A Behavioral Finance Explanation for the Success of Low Volatility Portfolios

Malcolm Baker
Harvard Business School and NBER

Brendan Bradley
Acadian Asset Management

Jeffrey Wurgler
NYU Stern School of Business and NBER

December 18, 2009

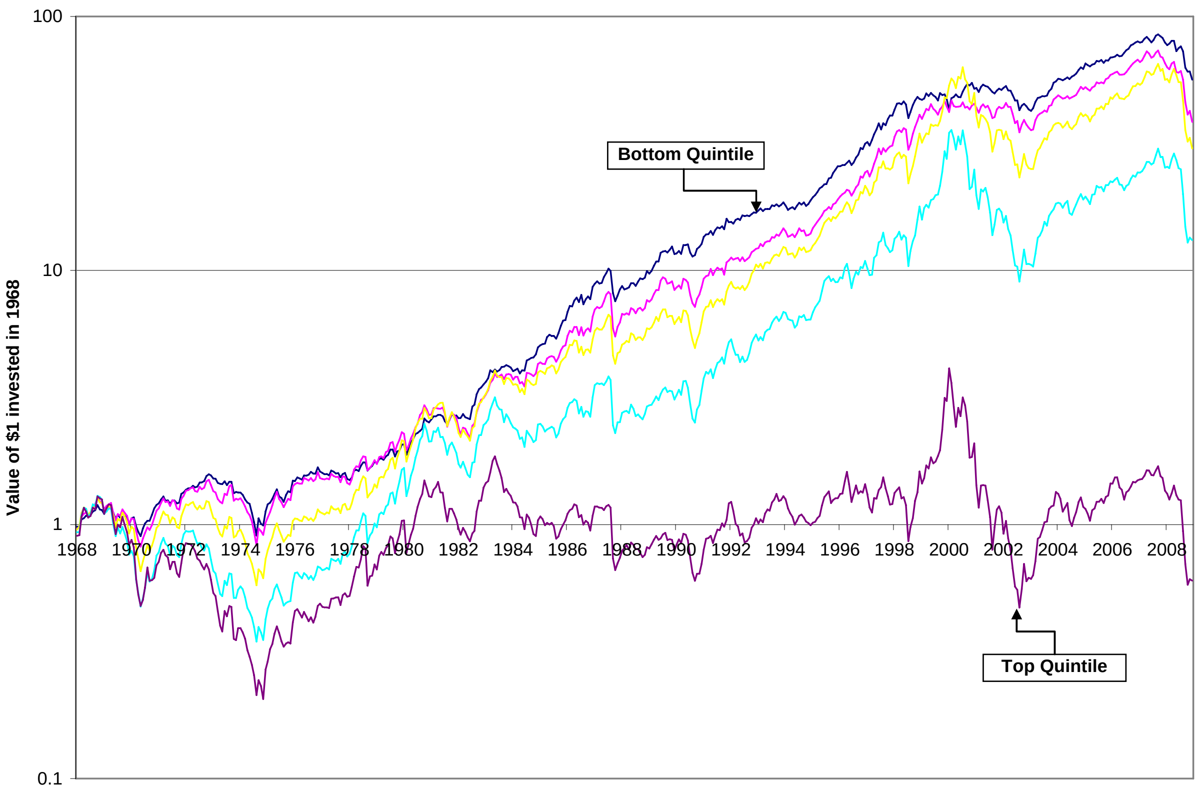
Abstract

Arguably the most remarkable anomaly in finance is the violation of the risk-return tradeoff within the stock market: Over the past 40 years, high volatility and high beta stocks in U.S. markets have substantially underperformed low volatility and low beta stocks. We propose an explanation that combines the average investor's preference for risk and the typical institutional investor's mandate to maximize the ratio of excess returns to tracking error relative to a fixed benchmark (the information ratio) rather than the Sharpe ratio. Models of delegated asset management show that such mandates discourage arbitrage activity in both high alpha, low beta stocks and low alpha, high beta stocks. This explanation is consistent with several aspects of the low volatility anomaly including why it has only strengthened even as institutional investors have become more numerous.

Introduction

Over the past 40 years, low volatility portfolios have offered an enviable combination of high average returns and small drawdowns. We use the principles of behavioral finance to examine the source of this anomalous performance and to consider the chance that it will persist. Behavioral models of security prices combine two ingredients: the irrationality of some market participants and the limits to arbitrage – why the smart money does not offset any irrational demand. Specifically, we believe that a preference for lotteries and well established biases of representativeness and overconfidence lead to an irrational demand for higher volatility stocks. Moreover, most institutional investors that are in a position to offset this irrational demand have fixed benchmark mandates, which by their nature discourage investments in low volatility stocks. Applying the logic of Brennan’s (1993) model of agency and asset prices, we show that fixed benchmark mandates cause institutional investors to pass up the superior risk-return tradeoff of low volatility portfolios. For that reason, the strong performance of these strategies is likely to persist.

