constraints. Therefore, we can write Gross Consumer Surplus as a function, $V^{\mathcal{I}} : (0,1) \times [0,1]$, of the side $\mathcal{I}$ coarse allocation, given by the solution to the following constrained maximization problem

$$V^{\mathcal{I}}\left(\widetilde{\boldsymbol{N}^{\mathcal{I}}}, \boldsymbol{N}^{\mathcal{J}}\right) \equiv \max_{\mathscr{M}^{\mathcal{I}} \in \left\{\mathscr{M}^{\mathcal{I}} : \mathrm{N}^{\mathcal{I}}[\mathscr{M}^{\mathcal{I}}, \sigma] = \widetilde{\mathrm{N}^{\mathcal{I}}}\right\}} \sum_{\mathscr{X} \in \wp(\mathscr{M})} \int_{\theta^{\mathcal{I}} : \mathscr{M}^{\mathcal{I}}(\theta^{\mathcal{I}}, [\sigma]) = \mathscr{X}} v^{\mathcal{I}}(\mathscr{X}, \boldsymbol{N}^{\mathcal{J}}, \boldsymbol{\theta}) f(\boldsymbol{\theta}) d\boldsymbol{\theta} \qquad (3)$$

Lemma 1 regards the differentiability of this function with respect to elements, $\widetilde{N^{\mathcal{I},j}}$, of the "own-side" coarse allocation, $\widetilde{\boldsymbol{N}^{\mathcal{I}}}$.

Lemma 1 (Differentiability)**.** *The derivative $\partial V^{\mathcal{I}} / \partial \widetilde{N^{\mathcal{I},j}}$ exists and is equal to $\sigma^{\mathcal{I},j}(\boldsymbol{N}^{\mathcal{J}})$.*

Proof. See Appendix A. ◻

Theorem 1 (Allocation Implies Price)**.** *Let $\widetilde{\mathscr{M}^{*}}$ and $\widetilde{\mathscr{M}^{**}}$ be respective CNE strategy profiles of Consumer Games $\boldsymbol{\sigma}^{*}$ and $\boldsymbol{\sigma}^{**}$. If $\boldsymbol{N}[\widetilde{\mathscr{M}^{*}}; \boldsymbol{\sigma}^{*}] = \boldsymbol{N}[\widetilde{\mathscr{M}^{**}}; \boldsymbol{\sigma}^{**}]$, then $\boldsymbol{\sigma}^{*}(\boldsymbol{N}[\widetilde{\mathscr{M}^{*}}; \boldsymbol{\sigma}^{*}]) = \boldsymbol{\sigma}^{**}(\boldsymbol{N}[\widetilde{\mathscr{M}^{**}}; \boldsymbol{\sigma}^{**}])$.*

Proof. This result follows trivially from Lemma 1. ◻

Theorem 1 suggests that it is more sensible to frame the platform's problem as, "Which of the feasible allocations, given the strategies of other platforms, is it most profitable to implement?" The alternative approach, which has been prevalent in the literature asks, "Which price structure maximizes profits, given the price structures of other platforms and *given an arbitrary assumption on consumers' reactions?*"

5 Insulated Equilibrium

In this section, we consider the first stage of the game, in which platforms move. The previous section suggests that it is natural to think of platforms' problem is as a choice of *allocation*. We briefly expound upon this idea in abstract terms, before defining our solution concept of *Insulated Equilibrium*.

An individual platform j, taking as given the strategies of other platforms, can identify a set of coarse allocations that can feasibly occur as CNE in the subsequent Consumer Game. Moreover, since each platform's profits can be written as a direct function of the CNE coarse allocation, platform j can identify the subset of feasible coarse allocations that, for it, are profit-maximizing. For the sake of exposition, let us suppose that this subset is single-valued and call its member $\boldsymbol{N}^{*,j}$. Clearly, given the non-cooperative nature of the game, platform j fares strictly better if it is able to implement $\boldsymbol{N}^{*,j}$ than it does if some other coarse allocation arises. In light of this observation, the obvious issues are whether and how platform j can "robustly" implement its desired coarse allocation.

Holding fixed the strategies of the other platforms, there is an infinite set of strategies platform j can choose in order to implement $\boldsymbol{N}^{*,j}$ as *one of the* CNE in the subsequent Consumer Game. However, an arbitrary strategy that weakly implements $\boldsymbol{N}^{*,j}$ can also lead to other, less profitable allocations for platform j. This multiplicity of CNE arises from the fact that a coordination game takes place between consumers on opposite sides of the market. In the rest of this section, we show that by shaping its tariffs properly, a platform can completely eliminate the coordination game among consumers on opposite side of the market.

5.1 Definition

We now formally define *Residual Insulating Tariffs*,[9] which Weyl (2010) introduces in the case of a monopoly platform. In the context of competing platforms, they work in the following way. In order to "pick" a coarse allocation on side $\mathcal{J}$, a platform charges an insulating tariff on side $\mathcal{I}$ and thus guarantees that, on the latter side, a particular coarse allocation prevails.

Definition 3. *Given a profile of strategies of other platforms, $\boldsymbol{\sigma}^{-j}$, platform j is said to charge a* Residual Insulating Tariff *on side $\mathcal{I}$ if, $\forall \mathbf{N}^{\mathcal{J}}, \widetilde{\mathbf{N}^{\mathcal{J}}} \in [0,1]$,*

$$N^{\mathcal{I},j}[\mathscr{M}^{\mathcal{I}*}(\boldsymbol{\theta}^{\mathcal{I}}, \mathbf{N}^{\mathcal{J}}, [\sigma]), \boldsymbol{\sigma}] = N^{\mathcal{I},j}[\mathscr{M}^{\mathcal{I}*}(\boldsymbol{\theta}^{\mathcal{I}}, \widetilde{\mathbf{N}^{\mathcal{J}}}, [\sigma]), \boldsymbol{\sigma}]$$

For a firm j to charge an insulating tariff on side $\mathcal{I}$, it must choose a price function, $\sigma^{\mathcal{I},j}(\mathbf{N}^{\mathcal{J}})$, that, given the strategies of the other platforms, preserves the coarse allocation on side $\mathcal{I}$, regardless of the strategy profile adopted by side $\mathcal{J}$ consumers. To see how such a function operates, consider the demand for platform j among side $\mathcal{I}$ consumers,

[9]Hereafter, when discussing them informally, we typically drop the term "residual" when speaking of such tariffs.

$N^{\mathcal{I},j}$. It can be written

$$N^{\mathcal{I},j} = N^{\mathcal{I},j}(\sigma^{\mathcal{I},j}(\boldsymbol{N}^{\mathcal{J}}), \boldsymbol{\sigma}^{\mathcal{I},-j}(\boldsymbol{N}^{\mathcal{J}}), \boldsymbol{N}^{\mathcal{J}})$$

An insulating tariff, charged by firm j on side $\mathcal{I}$ is thus a function, $\sigma^{\mathcal{I},j}(\cdot)$, that takes into account the shape of $N^{\mathcal{I},j}(\cdot,\cdot,\cdot)$ and the shape of other firms' side $\mathcal{I}$ price functions, denoted by the vector $\boldsymbol{\sigma}^{\mathcal{I},-j}(\cdot)$, in order to ensure that the output of $N^{\mathcal{I},j}$ is constant. Lemma 2 establishes the existence and uniqueness of an insulating tariff for firm j on side $\mathcal{I}$, given the side $\mathcal{I}$ price functions announced by other firms and the coarse allocation on the other side of the market, $\boldsymbol{N}^{\mathcal{J}}$.

Lemma 2 (Existence and Uniqueness of a Residual Insulating Tariff)**.** *There exists a unique function,* $\overline{P^{\mathcal{I},j}}(\boldsymbol{N}^{\mathcal{J}}; \widetilde{N}, [\boldsymbol{\sigma}^{\mathcal{I},-j}(\boldsymbol{N}^{\mathcal{J}})])$*, such that,* $\forall \boldsymbol{N}^{\mathcal{J}}$, $\forall \widetilde{N} \in (0,1)$, $\forall \boldsymbol{\sigma}^{\mathcal{I},-j}$

$$N^{\mathcal{I},j}(\overline{P^{\mathcal{I},j}}(\boldsymbol{N}^{\mathcal{J}}; \widetilde{N}, [\boldsymbol{\sigma}^{\mathcal{I},-j}(\boldsymbol{N}^{\mathcal{J}})]), \boldsymbol{\sigma}^{\mathcal{I},-j}(\boldsymbol{N}^{\mathcal{J}}), \boldsymbol{N}^{\mathcal{J}}) = \widetilde{N}$$

Moreover, $\overline{P^{\mathcal{I},j}}$ *is* C^2 *in all dimensions of its first argument.*

Proof. For existence, note that (i) $N^{\mathcal{I},j}(\cdot,\cdot,\cdot)$ is continuous in its first argument, since it is the integral of a smooth set, and (ii) $\forall \boldsymbol{N}^{\mathcal{J}}, \ \forall \boldsymbol{\sigma}^{\mathcal{I},-j}$, $\lim_{P^{\mathcal{I},j} \to -\infty} N^{\mathcal{I},j}(P^{\mathcal{I},j}, \boldsymbol{P}^{\mathcal{I},-j}, \boldsymbol{N}^{\mathcal{J}}) = 1$ (and $\lim_{P^{\mathcal{I},j} \to \infty} N^{\mathcal{I},j}(P^{\mathcal{I},j}, \boldsymbol{P}^{\mathcal{I},-j}, \boldsymbol{N}^{\mathcal{J}}) = 0$), since $\forall \boldsymbol{\theta}^{\mathcal{I}}, \ \forall \boldsymbol{N}^{\mathcal{J}}, \ \forall \boldsymbol{\sigma}^{\mathcal{I},-j}, \exists P^{\mathcal{I},j}$ such that

$$\max_{\mathscr{X}: j \in \mathscr{X}} \left\{ v^{\mathcal{I}}(\mathscr{X}, \boldsymbol{N}^{\mathcal{J}}, \boldsymbol{\theta}^{\mathcal{I}}) - \widehat{P^{\mathcal{I},\mathscr{X}}} \right\} > (<) \max_{\mathscr{Y}: j \notin \mathscr{Y}} \left\{ v^{\mathcal{I}}(\mathscr{Y}, \boldsymbol{N}^{\mathcal{J}}, \boldsymbol{\theta}^{\mathcal{I}}) - \widehat{P^{\mathcal{I},\mathscr{Y}}} \right\}$$

For uniqueness, note that $N^{\mathcal{I},j}(\cdot,\cdot,\cdot)$ is decreasing in its first argument, since it is the sum of a set of nonincreasing functions, some of which are decreasing.

