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Working Paper #10-03

May 2010

Measuring Network Effects in a Dynamic Environment

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Measuring Network Effects in a Dynamic Environment *

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May 24, 2010

Abstract

This paper proposes methods for identifying indirect network effects with dynamically optimizing consumers purchasing a durable hardware good and associated software. We apply this model to a data drawn from the DVD player and titles markets. We observe model-level prices, sales and characteristics of DVD players and sales and availability of DVDs at the level of the month for 10 years. We augment these aggregate data with household survey data on player holdings. In our model, forward looking consumers buy possibly multiple DVD players over time and benefit from the evolution of the titles market. We address issues of clustering, spurious correlation and endogeneity.

1 Introduction

This paper proposes methods for identifying indirect network effects with dynamically optimizing consumers purchasing a durable hardware good and associated software. We apply these methods to data drawn from the DVD

*We with to thank the NPD Group, ICR-Centris, Home Media Magazine, and the DVD Release Report for help in securing data. Da Young Sin, Danny Chan, David Rapson, Kathleen Nosal and Jonathan Smith provided excellent research assistance. We thank the NET Institute for financial support, as well as NSF Grant SES-0551348.

player and titles markets. We observe model-level prices, sales and characteristics of DVD players and sales and availability of DVDs at the level of the month for 10 years. We augment these aggregate data with household survey data on player holdings. In our model, forward looking consumers buy possibly multiple DVD players over time and benefit from the evolution of the titles market. We address issues of clustering, spurious correlation and endogeneity.

Our work builds on the literature that has considered the estimation of network effects. The most successful of these papers focus on static environments and exploit cross-sectional variation in data (see Saloner & Shepard, 1995; Rysman, 2004; Ackerberg & Gowrisankaran, 2006; Gowrisankaran & Stavins, 2004). [1] However, much of the motivation and impetus for studying network effects has been with regards to product diffusion over time, usually with high technology products. Addressing estimation in this environment is the goal of this paper. A number of papers have taken on these issues before us, studying for example the diffusion of video cassette recorders, compact disc players, and video games. Early examples are Park (2004) and Ohashi (2003) for VCRs and Gandal, Kende & Rob (2000) for CD players. [2] These papers do not address several issues that we view as important. Typically, these papers use static demand models even though the goods in question are durable. While a few papers have a dynamic interpretation, they do not address the time series feature of the data or do not account for the mismatch between a dependent variable that varies cross-sectionally and an independent variable that varies only in the time-series.

Arguably, the closest paper to ours is Lee (2008), which like us specifies a dynamic model of demand for hardware (in this case, video game consoles). Lee differs from us in that he also specifies a structural model of demand for the complementary good, video games. This setup is appropriate for the questions of interest in the paper, which center around exclusive dealing. However, the strong assumptions on the software side of the market preclude flexibly studying the time series structure of the data in the way we envision, and Lee does not directly address the endogeneity of the two markets. Also,

[1] More recent examples of explicitly static demand systems with an element of positive feedback loops are Fan (2009) for newspapers, Jeziorski (2009) for radio stations and Rysman (2007) for payment cards. An important early citation on newspapers is Rosse (1970).

[2] There is now a relatively large literature estimating models of diffusion in markets with indirect network effects. A partial list of more recent examples is Clements & Ohashi (2005), Derdenger (2009), and Corts & Lederman (2009) for video games, and Nair, Chintagunta & Dube (2004) for personal digital assistants.

Inceoglu & Park (2004) and Park (2008) provide earlier attempts to address time series issues in DVD diffusion.

At this stage, our results are highly preliminary and frankly, not very supportive of our basic hypotheses. We present our these preliminary results and discuss potential issues that we hope to address in the near future.

2 Overview

We identify four important econometric problems with estimating network effects in a dynamic durable-goods environment, and then we propose methods for addressing these problems. The issues are as follows:

1. Dynamics: An appropriate model recognizes the dynamic nature of consumer decision-making. Consumer choice is affected by the durability of the goods and the fact that consumers can wait until a future date, and typically obtain similar quality for a lower price and realize higher values of the complementary good.

2. Clustering: In most econometric analysis of markets with network effects, we observe a panel of hardware products but we have only limited variation in the measure of the complementary good. For instance, if we believe sales of DVD players are affected by the number of titles available, we may observe sales for 200 players in a month but the number of titles varies only in the time series. If we are comparing DVD sales to VCR sales, perhaps we observe two measures of titles per period but the issue remains largely the same.

3. Spurious correlation: Under almost any diffusion process, we would expect sales of DVD players and DVD titles to increase over time even if they did not have a causal relationship. Since sales of both are correlated in time, a naive regression of one on the other will find a positive coefficient and falsely conclude a causal relationship.

4. Endogeneity: Since sales of DVD titles and players are determined simultaneously and endogenously, we expect any regression to exhibit problems of endogeneity. For instance, an unobserved shock to the demand for DVD players may lead movie producers to introduce more DVD titles, creating reverse causality in our estimation equation. [3]

[3] To clarify, we view spurious correlation and endogeneity as separate and distinct problems. For instance, sales of Commodore 64 computers and mini-vans exhibit spurious

We propose a method that addresses these four issues. In order to address the first problem, we use a structural dynamic model of consumer behavior. In particular, we adapt the model of Gowrisankaran & Rysman (2009) to our context. Gowrisankaran & Rysman (2009) allows for persistently heterogeneous consumers to purchase one of the available products or wait based on rational expectations about the future evolution of market characteristics. The model is designed to be applied to aggregated data such as ours and allows for endogenous prices and changes in the number of products over time. We adapt the model to allow for a complementary good and importantly for our purposes, to allow consumers to hold multiple products, whereas Gowrisankaran & Rysman (2009) requires consumers to hold no more than one unit of a product at a time.

To address the second problem (clustering), we recognize that it is akin to the problem confronted in the treatment effects literature, in which researchers often employ panels with thousands of households to study policy changes that vary only across states. Moulton (1990) and Donald & Lang (2007) show that common state-time shocks make the proper construction of standard errors challenging in this context. Donald & Lang (2007) recommend estimating with state-time dummies in a first stage and then regressing the dummies on the policy variables in the second stage, and show that although the second stage has many fewer observations than the first, it actually gives the correct standard errors.

