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A Steady State Approach to a Network Externality Market With Switching Costs

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A steady state approach to a network externality market with switching costs*

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Abstract

We study duopoly pricing in the market for mobile phone service, which features network externalities, switching costs, and consumer heterogeneity. We introduce a steady state approach that enables a tractable analysis without endgame effects. The model can generate a variety of testable predictions, of which we focus on the comparative statics with respect to switching costs. Using data on the mobile phone service industries in 52 countries, we use the variation in market structure at the time switching costs were suddenly reduced by the regulatory imposition of mobile number portability (MNP). Firms that grew more rapidly prior to MNP respond to MNP by pricing more aggressively; firms facing large competitors respond less aggressively. Exploration of the model and its implications is an object of ongoing research.

Keywords: Oligopoly, network externalities, switching costs, mobile number portability. *JEL Classifications:* L13.

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1 Introduction

Markets for mobile phone service are interesting not only for their economic and technological importance, but also because they exhibit network externalities and switching costs. One particular switching cost—the cost of changing one's phone number when one switches service providers—has been eliminated in many countries by the regulatory imposition of mobile number portability (MNP). The fact that MNP was introduced in different countries at different times, and to different market structures, provides a kind of natural experiment that can help inform our understanding of network externalities and switching costs. There is evidence that introducing MNP decreases market concentration; surprisingly, however, it actually increases the disparity in profitability between the leader and other firms, although the effect is mitigated in markets that are growing particularly quickly (Wei and Zhu, 2010).

Our study is inspired by such a conundrum, and seeks to further explore its underlying mechanisms. Our preliminary empirical results show that when MNP is introduced, although prices trend downward, the leader firm tends to reduce its prices less than other firms. Whereas the finding is consistent with market shares convergence and profitability divergence described above, it raises an important question: Why, when switching costs suddenly decrease, does the leader not compete as aggressively as the other firms? This empirical result is counter-intuitive, because at first one tends to think that the leader is the firm most privileged by the absence of MNP, and therefore the firm most likely to respond aggressively when MNP is introduced. To make sense of this puzzle, we hypothesize that the effect of MNP differs across firms due to consumer heterogeneity. If different consumers have different tastes, then on the eve of MNP each firm is catering to a different set of consumers. Due to high switching costs, some of those consumers may have signed up for service contracts at a time when the market structure differed, and now that MNP is arriving they are planning to switch firms.

On the eve of MNP, we can categorize consumers into three groups, based on their sensitivity to price changes after MNP is introduced.

- *Inframarginal* consumers will stay with their existing firm if prices change only marginally;
- *Marginal* consumers are close to indifference over whether to switch, and will be sensitive to marginal price changes;
- *Supramarginal* consumers plan to switch firms at the current prices, and marginal price changes will not affect their decision.

In a market inhabited by a dominant firm and some upstart entrants, the dominant firm may have many supramarginal customers who signed up before the entrants were viable. In order to retain these supramarginal customers, the firm would have to significantly decrease its prices. But a dominant firm probably also has many inframarginal customers, and a large price decrease will significantly reduce the profitability of serving them. So the large firm has to weigh the tradeoff between supramarginal and inframarginal customers. A small upstart firm, however, probably has few inframarginal customers; meanwhile, it is going to attract relatively many of the dominant firm's supramarginal customers. Hence a small firm mainly weighs the profitability of serving the supramarginal switchers against the benefit of attracting additional marginal customers away from the dominant firm. Depending on the composition of the firms' customer bases on the eve of MNP, these decisions can lead to the pricing behavior that we document.

Since we cannot observe consumer valuations, to understand the division of consumers across firms we must look at the historical development of each market prior to MNP. For instance, in a two-firm market with roughly equal customer bases on the eve of MNP, the allocation of consumers to firms will be different across the following historical settings:

- If market saturation has been relatively constant and firm 1 was initially dominant but has recently lost market share to upstart firm 2, we might expect firm 1 to have many supramarginal customers waiting to switch, while firm 2 should have few supramarginal customers to lose. Hence firm 2 may actually have more inframarginal customers than firm 1.

- If market saturation has been increasing, and both firms have been growing quickly with roughly equal market shares all along, we might expect that most consumers signed up with whichever firm was better for them, so neither firm has very many supramarginal customers, and both primarily serve inframarginal customers.

- If market saturation is still low, even if one firm was historically dominant, both firms may have large numbers of marginal consumers who recently signed up, and relatively few supramarginal or inframarginal customers. Furthermore, many consumers who have never signed with either firm may be responsive to marginal price decreases.

These kinds of intuitions support a variety of exploratory hypotheses. We consider two in particular:

1. The pricing response to MNP depends on the historical growth paths of the firms, in addition to their sizes;

2. The pricing response to MNP of a given firm depends on the size of its largest competitor.

We address these hypotheses using a global dataset covering the mobile phone service industries of 52 countries over the period 2003–2009. We find that firms that grew more rapidly in the 3–8 quarters prior to MNP respond to MNP more aggressively; such firms probably have relatively few supramarginal customers to lose but may be able to attract many additional marginal customers after MNP. We also find that firms facing large competitors respond to MNP less aggressively. Combined with the observation that large firms respond to MNP less aggressively, this finding suggests that a firm may expect to attract many supramarginal customers away from a large competitor after MNP is implemented, and that marginal customers may be relatively unimportant.

We consider the above findings suggestive and exploratory for now. To develop formal testable implications, we construct a model of dynamic duopoly that incorporates both network externalities and switching costs. Each consumer has an idiosyncratic valuation for each firm's product, and consumers are distributed uniformly on the two-dimensional unit simplex. In addition, all consumers share a common concern for the size of their product's network. The game starts with an arbitrary inherited market structure, in which consumers are already attached to particular firms (or the "null firm," for unattached consumers) in a manner consistent with some historical price path. Once a consumer has chosen a product from one of the firms, he faces a cost of switching—either to the other firm or to exiting the market; this switching cost is reduced when MNP arrives. Then firms can set new prices, and consumers may switch.

First, we show how to compute the current market structure under duopoly for any historical price path under switching costs. Given current prices, the current market can always be characterized by the lowest-value consumer served by each firm, and the boundary dividing the customers attached to each firm. In the two-dimensional space representing consumer valuations for the two firms' products, this boundary between the two firms' customers is piecewise linear with slope either 0, 1, or ∞ almost everywhere, and is contained in a band of width twice the switching cost. These results characterize the entire class of possible inherited market structures.