To show that $\overline{P^{\mathcal{I},j}}$ is C^2 in all dimensions of its first argument, we note that, in response to a change in the value of an arbitrary element of $\boldsymbol{N}^{\mathcal{J}}$, $N^{\mathcal{J},k}$, in order to be insulating $\overline{P^{\mathcal{I},j}}$ must change, globally, at a smooth rate given by

$$-\frac{\sum_{l \neq j} \frac{\partial N^{\mathcal{I},j}}{\partial \sigma^{\mathcal{I},l}} \frac{\partial \sigma^{\mathcal{I},l}}{\partial N^{\mathcal{J},k}} + \frac{\partial N^{\mathcal{I},j}}{\partial N^{\mathcal{J},k}}}{\frac{\partial N^{\mathcal{I},j}}{\partial N^{\mathcal{J},k}}}$$

which is, itself, differentiable in all elements of $\boldsymbol{N}^{\mathcal{J}}$. □

We now introduce vocabulary to describe the case when all platforms charge insulating tariffs.

Definition 4. *An* Insulating Tariff System (ITS) *on side* $\mathcal{I}$, $\overline{\boldsymbol{P}^{\mathcal{I}}}(\boldsymbol{N}^{\mathcal{J}}; \widetilde{\boldsymbol{N}^{\mathcal{I}}})$*, is a profile of insulating tariffs, parameterized by the coarse allocation it induces,* $\widetilde{\boldsymbol{N}^{\mathcal{I}}}$*. We say that* $\overline{\boldsymbol{P}^{\mathcal{I}}}$ *is "anchored" at* Reference Allocation $\widetilde{\boldsymbol{N}^{\mathcal{I}}}$*. We denote a marketwide ITS by* $\overline{\boldsymbol{P}}(\widetilde{\boldsymbol{N}}) \equiv \left(\overline{\boldsymbol{P}^{\mathcal{A}}}(\boldsymbol{N}^{\mathcal{B}}; \widetilde{\boldsymbol{N}^{\mathcal{A}}}), \overline{\boldsymbol{P}^{\mathcal{B}}}(\boldsymbol{N}^{\mathcal{A}}; \widetilde{\boldsymbol{N}^{\mathcal{B}}}) \right)$.

We can now define our solution concept. Insulated Equilibria are particular Subgame Perfect Equilibria. We first define the latter in the context of our game and then we state the definition of IE. Given a consumer strategy profile, $\widetilde{\mathscr{M}}$, and a profile of strategies adopted by other firms, $\boldsymbol{\sigma}^{-j}$, denote firm j's profits by

$$\Pi^j[\sigma^j, \boldsymbol{\sigma}^{-j}; \widetilde{\mathscr{M}}] \equiv \sum_{\mathcal{I}=\mathcal{A},\mathcal{B}} \sigma^{\mathcal{I},j}(N^{\mathcal{J},j}[\widetilde{\mathscr{M}}, \boldsymbol{\sigma}])N^{\mathcal{I},j}[\widetilde{\mathscr{M}}, \boldsymbol{\sigma}] - C^j(N^{\mathcal{A},j}[\widetilde{\mathscr{M}}, \boldsymbol{\sigma}], N^{\mathcal{B},j}[\widetilde{\mathscr{M}}, \boldsymbol{\sigma}]) \tag{4}$$

Definition 5. *In a particular platform game, defined by* $\widetilde{\mathscr{M}}$*, a platform strategy profile,* $\boldsymbol{\sigma}$*, forms a* Platform Nash Equilibrium (PNE) *if* $\sigma^j \in \arg\max_{x\in\Sigma} \Pi^j(x, \boldsymbol{\sigma}^{-j}; \widetilde{\mathscr{M}}),\ \forall j \in \mathscr{M}$.

Definition 6. *A set containing a profile of strategies for platforms and for consumers on each side,* $\{\boldsymbol{\sigma}^*, \{\mathscr{M}^{\mathcal{I}}\}_{\mathcal{I}=\mathcal{A},\mathcal{B}}\}$*, forms a* Subgame Perfect Equilibrium (SPE) *if* $\boldsymbol{\sigma}^*$ *forms a PNE given* $\{\mathscr{M}^{\mathcal{I}}\}_{\mathcal{I}=\mathcal{A},\mathcal{B}}$ *and* $\mathscr{M}^{\mathcal{I}} = \mathscr{M}^{\mathcal{I}*}(\boldsymbol{\theta}^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}[\mathscr{M}^{\mathcal{J}}, \boldsymbol{x}]; [\boldsymbol{x}]), \forall \theta^{\mathcal{I}} \in \Theta^{\mathcal{I}},\ \forall \boldsymbol{x} \in \Sigma^m$.

We now state the definition of an Insulated Equilibrium.

Definition 7. *Let* $\{\boldsymbol{\sigma}^*, \widetilde{\mathscr{M}^*}\}$ *be an SPE with coarse allocation* $\boldsymbol{N}^* = (\boldsymbol{N}^{\mathcal{A}^*}, \boldsymbol{N}^{\mathcal{B}^*})$*. The SPE* $\{\boldsymbol{\sigma}^*, \widetilde{\mathscr{M}^*}\}$ *is an* Insulated Equilibrium (IE) *if* $\boldsymbol{\sigma}^* = \overline{\boldsymbol{P}}(\widetilde{\boldsymbol{N}^*})$.

In a Subgame Perfect Equilibrium, platforms select their strategies *as if they had complete certainty* of the outcome of the continuation Consumer Game, even when the particular consumer game that they induce has multiple Consumer Nash Equilibria. Thus, one must speak of platforms' profits as function of *both* platforms' strategies *and* of consumers' strategies. ***Under Insulated Equilibrium, on the other hand, the particular strategy profile adopted by consumers is of no consequence***, since, when the platforms' strategy profile amounts to an Insulating Tariff System, in the subsequent Consumer Game, there is a unique Consumer Nash Equilibrium.

5.2 A Special Property of Insulating Tariff Systems

An instructive lens through which to consider an Insulating Tariff System is through that of a *Representative Consumer (RC)*.[10] Suppose that on side $\mathcal{I}$ there is a single "superagent" in charge of choosing quantities, or "slots" on platforms, for his constituent consumers on side $\mathcal{I}$ to efficiently allocate among themselves, and that the RC's objective is to maximize the sum of constituents' utility. The RC's objective function can thus be written

$$V^{\mathcal{I}}(\boldsymbol{N}^{\mathcal{I}}_{RC}, \boldsymbol{N}^{\mathcal{J}}) - \boldsymbol{\sigma}^{\mathcal{I}}(\boldsymbol{N}^{\mathcal{J}}) \cdot \boldsymbol{N}^{\mathcal{I}}_{RC}$$

[10]See Anderson et al. (1992), particularly chapter 3, for foundations of the representative consumer approach in a one-sided discrete choice setting.

where "·" denotes the inner-product operator and where, as defined in (3), $V^{\mathcal{I}}$ denotes Gross Consumer Surplus on side $\mathcal{I}$, which, here, can be interpreted as the gross payoff to the representative consumer. Consider the following *Representative Consumer Game*, defined by the strategy profile, $\boldsymbol{\sigma}$, announced by platforms. On side $\mathcal{I}$, the RC chooses coarse allocation $\mathbf{N}^{\mathcal{I}}_{RC}$; activity among side $\mathcal{J}$ consumers occurs as before. We can now state Theorem 2.

Theorem 2. *In an RC game, $\boldsymbol{\sigma}$, it is a strictly dominant strategy for the Representative Consumer to select $\mathbf{N}^{\mathcal{I}}_{RC} = \mathbf{N}^{\mathcal{I}^*}$ if and only if the platforms' side $\mathcal{I}$ strategy profile is the Insulating Tariff System anchored at $\mathbf{N}^{\mathcal{I}^*}$. Formally,*

$$V^{\mathcal{I}}(\mathbf{N}^{\mathcal{I}^*}, \mathbf{N}^{\mathcal{J}}) - \boldsymbol{\sigma}^{\mathcal{I}}(\mathbf{N}^{\mathcal{J}}) \cdot \mathbf{N}^{\mathcal{I}^*} > V^{\mathcal{I}}(\widetilde{\mathbf{N}^{\mathcal{I}}}, \mathbf{N}^{\mathcal{J}}) - \boldsymbol{\sigma}^{\mathcal{I}}(\mathbf{N}^{\mathcal{J}}) \cdot \widetilde{\mathbf{N}^{\mathcal{I}}}, \;\; \forall \mathbf{N}^{\mathcal{J}}, \forall \widetilde{\mathbf{N}^{\mathcal{I}}} \neq \mathbf{N}^{\mathcal{I}^*}$$
$$\Leftrightarrow \boldsymbol{\sigma} = \left(\overline{\mathbf{P}^{\mathcal{I}}}(\mathbf{N}^{\mathcal{J}}; \mathbf{N}^{\mathcal{I}^*}), \boldsymbol{\sigma}^{\mathcal{J}}(\mathbf{N}^{\mathcal{I}})\right)$$

Proof. First note that

$$V^{\mathcal{I}}(\mathbf{N}^{\mathcal{I}}_{RC}, \mathbf{N}^{\mathcal{J}}) - \boldsymbol{\sigma}^{\mathcal{I}}(\mathbf{N}^{\mathcal{J}}) \cdot \mathbf{N}^{\mathcal{I}}_{RC} =$$
$$\max_{\mathscr{M}^{\mathcal{I}} \in \left\{\mathscr{M}^{\mathcal{I}} : \mathbf{N}^{\mathcal{I}}[\mathscr{M}^{\mathcal{I}}, \sigma] = \mathbf{N}^{\mathcal{I}}_{RC}\right\}} \sum_{\mathscr{X} \in \wp(\mathscr{M})} \int_{\boldsymbol{\theta}^{\mathcal{I}} : \mathscr{M}^{\mathcal{I}}(\boldsymbol{\theta}^{\mathcal{I}}, [\sigma]) = \mathscr{X}} \left(v^{\mathcal{I}}(\mathscr{X}, \mathbf{N}^{\mathcal{J}}, \boldsymbol{\theta}) - \widehat{P^{\mathcal{I}, \mathscr{X}}}(\mathbf{N}^{\mathcal{J}})\right) f(\boldsymbol{\theta}) d\boldsymbol{\theta}$$
$$\leq \sum_{\mathscr{X} \in \wp(\mathscr{M})} \int_{\boldsymbol{\theta}^{\mathcal{I}} : \mathscr{M}^{\mathcal{I}*}(\boldsymbol{\theta}^{\mathcal{I}}, \mathbf{N}^{\mathcal{J}}; [\sigma]) = \mathscr{X}} \left(v^{\mathcal{I}}(\mathscr{X}, \mathbf{N}^{\mathcal{J}}, \boldsymbol{\theta}) - \widehat{P^{\mathcal{I}, \mathscr{X}}}(\mathbf{N}^{\mathcal{J}})\right) f(\boldsymbol{\theta}) d\boldsymbol{\theta} \quad (5)$$

where, by revealed preference, the inequality in (5) is strict if and only if $\mathbf{N}^{\mathcal{I}}_{RC} \neq \mathbf{N}^{\mathcal{I}}[\mathscr{M}^{\mathcal{I}*}(\boldsymbol{\theta}^{\mathcal{I}}, \mathbf{N}^{\mathcal{J}}; [\sigma]), \sigma]$. Second, note that, $\mathbf{N}^{\mathcal{I}}[\mathscr{M}^{\mathcal{I}*}(\boldsymbol{\theta}^{\mathcal{I}}, \mathbf{N}^{\mathcal{J}}; [\sigma]), \sigma] = \mathbf{N}^{\mathcal{I}^*}$, $\forall \mathbf{N}^{\mathcal{J}}$, if and only if $\boldsymbol{\sigma}^{\mathcal{I}}(\cdot) = \overline{\mathbf{P}^{\mathcal{I}}}(\cdot; \mathbf{N}^{\mathcal{I}^*})$, by the definition of the Insulating Tariff System. This establishes our claim. □