Our approach is analogous. Since our concern is with a "policy" variable that varies only in the time series, we introduce time dummy variables. As we show formally below, the month dummies can be interpreted as the expected current and future network benefits to a consumer at a given time, plus any other features that vary only in time. Importantly, we construct our structural model so that the addition of month dummies does not significantly increase the computational time of our model. Because titles, and hence expectations about current and future titles, vary only over time and not cross-sectionally, the time dummies in our model capture the complementary goods part of utility.

Thus, the structural model is a "first stage" in our estimation procedure, designed to provide us with a set of coefficients on time dummies. In our "second stage", we regress this sequence of dummy variable coefficients on variables from the titles market using standard time series techniques. Note

correlation. They were introduced at similar times and growing sales over time, although there was no endogeneity between them. In contrast, endogeneity could be realized in a purely cross-sectional data set, but not spurious correlation.

that this second stage will have many fewer observations than the structural estimation. This two-stage approach generates appropriate standard errors for the relationship between the titles and player market, following Donald & Lang (2007). It also allows us to deal with the third problem, spurious correlation. The second stage is a purely time series regression so we can incorporate standard tools from time series econometrics to address spurious correlation. We test for integration and heterogeneity of various orders and in particular, cointegration between the time dummy coefficients and titles variables. There is a growing recognition that time series issues should play an important role in microeconometric studies, and this paper contributes to that stream of research (see Bertrand, Duflo & Mullainathan, 2004; Angrist & Pischke, 2009).

The fourth issue is endogeneity. We turn to the feature film market to provide instruments. At least early in the product life when DVD sales were relatively small, activity in the film market can be characterized as exogenous to the DVD market. Chiou (2008) shows that the time period between a film's introduction and the release of a DVD varied between 4 and 6 months over our time period, and we have independent data to study this. Hence, intuitively our instrumenting assumption is that if we see sales of DVD players shifting up 4 to 6 months after a big weekend at the box office, we assume that this is happening through the titles market and is evidence of a network effect. That is, the box office affects titles but is otherwise excluded from affecting the player market.

Our goal is to be very flexible about the way that the titles market might affect the player market. An advantage of our approach is that the only computationally expensive step is the structural first stage. The step we wish to be flexible in, the relationship between the time dummy coefficients and the variables capturing the titles market, is computationally cheap. Not only can we try many forms of time series processes, but we can also experiment with different summary statistics from the titles market. A common question in this literature is about the appropriate measure of activity in the titles market: Is what matters sales of titles, the number of titles, the presence of a big hit or multiple big hits? Since we have data on each of these variables and specifications are computationally cheap – and yet consistent with dynamic optimization – we can explore all of these.

Overall, estimation of network effects models in the canonical dynamic, durable goods setting presents serious econometric challenges. We propose a polyglot method, drawing on ideas from structural micro-econometrics, treatment effects, time series and instrumental variables to address these problems. Our method addresses each of the important problems that we have identified and allows the researcher a great deal of flexibility in studying the role of network effects.

3 Data

Our data set is drawn from a variety of sources. The centerpiece comes from the NPD Group and contains monthly level observations on price and sales for DVD players, at the level of the model. We have data from March 1997 to October 2006, a long panel that reaches back to what was essentially the start of the industry. These data are drawn from relationships that NPD has with a large set of consumer electronics retailers, but unfortunately does not include WalMart or on-line sales.[4] For each model, we collected characteristics by hand based on web searches. For DVD players, characteristics are typically dummy variables for features, such as progressive scan or DTS audio capability. We also collected volume and weight although we restrict ourselves to console DVD players, as opposed to portable DVD players so this should be less important. We do not have data on other items, such as personal computers, that also have DVD capability.

Figure 1 shows the number of models that appear in our data each month over time. The growth in the number of products is startling. We observe 9 products in the first month of the data, which increases almost monotonically to show more than 350 products throughout 2006. Figure 2 shows sales (in units) by month. Like many consumer electronics products, a great deal of sales of DVD players takes place during the holiday shopping season in the fourth quarter. Conditional on that, DVD sales climb from 1997 to 2004. Interestingly, sales level off and begin to decline after 2004. That is, we observe a sort of maturation of the DVD market in our data set. Prices make a dramatic decline. Figure 1 graphs the sales-weighted average price normalized to 2000 dollars. It reaches a high in the third month of data at $766.30 and drops just below $100 in the final year of data.

DVD players are only useful with DVDs. We have obtained a monthly time series from January 2001 to September 2008 on sales of pre-recorded DVDs from the Research Department of Home Media Magazine, which uses information from Nielson VideoScan. Like NPD, Nielson's information comes from relationships with retailers, but does not include WalMart. In this case, Home Media infers WalMart sales based on their research. Fig-

[4]To be specific, NPD imputes sales at retailers that are not part of its survey, but does not attempt to impute sales at WalMart or wholesale clubs such as Costco, or on-line sales.

