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Identity, Community and Segregation

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Abstract

I develop a framework to explain why identity divides some communities and not others. An identity group is defined as a group of individuals with the same 'culture'. A community is divided when different identities are socially segregated; a community is integrated when there is no social segregation between different identities. I find three possible outcomes for a community: assimilation, where groups socially integrate and one group conforms to the culture of another; non-assimilative integration, where groups integrate but individuals retain their own identity; and segregation, where groups socially segregate and retain their own culture. I find that certain community environments encourage segregation: (*i*) communities with similar sized identity groups; (*ii*) larger communities; (*iii*) communities with greater cultural distance between identities. Further, when segregation occurs, the cultural divide between the two groups can increase endogenously beyond ex-ante differences.

1 Introduction

Daniel Posner (2004) describes the different fates of the Chewa and Tumbuka peoples divided by the border between Zambia and Malawi. The border, drawn purely for administrative purposes by the British South African Company in 1891, divided the groups such that about two-thirds of each of the Chewa and Tumbuka peoples were in Malawi, and a quarter of the Chewa and a third of the Tumbuka were in Zambia - the remainder of the Chewa resided in Zimbabwe and Mozambique. Posner undertakes surveys in two villages on each side of the border and shows that objective cultural differences between the two groups remain similar in both countries. However, relations between the Chewa and Tumbuka are completely different in Malawi and Zambia. In Malawi the attitude of each group towards the other was antagonistic and 55% of those questioned said they would not marry a member of the other group. While in Zambia, when asked what differentiates the two groups, most members of

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both groups said 'We are the same.' In Zambia only 24% said they would not marry a member of the opposite group. Thus, despite the same cultural differences between the two groups in both countries, in the Malawian villages the cultural cleavage between the two groups is much more socially salient.[1]

Why is ethnic identity so much more divisive in the villages on the Malawian side of the border? Similar questions arise in other parts of the world. Moody (2001), and Fryer & Echenique (2006) examine friendship segregation between different races in US middle and high schools. Moody (2001) finds a huge range in the level of friendship segregation between race groups across different high schools: some schools have 'near perfect integration' between different races, in the sense that a child is just as likely to have a different-race friend as a same-race friend; while in about 8% of the sample the probability of having a same-race friend is at least five times the probability of having a cross-race friend. Fryer & Echenique (2006) look at the level of segregation of a particular race group (Asian, Black, Hispanic, White) within a school. They find that Hispanic students, for example, might be integrated in one school while in another school the group of Hispanic students has almost no non-Hispanic friends. Why is it that race is not salient to friendship in some school communities while in other schools race draws a dividing line across which few friendships are formed?

The goal of this paper is to provide a model of a diverse community with which to examine why and when identity (such as race or ethnicity) divides a community. I examine what type of community environments encourage division and what conditions contribute to an integrated community where identity plays no divisive role. Further, I examine why certain features of community environments foster divisions. The type of community division that I analyze is social segregation. Do different identity groups interact with each other or is there social segregation between groups? The definition we use here to identify an integrated versus a divided community is this: if individuals show no preference for forming same-identity relationships over different-identity relationships, then we say the community is integrated; if individuals show a preference for same-identity relationships such that social interaction is restricted to some extent to same-identity individuals, then we say identity divides.

An identity group is defined as a group of individuals with the same prescribed 'culture'. This culture could involve particular behavior, attitudes, and activities such as listening to certain kinds of music or playing certain sports. For example, gender is an identity if the prescribed male and female norms of behavior and activities differ to some extent. Note that I say an identity group is 'prescribed' a culture to make clear that an individual can choose not to follow her prescribed culture. However, when individuals do not follow their prescribed culture and instead adhere to the culture of another group they face a cost. This cost could

[1] For a review of ethnic diversity see Alesina and La Ferrara (2005).

arise in numerous ways. For example, culture can be so deeply entrenched that individuals find it psychologically costly to adhere to behaviors or attitudes that differ from the culture one has grown up with. Culture could represent social norms for which there are costs to breaking. Culture could be different activities such as listening to and playing different types of music or sport. Listening to, talking about, or playing the sport or music of another group can be costly because it is unfamiliar. Identity as presented here is also consistent with the introduction of identity into an individual's utility function proposed by Akerlof and Kranton (2000), whereby undertaking behavior different from that prescribed to one's own identity causes psychological discomfort.[2]

The model of community proposed in this paper is based on the premise that a social tie between two individuals requires coordination on shared activities and behaviors. For example, friendship requires coordination on what to talk about, what to do together, when to meet up, how to behave towards one another and to third parties, how to speak to each other, and how polite to be to each other. Studies of child and adolescent interaction between friends and non friends show such coordination to be a central feature distinguishing friendship relationships from non friendship relations (see Rubin, Bukowski, & Laursen 2008). The fundamental nature of coordination on shared activities and behaviors to the existence of a social tie suggests that individuals may wish to associate with other individuals who share the same culture in order to facilitate coordination.

Individuals make two choices: they choose their social ties from within the community and they choose a set of activities, which includes the choice of whether to retain their own culture or adopt the culture of another group. A social tie between two individuals requires coordination on shared activities and behavior. This could be anything from participating in sports together to enjoying conversation, but it requires both individuals to coordinate on what they want to do, what they want to talk about, and norms of behavior. It follows that if two individuals of different identities with different cultures want to form a social tie then one of them may have to adopt the culture of the other. Thus the key trade-off in this model involves whether to compromise on culture and interact with more people or whether to retain one's own culture and so potentially restrict social interaction. Segregation occurs when the cost involved in adjusting culture to coordinate with an individual of a different identity is too great relative to the benefits of interacting with that individual.

We find three possible outcomes for a community: assimilation, where one group integrates with another by conforming to the culture of that group; non-assimilative integration, where groups integrate but individuals retain their own culture; and segregation where groups seg-

[2]We do not examine the other side of identity discussed in Akerlof and Kranton (2000) where deviation from the behavior prescribed by one's own identity has a direct effect on the utility of others.

regate and individuals retain their own culture. Note that when assimilation occurs there is complete conformity of culture and other activities. When non-assimilative integration occurs there is differentiation of culture but conformity on all other activities in the society. In a sense, when non-assimilative integration occurs individuals retain their own culture but also develop a shared community culture which allows for an integrated society. When segregation occurs it is sometimes accompanied by increased polarization of culture between groups. Groups react to a shared community by further differentiating themselves to avoid integration.

I also examine the community conditions under which each of the three outcomes will occur, in particular, when communities will want to integrate versus segregate. First, segregation is more likely to occur in communities where identity groups are more similar in size. To be precise, there is a threshold property such that segregation occurs in communities only once groups are sufficiently similar in size; before this threshold, when groups are dissimilar sizes, the community integrates. This is a result of the trade-off facing the minority group: interact with the minority group without compromising on culture, or interact with more people in the majority group by adopting the majority culture. Second, segregation is more likely to occur in larger communities. This is because relative size differences matter less when groups are larger. For example, having 2 people to interact with versus 5 people is much more salient than 20 people versus 50. Third, segregation is more likely to occur when the cultural distance between groups is larger.

I also conduct a welfare analysis of the different community outcomes. I find that under certain parameters segregation can be both welfare maximizing and Pareto optimal. Segregation is optimal when the only alternative is assimilation and the cost to the assimilating group of losing their own culture and adopting the culture of the other group is high compared to the benefits of interacting with that other group. Segregation however is never optimal when non-assimilative integration is an alternative.

2 Related Literature

This paper examines why and when diverse communities are cohesive or divided. A related paper by Caselli and Coleman (2006) examines the relationship between ethnic conflict (over the resources available to a society) and group sizes, the degree of ethnic differences, income and resources. At a general level both papers examine why and when differences between groups become divisive, but they do so in two very different situations. Caselli and Coleman examine the situation where one group can expropriate resources from another group and when they will want to do so. While in this paper I examine a situation where groups with different cultures share the same community and can socially interact or segregate and when they will want to do so.

The work here is motivated by an empirical literature looking at the relationship between population heterogeneity and social outcomes such as segregation, trust, and participation in social activities. Work by Moody (2001), Echenique, Fryer and Kaufman (2006), and Echenique and Fryer (2007) examines the empirical relationship between racial heterogeneity and segregation in schools. Both find a non-linear (increasing) relationship between the proportion of a race group in a school and segregation. Echenique and Fryer (2007) also find some evidence of greater polarization of behavior in segregated schools. Alesina and La Ferrara (2000) analyze both theoretically and empirically the relationship between community heterogeneity (racial, ethnic, income) and participation in associational activities, such as religious groups, sports/hobby clubs, and educational groups. They find that for income, race and ethnicity, individuals living in more heterogeneous communities are less likely to participate in associational activities. In another paper, Alesina and La Ferrara (2002) find that racial heterogeneity in a community is associated with low levels of trust in that community.