5.3 Marginal Costs Are Identified Under Insulated Equilibrium

We propose Insulated Equilibrium as a refinement of Subgame Perfect Equilibrium. In doing so, we claim, as motivation *for this particular refinement*, that platforms can reasonably be expected to charge insulating tariffs. Independently, however, of the issue of multiplicity of CNE, there is another, perhaps more damning problem with SPE as a solution concept for our class of games, namely that it is largely vacuous. This issue of multiplicity of *Platform Equilibria*, holding fixed consumers' strategy profile, is discussed by Armstrong (2006), in Proposition 3 and in the discussion thereafter. The basic issue, which we refer to as *Armstrong's Paradox*, follows from the multiplicity of supply function equilibria in a deterministic setting, analyzed by Klemperer and Meyer (1989).

It can be understood by observing expression (4) of the profits of a platform j. Suppose there is some set, $\{\sigma^{\mathcal{A},j}(N^{\mathcal{B},j}), \sigma^{\mathcal{B},j}(N^{\mathcal{A},j}), N^{\mathcal{A},j}, N^{\mathcal{B},j}\}$, that uniquely maximizes j's profits, given the strategies of the other platforms and given the consumers' strategy profile. Then, j's equilibrium strategy, σ^j, must, in one way or another, include this set. However, the functions $\sigma^{\mathcal{I},j}(\cdot)$ are not pinned down when evaluated at non-equilibrium quantities $\widetilde{N}^{\mathcal{J},j} \neq N^{\mathcal{J},j}$. As a result, we conjecture that it is possible to construct a set of platform strategies that support, as an SPE, any coarse allocation in which all platforms make positive profits.[11] However, regardless of whether this is precisely true, the set of subgame perfect equilibria is very large.

Thus, if a solution concept for this class of games is to have significant predictive power, it must be stronger than SPE. Here we show that under the IE solution concept, the issue of multiplicity of equilibria is reduced to the point where it takes the same form as in traditional "one-sided" models of imperfect competition typically used in industrial organization. Theorem 3 states this from the perspective of an econometrician who observes prices and quantities and has estimated the platforms demand but does not observe platforms' marginal costs.

Theorem 3 (Under Insulated Equilibrium, Marginal Cost is Identified)**.** *Suppose $\{\widetilde{\mathscr{M}}^*, \boldsymbol{\sigma}^*\}$ is an IE with coarse allocation $\boldsymbol{N}^*$, with generic elements $N^{\mathcal{I},j*}$. Then, the vector of platform marginal costs is identified jointly by the vector of prices, $\{\boldsymbol{P}^{\mathcal{I}}\}_{\mathcal{I}=\mathcal{A},\mathcal{B}}$, the coarse allocation, the payoff functions $\{v^{\mathcal{I}}\}_{\mathcal{I}=\mathcal{A},\mathcal{B}}$ and the distribution of types $\{f^{\mathcal{I}}\}_{\mathcal{I}=\mathcal{A},\mathcal{B}}$.*

Proof. Since $\{\widetilde{\mathscr{M}}^*, \boldsymbol{\sigma}^*\}$ is an IE with coarse allocation $\boldsymbol{N}^*$, the equilibrium profile of platform strategies is $\boldsymbol{\sigma}^* = \overline{\boldsymbol{P}}(\boldsymbol{N}^*)$. Thus, platform j's profit maximization problem can be written

$$\max_{\{N^{\mathcal{A},j}, N^{\mathcal{B},j}\}} \sum_{\mathcal{I}=\mathcal{A},\mathcal{B}} N^{\mathcal{I},j} \cdot P^{\mathcal{I},j}(N^{\mathcal{I},j}, N^{\mathcal{J},j}) - C^j(N^{\mathcal{A},j}, N^{\mathcal{B},j}) \tag{6}$$

where

$$P^{\mathcal{I},j}(N^{\mathcal{I},j}, N^{\mathcal{J},j}) \equiv \overline{P^{\mathcal{I},j}}\left(\boldsymbol{N}^{\mathcal{J}}; N^{\mathcal{I},j}, [\overline{\boldsymbol{P}^{\mathcal{I},-j}}(\boldsymbol{N}^{\mathcal{J}}, \boldsymbol{N}^{\mathcal{I}^*})]\right)$$

and

$$\boldsymbol{N}^{\mathcal{J}} = \boldsymbol{N}^{\mathcal{J}}\left(\overline{P^{\mathcal{J},j}}(\boldsymbol{N}^{\mathcal{I}}; N^{\mathcal{J},j}, [\overline{\boldsymbol{P}^{\mathcal{J},-j}}(\boldsymbol{N}^{\mathcal{I}}; \boldsymbol{N}^{\mathcal{J}^*})]), \overline{\boldsymbol{P}^{\mathcal{J},-j}}(\boldsymbol{N}^{\mathcal{I}}; \boldsymbol{N}^{\mathcal{J}^*}), \boldsymbol{N}^{\mathcal{I}}\right)$$

[11] We are working on a formal proof of this conjecture.

The values that maximize (6), $N^{\mathcal{A},j*}$ and $N^{\mathcal{B},j*}$, satisfy first-order condition

$$P^{\mathcal{I},j} + N^{\mathcal{I},j*}\frac{\partial P^{\mathcal{I},j}}{\partial N^{\mathcal{I},j}} + N^{\mathcal{J},j*}\frac{\partial P^{\mathcal{J},j}}{\partial N^{\mathcal{I},j}}$$

$$= \overline{P^{\mathcal{I},j}} + N^{\mathcal{I},j*}\frac{\partial \overline{P^{\mathcal{I},j}}}{\partial N^{\mathcal{I},j}} + N^{\mathcal{J},j*}\frac{\partial \overline{P^{\mathcal{J},j}}}{\partial \mathbf{N}^{\mathcal{I}}} \cdot \frac{\partial \mathbf{N}^{\mathcal{I}}}{\partial \overline{P^{\mathcal{I},j}}}\frac{\partial \overline{P^{\mathcal{I},j}}}{\partial N^{\mathcal{I},j}} = \frac{\partial C^{j}}{\partial N^{\mathcal{I},j}} \quad (7)$$

and thus a unique vector of marginal costs is consistent with a given Insulated Equilibrium.

□

6 Pricing Under Insulated Equilibrium

In this section, we characterize platforms' pricing under Insulated Equilibrium. Let us denote by $C^{j}_{\mathcal{I}} \equiv \frac{\partial C^j}{\partial N^{\mathcal{I},j}}$ platform j's marginal cost of serving an additional consumer on side $\mathcal{I}$; let $\mu^{\mathcal{I},j} \equiv -N^{\mathcal{I},j}/\frac{\partial N^{\mathcal{I},j}}{\partial \overline{P^{\mathcal{I},j}}}$ denote platform j's *Side $\mathcal{I}$ Market Power*. Rearranging expression (7), we obtain

$$P^{\mathcal{I},j} = C^{j}_{\mathcal{I}} + \mu^{\mathcal{I},j} - N^{\mathcal{J},j}\gamma^{\mathcal{J},j} \quad (8)$$

where

$$\gamma^{\mathcal{J},j} \equiv \frac{d\overline{P^{\mathcal{J},j}}}{dN^{\mathcal{I},j}} = \frac{\partial \overline{P^{\mathcal{J},j}}}{\partial \mathbf{N}^{\mathcal{I}}} \cdot -\mathbf{D}^{\mathcal{I}}_{\cdot,j}$$

The formula given in equation (8) says that platform j's price on side $\mathcal{I}$ is equal to its marginal cost of serving an additional side $\mathcal{I}$ consumer, plus its side $\mathcal{I}$ market power, minus the total "cross externality" an additional side $\mathcal{I}$ consumer contributes to j's interaction with consumers on side $\mathcal{J}$.

The first factor in $\gamma^{\mathcal{J},j}$, $\frac{\partial \overline{P^{\mathcal{J},j}}}{\partial \mathbf{N}^{\mathcal{I}}}$, is the j^{th} row of the Jacobian matrix of $\overline{\boldsymbol{P}^{\mathcal{J}}}(\mathbf{N}^{\mathcal{I}}; \mathbf{N}^{\mathcal{J}^*})$, the side $\mathcal{J}$ Insulating Tariff System anchored at the IE coarse allocation. The second factor in $\gamma^{\mathcal{J},j}$, as defined in Section 2.1, is the j^{th} column of the diversion ratio matrix of the side $\mathcal{I}$ demand system. To solve for the Insulating Tariff System, we note

$$\underbrace{\mathbf{0}}_{m\times m} = \left[\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{N}^{\mathcal{I}}}\right] + \left[\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \overline{\boldsymbol{P}^{\mathcal{J}}}}\right]\left[\frac{\partial \overline{\boldsymbol{P}^{\mathcal{J}}}}{\partial \mathbf{N}^{\mathcal{I}}}\right]$$

$$\Leftrightarrow \left[\frac{\partial \overline{\boldsymbol{P}^{\mathcal{J}}}}{\partial \mathbf{N}^{\mathcal{I}}}\right] = -\left[\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \overline{\boldsymbol{P}^{\mathcal{J}}}}\right]^{-1}\left[\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{N}^{\mathcal{I}}}\right] \quad (9)$$

Equation (9) shows the relationship between the Insulating Tariff System and the underlying demand system. In Section 6.2, we further characterize the latter.

6.1 A Monopoly Platform

To develop an intuition for the pricing formula under competition, it is useful to first examine the monopoly case.[12] As a benchmark, we first consider prices that are consistent with the socially optimal allocation. Let $\overline{v^{\mathcal{I}}_{\mathcal{J}}}$ denote the average *interaction value*, or valuation for interacting with an additional consumer on the other side of the market,[13] among the *entire set* of consumers on side $\mathcal{I}$ that join the platform. Let $\mathring{N}^{\mathcal{I}}$ denote the socially optimal level of participation on side $\mathcal{I}$. The socially optimal price on side $\mathcal{I}$ is given by

$$P^{\mathcal{I}} = C_{\mathcal{I}} - \mathring{N}^{\mathcal{J}} \overline{v^{\mathcal{J}}_{\mathcal{I}}} \tag{10}$$

Under optimal pricing, a consumer on side $\mathcal{I}$ pays the direct marginal cost he imposes minus the *entire* externality he emits to consumers on the other side of the market.