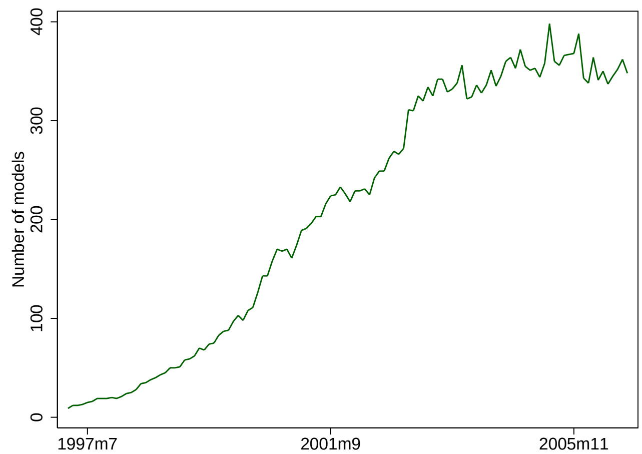

Figure 1: Number of models by month

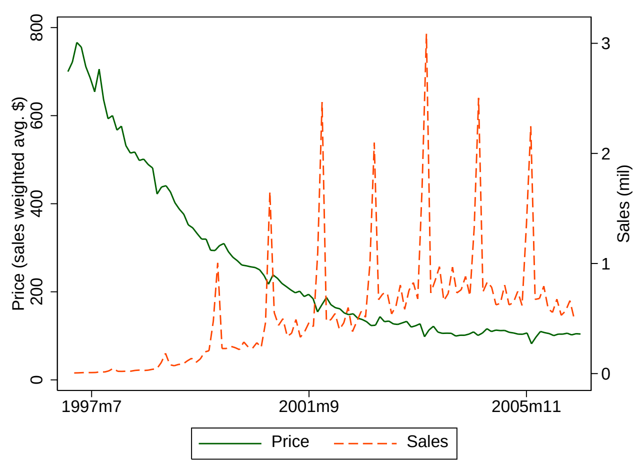

Figure 2: Number of units sold by month and average price

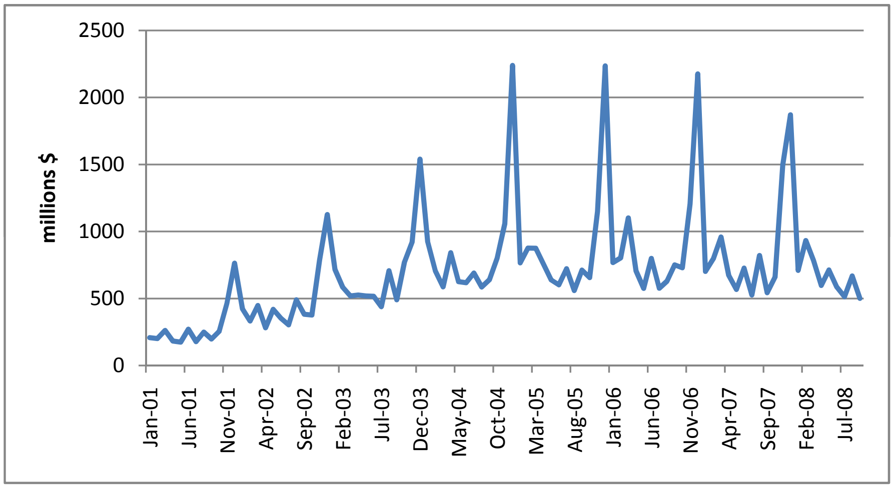

Figure 3: Sales of DVD titles by month

ure 3 graphs this time series. Similar to DVD players, the series exhibits exaggerated holiday sales and a leveling off of sales growth around 2004. The titles data overlaps with our player data from six years, from 2001 to 2006. We have also obtained data on counts of the number of available titles. In fact, we have a comprehensive data set on the release date of each title, as well as some characteristics such as genre. Hence, we know not only the number of DVD titles but their identity.

Sales of DVD titles are likely to be endogenous to sales of DVD players. As an instrument, we use outcomes from the cinema release market. We have obtained box office revenue and the number of movies released from Box Office Guru, an on-line source of movie information. We observe weekly data from the last week of 1995 to the 7th week of 2008. Figure 4 displays this variable. It is highly variable from week to week and displays less seasonal variation than the other variables.

Finally, households that make multiple purchases play an important role in our model. However, it is questionable whether one can infer the prevalence of multiple purchases from aggregate data on sales. To make progress, we use a data set from Centris of ICR, a market research firm. Centris performs a telephone survey based on random digit dialing of consumer holdings of consumer electronics. They complete about 4,000 surveys per month. They specifically ask each household how many console DVD players they hold, and they report the percentage of households that hold 0, 1, 2 or more than 2 console players. We obtained data for the third quarter

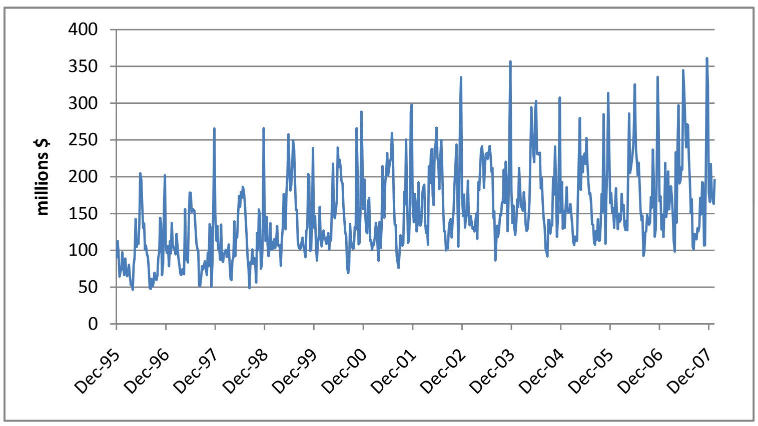

Figure 4: Box office revenue for films by week

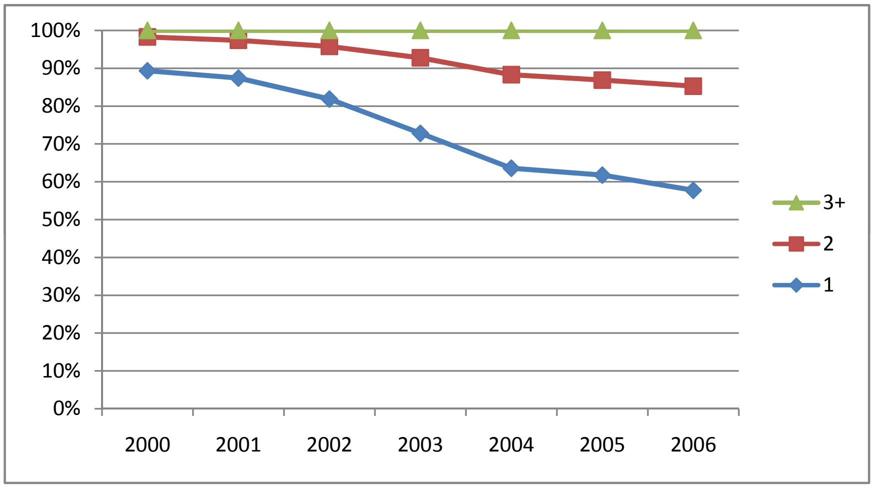

Figure 5: Number of DVD players in a household among those that have at least one

of each year from 2000 to 2006. That data appears as a stacked line chart in Figure 5. Among households that have at least one DVD player, 87.9% have only one in 2000. This number drops to 56% by 2006, with the number reporting that they have more than 2 climbing from less than 2% to 14.3%.