The work here brings together two different literatures: a literature that examines friendship segregation and a literature that examines choice of identity or culture. In this paper, the choice of identity and choice of social interaction are entwined. Allowing individuals to make both choices brings a range of possibilities and results unexplored in the two literatures individually.

Literature that examines friendship segregation includes network literature on homophily (Currarini, Jackson, and Pin (2009), Bramoulle and Rodgers (2009), Currarini and Vega Redondo (2010)). These three papers model the formation of social ties using random search and assume a bias towards meeting same-type individuals or cost to searching for different-type friendships. They analyze the extent to which race/gender groups exhibit a tendency towards same race/gender friendships. The results on segregation presented below in section 4 match many of the theoretical and empirical findings on segregation in these three papers. Interestingly, all three papers share the same result with this paper that homophily is highest in middle-sized groups. Again examining friendship segregation but from a different perspective, a paper by de Marti and Zenou (2009) looks at network formation assuming two different groups. The model supposes lower costs of linking to one's own group than to the other group; however, the more an individual interacts with the other group, the lower the costs of interacting with the other group become. They find multiple equilibria where communities can be segregated or integrated as well as a range of intermediate cases where there are some but fewer links between groups and where some minority members interact mostly with the majority group instead. This paper contributes to the literature on social segregation in the following ways. First, we add a reason for potential bias towards same-type individuals:

we assume each individual has a culture and adopting a different culture has a cost. This structure allows us to look in greater detail than previous papers at what kind of community environments encourage social segregation versus integration. It also allows us to examine in detail the type of segregation or integration occurring. Indeed, we find that segregation and integration can take on different forms with some of the forms 'better' than others. The merits of the different forms of segregation and integration also depend on the community environment. Our framework allows us to undertake detailed welfare analysis of the different types of integration and segregation and in different settings.

The paper brings together this literature on friendship segregation with a literature on identity, pioneered by Akerlof and Kranton (2000). Akerlof and Kranton (2000) propose a framework with which to introduce identity into economic models. I incorporate an individual's identity into the model in a way consistent with the simplest version of the framework suggested by Akerlof and Kranton (2000). An individual has a given identity (or social category), and that identity is associated with particular behaviors deemed appropriate to that social category. The individual's utility depends on both her identity and how well she conforms to the behaviors associated with her given identity.[3]

In particular, within work on identity the work here is related to a literature that examines choice of culture or identity often by minority groups (Akerlof and Kranton (2000), Bisin and Verdier (2000),Bisin, Patacchini, Verdier and Zenou (2009*a*), (2009*b*), (2010),) . The closest of these papers is Bisin, Patacchini, Verdier and Zenou (2009*a*). They provide a dynamic framework to examine the choice by minority individuals from one generation to the next of promoting a mainstream identity versus minority identity. Individuals can also decided the intensity of the allegiance to their mainstream or minority identity. They examine what factors sustain minority culture in the long run and increase the intensity of allegiance to the minority culture. The papers listed above tend to assume a binary choice, a group either assimilates to the culture of the mainstream or retains their own culture. My paper looks at the choice of an individual along two dimensions, whether to adopt his own culture or that of the other group and whether to socially segregate from the other group or integrate with the rest of the community. This allows us to differentiate between cultural integration and social integration which allows for the result that communities composed of different groups have other possibilities beyond cultural assimilation or cultural segregation. The framework I present here also allows for the possibility that culture is endogenous and provides an analysis of the new cultures that can emerge in a diverse community and under what conditions they

[3]I say my incorporation of identity is consistent with the simplest version of their framework because I allow people to 'have identity-based payoffs derived from their own actions'; while their framework also allows people to 'have identity-based payoffs derived from others' actions' as well as allowing for the possibility of third parties generating changes in these payoffs and some people being able to choose their identity.

will emerge.

In this paper I examine choice of social interactions simultaneously with choice of actions. The methodology used in this paper thus builds on theory examining network formation and coordination games (Goyal and Vega-Redondo 2005, Hojman and Szeidl 2006, Jackson and Watts 2002 amongst others). It is also related to work on coordination on networks (Ellison 1993, Morris 2000 amongst others). In contrast to the existing methodology, I allow for both heterogeneity of the population and multiple games.

3 Model

3.1 Two Groups

I examine in detail the case of a community comprising two identity groups. I then extend it to allow for multiple identity groups in section 6. A community comprises n individuals, $N = \{1, 2, ..., n\}$. Individuals can have exogenous identity A or B, where there are $n_A \in \mathbb{N}$ individuals of identity A and $n_B \in \mathbb{N}$ individuals of identity B ($n_A, n_B \geq 2$, and $n_A + n_B = n$). Identity is common knowledge.[4]

Individuals choose who to form social ties with from within the population. If individual i forms a social tie with individual j, we denote this by $g_{ij} = 1$. If individual i does not form a social tie with j we denote this by $g_{ij} = 0$. Player i's choice of social ties can be represented by a vector of 0's and 1's: $g_i = (g_{i1}, g_{i2}, ..., g_{in})$, where $g_{ii} = 0$.

Individual i in the community chooses a set of actions, a_i. This involves choosing a culture x_i, and M other actions $y_{1i}, ..., y_{Mi}$ (where M is a positive integer or zero), such that $a_i = \{x_i, y_{1i}, ..., y_{Mi}\}$. The choice of culture is a choice between x^A which represents the culture associated with identity A, and x^B representing the culture associated with identity B, $x_i \in \{x^A, x^B\}$. Note that either type can choose either culture; however, an individual's utility may depend on her type and the choice of culture. This is explained in detail below. Similarly for each of the $m = 1, ..., M$ other actions, individual i has a choice between two possibilities, $y_{mi} \in \{y_m^1, y_m^2\}$. The utility from these choices are unrelated to identity, as is explained below. A player's strategy thus consists of choosing who to link with and what actions to play: $s_i = (g_i, a_i)$.

The total utility an individual receives depends on her type, her strategy s_i and the strategy of the other $n-1$ individuals in the community, s_{-i}. An individual i receives utility from each

[4] In reality people may be members of more than one of these small populations: an individual may function within a workplace population during the week and a social club at weekends. In our model we look at a single population; we therefore make the implicit assumption that either there are no externalities between communities or that externalities are captured in the exogenous parameters.

social tie, where the utility i receives from a social tie with j is denoted $u_i(a_i, a_j)$. The utility received from this social tie with individual j depends on i's identity (because within groups individuals are homogeneous, with a slight abuse of notation we let i represent both the individual and her type so that $i \in \{A, B\}$), individual i's set of actions a_i, and the actions of her social tie, a_j.[5]

Individual i receives a positive utility from coordinating on the same action as social tie j and a negative utility for mis-coordination. Specifically, when $x_i = x_j$ individual i receives a positive contribution of α_x towards her utility, and for each $y_{mi} = y_{mj}$, for $m = 1, ..., M$, individual i receives a further positive contribution of α_m. When $x_i \neq x_j$, individual i receives a negative contribution of $-\gamma_x$ and for each $y_{mi} \neq y_{mj}$, for $m = 1, ..., M$, individual i receives a negative contribution of $-\gamma_m$ towards her utility. This feature of the model captures the fundamental nature of coordination on shared activities and behavioral norms to the existence of a social tie.

The notion of identity enters the model in the following way. If individual i is identity A and i and j coordinate on the culture associated with identity B, x^B, then individual i pays a cost, c_{AB}. This cost is the cost of an individual of type A adopting the other group's culture, as discussed in the introduction. Symmetrically, if individual i is of identity B and i and j coordinate on the culture associate with identity A, x^A, then i pays the cost c_{BA}. This is the cost of an individual of identity B adopting the other group's culture. Note that while the payoffs from culture are associated with identity, the payoffs from the other M choices are unrelated to identity. I will therefore refer to these M choices as identity-independent actions.

The total utility individual i receives from a social tie with j is equal to the sum of the positive and negative contributions from whether i and j coordinate for x and each of the choices $y_1, y_2, ..., y_M$, plus a cost c_{AB} or c_{BA} if relevant. I assume that forming a social tie is costless. As an example suppose i forms a social tie with j. Suppose they both adopt culture x_A, and there are two identity independent actions ($M = 2$) where for action choice y_1 they both choose y_1^1 but for action choice y_2 individual i chooses y_2^1 and j chooses y_2^2. Suppose individual i has identity B. Thus the utility individual i receives from the social tie with j is $u_i(a_i, a_j) = \alpha_x - c_{BA} + \alpha_1 - \gamma_2$.