We find that a fully dynamic, multiperiod version of this model, with fully rational, forward-looking firms and consumers, is analytically intractable even without switching costs. We show where analytical tractability breaks down, and propose several conjectures that could be studied numerically. However, since our main focus is on the effect of switching costs, we simplify the solution concept in order to focus on the most relevant issues. Specifically, we assume that both firms and consumers act myopically. Consumers purchase whichever product they find most advantageous at that moment, without considering future switching costs that they might incur. Similarly, firms set prices to maximize their profits in the current period, without accounting for the way the current period outcome may lead to future changes. The main benefit of these assumptions is tractability; of course the drawback is that dynamic strategic considerations are eliminated. In a sense, our solution concept can be viewed as a kind of steady state analysis, since in a steady state, myopic beliefs are correct along the equilibrium path. Although myopic beliefs are typically wrong regarding the dynamic consequences of deviating, since a myopic deviator believes that its opponent will not respond to the deviation, deviations will tend to be more attractive under myopic beliefs than under fully rational beliefs. Therefore a steady state equilibrium under myopic beliefs will also tend to be a steady state equilibrium under fully rational beliefs. (To be able to make such a claim formally, we will need to establish that the game satisfies appropriate monotonicity properties; this is an issue for future research.)

This model has several distinguishing features. First, it has *both* network externalities and switching costs. Network externalities and switching costs have been studied intensively but separately in the literature (Farrell and Klemperer 2007). Surprisingly, little work has been done to understand their interplay (except Doganoglu and Grzybowski 2005, Suleymanova and Wey 2008 and Chen 2010). While the literature on switching costs usually does not consider any network effects, the literature on network effects usually presumes infinite switching costs that induce lock-in. In fact, customer switching is a salient feature that distinguishes network industries from others (Shy 2001); the co-existence and interaction between switching costs and network effects would have important implications for market competition in these industries. By explicitly modeling finite switching costs with network externalities, our model provides general insights, and is applicable to understanding switching costs and network effects in various industries.

Second, our model contributes to the understanding of market dynamics and industry evolution in the presence of network externalities and switching costs. The very few studies on network externalities and switching costs rely on finite-period models, such as Doganoglu and Grzybowski (2005) and Suleymanova and Wey (2008). These models inevitably involve end-of-game effects in the last period, whereas we distill an infinite-horizon model down to a steady-state question without endgame effects. One notable exception is Chen (2010), who studies a dynamic oligopolistic model with symmetric Markov perfect equilibria. Finally, previous works have generally assumed a one-dimensional type space where a consumer's preference for one firm is inversely related to her preference for the other firm. This means that if the market is unsaturated, the firms have no strategic interaction. Therefore, most of these studies assume the market is saturated. In contrast, our assumption of a two-dimensional type space admits the possibility that the market is unsaturated, which accords with empirical evidence.

We are still in the early stages of working with this model. Our steady state solution concept is easy to work with numerically, and so far appears to be analytically tractable as well. Testable implications will arise from relating market structure of the eve of MNP to the historical price path, and from relating the inherited market structure to the pricing behavior that arises in equilibrium once MNP is instituted. We think that both the model and the data sets are well suited to this purpose, and expect to make rapid progress over the coming year.

In sum, we develop a unique model in that it incorporates both network externalities and switching costs in growing markets under a dynamic oligopolistic setting. This fills an important gap in the literature. Furthermore, with MNP as a valuable experiment in the wireless industry, our empirical study provides useful insights into switching costs related policymaking in network industries.

2 Model

In this section, we describe the model. We analyze a discrete time model of a duopolistic industry with finite switching costs and network externalities. To simplify the analysis, we assume all agents are myopic.

2.1 Consumers

We model consumers using a overlapping generations framework. Each consumer maximizes his payoff over a two period horizon, which then repeats. There is a unit mass of consumers evenly divided between "young" and "old" in each period, with age independent of preferences. In order to purchase from a firm, a consumer must sign a two-period contract when young that obligates the consumer to purchase from that firm in both period of her life at the same price in both periods. Consumers are not allowed to enter into a contract when old.

Consumers have a maximum demand of one unit. Let $j \in \mathcal{J} = \{A, B, 0\}$ be each consumer's action set at any moment in time, where $j = A$ or B corresponds to purchasing from firm A or B, and $j = 0$ corresponds to the null firm, or not purchasing the good. A consumer i is distinguished by her preference $\theta^{i,j}$ for firm j's product, where $\theta^i = (\theta^{i,A}, \theta^{i,B})$ is distributed on $\Theta = \{\theta \in \mathbb{R}^2_{0+} | \ \theta^A + \theta^B \leq 1\}$ with a pdf $f(\theta)$. For convenience, we refer to $\theta^{i,j}$ as a j-type.

Consumers also have preferences over the number of other individuals s^j_t who purchase the good from firm j at the same time, which is the market share or the size of the network. This preference is proportional to the size of the network at rate $\phi^j < 1$. Let the number of consumers signing a new contract with firm j at time t be n^j_t. Because consumers sign two-period contracts, the size of firm j's network at time t is $s^j_t = n^j_{t-1} + n^j_t$.

Additionally, if at any time a consumer purchases the good from a different firm then they did in the previous generation, they incur a switching cost of β. For simplicity, we assume that consumers' preferences are additively separable in direct utility, the network externality, the switching cost, and prices.

Because the fully dynamic problem with forward looking firms and consumers is intractable, we limit the scope of consumers' foresight. First, each consumer is myopic in that she purchases from the firm that maximizes current surplus and does not consider future switching costs. Second, recall that a consumer's payoff from a firm depends on the her expectations of that firm's market share next period. We assume that each consumer expects the same number of consumers to sign a contract this period and next period as did last period; that is, $\mathbf{E}[n^j_{t+1}] = \mathbf{E}[n^j_t] = n^j_{t-1}$ for all t and j so that $\mathbf{E}[s^j_t + \delta s^j_{t+1}] = 2(1+\delta)n^j_{t-1}$.

A consumer with preference θ^i purchases from firm $j \in \mathcal{J}$ at time t if $J(\theta^i, t) = j$. This function is the solution to the following problem:

$$J(\theta^i, t) \equiv \arg\max_{j \in \mathcal{J}} (1+\delta)[\theta^{i,j} - p^j_t + 2\phi^j n^j_{t-1}] - \beta h(j, J(\theta^i, t-2))$$

$$\text{where } h(j, J(\theta^i, t-2)) = \begin{cases} 0 & \text{if } j = J(\theta^i, t-2) \text{ or } J(\theta^i, t-2) = 0 \text{ or } t \leq 2 \\ 1 & \text{otherwise} \end{cases}$$

In the first two periods, the switching cost term disappears. Clearly, $p^0_t = 0$ for all times t, $\phi^0 = 0$, and $\theta^i, 0 = 0$ for each consumer i. For simplicity, we assume $\phi^A = \phi^B = \phi$.

2.2 Evolution of Market Shares

We begin by presenting an example of how market shares can potentially evolve. This is a complicated process because it depends not only on how many consumers have already purchased from each firm and prices, but *which* consumers purchase from each firm. There are two additional pieces of notation that play a critical role in determining which consumers purchase from which firm. First, let ψ^j_t be the lowest j-type who purchases from firm j at time t. By definition, the only consumers who do not purchase from either firm are those with sufficiently weak preferences (i.e. $\theta^j < \psi^j_t$ for $j = A, B$). We need to characterize the partition of consumers between firm A and firm B. Second, let $\underline{\theta}^j$ be the j-type who receives 0 surplus when purchasing from firm j if she does not have to pay a switching cost. This provides a simple cutoff for when consumers wish to purchase from a given firm. This will be sufficient even when considering consumers who face a switching cost because payoffs are additively separable in direct preferences and switching costs.