At the monopolist's privately optimal allocation, $\boldsymbol{N}^*$, the price on side $\mathcal{I}$ is given by

$$P^{\mathcal{I}} = C_{\mathcal{I}} + \mu^{\mathcal{I}} - N^{\mathcal{J}*} \widetilde{v^{\mathcal{J}}_{\mathcal{I}}} \tag{11}$$

where $\widetilde{v^{\mathcal{I}}_{\mathcal{J}}}$ denotes the average interaction value of the set of side $\mathcal{J}$ consumers that are *on the margin* between joining the platform and dropping out. Two forces govern the relationship between (10) and (11). The first is the classical market power distortion, captured by μ^I and well-known from one-sided analysis. The second force is the *Spence distortion*, whose name is inspired by the analysis in Spence (1975) of a traditional monopolist's choice of quality. The platform's allocation on side $\mathcal{I}$ determines the quality of the platform for consumers on side $\mathcal{J}$. As in Spence's model of a one-sided monopolist, the quality that a two-sided monopolist provides to consumers on each side is mediated by the willingness to pay for quality of *marginal* consumers, rather than of the entire set of marginal and inframarginal consumers. Correspondingly, the price the monopolist charges on side $\mathcal{I}$ takes into account the influence an additional side $\mathcal{I}$ consumer has on the *profit* it can garner from side $\mathcal{J}$. Appendix B provides a detailed illustration of the relationship between the Spence distortion in one- and two-sided monopoly settings.

[12] When $m = 1$, our model corresponds to the version of the model in Section III of Weyl (2010) with general utility functions, two groups of consumers and only intergroup network effects. That section considers a monopolist platform with M sides and with potential within-side network effects.

[13] In Section 6.2, we define this notion precisely.

6.2 Decomposition

We now derive the relationship between the Insulating Tariff System, when there are *m* platforms, and the underlying demand system. The matrices on the right-hand side of equation (9) are comprised of two sorts of quantities. First, they involve densities of consumers that are *on the margin* between two sets of platforms. Second, they involve the aforementioned *interaction values*, or valuations for "interacting", on a given platform, with an additional consumer from the opposite side, averaged over sets of such marginal consumers. In order to express these elements of the demand system, we first define sets that contain the relevant sorts of marginal consumers.

First, let $\widetilde{\mathbf{\Theta}}^{\mathcal{I}}_j$ denote the entire set of consumers on side $\mathcal{I}$ that are indifferent between consuming some bundle of platforms, $\mathscr{X}$, containing platform j, and consuming some other bundle, $\mathscr{Y}$, not containing platform j.

$$\widetilde{\mathbf{\Theta}}^{\mathcal{I}}_j \equiv \left\{ \boldsymbol{\theta}^{\mathcal{I}} \in \mathbf{\Theta}^{\mathcal{I}} : \exists \mathscr{X}, \mathscr{Y} \in \underset{\mathscr{Z} \in \wp(\mathscr{M})}{\arg\max} \left\{ v^{\mathcal{I}}(\mathscr{Z}, \mathbf{N}^{\mathcal{J}}, \boldsymbol{\theta}^{\mathcal{I}}) - \widehat{P^{\mathcal{I},\mathscr{Z}}} \right\} \text{s.t. } j \in \mathscr{X}, j \notin \mathscr{Y} \right\}$$

Second, let $\widetilde{\mathbf{\Theta}^{\mathcal{I}}}_{j,k}$ denote the set of consumers on side $\mathcal{I}$ that are indifferent between consuming some bundle of platforms, $\mathscr{X}$, containing platform j and not containing platform k, and consuming some other bundle $\mathscr{Y}$, containing platform k and not containing platform j.

$$\widetilde{\mathbf{\Theta}^{\mathcal{I}}}_{j,k} \equiv \left\{ \boldsymbol{\theta}^{\mathcal{I}} \in \mathbf{\Theta}^{\mathcal{I}} : \exists \mathscr{X}, \mathscr{Y} \in \underset{\mathscr{Z} \in \wp(\mathscr{M})}{\arg\max} \left\{ v^{\mathcal{I}}(\mathscr{Z}, \mathbf{N}^{\mathcal{J}}, \boldsymbol{\theta}^{\mathcal{I}}) - \widehat{P^{\mathcal{I},\mathscr{Z}}} \right\} \text{s.t. } j \in \mathscr{X}, j \notin \mathscr{Y}, k \in \mathscr{Y}, k \notin \mathscr{X} \right\}$$

Finally, let $\widetilde{\mathbf{\Theta}^{\mathcal{I}}}_{\{jk\},\{-jk\}}$ denote the set of consumers on side $\mathcal{I}$ that are indifferent between consuming some bundle of platforms, $\mathscr{X}$, containing both platforms j and k, and consuming some bundle of platforms, $\mathscr{Y}$, containing neither platform j nor k.

$$\widetilde{\mathbf{\Theta}^{\mathcal{I}}}_{\{jk\},\{-jk\}} \equiv \left\{ \boldsymbol{\theta}^{\mathcal{I}} \in \mathbf{\Theta}^{\mathcal{I}} : \exists \mathscr{X}, \mathscr{Y} \in \underset{\mathscr{Z} \in \wp(\mathscr{M})}{\arg\max} \left\{ v^{\mathcal{I}}(\mathscr{Z}, \mathbf{N}^{\mathcal{J}}, \boldsymbol{\theta}^{\mathcal{I}}) - \widehat{P^{\mathcal{I},\mathscr{Z}}} \right\} \text{s.t. } j,k \in \mathscr{X}, j,k \notin \mathscr{Y} \right\}$$

The matrix $\left[\frac{\partial N^{\mathcal{I}}}{\partial P^{\mathcal{I}}}\right]$ is simply the Slutsky matrix of the side $\mathcal{I}$ demand system that arises,given the coarse allocation on the opposite side, $\mathbf{N}^{\mathcal{J}^*}$. Let us denote the density of a set of marginal consumers, $\mathbf{\Theta}$, by $F[\mathbf{\Theta}] \equiv \int_{\mathbf{\Theta}} f(\boldsymbol{\theta}) d\boldsymbol{\theta}$. The elements of the Slutsky matrix are thus

$$\frac{\partial N^{\mathcal{I},j}}{\partial \overline{P^{\mathcal{I},k}}} = \begin{cases} -F[\widetilde{\boldsymbol{\Theta}^{\mathcal{I}}_{j}}], & \text{if } j = k \\ F[\widetilde{\boldsymbol{\Theta}^{\mathcal{I}}}_{j,k}] - F[\widetilde{\boldsymbol{\Theta}^{\mathcal{I}}}_{\{jk\},\{-jk\}}], & \text{if } j \neq k \end{cases}$$

Terms on the diagonal of this matrix simply reflect the number of consumers a platform loses when it increases its price by a small amount. Terms on the off-diagonal are slightly more complicated as they reflect both the number of consumers that switch *to* a bundle containing platform j when another platform k increases its price as well as the number of consumers that switch *away from* a bundle containing platform j, when platform k increases its price. Note that consumers in the former of these two groups consider platforms j and k to be net substitutes, while, for consumers in the latter group, platforms j and k are net complements.

The *Interaction Matrix*, $\left[\frac{\partial N^{\mathcal{I}}}{\partial N^{\mathcal{J}}}\right]$, mirrors the Slutsky matrix except that it is weighted by the average over *marginal consumers' valuations for additional interaction, on a particular platform*, with consumers on the opposite side of the market. For a set of consumers, $\boldsymbol{\Theta}$, defined in terms of bundle of platforms, $\mathscr{X}$, we denote the average, over $\boldsymbol{\Theta}$, of such interaction values by $v_k^{\mathcal{I},\mathscr{X}}[\boldsymbol{\Theta}]$, where

$$v_k^{\mathcal{I},\mathscr{X}}[\boldsymbol{\Theta}] \equiv \frac{\int_{\boldsymbol{\Theta}} \frac{\partial v^{\mathcal{I}}(\mathscr{X},\mathbf{N}^{\mathcal{J}},\boldsymbol{\theta})}{\partial N^{\mathcal{J},k}} f(\boldsymbol{\theta}) d\boldsymbol{\theta}}{F[\boldsymbol{\Theta}]}$$

The elements of the interaction matrix are thus

$$\frac{\partial N^{\mathcal{I},j}}{\partial N^{\mathcal{J},k}} = \begin{cases} v_k^{\mathcal{I},\mathscr{X}}[\widetilde{\boldsymbol{\Theta}^{\mathcal{I}}_{j}}] \cdot F[\widetilde{\boldsymbol{\Theta}^{\mathcal{I}}_{j}}], & \text{if } j = k \\ -v_k^{\mathcal{I},\mathscr{Y}}[\widetilde{\boldsymbol{\Theta}^{\mathcal{I}}}_{j,k}] \cdot F[\widetilde{\boldsymbol{\Theta}^{\mathcal{I}}}_{j,k}] + v_k^{\mathcal{I},\mathscr{X}}[\widetilde{\boldsymbol{\Theta}^{\mathcal{I}}}_{\{jk\},\{-jk\}}] \cdot F[\widetilde{\boldsymbol{\Theta}^{\mathcal{I}}}_{\{jk\},\{-jk\}}], & \text{if } j \neq k \end{cases}$$

Note first that the signs of the terms in this matrix correspond to the signs of marginal consumers' average "interaction valuations". Thus, in the case where consumers have positive interaction values, the signs of the terms in this matrix are the reverse of those in the Slutsky matrix. Second, note that the first argument of $\frac{\partial v^{\mathcal{I}}(\cdot,\mathbf{N}^{\mathcal{J}},\boldsymbol{\theta}^{\mathcal{I}})}{\partial N^{\mathcal{J},k}}$ in the various terms, corresponds to the subset to which the platform on which there is a change in allocation belongs. In the case of the sets $\widetilde{\boldsymbol{\Theta}^{\mathcal{I}}_{j}}$ and $\widetilde{\boldsymbol{\Theta}^{\mathcal{I}}}_{\{jk\},\{-jk\}}$, the change in coarse allocation being contemplated, $\partial N^{\mathcal{J},k}$, occurs on a platform that forms part of the bundle, $\mathscr{X}$, of which platform j is a member. In contrast, in the case of the set $\widetilde{\boldsymbol{\Theta}^{\mathcal{I}}}_{j,k}$, the change under consideration occurs on a platform that is part of a bundle, $\mathscr{Y}$, to which platform j does not belong.