I do not allow an individual to differentiate their behavior according to each social tie. An individual must play the same set of actions, a_i, against all social ties. This requires an individual to be 'consistent' in their behavior within a given community. This requirement could also be the result of time constraints. In conclusion, the total utility player i receives is

[5]Note, this utility does not depend on the identity of i's social tie.

equal to the sum of the utilities she receives from each of her social ties, $u_i(a_i, a_j)$:

$$U_i(s_i, s_{-i}) = \sum_{j \in N} u_i(a_i, a_j) g_{ij}.$$

We see that when i has a link with j, $g_{ij} = 1$ and so the utility from a social tie with j, $u_i(a_i, a_j)$, contributes to i's total utility. Note that ties are uni-directional, so for i to receive utility from the social tie with j all we require is that i forms a social tie with j. We can view this as the direction of influence. If i forms a tie with j then i is influenced by j's behavior. In equilibrium however, if i forms a social tie with j then j will always form a social tie with i.

A Nash equilibrium exists when no player wants to change her action, social ties, or any combination of the two given the actions and social ties of the other players in the population.

Definition 1 *The strategy profile* $(s_1^*, s_2^*, ..., s_n^*)$ *is a Nash equilibrium if* $U_i(s_i^*, s_{-i}^*) \geq U_i(s_i, s_{-i}^*)$ $\forall i \in N$ *and* $\forall s_i \in S_i$.

4 Results

The following two propositions characterize the equilibria and provide the conditions under which each equilibrium is sustainable. First, we must distinguish social integration and segregation from cultural integration and segregation. Social integration occurs when a community shows no preference for forming social ties with individuals of the same identity over individuals of a different identity. Social segregation occurs in a community in which individuals show a preference for forming social ties with others of the same identity. Cultural integration occurs when individuals in a community of different identities adopt the same culture and cultural segregation occurs when individuals of different identities retain different cultures. I define three possible community outcomes relating to community cohesion and culture.

Definition 2 *Assimilation, non-assimilative integration, and segregation are defined as follows:*

(i). ***Assimilation****: The whole population integrates. All individuals coordinate on the same culture* x *and same actions* $(y_1, y_2, ..., y_M)$.

(ii). ***Non-assimilative integration****: The whole population integrates. Individuals retain their own culture but everyone coordinates on the same actions for* $(y_1, y_2, ..., y_M)$.

(iii). ***Segregation****: The two groups do not interact. Individuals retain their own culture.*

Assimilation involves one group adopting the culture of the other group in order to interact with the other group in the community. Segregation occurs when the two groups restrict social interaction to their own group in order to retain their own culture. Non-assimilative integration describes an integrated community where individuals retain their distinct culture.

For proposition 1 assume the following inequality holds

$$\gamma_x > \sum_{m=1}^{M} \alpha_m. \tag{1}$$

Proposition 1 *There are two distinct forms of Nash equilibria:*

(i). ***Assimilation.*** *Assimilation is an equilibrium under all parameter ranges.*

(ii). ***Segregation****. Segregation is an equilibrium when:*

$$\frac{n_A - 1}{n_B} \geq \frac{\alpha_x - c_{AB} + \sum_{m=1}^{M} \alpha_m}{\alpha + \sum_{m=1}^{M} \alpha_m}$$

$$\frac{n_B - 1}{n_A} \geq \frac{\alpha_x - c_{BA} + \sum_{m=1}^{M} \alpha_m}{\alpha + \sum_{m=1}^{M} \alpha_m}$$

There are two forms of equilibria possible when inequality (1) holds: assimilation and segregation. The possible equilibria are either an integrated community formed at the cost of one group adopting the culture of the other group, and a segregated community where both groups retain their own culture. Note that when the two groups segregate there are no restrictions on the other M actions that are chosen by either group, any combination is possible in equilibrium.

Assimilation is an equilibrium under all parameters. In other words, assimilation can be sustained in any community environment. However, when the minority group is small relative to the majority group, assimilation is the unique equilibrium. The minority group does not want to segregate because they would then be restricted to a small group of individuals with whom to interact relative to the much larger majority group.

Segregation can be sustained when a minority group individual is better off interacting with the minority group and retaining her own culture than interacting with a larger number of individuals in the majority group but conforming to the majority group's culture. Let B be the minority group. A type B individual is better off interacting with the minority group when:

$$\frac{n_B - 1}{n_A} \geq \frac{\alpha_x - c_{BA} + \sum_{m=1}^{M} \alpha_m}{\alpha_x + \sum_{m=1}^{M} \alpha_m}. \tag{2}$$

Figure 1 illustrates this inequality graphically (and the inequality for group A in the minority). The vertical axis denotes the number of type A individuals and the horizontal axis

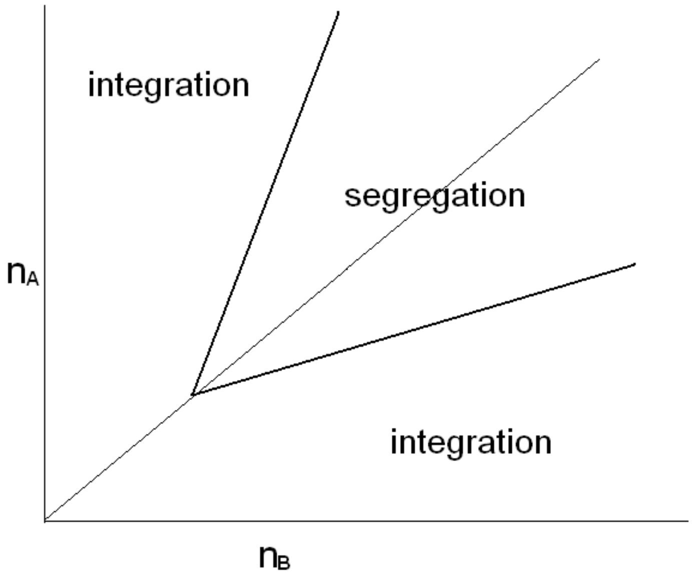

Figure 1: Segregation is an equilibrium in the triangular area in the top right of the graph encompassing the upper part of the 45° line.

denotes the number of type B individuals, so that any point in the area of the graph gives the number of type A individuals in the population, the number of type B individuals in the population, and thus the population size. The 45° line denotes when groups are of equal size. To the left of the 45° line type A's are in the majority and to the right of the 45° line type B's are in the majority. Moving up and right on the graph denotes increasing population size. Assimilation is an equilibrium on the whole graph. Segregation is an equilibrium only in the top-right triangular region denoted. I discuss what kind of community environment this triangular area represents and summarize in corollary 1.

Inequality (2) implies that a minority group individual will want to remain segregated from the majority group only when the minority group is not 'too small' relative to the majority group. Intuitively, the smaller the difference in size between the minority and majority group, the smaller the gain to a minority group member from interacting with the larger group and the less likely this gain is to outweigh the cost of conforming to majority culture. To put inequality (2) in other words, segregation can be sustained only when the population is sufficiently heterogeneous holding all other parameters fixed. Heterogeneity is defined by the following Herfindahl-based index, H, which denotes the probability that two randomly drawn individuals come from different groups. It is commonly used as a measure of heterogeneity in a population. In the case of two groups, heterogeneity is increasing and reaches a maximum as the minority group approaches 50% of the population. Conversely, as the minority group becomes a smaller and smaller fraction of the population, the population is becoming more homogeneous.

Definition 3 *Heterogeneity in a population is given by the following Herfindahl-based index*

$$H = 1 - \sum_k s_k^2$$

where s_k is the proportion of group k in the population.

Inequality (2) also implies that segregation can be sustained in more homogeneous populations when the community is larger. Indeed figure 1 shows that segregation cannot be sustained at all in very small communities. As the size of the community increases, the area in which segregation can be sustained is increasing. The intuition for this result comes from realizing that the relative cost to the minority group of not interacting with the majority group is higher the smaller the population. This is because relative size differences matter more when groups are small than large. For example, having 2 people to interact with versus 5 people is much more salient than 20 people versus 50.

Note two final points. When the costs to the minority group of adopting majority group culture (c_{BA}) are higher, segregation can be sustained in more homogeneous populations. Intuitively, the more costly it is for a minority group member to adopt the culture of the majority group, the larger the required benefits from interaction with the majority group. When the importance of culture is diminished relative to identity-independent actions (higher $\sum_{m=1}^{M} \alpha_m$), the level of heterogeneity required for segregation to be sustainable is lower. I summarize the above discussion in the following corollary. Note this discussion assumes keeping the parameter α fixed.