Consider the behavior of consumers who have previously not purchased the good. Ignoring indifference, their purchasing strategy is:

$$J(\theta^i, t|J(\cdot, t-2) = 0) = \begin{cases} A & \text{if } \theta^{i,A} \geq \underline{\theta}^A \text{ and } \theta^{i,B} \leq \theta^{i,A} + [\underline{\theta}^B - \underline{\theta}^A] \\ B & \text{if } \theta^{i,B} \geq \underline{\theta}^B \text{ and } \theta^{i,B} \geq \theta^{i,A} + [\underline{\theta}^B - \underline{\theta}^A] \\ 0 & \text{if } \theta^{i,A} \leq \underline{\theta}^A \text{ or } \theta^{i,B} \leq \underline{\theta}^B \end{cases}$$

Now consider the behavior of consumers who have previously purchased from firm A. Their purchasing decision is:

$$J(\theta^i, t|J(\cdot, t-2) = A) = \begin{cases} A & \text{if } \theta^{i,A} \geq \underline{\theta}^A - \beta \text{ and } \theta^{i,B} \leq \theta^{i,A} + [\underline{\theta}^B - (\underline{\theta}^A - \beta)] \\ B & \text{if } \theta^{i,B} \geq \underline{\theta}^B \text{ and } \theta^{i,B} \geq \theta^{i,A} + [\underline{\theta}^B - (\underline{\theta}^A - \beta)] \\ 0 & \text{if } \theta^{i,A} \leq \underline{\theta}^A - \beta \text{ or } \theta^{i,B} \leq \underline{\theta}^B \end{cases}$$

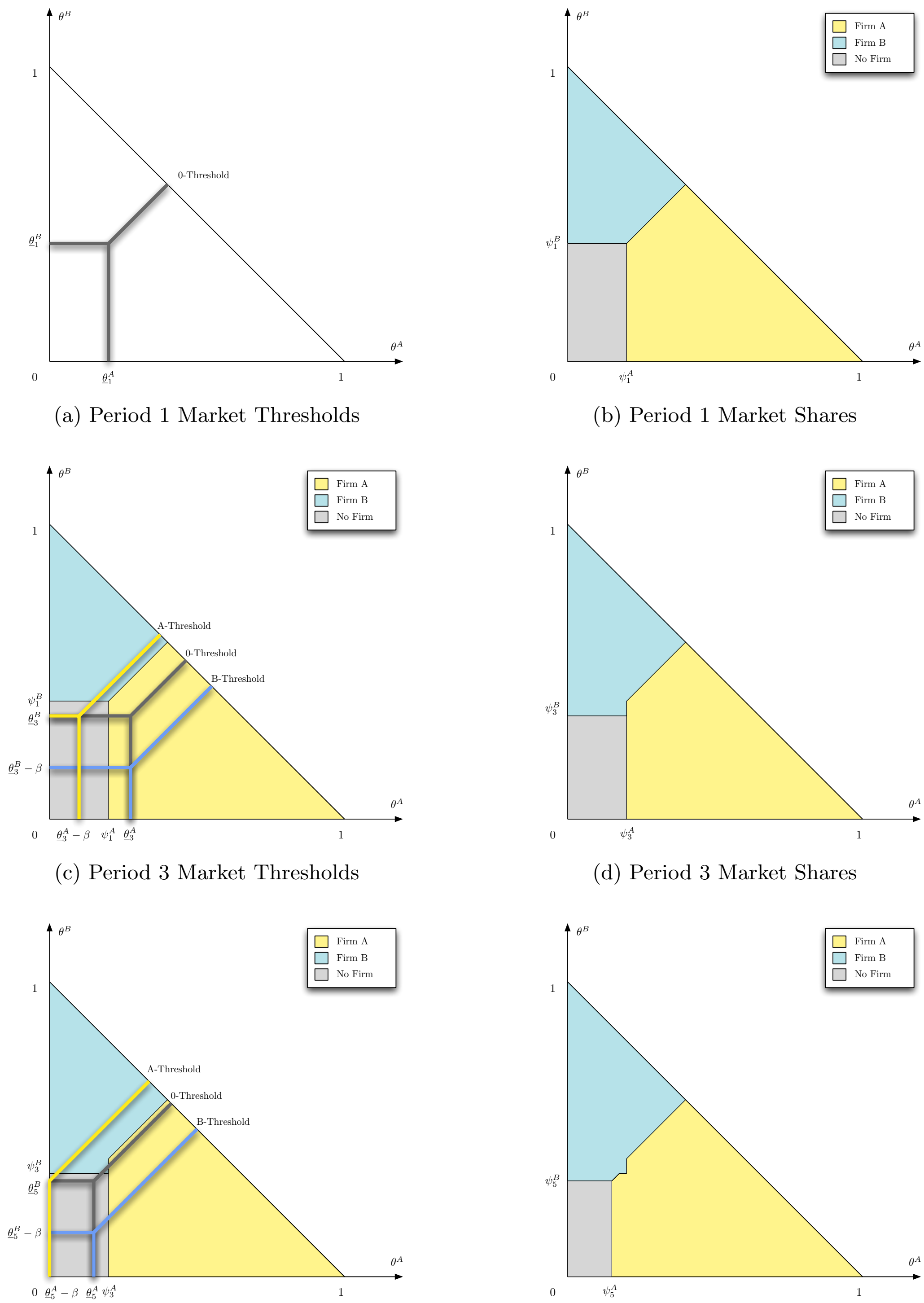
The behavior of consumers who have previously purchased from firm B is analogous, where the switching cost appears with $\underline{\theta}^B$ rather than $\underline{\theta}^B$.

No matter what previous decision a consumer has made, the set of types who are indifferent between firm A and firm B given this decision are on a line of slope 1. We call the lines partitioning the consumer space conditional on having previously purchased from firm j the j-threshold. Using these thresholds, we graphically show the distribution of a generation of consumers between firms in Figure 1 starting at $t = 1$.

We begin by constructing the division of consumers who have not previously purchased from any firm (1a), which leads to a distribution of consumers (1b). This demonstrates one of the key features of this model; because direct preferences for the two firms are not perfectly correlated, it is possible to have strategic interaction between the firms without requiring a saturated market.

At $t = 3$, when these consumers again make a decision, there are three thresholds to consider–one for each previous decision a consumer could have made. These thresholds are constructed as detailed in the above equations (1c) and all have the shape. In turn, this leads to a new distribution of consumers between firms (1d). For the illustrated values, no consumers who previously purchased the good switch firms. No customers begin purchasing from firm A, while Firm B attracts new consumers (consumers with $\theta^{i,B} \in [\underline{\theta}_3^B, \psi_1^B]$ and $\theta^{i,A} \leq \psi_1^A$ begin purchasing from firm B).