4 Structural Model

Here, we present our model of consumer demand that allows us to account for the issues we describe above. The model builds on Gowrisankaran & Rysman (2009) by extending it to allow for complementary goods and for households to hold multiple products. Our model starts with the introduction of a new consumer durable good at time $t = 0$. The unit of observation is a month and there is a continuum of heterogeneous potential consumers indexed by i. Consumers have infinite horizons and discount the future with a common factor β. Consumer i chooses one of among J_t products in each period t or chooses to purchase no product in the current period. From these $J_t + 1$ choices, the consumer chooses the option that maximizes the sum of the expected discounted value of future expected utilities conditional on her information at time t.

Product j at time t is characterized by observed characteristics x_{jt}, price p_{jt}, an "environmental variable" N_t and an unobserved (to the econometrician) characteristic ξ_{jt}. For DVD players, observed characteristics include the presence of advanced sound and display features such as Dolby audio and progressive scan. In our context, the environmental variable describes the title market, for instance the number of titles available at time t. The environmental variable that the consumer obtains from purchase is allowed to change over time. We assume that consumers and firms know all time t information when making their time t decisions. The additional flow utility to consumer i who purchases product j in period t is:

$$u_{ijt} = x_{jt}\alpha_i^x - \alpha_i^p p_{jt} + \theta N_t + \xi_{jt} + \varepsilon_{ijt}$$

Here, ε_{ijt} is distributed independently across consumers, products and time according to the Type I Extreme Value distribution, creating the familiar logit demand system. Consumers know only their current set of ε_{ijt}, not future values. Consumers are characterized by their demand parameters $\alpha_i = \{\alpha_i^x, \alpha_i^p\}$, which stay constant over time.

Now we turn to the dynamics of our problem. We assume that products are infinitely durable. Let $\delta_{ijt}^f = x_{jt}\alpha_i^x + \xi_{jt}$, the permanent part of the flow utility (we use the superscript f to refer to *flow* utility). Consumer i who purchases product j in period t receives $\delta_{ijt}^f + \theta N_\tau$ in all periods $\tau > t$. Notice that the value of the environmental variable can change over time whereas the value of δ_{ijt}^f cannot. Let δ_{it}^{0f} be the accumulated flow utility from all products the consumer has purchased up to time t (as in GR, we use the superscript 0 to represent the quality of the consumer's holdings):

$$\delta_{it}^{0f} = \sum_{\tau=1}^{t-1} \sum_{j}^{J_t} \delta_{ij\tau}^f \mathbf{1}\{d_{ij\tau} = 1\}$$

where $d_{ij\tau} = 1$ if consumer i bought product j in time τ, and $\mathbf{1}\{.\}$ is an indicator function.

Also, we allow for declining marginal value from holding multiple goods. Let $\hat{\psi}_n$ to be a discount to utility for holding n goods. [5] Let Ω_t denote the state of the market, which is made up of current product attributes and any other factors that influence future product attributes. We assume that Ω_{t+1} evolves according to some Markov process $P(\Omega_{t+1}|\Omega_t)$ that accounts for firm optimizing behavior. Thus, the relevant state variables for the consumer are the number of players a consumer holds, n_{it}, their accumulated quality δ_{it}^{0f}, the draws $\varepsilon_{i.t}$ and the state of the market, Ω_t. The Bellman equation is:

$$V(\delta_{it}^{0f}, n_{it}, \Omega_t, \varepsilon_{i.t}) = \max$$

[5] There is an important assumption in this statement, which is that the decline does not depend on the flow utility of the products that the consumer holds. We return to this point below.

$$\max_{j=1,\ldots,J_t} \delta_{it}^{0f}+\delta_{ijt}^{0f}+(n_{it}+1)\theta N_t+\widetilde{\psi}_{n_{it}+1}+\varepsilon_{ijt}+\beta E\left[V\left(\delta_{it+1}^{0f}, n_{it}+1, \Omega_{t+1}, \varepsilon_{i.t+1}\right)|\delta_{it}^{0f}, n_{it}, \Omega_t, \varepsilon_{i.t}\right], \quad (1)$$

$$\delta_{it}^{0f}+n_{it}\theta N_t+\widetilde{\psi}_{n_{it}}+\varepsilon_{i0t}+\beta E\left[V\left(\delta_{it+1}^{0f}, n_{it}+1, \Omega_{t+1}, \varepsilon_{i.t+1}\right)|\delta_{it}^{0f}, n_{it}, \Omega_t, \varepsilon_{i.t}\right] \quad (2)$$

Line 1 represents the value of buying and line 2 represents the value of not buying.

The value function $V(\delta_{it}^{0f}, n_{it}, \Omega_t, \varepsilon_{i.t})$ is too large for us to work with numerically, so we use various techniques to simplify it. First, note that δ_{it}^{0f} and $n_{it}\theta N_t$ enter both the value of buying and the value of not buying. Thus, we can subtract them from both values and write a decision problem that generates the same purchase decisions as the original problem. This step eliminates the state variable δ_{it}^{0f}. Furthermore, subtract $\hat{\psi_{n+1}}$ from both problems and let $\psi_n = \hat{\psi}_n - \hat{\psi}_{n+1}$. Then we have:

$$V(n_{it}, \Omega_t, \varepsilon_{i.t}) = \max$$

$$\max_{j=1,\ldots,J_t} \delta_{ijt}^{f} + \theta N_t + \varepsilon_{ijt} + \beta E\left[V\left(n_{it}+1, \Omega_{t+1}, \varepsilon_{i.t+1}\right)|n_{it}, \Omega_t, \varepsilon_{i.t}\right],$$

$$\psi_{n_{it}} + \varepsilon_{i0t} + \beta E\left[V\left(n_{it}+1, \Omega_{t+1}, \varepsilon_{i.t+1}\right)|n_{it}, \Omega_t, \varepsilon_{i.t}\right]$$

In words, flow utility from products the consumer already holds does not affect future decision-making. A critical assumption to get this result is that ψ_n depends only on the number of products, not δ_{it}^{0f}. It is restrictive, but greatly simplifies our computational problem. Similarly, we rule out that ψ_n depends on N_t, so it affects decision-making only to the extent that it affects the value of purchasing a new good. Thus, consumers care only about the number of DVD players they hold, not the characteristics of those DVD players. It is as if consumers get these benefits "up front" at the time of purchase, and afterwards, the consumer gets only the benefits of holding n goods, captured by ψ_n. [6] Because we expect that $\hat{\psi}_{n+1} < \hat{\psi}_n$, we expect that $\psi_{n+1} > \psi_n$.