Corollary 1 *Segregation occurs only once the community is sufficiently heterogeneous, given by inequality (2). The level of heterogeneity at which segregation can be sustained is*

(i). Decreasing in community size, n;

(ii). Decreasing in cultural cost, c_{BA};

(iii). Increasing in $\sum_{m=1}^{M} \alpha_m$, the importance of identity-independent actions relative to culture.

For proposition 2 we reverse inequality (1) and suppose

$$\sum_{m=1}^{M} \alpha_m \geq \gamma_x. \tag{3}$$

For notational purposes denote by S the subset of $\{y_1, .., y_M\}$ where types A and B coordinate on different actions, $S = \{m : y_{mA} \neq y_{mB}, \text{for} \quad m = 1, ..., M\}$.

Proposition 2 *There are three distinct forms of Nash equilibria:*

(i). ***Assimilation****. Assimilation is an equilibrium under all parameter ranges.*

(ii). ***Non-assimilative integration****. Non-assimilative integration is an equilibrium when:*

$$\frac{n_A - 1}{n_B} \geq \frac{\alpha_x - c_{AB} + \gamma_x}{\alpha_x + \gamma_x}$$
$$\frac{n_B - 1}{n_A} \geq \frac{\alpha_x - c_{BA} + \gamma_x}{\alpha_x + \gamma_x}$$

(iii). ***Segregation****. Segregation is an equilibrium when:*

$$\frac{n_A - 1}{n_B} \geq \frac{\alpha_x - c_{AB} + \sum_{m=1}^{M} \alpha_m}{\alpha_x + \sum_{m=1}^{M} \alpha_m}$$
$$\frac{n_B - 1}{n_A} \geq \frac{\alpha_x - c_{BA} + \sum_{m=1}^{M} \alpha_m}{\alpha_x + \sum_{m=1}^{M} \alpha_m}$$

and the two types differ in their choice of other actions, $(y_1, y_2, ..., y_M)$*, such that*

$$\sum_{s' \notin S} \alpha_{s'} < \gamma_x + \sum_{s \in S} \gamma_s.$$

It seems natural that a social tie might exist where two individuals have some activities in common but differ in other aspects. When inequality (3) holds we find an equilibrium with interaction across identity groups where interaction across groups is based on shared activities and behavioral norms that are independent of identity, while at the same time individuals retain the diverse activities and behavioral norms associated with identity. We label this equilibrium, non-assimilative integration. We can view this as a situation where a community adopts a shared 'community culture' by coordinating on the M identity-independent actions and this allows the community to integrate while at the same time the different identity groups retain their own culture. Note that without some form of shared culture, integration cannot be sustained.

In the previous proposition inequality (3) was reversed. The reversed inequality implies that if two individuals adopt different cultures, the cost of not conforming to the same culture is so high that even coordinating on all of the M other activities together is unsustainable. This can be interpreted in two ways. First, cultural activities are so dominant relative to other activities that not adopting the same culture leaves little room for a friendship to form. Second, the two cultures are so polarized that they cannot exist side by side, if an individual chooses not to assimilate and to retain her own culture then this culture is so at odds with the other culture that she cannot form a social tie with someone of the other culture.

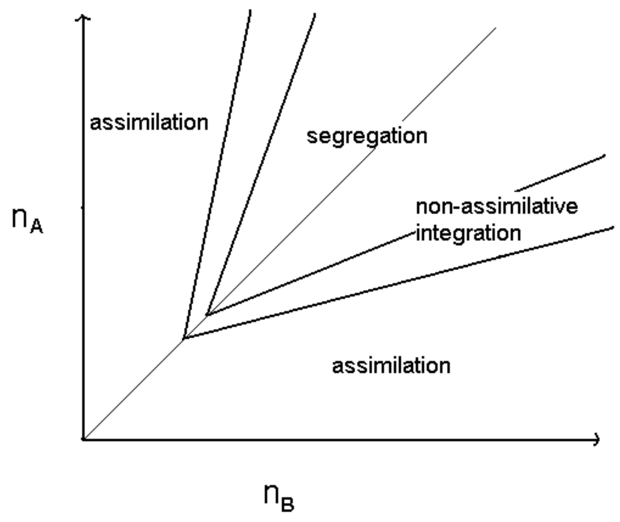

Figure 2: Segregation is an equilibrium in the triangular area in the top right of the graph encompassing the upper part of the 45° line. Non-assimilative integration is an equilibrium in the larger triangular area surrounding it.

This highlights the important difference between c_{AB}, c_{BA} and γ_x. Inequality (3) implies that undertaking the M activities together is enough to sustain a social tie when the two individuals have adopted different cultures.[6] This is reminiscent of the saying 'bad times bring people together', expressing the idea that when other things are important enough differences can be ignored.

The community environments that sustain assimilation and segregation are the same as proposition 1. Interestingly, non-assimilative integration is sustained as an equilibrium under roughly the same community environments as segregation. This is seen in figure 2.

Segregation in this framework can in fact also be divided into two forms. The first occurs when non-assimilative integration is not possible. This type of segregation simply places the requirements on the community environment discussed above. The second type of segregation occurs when non-assimilative integration is possible. When inequality (3) holds this allows for non-assimilative integration; however, it also allows for the possibility of segregation where the community endogenously becomes more 'culturally polarized'. When segregation occurs under inequality (3), A and B types must differentiate 'sufficiently' their choices of the other M actions such that

$$\sum_{s' \notin S} \alpha_{s'} < \gamma_x + \sum_{s \in S} \gamma_s$$

[6]Note that the equilibrium specifies that in non-assimilative integration individuals from both groups will adopt the same actions for each of the M choices. Thus inequality (3) suffices to sustain non-assimilative integration as an equilibrium.

where S is the subset of $\{y_1, .., y_M\}$ where types A and B coordinate on different actions.

Ex-ante we see that identity is related to culture but choices of actions $\{y_1, .., y_M\}$ are unrelated to identity. In equilibrium however, the individuals choice for y_m becomes associated with identity. Suppose in a community the culture of identity A involves doing a particular activity on a Monday night (x^A) and the culture of identity B involves doing a different activity on a Monday night (x^B). On Tuesday night, however, individuals make a choice (y_m) between joining the local football team (y_m^1) or joining the local dance class (y_m^2). Payoffs from joining the local football team or dance class are unrelated to identity; however, in equilibrium, individuals of identity A choose to play football and individuals of identity B choose to go to the dance class. Thus in equilibrium, football is associated with identity A and dance class with identity B. The 'cultural polarization' defined by the differences in actions/behaviours between the two groups increases in equilibrium beyond simply the associated cultures x_A and x_B.

Intuitively, if groups did not differentiate their activities beyond culture the two groups would have so many activities and behavioral norms in common it would be beneficial for individuals to form social ties across groups. The two groups endogenously take on different actions which raises the cost to an individual of interacting with the other group, so much so that segregation can be sustained. The actions $(y_1, y_2, ..., y_M)$ which are ex-ante identity-independent become relevant to identity in equilibrium. This is one explanation for why some segregated groups seem to have many differences in activities and behavioral norms, and why these differences can sometimes seem arbitrary.

4.1 Equilibrium Refinement

When assimilation is an equilibrium, one community does the assimilating (adopts the culture of the other group). Individuals in the assimilating group face lower utility as they must pay the cost of adopting the other group's culture and face the loss of their own culture. For assimilation to be a Nash equilibrium it requires only that no individual wants to deviate given everyone in the community is playing the same strategy. This means that it is a Nash equilibrium both for the minority group to assimilate and for the majority group to assimilate. I would like to determine which of these equilibria are more likely to arise.