This also shows the effect of switching costs on consumers. Some consumers purchasing from firm A (those with $\theta^{i,A} \in [\psi_1^A, \underline{\theta}_3^A]$) are actually receiving negative surplus. These consumers would prefer to exit the market if they did not have to pay the switching cost. In addition, some consumers purchasing from firm A would prefer to purchase from firm B instead of firm A. The threshold for consumers who did not purchase in the previous period



(a) Period 1 Market Thresholds

(b) Period 1 Market Shares

(c) Period 3 Market Thresholds

(d) Period 3 Market Shares

(e) Period 5 Market Thresholds

(f) Period 5 Market Shares

FIGURE 1. Possible Distribution of Consumers with a Dominant Firm

represents what consumers would do if they did not have to pay a switching cost. Also, note that the partition of consumers between firm A and firm B is not a function; a consumer with an A-type of ψ_3^A is indifferent between purchasing from the two firms if $\theta^B \in [\underline{\theta}_3^B, \psi_1^B]$. In order to capture the possibility of vertical segments, the partition must be represented by a correspondence.

Repeating this process again at $t = 5$ yields new thresholds (1e) and yet another partition of consumers between firms (1f). Again, no consumers who previously purchased the good switch firms. However, both firms now attract new consumers. The partition of consumers now contains a horizontal segment. As we continue to repeat this process, the partitioning correspondence will be continuous, differentiable almost everywhere, and made up of segments of slope $0, 1$, or ∞. Therefore, given a nondecreasing correspondence[1] we can find a series of prices as $t \to \infty$ such that the partition of consumers between firms approximates the correspondence arbitrarily closely.

Let $g_t(\cdot) : [\psi^A, 1] \rightrightarrows [\psi^B, 1]$ be a such a nondecreasing correspondence between A-types and B-types. At any time t, the relevant information for determining the evolution of the distribution of consumers between firms consists of (A) the number of old consumers that have contracts with each firm, and (B) the distribution of currently young consumers between firms at $t-2$. As a shorthand, we will write $h^t = \{\psi_{t-2}^A, \psi_{t-2}^B, g_{t-2}(\cdot); n_{t-1}^A, n_{t-1}^B\}$.

Then, we can partition the space of consumers as follows:

$$J(\theta, t) \equiv J(\theta | h^t, p_t^j, p_t^{-j}) \begin{cases} A & \text{if } \theta^A \geq \psi_t^A \text{ and } \theta^B < \inf g_t(\theta^A) \\ B & \text{if } \theta^B \geq \psi_t^B \text{ and } \theta^B > \sup g_t(\theta^A) \\ 0 & \text{if } \theta^A < \psi_t^A \text{ and } \theta^B < \psi_t^B \end{cases}$$

If $\theta^B \in g(\theta^A)$, then a consumer with these preferences is indifferent between purchasing from the two firms. The set of such consumers are of measure 0, so we ignore them. We call $g_t(\cdot)$ the partitioning function.

We can then write the number of consumers who sign with firm j at time t as

$$n_t^j(p_t^j, p_t^{-j}, h^t) = \int_{\Theta} \mathbf{1}\{J(\theta | h^t, p_t^j, p_t^{-j})) = j\} f(\theta) d(\theta)$$

2.3 Firms

As with consumers, we place limitations on the firms' farsightedness. In particular, we assume firms are myopic and only maximize their profit in the current period. In contrast to consumers, however, firms correctly forecast the number of consumers who sign a contract

[1] That is, if $x' > x$, then $\sup g(x) \leq \inf g(x')$.

in the current period.

In addition, firms incur a cost that depends on the number of consumers the firm must serve in each period that potentially varies over time and between firms. Unless otherwise noted, we will typically assume that firms have constant marginal costs that do not vary over time and no fixed costs; that is, $c_t^j(s) = c^j s$ for all t with $c^j \geq 0$. If marginal costs were decreasing (or increasing), firms would have economies (or diseconomies) of scale. This would only accentuate (or counteract) the firms' existing incentive to attract a larger market share due to the network externality.

Therefore, after history h^t, firm j solves the following problem:

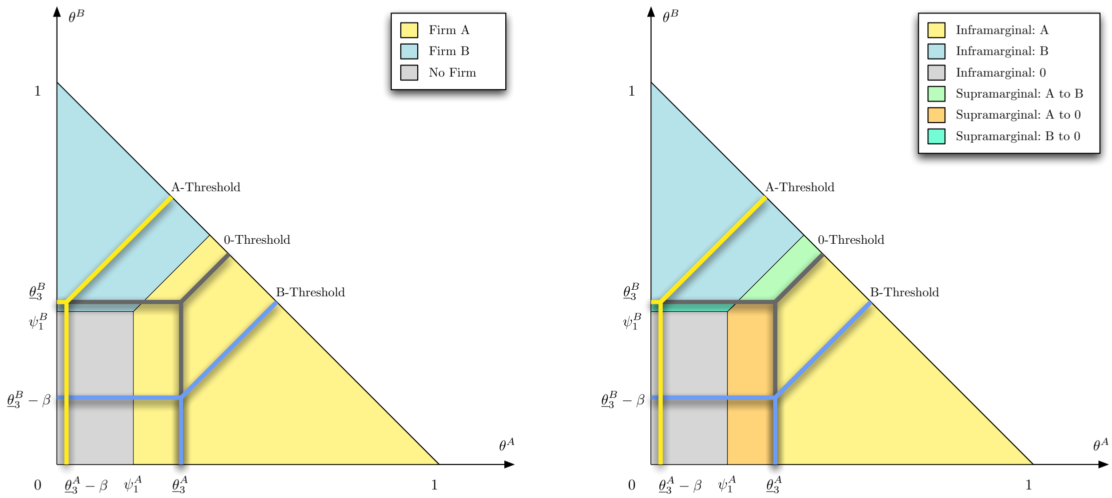
$$\max_{p^j} \Pi^j(p^j, p^{-j}, h^t) = p^j \cdot n^j(p_t^j, p_t^{-j}, h^t) - c_t^j \left(n_{t-1}^j + n^j(p_t^j, p_t^{-j}, h^t) \right)$$

2.4 Solution Concept

Given the static nature of the model, the appropriate solution concept is a Nash equilibrium in prices given any history. We look for steady states in equilibrium A steady state is a history in which the market partition is the same in the prior two periods, and does not change in equilibrium. In a sense, our solution concept can be viewed as a kind of dynamic steady state analysis, since in a steady state, myopic beliefs are correct along the equilibrium path. However, myopic beliefs are typically wrong regarding the dynamic consequences of deviating, since a myopic deviator believes that neither he nor his opponent will respond to the deviation.

Figure 2(a) illustrates a steady state equilibrium. At the current prices, no consumer wishes to switch; the threshold for each firm's existing consumers does not intersect the set of consumers who are attached to that firm.

