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Abstract

We present a theory for why it might be rational for a platform to limit the number of applications available on it. Our model is based on the observation that even if users prefer application variety, applications often also exhibit direct network effects. When there are direct network effects, users prefer to consume the same applications to benefit from consumption complementarities. We show that the combination of preference for variety and consumption complementarities gives rise to (i) a commons problem (users have an incentive to consume more applications than the social optimum to better satisfy their preference for variety); (ii) an equilibrium selection problem (consumption complementarities often lead to multiple equilibria); and (iii) a coordination problem (lacking perfect foresight, it is unlikely that users will end up buying the same set of applications). The analysis shows that the platform can resolve these problems by limiting the number of applications available. By limiting choice, the platform may create new equilibria (including the socially efficient allocation), destroy Pareto-dominated equilibria, and reduce the severity of the coordination problem faced by users.

1 Introduction

Platforms, such as video game consoles or personal computers, bring together third-party application developers and users who demand a variety of these applications. Because platforms connect two (or more) sides of a market, they are characterized by the presence of *indirect* network effects: the larger the number of platform users, the more applications are likely to be developed for it, which, in turn, increases users' valuation of the platform. For example, developers' desire to write Windows applications grows with the number of users who are expected to adopt that operating system; likewise, the larger the number of applications expected to run on Windows, the more willing users are to adopt it. Naturally, indirect network effects have played a prominent role in models of platforms, beginning, at least, from the pioneering work of Church and Gandal (1992) and Chou and Shy (1996) and spanning to recent contributions such as Hagiu (2009) or Weyl (2010).

When the value of a platform increases with the number of applications available, it is a profit-maximizing strategy to offer as many applications as possible so as to exploit indirect network effects to the maximum possible extent. What's more, suboptimal exploitation of indirect network effects may have disastrous consequences as technically superior platforms may perish in their competition against second-rate alternatives. For example, it is widely believed that Apple lost its battle against the PC in the late 1980s because of a dearth of applications. While Microsoft aggressively evangelized independent software vendors and provided them with tools and support, Apple based its approach on in-house development of a small number of applications. By the early 1990s, the number of applications available for the Mac was a small fraction to that for the PC. Likewise, in the 1980s, Sony lost its battle against JVC whose VHS standard was inferior to Betamax, due, largely, to reduced movie availability for Sony's standard.

Given the wealth of evidence suggesting that maximizing the number of applications available on a platform is a good idea, it is puzzling that firms such as Nintendo or Apple appear to have actively limited the number of applications on their platforms. In the late 1980s, for example, Nintendo restricted to five the number of new games that developers were allowed to produce each year for the Nintendo Entertainment System (NES).[1] The company also restricted the number of developers who could sell games for the NES. More recently, Apple restricts the number of applications available for the iPhone beyond just controlling for quality. This evidence runs counter the conventional wisdom that "more is

[1] The NES was the leading second-generation (8-bit) game console. Nintendo's global market share for 8-bit consoles was greater than 90%.

always better."

In this paper, we ask: *Why might it be rational for a platform to limit the number of applications when indirect network effects are at play?* Our answer is that by limiting the number of applications, the platform can help rise users' utility by resolving:

i. a *commons* problem;

ii. an *equilibrium selection* problem; and set)

iii. a *coordination* problem.

Our theory is based on the observation that even if platforms enjoy indirect network effects, on many occasions, applications also exhibit *direct* network effects, i.e., users are better off consuming the same applications as other users.[2,3] When users have limited resources (such as finite time to enjoy applications or an income constraint) and there are many applications available, they must pick and choose which ones to use. And if direct network effects are at play, users are better off if they purchase and consume the same limited set of applications.

We show that when users like application variety but also benefit from consumption complementarities, three issues may arise. First, the socially optimal number of applications may not be part of an equilibrium as users may find it unilaterally optimal to deviate to buy a larger number of applications so as to better satisfy their craving for variety. Second, multiple equilibria often arise. With the usual assumption that users have perfect foresight, any one of those equilibria could, in principle, be selected. However, some equilibria lead to higher user utility than others. Third, if users have no perfect foresight on other users' choices in equilibrium, it is unlikely that they will end up with the exact same set of applications, but such coordination is necessary to fully exploit consumption complementarities.

Our analysis demonstrates that by limiting choice, the platform can accomplish three tasks. First, it can create new equilibria that did not exist when application choice was broad. Second, it can eliminate socially inferior equilibria. Third, it can reduce the severity of the coordination problem faced by users when they do not know other users' choices in equilibrium. We conclude that when direct and indirect network effects are at play, an

[2]For example, gamers often prefer to play the same video games as other gamers as they can then discuss strategies to beat the game, which makes playing more enjoyable. And in the case of massively multiplayer online games (MMOG), such as World of Warcraft, the richness of the experience is based directly on the interactions between players; MMOGs are not fun if played alone.

[3]The notion of direct network effects was central to the early literature on systems competition, e.g. Katz and Shapiro (1985), but for the most part has been sidestepped by the literature on platforms.

important governance decision by platforms is the choice of how many applications should to be allowed to run on them.

Specifically, we model a platform that connects application developers and users of applications. We assume that all users are identical and derive utility from product variety and from consuming the same applications as other users. Preference for variety, modeled as in Dixit and Stiglitz (1977), gives rise to indirect network effects. Preference for consuming the same applications as other users, modeled as consumption complementarity (i.e., the marginal effect of consuming more of a given application increases with other users' consumption of the same application), gives rise to direct network effects. The model is flexible in that, depending on the parameter values, there may be direct network effects only, indirect network effects only, or both types of network effects. We assume that users have a time budget and that applications are sold at positive prices.

Users play the game where, in a first stage, they decide simultaneously which applications to purchase and, in a second stage, choose how much consumption time to allocate to each application. We study subgame-perfect Nash equilibria. Contrary to the recent literature on platforms (e.g., Rochet and Tirole 2003; Caillaud and Jullien 2003; Armstrong 2006; Hagiu 2009; Casadesus-Masanell and Ruiz-Aliseda 2009), which for the most part studies access prices and developer entry, we focus on the behavior of users. Thus, we consider elements such as application prices and qualities as given. And while the platform and application developers are passive in our model, we study how changing the number of applications available on the platform affects equilibria.

We find that under pure direct network effects, in every equilibrium users consume only one application which is the same for all users so that consumption complementarity is exploited to the maximum possible extent. There are as many equilibria as applications available on the platform, but since all equilibria lead to the same utility, there is no way for the platform to improve on users' utility by limiting or extending choice.

Under pure indirect network effects, users consume a large number of applications to satisfy their preference for variety. Since there are no consumption complementarities, users' utility functions are not interdependent and the game is simply a collection of independent optimization problems (one per user). While there are many equilibria, they all lead to the same level of utility. Limiting the number of applications available on the platform when users care exclusively about application variety, can only reduce users' equilibrium utility.

When both types of network effects are at play, we find that just as in the case of pure direct network effects, users consume the same set of applications in equilibrium in order to exploit consumption complementarities. We also find that there are multiple equilibria. Multiplicity has two distinct natures in this case. First, there is multiplicity in the sense that given a number of applications consumed, the specific identity of the applications consumed is irrelevant (so long as all users consume the same applications). This is the type of multiplicity that occurs with pure direct or pure indirect network effects. Second, there is multiplicity in the sense that the set of applications consumed (which is the same for all users) has varying cardinality. This type of multiplicity is interesting because user utility varies across equilibria. This type of multiplicity does not exist with pure direct or pure indirect network effects. (In what follows, by "different equilibria" we mean equilibria with different cardinalities of applications consumed.)

We find that there is always a set of different equilibria close to the (generally large) number of applications that users would choose if only indirect network effects were at play. And if consumption complementarity is sufficiently strong, another set of equilibria emerges around consuming one application, which is the equilibrium number of applications consumed under pure direct network effects. Therefore, what is an equilibrium in the case of combined direct and indirect network effects depends on the strength of consumption complementarity relative to that of preference for variety. Moreover, in this case new equilibria emerge which are not an equilibrium under pure network effects of either type.

When preference for variety is more important than consumption complementarity, all equilibria have a number of applications larger than the socially optimal number. Thus users face a *commons problem.* They would be better off if they all consumed fewer applications (as they would exploit consumption complementarities more fully) but the strong preference for variety provides incentives to deviate upwards and consume more applications.

When consumption complementarity is stronger than preference for variety, the socially optimal number of applications is one and there are equilibria where users consume one application only. However, there are also other equilibria with application cardinalities greater than one. Thus users face an *equilibrium selection problem* because, with perfect foresight, any one of those equilibria could be selected.

Having identified the commons and the equilibrium selection problems, we explore whether the platform may be able to fix these by choice of the number of applications offered. We show that if the platform constrains the number of applications to the social optimum, then in every equilibrium, users consume the socially efficient number of applications. Put differently, when consumption complementarity is more important than preference for variety, by limiting the number of applications the platform gets rid of Pareto dominated equilibria. And when preference for variety is more important than consumption complementarity, by limiting the number of applications the platform can also create new, Pareto dominant equilibria.

The final part of the paper we relax an important standard modeling assumption that, we believe, has little empirical appeal in our context: perfect foresight about other users' choices in equilibrium. Since users choose simultaneously which applications to purchase, it does not seem sensible to assume that they know other users' choices when they make their own. It would seem more reasonable to assume that users have no way of knowing which applications other users will consume when they make their own choices. Thus we study the game under the assumption that users have no foresight about other users' choices in equilibrium.

We find that when direct network effects are at play, users face a *coordination problem.* While users will want to purchase and consume the same applications as other users, they will generally not be able to do so because they have no way of knowing which applications are being bought by others. The implication is that some of the benefit of direct network effects is lost as consumption complementarities cannot be fully exploited when users lack foresight about other users' choices. In this case, we show that users may benefit substantially when the number of applications is limited by the platform because this curbs the severity of the coordination problem that they face. With a smaller choice set, it is more likely that they will end up purchasing and consuming the same applications and thus more likely that they will enjoy consumption complementarities.

Our paper contributes to the literature on platforms and two-sided markets, a literature that has flourished on the basis of industry-specific models. Rochet and Tirole (2003), for example, is inspired by the credit card industry, Armstrong (2006) captures well the economics of newspapers, and Hagiu (2009) competition between video game systems. Likewise, our model fits well hardware/software platforms such as PDAs or video game systems. While most of the literature on hardware/software platforms has studied questions related to pricing, our focus is on one aspect of platform governance that has received little attention thus far: the effect of limiting the number of applications available on user behavior and, ultimately, on the value created by the platform.

The only two papers we are aware of that are directly related to the question that we address here are Zhao (2010) and Hałaburda and Piskorski (2010). Zhao (2010) studies hardware/software platforms and explores the effects of quantity constraints on product quality and variety on a monopolistic two-sided platform where quality is uncontractible. He finds that when users cannot perfectly observe application quality, developers underinvest in application quality and that the platform can then use quantity restraints to help mitigate free-riding and increase overall application quality. While Zhao (2010) studies the effects of quantity limitations on the behavior of developers, we study the effects on the behavior of users. A second point of differentiation is that while he provides an explanation for why it may make sense for the platform to limit the number of applications per developer, in his theory the platform gains nothing from limiting the number of developers. Therefore, contrary to ours, his theory is silent about the benefits of limiting the overall number of applications offered by the platform.

Hałaburda and Piskorski (2010) studies a two-sided platform model based on dating markets. Without any difference in quality, men prefer market with a larger number of women, and women prefer a market with more men. It is an environment that gives rise to indirect network effects. Nonetheless, Hałaburda and Piskorski (2010) shows that users may benefit when a platform limits the number of candidates. This is because a dating platform limits the number of candidates on both sides. So, even though it limits the choice, it also limits the competition. Some agents prefer a platform with less choice, because it increases the probability that they will find a match. The current paper differs from Hałaburda and Piskorski (2010) in two ways. First, Hałaburda and Piskorski (2010) is the best suited for markets with one-to-one matching, like dating or housing markets. The current paper focuses on markets where users can consume large number of applications. Moreover, the applications are infinitely duplicable: When one user consumes an application, it does not limit the availability of the same application to other users. Second, in the applications market, there is no competitive effect that drives the result in Hałaburda and Piskorski (2010). To the contrary: as the result of consumption complementarity, the direct network effect is positive. Users gain if more users (on the same side of the market) consume the same applications. In the applications market users benefit when the platform restricts choice because they want to coordinate consumption instead of avoiding competition.

The paper is organized as follows. In Section 2 we present the game with perfect foresight, and solve for equilibria under direct and indirect network effects and discuss the utility implications of the platform limiting choice. In Section 3 we recast the model as one where users have no foresight about other users' choices in equilibrium, and solve for equilibria under the different types of network effects and study the implications for users of the platform limiting choice. Section 4 concludes.

2 Game with perfect foresight

We consider a platform which brings together developers and users of applications. There is a set $\mathcal{A}$ of available applications and N users. The number of users is exogenous. We denote the cardinality of $\mathcal{A}$ by A. Since our focus is on the user side, we treat $\mathcal{A}$ also as exogenous.

Let x_a^k denote user k's consumption of application a. The consumption utility that user k derives from consuming $\mathbf{x}^k = (x_1^k, x_2^k, \ldots, x_A^k)$ applications is given by

$$u(\mathbf{x}^k; \{\mathbf{x}^l\}_{l\neq k}) = \left(\sum_{a\in\mathcal{A}} \left(x_a^k\right)^{1/R}\right)^R + \alpha \sum_{a\in\mathcal{A}} \left(x_a^k \sum_{l\neq k} x_a^l\right),$$

where $\alpha \geq 0$ captures the value of consumption complementarity, and $1 \leq R < 2$ captures the intensity of the user's preference for variety.[4] The larger is R, the more the users prefer product variety. When $R = 1$, there is no preference for variety.

Consumption utility u allows to capture both the direct and indirect network effects. *Indirect* network effects originate from users' preference for variety: users prefer platforms with more users because it is more likely that more applications will be developed for that platform. Therefore, the larger is R, the stronger is the source of indirect network effects. When $R = 1$, however, users have no preference for variety and, therefore, there are no indirect network effects.

Direct network effects are present when a user's utility from consuming an application increases with other users' consumption levels of the same application. For example, users of video games enjoy a given game more if their friends also use the same game, as they can discuss strategies to beat the game. Direct network effects are captured by the term $\alpha \cdot x_a^k \cdot \sum_{l\neq k} x_a^l$: user k's enjoyment of her consumption of application a is larger the more the other users ($l \neq k$) consume application a. We let $\alpha \geq 0$. When $\alpha = 0$, there are no direct network effects and as α increases, direct network effects become stronger. In summary, user preferences may exhibit direct or indirect network effects, depending on the value of parameters α and R. Table 1 illustrates how the presence of network effects depends on the parameter values.

We assume that users have a budget of X units of time to consume the applications and interpret $x_a^k \geq 0$ as the amount of time that user k spends consuming application a. Thus, if user k consumes $\mathcal{Q} \subseteq \mathcal{A}$ applications, she must satisfy the budget constraint: $X \geq \sum_{a\in\mathcal{Q}} x_a^k$. Applications are sold at exogenous price $p > 0$ each, regardless of how much users consume

[4]Note that when $\alpha = 0$, preferences are as in Dixit and Stiglitz (1977).

		Direct network effects	
		Yes	No
Indirect network effects	Yes	R > 1 α > 0	R > 1 α = 0
	No	R = 1 α > 0	R = 1 α = 0

Figure 1: Types of network effects depending on parameter values.

them.[5] To make the problem nontrivial, we assume that $p < X$. This guarantees that p is sufficiently low for users to find it desirable to consume at least one application. Therefore, user k's net utility from consuming $\mathbf{x}^k$ when price is p is given by

$$U(\mathbf{x}^k; \{\mathbf{x}^l\}_{l \neq k}) = u(\mathbf{x}^k; \{\mathbf{x}^l\}_{l \neq k}) - p \cdot \sum_{a \in \mathcal{A}} \mathbf{1}(x_a^k), \tag{1}$$

where $\mathbf{1}(\cdot)$ is an indicator function taking value 1 when its argument is different from zero.