For this I use stochastic stability as an equilibrium refinement (for details see seminal papers by Kandori et al. 1993; Young 1993). Stochastic stability has been adopted in related models as a means of equilibrium refinement (Goyal and Vega-Rendondo, 2005). To use stochastic stability, a form of dynamic adjustment within the population is assumed. I suppose the community is at some state in period t where a state is given by a strategy for each individual, $(s_1^t, s_2^t, ..., s_n^t)$. This state need not be an equilibrium and utilities for each individual in each state, $(U_1^t, U_2^t, ..., U_n^t)$, are calculated as described in section 3.1. At period $t+1$ each individual is picked with independent probability $p \in (0,1)$ to change her strategy. If an individual is given the opportunity to change her strategy she maximizes her payoff given the strategies of the others at period t,

$$s_i^{t+1} \in \arg\max_{S_i} U_i(s_i^{t+1}, s_{-i}^t).$$

When an individual has the opportunity to change her strategy, with probability ϵ she makes a mistake and chooses some other strategy randomly. This process defines a Markov chain with a unique invariant probability distribution denoted μ_ϵ. As $\epsilon \to 0$ we define $lim_{\epsilon \to 0} \mu_\epsilon = \hat{\mu}$, where any state s such that $\hat{\mu}(s) > 0$ is stochastically stable. Roughly speaking, a stochastically stable state is a strict Nash equilibrium state which, supposing the above dynamics, is the easiest to end up at and hardest to move away from. See Kandori et al. (1993) and Young (1993) for details. To simplify I assume inequality (1) holds, that $c_{AB} = c_{BA} = c$, and that the number of identity independent actions, M, is zero. Because we assume inequality (1) holds we restrict ourselves to two equilibria: assimilation and segregation.

Proposition 3 *The unique stochastically stable state depends on the community parameters as follows:*

(i). Segregation is stochastically stable when

$$\frac{n_B - 1}{n_A} \geq \frac{2\alpha_x - 3c}{2c}$$

and

$$\frac{n_A - 1}{n_B} \geq \frac{2\alpha_x - 3c}{2c}$$

(ii). Otherwise, the stochastically stable equilibrium is assimilation where everyone adopts the culture of the majority group. [7]

When assimilation occurs, the minority group adopts the behavior of the majority group. This is the unique stochastically stable state in more homogeneous populations. I find a switch in the unique stochastically stable equilibrium once the population is sufficiently heterogeneous such that segregation is the unique stochastically stable state. Using stochastic stability as a refinement process suggests that when assimilation occurs it is the minority group that assimilates.

[7] When both groups are equal size, the complete social network where all play x_A or all play x_B are both stochastically stable equilibria outside the area where segregation is stochastically stable.

5 Welfare

In this section when analyzing the assimilation equilibrium I assume that the minority group (taken to be the identity B group) assimilates to the majority group culture. I assess social welfare by summing individual utilities. In order to understand welfare differences between minority and majority groups I also analyze the Pareto optimal equilibria.

Suppose inequality (1) holds.

Proposition 4 *Segregation maximizes social welfare when*

$$\frac{n_B - 1}{n_A} \geq \frac{2(\alpha_x + \sum_{m=1}^{M} \alpha_m) - c_{BA}}{c_{BA}}.$$

Otherwise assimilation maximizes social welfare.

The first point to make here is that segregation can be welfare maximizing. This occurs because segregation allows the minority group to avoid the utility loss from adopting majority culture. This loss can be so large that segregation is social-welfare maximizing. Segregation maximizes social welfare in more heterogeneous populations. The level of heterogeneity required for segregation to be social welfare maximizing is decreasing in the size of the population, decreasing in cultural costs and decreasing as culture becomes more important.[8]

Suppose inequality (1) continues to hold.

Proposition 5 *Segregation is Pareto optimal when*

$$\frac{n_B - 1}{n_A} \geq \frac{\alpha_x + \sum_{m=1}^{M} \alpha_m - c_{BA}}{c_{BA}}.$$

Assimilation by the minority is Pareto optimal under all parameters.

In proposition 5 we see that segregation can also be Pareto optimal. Again this is a result of the cost to the minority group of assimilating. It can be more costly for the minority group to assimilate and lose their culture than to segregate and retain their culture. It is also important to note that assimilation is always Pareto optimal. This is because assimilation maximizes utility for the majority group. This suggests caution over promoting assimilation and recognition that in the context of this model assimilation may neither be in the best interests of the minority group nor maximize social welfare.

Propositions 4 and 5 uncover a tension between segregation maximizing social welfare, segregation being Pareto optimal and segregation being a Nash equilibrium. Segregation becomes Pareto optimal at a higher level of heterogeneity than that at which segregation

[8]The same qualitative conditions for segregation to be a Nash equilibrium.

becomes an equilibrium. The minority group segregates in equilibrium before it is Pareto optimal to do so. This is because an individual from the minority group compares whether they are better off interacting with the less numerous minority group but keeping their own culture or joining the majority group and adopting its culture. They do not compare whether the minority group as a whole might be better off assimilating to the majority group. Segregation maximizes social welfare at a higher level of heterogeneity than that at which segregation is Pareto optimal. This is because social welfare maximization also takes into account how segregation reduces the utility of the majority group relative to to the assimilation whereas Pareto optimality only takes into account the minority group.

Assume inequality (3) holds.

Proposition 6 *Non-assimilative integration maximizes social welfare when*

$$\frac{n_B - 1}{n_A} \geq \frac{2(\alpha_x + \gamma_x) - c_{BA}}{c_{BA}}.$$

Otherwise assimilation maximizes social welfare. Segregation never maximizes social welfare.

Proposition 7 *Non-assimilative integration is Pareto optimal when*

$$\frac{n_B - 1}{n_A} \geq \frac{\alpha_x + \gamma_x - c_{BA}}{c_{BA}}.$$

Assimilation is always Pareto optimal. Segregation is never Pareto optimal.

Assimilation is always Pareto optimal because the majority can do no better than have the minority assimilate. The majority does better if the minority assimilates rather than integrates non-assimilatively. Non-assimilative integration does maximize social welfare and is Pareto optimal when the minority group is large enough. When the minority group is sufficiently small, they would do better by assimilating.

6 Multiple Groups

Suppose we have a population of n individuals, where each individual has one of T identities labeled $t = 1, ..., T$. The corresponding group sizes are denoted $n_1, n_2, ..., n_T$, where $n_t \geq 2, \forall t \in 1, .., T$ and $n_1 + n_2 + ... + n_T = n$. The corresponding culture for each group is respectively $x^1, x^2, ..., x^T$. Cultural distance between identity groups i and j (again abusing notation slightly) is denoted c_{ij}, where this is the cost of an identity i individual conforming to culture, x^j, with any given social tie. Each individual now chooses his social ties and an action from the larger set of possible cultures $a_i \in \{x^1, x^2, ..., x^T\}$. As before, an individual of identity i receives α_x from a social tie where he coordinates on culture and $-\gamma_x$ when he does not coordinate with that social tie. If he coordinates with the social tie on a culture, j, other than his own he pays c_{ij}. Apart from the increased choice of actions and and accompanying costs, the model follows as in the two group case above. For simplicity however, we assume $M = 0$ (so there are no identity-independent actions) and inequality (1) holds.

Define a coalition as a set of identity groups who all form social ties with all others in the set (integrate). With multiple groups, it is possible that some identity groups form coalitions while segregating from other coalitions. Using a community composed of three identity groups as an example, there is the possibility of a partition such that two groups, t_1 and t_2, integrate to form a coalition while segregating from the third group, t_3. Thus the three identity groups are partitioned into two elements (here denoted coalitions): $\{\{t_1, t_2\}, \{t_3\}\}$. There are five possible partitions of the three identity groups, where an element of a partition is a coalition: $P_1 = \{\{t_1\}, \{t_2\}, \{t_3\}\}$; $P_2 = \{\{t_1, t_2\}, \{t_3\}\}$; $P_3 = \{\{t_1\}, \{t_2, t_3\}\}$; $P_4 = \{\{t_1, t_3\}, \{t_2\}\}$; $P_5 = \{\{t_1, t_2, t_3\}\}$.

For T identity groups let $T' \in \mathbb{N}$ be the number of possible partitions of the set of T identity groups where each partition is denoted $P_1, P_2, ..., P_{T'}$.[9] Let P_r, $r \in 1, ..., T'$, denote a particular partition. Recall that the elements of a partition are coalitions. Let m_r be the number of coalitions in the particular partition P_r where each coalition is labeled $p_{1r}, p_{2r}, ..., p_{m_r r}$. Let $n_{1r}, n_{2r}, ..., n_{m_r r}$ be the number of individuals in each coalition $p_{1r}, p_{2r}, ..., p_{m_r r}$ respectively. Let $x_{1r}, x_{2r}, ..., x_{m_r r} \in x_1, x_2, ..., x_T$, be the culture adopted respectively by each coalition $p_{1r}, p_{2r}, ..., p_{m_r r}$.