Next, we note that because $\varepsilon_{i.t}$ is *iid*, it satisfies the assumption of conditional independence (Rust, 1987) and may be integrated out. We work with $EV(n_{it}, \Omega_t)$, where:

$$EV(n_{it}, \Omega_t) = \int_{\varepsilon} V(n_{it}, \Omega_t, \varepsilon) f(\varepsilon) d\epsilon$$

[6] This assumption is similar to one made in Hendel & Nevo (2006). They assume that only the amount of laundry detergent a household holds affects dynamic decision making but that brand characteristics still affect the household at the time of purchase.

Finally, we turn towards simplifying Ω_t. Define δ_{it}^f to be the expected value of the flow utility (u_{ijt}) to consumer i that chooses among the products available in period t. Because we have assumed logit errors, this expected value takes on a convenient closed form, known as the inclusive value:

$$\delta_{it}^f = \ln\left(\sum_{j=1}^{J_t} \exp(\delta_{ijt}^f - \alpha_i^p p_{jt} + \theta N_t)\right). \tag{3}$$

If a consumer knew current and future values of δ_{it}^f, the consumer would have enough information to optimally choose when to make her next purchase. The consumer does not need to know Ω_t, which simplifies the value function. That is, n_{it}, δ_{it}^f and the contingent path of δ_{it}^f are sufficient statistics to define $EV_i\left(n_{it}, \Omega_t\right)$. Formally,

$$EV_i\left(n_{it}, \Omega_t\right) = EV_i\left(n_{it}, \delta_{it}^f, P\left[\delta_{i,\tau+1}^f | \Omega_\tau\right]\right). \tag{4}$$

Gowrisankaran & Rysman (2009) and Melnikov (2001) prove this point formally. This result follows from assuming the logit functional form for ε_{ijt} and does not require further assumptions.

Unfortunately, Equation 4 does not generate a numerical simplification since consumers should still predict future values of δ_{it}^f using all of Ω_t. In order to make progress, we make an important simplifying assumption on how consumers make predictions. In particular, we assume that consumers use only the current value of δ_{it}^f to predict Ω_t. Following Gowrisankaran & Rysman (2009), we refer to this as the assumption of Inclusive Value Sufficiency.

Assumption 1 *Inclusive Value Sufficiency (IVS)*

If two states Ω_t and Ω_t' generate the same value of δ_t^f, then $P_i\left(\delta_{it+1}^f)|\Omega_t\right) = P_i\left(\delta_{it+1}^f)|\Omega_t'\right)$ for all t and Ω_t, Ω_t'.

The assumption of IVS implies that all states with the same n_{it} and δ_{it}^f have the same continuation value, and so Ω_t become unnecessary. Thus, the state space is reduced to two dimensions. The IVS assumption can be interpreted as an assumption that consumers are boundedly rational and use only a subset of the data potentially available to them in forming their predictions. The assumption is restrictive. For example, δ_{it}^f could be high either because there are many products in the market all with high prices or because there is a single product in the market with a low price. While dynamic profit maximization might lead these two states to have different patterns of industry evolution, consumers in our model will lump them into the same state.[7]

For our specifications we assume that consumer i perceives $P_i\left(\delta^f_{i,t+1}|\delta^f_{it}\right)$ as its actual empirical density fitted to a simple functional form and use a simple linear autoregressive specification,

$$\delta^f_{i,t+1} = \gamma_{1i} + \gamma_{2i}\delta^f_{it} + \nu_{it}, \tag{5}$$

where ν_{it} is normally distributed with mean 0 and γ_{1i} and γ_{2i} are incidental parameters specific to each consumer i. By assuming that consumers make predictions based on the parameters from (5) derived from the realized values of δ_{it}, we are assuming that consumers have rational expectations, conditional on the restriction in (5).

Note that our IVS assumption and Equation 5 are statements only about exogenous items such as the current numbers of products, prices, and characteristics. In this sense, our assumptions are more similar to those in Melnikov (2001) and Hendel & Nevo (2006) than Gowrisankaran & Rysman (2009). This follows from our model in which only the number of products has dynamic content, not the characteristics of those products. This assumption also makes computation much easier. Without this assumption, the characteristics of the product would affect not only which product the consumer chooses today but also future decision-making, and so must play a role in the value function. Thus, N_t, the features of the titles market, would affect the state of the consumers, which means we could not write the time dummies as a linear function of mean utilities and we would have to search over time coefficients non-linearly (for more on this, see Section 5), which would be infeasible. Hence, the assumption that the value function depends on the number of products held and not their characteristics generates a major computational savings. We also believe it is a reasonable assumption, but we discuss this more later.

An implication of (5) is that, for $0 < \gamma_{2i} < 1$, a graph of mean δ^f_{it} against time finds a concave line with an asymptote that is approached from below. This asymptote is important in our model since it represents a steady state in the evolution of product characteristics that the consumer expects to approach. The eventual arrival of a steady state is what allows us to treat the consumer as facing a stationary environment, even though observed choices are evolving quickly. In practice, we estimate $0 < \gamma_{2i} < 1$

[7] Hendel & Nevo (2006) and Gowrisankaran & Rysman (2009) provide a similar discussion of the implications of Assumption 1.

for all types i, and we find that consumers view the current value of δ_{it}^f as substantially below the asymptote, so that consumers believe the market is improving for well after the time frame of our data set.

The logit assumption on ε_{ijt} generates a convenient closed for solution for the Bellman Equation that we exploit when solving the problem:

$$\begin{aligned} EV_i\left(n_{it}, \delta_{it}^f\right) &= \ln\Big(\exp\left(\delta_{it}^f + \beta E\left[EV_i(n_{it}+1, \delta_{it+1}^f)|n_{it}, \delta_{it}^f\right]\right) \\ &\quad + \exp\left(\psi_{n_{it}} + \beta E\left[EV_i\left(n_{it}, \delta_{it+1}^f\right)\Big| n_{it}, \delta_{it}^f\right]\right) + \gamma.\Big) \end{aligned}$$

Notice how our method casts the dynamic decision as a single binary choice about when to buy, similar to an optimal stopping problem. Conditional on buying, the consumer chooses what to buy, but we can abstract away from this choice in the dynamic problem.