6.3 General Pricing

Having characterized the role of the demand system in determining the Insulating Tariff System, we now further discuss the economic forces that determine pricing. As a benchmark, we first extend the analysis of socially optimal pricing to the case of m platforms. Then, we turn to pricing under Insulated Equilibrium.

Socially Optimal

When there are multiple platforms, the forces determining the optimal allocation are no different from those at play in the case of a monopolist. The benevolent planner solves

$$\max_{\boldsymbol{N}^{\mathcal{A}},\boldsymbol{N}^{\mathcal{B}}} \sum_{\mathcal{I}=\mathcal{A},\mathcal{B}} V^{\mathcal{I}}(\boldsymbol{N}^{\mathcal{I}},\boldsymbol{N}^{\mathcal{J}}) - \sum_{j\in\mathcal{M}} C^j(N^{\mathcal{A},j},N^{\mathcal{B},j})$$

which can be rewritten as

$$\max_{\boldsymbol{N}^{\mathcal{A}},\boldsymbol{N}^{\mathcal{B}}} \left\{ \sum_{\mathcal{I}=\mathcal{A},\mathcal{B}} \left(\sum_{\mathscr{X}\in\wp(\mathcal{M})} \int_{\theta^{\mathcal{I}}:\mathcal{M}^{\mathcal{I}*}(\theta^{\mathcal{I}},\boldsymbol{N}^{\mathcal{J}};[\overline{\boldsymbol{P}}(\boldsymbol{N})])=\mathscr{X}} \left(v^{\mathcal{I}}(\mathscr{X},\boldsymbol{N}^{\mathcal{J}},\boldsymbol{\theta}) - \widehat{P^{\mathcal{I},\mathscr{X}}}\right) f(\boldsymbol{\theta})d\boldsymbol{\theta} + \overline{\boldsymbol{P}^{\mathcal{I}}}\cdot \boldsymbol{N}^{\mathcal{I}} \right) \right.$$
$$\left. - \sum_{j\in\mathcal{M}} C^j(N^{\mathcal{A},j},N^{\mathcal{B},j}) \right\} \quad (12)$$

To express the first-order condition, let $\boldsymbol{\Theta}_j^{\mathcal{I}}$ denote the entire set of consumers on side $\mathcal{I}$ that select a bundle of platforms that includes j.

$$\boldsymbol{\Theta}_j^{\mathcal{I}} \equiv \left\{ \boldsymbol{\theta}^{\mathcal{I}} \in \boldsymbol{\Theta}^{\mathcal{I}} : j \in \mathcal{M}^{\mathcal{I}*}(\boldsymbol{\theta}^{\mathcal{I}},\boldsymbol{N}^{\mathcal{J}},[\overline{\boldsymbol{P}}(\boldsymbol{N})]) \right\}$$

Also, let $\overline{v_{\mathcal{J},j}^{\mathcal{I},j}}$ denote the *average interaction value*, among all consumers in $\boldsymbol{\Theta}_j^{\mathcal{I}}$, for an additional $\mathcal{J}$ consumer on platform j.

$$\overline{v_{\mathcal{J},j}^{\mathcal{I},j}} \equiv \frac{\int_{\boldsymbol{\Theta}_j^{\mathcal{J}}} \frac{\partial v^{\mathcal{I}}(\mathscr{X},\boldsymbol{N}^{\mathcal{J}},\boldsymbol{\theta}^{\mathcal{I}})}{\partial N^{\mathcal{J},j}} f(\boldsymbol{\theta})d\boldsymbol{\theta}}{F[\boldsymbol{\Theta}_j^{\mathcal{I}}]}$$

Note from the first line in expression (12) that, when some element of the coarse allocation, $N^{\mathcal{I},j}$, increases by a small amount, all new consumers following this change are marginal. Consequently, $\frac{\partial V^{\mathcal{I}}}{\partial N^{\mathcal{I},j}} = P^{\mathcal{I},j}$. The planner's first-order condition, with respect to

$N^{\mathcal{I},j}$ is thus

$$P^{\mathcal{I},j} = C^j_{\mathcal{I}} - \mathring{N}^{\mathcal{J},j}\overline{v^{\mathcal{J},j}_{\mathcal{I},j}} \tag{13}$$

The intuition for equation (13) is essentially the same as in the case of the monopoly, with the caveat that the behavior of $\overline{v^{\mathcal{J},j}_{\mathcal{I},j}}$, as the number of platforms changes, is ambiguous. In particular, as we discuss further in Section 6.4, an important issue is the correlation between consumers' interaction values and their "horizontal" preferences for platforms. If, for instance, on side $\mathcal{I}$ consumers with a strong preference for one platform have high interaction values while consumers with a strong preference for another platform have low interaction values, then, it could be optimal for the former platform to subsidize participation by consumers on side $\mathcal{J}$ and for the latter to tax it. Another issue that could have significant bearing on the value of $\overline{v^{\mathcal{J},j}_{\mathcal{I},j}}$ is the prevalence of consumers' multi-homing. Holding fixed the total number of opposite side counterparts to which a consumer on side $\mathcal{J}$ of platform j has access, one might expect the value she places on an additional interaction *on platform* j to increase with the number of platforms of which she is a member.

Under Insulated Equilibrium

Combining equations (8) and (9) gives the full expression for Insulated Equilibrium pricing. Platform j's price on side $\mathcal{I}$ is given by

$$P^{\mathcal{I},j} = C^j_{\mathcal{I}} + \mu^{\mathcal{I},j} - N^{\mathcal{J},j} \underbrace{\underbrace{\left(\left[-\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \overline{\mathbf{P}^{\mathcal{J}}}}\right]^{-1}\left[\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{N}^{\mathcal{I}}}\right]\right)_{j,\cdot}}_{j^{th}\text{ Row of Side }\mathcal{J}\text{ ITS}} \cdot -\mathbf{D}^{\mathcal{I}}_{\cdot,j}}_{\gamma^{\mathcal{J},j}} \tag{14}$$

To interpret the price in expression (14), recall the market power distortion and the Spence distortion, discussed in Section 6.1, regarding a monopolist platform. Under Insulated Equilibrium, firms engage in the first distortion, as represented by the term $\mu^{\mathcal{I},j}$. As is well know from one-sided analysis, this term decreases as competition increases, through an increase in the number of platforms and/or an increase in their substitutability.

Regarding the Spence distortion, note first that, under competition, $\gamma^{\mathcal{J},j}$ plays a role that is analogous to that of $\widetilde{v^{\mathcal{J}}_{\mathcal{I}}}$, under monopoly. The term $\gamma^{\mathcal{J},j}$ has two component vectors and can be understood as follows. The diversion ratio vector on the right, $-\mathbf{D}^{\mathcal{I}}_{\cdot,j}$, describes the reallocation that occurs, among *all* side $\mathcal{I}$ consumers, when platform j increases the number of side $\mathcal{I}$ consumers that *it serves* by a small amount. This "reshuffling" of side $\mathcal{I}$

consumers then triggers the response of the left-hand vector, the side $\mathcal{J}$ ITS, which keeps intact the coarse allocation on side $\mathcal{J}$. In view of this set of moving parts, it is clear that analyzing the effects of competition on the Spence distortion requires taking into account a substantial number of factors. Section 6.4, below, takes natural a first step in this direction.

6.4 The 2 Platform Case

In this section, we set m, the number of platforms, to 2. In doing so, we first consider a pair of instructive special cases before stating a general 2 platform pricing formula.

Consider an Insulated Equilibrium in which the number of consumers on either side that multi-home is negligible. Morevover, suppose that on side $\mathcal{I}$, the number of consumers on the margin between joining platform 1 and platform 2 is negligible. Under these assumptions, expression (8), of platform j's price on side $\mathcal{I}$ becomes

$$P^{\mathcal{I},j} = C^j_{\mathcal{I}} + \mu^{\mathcal{I},j} - N^{\mathcal{J},j}\frac{\partial \overline{P^{\mathcal{J},j}}}{\partial N^{\mathcal{I},j}} \tag{15}$$

The right-most term in (15), is the partial derivative of firm j's side $\mathcal{J}$ insulating tariff, with respect to its own coarse allocation. Note that (15) is general expression for firm j's side $\mathcal{I}$ price, provided that it can change its side $\mathcal{I}$ allocation without affecting the side $\mathcal{I}$ allocation of other firms.

Under the current assumptions, on side $\mathcal{J}$, there are three margins – one "cannibalization margin" between firms 1 and 2 and one "market expansion margin" between each firm and $\emptyset$. The second factor of the last term in (15) simplifies to

$$\frac{\partial \overline{P^{\mathcal{J},j}}}{\partial N^{\mathcal{I},j}} = \left((1-\kappa)\cdot v_j^{\mathcal{J},\mathscr{X}}[\widetilde{\Theta^{\mathcal{J}}}_{j,\emptyset}] + \kappa \cdot v_j^{\mathcal{J},\mathscr{X}}[\widetilde{\Theta^{\mathcal{J}}}_{j,k}]\right) \tag{16}$$

where

$$\kappa \equiv 1/\left(1 + F[\widetilde{\Theta^{\mathcal{J}}}_{j,\emptyset}]\left(\frac{1}{F[\widetilde{\Theta^{\mathcal{J}}}_{j,k}]} + \frac{1}{F[\widetilde{\Theta^{\mathcal{J}}}_{k,\emptyset}]}\right)\right)$$

$\frac{\partial \overline{P^{\mathcal{J},j}}}{\partial N^{\mathcal{I},j}}$ is thus a weighted average of the average interaction values for an additional interaction on platform j of side $\mathcal{J}$ consumers along its own two margins. This weighting, governed by κ, depends on the relative measures of consumers on each of the three side $\mathcal{J}$ margins.

When firm j's market expansion margin is more crowded, then κ is small and firm j behaves similarly to a monopoly. In particular, it is analogous to a monopoly in that it sets its quality on side $\mathcal{J}$ to cater to consumers on the market expansion margin, on which the consumers would likely be similar to those on the margin of a monopolist.

On the other hand, when the cannibalization margin is heavier, then κ is larger, and platform j caters more to consumers on this margin. Consumers on the cannibalization margin are quite different from those on the market expansion margin. Crucially, with respect to the overall decision of whether or not to join *some* platform, they are infra-marginal – and to all different degrees. As a result, under certain conditions, the average interaction value of consumers on the cannibalization margin, $v_j^{\mathcal{J},\mathcal{X}}[\widetilde{\Theta^{\mathcal{J}}}_{j,k}]$, will be closer than the average interaction value of consumers on j's expansion margin, $v_j^{\mathcal{J},\mathcal{X}}[\widetilde{\Theta^{\mathcal{J}}}_{j,\emptyset}]$, to the average interaction value among *all* of platform j's consumers on side $\mathcal{J}$. As Figure 1 illustrates, under circumstances such as those where the two platforms' primary dimension of differentiation, on side $\mathcal{J}$, is *horizontal* in membership benefits, the former group of consumers on the *cannibalization margin* constitutes a more representative sample than the latter group of consumers on the *market expansion* margin.