Since the focus of our analysis is on the value of limiting choice, we also assume that absent action by the platform to constrain the set of available applicaitons, the cardinality of $\mathcal{A}$ is large. Specifically, we assume that $A \geq \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$. We will show that this guarantees that there are sufficiently many different applications available for users to satisfy their preference for variety.

We consider the following two-stage game: In the first stage, all users decide simultaneously which games to purchase at price p. In the second stage, users decide simultaneously how to allocate their time budget X across the applications they have purchased. We solve for the subgame-perfect Nash equilibria in pure strategies and follow Katz and Shapiro (1985) in assuming that expectations are fulfilled in equilibrium.

Specifically, given that user k has already purchased set of applications $\mathcal{Q}^k$, in the second stage she chooses consumption $\mathbf{x}^k$ to maximize her own consumption utility u given the expected consumption of all other $N-1$ users, $\mathbf{x}^l$ for $l \neq k$:

$$\max_{x_a^k,\, a \in \mathcal{Q}^k} u(\mathbf{x}^k; \{\mathbf{x}^l\}_{l \neq k}) \quad \text{subject to} \quad X \geq \sum_{a \in \mathcal{Q}^k} x_a^k. \tag{2}$$

[5] We present the results for $p = 0$ in an appendix.

In the first stage, users choose the set of applications to purchase, $\mathcal{Q}^k \subseteq \mathcal{A}$, anticipating their own consumption and that of all other users in the second stage. User k's objective is to maximize her own net utility U.

We end the description of the model by presenting two definitions that are helpful for the discussion of equilibria.

Definition 1 (balanced strategy) *Let $\mathcal{Q}^k$ be the set of applications consumed by user k. And let Q^k be the cardinality of $\mathcal{Q}^k$. We say that user k's strategy is balanced if $x_a^k = \frac{X}{Q^k}$ for $a \in \mathcal{Q}^k$, and $x_a^k = 0$ for $a \notin \mathcal{Q}^k$.*

Thus, a balanced strategy is one where the user allocates her time budget equally across all the applications she consumes. Note that balanced strategies are pure strategies and that for any $\mathcal{Q}^k$ there is a unique balanced strategy.

Definition 2 (balanced equilibrium) *An equilibrium is balanced if all users play balanced strategies.*

In this section, we solve the game under the assumption that users have a perfect foresight about other users' choices in equilibrium. This is a classic assumption of rational beliefs, a part of Nash equilibrium. Later, in Section 3 we relax the perfect-foresight assumption.

In the remainder of this section, we investigate separately each type of network effects before considering the interplay of both types together. We first study the model with direct network effects and find that users consume one single application so as to take full advantage of consumption complementarities (Section 2.1). Then, we move on to studying the model with pure indirect network effects and find that users choose to consume a large number of applications driven by their preference for variety (Section 2.2). Next, we study the interplay between the two types of network effects and find that there is a tradeoff between harnessing consumption complementarities and the utility gains from product variety. In the Pareto-optimal equilibrium, users always consume a smaller number of applications than under pure indirect network effects (Section 2.3). Finally, we show that the platform can create value by limiting the number of applications available even if though users have perfect foresight about each others' purchase and consumption decisions (Section 2.4).

2.1 Direct network effects

There are pure direct network effects when users derive utility from consuming the same applications as other users but not from product variety. Therefore, consumption utility

u exhibits pure direct network effects when $R = 1$ and $\alpha > 0$. In this case, user k's net utility (1) takes the form

$$U_D(\mathbf{x}^k; \{\mathbf{x}^l\}_{l \neq k}) = \sum_{a \in \mathcal{A}} x_a^k + \alpha \sum_{a \in \mathcal{A}} \big(x_a^k \sum_{l \neq k} x_a^l \big) - p \cdot \sum_{a \in \mathcal{A}} \mathbf{1}(x_a^k).$$

User k's consumption of application a in an equilibrium is denoted by $\hat{x}_a^k$. Let $\mathcal{Q}_D^k \subseteq \mathcal{A}$ be a set of applications that user k consumes in equilibrium in an environment with pure direct network effects. Then, the cardinality of $\mathcal{Q}_D^k$ is $Q_D^k = \sum_{a \in \mathcal{A}} \mathbf{1}(\hat{x}_a^k)$. The following proposition characterizes the equilibria in this case.

Proposition 1 *When $R = 1$ and $\alpha > 0$, in every equilibrium $\mathcal{Q}_D^k = \mathcal{Q}_D$ for all k and the number of applications consumed is $Q_D^k = Q_D = 1$ for all k. There are A equilibria. All equilibria are balanced and Pareto optimal.*

Proof. See Appendix A, page 30.

Because $R = 1$, users derive no utility from product variety. However, because $\alpha > 0$ they derive utility from other users consuming the same applications for longer periods of time. Indeed, user k's marginal utility of consuming application a is increasing in other users' aggregate consumption of a,

$$\frac{\partial\, u_D(\mathbf{x}^k; \{\mathbf{x}^l\}_{l \neq k})}{\partial x_a^k} = 1 + \alpha \cdot \sum_{l \neq k} x_a^l \, .$$

Therefore, the more other users consume application a, the more user k desires to consume a. Since the same applies to all users, in equilibrium all users consume the same application. Users could coordinate on any one of the A applications available, since all users and all applications are homogeneous.

2.2 Indirect network effects

There are pure indirect network effects when users derive utility from product variety but not from consuming the same applications as other users. Therefore, consumption utility u exhibits pure indirect network effects when $1 < R < 2$ and $\alpha = 0$. In such a case, user k's net utility (1) takes the form

$$U_I(\mathbf{x}^k; \{\mathbf{x}^l\}_{l\neq k}) = \left(\sum_{a\in\mathcal{A}} \left(x_a^k\right)^{1/R}\right)^R - p \cdot \sum_{a\in\mathcal{A}} \mathbf{1}(x_a^k). \tag{3}$$

Note that (3) is essentially the same as the setup in Dixit and Stiglitz (1977), with two exceptions. First, the cost of time spent using application a is set in our model to 1 for all $a \in \mathcal{A}$. Second, we impose a price $p > 0$ that users must pay to use an application.[6] Proposition 2 characterizes the equilibria in this case.

Proposition 2 *Assume* $1 < R < 2$ *and* $\alpha = 0$. *Let* $q_I = \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$. *In every equilibrium the number of applications consumed is* $Q_I^k = Q_I$ *for each user* k, *where* $Q_I = \max\{1, q_I\}$. *All equilibria are balanced and Pareto optimal.*[7]

Proof. See Appendix A, page 31.

To understand this result, notice that Dixit and Stiglitz (1977) implies that when $\alpha = 0$ and $p \to 0$, the solution to optimization problem (2) is $Q_I \to \infty$ and $\hat{x}_a^k = \frac{X}{Q_I} \to 0$. Users derive utility from product variety and find it optimal to consume as many applicaitons as possible in equal proportions. The result is driven by the fact that, as long as $R > 1$, applications have infinite marginal consumption utility around zero:

$$\lim_{x_a^k \to 0} \frac{\partial\, u_I(\mathbf{x}^k; \{\mathbf{x}^l\}_{l\neq k})}{\partial x_a^k} = \infty$$

and that this marginal utility decreases as consumption increases. Therefore, spreading the time budget evenly across $Q + 1$ applications yields more utility than spreading the same time budget across Q applications.

To determine how many applications to purchase, users must compare the additional benefit from consuming an additional application and the price p that they must pay for that application. Specifically, if Q applications are consumed by a user in optimal consumption

[6]More precisely, our cost of time (which we normalize to 1) corresponds to the application prices in the original Dixit-Stiglitz's formulation. In contrast to Dixit-Stiglitz, we assume that users must pay a fixed price for access to each application she consumes, $p > 0$. This price is independent of the usage. For example, when users buy a particular videogame title, they pay for it once regardless of the usage, and then they allocate scarce time to playing the game. In our model, the price of the game is p and the opportunity cost of time allocated to playing the game is 1.

[7]There are $\frac{N\cdot A!}{Q_I!(A-Q_I)!}$ pure-strategy subgame-perfect Nash equilibria and continuum mixed strategy equilibria.

schedule, her utility is $\left(Q\left(\frac{X}{Q}\right)^{\frac{1}{R}}\right)^R - p\,Q = Q^{R-1}X - p\,Q$. Therefore, the marginal benefit from increasing Q is $(R-1)Q^{R-2}X$. The marginal cost of an additional application is p. Equating marginal benefit and marginal cost, we find that the optimal number of applications consumed under pure indirect network effects is $q_I = \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$. With $p < X$, it is always worth for a user to consume at least one application. We assume that the user cannot consume less than one application. Thus, if $q_I < 1$, the user consumes one application. Therefore, the optimal consumption is characterized by $Q_I = \max\{1, q_I\}$.

Let $\mathcal{Q}_I^k \subseteq \mathcal{A}$ be the set of applications that user k consumes in equilibrium in an environment with pure indirect network effects. Proposition 2 states that all users consume the same number of applications in equilibrium, i.e., $Q_I^k = Q_I$ for all k. However, it does not need to be that users consume the same applications, i.e. we may have $\mathcal{Q}_I^k \neq \mathcal{Q}_I^l$ for k and $l \neq k$. This is because users gain no utility from consuming the same applications as others. Thus, any N subsets of $\mathcal{A}$ with cardinality Q_I constitutes an equilibrium.

2.3 Interplay between direct and indirect network effects

Now we investigate what happens when users in the platform experience both direct and indirect network effects, so that they derive utility from product variety and from consuming the same applications as other users. In such a case, $1 < R < 2$ and $\alpha > 0$. Let $\mathcal{Q}_{DI}^k \subseteq \mathcal{A}$ be a set of applications that user k consumes in equilibrium in an environment with direct and indirect network effects, and let Q_{DI}^k be the cardinality of $\mathcal{Q}_{DI}^k$. Note that in the cases of pure direct and of pure indirect network effects, in all equilibria users consumed exactly the same number of applications, i.e., $Q_D^k = 1$ and $Q_I^k = \max\{1, q_I\}$. However, when both direct network effects are present, multiple values of Q_{DI}^k are possible.

The study of this hybrid specification is substantially more complex than the cases of pure direct and pure indirect network effects. Therefore, we present the analysis in parts. We begin by introducing two helpful lemmas.

Lemma 1 *Assume that $1 < R < 2$ and $\alpha > 0$. In every balanced equilibrium $\mathcal{Q}_{DI}^k = \mathcal{Q}_{DI}$ for all k.*

Proof. See Appendix A, page 33.

Lemma 2 *Assume that $1 < R < 2$ and $\alpha > 0$. If Q_{DI} is the cardinality of the consumption set in a balanced equilibrium, then any set of applications $\mathcal{Q}_{DI} \subseteq \mathcal{A}$ of cardinality Q_{DI} constitutes a balanced equilibrium.*

Proof. See Appendix A, page 35.

The lemmas imply that in the case of $1 < R < 2$ and $\alpha > 0$ we may completely characterize balanced equilibria by simply stating equilibrium cardinalities Q_{DI}. Lemma 1 says that in every balanced equilibrium all users consume the same applications. Lemma 2 says that if Q_{DI} is the number of applications consumed in a particular balanced equilibrium, then there are $C^{A}_{Q_{DI}}$ equilibria with the same number of applications consumed. For example, if $A = \{1, 2, 3, 4\}$, $Q_{DI} = 2$ characterizes six balanced equilibria: $\mathcal{Q}_{DI_1} = \{1, 2\}$; $\mathcal{Q}_{DI_2} = \{1, 3\}$; $\mathcal{Q}_{DI_3} = \{1, 4\}$; $\mathcal{Q}_{DI_4} = \{2, 3\}$; $\mathcal{Q}_{DI_5} = \{2, 4\}$; and $\mathcal{Q}_{DI_6} = \{3, 4\}$. It is easy to see that users derive the same utility in all of these equilibria and, thus, we think of them as equivalent. For clarity of exposition, we refer to balanced equilibria by just indicating their cardinality.

To study balanced equilibria, it is helpful to define the following function:

$$V(Q) = Q^{R-1}X + \alpha\frac{X^2}{Q}(N-1) - p\,Q. \tag{4}$$

Suppose that all users play balanced strategies and consume the same set of applications of cardinality Q. Then, each user's net utility (1) is given by $V(Q)$. Lemma 1 implies that the net utility in every balanced equilibrium must be on $V(Q)$. Figure 2 illustrates the shape of V for different values of α.

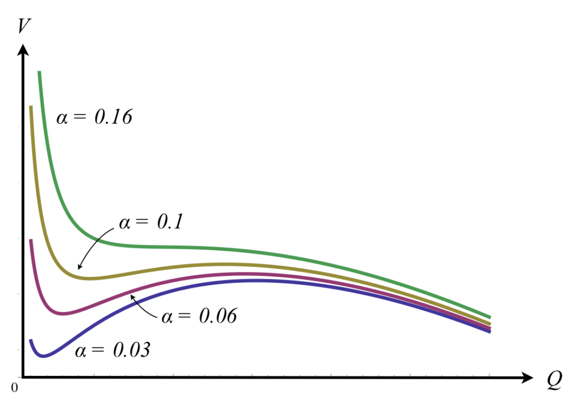


Figure 2: Shape of V for different values of α. ($R = 1.7135$, $A = 30$, $X = 2$, $N = 16$, $p = 0.646$.)

The shape of V is driven by the tradeoff between consumption complementarity and preference for variety. As shown by Proposition 1 consumption complementarity and the resulting direct network effects induce users to consume one application only. Proposition 2, however, shows that preference for variety and the resulting indirect network effects induce users to consume more applications. The figure shows that when users have strong preference for variety as compared to consumption complementarity (low α), indirect network effects outweigh direct network effects. When preference for variety is weak, however, direct network effects outweigh indirect network effects.

Let

$$\widehat{Q} = \max\left\{1,\ Q \text{ such that } \frac{\mathrm{d}\,V}{\mathrm{d}\,Q} = 0\right\}.$$

If V has interior maxima, then $\widehat{Q}$ is the unique interior maximum. Otherwise, V reaches its maximum at $\widehat{Q} = 1$. As we can see in Figure 2, when α is large, $\widehat{Q} = 1$ (cf. $\alpha = 0.16$ in the figure). Otherwise, $\widehat{Q} > 1$ (other values of α in the figure). The value $\widehat{Q}$ is important for the shape of V. Specifically, for $Q > \widehat{Q}$, V is always decreasing. However, for $\widehat{Q} > 1$, when $Q < \widehat{Q}$, V first decreases and then increases. It is possible for some Qs less than $\widehat{Q}$ that $V(Q) > V(\widehat{Q})$. Let $Q_\star$ be $Q < \widehat{Q}$ such that $V(Q_\star) = V(\widehat{Q})$, when $\widehat{Q} > 1$.

The following remark states that $\widehat{Q}$ is lower than Q_I, the equilibrium number of applications consumed when there are no direct network effects (as defined in Proposition 2).

Remark 1 *When $Q_I \geq 1$, then $\widehat{Q} < Q_I$.*

Proof. See Appendix A, page 35.

Intuitively, the presence of direct network effects prompts users to allocate their limited time budget to fewer applications. The fact that other users consume the same applications compensates for the loss of application variety. The condition $Q_I \geq 1$ guarantees that the comparison between $\widehat{Q}$ and Q_I is nontrivial.[8]

The values $\widehat{Q}$, Q_I and $Q_\star$ are very important for characterization of balanced equilibria when both direct and indirect network effects are present. Following two proposition show that there is a large set of Qs that cannot characterize balanced equilibria. The results are helpful because they significantly constrain the set of Qs that may characterize equilibria.

Proposition 3 *Assume that $1 < R < 2$ and $\alpha > 0$. If $\widehat{Q} > 1$, then $\widehat{Q}$ is not a balanced equilibrium.*

Proof. See Appendix A, page 36.

[8]For $Q_I = 1$, $\widehat{Q} = Q_I = 1$.