5 Inference

This section discusses the parametrization and estimation of the model. Our methods for estimating the model follow closely those in Gowrisankaran & Rysman (2009) and so we cover them only briefly here. Integrating over consumers i does not generate a closed-form solution for the market shares for products. Hence, we simulate consumers by drawing consumer deviations. In practice, we assume that $\alpha_i \sim \mathbb{N}(\alpha, \Sigma)$, where Σ is non-zero only on the diagonal of the matrix. We draw from the standard normal to represent consumer deviation from the mean and estimate α and Σ to create each consumer's α_i.

We do not attempt to estimate β because it is widely understood to be unidentified in dynamic decision models (see Magnac & Thesmar, 2002). This is particularly true for our model, where substantial consumer waiting can be explained by either little discounting of the future or moderate preferences for the product. Thus, we set $\beta = .99$ at the level of the month.

For purposes of this section, we define mean utilities to consumers and products:

$$\begin{aligned} \delta_{ijt} &= x_{jt}\alpha_i^x - \alpha_i^p p_{jt} + \theta N_t + \xi_{jt} \\ \delta_{jt} &= x_{jt}\alpha^x - \alpha^p p_{jt} + \theta N_t + \xi_{jt}. \end{aligned}$$

Here, α^x and α^p are the mean values of α_i^x and α_i^p. Because δ_{ijt} and δ_{jt} are a flow utilities, they should be supercripted with "f" by our notation, but we leave it off for simplicity. Because we can write α and ξ_{jt} as a linear functions of δ_{jt}, we can "concentrate out" α, as in Nevo (2000). Hence, our estimation algorithm solves for δ_{jt} as a function of the remaining parameters Σ and ψ_n, and then constructs moments of ξ_{jt} based on matrix algebra techniques, so we search over only Σ and ψ_n. In estimation, the vector α includes a set of time dummy coefficients. Being able to solve for the time dummies coefficients in this way, as opposed to searching for them non-linearly, is important since there are a great number of them.

For any given set of parameters Σ and ψ_n, we start with a guess of δ_{jt}. Based on this, we construct individual flow utilities δ_{ijt} using the draws of consumer deviations from the mean. We then construct δ_{it}^f based on Equation 3. Then, we perform the AR(1) regression Equation 5 for each consumer i separately, thereby recovering belief parameters γ_i. Because we discretize the state space, we convert the parameters to a transition matrix following Tauchen (1986). Then, for each consumer separately, we guess a starting value for the value function and solve the Bellman equation (Equation 4) by successive approximations.

Once we have value function $EV_i(n_{it}, \delta_{it}^f)$, we are ready to solve for conditional and unconditional probabilities of purchase. Conditional probabilities of purchase are as follows. For consumer i in period t who holds n_{it} products and faces a market with δ_{it}^f, the probability of purchase is:

$$P_{it}(n_{it}, \delta_{it}^f) = \frac{\exp\left(\delta_{it}^f + \beta E\left[EV_i(n_{it}+1, \delta_{it+1}^f)|n_{it}, \delta_{it}^f\right]\right)}{\exp\left(\delta_{it}^f + \beta E\left[EV_i(n_{it}+1, \delta_{it+1}^f)|n_{it}, \delta_{it}^f\right]\right) + \exp\left(\psi_{n_{it}} + \beta E\left[EV_i\left(n_{it}, \delta_{i,t+1}^f\right)\middle| n_{it}, \delta_{it}^f\right]\right)}$$

Conditional on purchasing in period t, consumer i picks product j with probability:

$$P_{ij|t} = \frac{\delta_{ijt}}{\sum_{k=1}^{J_t} \exp(\delta_{ikt})}.$$

In order to compute the unconditional probabilities, the market shares, define the $(T+1) \times (\overline{n}+1)$ matrix s_i for each consumer i. Here, T is the number of periods in the data set, $\overline{n}$ is the maximum number of products a consumer may hold, and s_i is the share of consumers of type i holding each number of products at each period. We index s_i from 0 to T and from 0 to $\overline{n}$. We assume that for each i, the first element row is a vector of zeros, with the first element being 1. That is, everyone holds zero products in period 0. [8] Then, we can use P_{it} to successively fill out each row of s_i. For instance,

[8] This is reasonable because our data set reaches back to what is essentially the onset of the industry. For an alternative approach, see Schiraldi (2009) who estimates an initial distribution in the used car market.

$s_i[1,1] = 1 - P_{it}(0, \delta^f_{i1})$ and $s_i[1,2] = P_{it}(0, \delta^f_{i1})$. [9] Because consumers cannot buy more than one product in a period, $s_{it}[1,n] = 0$ for $n > 2$. Element t, n of s_{it} is $P_{it}s_{it}[n-1, t-1] + (1 - P_{it})s_{it}[n, t-1]$, the sum of purchasers who held $n-1$ products and non-purchasers who held n products in period $t-1$.

With these elements, we can compute market shares. The market share predicted by the model of product j in period t is:

$$\widehat{s_{jt}} = \sum_{i=1}^{ns} P_{ij|t} \left(\sum_{n=0}^{\overline{n}} P_{it}(n, \delta^f_{it}) s_i[t-1, n] \right).$$

Here, ns is the number of consumer types that we sample. That is, we sum over each consumer type the set of consumers holding each number of products in the previous period multiplied by the probability of choosing product j.

We use the fixed point equation of Berry, Levinsohn & Pakes (1995) to generate a new guess for δ_{jt}. In vectors, where s^0 is the observed data, δ is the vector of elements δ_{jt} and $\widehat{s(\delta)}$ is the resulting market shares:

$$\delta' = \delta + \ln(s^0) - \ln(\widehat{s(\delta)})$$

Thus, we iteratively compute δ until we find one that generates the observed market shares. Although we cannot prove that there is a unique solution, we have not had any problems with convergence. Gowrisankaran & Rysman (2009) discusses this issue further.