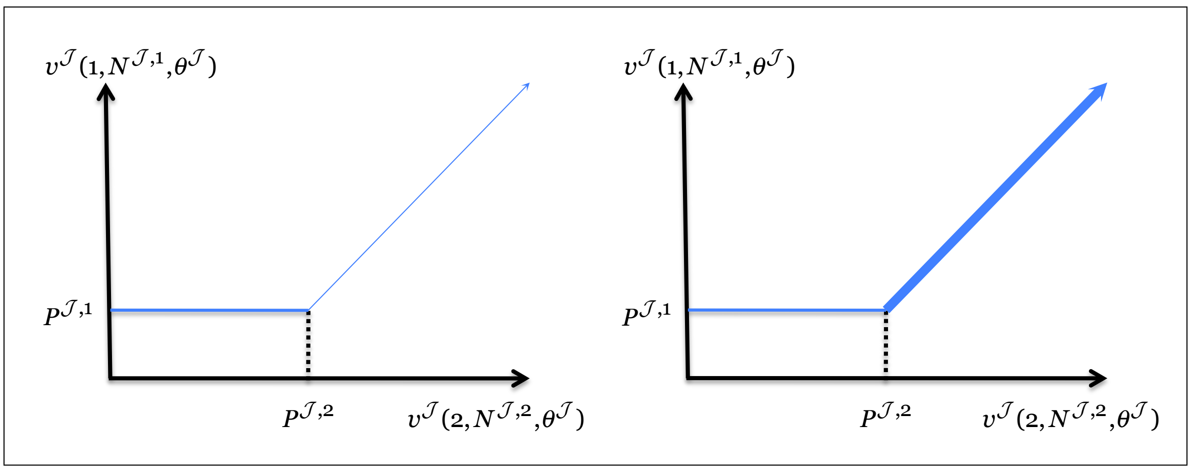

Figure 1: On the left, the thin diagonal line represents a "thin margin" between platforms 1 and 2 on side $\mathcal{J}$ and thus a low value of κ; on the right, the thick diagonal line represents a "thick margin" between platforms and thus high value of κ.

This scenario thus represents a mechanism through which competition among platforms can reduce the Spence distortion. This need not be the case, however. Continuing with the same assumptions, suppose, instead, that the primary dimension of differentiation on side $\mathcal{J}$ is *vertical*. To fix ideas, assume that demand system and platform cost functions lead to an equilibrium allocation on side $\mathcal{I}$ that is such that $N^{\mathcal{I},j*} > N^{\mathcal{I},k*}$. Furthermore, suppose that consumers on $\mathcal{J}$ differ significantly from one another in both the membership and interaction benefits they perceive but that these preferences are, for most consumers, very stable across platforms. Formally, such preferences can be straightforwardly represented by the utility function giving a payoff

$$B_i^{\mathcal{J}} + b_i^{\mathcal{J}} N^{\mathcal{I},j} + \epsilon_i^{\{j\}} - \sigma^{\mathcal{J},j}\left(\boldsymbol{N}^{\mathcal{J}}\right)$$

to consumer i on side $\mathcal{J}$, when he joins the set containing only platform j, where $\epsilon_i^{\mathscr{X}}$ is a bundle-specific idiosyncratic term, $B_i^{\mathcal{J}}$ denotes consumer i's membership value and $b_i^{\mathcal{J}}$ denotes consumer i's interaction value. As implied by the description above, suppose that consumers' values of $\epsilon^{\mathscr{X}}$ are heavily concentrated around some value, normalized to zero.

In this setup, provided appropriate cost functions, under Insulated Equilibrium, both platform j and platform k attract a significant number of side $\mathcal{J}$ consumers. Furthermore, the two platforms are, in effect, vertically differentiated from one another in a manner analogous to that of Shaked and Sutton (1982). Platform j charges its consumers a higher price than does platform k, while also allowing for interaction with a larger number of side $\mathcal{I}$ consumers. Accordingly, (ignoring noise term, $\epsilon^{\mathscr{X}}$), we can define a threshold interaction value, $\widetilde{b_{j,k}^{\mathcal{J}}} \equiv \frac{\sigma^{\mathcal{J},j} - \sigma^{\mathcal{J},k}}{N^{\mathcal{I},j} - N^{\mathcal{I},k}}$, which represents the interaction value of all side $\mathcal{J}$ consumers that lie on the cannibalization margin between platforms j and k.

Recall the first-order condition in expression (16), and note that as the mass of consumers with an interaction value of $\widetilde{b_{j,k}^{\mathcal{J}}}$ increases, so does κ. As a result, if such an increase were to occur, each platform would have an incentive to adjust its allocation on side $\mathcal{I}$ so as to cater more to consumers on this cannibalization margin. In contrast to the previous example, however, this *exacerbates* the Spence distortion on side $\mathcal{J}$ inflicted by each of the two platforms. This is because, on the one hand, (except for an arbitrarily small measure) all of platform j's side $\mathcal{J}$ consumers have interaction values greater than $\widetilde{b_{j,k}^{\mathcal{J}}}$, while all of platform k's side $\mathcal{J}$ consumers have interaction values less than $\widetilde{b_{j,k}^{\mathcal{J}}}$.

Thus far in this section, we have "turned off" the competition among platforms on side $\mathcal{I}$ by assuming that the cannibalization margin on this side is negligible. We now activate this feature of the model and examine the general case of competition between two platforms. Let $\widetilde{D_{jk}^{\mathcal{I}}} \equiv \frac{\partial N^{\mathcal{I},k}}{\partial P^{\mathcal{I},j}} / - \frac{\partial N^{\mathcal{I},j}}{\partial P^{\mathcal{I},j}}$ denote the side $\mathcal{I}$ diversion ratio, i.e., the fraction of platform j's side $\mathcal{I}$'s marginal consumers, who, in response to a price increase by j would add platform k to the bundle they of platforms they join. We have

$$P^{\mathcal{I},j} = C^j_{\mathcal{I}} + \mu^{\mathcal{I},j} - N^{\mathcal{J},j}\left((1-\kappa)\cdot v_j^{\mathcal{J},\mathscr{X}}[\widetilde{\Theta^{\mathcal{J}}}_{j,\emptyset}] + \kappa\left(v_j^{\mathcal{J},\mathscr{X}}[\widetilde{\Theta^{\mathcal{J}}}_{j,k}] + \overbrace{D^{\mathcal{I}}_{jk}(v_k^{\mathcal{J},\mathscr{X}}[\widetilde{\Theta^{\mathcal{J}}}_{k,j}] - v_k^{\mathcal{J},\mathscr{X}}[\widetilde{\Theta^{\mathcal{J}}}_{k,\emptyset}])}^{\star}\right)\right) \tag{17}$$

Note, first, the terms in (17), indicated by $\star$, that do not enter the prior first-order condition, (16). These appear, since, when there is competition on side $\mathcal{I}$ and j changes its quantity on this side, this affects the number of consumers that k serves as well. The diversion ratio on side $\mathcal{I}$ represents the significance of this side $\mathcal{I}$ reallocation. This overall reallocation of side $\mathcal{I}$ consumers influences the perceived quality by side $\mathcal{J}$ consumers of not only platform j but also of platform k. As a result, in order to hold fixed its quantity on side $\mathcal{J}$, j must take into account the interaction values of k's marginal consumers for an additional interaction on that platform.

In particular, the relevant quantity for this purpose is $v_k^{\mathcal{J},\mathscr{X}}[\widetilde{\Theta^{\mathcal{J}}}_{k,j}] - v_k^{\mathcal{J},\mathscr{X}}[\widetilde{\Theta^{\mathcal{J}}}_{k,\emptyset}]$, the *difference* between the value for an additional interaction on platform k of the side $\mathcal{J}$ consumers on k's cannibalization margin and those consumers on the market expansion margin. Thus, the extent to which j distorts the quality it provides to its side $\mathcal{J}$ consumers depends not only on the divergence between the interaction values of its own marginal versus average consumers but also on the distribution of such valuations among consumers on other platforms. As competition on side $\mathcal{I}$ toughens through an increase in $D^{\mathcal{I}}_{jk}$, in determining its quality on side $\mathcal{J}$, platform j puts more weight on the preferences of consumers on the cannibalization margin.

6.5 *m* Symmetric Platforms

We now consider a symmetric Insulated Equilibrium among m identical platforms. Let $F_k^{\mathcal{J}}$ and $v_k^{\mathcal{J}}$ denote, respectively, the mass and average interaction value of side $\mathcal{J}$ consumers on a given platform's cannibalization margin, and let $F_\emptyset^{\mathcal{J}}$ and $v_\emptyset^{\mathcal{J}}$ denote the mass and interaction value of $\mathcal{J}$ consumers on a given platform's market expansion margin. The side $\mathcal{I}$ first-order condition for platform j is given by

$$P^{\mathcal{I},j} = C^j_{\mathcal{I}} + \mu^{\mathcal{I},j} - N^{\mathcal{J},j}\left((1-\kappa^{sym})v_\emptyset^{\mathcal{J}} + \kappa^{sym}v_k^{\mathcal{J}}\right) \tag{18}$$

where

$$\kappa^{sym} \equiv \frac{\frac{(m-1)F_k^{\mathcal{J}}}{F_\emptyset^{\mathcal{J}}+mF_k^{\mathcal{J}}}}{1-\frac{F_k^{\mathcal{I}}}{F_\emptyset^{\mathcal{I}}+mF_k^{\mathcal{I}}}}$$

Expression (18) reinforces the themes we discuss in Section 6.4. As in expression (16), the extent to which the quality provided to consumers on side $\mathcal{J}$ depends on the characteristics of consumers on the two types of margins and on the weight the platform attributes to each of these margins. In particular, since κ^{sym} is increasing in $F_k^{\mathcal{J}}$, the mass of consumers on the side $\mathcal{J}$ cannibalization margins, such an increase in competition on side $\mathcal{J}$ reduces the Spence distortion experienced by consumers on that side if and only if the average interaction value of consumers on the cannibalization margin is closer than that of the consumers on the market expansion margin to the average interaction value of all consumers.