The proposition states that when $\widehat{Q}$ in interior, it cannot be a balanced equilibrium. This is because there is a profitable upward deviation, i.e., users have incentive to consume more applications. By the definition of $\widehat{Q}$, if $\widehat{Q} > 1$, it must be that $\frac{\partial V(Q)}{\partial Q}\big|_{Q=\widehat{Q}} = 0$. Therefore, an incremental balanced deviation upwards to $\widehat{Q} + \varepsilon$ has no effect on the utility of the deviator. However, the optimal unilateral deviation upward is *not* balanced.[9] The optimal upward deviation is strictly more beneficial to the user than the balanced deviation. Therefore, an optimal upward deviation is strictly profitable.

Proposition 4 *Assume that* $1 < R < 2$ *and* $\alpha > 0$*. Then for any* Q *such that* $\max\{1, Q_\star\} \leq Q < \widehat{Q}$ *or* $Q > Q_I$*,* Q *cannot characterize a balanced equilibrium.*

Proof. See Appendix A, page 37.

Figure 3 illustrates Proposition 4.

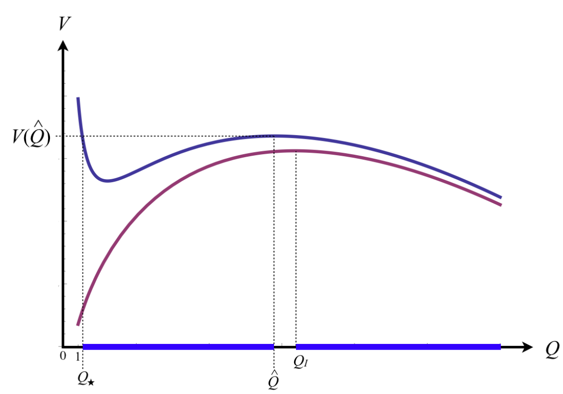


Figure 3: Intervals of Q that cannot be an equilibrium as described in Proposition 4.

To understand this result, consider first $Q > Q_I$. Given that all other users consume Q applications, any user has incentive to deviate downwards to Q_I. The utility for user k from deviating to $Q^k < Q$ is

$$U_{DI}(Q^k \leq Q) = \left(Q^k\right)^{R-1} X + \underbrace{\cancel{Q^k} \alpha \frac{X}{\cancel{Q^k}} (N-1) \frac{X}{Q}}_{\text{consumption complementarity}} - p\, Q^k \,.$$

[9]In the case of deviation upward, the deviator consumes some applications that no other user consumes. Due to consumption complementarity, an optimal consumption schedule then calls for more consumption of those applications that other users consume, and less (but positive) consumption of applications that only the deviator consumes.

Note that the consumption complementarity term is independent of Q^k. Since she consumes $Q^k < Q$, the deviator consumes only applications also consumed by the other users.[10] Each of those applications is consumed by all other users at the level of $(N-1)\frac{X}{Q}$. The deviator divides her time budget X amongst the Q^k applications that she consumes, $Q^k \cdot \frac{X}{Q^k}$. Therefore, the benefit of the direct network effect is constant, no matter what $Q^k < Q$ the deviator chooses. However, the net benefit of variety $(Q^k)^{R-1}X - p\,Q^k$ is maximized at Q_I which is lower than Q. As a consequence, the deviator would want to deviate to Q_I. We conclude that $Q > Q_I$ may not be an equilibrium. Intuitively, consuming more than Q_I applications leads to "too much" application variety for the price. Moreover, if it had an effect, consumption complementarity would push users to consume fewer applications also.

Now we argue that for $Q \in [\max\{1, Q_\star\}, \widehat{Q})$ there is a profitable deviation upwards. In what follows we impose that the deviator balances his time budget across all the applications that she consumes. Even though this is not the optimal deviation, we show that it is a profitable deviation (and therefore, the optimal deviation is also profitable). Given that all other users consume Q applications in a balanced way, the utility of the deviator from a balanced consumption of Q^k applications is:

$$U_{DI}(Q^k \geq Q) = \left(Q^k\right)^{R-1} X + \underbrace{\cancel{Q}\,\alpha\,\frac{X}{Q^k}(N-1)\frac{X}{\cancel{Q}}}_{\text{consumption complementarity}} - p\,Q^k\,. \tag{5}$$

Note that $U_{DI}(Q^k \geq Q)$ is the same function of Q^k as V in equation (4) which has a local maximum at $\widehat{Q} > Q$. Moreover, for all $Q \in [\max\{1, Q_\star\}, \widehat{Q})$, $U_{DI}(\widehat{Q} > Q) > U_{DI}(Q)$. Thus, for all those values of Q, there is a profitable upwards deviation. We conclude that $Q \in [\max\{1, Q_\star\}, \widehat{Q})$ may not be an equilibrium.

Intuitively, consuming more applications satisfies the deviator's preference for variety to a greater extent. However, consuming less of each application consumed by other users means that the utility from consumption complementarity is lower. When $Q \in [Q_\star, \widehat{Q}]$ the tradeoff is resolved in favor of consuming more applications.

Note that for $Q \in [1, Q_\star]$ and $Q \in [\widehat{Q}, Q_I]$ the same trade off is at play. However, it is possible that the trade off is resolved in favor of consumption complementarity which means that it is not worth for users to deviate upwards. In combination with Lemma 3

[10] Consider user k and suppose that all other users play balanced strategies consuming the same set of applications $\mathcal{Q}$. Directly, we can see that if user k decides to also consume Q applications, she consumes exactly the applications in $\mathcal{Q}$ and not other. Moreover, it is optimal for her to consume them in equal amounts, i.e., she consumes them according to a balanced strategy. However, user k may also consider deviations that involve consuming a different number of applications.

this implies that equilibria are possible in these intervals. Propositions 4 and 5 show that multiple such equilibria exist. We show that there are two aspects to this multiplicity. First, as described in Lemma 2, for any given cardinality Q_{DI} there may exist multiple sets $\mathcal{Q}_{DI}$ each constituting a separate equilibrium. Second, there may exist many different values of Q_{DI} that characterize equilibria. The former type of multiplicity is of no consequence to user utility while the latter has important utility implications. Thus we focus on the second type of multiplicity in our analysis.

The following lemma assures that so long as $Q \leq Q_I$, it is never beneficial for user k to deviate to a strategy with a lower number of applications. Thus, in searching for balanced equilibria, we need to focus only on deviations to a larger number of applications.

Lemma 3 *Assume that $1 < R < 2$ and $\alpha > 0$. If all users play balanced strategy $\mathcal{Q}$ with cardinality $Q \leq Q_I$, then any unilateral deviation by user k to any other strategy with $Q^k < Q$ leads to lower utility for player k.*

Proof. See Appendix A, page 38.

To understand this result, suppose that all users are consuming $Q \leq Q_I$ and consider a deviation to $Q^k < Q$. At Q, users have no incentive to deviate downward. The utility for the deviator is

$$U_{DI}(Q^k \leq Q) = \left(Q^k\right)^{R-1} X + \underbrace{\alpha \cancel{Q^k} \frac{X}{\cancel{Q^k}}(N-1)\frac{X}{Q}}_{\text{consumption complementarity}} - p\,Q^k\,.$$

Note that $U_{DI}(Q^k \leq Q)$ is increasing for all $Q^k \leq Q$ and therefore it is maximized at $Q^k = Q$. Thus there is no incentive to deviate downward.

Intuitively, consuming fewer applications satisfies user k preference for variety to a lesser extent. At the same time, there is no benefit from consumption complementarity. The reason is that each of the applications used by the deviator are consumed by all other users at the level of $(N-1)\frac{X}{Q}$. The deviator divides her time budget X amongst the Q^k applications that she consumes $Q^k \cdot \frac{X}{Q^k} = X$. Therefore, the benefit of the direct network effect is constant, no matter what Q^k the deviator chooses.

Proposition 5 states that there always exists a balanced equilibrium where all users consume Q_I applications and that Qs close but lower than Q_I also characterize equilibria.

Together with Proposition 4, Proposition 5 indicates that Q_I is the equilibrium with the largest number of applications consumed.

Proposition 5 *When $1 < R < 2$ and $\alpha > 0$, there always exist balanced equilibria with $Q_{DI} = Q_I$, where Q_I is defined in Proposition 2. Furthermore, if $Q_I > 1$ there is $Q^o < Q_I$ such that any $Q \in [Q^o, Q_I]$ characterizes balanced equilibria, i.e., $Q = Q_{DI}$.*

Proof. See Appendix A, page 39.

Figure 4a illustrates this result. The proposition states that so long as users exhibit preference for variety, no matter how small, there are balanced equilibria with the same number of applications, Q_I, that users would choose to consume if there were no direct network effects.

To understand why Q_I is an equilibrium, by Lemma 3 we need only consider deviations upward. By the same argument to that following equation (5), a deviation upward cannot improve the utility from consumption complementarity. Moreover, Q_I maximizes utility from preference for variety. Therefore, there are no incentives to deviate and Q_I is an equilibrium.

A deviation upwards always decreases utility from consumption complementarity. Notwithstanding, for $Q < Q_I$ there is some benefit from increased variety. For Q less than but close to Q_I, however, this benefit is infinitesimally small (the FOC is satisfied at Q_I) and it is outweighed by the utility loss from consumption complementarity. Therefore, Q less than but close to Q_I also characterize equilibria.

Proposition 6 shows that there may also exist other equilibria.

Proposition 6 *Assume that $1 < R < 2$ and $\alpha > 0$. There exist parameter values such that $Q_{DI} = 1$ and $Q_I > 1$.*

Proof. Suppose that $Q_I > 1$, which implies that $q_I > 1$ or $(R-1)X > p$. When all users consume one application only, their consumption utility is:

$$u(Q{=}1) = X + \alpha X^2(N-1).$$

Now, if a user deviates to consume y of second application, her consumption utility is:

$$u(Q{=}2) = \left((X-y)^{\frac{1}{R}} + y^{\frac{1}{R}}\right)^R + \alpha X(N-1)(X-y).$$

The optimal level of deviation y^* is characterized by the first order condition:

$$\frac{\partial u(Q=2)}{\partial y} = \left((X-y^*)^{\frac{1}{R}} + {y^*}^{\frac{1}{R}}\right)^{R-1} \left(\left(\frac{1}{y^*}\right)^{1-\frac{1}{R}} - \left(\frac{1}{X-y^*}\right)^{1-\frac{1}{R}}\right) - \alpha X(N-1) = 0.$$

Notice that y^* decreases with N and $y^* \to 0$ as $N \to \infty$. Therefore, as N increases, y^* decreases but Q_I is not affected.

To find out if the value of the optimal deviation is larger than the price of the second application, we compute:

$$\begin{aligned} &u(Q=2|\, y=y^*) - u(Q=1) = \\ &\quad = \left((X-y^*)^{\frac{1}{R}} + {y^*}^{\frac{1}{R}}\right)^{R} + \alpha X(N-1)(X-y^*) - (X + \alpha X^2(N-1)) < \\ &\quad\quad < \left((X-y^*)^{\frac{1}{R}} + {y^*}^{\frac{1}{R}}\right)^{R} - X. \end{aligned}$$

Note that $((X-y^*)^{\frac{1}{R}} + {y^*}^{\frac{1}{R}})^R - X$ is continuous, takes value zero at $y^* = 0$ and it is strictly increasing in y^*. Therefore for any price p, we can find N large enough so that y^* is low enough so that

$$u(Q=2|\, y=y^*) - u(Q=1) < p,$$

and the deviation is not profitable.

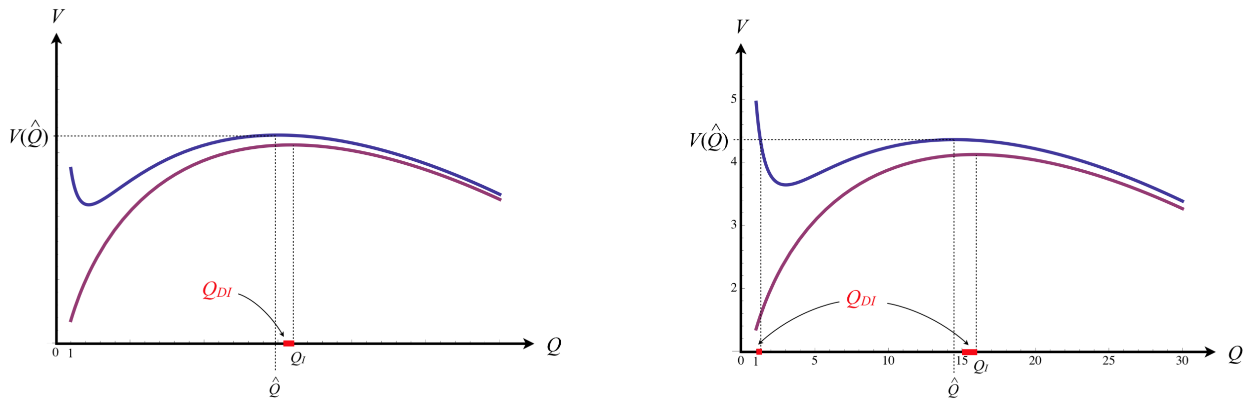
This completes the proof of Proposition 6. ■

Notice that it is necessary that $V(Q=1) > V(\widehat{Q})$ for $Q=1$ to be an equilibrium. It follows from Proposition 4.

By similar arguments as in the proof pf Proposition 6, we can show that an y environment around $Q=1$ may also be an equilibrium.

The result of Proposiiton 6 is illustrated in Figure 4b. There we can see that equilibria exist in two disconnected intervals: one interval around $Q=1$ (recall that by Proposition 1 $Q_D = 1$ is the equilibrium under pure direct network effects) and the other one around Q_I. In the interval around $Q_D = 1$, the strong consumption complementarities (users consume the same few applications intensely) guarantee that users do not want to deviate to consume more applications. In the interval around Q_I, the weak consumption complementarities (users consume little of many applications) guarantee that users do not want to deviate to consume fewer applications.

We have seen that there are multiple equilibria. The final result in this subsection Pareto



(a) (b)

Figure 4: Intervals of Q_{DI}.

ranks the equilibria.

Proposition 7 *When there exist multiple balanced equilibria with different values of Q_{DI}, equilibria with a smaller Q_{DI} Pareto dominate equilibria with larger Q_{DI}.*

Proof. See Appendix A, page 41.

Function $V(Q)$ is user utility in a situation where every user consumes Q applications in a balanced way. Therefore, for Q_{DI} that constitute a balanced equilibrium, $V(Q_{DI})$ gives the utility that users obtain in equilibrium. As follows from Proposition 4 and illustrated by Figure 3, whenever $V(Q)$ is increasing, such Qs cannot be equilibria. Therefore, equilibrium utility must be decreasing in Q_{DI}.

2.4 On the role of the platform: Creating value by limiting choice

We conclude Section 2 by showing that users may benefit when platform limits the number of applications available; but only when both direct and indirect network effects are present. To examine the platform's choice of the number of applications available, A, we relax the assumption that $A \geq \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$.

First, notice that when pure direct network effects are present, the users achieve the same net utility in an equilibrium, for any $A \geq 1$. The platform cannot improve on this.

Corollary 1 *Assume that $R = 1$ and $\alpha > 0$. Then the platform cannot change the net utility that users achieve in an equilibrium, for any $A \geq 1$.*

From Section 2.2 we know that when $A \geq q_I$, under pure indirect network effects, in an equilibrium, every user consumes Q_I applications. As Corollary 2 states, when the platform sets $1 < A < q_I$, the users consume all available applications. But they yield lower utility than from consuming Q_I applications.

Corollary 2 *Assume that $1 < R < 2$ and $\alpha = 0$. If $1 < A < q_I$, then there exists a unique equilibrium. This is a balanced equilibrium where all users consume all A applications. The net utility of users in equilibrium strictly increases with A for $A < q_I$.*

Therefore, the platform can only decrease users' utility when limiting the number of available applications. When $A < q_I$ users strictly gain from access to a larger number of applications. And when $A \geq q_I$, the users do not gain or lose by having more applications available.