Based on the resulting vector δ, we compute $\xi_{jt} = \delta_{jt} - (x_{jt}\alpha^x - \alpha^p p_{jt})$. We can solve for ξ_{jt} using matrix algebra techniques as described in Nevo (2000). We form moments with the resulting vector ξ_{jt} using instruments z_{jt}. Thus, our objective function is:

$$\left(\hat{\alpha}, \hat{\Sigma}, \hat{\psi}\right) = \arg\min_{\alpha,\Sigma,\psi} \left(z'\xi(\alpha, \Sigma, \psi)\right)' W \left(z'\xi(\alpha, \Sigma, \psi)\right), \tag{6}$$

where W is a weighting matrix. As is standard, we obtain GMM estimates in two steps. We first set $W = z'z$, which is efficient under the assumption of homoskedastic errors and generates consistent estimates, and then we construct the efficient weighting matrix allowing for arbitrary heteroskedasticity.

In practice, we draw 48 consumers ($ns = 48$). We discretize δ_{it} into 50 bins stretching from -40 to 0, which is much greater than the span of what we

[9] We use the notation $s_i[l, m]$ to denote the element in row l and column m of matrix s_i.

observe in our model. We set the maximum number of products a household can hold to 4 ($\overline{n} = 4$). In our results, less than 2% of households hold four DVD players at the end of the sample. We use importance sampling as described in Gowrisankaran & Rysman (2009) to reduce sampling error. We assume there are 100 million households in the United States during this time period, although in practice this changes from about 95 million to 105 million. Incorporating a growing market is straightforward but we have not done this yet.

We normalize $\psi_0 = 0$ and $\psi_1 = 0$ and parameterize

$$\psi_n = \psi(n-1) \quad \texttt{for} \quad n > 1.$$

Normalizing $\psi_0 = 0$ is the standard normalization in any discrete choice model that one option must have zero utility. We normalize $\psi_1 = 0$ because it is collinear with the constant term.

In order to identify the ψ parameter, we incorporate micro-moments in the spirit of Petrin (2002) and Berry, Levinsohn & Pakes (2004). We do not use the survey data to establish how many households have purchased, as we are concerned that because our data set does not cover all retailers, it may mismatch in this dimension. Instead, we use survey data to determine holdings among households that hold at least one player. We use the ICR survey mentioned above to identify 2 moments at 7 time periods for 14 moments: the percentage of households holding one console DVD player amongst those holding a console DVD player annually from March 2000 to March 2006, and the percentage holding two. The remaining households hold three or more. We compute the equivalent moments by summing over consumer types with the appropriate row of s_i. We include the difference between the models predictions and the ICR data as moments, vertically concatenated onto $z'\xi$ in Equation 6. We expand the weighting matrix by 14 elements in each dimension. The diagonal elements of the weighting matrix should be the inverse of the variance of the moment. For variance, we use $(p)(1-p)/4000$, where p is the value of the moment in the data, and 4000 is the approximate number of households sampled in each period. As this variance is very small, our weighting matrix puts a high weight on the micromoments so our estimation algorithm attempts to match these very closely.

We search using non-derivative methods such as the Nelder-Mead algorithm and direct search techniques. All programs are available on request.

5.1 Second Stage

Now consider θ. Rather than identify θ from the structural model, we identify θ from correlation between the month dummy coefficients and the exogenous variables representing the DVD titles and feature film markets. In this second stage, we plan to consider complicated dynamic processes for the coefficients and N_t and their relationship, apply appropriate tests for co-integration and other issues and introduce instruments for N_t. Little is known about how the titles markets affect player markets, and hence we propose a model that allows allows maximum flexibility and low-cost specification searching over this issue while still capturing dynamic consumer behavior appropriately over this durable and rapidly changing good. In theory, whatever specification we find to be superior could be imposed in the structural model and we could estimate θ in one step.

6 Results

In this section, we present the results of our model. Our results are preliminary and, in some, respects, problematic. We discuss these issues and propose some possible problems in our approach so far.

We estimate the model described above. At our estimated parameters, our model predicts that less than 2% of consumers hold 4 DVD players so we do not view this as a binding constraint. We set the discount rate to 0.9, which is too low to reflect monthly discounting. Our current experiments suggest that results do not change much as we increase the discount rate. We plan to use a higher discount rate in our final version of the paper.

We include brand dummies in our model. In practice, we aggregate brands with less than 70 observations (for instance, 5 models for one year would be 60 observations) into a single brand. This aggregate brand still accounts for less than 5% of the data.

Results appear in Table 1. We include several dummies for quality in the linear specification of utility. They indicate whether the DVD player has an S-video outlet, whether it can use the DTS sound standard, whether it can play compact disks that use read or re-write states (CD-R and CD-RW), whether the player can play MP3 files, whether it is "dual" player that can also play VHS tapes, whether it uses Progressive Scan technology and whether it can record DVDs. We find all of the coefficients are positive and significant.

We include in price in logs. Note that it is difficult to justify log price in a utility function and most other similar papers use price in levels, for instance

Constant	0.773	(1.411)
ln Price	-2.251	(0.358)
S-Video outlet	0.354	(0.051)
DTS sound capable	0.208	(0.050)
Plays CD-R/RW	0.124	(0.032)
Plays MP3	0.584	(0.070)
Plays VHS	1.139	(0.085)
Uses Progressive Scan	0.891	(0.110)
Records to DVD	1.357	(0.238)
Random coefficient:		
ln price	0.187	(0.118)
Psi	0.027	(0.106)
Observations	11,534	

Table 1: Results from structural estimation

Berry et al. (1995). However, logit based models can be interpreted as log-linear models (see Berry, 1994), log independent variables seem natural, and we find that price in logs seems to fit the data better. We plan to experiment with price in levels as well. Logged price is negative and significant, with a coefficient of -2.251. We allow for only one random coefficient, and this is on price. We find a find a coefficient of 0.187, which is significant but particularly large relative to the level price.

We estimate one coefficient ψ, which indicates how much the flow utility increases each time a consumer adds a product. We find a value 0.027, although it is imprecisely measured. This is a permanent flow, so it can multiplied by $1/(1-\beta)$ in order to compare to price. Thus, it is not economically large. Note that in our model, this parameter is the only source of dynamics.

Finally, we estimate time dummies. These appear in Figure 6. The most striking aspect of the time dummies is their marked downward slope. That contrasts with actual sales, which we know were increasing during this time. We believe that a prime issue here is the rapid increase in the number of products. Since the number of products increases faster than sales, sales per product falls over time, which is what the time dummies match. Thus, it will be important to include the number of products in a period as an explanatory variable in our second-stage estimation.