Proposition 8 *With T groups there exists a Nash equilibrium where*

(i). The whole population integrates and all coordinate on the same culture, x_t*;*

(ii). The T groups form a proper partition, P_r.[10] *Every individual in a given coalition adopts the same culture, while different coalitions adopt distinct cultures.*

This is an equilibrium when

$$\frac{n_{kr} - 1}{n_{lr}} \geq \frac{\alpha_x - c_{tx_{lr}}}{\alpha_x - c_{tx_{kr}}} \tag{4}$$

$\forall\ k, l \in 1, ..., m_r$ *and* $\forall t \in T$ *where* t *is an identity group in the coalition* p_{kr} *and where* $c_{tx_{kr}}$ *represents the cost to an individual of identity* t *of adopting the culture* x_{kr} *adopted by coalition* p_{kr}*, and* $c_{tx_{lr}}$ *represents the cost to an individual of identity* t *of adopting the culture* x_{lr} *adopted by coalition* p_{lr}*.*

[9] The number of partitions of the set of identity groups $T = 1, ..., T$, is given by the Bell number B_n which satisfies the recursion $B_n = \sum_{k=0}^{n} \binom{n}{k} B_k$, where $B_0 = 1, B_1 = 1$.

[10] I define a proper partition where $m_r > 1$.

Expression (4) says that for each member of each coalition, they must be at least as well off adopting the culture of their own coalition and interacting within that coalition as they would be if they adopted the culture of another coalition and interacted with that coalition. For this to hold it must be true that the relative number of people they interact with in their coalition compared to the other is sufficiently large to outweigh the costs of conforming to the culture of their coalition rather than the other.

When there are more than two groups, the features of equilibrium in the two group case continue to hold but in a slightly more complex fashion. First note that it remains an equilibrium for all T groups to interact and conform to one culture. This is an equilibrium under all community parameters as before.

It is also an equilibrium for all T groups to segregate and adopt their respective cultures. As in the two-group case, this is an equilibrium when the T groups are of a sufficiently similar size (given the population size and exogenous cultural distances between groups, c_{kl} where $k, l \in 1, ..., T$). If we do not assume symmetric costs, then the similarity in size between the different groups that is required for segregation will be different between groups depending on the costs. The more costly it is for an identity i individual to adopt the culture of identity j then segregation can be sustained as an equilibrium when group i is smaller relative to group j. As before, all else equal, the similarity of size between each pair of groups required for segregation to be an equilibrium is lower the larger the total size of the community.

In the multiple-group case there is another type of equilibrium where groups are partitioned such that a subset of the groups integrate (a coalition) while segregating from other integrated subsets (coalitions). Each coalition conforms to a single culture and different coalitions adopt different cultures. The reason coalitions arise is because costs vary between different cultures, for example it may be much more costly for group i to adopt group j's culture than to adopt group h's culture. Two coalitions k and j can be sustained when those identity groups in coalition k find it less costly to conform to the culture of coalition k relative to coalition l given that k is sufficiently large relative to l and similarly for those identity groups in coalition l. Thus we see that coalitions can be sustained in equilibrium when the coalitions are sufficiently similar in size taking into account the cultural costs of the coalition members.

A new feature of equilibrium arises when there is segregation between coalitions of groups. In this case, when an identity group joins a coalition that identity group may have to pay the cost of conforming to a different culture (that adopted by the coalition). Because some groups within a coalition will have to pay the cost of conforming to the culture of that coalition, this makes the segregation equilibrium between coalitions weaker in a sense than segregation between identity groups. Comparing two communities of equal size where one community is composed of two segregated groups and the other composed of two segregated coalitions, the restrictions on group/coalition sizes (all else equal) will be stricter for the community with two coalitions. This is because of the extra constraint involved in a coalition, whereby at least one group must also pay a cost of adopting the culture of the coalition they belong to.

7 Conclusion

The paper proposes a model of community in which identities segregate in order to retain their own culture and avoid paying the costs of conforming to a different culture. I find three possible outcomes for a diverse community: assimilation, non-assimilative integration, and segregation. Assimilation involves complete conformity usually by the minority group. Non-assimilative integration allows groups to retain their diverse cultures while maintaining a cohesive community. Segregation involves social and cultural separation. Indeed segregation can be accompanied by an increased cultural polarization between groups as a reaction to sharing a community environment.

There are three effects that determine whether a community is segregated: community size, group size, and cultural distance between groups. There is a threshold property such that segregation only occurs at a high enough level of population heterogeneity. Increased population size or increased cultural distance between groups decreases the threshold level of population heterogeneity required for segregation.

Depending on the community parameters, all three outcomes can be both welfare maximizing and Pareto-optimal. However, we should take note that the majority group always does better under assimilation even when it is not the welfare-maximizing outcome, and so we should be wary of calls for assimilation by dominant groups. Nevertheless, when the minority group is very small, assimilation can be the utility-maximizing thing to do for members of the minority group. Finally, when non-assimilative integration is feasible, segregation is never welfare-maximizing nor Pareto-optimal.

Note that we assume only the possibility of diversity along one dimension. An important extension would allow individuals to differ along more than one dimension. For example, by race and gender, or gender and religion. This could give some insight into the conditions under which one identity becomes relevant, say gender, as opposed to a different identity, say religion.

8 Appendix

Proof of Proposition 1 I first show that the equilibria (i) and (ii) are equilibria. Then prove that no other equilibria exist.

The complete network where all players coordinate on one of the two actions is an equilibrium. Suppose types A and B form a complete network and WLOG play action x^A and the same action y_m for all $m = 1, ..., M$. Then the payoff to a type A is $(\alpha_x + \sum_{m=1}^{M} \alpha_m)(n_A + n_B - 1)$. The payoff to a type B is $(\alpha_x - c_{BA} + \sum_{m=1}^{M} \alpha_m)(n_A + n_B - 1)$. A player can deviate by reducing the number of ties he has and/or changing action. Since we assumed $\alpha_x, \alpha_x - c_{AB}, \alpha_x - c_{BA}\alpha_m > 0$ he does strictly worse by deviating. Therefore the complete network where all players coordinate on one of the two actions is an equilibrium.

I next show that there exists an equilibrium consisting of two complete components (a complete component is a fully linked group with no links to anyone outside the component) playing different cultures where each identity is a member of the component playing their associated culture. For the other M actions each component plays the same action for all choices y_m, $m = 1, ..., M$. There are no other restrictions on the M identity-independent actions. Suppose type A's play x^A and form a complete component and type B's play x^B and form a complete component. No player will want to change action within his current component or reduce his ties since he gets a strictly worse payoff. No player will want to continue with the same action and form ties with the other component since his payoff from a tie with a member of the other component is $-\gamma_x + \chi$ where $\chi \leq \sum_{m=1}^{M} \alpha_m$ but then $-\gamma_x + \chi < 0$ since from inequality (1) we assume $-\gamma + \sum_{m=1}^{M} \alpha_m < 0$. Similar arguments show no player will want to do any combination of these things. Finally, for this to be an equilibrium, no player must want to leave his component and join the other component and play their action. This is true when the following inequalities hold:

$$(\alpha_x + \sum_{m=1}^{M} \alpha_m)(n_A - 1) \geq (\alpha_x - c_{AB} + \sum_{m=1}^{M} \alpha_m)n_B$$

$$(\alpha_x + \sum_{m=1}^{M} \alpha_m)(n_B - 1) \geq (\alpha_x - c_{BA} + \sum_{m=1}^{M} \alpha_m)n_A.$$

These can be rewritten as:

$$\frac{n_A - 1}{n_B} \geq \frac{\alpha_x - c_{AB} + \sum_{m=1}^{M} \alpha_m}{\alpha_x + \sum_{m=1}^{M} \alpha_m}$$

$$\frac{n_B - 1}{n_A} \geq \frac{\alpha_x - c_{BA} + \sum_{m=1}^{M} \alpha_m}{\alpha_x + \sum_{m=1}^{M} \alpha_m}$$

I now show that no other equilibria exist. First, no two individuals of the same type can be playing different strategies since if they are undertaking different strategies, one individual must be doing weakly better than the other and the player who is doing weakly worse can mimic the player who is doing weakly better (plus link with him) and do strictly better.

Second, in equilibrium we cannot have two individuals in the same component coordinating on different actions. If two individuals in a single component are playing different actions then there must exist a link between two individuals who are playing different actions. If those two players are coordinating on different cultures then from inequality (1) we see that any individual sponsoring that link must be getting a negative payoff from the link. Suppose at least two players in the same component are playing different actions for at least one of the identity-independent actions. Then one player must be doing weakly better than the other in his payoff from the identity-independent actions. The player doing weakly worse can change actions to mimic the player doing weakly better and will strictly improve his payoff. Third, in equilibrium there cannot be more than one component playing a given set of actions. If there are two components playing the same set of actions, a member of one component can do strictly better by linking with a member of the other component. Fourth, in equilibrium each component is completely connected. This occurs because the whole component is playing the same action, if any two individuals in a component are not connected, they can do strictly better by both linking to each other.