Now we turn to the case where both direct and indirect network effects are present. The following definition is helpful for the arguments in this case. Let

$$Q^{**} = \arg\max V(Q). \tag{6}$$

From the shape of V follows that Q^{**} may be either 1 or $\widehat{Q}$. In both cases $Q^{**} \leq \widehat{Q} \leq Q_I$ (the latter equality holds only when $Q_I = 1$). When $Q^{**} = \widehat{Q} > 1$, then by Proposition 3, Q^{**} never characterizes a balanced equilibrium. When $Q^{**} = 1$ (it may, but does not have to be when $\widehat{Q} = 1$), it may characterize a balanced equilibrium (as Proposition 6 shows), but it not always does.[11]

There are two ways in which users may benefit from the platform limiting choice. First, when Q^{**} is not an equilibrium, by limiting choice the platform is helping users solve a "commons" problem. Second, even if Q^{**} is an equilibrium, it is one of many equilibria and other equilibria are Pareto inferior to Q^{**}. In this case, the platform may help users solve an "equilibrium selection" problem by limiting choice.

The following proposition shows that regardless of whether Q^{**} is in the equilibrium set of the original game, the platform can ensure that Q^{**} becomes the only equilibrium of the game by restricting A to Q^{**}.

Proposition 8 *Assume that $1 < R < 2$ and $\alpha > 0$. If the platform sets $A = Q^{**}$, then there exists a unique balanced equilibrium where all users consume Q^{**} applications.*

[11] Notice that whether $Q^{**} = 1$ or $Q^{**} = \widehat{Q} > 1$ depends on the value of α. For small α (as $\alpha = 0.03$ in Figure 2) $Q^{**} = \widehat{Q} > 1$. For larger, $\widehat{Q} > 1$, but $Q^{**} = 1$ (cf. $\alpha = 0.06$ and $\alpha = 0.1$) in the figure). For even larger α, $Q^{**} = \widehat{Q} = 1$ (as for $\alpha = 0.16$ in the figure).

Proof. See Appendix A, page 42.

The proposition implies that the equilibrium set may change with changes in A. In particular, when the platform sets $A = Q^{**}$, Q^{**} becomes the unique equilibrium. Therefore, when Q^{**} is in the original equilibrium set, if the platform constrains A to be equal to Q^{**}, it eliminates all the Pareto-inferior equilibria and eliminates the possibility that users select inferior equilibria. Thus,

Corollary 3 *Assume that $1 < R < 2$ and $\alpha > 0$. When Q^{**} is in the equilibrium set, users may benefit when platform restricts the number of available applications to Q^{**}.*

On the other hand, if Q^{**} is not in the original equilibrium set, when the platform constrains A to be equal to Q^{**}, it creates a new equilibrium that Pareto dominates all the original equilibria. Thus,

Corollary 4 *Assume that $1 < R < 2$ and $\alpha > 0$. When Q^{**} is not in the equilibrium set, users strictly benefit when platform restricts the number of available applications to Q^{**}.*

Intuitively, when Q^{**} is not an equilibrium, users face a commons problem. They would all be better off if they limited their consumption to Q^{**}. However, they all have an incentive to deviate towards higher consumption which ends up reducing all users utility.

We note that in the environment with perfect foresight, the platform cannot create value by restricting choice when there are only direct or only indirect network effects. With pure direct network effects there are multiple equilibria but they are all Pareto optimal. In the case of pure indirect network effects, a user's optimal choice does not depend on other users' choices. Therefore, users do not face neither a commons problem nor an equilibrium selection problem when network effects are of one type only.

3 Game with no foresight

Whenever direct network effects are present, the equilibria we have studied in Section 2 require that users know exactly which applications are consumed by all other users. That is, we assumed that when making their choices, users had perfect foresight about other users' choices. In many environments, such perfect foresight may be difficult to achieve.

In this section we analyze a similar but distinct game to that of the previous section. The difference is that we change the assumption of perfect foresight and study equilibria where users have no way of knowing which applications other users will consume when they make their own choices.

Perfect foresight assumes that in equilibrium player k knows the cardinality and the identity of the applications that all other users with consume. Moreover, she also knows the how much of each application other users consume. As an alternative, now we assume *no foresight* by which we mean that users initially assign equal probability to any feasible strategy of other users. However, they refine their beliefs by Bayesian updating and eventually reach equilibrium beliefs.

In the game with no foresight, users face a coordination problem. Since users do not know which applications are consumed by other users, some of the benefit to the direct network effects is lost. The utility that users can achieve in this environment is lower than in the environment with perfect foresight, because users cannot exploit consumption complementarities as well due to lack of coordination.

This coordination problem arises only when the direct network effects are present ($\alpha > 0$). Section 3.1 shows this effect for pure direct network effects and Section 3.3 shows that the effect is also present when there are both types of network effects. In those cases, the platform can create value by limiting the number of available applications. By providing fewer applications, the platform alleviates the coordination problem faced by users. Without direct network effects, there are no consumption complementarities and thus coordination problems do not lead to loss in utility as shown in Section 3.2.

We now describe the game with no foresight. The only difference with the game in Section 2 is what we assume about users' beliefs. Recall that $\mathbf{x}^k = \{x_1^k, x_2^k, \ldots, x_A^k\}$ such that $\sum_{a\in\mathcal{A}} x_a^k = X$ denotes a feasible consumption vector. We use $\mathbf{x}^k$ to also denote a pure strategy. Because all users are identical, they all have access to the same set of pure strategies. Let $\mathcal{X}$ denote the set of all pure strategies for any given user. Let $\phi_l^k \sim U[\mathcal{X}]$ denote user k's beliefs on user l's choice of pure strategy. Let $\boldsymbol{\phi}^k = \{\phi_l^k\}_{l\neq k}$ be a vector that denotes user k's beliefs on all other users' choices of pure strategy.

With this, user k's utility from consuming vector $\mathbf{x}^k$ is:

$$\mathbb{E}_{\boldsymbol{\phi}^k} u(\mathbf{x}^k) = \left(\sum_{a\in\mathcal{A}} \left(x_a^k\right)^{1/R}\right)^R + \alpha \sum_{a\in\mathcal{A}} \left(x_a^k \, \mathbb{E}_{\boldsymbol{\phi}^k} \sum_{l\neq k} x_a^l\right),$$

and the optimization problem (2) becomes

$$\max_{x_a^k,\, a\in\mathcal{A}} \left\{\mathbb{E}_{\boldsymbol{\phi}^k} u(\mathbf{x}^k) - p \cdot \sum_{a\in\mathcal{A}} \mathbf{1}(x_a^k)\right\} \quad \text{subject to} \quad X \geq \sum_{a\in\mathcal{A}} x_a^k. \tag{7}$$

As Lemma 4 shows, under no foresight the expectation over consumption of any application a by any other user $l \neq k$ significantly simplifies.

Lemma 4 *For every l and k and a,*

$$\mathbb{E}_{\phi_l^k} x_a^l = \frac{X}{A}.$$

Proof. See Appendix A, page 42.

Therefore,

$$\mathbb{E}_{\boldsymbol{\phi}^k} \sum_{l\neq k} x_a^l = \sum_{l\neq k} \mathbb{E}_{\phi_l^k} x_a^l = \sum_{l\neq k} \frac{X}{A} = (N-1)\frac{X}{A}. \tag{8}$$

Note that this expectation does not depend on how many applications or which applications all other users consume, therefore there is no interdependence between users' choices. Given this, we now can find the optimal choice by user k (which in our setting is independent of what all other users do). Whatever is the number of applications G^k that user k wishes to consume, her optimal consumption pattern is the following balanced consumption: divide the time budget equally among the applications consumed. Once user k decides that G^* is the optimal number of applications for her to consume, it does not matter which subset of $\mathcal{A}$ she chooses, as all yield the same utility. Therefore, G^* fully describes k's set of best responses.

Moreover, G^* identifies what are the dominated strategies for user k: any strategy with cardinality different from G^* and any strategy with cardinality equal to G^* but with non-balanced consumption pattern. User k knows that all other users are the same and she knows that all other users are rational. Therefore, user k cannot believe that other users will play dominated strategies. Because users are homogeneous, user k can infer dominated strategies of other users. When user k finds an optimal number of applications for her to consume, G^*, she knows that that number is the same for all other users and also that all users are going to consume the G^* applications in a balanced way.

User k updates her beliefs using Bayes' Rule. Therefore, she assigns zero probability to dominated strategies and equal probability to undominated strategies. Since with no foresight she does not know which applications they consume, she believes that every subset of $\mathcal{A}$ with cardinality G^* is equally likely to be consumed by user $l \neq k$. That is, the updating does not tell her which precise applications other users will consume.

Finally, user k recalculates her best response under the updated beliefs. This recalculated best response is exactly the same as the original best response. This is because under the new beliefs the expected consumption of any application a by agent $l \neq k$ is still $\frac{X}{A}$, as in Lemma 4. If every user behaves this way, beliefs are consistent with strategies and this constitutes a ***no-foresight equilibrium***.

3.1 Direct network effects

As in Section 2.1, to capture pure direct network effects, we set $R = 1$ and $\alpha > 0$. Recall that $\hat{x}_a^k$ denotes user k's equilibrium consumption of application a. Let G_D^k be the number of applications consumed by user k in a no-foresight equilibrium in an environment with pure direct network effects.

Proposition 9 *Assume $R = 1$ and $\alpha > 0$. In every no-foresight equilibrium, user k consumes one application, i.e., $G_D^k = 1$ for every k.*

Proof. See Appendix A, page 43.

The intuition is as follows. Suppose that user k consumes G^k applications under no-foresight. Using expression (8), we find that for $R = 1$, the expected consumption utility for every user k is:

$$\mathbb{E}\, u_D(G^k) = X + \alpha X(N-1)\frac{X}{A}.$$

Because that u_D does not depend on how many applications the user consumes, for $p > 0$ there is no point for any of the agents to buy more than one application.

Notice that the equilibrium net utility $\mathbb{E}\, U_D(G_D^k = 1) = \mathbb{E}\, u_D - p$ decreases with A. Clearly, the platform can increase utility by decreasing A. In fact, user utility can be raised to $X(1 + \alpha(N-1)X) - p$ by limiting the number of applications to $A = 1$. This is the maximum utility that users may achieve given those preferences.

3.2 Indirect network effects

As in Section 2.2, to capture pure indirect network effects, we set $1 < R < 2$ and $\alpha = 0$. Let G_I^k be the number of applications consumed by user k in a no-foresight equilibrium in an environment with pure indirect network effects.

Proposition 10 *Assume* $1 < R < 2$ *and* $\alpha = 0$. *In every no-foresight equilibrium, every user* k *consumes* $G_I^k = G_I = Q_I$ *applications in equal amount, where* Q_I *is defined in Proposition 2.*

Proof. See Appendix A, page 43.

Intuitively, when there are pure indirect network effects users do not gain anything from coordinating consumption. Therefore, a user's beliefs on other users' choices have no bearing on her utility. Thus the equilibrium behavior is the same as in the case of perfect foresight. As a consequence, there is no gain to users if the platform limits the number of applications available.

3.3 Interplay between direct and indirect network effects

Finally, we investigate what happens when users experience both direct and indirect network effects. In such a case, $1 < R < 2$ and $\alpha > 0$. Let G_{DI}^k be the number of applications consumed by user k in a no-foresight equilibrium in an environment with direct and indirect network effects.

Proposition 11 *Suppose* $1 < R < 2$ *and* $\alpha > 0$. *In every no-foresight equilibrium, every user* k *consumes* $G_{DI}^k = G_{DI} = Q_I$ *applications in equal amount, where* Q_I *is defined in Proposition 2.*

Proof. See Appendix A, page 44.

Note that these equilibria are the same as the ones described in Proposition 10. However, using equation (8), we find that in this case the expected equilibrium net utility can be expressed as

$$\mathbb{E}\, U_{DI}(G_{DI}) = G_{DI}^{R-1} X + \alpha\, X(N-1)\frac{X}{A} - p\, G_{DI}, \tag{9}$$

which depends on A. Therefore, unlike in the case of pure indirect network effects (Proposition 10), with both types of network effects equilibria are not efficient and the platform may improve efficiency by limiting A.

Equation (8) shows that $\mathbb{E}_{\phi^k} \sum_{l \neq k} x_a^l = (N-1)\frac{X}{A}$. This says that the expected consumption of any application by other users does not depend on the number of applications consumed. Therefore, the direct network effect does not influence the number of applications consumed in equilibrium, i.e., $G_{DI} = G_I = Q_I$. Still, the direct network effects affect the expected utility achieved in equilibrium (eq. 9).

Combining this result with Propositions 4 and 5, we see that the number of applications consumed in equilibria with perfect foresight is weakly lower than in the no-foresight equilibrium, i.e., $Q_{DI} \leq G_{DI}$. Moreover, as shown in Proposition 7, when $Q_{DI} = G_{DI}$ this is the worst of the equilibria with perfect foresight in terms of Pareto efficiency. Therefore, the platform can create value by limiting the number of applications available. We turn to this issue in the next subsection.

3.4 On the role of the platform: Creating value by limiting choice

In Section 2 we have shown that with perfect foresight and in the presence of direct and indirect network effects, users may benefit when the platform limits the set of applications available. Specifically, users always face an *equilibrium selection* problem and may also face a *commons problem*. Both of these issues can be resolved by the platform limiting the number of applications available.

We now show that under no foresight there is a different reason why users benefit from limited choice: resolving the *coordination problem* that users face when direct network effects are at play. We note that under no foresight, users do not face neither equilibrium selection nor commons problems.

Proposition 12 *Suppose that $\alpha > 0$, $1 \leq R < 2$. Under no foresight, the platform maximizes users' net utility by setting the number of available applications to $A = Q^{**}$, where Q^{**} is given by* (6)*.*

Proof. See Appendix A, page 44.

Note that the result applies whenever direct network effects are at play, regardless of the preference for variety. Intuitively, by reducing A the platform alleviates the coordination problem as it is more likely that users consume the same applications and gain utility from direct network effects.

So long as $A > Q^{**}$, the equilibrium is inefficient, specially when A is large. Only when $A = Q^{**}$ the efficient outcome is an equilibrium. The platform creates value by creating a new equilibrium.

4 Concluding remarks

We have shown that when users enjoy application variety but also benefit from consumption complementarities, three problems may arise: the socially optimal number of applications may not be part of an equilibrium; multiple equilibria ensue; and, users will likely find it hard to coordinate consumption. The analysis has demonstrated that by limiting the number of available applications (i.e., limiting choice) the platform can provide a fix to each one of these problems. Specifically, by limiting choice the platform may create new equilibria that do not exist when application choice is broad. In addition, it can eliminate socially inferior equilibria. Moreover, it can reduce the severity of the coordination problem faced by users.

The overall conclusion is that when direct and indirect network effects are at play, an important governance decision by platforms is the choice of how many applications should to be allowed to run on them. To implement such a choice, the platform may directly suppress access to developers and impose quantity constraints (as Nintendo did in the mid-1980s), or it may limit the number of applications indirectly through setting high prices for developers to access the platform.

While we have shown that the platform may create value by limiting choice, the recommendation to practitioners is not "provide as few applications as possible." Rather, it is that even in settings where users have a strong preference for variety, the platform should actively manage the number of applications that it offers as there is a number beyond which offering more applications will decrease users utility and, thus, overall platform value. This recommendation is in stark contrast to the conventional wisdom that platforms should encourage the development of complements to the maximal possible extent.

The obvious next step to gain further insight on the value that a platform may create by acting as a gatekeeper is endogenizing the price at which applications are sold and the number of applications that would be provided absent intervention by the platform. Likewise, it would be valuable to embed our model into a setting with competing platforms. Given the complexity of the analysis when users are the only strategic players, we expect these extensions to be challenging. We hope, however, to have provided a solid first step on which to build general theories of platform competition that will shed further light on the value that platforms create by limiting choice.

Appendix

A Proofs

Proof of Proposition 1 (page 10)

Proof. Let $R = 1$ and $\alpha > 0$. Suppose that users $l \neq k$ play pure strategies $\mathbf{x}^l$. The proof proceeds in following steps: First, we find the optimal consumption patterns, given that user k has access to some set $\mathcal{Q} \subseteq \mathcal{A}$ of applications, where the cardinality of $\mathcal{Q}$ is $Q \geq 1$. Second, given the consumption pattern, we find the optimal set of applications consumed, $\mathcal{Q}_D^k$. We characterize subgame-perfect Nash equilibria where all users decide which applications to consume and at which level.