7 Many Sides of the Market

Thus far, for expository purposes, we have focused on market configurations with two "sides" or groups of consumers. The model easily extends to accommodate an arbitrary number of sides. To see this, suppose there are S groups of consumers, indexed by $\mathcal{I} = \mathcal{A}, \mathcal{B}, \mathcal{C}, ...$, and let the gross payoff of joining a bundle of platforms, $\mathscr{X}$, to a consumer of type $\boldsymbol{\theta}^{\mathcal{I}}$ on side $\mathcal{I}$ be $v^{\mathcal{I}}(\mathscr{X}, \boldsymbol{N}^{-\mathcal{I}}, \boldsymbol{\theta}^{\mathcal{I}})$, where $v^{\mathcal{I}} : \wp(\mathscr{M}) \times [0,1]^{m(S-1)} \times \Theta^{\mathcal{I}} \to \mathbb{R}$ now depends on $\boldsymbol{N}^{-\mathcal{I}} \in [0,1]^{S-1}$, the coarse allocation on the $S-1$ other sides of the market apart from side $\mathcal{I}$. Also, let platform j's strategy now be given by $\sigma^j \equiv \left(\sigma^{\mathcal{A},j}(\boldsymbol{N}^{-\mathcal{A}}), \sigma^{\mathcal{B},j}(\boldsymbol{N}^{-\mathcal{B}}), \sigma^{\mathcal{C},j}(\boldsymbol{N}^{-\mathcal{C}}), ...\right)$, where $\sigma^{\mathcal{I},j} : [0,1]^{m(S-1)} \to \mathbb{R}$ maps from $\boldsymbol{N}^{-\mathcal{I}} \in [0,1]^{m(S-1)}$ to a total price that side $\mathcal{I}$ consumers pay to join platform j.

It is straightforward to see that, when the model is extended in this way, none of the arguments made thus far in the paper depend on the presence of only two sides. In particular, the result of Theorem 1, that a CNE coarse allocation implies a price vector, continues to hold. Thus, the simplest way to consider a platform's profit maximization problem continues to be as a choice of allocation, holding fixed the strategies of the other platforms

$$\max_{\{N^{\mathcal{A},j}, N^{\mathcal{B},j}, N^{\mathcal{C},j}, ...\}} \sum_{\mathcal{I}} N^{\mathcal{I},j} P^{\mathcal{I},j}\left(N^{\mathcal{I},j}, \boldsymbol{N}^{-\mathcal{I}}\right) - C^j\left(N^{\mathcal{A},j}, N^{\mathcal{B},j}, N^{\mathcal{C},j}, ...\right)$$

Analogously to the results of Section 6.3, the prices that implement the socially optimal allocation satisfy

$$P^{\mathcal{I},j} = C^j_{\mathcal{I}} - \sum_{\mathcal{J}=-\mathcal{I}} N^{\mathring{\mathcal{J}},j} \overline{v^{\mathcal{J},j}_{\mathcal{I},j}}$$

and the platforms' prices under Insulated Equilibrium satisfy

$$P^{\mathcal{I},j} = C^j_{\mathcal{I}} + \mu^{\mathcal{I},j} - \sum_{\mathcal{J}=-\mathcal{I}} \left(\left[-\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \overline{\mathbf{P}^{\mathcal{J}}}} \right]^{-1} \left[\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{N}^{\mathcal{I}}} \right] \right)_{j,\cdot} \cdot -\mathbf{D}^{\mathcal{I}}_{\cdot,j}$$

These expressions are completely analogous to those discussed in Section 6. The only difference in the case where there are S sides of the market is that the number of consumers on side $\mathcal{I}$ affects the payoffs of consumers on all of the $S-1$ other sides. Consequently, both the socially optimal and platforms' equilibrium prices take into account the sum of such externalities.

8 Conclusion

This paper makes two contributions in its current form. First it provides what we believe is close to the maximally general model of multi-sided platforms that is also tractable. Second, it proposes tools, particularly the solution concept of *Insulated Equilibrium*, that allow this broad model to be analyzed. While we believe this constitutes perhaps *the* critical next step for the literature on multi-sided platforms it is certainly no more than that: much remains to be done, both for us and others. We therefore now briefly discuss both some extensions we plan to develop in the final version of this paper, as well as some of what we consider the most promising directions for future research.

8.1 Plans for Final Draft

In the upcoming draft of this paper we plan to:

1. Include an extension that allows for within-side externalities.

2. Characterize the conditions for existence, uniqueness and stability of Insulated Equilibrium.

3. Formalize Armstrong's Paradox through a result analogous to that of Klemperer and Meyer (1989) showing that any allocation with weakly positive profits can be an equilibrium, or not, depending on the "details" of firm strategies.

4. Extend Jaffe and Weyl (2010b)'s formulation of the first-order approach to merger analysis that appears in the new DOJ-FTC draft merger guidelines (Federal Trade Commission, 2009) to multi-sided platforms in Insulated Equilibrium. This will be something of a culmination of our work, allowing a full extension of the "standard paradigm" of merger analysis to multi-sided platforms.

In subsequent work, we plan to develop techniques for computing the first-order conditions of Insulated Equilibrium given common parametric demand models, such as Berry et al. (1995). We hope this will make it feasible to extend the standard static industrial organization empirical analysis to multi-sided platforms, allowing inference on cost functions and the computation of counterfactual policy outcomes. We also hope to obtain results on the instruments and variation sufficient to first-order (Weyl, 2009) or fully non-parametrically identify marginal costs and other quantities of interest. We may, in addition, develop software for implementing some of the latter functions computationally.

8.2 Directions for Future Research

Beyond these direct avenues for extension, our paper suggests many natural directions for future research. Most clearly, relaxing the assumptions (the "macroness" of the model, homogeneous quality, no price discrimination) we discussed in Section 2 is important for the literature to progress. Our solution concept also seems naturally connected to a number of other problems in economics; elucidating these connections would help unify these areas. Most clearly, White is currently constructing a model, with Germain Gaudin, that builds on the techniques developed in this paper to study the effect of competition on the quality provision by one-sided firms. Similarly by the Bulow and Roberts (1989) equivalence, Weyl (2010) is equivalent to Segal (1999)'s general model of contracting with externalities with asymmetric information. Thus it seems natural that our model should be closely related to common agency with externalities and asymmetric information. It would therefore be interesting to consider whether Insulated Equilibrium has a natural analogy to solution concepts invoked in that literature, or whether it offers a potential alternative concept.

At a deeper theoretical level, it would be interesting to understand more clearly the dynamic incentives of multi-sided platforms, in the spirit of Chen et al. (2009) and Cabral (forthcoming), and whether these lead to price paths resembling insulating tariff systems. Also the intersection of profit maximization and matching market design (Roth, 2002) is conspicuously limited but very promising; see Gomes (2009) for an exception proving the rule.

On the applied side, we believe our paper offers a number of tools that make possible a range of interesting empirical analyses of multi-sided platforms, measuring market power and Spence distortions and predicting counter-factual effects of policy interventions, which we hope will develop in coming years. Making the theory of multi-sided platforms useful to policy makers will also require enriching our model to consider issues that are beyond the scope of this paper such as interconnection, vertical restraints, bundling, predation and regulatory design. Perhaps soon the theory and measurement of multi-sided platform industries will finally fulfill their promise in helping to clarify some of the most heated and ideological industrial policy debates of our generation.

Appendices

A Proof of Lemma 1

Proof. The continuity and strict sign assumptions on $f^{\mathcal{I}}$ as well as the smoothness and full coverage assumptions on $v^{\mathcal{I}}$ imply that, given any "other-side" coarse allocation, $\boldsymbol{N}^{\mathcal{J}}$, and platform strategy profile on side $\mathcal{I}$, evaluated at this coarse allocation, $\boldsymbol{\sigma}^{\mathcal{I}}(\boldsymbol{N}^{\mathcal{J}})$, there is a diffuse distribution over $\mathbb{R}^{2^m-2}$ of net payoff vectors, $\boldsymbol{U}_i^{\mathcal{I}}$, that is strictly positive throughout its domain. Denote this family of distributions by $g^{\mathcal{I}}(\boldsymbol{U}_i^{\mathcal{I}};\boldsymbol{N}^{\mathcal{J}},\boldsymbol{\sigma}^{\mathcal{I}}(\boldsymbol{N}^{\mathcal{J}}))$.

For any platform, j, for any $\boldsymbol{N}^{\mathcal{J}}$, and for any vector of prices, $\boldsymbol{\sigma}^{\mathcal{I}}(\boldsymbol{N}^{\mathcal{J}})$, facing consumers on side $\mathcal{I}$, we can thus pick out a *j-Isolated Straddling Set*, $\mathscr{S}$, that is open and convex and which satisfies the conditions

(a) *j-Isolation*: Contains only consumers who strictly prefer, compared to all other bundles, both the empty set and the bundle containing only platform j. Formally, these are consumers for whom

$$\min\left\{0,U_i^{\mathcal{I}}(\{j\};\boldsymbol{N}^{\mathcal{J}},\boldsymbol{\sigma}^{\mathcal{I}}(\boldsymbol{N}^{\mathcal{J}}))\right\} > U_i^{\mathcal{I}}(\mathscr{X};\boldsymbol{N}^{\mathcal{J}},\boldsymbol{\sigma}^{\mathcal{I}}(\boldsymbol{N}^{\mathcal{J}})),\ \ \forall\mathscr{X}\in\wp(\mathscr{M})\backslash\{\emptyset,\{j\}\}$$

(b) *j-Straddling*: Contains some consumers who strictly prefer $\{j\}$ to $\emptyset$, some who strictly prefer $\emptyset$ to $\{j\}$ and some *Indifferent Consumers*, between the two.

We now show that $\partial V^{\mathcal{I}}/\partial\widetilde{N^{\mathcal{I},j}}$ exists and is equal to $\sigma^{\mathcal{I},j}(\boldsymbol{N}^{\mathcal{J}})$. Pick a particular indifferent consumer in this set and denote her net payoff vector by $\widetilde{\boldsymbol{U}^{\mathcal{I}}}$, and pick any $\delta>0$ that is small enough such that, $\{i:\|\boldsymbol{U}_i^{\mathcal{I}}-\widetilde{\boldsymbol{U}^{\mathcal{I}}}\|<\delta\}\subset\mathscr{S}$. Denote this δ-ball about $\widetilde{\boldsymbol{U}^{\mathcal{I}}}$ by $B_\delta\left(\widetilde{\boldsymbol{U}^{\mathcal{I}}}\right)$. Denote by $\underline{B_\delta}\left(\widetilde{\boldsymbol{U}^{\mathcal{I}}}\right)\equiv B_\delta\left(\widetilde{\boldsymbol{U}^{\mathcal{I}}}\right)\cap\left\{i:0>U_i^{\mathcal{I}}(\{j\};\boldsymbol{N}^{\mathcal{J}},\boldsymbol{\sigma}^{\mathcal{I}}(\boldsymbol{N}^{\mathcal{J}}))\right\}$ the set of consumers in this ball who strictly prefer $\emptyset$ to $\{j\}$, and denote by $\overline{B_\delta}\left(\widetilde{\boldsymbol{U}^{\mathcal{I}}}\right)\equiv B_\delta\left(\widetilde{\boldsymbol{U}^{\mathcal{I}}}\right)\cap\left\{i:U_i^{\mathcal{I}}(\{j\};\boldsymbol{N}^{\mathcal{J}},\boldsymbol{\sigma}^{\mathcal{I}}(\boldsymbol{N}^{\mathcal{J}}))>0\right\}$ the set of consumers in this ball who strictly prefer $\{j\}$ to $\emptyset$.