Taking these four conditions, the only equilibria possible are the complete network where a single set of actions are played by everyone, and the network where two complete components play a different set of actions in each component. It suffices to show that the only equilibrium with two components has all the type A's in one component playing x^A and all the type B's in the other component playing x^B. The only other possible equilibrium is that the A's form a component and play x_B and the B's play x_A; but if a B weakly prefers to be in the component playing x_A then an A must strictly prefer to be in this component. Thus the only equilibrium left is equilibrium (*ii*). □

Proof of Corollary 1 Suppose B is the minority group, then heterogeneity is increasing as $n_B/n_A \to 1$.

(i). Segregation occurs only once the community is sufficiently heterogeneous, all else fixed. Segregation occurs when:

$$\frac{n_B - 1}{n_A} \geq \frac{\alpha_x - c_{BA} + \sum_{m=1}^{M} \alpha_m}{\alpha_x + \sum_{m=1}^{M} \alpha_m}.$$

Rewrite $\frac{n_B-1}{n_A} = \frac{n_B-1}{n-n_B}$. Let $\bar{n_B}$ be such that $\frac{\bar{n_B}-1}{n-\bar{n_B}} = \frac{\alpha - c_{BA} + \sum_{m=1}^{M} \alpha_m}{\alpha + \sum_{m=1}^{M} \alpha_m}$. Then segregation occurs only once $n_B \geq \bar{n_B}$. Fix n then $n_B/n_A = n_B/(n - n_B)$. Thus n_B/n_A is increasing in n_B. Thus segregation occurs only once the population is sufficiently heterogeneous.

(ii). The larger the community size, the lower the level of heterogeneity required for segregation. Fix n then we saw that segregation occurs when $n_B \geq \bar{n_B}$. But if we increase n to

βn where $\beta > 1$ then $\frac{\bar{n_B}-1}{n-\bar{n_B}} = \frac{\beta\bar{n_B}-1\beta}{\beta n-\beta\bar{n_B}} \leq \frac{\beta\bar{n_B}-1}{\beta n-\beta\bar{n_B}}$ which implies if we increase population size to βn the number of B types required for the equality to continue to hold is less than $\beta\bar{n_B}$ which implies a lower level of heterogeneity.

(iii). The larger the cultural cost, the lower the level of heterogeneity required for segregation. The right hand side of $\frac{\bar{n_B}-1}{n-\bar{n_B}} = \frac{\alpha_x - c_{BA} + \sum_{m=1}^{M}\alpha_m}{\alpha_x + \sum_{m=1}^{M}\alpha_m}$ is decreasing in c_{AB} thus $\bar{n_B}$ must decrease as well for the equality to hold.

□

Proof of Proposition 2 Note that all individuals of the same identity must play the same strategy by a previous argument. Further, if an individual has a link to a player playing a particular action then he has a link to everyone else playing that action. If he retains a link to a player in equilibrium that link must give positive utility, therefore he does strictly better by forming a link to everyone else who is following the same strategy.

There are three possible types of equilibria.

(i). All individuals play the same actions in all games and everyone forms a social tie with everyone else. This follows by the same argument as for proposition 1. This is the only equilibrium possible where all individuals adopt the same culture. Suppose not. Then either all individuals are integrated but the two groups play different actions in some of the other M games or the two groups are segregated and play different actions in the M games. An individual in one group must be doing weakly better in the M identity-independent games than an individual in the other group. Since the payoff from a single link is the same whether one adopts the actions of one's own group or the other group, an individual does strictly better by mimicking the player who is doing weakly better and also forming a link to him.

(ii). The only alternative equilibria are thus when type A individuals play x^A and type B individuals play x^B. Suppose some individuals play action x_A and some play action x_B. Then all the type A's must be playing action x_A and all the type B's playing action x_B. Suppose not, then a B must be playing x_A , but then if a B does better by playing x_A rather than x_B an A must do better by playing x_A as well. There are then two possibilities, either all individuals form social ties with all others or the two types segregate.

Any individuals who form a social tie must coordinate on the same actions in all of the M generic coordination games. Since all individuals of the same type must play the same strategy suppose a tie exists between two individuals of different types A and B. Suppose an A and B have at least one game in the generic coordination game on which they do not coordinate. Since a social tie exists between A and B we know the sum of the utility from this social tie must be positive. Since we assume symmetry of payoffs with the generic coordination games, switching actions in the M generic coordination games to match the actions of the different type individual means that the utility from a tie with a same type individual falls, but remains positive (because it was positive when coordinating on that subset of the M games with the different type) while the utility from a different type social tie increases. Finally note that for the M generic coordination games one of A or B must be earning a weakly higher payoff, then then the individual who is earning a weakly lower payoff can do strictly better by switching and mimicking the actions of the individual earning a weakly higher payoff without losing any social ties. Thus any individuals with a social tie must be playing the same actions in all the generic coordination games.

(a) Thus the only equilibrium where A and B play their own culture and integrate involves them coordinating on the same actions for each of the M generic coordination games. This is an equilibrium when

$$(\alpha_x+\sum_{m=1}^{M}\alpha_m)(n_A-1)+(\sum_{m=1}^{M}\alpha_m-\gamma_x)n_B \geq (-\gamma_x+\sum_{m=1}^{M}\alpha_m)(n_A-1)+(\sum_{m=1}^{M}\alpha_m+\alpha_x-c_{AB})n_B$$

$$(\alpha_x+\sum_{m=1}^{M}\alpha_m)(n_B-1)+(\sum_{m=1}^{M}\alpha_m-\gamma)n_A \geq (-\gamma_x+\sum_{m=1}^{M}\alpha_m)(n_B-1)+(\sum_{m=1}^{M}\alpha_m+(\alpha_x-c_{BA}))n_A$$

$$\frac{n_A-1}{n_B} \geq \frac{\alpha_x+\gamma_x-c_{AB}}{\alpha_x+\gamma_x}$$

$$\frac{n_B-1}{n_A} \geq \frac{\alpha_x+\gamma_x-c_{BA}}{\alpha_x+\gamma_x}$$

(b) Alternatively, the two types play their own culture and segregate. This is an equilibrium when

$$(\alpha_x+\sum_{m=1}^{M}\alpha_m)(n_A-1) \geq ((\alpha_x-c_{AB})+\sum_{m=1}^{M}\alpha_m)n_B$$

$$(\alpha_x+\sum_{m=1}^{M}\alpha_m)(n_B-1) \geq (\alpha_x-c_{BA}\sum_{m=1}^{M}\alpha_m)n_A$$

$$\frac{n_A-1}{n_B} \geq \frac{\alpha_x-c_{AB}+\sum_{m=1}^{M}\alpha_m}{\alpha_x+\sum_{m=1}^{M}\alpha_m}$$

$$\frac{n_B-1}{n_A} \geq \frac{\alpha_x-c_{BA}+\sum_{m=1}^{M}\alpha_m}{\alpha_x+\sum_{m=1}^{M}\alpha_m}$$

However, there is also a second condition that requires type A's and B's do not wish to form social ties with each other while retaining their own culture:

$$\sum_{s'} \alpha_{s'} < \gamma + \sum_{s} \gamma_s$$

where S is the subset of $\{y_1, .., y_M\}$ where types A and B coordinate on different actions, $S = \{m : y_{mA} \neq y_{mB}, \text{for} \quad m = 1, ..., M\}$.

□

Proof of Proposition 3 Note firstly that only strict Nash equilibria are absorbing. Second, I am going to assume continuity of k, n, n_A, and n_B - that is k, n, n_A, $n_B \in \mathbb{R}$ and not integers. This is for simplicity and affects the results only in the sense that when we show that one equilibrium is stochastically stable under certain parameter values then around the margin of these parameter values, because k, n_A, n_B, n are in fact integers, it is possible that another equilibrium which also borders these parameter values is also stochastically stable. Third, note that I will look at stochastic stability when type A group is in the majority. The results when identity group B makes up the majority of the population are symmetric. Finally I will consider stochastic stability when $n_A = n_B$.