Suppose that user k has access to a set $\mathcal{Q} \subseteq \mathcal{A}$ applications. Given $\mathcal{Q}$, user k's objective is to allocate the consumption in order to maximize her utility, i.e.,

$$\max_{x_a^k, a\in\mathcal{Q}} \left\{ \sum_{a\in\mathcal{Q}} x_a^k + \alpha \sum_{a\in\mathcal{Q}} \left(x_a^k \sum_{l\neq k} x_a^l \right) \right\} \qquad \text{s.t. } X \geq \sum x_a^k .$$

The Lagrangian of this maximization problem, including the constraint is

$$\mathcal{L} = \sum_{a\in\mathcal{Q}} x_a^k + \alpha \sum_{a\in\mathcal{Q}} \left(x_a^k \sum_{l\neq k} x_a^l \right) + \lambda (X - \sum x_a^k) .$$

The derivative of the Lagrangian with respect to x_a^k is

$$\frac{\partial \mathcal{L}}{\partial x_a^k} = 1 + \alpha \sum_{l\neq k} x_a^l - \lambda .$$

This derivative does not depend on x_a^k. Let a' be an application such that $\sum_{l\neq k} x_{a'}^l \geq \sum_{l\neq k} x_a^l$ for all $a \in \mathcal{Q}$. There may be one or more such applications. Those applications yield the largest derivative $\frac{\partial \mathcal{L}}{\partial x_{a'}^k}$, i.e., additional consumption of those applications brings more additional consumption utility than other applications. In equilibrium, user k does not consume other applications than a'. If there is only one a', the best response of user k is to consume only this one application, i.e., $x_{a'}^k = X$ and $x_a^k = 0$ for $a \neq a'$. If there is more than one a', any allocation of time budget X across all those applications yields exactly the same consumption utility. A special case of such allocation is allocating whole X to one application a'.

Given this consumption pattern, user k needs to decide on the set of applications that she consumes, $\mathcal{Q}$, in order to maximize her net utility. If there exists unique $a' \in \mathcal{A}$ such that $\sum_{l\neq k} x^l_{a'} \geq \sum_{l\neq k} x^l_a$ for all $a \in \mathcal{A}$, then the optimal set of applications consumed by user k is a singleton $\mathcal{Q}^k_D = \{a'\}$. Notice that it leads to an equilibrium, where all users allocate their whole time budget to the same application, i.e. $\mathcal{Q}^k_D = \mathcal{Q}_D = \{a'\}$ and $x^k_{a'} = X$ for all k. Therefore, it is a balanced equilibrium. Since any $a' \in \mathcal{A}$ would constitute such an equilibrium, there are A equilibria of this form. Every user's net utility in such an equilibrium is $X + \alpha X^2(N-1) - p$. The following paragraph shows that no other equilibrium exists. In particular, there is no equilibrium that Pareto dominates an equilibrium with Q_D. Therefore, all those equilibria are Pareto efficient.

Suppose now that there is more than one a' such that $\sum_{l\neq k} x^l_{a'} \geq \sum_{l\neq k} x^l_a$. Since the price $p > 0$, user k's best response is to consume only one of a' applications. This is because consuming more of those applications yields exactly the same consumption utility, but user k needs to pay additional price p for each additional application. The net utility is lower when more applications are consumed. Therefore, there cannot be an equilibrium with $Q_D \geq 2$. Moreover, since $p < X$, it is always better for any user to consume one application to none. Therefore, in each equilibrium exactly one application is consumed by all the users.

This completes the proof of Proposition 1. ■

Proof of Proposition 2 (page 11)

Proof. Suppose that $1 < R < 2$ and $\alpha = 0$. User k's consumption utility (and net utility) does not depend on other users' consumption, due to $\alpha = 0$. Thus, the equilibrium consumption decision of user k does not depend on the decisions of other users (i.e., the equilibrium strategy is simply the optimization result of each user).

The proof proceeds in two steps: First, we find the optimal consumption pattern, given that user k has access to some set $\mathcal{Q}$ of applications, where cardinality of $\mathcal{Q}$ is $Q \geq 1$. Second, given the consumption pattern, we find the optimal set of applications consumed, $\mathcal{Q}^k_I$.

Suppose that user k has access to set $\mathcal{Q} \subseteq \mathcal{A}$ of applications. Given $\mathcal{Q}$, user k's objective is to allocate the consumption in order to maximize her utility, i.e.,

$$\max_{x^k_a, a\in\mathcal{Q}} \left(\sum_{a\in\mathcal{Q}} (x^k_a)^{\frac{1}{R}}\right)^R \qquad \text{s.t. } X \geq \sum_{a\in\mathcal{Q}} x^k_a \,.$$

The Lagrangian associated with this problem, including the constraint is

$$\mathcal{L} = \Big(\sum_{a\in\mathcal{Q}} (x_a^k)^{\frac{1}{R}}\Big)^R + \lambda(X - \sum_{a\in\mathcal{Q}} x_a^k)\,.$$

The first order condition for a particular application $a' \in \mathcal{Q}$, $\frac{\partial\mathcal{L}}{\partial x_{a'}^k} = 0$ yields

$$\Big(\sum_{a\in\mathcal{Q}} (x_a^k)^{\frac{1}{R}}\Big)^{R-1} \cdot (x_{a'}^k)^{\frac{1}{R}-1} = \lambda \quad\iff\quad x_{a'}^k = \frac{\Big(\sum_{a\in\mathcal{Q}} (x_a^k)^{\frac{1}{R}}\Big)^R}{\lambda^{\frac{R}{R-1}}}\,, \qquad \forall a' \in \mathcal{Q}\,.$$

Thus, in the consumption schedule that maximizes the consumption utility, every application is consumed in the same amount, i.e., $\hat{x}_a^k = \hat{x}$ for all $a \in \mathcal{Q}$. To reach the maximum the constraint $X \geq \sum_{a\in\mathcal{Q}} x_a^k$ needs to bind. Therefore $Q\cdot\hat{x} = X$ and $\hat{x} = \frac{X}{Q}$. That implies that every equilibrium must be a balanced equilibrium.

With $\hat{x} = \frac{X}{Q}$ the maximal consumption utility given $\mathcal{Q}$ is

$$u_I(\hat{x};\mathcal{Q}) = \left(\sum_{a\in\mathcal{Q}} \left(\frac{X}{Q}\right)^{\frac{1}{R}}\right)^R = \left(Q\left(\frac{X}{Q}\right)^{\frac{1}{R}}\right)^R = Q^{R-1}X\,.$$

This consumption utility is the same for any set $\mathcal{Q}$ of cardinality Q. The net utility also depends solely on the cardinality of set $\mathcal{Q}$. For any set $\mathcal{Q}$ with cardinality Q, the user k's maximal net utility is

$$U_I(\hat{x};Q) = Q^{R-1}X - pQ\,.$$

For $p > 0$, the optimal number of applications consumed by user k is characterized by the first order condition

$$(R-1)\,Q^{R-2}X = p \iff Q = \left(\frac{X(R-1)}{p}\right)^{\frac{1}{2-R}}\,. \tag{10}$$

Let $q_I = \left(\frac{X(R-1)}{p}\right)^{\frac{1}{2-R}}$.

The number of applications consumed cannot be greater than A or smaller than 1. We have assumed that the number of applications is large enough.[12] Specifically, we have assumed that $A \geq q_I$. Therefore, we need to assure that the number of applications consumed

[12] If we had allowed for $A < q_I$, it would be optimal for a user to consume all A applications. This is because the derivative of $U_I(\hat{x};Q)$ is strictly positive for all $Q < q_I$. So it would be positive on the whole domain $[1, A]$ for $A < q_I$.

is not lower than 1. Therefore, the optimal number of applications consumed by any user k is $Q_I^k = \max\{1, q_I\}$.

Since the optimal number of applications consumed is the same for all users, let Q_I denote Q_I^k for any k. Each user is indifferent between consumption of any subset with cardinality Q_I. Any collection of sets $\{\mathcal{Q}_I^1, \ldots, \mathcal{Q}_I^N\}$ constitutes an equilibrium, as long as for any k, cardinality of $\mathcal{Q}_I^k$ is Q_I. There are $\frac{A!}{Q_I!(A-Q_I)!} \cdot N$ such collections of sets. Therefore there is that many pure strategy Nash equilibria. There is also a continuum of mixed strategy equilibria: any probability distribution over all the pure strategies described above constitutes an equilibrium strategy for user k (given that all the subsets have the same cardinality, any of such mixed strategies yields the same utility as a pure strategy).

If there existed any other equilibrium, it would involve some users consuming other number of applications than Q_I with a positive probability. That is suboptimal strategy for those users. Therefore, there are no other equilibria.

Since in all subgame perfect Nash equilibria every user consumes Q_I applications, each in equal amount, all the equilibria yield the same net utility to all users. Moreover, there does not exist an equilibrium where some users could achieve a higher net utility. Thus, all equilibria are Pareto optimal.

This completes the proof of Proposition 2. ■

Proof of Lemma 1 (page 12)

Proof. Assume that $1 < R < 2$ and $\alpha > 0$. Suppose, to the contrary, that in some equilibrium $\mathcal{Q}^k \neq \mathcal{Q}^l$ for some l and k (we drop the subscript DI in this proof for clarity of exposition). We show that this cannot be an equilibrium.

First, consider the case where $Q^k = Q^l$, i.e., user k and user l consume the same amount of applications, but different ones. This cannot be an equilibrium. Take an application a' that k consumes, but l does not, and application a'' that l consumes but k does not. Suppose, without loss of generality that $\sum_{j \neq l,\, k} x_{a'}^j \leq \sum_{j \neq l,\, k} x_{a'}^j$ (otherwise, we switch k and l). User k's net utility in such a candidate equilibrium is

$$\Big(\sum_{a \in \mathcal{Q}^k} \left(x_a^k\right)^{\frac{1}{R}} \Big)^R + \alpha \sum_{a \in \mathcal{Q}^k \setminus \{a'\}} \Big(x_a^k \sum_{j \neq k} x_a^j \Big) + \alpha\, x_{a'}^k \sum_{j \neq k,\, l} x_{a'}^j - p\, Q^k \,.$$

If user k spends $x_{a'}^k$ consuming application a'' instead of a' (without changing anything else), she increases her utility to

$$\Big(\sum_{a\in\mathcal{Q}^k}\big(x_a^k\big)^{\frac{1}{R}}\Big)^R+\alpha\sum_{a\in\mathcal{Q}^k\setminus\{a'\}}\Big(x_a^k\sum_{j\neq k}x_a^j\Big)+\alpha\,x_{a'}^k\underbrace{\Big(\sum_{j\neq k,\,l}x_{a''}^j+x_{a''}^l\Big)}_{>\sum_{j\neq k,\,l}x_{a'}^j}-p\,Q^k\,.$$

Therefore, it is not an equilibrium for users to consume different application, since $\alpha>0$.

For the same reason, if $Q^k<Q^l$, user k consumes only applications that l consumes, i.e., $\mathcal{Q}^k\subset\mathcal{Q}^l$. However, $Q^k<Q^l$ cannot be an equilibrium.

Suppose, to the contrary, that in a balanced equilibrium $Q^l>Q^k$ and $\mathcal{Q}^k\subset\mathcal{Q}^l$. Since they place balanced strategies, $x_a^l=\frac{X}{Q^l}$ for $a\in\mathcal{Q}^l$, and $x_a^k=\frac{X}{Q^k}$ for $a\in\mathcal{Q}^k$ and $x_a^k=0$ for all other applications, especially for $a\in\mathcal{Q}^k\setminus\mathcal{Q}^k$. The consumption of all other users is $\sum_{j\neq k,\,l}x_a^j$ for all $a\in\mathcal{A}$. For k, it is optimal to consume Q^k. Such consumption yields the net utility

$$\Big(\sum_{a\in\mathcal{Q}^k}\big(x_a^k\big)^{\frac{1}{R}}\Big)^R+\sum_{a\in\mathcal{Q}^k}x_a^k\Big(\sum_{j\neq k,\,l}x_a^j+x_a^l\Big)-p\,Q^k\,.$$

And after substituting $x_a^k=\frac{X}{Q^k}$ and $x_a^l=\frac{X}{Q^l}$

$$(Q^k)^{R-1}X+\alpha\frac{X}{Q^k}\sum_{a\in\mathcal{Q}^k}\Big(\sum_{j\neq k,\,l}x_a^j\Big)+\alpha\frac{X}{Q^k}\,Q^k\frac{X}{Q^l}-p\,Q^k.$$

In particular, consuming Q^k applications yields higher utility for user k than consuming the same Q^l applications as user l, i.e.[13]

$$\begin{aligned}&(Q^l)^{R-1}X+\alpha(\frac{X}{Q^l})^2Q^k+\alpha\frac{X}{Q^l}\sum_{a\in\mathcal{Q}^l}\Big(\sum_{j\neq k,\,l}x_a^j\Big)-p\,Q^l\le(Q^k)^{R-1}X+\alpha\,\frac{X^2}{Q^l}+\alpha\frac{X}{Q^k}\sum_{a\in\mathcal{Q}^k}\Big(\sum_{j\neq k,\,l}x_a^j\Big)-p\,Q^k\implies\\ &\implies X\Big((Q^l)^{R-1}-(Q^k)^{R-1}\Big)+\alpha\,X\Big(\frac{1}{Q^l}\sum_{a\in\mathcal{Q}^l}\Big(\sum_{j\neq k,\,l}x_a^j\Big)-\frac{1}{Q^k}\sum_{a\in\mathcal{Q}^k}\Big(\sum_{j\neq k,\,l}x_a^j\Big)\Big)-p\Big(Q^l-Q^k\Big)\;\le\;\alpha\,\frac{X^2}{Q^l}\Big(1-\frac{Q^k}{Q^l}\Big)\,.\end{aligned}\tag{11}$$

[13]The utility if user k would consume Q^l applications in a balanced strategy is calculated with the following formula:

$$(Q^l)^{R-1}X+\alpha\frac{X}{Q^l}\Big(Q_{DI}^K\,\frac{X}{Q^k}+\sum_{a\in\mathcal{Q}_{DI}^k}\Big(\sum_{j\neq k,\,l}x_a^j\Big)\Big)+\alpha\frac{X}{Q^l}\sum_{a\in\mathcal{Q}^k\setminus\mathcal{Q}^l}\Big(\sum_{j\neq k,\,l}x_a^j\Big)-p\,Q^l\,.$$

For l it is optimal to consume

$$\Big(\sum_{a\in\mathcal{Q}^l}(x_a^l)^{\frac{1}{R}}\Big)^R+\sum_{a\in\mathcal{Q}^k}x_a^l\Big(\sum_{j\neq k,l}x_a^j+x_a^k\Big)+\sum_{a\in\mathcal{Q}^l\setminus\mathcal{Q}^k}x_a^l\Big(\sum_{j\neq k,l}x_a^j\Big)-p\,Q^l\;=$$

$$=\;(Q)^{R-1}+\alpha\,\frac{X}{Q^l}\,X+\alpha\frac{X}{Q^l}\sum_{a\in\mathcal{Q}^l}\Big(\sum_{j\neq k,l}x_a^j\Big)-p\,Q^l\,.$$

In particular, consuming Q^l applications yields higher utility for user l than consuming only Q^k applications, i.e.,

$$X\Big((Q^l)^{R-1}-(Q^k)^{R-1}\Big)+\alpha\,X\bigg(\frac{1}{Q^l}\sum_{a\in\mathcal{Q}^l}\Big(\sum_{j\neq k,l}x_a^j\Big)-\frac{1}{Q^k}\sum_{a\in\mathcal{Q}^k}\Big(\sum_{j\neq k,l}x_a^j\Big)\bigg)-p\Big(Q^l-Q^k\Big)\;\geq\alpha\,X^2\left(\frac{1}{Q^k}-\frac{1}{Q^l}\right)\,. \tag{12}$$

However, for $Q^l>Q^k\geq 1$, $\alpha\,\frac{X^2}{Q^l}\Big(1-\frac{Q^k}{Q^l}\Big)\;<\;\alpha\,X^2\left(\frac{1}{Q^k}-\frac{1}{Q^l}\right)$. Therefore, both inequalities (11) and (12) cannot be satisfied at the same time. Thus, it cannot be that there is a balanced equilibrium where $Q^l>Q^k$.