As a preview of later work on comparative statics with respect to switching costs, Figure 2(b) illustrates our classification of consumers after switching costs fall to 0 (as in MNP). If prices remain the same, some consumers are *inframarginal* and will continue to purchase from the firm they are already attached to, even if prices change marginally. Some consumers are *supramarginal* and will switch firms, even if prices change marginally. As drawn, some consumers are inframarginal between A & B; A & 0; and B & 0. Finally, some consumers are *marginal* and will change their choice of firm if prices change marginally. These consumers are not labeled in the figure, but are simply those types on the boundary between the supramarginal and inframarginal regions.



(a) A Steady State Equilibrium (b) Consumer Classification after MNP

FIGURE 2. Marginal, Inframarginal, and Supramarginal Consumers

3 Theoretical Analysis

This is a highly nonlinear optimization problem. At this time, we have been unable to fully analyze the proposed model from Section 2. We first present two benchmarks without switching costs in Section 3.1 to highlight the difference between the dynamic steady state and static equilibrium. Then, we examine two examples with positive but finite switching costs and linear partition correspondences in Section 3.2 to illustrate the basic workings of the model. Finally, we describe our plan for analyzing the model in full generality in Section 3.3.

Because the partition correspondence is linear both in the benchmarks (because switching cost are 0) and examples (by assumption), we can simplify the problem by translating it from a choice of prices to a choice of the $\underline{\theta}^j$'s, the lowest j-type that is willing to purchase from firm j without switching costs. Solving for prices in terms of consumer types yields $p_t^j = \underline{\theta}_t^j + 2\phi n_{t-1}^j$.

3.1 Benchmarks

In this section, we compare the steady state of a dynamic model with forward looking firms but myopic consumers with the steady state of the static model described in Section 2, assuming there are no switching costs.

Note that without switching costs, the partition correspondence always has slope 1 and $\underline{\theta}_t^j = \psi_t^j$. We can then express the number of consumers who sign a contract with each firm solely as a function of the $\underline{\theta}^j$'s because history only matters through the number of consumers who signed a contract with each firm in the previous period. This is:

$$n^j(\underline{\theta}^j, \underline{\theta}^{-j}) = \begin{cases} \frac{1}{2}[(\underline{\theta}^j - 1)^2 - (\underline{\theta}^{-j} - 1)^2 + 1 - 2\underline{\theta}^j\underline{\theta}^{-j}] & \text{if } \underline{\theta}^j + \underline{\theta}^{-j} \geq 1 \\ (1 - \underline{\theta}^j)^2 & \text{if } \underline{\theta}^j + \underline{\theta}^{-j} \geq 1 \end{cases}$$

We then investigate the static model, maintaining the same assumptions. This allows us to compare the first order condition that determines a steady state in the variant with forward looking firms to the first order condition that solves the static model.

3.1.1 Dynamic Model without Switching Costs

In the dynamic model, firm j's must choose a sequence of consumer types. His problem is:

$$\max_{\{\underline{\theta}_t^j\}} \Pi^j_{\text{dyn}}[\{\underline{\theta}_t^j\}, \{\underline{\theta}_t^{-j}\}] \equiv \sum_{t=1}^{\infty} \delta^t (1+\delta) \left[(\underline{\theta}_t^j + 2\phi n_{t-1}^j - c^j) n_t^j \right] \tag{1}$$
$$\text{s.t. } \underline{\theta}_t^j \in [0,1] \; \forall t$$

Because consumers sign a two period contract, firms collect revenues from these consumers in two periods. With constant marginal costs, firms incur the same costs for serving these consumers during both periods of their contract. Therefore, each generation's profits are weighted by $1 + \delta$. While this affects the level of profits, it does not affect the solution.

The general first order condition is:

$$\frac{\partial \Pi^j_{\text{dyn}}}{\partial \underline{\theta}_t^j} = n_t^j + \left(\underline{\theta}_t^j + 2\phi[n_{t-1}^j + \delta n_{t+1}^j] - c^j \right) \frac{\partial n_t^j}{\partial \underline{\theta}_t^j} \equiv 0$$

In the steady state, when $\underline{\theta}_t^j = \underline{\theta}^j$ and $n_t^j = n^j$ for all t, this condition is:

$$n^j + \left(\underline{\theta}^j + 2(1+\delta)\phi n^j - c^j \right) \frac{\partial n^j}{\partial \underline{\theta}^j} \equiv 0 \tag{2}$$

3.1.2 Static Model without Switching Costs

In the static model, firm j must choose a single consumer type. He solves the following problem:

$$\max_{\underline{\theta}^j} \Pi^j(\underline{\theta}^j, \underline{\theta}^{-j}, h^t) \equiv (\underline{\theta}_t^j + 2\phi n_{t-1}^j - c^j) n^j \tag{3}$$
$$\text{s.t. } \underline{\theta}_t^j \in [0,1] \; \forall t$$

The first order condition is:

$$n^j + \left(\underline{\theta}^j + 2\phi n^j_{t-1} - c^j\right)\frac{\partial n^j}{\partial \underline{\theta}^j} \equiv 0$$

In a steady state, $n^j_{t-1} = n^j_t = n^j$, so the first order condition is

$$n^j + \left(\underline{\theta}^j + 2\phi n^j - c^j\right)\frac{\partial n^j}{\partial \underline{\theta}^j} \equiv 0 \tag{4}$$

Note that Eq. 2 and Eq. 4 are nearly identical; the only difference is an extra $(1+\delta)$ term multiplying the $n^j \frac{\partial n^j}{\partial \underline{\theta}^j}$ term in the former. In the dynamic model, firms consider the effect of increasing their market share today on profits tomorrow. This appears as a more heavily weighted network externality coefficient that depends on the firm's patience. A similar result holds for the second derivatives as well. Because the structure of the optimization problem are therefore the same, the solution to Eq. 1 and Eq. 3 will share the same general comparative statics with respect to ϕ and c^j. In fact, if $\delta = 0$, the solutions coincide. Therefore, static model is a reasonable approximation of the dynamic model.

3.1.3 Comparative Statics without Switching Costs

Unfortunately, solving for an equilibrium analytically for all parameter values is intractable. Solving the system of equations given by the first order conditions for the two firms in either model requires finding the roots of a sixth degree polynomial. For now, we posit several conjectures based on numerical investigation.

Conjecture 1. *There is at most one equilibrium in which the firms have strategic interaction (that is, $\theta^A + \theta^B < 1$).*

Conjecture 2. *If the rate of preference for the network externality ϕ increases, then both firms reduce their price and see their market share and profits increase.*

Conjecture 3. *As marginal cost of one firm increases, then (a) that firm increases its price, and its market share and profits decrease; and (b) the other firm reduces its price while its market share and profits increase.*

3.2 Examples with Switching Costs

To capture the intuition that the age of the firms matter, we consider two extreme situations. We begin by examining the steady state when one firm is considerably older than the other. We represent this with a vertical market partition correspondence. This represents a scenario where firm A is much older than firm B.[2] Figure 3 presents a graphical depiction of how this might happen. Suppose firm A is a monopolist. Then, the set of consumers who buy from firm A are those whose A-type is larger than some cutoff value; that is, there is a vertical partition correspondence between those who purchase from firm A and those who do not. Now consider what happens when firm B enters the market. The first consumers to switch from firm A to firm B are those with high B-types and low A-types. The market partition correspondence will still be vertical except near the edge of the consumer space.