Figure 1. Returns by volatility quintile.



Source: Acadian calculation using data from the Center for Research on Security Prices (CRSP). In each month, we sort all publicly traded stocks tracked by CRSP with at least 24 months of return history into five equal quintiles according to trailing volatility. Volatility is measured as the standard deviation of up to 60 months of trailing returns. In January 1968, $1 is invested, according to capitalization weights, in the stocks of, for example, the bottom quintile. At the end of each month, each portfolio is rebalanced, with no transaction costs included.

Behavioral Finance and Low Volatility Portfolios

In an efficient market, investors only realize above average returns by taking above average risks. In other words, risky stocks have high future returns on average, and safe stocks do not. This simple empirical proposition has been hard to find in the history of US stock returns. Most intuitive measures of risk go in the wrong direction. For example, we take the last 40 years of data from the Center for Research on Security Prices (CRSP) and divide all stocks into five groups in each month according to trailing volatility. **Figure 1** shows that a dollar invested in the highest risk portfolio in January 1968 would be worth \$0.61 at the end of December 2008, assuming no transaction costs. A dollar invested in the lowest risk portfolio would be worth \$56.38. The path to that higher dollar value is also considerably smoother. Adding to the puzzle, much of the gap comes in recent years after 1983 – over a period when institutional investment managers have become more numerous, better capitalized, and more quantitatively sophisticated.

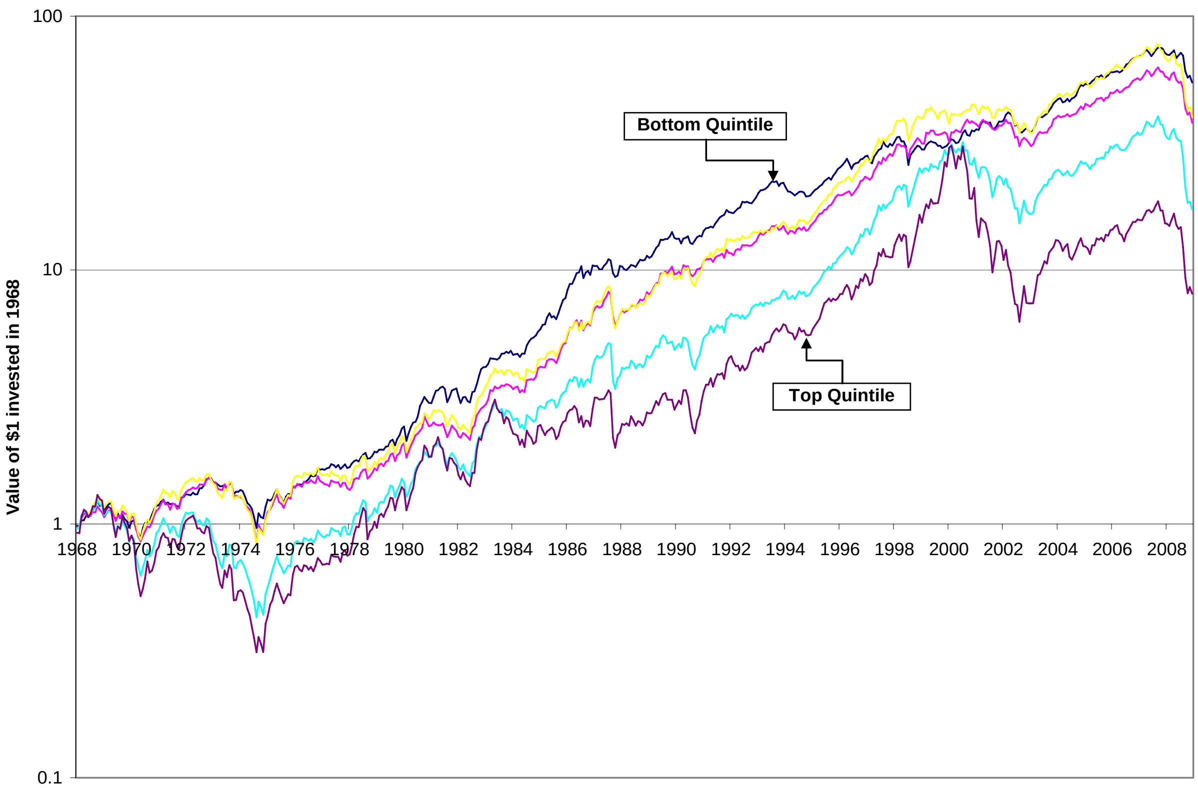
What Explains This Difference in Performance?