This completes the proof of Lemma 1. ■

Proof of Lemma 2 (page 12)

Proof. Suppose that in a balanced equilibrium user k consumes $Q^k_{DI}=Q_{DI}$ applications. By Lemma 1, we know that all users consume the same Q_{DI} applications, where $\mathcal{Q}_{DI}$ denotes the consumption set. Notice that the net utility of users does not depend on the identity of the applications. The net utility is the same as long as all users consume the same Q_{DI} applications, whichever they are. Therefore, any subset of applications $\mathcal{Q}_{DI}$ of cardinality Q_{DI} constitutes an equilibrium. ■

Proof of Remark 1 (page 14)

The remark directly follows from Lemma 5.

Lemma 5 *For all parameters $\alpha\geq 0$ and $1\leq R<2$, $Q_I\geq\widehat{Q}$. Moreover when $Q_I>1$, then $\widehat{Q}<Q_I$, and when $Q_I=1$ then $\widehat{Q}=Q_I$.*

Proof. Recall that Q_I is defined based on the solution (q_I) to the following first order condition

$$\underbrace{(R-1)Q^{R-2}X-p}_{D_I(Q)}=0 \tag{13}$$

For $Q \to 0^+$ the derivative $D_I \to \infty$. Moreover, the derivative is always decreasing. For any $p > 0$, the first order condition $D_I(Q) = 0$ has exactly one solution, at q_I.

Similarly, $\widehat{Q}$ is defined based on the solution to another first order condition

$$\widehat{D}(Q) = \underbrace{(R-1)Q^{R-2}X - p}_{D_I(Q)} - \underbrace{\alpha X(N-1)\frac{X}{Q^2}}_{+} = 0 \tag{14}$$

For any Q the derivative $\widehat{D}$ is smaller than the derivative D_I. Therefore, whenever $\widehat{D} = 0$ for some $\widetilde{Q}$, then $D_I > 0$ for this $\widetilde{Q}$. Moreover, since the derivative D_I is decreasing, $D_I = 0$ for a larger Q than $\widetilde{Q}$. Therefore, the solution (q_I) to the first order condition (13) is always larger than any solution to the first order condition (14), if the solution to the latter exists.

After establishing this fact, the proof proceeds to analyze two cases: $Q_I > 1$ and $Q_I = 1$.

Suppose first that $Q_I > 1$. This happens when $q_I > 1$. The value of $\widehat{Q}$ is either a solution to (14) or 1. In either case $\widehat{Q} < q_I = Q_I$.

Suppose now that $Q_I = 1$. This happens when $q_I < 1$. Therefore, any solution to (14) must also be smaller than 1. Then $\widehat{Q} = 1 = Q_I$.

This completes the proof of Lemma 5. ■

Proof of Proposition 3 (page 14)

Proof. The optimal upward deviation involves non-balanced consumption. It yields strictly higher utility than an upward deviation with balanced consumption. Suppose that $\widehat{Q} > 1$. Then $V'(\widehat{Q}) = 0$. Note that the payoff from an upward deviation to Q^k under balanced consumption is the same as $V(Q^k)$. Therefore, an incremental upward deviation with balanced consumption from $\widehat{Q}$ yields 0 benefit. But that means that the optimal upward deviation from $\widehat{Q}$ yields strictly positive benefit. Thus, $\widehat{Q}$ is not a balanced equilibrium.

This completes the proof of Proposition 3.[14] ■

[14] Notice the implication of this result for the incentives in the market: Suppose that the platform limits the number of applications to $\widehat{Q}$, and $\widehat{Q}$ is optimal. Thus, the platform guarantees users the best equilibrium outcome. Nonetheless, the users are not happy with this restriction. They may believe (because they look at their profitable deviation upward) that if one more application would be available, they would be better off. But, of course, in an equilibrium they wouldn't.

Proof of Proposition 4 (page 15)

Proof. Let $1 < R < 2$ and $\alpha > 0$. Suppose that all users play a balanced strategy, where they consume a set of applications $\mathcal{Q}$ with cardinality Q.

The proof proceeds in two steps: In the first step, we show that for $Q > Q_I$ any user has incentive to deviate from this strategy and consume fewer applications. In the second step, we show that for Q such that $\max\{1, Q_\star\} \leq Q < \widehat{Q}$ any user has incentive to deviate and consume more applications. Therefore, no Q in those two intervals can characterize a balanced equilibrium.

Suppose that $Q > Q_I$. If user k consumes Q or fewer applications, i.e., $Q^k \leq Q$, he consumes the same applications as other users, i.e., $\mathcal{Q}^k \subseteq \mathcal{Q}$. This is because, due to direct network effects ($\alpha > 0$), user k's consumption utility would be lower if he consumed other applications instead. Moreover, if user k consumes $Q^k \leq Q$ applications, it is optimal for her to consume them according to a balanced consumption schedule: $\frac{X}{Q^k}$ of each. This is because each application presents the same benefit through consumption complementarity. Therefore, the net utility when user k consumes $Q^k \leq Q$ applications is

$$U_{DI}(Q^k \leq Q) = \left(Q^k\right)^{R-1} X + \cancel{Q^k} \alpha \frac{X}{\cancel{Q^k}} (N-1) \frac{X}{Q} - p\, Q^k \,.$$

Since $p > 0$, the optimal number of applications that user k would like to consume is characterize by the first order condition

$$\frac{\partial U_{DI}(Q^k \leq Q)}{\partial Q^k} = (R-1) \, \left(Q^k\right)^{R-2} X - p = 0 \,. \tag{15}$$

Note that this is the same condition as (10) in the proof of Proposition 2. So $Q^k = q_I$ is the only positive value satisfying this condition. Moreover, for any $Q^k > q_I$, the derivative in (15) is negative. Therefore, for any $Q > Q_I$, user k can profitably deviate to consuming Q_I applications.

In the second step of the proof, we turn to Q such that $\max\{1, Q_\star\} \leq Q < \widehat{Q}$, and we show that any user can profitably deviate by consuming more applications. When user k consumes more applications than Q, she consumes all applications in $\mathcal{Q}$, and $Q^k - Q$ applications that no other user consumes. The optimal consumptions schedule in such deviation is *not* a balanced consumption schedule. If we impose the balanced consumption schedule on the upward deviation, it yields lower utility than the optimal deviation. Even though it is not the optimal deviation, we show that an upward deviation with a balanced consumption schedule is profitable for any user. The net utility from user k's balanced consumption of $Q^k \geq Q$ applications is

$$U_{DI}(Q^k \geq Q) = \left(Q^k\right)^{R-1} X + \not{Q}\alpha \frac{X}{Q^k}(N-1)\frac{X}{\not{Q}} - p\,Q^k\,.$$

Note that $U_{DI}(Q^k \geq Q)$ is the same as $V(Q)$ in equation (4) which has a local maximum at $\widehat{Q} > Q$. Moreover, if there does not exist $Q_\star \leq 1$, then for any $Q \in [1, \widehat{Q})$, and when $Q_\star \leq 1$ exists, then for any $Q \in (Q_\star, \widehat{Q})$, $U_{DI}(\widehat{Q} > Q) > U_{DI}(Q)$. That is, it is strictly profitable for a user to deviate upwards (to $\widehat{Q}$ from those Qs). It reminds to show that there exists a profitable deviation away from $Q_\star \leq 1$. By the definition of $Q_\star$, $U_{DI}(\widehat{Q} > Q) = U_{DI}(Q_\star)$. The most profitable deviation, however involves a non-balanced consumption schedule, and yields strictly higher utility than $U_{DI}(Q^k > Q)$. Therefore, the optimal deviation away from $Q_\star$ is profitable.

This completes the proof of Proposition 4. ■

Proof of Lemma 3 (page 17)

Proof. Let $1 < R < 2$ and $\alpha > 0$. Suppose that all other users $l \neq k$ play a balanced strategy where they consume a set of applications $\mathcal{Q}$ with cardinality Q. If user k consumes Q or fewer applications, i.e. $Q^k \leq Q$, he consumes the same applications as other users, i.e. $\mathcal{Q}^k \subseteq \mathcal{Q}$. This is because, due to direct network effects ($\alpha > 0$), user k's net utility would be lower if he consumed other applications instead.

User k's consumption utility from consuming $Q^k \leq Q$ applications is

$$u(\mathbf{x}^k, Q^k; Q) = \Big(\sum_{a \in \mathcal{Q}^k} \left(x_a^k\right)^{\frac{1}{R}}\Big)^R + \alpha \underbrace{\sum_{a \in \mathcal{Q}^k} x_a^k}_{=X} (N-1)\frac{X}{Q}\,.$$

By usual arguments we find that the consumption schedule maximizing the consumption utility, under the constraint $\sum_{a \in \mathcal{Q}^k} x_a^k \leq X$ is balanced strategy, i.e., $x_a^k = \frac{X}{Q^k}$ for all $a \in \mathcal{Q}^k$.

Therefore, the net utility of user k from consuming Q^k applications is

$$U_{DI}(Q^k; Q) = \left(Q^k\right)^{R-1} X + \alpha\, X(N-1)\frac{X}{Q} - p\,Q^k\,. \tag{16}$$

Notice that this utility is strictly increasing for $Q^k < q_I$ and strictly decreasing for $Q^k > q_I$. if $q_I < 1$, then $Q_I = 1$ and it is not possible that $Q^k < Q_I$. Otherwise, suppose that $Q \leq Q_I$.

Then if $Q^k < Q \leq q_I$, then the utility in (16) increases with Q^k. That is, the user achieves a lower utility if he deviates from Q to $Q^k < Q$.

This completes the proof of Lemma 3. ■

Proof of Proposition 5 (page 18)

Proof. Let $1 < R < 2$ and $\alpha > 0$. Suppose that all other users play a balanced strategies and consume a set of applications $\mathcal{Q}_I$ with cardinality Q_I. By Lemma 3, it is enough to show that there is no profitable deviation upward to prove that Q_I is a balanced equilibrium.

Consider user k who consumes $Q^k > Q_I$ applications. When user k diverts part of her time y away from the Q_I applications that all other users consume, it is optimal for her to consume the same amount of each application in $\mathcal{Q}_I$, $\frac{X-y}{Q_I}$. Moreover, it is also optimal to consume the same amount of each application that user k consumes outside $\mathcal{Q}_I$, $\frac{y}{Q^k-Q_I}$. Then, the net utility of user k is

$$U_{DI}(Q^k > Q_I|\, y) = \left(Q_I \left(\frac{X-y}{Q_I} \right)^{\frac{1}{R}} + (Q^k - Q_I) \left(\frac{y}{Q^k - Q_I} \right)^{\frac{1}{R}} \right)^R + \alpha \frac{X(X-y)}{Q_I} (N-1) - p\, Q^k \,.$$

Consider first only the part of the net utility without the direct network effects:

$$\left(Q_I \left(\frac{X-y}{Q_I} \right)^{\frac{1}{R}} + (Q^k - Q_I) \left(\frac{y}{Q^k - Q_I} \right)^{\frac{1}{R}} \right)^R - p\, Q^k \,.$$

This is the same as the utility under pure indirect network effects. We know from the proof of Proposition 2 that for any Q^k, the utility maximizing consumption schedule is balanced. However, since $\alpha > 0$, in this case the optimal deviation upward must involve un-balanced consumption (in an optimal deviation user consumes more of each application that other users consume and less of each applications that she alone consumes), i.e., $y < \frac{X}{Q^k}(Q^k - Q_I)$. Therefore, if $Q^k > Q_I$, then

$$\left(Q_I \left(\frac{X-y}{Q_I} \right)^{\frac{1}{R}} + (Q^k - Q_I) \left(\frac{y}{Q^k - Q_I} \right)^{\frac{1}{R}} \right)^R - p\, Q^k < \left(Q^k \left(\frac{X}{Q^k} \right)^{\frac{1}{R}} \right)^R - p\, Q^k \,.$$

Recall that Q_I maximizes the net utility under pure indirect network effects. Therefore, for

$Q^k > Q_I$,

$$\left(Q^k\left(\frac{X}{Q^k}\right)^{\frac{1}{R}}\right)^R - p\,Q^k < \left(Q_I\left(\frac{X}{Q_I}\right)^{\frac{1}{R}}\right)^R - p\,Q_I\,.$$

Therefore,

$$\left(Q_I\left(\frac{X-y}{Q_I}\right)^{\frac{1}{R}} + (Q^k - Q_I)\left(\frac{y}{Q^k - Q_I}\right)^{\frac{1}{R}}\right)^R - p\,Q^k < \left(Q_I\left(\frac{X}{Q_I}\right)^{\frac{1}{R}}\right)^R - p\,Q_I\,.$$

Moreover, for any $y > 0$,

$$\alpha\frac{X(X-y)}{Q_I}(N-1) < \alpha\frac{X^2}{Q_I}(N-1)\,.$$

Therefore, any positive deviation, $y > 0$, toward consuming more applications, $Q^k > Q_I$, yields strictly worse net utility for user k,

$$\begin{aligned} U_{DI}(Q^k > Q_I|\,y) = \left(Q_I\left(\frac{X-y}{Q_I}\right)^{\frac{1}{R}} + (Q^k - Q_I)\left(\frac{y}{Q^k - Q_I}\right)^{\frac{1}{R}}\right)^R + \alpha\frac{X(X-y)}{Q_I}(N-1) - p\,Q^k < \\ < \left(Q_I\left(\frac{X}{Q_I}\right)^{\frac{1}{R}}\right)^R + \alpha\frac{X(X-y)}{Q_I}(N-1) - p\,Q_I < \\ < \left(Q_I\left(\frac{X}{Q_I}\right)^{\frac{1}{R}}\right)^R + \alpha\frac{X^2}{Q_I}(N-1) - p\,Q_I = U_{DI}(Q_I)\,. \end{aligned}$$

Therefore, any set of applications $\mathcal{Q}_I$ with cardinality Q_I constitutes a balanced equilibrium.

Notice that the optimal deviation y^* that maximizes the consumption utility is always positive.[15] That is if y^* satisfies the first order condition $\left.\frac{\partial U_{DI}(Q^k > Q_I|\,y)}{\partial y}\right|_{y=y^*} = 0$, it must be that $y^* > 0$. However, because the user needs to pay a positive price $p > 0$ for diverting even small y, it is not optimal to do so at Q_I. (As shown by declining net utility.) Below we show that if $Q_I > 1$, then also for Qs slightly smaller than Q_I, users have no incentive to deviate upward. And so those Qs constitute balanced equilibria.

[15] For formal proof of this property, see the proof of Proposition 15, for $p = 0$.

Suppose now that $Q_I > 1$. For $y > 0$, there exists $q^0 < Q_I$ such that[16]

$$(q^0)^{R-1}X + \alpha\frac{X^2}{q^0}(N-1) - p\,q^0 = Q_I^{R-1}X + \alpha\frac{X(X-y)}{q^0}(N-1) - p\,Q_I\,.$$

Let $Q^0 = \max\{1,\, q^0\}$. Then it is easy to check that

$$(Q^0)^{R-1}X + \alpha\frac{X^2}{Q^0}(N-1) - p\,Q^0 \geq Q_I^{R-1}X + \alpha\frac{X(X-y)}{Q^0}(N-1) - p\,Q_I\,. \tag{17}$$

Any deviation upward from Q^0 yields net utility

$$\begin{aligned} U_{DI}(Q^k > Q^0 |\, y > 0) = \left(Q^0\left(\frac{X-y}{Q^0}\right)^{\frac{1}{R}} + (Q^k - Q^0)\left(\frac{y}{Q^k - Q^0}\right)^{\frac{1}{R}}\right)^R + \alpha\frac{X(X-y)}{Q^0}(N-1) - p\,Q^k < \\ < (Q^k)^{R-1}X + \alpha\frac{X(X-y)}{Q^0}(N-1) - p\,Q^k \leq \\ \leq (Q_I)^{R-1}X + \alpha\frac{X(X-y)}{Q^0}(N-1) - p\,Q_I \leq \\ \leq (Q^0)^{R-1}X + \alpha\frac{X^2}{Q^0}(N-1) - p\,Q^0 = U_{DI}(Q^0)\,. \end{aligned}$$

Therefore, there exists $Q^0 < Q_I$ such that users have no incentive to deviate upward. Therefore, Q^0 constitutes a balanced equilibrium. Moreover, since for any Q such that $Q^0 \leq Q \leq Q_I$, inequality (17) holds, those also constitute balanced equilibria.