Then, we analyze the steady state when firms are of similar age. We proxy this by a market correspondence that has a slope of 1. This captures the idea that no firm was dominant during the early periods of the market, in contrast to the scenario described in the previous paragraph.

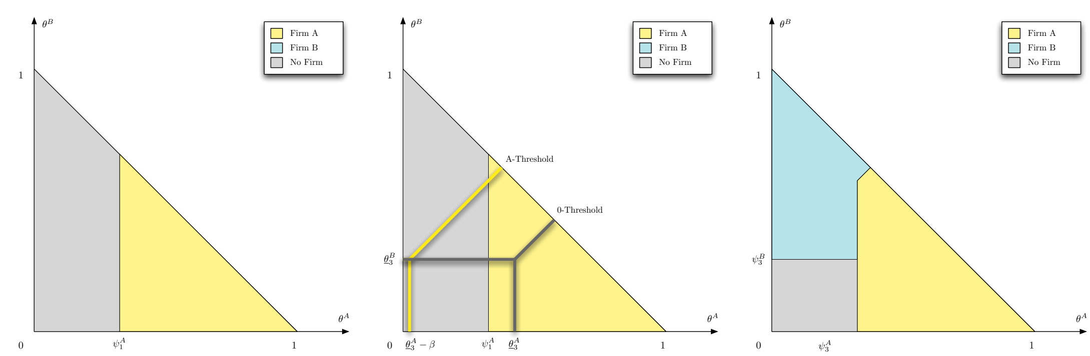
3.2.1 Firms of Different Ages

Market shares will remain constant if $\underline{\theta}^j - \beta \leq \psi^j \leq \underline{\theta}^j$ and $\underline{\theta}^A - \underline{\theta}^B \leq 1 + \beta - 2\psi^A$. Firm j would like to choose the highest price, or equivalently, the highest $\underline{\theta}^j$ as long as these constraints are satisfied. If $\psi^A + \psi^B + \beta \geq 1$, then the first set of constraints binds and firm j can choose $\underline{\theta}^j = \psi^j + \beta$. However, if $\psi^A + \psi^B + \beta < 1$ the second constraint binds and there will be a set of potential steady state equilibria.

Our analysis of this example is still in progress.

3.2.2 Firms of Similar Ages

Note that market shares will stay constant if $\underline{\theta}^j - \beta \leq \psi^j \leq \underline{\theta}^j$. Therefore, the only possible steady state equilibrium involves $\underline{\theta}^j - \beta = \psi^j$. Suppose to the contrary that both firms



(a) Consumers with Monopoly (b) Market Thresholds (c) Consumers with Duopoly

FIGURE 3. Possible Partition Correspondence with an Old Firm

[2]If firm B is much older than firm A, we use a horizontal market partition correspondence.

are choosing the $\underline{\theta}^j$'s so that market shares remain constant, but that one firm is setting $\underline{\theta}^j < \psi^j + \beta$. But then an increase in $\underline{\theta}^j$ is equivalent to increasing the price without affect the market share. Given that costs are low enough to warrant production, this is a profitable deviation.

Consider firm A's choice. Given that the firm B is setting $\theta^B = \psi^B + \beta$, there are several types of deviations. Firm A could choose to attract new consumers at the cost of lowering his price (or equivalently, $\underline{\theta}^A < \psi^A$). This requires a discrete drop in the price relative to the steady state price. If he lowers his price slightly, he will attract only previously unattached consumers, while if lowers his price dramatically, he will also attract consumers previously attached to firm B.

Firm A could instead choose to raise his price (that is, setting $\underline{\theta}^A > \psi^A + \beta$) at the cost of allowing some of his existing consumers to switch away from him. If he raises his price only slightly, he will just lose consumers to the null firm. If he raises his price by a moderate amount, he will lose consumers both to the null firm and to firm B, but there will be consumers who are indifferent between staying with firm A and switching to firm B. If he raises his price by a large amount, he will again lose consumers both to the null firm and firm B, but now there will be no consumers indifferent between the two firms. This is equivalent to being a monopolist over the highest A-types.

Firm A's profit function (conditional on $\underline{\theta}^B = \psi^B + \beta$) is a piecewise continuous function with six different segments. We need to find firm A's best response on each region, and look for the set of initial market shares (ψ^A, ψ^B) such that there is a steady state equilibrium.

Our work on analytically characterizing all possible steady states is still in progress for a given set of parameters. Preliminary numerical investigation yields the following findings. Our results are presented graphically in Figure 4, where the set of steady states (represented as (ψ^A, ψ^B)) for a given switching cost consists of all points within the corresponding line.

Start by considering the results with positive switching costs. Because the sets overlap, we cannot offer predictions on exactly how the steady state will change as a result in the decrease in switching costs. However, we can describe the movements in terms of a set ordering. A decrease in switching costs from $\beta = .3$ to $\beta = .2$ and then $\beta = .1$ has two effects. First, the steady states with the greatest degree of dispersion become less extreme. This is seen by the compression toward the center of the consumer space. Second, consider symmetric steady states. A symmetric steady state is more competitive than another if prices, or equivalently, the lowest consumer types are lower in the first steady state than the second. As switching costs fall, both the most competitive and least competitive symmetric steady states become less competitive. This is represented by the shift to the northeast.

There seems to be something unique about the model when there are no switching costs. First, the set of steady states when there are no switching costs has a radius of less than .0007, which is much smaller than the set of steady states associated with the other switching costs. Second, either there is a discontinuity in the direction of movement of the set of steady states as switching costs fall, or it reverses itself. We need to look at switching costs between $\beta = 0$ and $\beta = .1$ to see which is the case.

We need to investigate this example further to (A) pinpoint the reasons behind the movement of the set of steady states with positive switching costs, and (B) identify either what causes the discontinuity in the effect of decreasing switching costs or why the direction reverses for low switching costs.

We plan to derive the analytic conditions determining steady states to formally capture the mechanisms behind these phenomena.