We first find the minimum costs to transit from one equilibrium to another. We start with a transit from the assimilation equilibrium where all play x_A (*all A*) to the assimilation equilibrium where all play x_B (*all B*). Suppose all play x_A. What is the minimum number of mutations needed to trigger a transit from *all A* to *all B*? Let k be the minimum number of mutations. Note that an individual playing x_A will only link with those playing x_A, an individual playing x_B will only want to link with those playing x_B, and an individual maximizes her utility from linking to all others who are playing the same action. Thus when given the option to change strategy, an individual compares payoffs from linking with all those playing x_A or all those playing x_B. Since an A type individual gets a lower payoff than a B type individual when they both play action x_B and the same strategy, we need to examine how many mutations it takes for an A type individual to wish to switch and play x_B. This then implies a transition to *all B*. The minimum number of mutations required for transition from the *all A* equilibrium to *all B* is thus the minimum number of errors (where an error is that an individual playing x_A plays x_B instead) such that a type A who is currently playing x_A wants to switch and play x_B also:

$$(\alpha_x - c)k \geq \alpha(n - k - 1) \qquad \text{thus} \qquad k \geq \frac{\alpha_x(n-1)}{2\alpha_x - c}$$

However, for a B to want to change we require the following weaker condition

$$\alpha_x k \geq (\alpha_x - c)(n-k-1) \qquad \text{thus} \qquad k \geq \frac{(\alpha_x - c)(n-1)}{2\alpha_x - c}$$

If all type B's will change to play B, a type A will want to change to play B if

$$(\alpha_x - c)n_B \geq \alpha_x(n_A - 1) \qquad \text{thus} \qquad \frac{\alpha_x - c}{\alpha_x} \geq \frac{n_A - 1}{n_B}$$

Note that $\frac{\alpha_x - c}{\alpha_x} < 1$, and since A are in the majority $\frac{n_A - 1}{n_B} \geq 1$ so we have a contradiction (note that here we assume since A are in the majority $n_A - 1 \geq n_B$). Thus $k \geq \frac{\alpha_x(n-1)}{2\alpha_x - c}$ mutations are required to trigger a transit from *all A* to *all B*.

We next look at a transit from the equilibrium when all play x_B to the equilibrium where all play x_A. The minimum number of mutations required for a type B to choose to play x_A is:

$$(\alpha_x - c)k \geq \alpha_x(n-k-1) \qquad \text{thus} \qquad k \geq \frac{\alpha_x(n-1)}{2\alpha_x - c}$$

For a type A to choose to play x_A the minimum number of mutations is:

$$\alpha_x k \geq (\alpha_x - c)(n-k-1) \qquad \text{thus} \qquad k \geq \frac{(\alpha_x - c)(n-1)}{2\alpha_x - c}$$

If all type A's change and play x_A a type B will want to change only if

$$(\alpha_x - c)n_A \geq \alpha_x(n_B - 1) \qquad \text{thus} \qquad \frac{\alpha_x - c}{\alpha_x} \geq \frac{n_B - 1}{n_A}$$

These parameters denote the parameters for which segregation is not a Nash equilibrium (recall we ignore the case where $\frac{\alpha_x - c}{\alpha_x} = \frac{n_B - 1}{n_A}$ since there is no strict Nash equilibrium in this parameter range). Hence, under the parameters for which segregation is an equilibrium, the minimum number of mutations required to trigger *all A* is $k \geq \frac{\alpha_x(n-1)}{2\alpha_x - c}$. Whereas when we are in the parameter range where segregation is not an equilibrium, the minimum number of mutations required to trigger *all A* is $k \geq \frac{(\alpha_x - c)(n-1)}{2\alpha_x - c}$.

We next examine the transit from *all A* to the segregated equilibrium. Clearly, we are restricted here to the parameters for which segregation is an equilibrium. To transit to the segregation equilibrium we require that a type B wants to move from the group playing x_A and play x_B but the type A's wish to remain in the group playing x_A. A type B will want to switch and play x_B only if playing x_B and linking with those playing x_B gives more utility than playing x_A and linking with those playing x_A. Thus the minimum number of mutations required to transit to segregation is the minimum number of individuals playing x_B required for a type B to wish to switch under best response dynamics:

$$\alpha_x k > (\alpha_x - c)(n-k-1) \qquad \text{thus} \qquad k > \frac{(\alpha_x - c)(n-1)}{2\alpha_x - c}$$

We next look at a transit from segregation to *all A*. We want to find, k, the minimum number of individuals playing B who mutate and play x_A such that a type B playing x_B wants to play x_A under best response dynamics:

$$(\alpha_x - c)(n_A + k) \geq \alpha(n_B - k - 1) \qquad \text{thus} \qquad k \geq \frac{\alpha_x(n_B - 1) - (\alpha_x - c)n_A}{2\alpha_x - c}.$$

Symmetrically, for a transit from *all B* to segregation we require k mutations where:

$$k > \frac{(\alpha_x - c)(n - 1)}{2\alpha_x - c}$$

To transit from segregation to *all B*:

$$k \geq \frac{\alpha_x(n_A - 1) - (\alpha_x - c)n_B}{2\alpha_x - c}$$

We now calculate the minimum cost spanning tree/stochastic potential for each equilibrium. The stochastic potential of *all A* when segregation is an equilibrium is

$$\frac{\alpha_x(n-1)}{2\alpha_x - c} + \frac{\alpha_x(n_B - 1) - (\alpha_x - c)n_A}{2\alpha_x - c}$$

The stochastic potential of *all B* when segregation is an equilibrium is

$$\frac{\alpha(n-1)}{2\alpha_x - c} + \frac{\alpha_x(n_A - 1) - (\alpha_x - c)n_B}{2\alpha_x - c}$$

The stochastic potential of segregation is

$$\frac{2(\alpha - c)(n-1)}{2\alpha - c}.$$

We see immediately that the stochastic potential of *all A* is smaller than the stochastic potential of *all B* so we need to compare the stochastic potential of *all A* and segregation. The stochastic potential of segregation is smaller than the stochastic potential of *all A* when

$$\frac{\alpha_x(n-1)}{2\alpha_x - c} + \frac{\alpha_x(n_B - 1) - (\alpha_x - c)n_A}{2\alpha_x - c} \geq \frac{2(\alpha_x - c)(n-1)}{2\alpha_x - c}$$

$$\frac{n_B - 1}{n_A} \geq \frac{2\alpha_x - 3c}{2c}$$

The stochastic potential of *all A* when segregation is not an equilibrium is

$$\frac{(\alpha_x - c)(n-1)}{2\alpha_x - c}$$

The stochastic potential of *all B* when segregation is not an equilibrium is

$$\frac{\alpha_x(n-1)}{2\alpha_x - c}$$

The stochastic potential of *all A* is smaller.

Finally we must check what happens when $n_A = n_B$. When segregation is not stochastically stable then both $allA$ and $allB$ will be stochastically stable since all stochastic potential will be symmetric.

Proof of Proposition 8

(i). All T groups integrate and play the same action.This is a Nash equilibrium under all parameters since any individual earns a positive payoff and by deviating will strictly reduce their payoff.

(ii). The identity groups form a partition, P_r, where all individuals of the same identity are in the same coalition of the partition. Each individual in a given coalition plays the same culture but different coalitions play different cultures. The partition P_r is an equilibrium when

$$\frac{n_{kr} - 1}{n_{lr}} \geq \frac{\alpha_x - c_{tx_{lr}}}{\alpha_x - c_{t,x_{kr}}}$$

$\forall\ k, l \in 1, ..., m_r$ and $\forall t \in T$ where t is an identity group in the coalition p_{kr} and where $c_{t,x_{kr}}$ represents the cost to an individual of identity t of adopting the culture x_{kr} adopted by coalition p_{kr}, and $c_{t,x_{lr}}$ represents the cost to an individual of identity t of adopting the culture x_{lr} adopted by coalition p_{lr}.

That is each member of each coalition must be at least as well off adopting the culture of their own coalition and interacting within that coalition as they would be if they adopted the culture of another coalition and interacted with that coalition.

These are the only two equilibria. Individuals in the same identity group will play the same strategy this follows using a previous argument. Any segregated individuals must adopt different cultures and any linked individuals must adopt the same culture. If segregated individuals adopt the same culture they can do strictly better by forming a social tie. Individuals who adopt different cultures will not form a link as this link will give a strictly negative payoff. Thus the only form of equilibrium possible (apart from the complete network) is segregated coalitions where everyone in the coalition plays the same strategy and each coalition plays a different strategy.

When all identity groups segregate so that the partition has T elements, each group adopts its own culture. Suppose not. Since all groups must be playing different actions. At least two groups must not be playing there own action. One group not playing it's own action must be weakly larger than all other groups not playing their own action. But then the group whose culture is being played by the larger group would do better to swap and join this group.

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