This completes the proof of Proposition 5. ∎

Proof of Proposition 7 (page 20)

Proof. It follows directly from the shape of $V(Q)$ and Proposition 4: All possible equilibria need to be included in the interval $[1, Q_\star) \cup (\widehat{Q}, Q_I]$. (The set of equilibria is a strict subset of this interval). The utility obtained by every user in each equilibrium Q is $V(Q)$. Since $V(Q)$ is strictly increasing on the interval $[1, Q_\star) \cup (\widehat{Q}, Q_I]$, a lower equilibrium Q yields higher utility for every user than a higher equilibrium Q, i.e. lower equilibrium Q Pareto-dominates higher Q. ∎

[16]This is because $Q^{R-1}X - p\,Q$ is continuous and strictly increasing for $Q < Q_I$. And because for $y^* > 0$, $\frac{X-y^*}{q^0} < \frac{X}{q^0}$.

Proof of Proposition 8 (page 21)

Proof. The shape of V implies that either $Q^{**} = 1$ or $Q^{**} = \widehat{Q}$. The proof first considers $Q^{**} = \widehat{Q} > 1$, and then $Q^{**} = 1$.

Suppose that $Q^{**} = \widehat{Q} > 1$. Then, $Q_{\star}$ (as defined for Proposition 4) does not exist. Therefore, by Proposition 4, no $Q < \widehat{Q}$ may constitute a balanced equilibrium. As in the proof of Proposition 4, users are better off deviating upward to consuming $\widehat{Q}$ applications. When $A > \widehat{Q}$, then $\widehat{Q}$ is not a balanced equilibrium, by Proposition 3. This is because there exists profitable deviation upward, toward consuming larger number of applications. However, when $A = Q^{**} = \widehat{Q}$, such deviation is not possible. Therefore, consuming all $\widehat{Q}$ constitutes the only equilibrium.

Now, suppose that $Q^{**} = 1$. When platform sets $A = Q^{**} = 1$ then trivially, in the only equilibrium all users consume the only application in the market.

This completes the proof of Proposition 8. ■

Proof of Lemma 4 (page 24)

Proof. Suppose that l consumes G applications for some G. Given G, any application a is consumed by l with probability $\frac{G}{A}$. Since when every strategy is equally likely, any subset of cardinality G is equally likely to be consumed. The probability that particular application a is in a consumption set is

$$\frac{\text{how many subsets with } G \text{ can you choose from } \mathcal{A} \text{ that will include } a}{\text{how many subsets with } G \text{ can you choose from } \mathcal{A} \text{ overall}} =$$
$$= \frac{\text{choose } G-1 \text{ out of } A-1}{\text{choose } G \text{ out of } A} = \frac{(A-1)!}{(G-1)!\,(A-G)!} \Big/ \frac{A!}{G!\,(A-G)!} = \frac{G}{A}\,.$$

Now, we calculate the expected level of consumption $\mathbb{E}(x_a^l \,|\, x_a^l \in \mathcal{G})$ conditionally on a being in a given consumption set $\mathcal{G}$ of l. Since we know that all the consumption schedules over the set $\mathcal{G}$ satisfy $\sum_{a \in \mathcal{G}} x_a^l = X$, therefore

$$\mathbb{E}(\sum x) = X \implies \sum \mathbb{E}(x) = X\,,$$

due to the linearity of the sum.

But the applications are not distinct (they are interchangeable). If every consumption schedule is equally likely, the expected consumption of every a in the consumption set is the same.

Suppose to the contrary, that the expected consumption of some $a' \in \mathcal{G}$ is higher than some other application, a'', $\mathbb{E}(x^l_{a'}) > \mathbb{E}(x^l_{a''})$. Then, in the set of all possible consumption schedules, switch a' and a'' in every schedule. The set of all possible consumption schedules remains unchanged, but now $\mathbb{E}(x^l_{a''}) > \mathbb{E}(x^l_{a'})$. Hence, contradiction. Therefore, the expected consumption of every $a \in \mathcal{G}$ is the same $\mathbb{E}(x^l_a) = \frac{X}{G}$.

Then, the overall expected level of consumption of any application a is

$$\mathbb{E}_{\phi^k_l} x^l_a = \frac{G}{A} \cdot \frac{X}{G} = \frac{X}{A} \, .$$

(Technically, it is for a given G. But since the expectation for any G is the same, any probability distribution over G's gives the same expected value.)

This completes the proof of Lemma 4. ■

Proof of Proposition 9 (page 25)

Proof. Let $R = 1$ and $\alpha > 0$. Suppose that user k consumes G^k applications in a no-foresight environment. By Lemma 4, her expected consumption utility is

$$\mathbb{E}\, u_D(G^k) = X + \alpha X (N-1) \frac{X}{A} \, ,$$

and does not depend on the number of applications consumed, G^k (as long as the budget constraint is binding, $\sum x^k_a = X$).

Since $p > 0$, purchasing more than one application increases cost, without increasing consumption utility. Therefore, in equilibrium, every user k consumes only one application, $G^k_D = 1$.

This completes the proof of Proposition 9. ■

Proof of Proposition 10 (page 26)

Proof. The result of this proposition follows directly from Proposition 2.

Suppose $1 < R < 2$ and $\alpha = 0$. User k's consumption utility (and net utility) does not depend on other users's consumption, due to $\alpha = 0$. Thus, user k's optimization problem is exactly the same as under the perfect foresight (Proposition 2), and yields the same solution. In particular, the optimal number of applications consumed is the same, $G^k_I = Q_I$ for any k.

This completes the proof of Proposition 10. ■

Proof of Proposition 11 (page 26)

Proof. Let $1 < R < 2$ and $\alpha > 0$. Suppose that user k consumes G^k applications in a no-foresight environment. For any given number of applications, G^k, the optimal consumption schedule is a balanced consumption. This is because for any application, the expected level of consumption by other users is the same (Lemma 4). User k's expected net utility (using Lemma 4) is then

$$\mathbb{E}\, U_{DI}(G^k) = \left(G^k\right)^{R-1} X + \alpha\,(N-1)\frac{X^2}{A} - p\, G^k\,. \tag{18}$$

Note that the benefit from the direct network effect does not depend on G^k. This leads to a result similar to the one in Proposition 2: The above function U_{DI} is maximized by $G^k = q_I = \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$, for any k.

This completes the proof of Proposition 11. ■

Proof of Proposition 12 (page 27)

Proof. Let's consider separately the case for $R = 1$ and for $1 < R < 2$.

Suppose first that $\alpha > 0$ and $R = 1$. By Proposition 9, we know that when $A \geq \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$, the optimal number of applications consumed by any user k is $G_D^k = 1$. It is easy to show that for any $A \geq 1$, every user optimally consumes one application. The expected net utility for any user in an equilibrium for $A \geq 1$ is

$$\mathbb{E}\, U_D^* = X + \alpha(N-1)\frac{X^2}{A} - p\,.$$

Clearly, $\mathbb{E}\, U_D^*$ is maximized by $A = 1$. Moreover, for $\alpha > 0$ and $R = 1$, the unique maximum of $V(Q)$ is always $Q^{**} = 1$. Therefore, the platform maximizes users' net utility when it sets the number of available applications to $A = Q^{**} = 1$.

Suppose now that $\alpha > 0$ and $1 < R < 2$. By Proposition 11, we know that when $A \geq \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$, the optimal number of applications consumed by any user k is $G_{DI}^k = Q_I$, as this number maximizes $\mathbb{E}\, U_{DI}(G^k)$ in (18). We need to consider two cases: for $Q_I = 1$ and for $Q_I > 1$.

When $Q_I = 1$, the expected net utility of any user in equilibrium for $A \geq 1$ is

$$\mathbb{E}\, U_{DI}^*(Q_I\!=\!1) = X + \alpha(N-1)\frac{X^2}{A} - p\,,$$

which is maximized by $A = 1$. By Lemma 5, $\widehat{Q} = 1$ when $Q_I = 1$. Thus, the unique maximum of $V(Q)$ is always $Q^{**} = 1$. In result, the platform maximizes user's net utility when it sets the number of available applications to $A = Q^{**} = 1$.

The case of $Q_I > 1$ occurs only when $q_I > 1$. When $A \geq q_I = \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$, the expected net utility of a user in equilibrium is

$$\mathbb{E}\, U^*_{DI}(Q_I > 1 | A \leq Q_I) = (Q_I)^{R-1}\, X + \alpha\, (N-1) \frac{X^2}{A} - p\, Q_I\,.$$

On the possible range, this utility is maximized for $A = Q_I$.

Since $\mathbb{E}\, U_{DI}(G^k)$ strictly increases in G^k for $G^k < Q_I$, every user consumes all applications, if there is fewer applications available than Q_I. Thus, for $A \leq q_I = \left(\frac{(R-1)X}{p}\right)^{\frac{1}{2-R}}$, the expected net utility of a user in equilibrium is

$$\mathbb{E}\, U^*_{DI}(A | A \leq Q_I) = (A)^{R-1}\, X + \alpha\, (N-1) \frac{X^2}{A} - p\, A\,.$$

Note that this function of A is the same as V (with the exception that V is a function of Q). Moreover, when $Q_I > 1$, it must be that $Q^{**} < Q_I$. Since Q^{**} that maximizes V, then $A = Q^{**} < q_I$ also maximizes the expected net utility $\mathbb{E}\, U^*_{DI}(A|A \leq Q_I)$. Moreover, because $mathbbE\, U^*_{DI}(Q_I > 1|A \leq Q_I)$ is maximized at $A = Q_I$, $A = Q^* *$ maximizes the expected utility $\mathbb{E}\, U^*_{DI}$ on the whole range $A \geq 1$. Therefore, the platform maximizes users' net utility when it sets the number of available applications to $A = Q^{**}$.

This completes the proof of Proposition 12. ■

B Results for $p = 0$

Suppose that the access price for any application is $p = 0$. Then, the net utility is the same as the consumption utility, and every user chooses the set of consumed applications and the consumption schedule to maximize her consumption utility.

Note that under $p = 0$ the assumption on A does not make sense any more. It is not possible to make A "large enough". In this appendix, we allow for arbitrary A.

B.1 Game with perfect foresight

In this section, we assume that every agent knows (or correctly predicts) the number and identity of applications consumed by all other users (i.e., the user knows the consumption sets and consumption schedules of all other users) in equilibrium.

B.1.1 Game with perfect foresight: Direct network effects

Proposition 13 *Assume $R = 1$, $\alpha > 0$ and $p = 0$. For any $\mathcal{Q} \subseteq \mathcal{A}$, there exists a balanced equilibrium where all users consume the set of applications $\mathcal{Q}$. There is no other balanced equilibrium.*

Proof. Let $R = 1$, $\alpha > 0$ and $p = 0$. Suppose that all other users play a balanced strategy where they consume a set $\mathcal{Q} \subseteq \mathcal{A}$ applications. If user k consumes fewer applications than other users, she consumes the same applications as other users (due to consumption complementarity it would make sense otherwise). The optimal consumption schedule then is balanced. User k's net utility of consuming $Q^k \leq Q$ applications is

$$U_D(Q^k \leq Q) = \cancel{(Q^k)^{R-1}} X + \cancel{Q^k} \alpha \frac{X}{\cancel{Q^k}} (N-1) \frac{X}{Q} \,.$$

(The term $(Q^k)^{R-1}$ cancels because $R = 1$.) This utility does not depend on Q^k. Thus, user k does not have incentive to deviate downward, i.e., consume fewer applications than other users.

Now suppose that user k consumes more applications than other users. She diverts $y \leq X$ of her time to applications that no other users consume. It is optimal for her to still consume all $\mathcal{Q}$ applications that other users consume (hence $y < X$), and among those applications, each application is consumed at the same lever, $\frac{X-y}{Q}$. Because $R = 1$, independently of how many more applications user k consume, her net utility is

$$U_D(Q^k \geq Q) = X + \alpha X (N-1) \frac{X-y}{Q} \,.$$

This utility is strictly decreasing in y, and it reaches its maximum for $y = 0$, i.e., when user k does not divert *any* time to applications other than those consumed by other users. So, user k has no incentive to deviate upward, i.e., consume more applications.

Therefore, if all users consume $\mathcal{Q}$ applications in a balanced strategy, it constitutes a balanced equilibrium, for any $\mathcal{Q} \subseteq \mathcal{A}$.

To show that there is no other balanced equilibrium, notice that if other balanced equilibrium existed, users would need to consume different applications or different number of applications in an equilibrium. Suppose that $\mathcal{Q} \subseteq \mathcal{A}$ is a set of applications that is consumed at strictly positive level by at least one other user,

$$\mathcal{Q} = \{a, \exists l \neq k \text{ s.t. } x_a^l > 0\}.$$

If the aggregate consumption of all other users is not the same for all applications in $\mathcal{Q}$, the best response of user k is an un-balanced strategy, where she consumes larger levels of applications that have higher aggregate consumption levels by other users. Therefore, it cannot be a balanced equilibrium.

Suppose then that aggregate consumption by all other users is the same for all applications in $\mathcal{Q}$, $\sum_{l\neq k} x_a^l = \sum_{l\neq k} x_{a'}^l$ for all a, $a' \in \mathcal{Q}$. Then user k's best response is a balanced strategy where she consumes all $\mathcal{Q}$ applications. But if $x_a^l = x_{a'}^l$ for all l and all a, $a' \in \mathcal{Q}$, then it is a balanced equilibrium where all users consume $\mathcal{Q}$. To show that it cannot be any other balanced equilibrium, suppose that for some l and l' and some a, $x_a^l \neq x_a^{l'}$. But then for user l it is not true that $\sum_{j\neq l} x_a^j = \sum_{j\neq l} x_{a'}^j$. Therefore, it cannot be that user l's best response is a balanced strategy. Therefore, it cannot be a balanced equilibrium. The only balanced equilibrium is when all users consume all applications in $\mathcal{Q}$.

This completes the proof of Proposition 13. ■

Corollary 5 *Assume $R = 1$, $\alpha > 0$ and $p = 0$. Suppose that in an equilibrium users consume $\mathcal{Q}$ applications, where Q is cardinality of $\mathcal{Q}$. An equilibrium is Pareto-optimal if and only if $Q = 1$.*

Proof. Let $R = 1$, $\alpha > 0$ and $p = 0$, and suppose that in an equilibrium users consume $\mathcal{Q}$ applications, where Q is cardinality of $\mathcal{Q}$. The net utility of each user in an equilibrium with $Q \geq 1$ is

$$U_D^*(Q) = X + \alpha \sum_{a=1}^{Q} \left(\frac{X}{Q}(N-1)\frac{X}{Q} \right) = X + \alpha(N-1)\frac{X^2}{Q}.$$

This utility strictly decreases with Q, and is maximized when $Q = 1$. Whenever more than one application is consumed in equilibrium, the equilibrium is Pareto-inferior to an equilibrium where one application is consumed.

Therefore, only equilibria where $\mathcal{Q}_D^k = \mathcal{Q}_D = \{a\}$ for some $a \in \mathcal{A}$ and all k — i.e., equilibria where all users consume one and the same application — are Pareto optimal. ■

B.1.2 Game with perfect foresight: Indirect network effects

Proposition 14 *Assume $1 < R < 2$, $\alpha = 0$ and $p = 0$. There exists a unique equilibrium, where $\mathcal{Q}_I^k = \mathcal{A}$ each user k, i.e., every user k consumes all available applications. Moreover,* *the unique equilibrium is balanced.*

Proof. Let $1 < R < 2$, $\alpha = 0$ and $p = 0$.