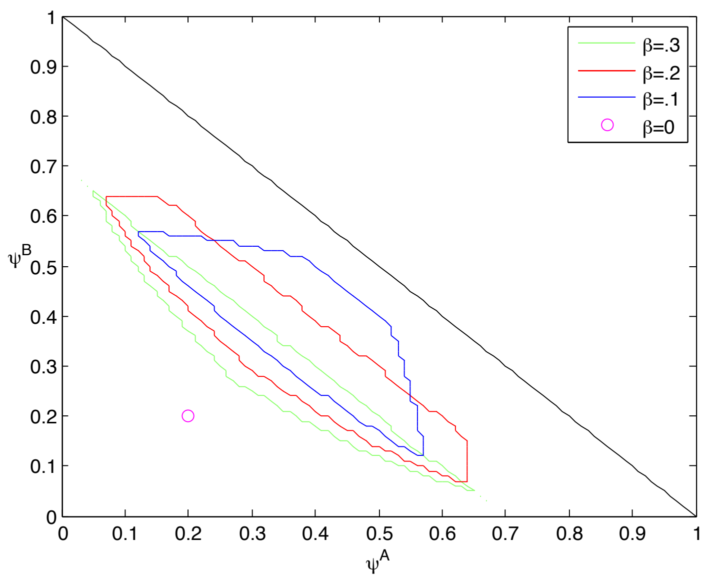


FIGURE 4. Comparative Statics with Respect to Switching Costs with Firms of Similar Ages

3.3 Plan for General Analysis

After finishing our analysis of the extreme examples presented in Section 3.2, we plan to generalize the above results to linear market partition correspondences with intermediate slopes between 1 and ∞. Again, our focus is on how a decrease in switching costs a la MNP affects the set of steady state equilibria.

Using the slope of the market partition correspondence as a proxy for age (with steeper slopes corresponding to an older firm A), we will be able to perform comparative statics on the effects of decreasing switching costs with respect to the relative age of the firms.

4 Empirical Evidence

To gauge the effect of switching costs on prices in the network industry, we use MNP policy in the cellular phone service market as a natural empirical setting. We test two major hypotheses: the effect of a decrease in switching costs induced by MNP depends on the age and size of firms, and the detailed market structure. We consider these two factors because in a market where firms other than the leader have arisen only recently, the leader should have more supramarginal customers to lose. These customers subscribed to the leader before alternative firms were available. Hence, the leader that has longer presence in the industry should respond to MNP less aggressively.

4.1 Data

We use two primary datasets in the empirical analysis. The first is a firm-level quarterly panel dataset from the Global Wireless Matrix, with 218 major wireless operators over 6 years (2003Q1-2009Q2) in 52 countries. During the sample period, 30 countries implemented MNP policy. Variables used in this study include number of subscribers, market share of subscribers, and monthly prices (based on average revenue per user).

In a cross-country study as ours, it is important to control for heterogeneity in industry characteristics and national demographics that may influence the relationships of major interest. Hence, the firm-level dataset is complemented by the Global Market Information Database with country-level variables. This includes cellular growth rate (*Growth Rate*), the number of cellular subscribers per 100 inhabitants (*Cellular Penetration*), substitute/complementary services such as *Fixed-line Telephone Penetration* and *Internet Penetration*, *GDP* per capita, as well as demographics variables including the percentages of age groups 13-19 (*Teen*), 20-29 (*Young*), and 30-49 (*Mid-age*), and people with higher education (*HiEdu*) out of the total population. Summary statistics of variables are reported in Table 1. A simple mean comparison (Columns 2 and 3) shows that the average market share increases after MNP (also the market concentration index HHI), while average prices decrease.

4.2 Analysis

Our empirical analysis is at an exploratory stage. As testable predictions of how switching costs would affect equilibrium prices have not been generated from the new analytical model, there is little guidance for the precise specification of the regression models. Hence, our analysis and interpretation of results are primarily based on conjectures.

To explore the possibility that firms might adjust prices in anticipation of MNP, we estimate prices as a function of lagged time trend. With a time trend of lagged 8 quarters

Table 1. Descriptive Statistics

	Full Sample	Before MNP	After MNP	Obs
Firm-level Variables				
Market Share	0.261	0.263	0.258	4809
	(0.171)	(0.183)	(0.154)	
Price	27.770	37.016	30.559	4161
	(16.874)	(13.645)	(18.050)	
Country-level Variables Cellular Market				
HHI	0.359	0.379	0.334	5294
	(0.099)	(0.111)	(0.074)	
Market Growth Rate	0.056	0.083	0.023	5082
	(0.082)	(0.093)	(0.048)	
Cellular Penetration	0.737	0.534	0.999	5336
	(0.376)	(0.334)	(0.239)	
Substitute/Complementary Markets				
Fixedline Penetration	0.325	0.209	0.481	5450
	(0.204)	(0.165)	(0.136)	
Internet Penetration	0.178	0.094	0.281	5450
	(0.141)	(0.113)	(0.097)	
Demographics				
GDP per capita($millions)	0.019	0.008	0.034	5450
	(0.018)	(0.001)	(0.014)	
High Education	0.137	0.100	0.183	5225
	(0.071)	(0.063)	(0.052)	
Teen	0.110	0.126	0.089	5450
	(0.027)	(0.023)	(0.016)	
Young	0.301	0.314	0.283	5450
	(0.027)	(0.022)	(0.023)	
Mid-age	0.135	0.124	0.149	5450
	(0.026)	(0.026)	(0.019)	

Standard deviations are in parentheses.

and its quadratic term, we find that the lags are not significant until 2 quarters before MNP, i.e., the coefficients on lagged 2 and 1 quarters are significantly negative (Column 1, Table 2). This may suggest that firms began to lower their prices 2 quarters before the policy implementation. MNP was usually implemented much later than its announcement. For example, in the U.S. MNP was postponed three times since its announcement in 1999 until 2003, due to strong resistance from large incumbents for fear of market share erosion. It is plausible that firms may strategically respond to the policy during this time gap, especially half a year before MNP, trying to sign up customers on longer contracts with lower prices, so as to mitigate the policy impact.

To examine whether and how market share respond before MNP implementation, we perform a similar analysis on market share. The regression estimation results (Column 2, Table 2) show that market share does not adjust significantly before MNP. This may be because many contracts are still in effect before MNP, hence customer base takes longer time to adjust.

The results on prices appear as an "Ashenfelter's Dip" (Ashenfelter 1978) before MNP, i.e., it involves a decrease in prices shortly before the policy presumably due to the anticipation of its implementation. If such a transitory decrease in prices does exist, the impact of MNP would have been overestimated if we do not consider this effect. Therefore, to account for possible actions by wireless operators before the policy that might bias the results, we exclude observations two quarters before MNP in the analysis hereafter.

Next, we investigate whether a firm's historical growth path—a proxy for its age—and its size would affect the MNP outcome. There are several steps involved: we estimate firms' pricing trend between lagged 8 to lagged 3 quarters before MNP, predict prices after MNP based on this trend, take the difference between actual prices and predicted prices, and calculate the difference as "price deviation" from the trend after MNP.