Figure 6 also shows DVD revenue of this time. It displays the same seasonality as the time dummies. Thus, it will be important to control for the month of the year. By way of comparison, we also graph new DVD titles against the time dummies in Figure 7.

Our second stage estimation appears in Table 2. The equation we estimate is:

$$\theta_t = \beta_0 + \beta_1\theta_{t-1} + \beta_2 \ln(J_t) + \beta_3 \ln(N_t) + \beta_4 X_t + u_t$$

In this regression, θ_t is a time dummy coefficient from the first stage estimation. Recall that J_t is the number of products available in a period. The DVD title market is captured by N_t. We experiment with three variables here, the number of new DVD titles, the number of new DVD titles associated with cinema-released movies, and DVD revenue. The X_t variables are other explanatory variables, which in particular are month-of-the-year dummy variables. In some specifications, we instrument for $\ln(N_t)$. For an excluded variable, we use the log of box office revenues from five months previous.

First, consider the three specifications with out instruments, in the top panel of Table 2. The autoregressive term is extremely large, greater than

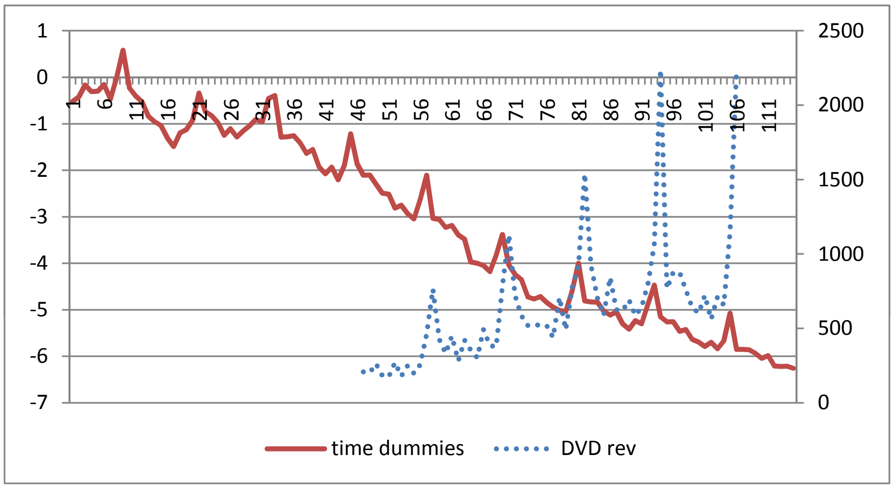

Figure 6: Time dummy coefficients, and DVD revenue

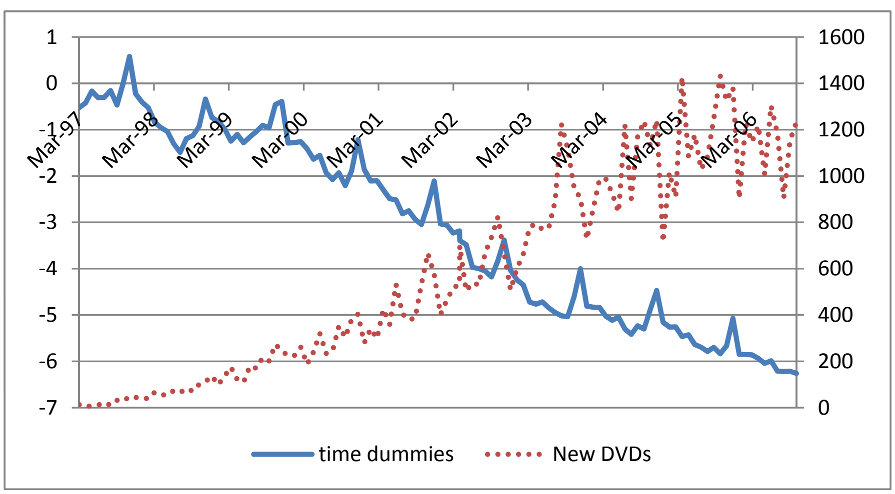

Figure 7: Time dummy coefficients, and the number of new DVD titles

lag time coefficient	2.193	(0.079)	2.274	(0.079)	2.900	(0.268)
# of players (ln)	0.285	(0.155)	0.150	(0.125)	1.212	(0.474)
new DVD titles (ln)	-0.495	(0.113)				
new DVD movie titles (ln)			-0.500	(0.111)		
DVD revenue (ln)					-0.344	(0.232)
lag time coefficient	2.292	(0.319)	2.181	(0.178)	5.499	(15.144)
# of players (ln)	-1.641	(5.747)	-0.765	(1.472)	-0.241	(8.505)
new DVD titles (ln)	1.054	(4.621)				
new DVD movie titles (ln)			0.451	(1.529)		
DVD revenue (ln)					2.937	(19.112)
Obs.	115		115		70	

In lower panel, we use Box Office Revenue (ln, 5 month lag) as an IV for DVD variables.
Standard errors are in parenthesis.

Table 2: Second stage results: Time dummy coefficients as the dependent variable

2 in each specification. The number of players is positive, which seems very surprising given their strong negative correlation. The three variables of interest, the measures of the titles market, all point the wrong way.

In contrast, results appear somewhat more as expected when we instrument for the titles market variables with box office revenue. In these regressions, the number of players on the market has a negative effect on the time coefficients. We believe that this is driven by simple arithmetic of sales per player varying in the number of players, but it may also reflect crowding effects discussed in Ackerberg & Rysman (2005). In these regressions, we find that the titles market has a positive effect on the time coefficients. However, the effects are very imprecisely measured, and are not significant.

While at this stage, our results are not very compelling, we see several paths to follow for continued work. Given our very small estimate of ψ, we are finding that demand is essentially static. That seems unlikely, and we plan on continued experimentation with the structural model to explore this issue. Furthermore, our implementation of the second-stage time series regression has so far been unsophisticated, and we believe that continued explorations with more thoughtful specifications will yield the expected results.

7 Conclusion

This paper proposes methods for estimating a network in a dynamic environment. We address a series econometric issues that have not been well-documented in the previous literature. Our (very) preliminary results do not find an important network during the time period of our data.

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