As in the proof for $p > 0$, we find that the optimal consumption schedule, given that user k has access to some set $\mathcal{Q}$ applications is to consume each of them in the amount of $\hat{x} = \frac{X}{Q}$. Therefore, every equilibrium is balanced equilibrium.

To find the equilibrium consumption set for user k, recall that her consumption utility given Q is

$$u_I(\hat{x}; Q) = \left(Q \left(\frac{X}{Q} \right)^{\frac{1}{R}} \right)^R = Q^{R-1} X \, .$$

This utility is always strictly increasing in Q, i.e., user k always prefers to consume as many applications as possible. In such a case, user k's consumption set is only limited by $\mathcal{A}$, i.e., she optimally consumes *all* applications available, each at the level $\hat{x} = \frac{X}{A}$.

This completes the proof of Proposition 14. ■

A Pareto-optimal equilibrium is an equilibrium that is not Pareto-dominated by any other equilibrium. A Pareto-optimal allocation is a feasible set of consumption schedules for all users such that is not Pareto-dominated by any other feasible set of consumption schedule. When an equilibrium is unique, it is clearly a Pareto-optimal equilibrium. However, it does not need to be a Pareto-optimal allocation (because a Pareto-optimal allocation does not have to be an equilibrium).

Corollary 6 *In the case for $1 < R < 2$, $\alpha = 0$ and $p = 0$, the unique balanced equilibrium is also the Pareto-optimal allocation.*

Proof. For $\alpha = 0$, the equilibrium utility does not depend on other users' consumption, and no other number of consumed application yields higher utility for user k than the equilibrium number, A. Therefore, this equilibrium is a Pareto-optimal allocation. ■

B.1.3 Game with perfect foresight: Interplay between direct and indirect network effects

Proposition 15 *Assume $1 < R < 2$, $\alpha > 0$ and $p = 0$. There exists a unique balanced equilibrium, where $\mathcal{Q}^k_{DI} = \mathcal{A}$ each user k, i.e., every user k consumes all available applications.*[17]

[17][] I am convinced that it is the unique equilibrium overall (not only unique balanced), but the current proof shows only that it is unique balanced. The original statement of the proposition was *Assume* $1 < R < 2$, $\alpha > 0$ *and* $p = 0$. *There exists a unique equilibrium, where $\mathcal{Q}^k_{DI} = \mathcal{A}$ each user k, i.e., every user k consumes all available applications. Moreover, the unique equilibrium is balanced.*

Proof. Let $1 < R < 2$, $\alpha > 0$ and $p = 0$. Suppose that all other users consume Q applications according to a balanced strategy. If user k consumes fewer applications than other users, $Q^k \leq Q$, it is optimal for her to consume them according to a balanced consumptions schedule. Then, the net utility of user k is

$$U_{DI}(Q^k \leq Q) = \left(Q^k\right)^{R-1} X + \cancel{Q^k}\frac{X}{\cancel{Q^k}}\alpha\frac{X}{Q}(N-1)\,.$$

This utility strictly increases with Q^k, and yields the highest value for $Q^k = Q$. Therefore, user k has no incentive do deviate downward, and consume fewer applications than other users.

Now, we show that if $Q < A$, then user k always has incentive to consume more applications than other users. Suppose that user k consumes one more application than other users. She diverts some y of her time to the new application, while it is optimal for her to consume at the same level all the applications that other users consume, $\frac{X-y}{Q}$. Then, user k's utility is

$$\left(Q\left(\frac{X-y}{Q}\right)^{\frac{1}{R}} + y^{\frac{1}{R}}\right)^{R} + \cancel{Q}\frac{X}{\cancel{Q}}(X-y)\alpha(N-1)\,.$$

Using additional application brings user k benefit due to preference for variety. However, diverting time from applications that are consumed by other users decreases user k payoff due to consumption complementarity (the direct network effect). The marginal "cost" of diverting consumption due to direct network effect is $\frac{\partial X(X-y)\alpha(N-1)}{\partial y} = \alpha X(N-1)$. The marginal benefit due to preference for variety is

$$\frac{\partial\left(Q^{1-\frac{1}{R}}(X-y)^{\frac{1}{R}} + y^{\frac{1}{R}}\right)^{R}}{\partial y} = \underbrace{\left(Q^{1-\frac{1}{R}}(X-y)^{\frac{1}{R}} + y^{\frac{1}{R}}\right)^{R-1}\left(y^{1-\frac{1}{R}} - Q^{1-\frac{1}{R}}(X-y)^{\frac{1}{R}-1}\right)}_{f(y)}\,.$$

Function $f(y)$ is strictly decreasing in y, and as $y \to 0^+$, $f(y) \to \infty$. Therefore, for any value of $\alpha X(N-1)$, there exists small enough y for which $f(y) > \alpha X(N-1)$. That means that there always exists a consumption schedule (characterized by y for which it is beneficial for user k to deviate from Q and consume one more application.[18]

[18] Notice that here it is fine to compare derivatives, while in the case of $p > 0$ it is not. Here for every little bit of y, we lose $y \cdot \alpha X(N-1)$, and we benefit more than $y \cdot f(y)$ (this is because it is an underestimation, for lower y's, $f(y)$ is higher). In the case of $p > 0$, user k needs to pay the whole p, even if using infinitesimally small amount of y.

Since for all $Q < A$ user k has incentive to deviate toward consuming more applications, such Q cannot characterize an equilibrium. When $Q = A$ a deviation upward is not feasible, and no user finds it profitable to deviate downward. Therefore, in a balanced equilibrium all applications are consumed by all users. There is only one such equilibrium.

This completes the proof of Proposition 15. ■

Corollary 7 *In the case for $1 < R < 2$, $\alpha > 0$ and $p = 0$, the unique balanced equilibrium may or may not be a Pareto-optimal allocation. When the equilibrium is not a Pareto-optimal allocation, then the allocation where all users consume* one *application is the Pareto-optimal allocation.*

Proof. Let $1 < R < 2$, $\alpha > 0$ and $p = 0$. In the unique balanced equilibrium all users consume all A available applications, which yields utility

$$U^*_{DI}(Q_{DI}\!=\!A) = A^{R-1}X + \alpha\frac{X^2}{A}(N-1)\,.$$

Now, suppose that all users would play a balanced strategy where they all consume a set of applications $\mathcal{Q}$ of cardinality Q. Then, user k's payoff is

$$\underbrace{Q^{R-1}X + \alpha\frac{X^2}{Q}(N-1)}_{V(Q)}\,,$$

which is the same function as V for $p = 0$. This function $V(Q)$ has only one optimum,[19] at $Q = \left(\frac{\alpha X(N-1)}{R-1}\right)^{\frac{1}{R}}$. This is a minimum.[20] So, if $\left(\frac{\alpha X(N-1)}{R-1}\right)^{\frac{1}{R}} \leq 1$, $V(Q)$ is increasing for all $A \geq 1$. When $\left(\frac{\alpha X(N-1)}{R-1}\right)^{\frac{1}{R}} > 1$, $V(Q)$ is first decreasing and then increasing. In such a

[19] The first order condition

$$\frac{\partial V(Q)}{\partial Q} = \frac{X}{Q^2}\left[(R-1)Q^R - \alpha X(N-1)\right] = 0$$

is satisfied only for $Q = \left(\frac{\alpha X(N-1)}{R-1}\right)^{\frac{1}{R}}$.

[20] This result may be obtained in two ways: First, it is enough to show that for Q lower than this threshold, the derivative is negative; and for Q higher than the threshold, the derivative is positive. In the second approach, we show that the second derivative of $V(Q)$ is negative for $Q = \left(\frac{\alpha X(N-1)}{R-1}\right)^{\frac{1}{R}}$.

$$V''(Q) = (R-1)\underbrace{(R-2)}_{-}Q^{R-3}X + 2\alpha\frac{X^2}{Q^3}(N-1) = 0 \iff Q^R = \frac{2\alpha X(N-1)}{(R-1)(2-R)}\,.$$

case, $V(Q)$ has two local maxima: at $Q = 1$ and at $Q = A$. It is possible that $V(1) > V(A)$, even though $Q = 1$ is not an equilibrium. When $V(A) \geq V(1)$, then the unique balance equilibrium is a Pareto-optimal allocation. But when when $V(A) < V(1)$, the equilibrium is Pareto-dominated by the allocation where $Q = 1$.

This completes the proof of Corollary 7. ■

B.1.4 Game with perfect foresight: On the role of the platform

With pure direct network effects there exist many possible equilibria. However, only equilibira where exactly one application is consumed are Pareto-optimal. Such equilibria always exist. But if more than one application is available, $A > 1$, there also exist Parto-inferior equilibria. The platform eliminates the Pareto-inferior equilibria by setting $A = 1$.

With pure indirect network effects there exists a unique equilibrium. This equilibrium is a Pareto-optimal allocation for a given A (i.e., in a given environment, users could not do better by consuming any other number of applications). However, the equilibrium net utility of each user: $A^{R-1}X$ increases with A. Therefore, the larger the number of applications the platform provides, the larger is users' net utility.

In the presence of both direct and indirect network effect there exists a unique equilibrium. But for a given A it may or may not be Pareto-optimal (Corollary 7). If the platform is bounded in setting A and cannot provide more applications than $\overline{A}$, then it needs to consider whether $V(1) \leq V(\overline{A})$ or not. When $V(1) \leq V(\overline{A})$, then the platform should set $A = \overline{A}$ to maximize users' net utility. But when $V(1) > V(\overline{A})$, then the platform should set $A = 1$.

If the platform is not bounded in setting A. It should set as large A as possible. This is because $U^*_{DI}(A) \to \infty$ when $A \to \infty$. Therefore, for large enough A, $U^*_{DI}(A) = V(A) > V(1)$, and then $U^*_{DI}(A)$ is only decreasing in A.

B.2 Game with no foresight

B.2.1 Game with no foresight: Direct network effects

Proposition 16 *Assume $R = 1$, $\alpha > 0$ and $p = 0$. Any set of consumption schedules constitutes a no-foresight equilibrium, as long as for any user k, $\sum_{a \in \mathcal{A}} x^k_a = X$.*

For $Q < \left(\frac{2\alpha X(N-1)}{(R-1)(2-R)}\right)^{\frac{1}{R}}$, $V''(Q) < 0$. And $\left(\frac{\alpha X(N-1)}{R-1}\right)^{\frac{1}{R}} < \left(\frac{2\alpha X(N-1)}{(R-1)(2-R)}\right)^{\frac{1}{R}}$, so the second derivative is negative where the first order condition is satisfied.

Proof. Let $R = 1$, $\alpha > 0$ and $p = 0$. Under the assumption of no-foresight, and $p = 0$, user k's net utility from consuming a set of applications $\mathcal{G}^k$ is

$$U_{DI}(\{x_a^k\}|x_a^k \in \mathcal{G}^k) = \sum_{x_a^k \in \mathcal{G}^k} x_a^k + \alpha(N-1)\frac{X^2}{A}\,.$$

Since a user does not maximize the consumption unless she uses all her time budget, $\sum_{x_a^k \in \mathcal{G}^k} x_a^k = X$ for any $\mathcal{G}^k$. Thus, the utility achieved by user k is $X + \alpha(N-1)\frac{X^2}{A}$, independently of a consumption schedule. Whatever is user k's consumption set and consumption schedule, she never has incentive to deviate. Therefore, whenever all users consume all their time budget, i.e., $\sum_{a \in \mathcal{A}} x_a^k = X$ for every k, a set of such consumption schedules constitute an equilibrium.

This completes the proof of Proposition 16. ■

B.2.2 Game with no foresight: Indirect network effects

In the case of pure indirect network effects, foresight plays no role. All the results, including results for the role of the platform, are the same for no-foresight as for perfect foresight.

B.2.3 Game with no foresight: Interplay between direct and indirect network effects

Proposition 17 *Assume that* $1 < R < 2$ *and* $\alpha > 0$. *There exists a unique no-foresight equilibrium, where* $\mathcal{G}_{DI}^k = \mathcal{A}$ *each user* k, *i.e., every user* k *consumes all available applications. Moreover, all users play balanced strategies in this equilibrium.*

Proof. Suppose that that $1 < R < 2$ and $\alpha > 0$. Under the assumption of no-foresight, and $p = 0$, user k's net utility from consuming a set of applications $\mathcal{G}^k$ is

$$U_{DI}(\{x_a^k\}|x_a^k \in \mathcal{G}^k) = \Big(\sum_{x_a^k \in \mathcal{G}^k} \left(x_a^k\right)^{\frac{1}{R}}\Big)^R + \alpha(N-1)\frac{X^2}{A}\,.$$

Maximizing $U_{DI}(\{x_a^k\}|x_i^k \in \mathcal{G}^k)$ under the constraint that $\sum_{x \in \mathcal{G}^k} x_a^k \leq X$, yields the same first order condition for every x_a^k. Therefore, the optimal consumption schedule is a balanced consumption.

Under balanced consumption, the utility of user k's from consuming G^k applications is $U_{DI}(G^k) = \left(G^k\right)^{R-1} X + \alpha(N-1)\frac{X^2}{A}$. This utility is strictly increasing in G^k. Therefore, every user finds it optimal to consume all A available applications, and in equilibrium all users play a balanced strategy and consume $\mathcal{G}^k_{DI} = \mathcal{A}$.

This completes the proof of Proposition 17. ■

B.2.4 Game with no foresight: On the role of the platform

Under pure direct network effects, when the platform provides $A \geq 1$ applications, each user's utility in any equilibrium is

$$U^*_D = X + \alpha(N-1)\frac{X^2}{A}\,.$$

This utility strictly decreases with A. The platform with the objective to maximize users' net utility should set $A = 1$.

Under pure indirect network effects, foresight plays no role. The cases of no-foresight and perfect foresight are the same: The users' utility strictly increases in A and, therefore, the platform should provide as many applications as possible.

When both network effects are present, the unique equilibrium under no-foresight equilibrium is the same as the unique balanced equilibrium under perfect foresight: All users consume all A available applications according to the balanced consumption schedule, and achieve the equilibrium utility of

$$U^*_{DI}(G^k_{DI}\!=\!A) = A^{R-1}X + \alpha\frac{X^2}{A}(N-1)\,.$$

Therefore, the same analysis as in the case of perfect foresight leads us to conclusion that if the platform is unbounded while setting A, it should set as large A as possible. When it is bounded by $\overline{A}$, it needs to consider whether $V(\overline{A}) \geq V(1)$ or not.

References

Armstrong, Mark. 2006. Competition in two-sided markets. *Rand Journal of Economics* **37**(3) 668–691.

Caillaud, Bernard, Bruno Jullien. 2003. Chicken & egg: Competition among intermediation service providers. *RAND Journal of Economics* **34**(2) 309–328.

Casadesus-Masanell, Ramon, Francisco Ruiz-Aliseda. 2009. Platform competition, compatibility, and social efficiency. NET Institute Working Paper.

Chou, Chien-fu, Oz Shy. 1996. Do consumers gain or lose when more people buy the same brand. *European Journal of Political Economy* **12**(1) 309–330.

Church, Jeffrey, Neil Gandal. 1992. Network effects, software provision and standardization. *Journal of Industrial Economics* **40**(1) 85–104.

Dixit, Avinash K., Joseph E. Stiglitz. 1977. Monopolistic competition and optimum product diversity. *American Economic Review* **67**(3) 297–308.

Hagiu, Andrei. 2009. Two-sided platforms: Pricing and social efficiency. *Journal of Economics & Management Strategy,* **18**(4).

Hałaburda, Hanna, Mikołaj Piskorski. 2010. Platforms and limits to network effects. Harvard Business School Working Paper.

Katz, Michael L., Carl Shapiro. 1985. Network externalities, competition, and compatibility. *American Economic Review* **75**(3) 424–440.

Rochet, Jean-Charles, Jean Tirole. 2003. Platform competition in two-sided markets. *Journal of European Economic Association* **1**(4) 990–1029.

Weyl, Glen. 2010. A price theory of multi-sided platforms. *American Economic Review* (forthcoming).

Zhao, Luyi. 2010. Quantity constraints on two-sided platforms. Working paper.