(a) **Right-hand limit**: for any such δ, we can find $\epsilon>0$, namely, the measure of the set of consumers in $\underline{B_\delta}\left(\widetilde{\boldsymbol{U}^{\mathcal{I}}}\right)$, such that

$$\frac{V^{\mathcal{I}}\left(\left(\widetilde{N^{\mathcal{I},1}},...,\widetilde{N^{\mathcal{I},j}}+\epsilon,...,\widetilde{N^{\mathcal{I},m}}\right),\boldsymbol{N}^{\mathcal{J}}\right)-V^{\mathcal{I}}\left(\widetilde{\boldsymbol{N}^{\mathcal{I}}},\boldsymbol{N}^{\mathcal{J}}\right)}{\epsilon}\geq$$

$$\frac{\int_{\theta_i^{\mathcal{I}}:i\in\underline{B_\delta}\left(\widetilde{\boldsymbol{U}^{\mathcal{I}}}\right)}v^{\mathcal{I}}(\{j\},\boldsymbol{N}^{\mathcal{J}},\boldsymbol{\theta})f(\boldsymbol{\theta})d\boldsymbol{\theta}+V^{\mathcal{I}}\left(\widetilde{\boldsymbol{N}^{\mathcal{I}}},\boldsymbol{N}^{\mathcal{J}}\right)-V^{\mathcal{I}}\left(\widetilde{\boldsymbol{N}^{\mathcal{I}}},\boldsymbol{N}^{\mathcal{J}}\right)}{\epsilon}\geq$$

$$\frac{\left(\widetilde{U_i^{\mathcal{I}}}\left(\{j\},\boldsymbol{N}^{\mathcal{J}},\boldsymbol{\sigma}^{\mathcal{J}}(\boldsymbol{N}^{\mathcal{J}})\right)+\sigma^{\mathcal{I},j}(\boldsymbol{N}^{\mathcal{J}})-\delta\right)\epsilon}{\epsilon}=\sigma^{\mathcal{I},j}(\boldsymbol{N}^{\mathcal{J}})-\delta$$

We also have that for any such δ, there exists $\epsilon > 0$ such that

$$\frac{V^{\mathcal{I}}\left(\left(\widetilde{N^{\mathcal{I},1}},...,\widetilde{N^{\mathcal{I},j}}+\epsilon,...,\widetilde{N^{\mathcal{I},m}}\right),\boldsymbol{N}^{\mathcal{J}}\right)-V^{\mathcal{I}}\left(\boldsymbol{N}^{\mathcal{I}},\boldsymbol{N}^{\mathcal{J}}\right)}{\epsilon}\leq\sigma^{\mathcal{I},j}(\boldsymbol{N}^{\mathcal{J}})+\delta \tag{19}$$

since, if this were not true, there must exist $k > 0$ such that, for all $\delta < k$, there exists no $\epsilon > 0$ satisfying the condition in (19). This implies that, under the original CNE that induces coarse allocation $\widetilde{\boldsymbol{N}^{\mathcal{I}}}$, there are consumers who *don't* consume $\{j\}$ but whose net payoff from consuming $\{j\}$ is strictly greater than their net payoff from consuming some other bundle. However, such an outcome violates the definition of a CNE.

(b) **Left-hand limit**: an analogous arguments holds in which the steps to establish the lower and upper bounds are reversed, and for the upper bound, one appeals to the properties of the set, $\overline{B_\delta}\left(\widetilde{\boldsymbol{U}^{\mathcal{I}}}\right)$.

□

B The One- and Two-Sided Spence Distortions

This appendix attempts to illustrate, in an intuitive manner, the precise way in which the quality distortion of a one-sided monopolist carries through to a two-sided setting.

Privately Optimal Pricing

A thought experiment that is instructive in understanding the "Spence term", $\overline{v_{\mathcal{I}}^{\mathcal{J}}}$, in the context of a monopoly two-sided platform is to think about consumers on side $\mathcal{I}$ *purely as inputs in the production of "quality" on side* $\mathcal{J}$. For the sake of argument, imagine that all side $\mathcal{I}$ consumers have willingness to pay to join the platform arbitrarily close to zero. Under such circumstances, the choice of $N^{\mathcal{I}}$ is equivalent to the one-sided monopolist's choice of quality in Spence's model, while the choice of $N^{\mathcal{J}}$ is equivalent to the monopolist's choice of quantity. Correspondingly, the monopolist sets $N^{\mathcal{I}}$ to satisfy $N^{\mathcal{J}}\widetilde{v_{\mathcal{I}}^{\mathcal{J}}} = C_{\mathcal{I}}$, thus equating the marginal cost of producing quality with the number of paying consumers, multiplied by the average valuation for an additional unit of quality *among the set of marginal paying consumers*. In the meantime, since the potential revenue from side $\mathcal{I}$ consumers is negligible, the monopolist is impervious to the quality they perceive and thus sets $N^{\mathcal{J}}$ to solve $P^{\mathcal{J}} - C_{\mathcal{J}} = \mu^{\mathcal{J}}$.

Extending the thought experiment slightly further affords a more general view of the relationship between the two-sided monopolist's two choices. Suppose that side $\mathcal{I}$ and side $\mathcal{J}$ consumers' preferences are denominated in side-specific "local currencies", but that their valuations for interacting with consumers on the other side are invariant with the exchange rate.[14] Let $\$^{\mathcal{I}}$ denote the value to the platform of the side $\mathcal{I}$ currency in terms of the side $\mathcal{J}$ currency, and let costs be measured in units of the latter. The platform's first-order conditions imply

$$\$^{\mathcal{I}} = \frac{C_{\mathcal{I}} - N^{\mathcal{J}}\widetilde{v_{\mathcal{I}}^{\mathcal{J}}}}{P^{\mathcal{I}} - \mu^{\mathcal{I}}} = \frac{\mu^{\mathcal{J}} - (P^{\mathcal{J}} - C_{\mathcal{J}})}{N^{\mathcal{I}}\widetilde{v_{\mathcal{J}}^{\mathcal{I}}}} \tag{20}$$

The case discussed above corresponds to the scenario in which the side $\mathcal{I}$ currency is worthless. In equation (20), as $\$^{\mathcal{I}}$ increases from zero, the monopolist cares more about setting the "right" level of quality on side $\mathcal{I}$. Also, as a result of such an increase, the platform chooses a quantity on side $\mathcal{I}$ that is designed more than before to raise revenue directly and less than before to optimize quality on side $\mathcal{J}$. Supposing, as would ordinarily be the case, that the values of consumers' "local currencies" are already incorporated into their preferences, we have $\$^{\mathcal{I}} = 1$, and thus the platform gives what may, in the appropriate sense, be called "equal weight" to the two problems.

Socially Optimal Pricing

The enduring insight of Spence's framework is that a one-sided monopolist engages in a distortion of quality, with respect to the social optimum, that is separate from and in addition to its well-known distortion of consumption level or "market power distortion". A one-sided monopolist chooses a level of quality that differs from the socially optimal level, because it fails to take into account the valuations for quality of its *infra-marginal*

[14]This example is designed to illuminate the mechanisms at play rather than to be realistic or applicable to a particular industry.

consumers. A *two-sided* monopolist's choice of quality on each side is biased, with respect to the social optimum, by the same lack of concern for infra-marginal consumers. Given the fixed relationship, however, between consumption on one side of the market and quality on the other, the interaction between these two distortions takes a particular form.

To understand this interaction, first reconsider, for a moment, the situation proposed above in which side $\mathcal{I}$ consumers have negligible willingness to pay and thus the problem is equivalent to Spence's one-sided model. Let $\overline{v^{\mathcal{I}}_{\mathcal{J}}}$ denote the average interaction value among the *entire set* of consumers on side $\mathcal{I}$ that join the platform, and let $\mathring{N}^{\mathcal{I}}$ denote the socially optimal level of consumption on side $\mathcal{I}$. Ignoring the (negligible) payoffs to side $\mathcal{I}$ consumers, a benevolent planner with a utilitarian social welfare function chooses consumption levels that solve

$$0 = C_{\mathcal{I}} - \mathring{N}^{\mathcal{J}}\overline{v^{\mathcal{J}}_{\mathcal{I}}} = C_{\mathcal{J}} - P^{\mathcal{J}}. \tag{21}$$

Note the absence of $\widetilde{v^{\mathcal{J}}_{\mathcal{I}}}$ and $\mu^{\mathcal{J}}$ in expression (21). Side $\mathcal{J}$ consumers face a price equal to the marginal cost of serving them. Meanwhile, the quality level is such that the cost, per consumer, of increasing it by a small amount is equal to the average valuation for such an increase among all side $\mathcal{J}$ consumers.

Now let us turn back to the "ordinary" two-sided case, taking into account the welfare of consumers on both sides. The socially optimal consumption levels solve

$$\frac{C_{\mathcal{I}} - \mathring{N}^{\mathcal{J}}\overline{v^{\mathcal{J}}_{\mathcal{I}}}}{P^{\mathcal{I}}} = \frac{C_{\mathcal{J}} - P^{\mathcal{J}}}{\mathring{N}^{\mathcal{I}}\overline{v^{\mathcal{I}}_{\mathcal{J}}}} = 1 \tag{22}$$

As the left-hand term in equation (22) shows, it is generically optimal *not* to satisfy the one-sided Spence first-order condition for quality on side $\mathcal{J}$, given by the left-hand equality in (21). This is because consumers on side $\mathcal{I}$ do not feel indifferently about their erstwhile role as quality inputs for $\mathcal{J}$. The optimal departure from the one-sided Spence first-order condition for quality on side $\mathcal{J}$ is proportional to the side $\mathcal{I}$ price, which is equal to the benefit or harm that marginal consumers on side $\mathcal{I}$ perceive in fulfilling this role.

Comparing equations (22) and (20) reveals the form that the interaction takes between the markup distortion and the Spence distortion induced by a two-sided monopolist. Consider the middle term of (20). The difference between $N^{\mathcal{J}}\widetilde{v^{\mathcal{J}}_{\mathcal{I}}}$ and $\mathring{N}^{\mathcal{J}}\overline{v^{\mathcal{J}}_{\mathcal{I}}}$ constitutes the "one-sided Spence distortion" and regards the matter of *which* set of side $\mathcal{J}$ consumers to cater to. In a two-sided setting, for any such set of side $\mathcal{J}$ consumers, the additional issue arises of how to *skew* the level of quality they receive. While the social planner skews in proportion to the preferences of marginal $\mathcal{I}$ consumers, the monopolist, unable to price discriminate to infra-marginal consumers on $\mathcal{I}$, skews in proportion to its *direct marginal revenue* from all side $\mathcal{I}$ consumers.

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