Then we specify and estimate the following model. We regress price deviation on the focal firm's growth rate (percentage change in market share) between lagged 8 to lagged 3 quarters before MNP, its size (market share) before MNP, and the size of the largest firm other than the focal firm before MNP:

$$\overline{p}_{ikt} = \alpha_1 + \sum_{\tau=1}^{5} \beta_\tau \Delta\sigma_{ik(t-\tau-2)} + \beta_6 \sigma_{ikt} + \beta_7 \sigma_{(-i)kt} + \phi Z_{kt} + \gamma_t + \eta_i + u_k + \varepsilon_{ikt}, \qquad ((1))$$

where $\overline{p}_{ikt}$ is the price deviation from the trend for firm i in country k at time t, $\Delta\sigma_{ik(t-\tau)}$ is the market share growth rate of firm i in lagged τ quarter, σ_{ikt} is the market share of firm i, and $\sigma_{(-i)kt} = \max\limits_{j \neq i} \sigma_{jkt}$ is the market share of firm i's largest competitor; Z_{kt} includes the country-level variables, and γ_t has quarterly time dummies, seasonality and country-specific time trend; η_i and u_k capture unobservable firm and country fixed effects,

Table 2. Time Trends before MNP

Dependent Variable	Prices	Market Share
Lag 1 quarter	-3.168**	-0.000
	(1.486)	(0.009)
Lag 2 quarter	-2.919**	-0.004
	(1.429)	(0.007)
Lag 3 quarter	-1.988	-0.007
	(1.244)	(0.009)
Lag 4 quarter	0.243	-0.002
	(0.858)	(0.008)
Lag 5 quarter	2.116	-0.004
	(1.314)	(0.010)
Lag 6 quarter	0.201	-0.003
	(1.226)	(0.008)
Lag 7 quarter	2.199	-0.001
	(1.566)	(0.009)
Lag 8 quarter	2.130*	-0.001
	(1.255)	(0.004)
Observations	2284	2509
R-squared	0.615	0.525

*, ** and *** denote statistical significance at 10%, 5% and 1% levels, respectively. Panel-corrected standard errors are in parentheses. All regressions include quadratic time trend, seasonality, country-specific time trend, country and firm fixed effects, and a constant term. These coefficients are not reported here. Other coefficients not reported: GDP per capita, Fixed-line Penetration, Internet Penetration, Teen, Young and Mid-age.

respectively; ε_{ikt} is the error term not captured by the regressors, which has zero conditional mean $E[\varepsilon_{ikt}|\Delta\sigma_{ikt}, \sigma_{ikt}, Z_{kt}, \gamma_t, \eta_i, u_k, \gamma_t] = 0$, under the assumption that it is uncorrelated with the exogenous regressors in each period after controlling for unobserved time-invariant heterogeneity. The parameters of primary interest are $\beta' s$, which measure how price deviation depends on a firm's previous growth pattern, its size, and its largest competitor's size before MNP.

The estimation results are reported in Table 3. First, as can be seen from Column 1, the growth rate of market share is significantly negative in lagged 3 quarters. It means that the faster a firm grows prior to MNP, the greater its price deviates below the trend after the policy. Specifically, a 1% increase in market share growth rate is associated with \$6.834 decrease (or 20% decrease) in price after MNP. Second, firm size before MNP is positively associated with the price deviation after MNP. That is, larger firms tend to price above the trend after the policy. Together, these two results provide evidence that the historical growth path and the size of a firm play important roles in the MNP effect. Whereas faster-growing and smaller firms respond to the policy more aggressively to attract customers, larger incumbents and firms growing more slowly tend to price less aggressively.

Third, the size of a focal firm's largest competitor is negatively associated with the focal firm's price. A 1% increase in market share of its largest competitor is associated with \$1.571 decrease (or 5% decrease) in its price after MNP. The intuition is that prices are strategic complements; if its largest competitor is a strong player with great network size, the firm will be more likely to avoid intense undercutting.

For a robustness check, we use the percentage change in lagged subscriber penetration (subscriber penetration is defined as the total number of a firm's subscribers relative to the national population), and the percentage change in lagged number of subscribers as regressors in (1), respectively. The firm size and the size of its largest competitor are measured by corresponding counterparts, i.e., subscriber penetration or number of subscribers. In both regressions, the coefficients on a firm's lagged growth rate and on the size of its largest competitor are have the same negative direction as before (Columns 2 and 3, Table 4).

For another set of robustness, we use the level changes in lagged market share / lagged subscriber penetration / lagged number of subscribers. Again, the evidence is consistent with the previous findings (Table 4).

To summarize, there are two major findings in the present analysis: (1) The historical growth pattern and the size of firms make a difference in MNP outcome; and (2) the size of the focal firm's largest competitor, which represents the detailed market structure, affects its price responsiveness to MNP negatively. Once the comparative statics on switching costs in the analytical model are derived, we will provide further analysis and more evidence for these effects.

Table 3. Price Deviation from Trend

	% Change in Market Share	% Change in Firm Penetration	% Change in Number of Subscribers
Lagged 3 quarter	-6.834***	-3.544**	-2.608
	(2.552)	(1.656)	(1.671)
Lagged 4 quarter	9.505	3.317	-3.202
	(6.627)	(5.001)	(3.614)
Lagged 5 quarter	7.627	7.533	5.213
	(5.462)	(5.940)	(6.699)
Lagged 6 quarter	3.688	6.090***	2.873
	(2.477)	(1.633)	(2.006)
Lagged 7 quarter	0.565	3.382	0.370
	(1.709)	(2.257)	(1.616)
Firm Size	13.142**	-5.270	0.179
	(7.057)	(3.372)	(0.128)
Size of Largest Competitor	-1.571**	-3.554**	-0.343
	(0.706)	(1.460)	(0.224)
Observations	199	199	199
R-squared	0.398	0.347	0.364

Dependent variable in all regressions is price deviation from time trend. *, ** and *** denote statistical significance at 10%, 5% and 1% levels, respectively. Panel-corrected standard errors are in parentheses. All regressions include quarterly dummy, seasonality, country-specific time trend, country and firm fixed effects, and a constant term. These coefficients are not reported here. Other coefficients not reported: GDP per capita, Fixed-line Penetration, Internet Penetration, Teen, Young and Mid-age.

TABLE 4. Robustness

	Change in Market Share	Change in Firm Penetration	Change in Number of Subscribers
Lagged 3 quarter	-16.921*	-47.737***	-0.588**
	(8.588)	(17.800)	(0.295)
Lagged 4 quarter	16.670	-21.349	-0.208
	(21.416)	(16.275)	(0.284)
Lagged 5 quarter	12.612	30.011*	0.516*
	(17.514)	(16.165)	(0.264)
Lagged 6 quarter	17.767	37.128	0.209
	(19.030)	(24.184)	(0.271)
Lagged 7 quarter	-3.882	-2.125	-0.083
	(14.073)	(29.787)	(0.240)
Firm Size	13.406	-45.825*	-0.385
	(26.461)	(25.294)	(0.379)
Size of Largest Competitor	-32.500***	-30.058***	-1.054***
	(11.881)	(8.498)	(0.256)
Observations	199	199	199
R-squared	0.340	0.326	0.315

Dependent variable in all regressions is price deviation from time trend. *, ** and *** denote statistical significance at 10%, 5% and 1% levels, respectively. Panel-corrected standard errors are in parentheses. All regressions include quarterly dummy, seasonality, country-specific time trend, country and firm fixed effects, and a constant term. These coefficients are not reported here. Other coefficients not reported: GDP per capita, Fixed-line Penetration, Internet Penetration, Teen, Young and Mid-age.

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