One possibility is that we have the wrong measure of risk. Maybe volatility is the wrong idea. Individual securities are not typically held in isolation. So, the right measure of risk is not the *level* of risk for a particular stock but its *contribution* to the risk of a diversified portfolio of stocks. This is the logic behind the Capital Asset Pricing Model (CAPM). The risk of a stock, or any other security for that matter, is its beta, which measures the contribution to risk in a broadly diversified market portfolio. This is a less intuitive, but theoretically more appealing notion of risk. Unfortunately, this refined measure of risk goes in the wrong direction, too. If instead we divide stocks into five groups in each month according to trailing beta, the picture looks better. But, not by much. **Figure 2** shows that a dollar invested in the highest risk portfolio returns \$8.07, while a dollar invested in the lowest risk portfolio returns \$54.78. With the exception of the technology bubble in the late 1990s, more of the gap again appears after 1983.

This is not entirely new. In the 1970s, Black (1972) and Black, Jensen, and Scholes (1972) noted that the relationship between risk and return was flatter than predicted by the CAPM. Fama and French (1992) extended this analysis out to 1992 and found that the relationship was flat, pronouncing that “beta is dead.” More recently, Ang, Hodrick, Ying, and Zhang (2006, 2009) set off a new wave of research, finding that high volatility stocks have “abysmally low returns” in both US and international data. If anything, the evidence for a risk-return tradeoff along the lines of the CAPM has deteriorated in the last 30 years.

These patterns are hard to explain with traditional, rational theories of asset prices. In principle, beta might simply be the wrong measure of risk, too. The CAPM is just one equilibrium model of risk and return, with unrealistic assumptions. For the past few decades, finance academics have devoted considerable energy to developing rational models, searching for the "right" measure of risk. Most of these newer models make the mathematics of the CAPM look quaint. But, despite superior computational firepower, this is a steep uphill battle. After all, the task is to prove that high volatility stocks are much *less* risky. Granted a less risky stock might not be less volatile, but it must at least provide insurance against bad events. A low average return on homeowner's insurance is acceptable, to make an analogy, because it provides a very high return at the right moment.

Figure 1. Returns by beta quintile.



Source: Acadian calculation using data from the Center for Research on Security Prices (CRSP). In each month, we sort all publicly traded stocks tracked by CRSP with at least 24 months of return history into five equal quintiles according to trailing beta. Volatility is measured as the standard deviation of up to 60 months of trailing returns. In January 1968, $1 is invested, according to capitalization weights, in the stocks of, for example, the bottom quintile. At the end of each month, each portfolio is rebalanced, with no transaction costs included.

A closer look at the pattern of returns reveals why this is such a hard argument to make for high volatility stocks. The top quintile as a group provided a very low return in precisely those periods when an insurance payment would have been most welcome. Underperformance is visually apparent in the downturns of 1972-1974 and 2000-2002, the crash of 1987 and the financial crisis in the fall of 2008. The ingenuity of financial economists applying axioms of rationality should never be underestimated. But, they seem unlikely to be able to explain **Figure 1**, where investors appear to be paying an insurance premium each month only to lose even more when the market and the economy as a whole are melting down.

A Behavioral Explanation

If the answer is not risk, then what is going on? We think three things are behind the data here: less than fully rational investor behavior, the limits to arbitrage, and the simple law of compounding.

We start with the first two and come to the more mundane issue of compounding later. In an inefficient market, mispricing comes from the combination of two things. First, some investors are not fully rational. This assumption is not hard to swallow, with a little inspection or introspection. Call these investors noise traders. Second, there must be limits to arbitrage. That means that the smart money must be less than fully competitive in taking advantage of the mispricing created by the noise traders. This basic logic is the foundation of behavioral finance, laid out in surveys like Shleifer (2000), Barberis and Thaler (2003), and Baker and Wurgler (2007).

It is easy to say that noise traders and the limits to arbitrage are to blame for any anomaly in the data. It is somewhat harder to explain the specific mechanism. What is the underlying psychology that leads to a preference for volatile stocks? And, why don’t smart institutions fill the gap, overweighting low volatility stocks and underweighting high volatility stocks by just enough to offset any irrational demand?

Investor Biases That Lead to a Preference for Volatile Stocks

We think that three biases are at work here: a preference for lotteries, representativeness, and overconfidence. Each has its roots in the early work of the Nobel-prize-winning psychologists Kahneman and Tversky (KT). For our purposes here, each leads to a less than fully rational increase in the demand for high volatility stocks.

Preference for lotteries. The preference for lotteries is the simplest. Would you take a gamble where you lose $100 with 50% probability and win $110 with 50% probability? Most people say no. Despite the positive expected payoff, the possibility of losing $100 is enough to deter participation, even when $100 is trivial in the context of wealth or income. KT called this “loss aversion.” Taken on its own, loss aversion suggests investors would shy away from volatility for fear of realizing a loss. But, something strange happens as the probabilities shift. Suppose instead the choice is between a certain gain of $5 or a large payoff of $5,000 with only a 0.1% probability. Most people take the gamble. The amount spent on lotteries and roulette wheels is a clear manifestation of this tendency.

To be statistically precise, this is more about positive skewness, where large positive payoffs are more likely than large negative ones, than it is about volatility. But, Mitton and Vorkink (2007) have pointed out that volatile individual stocks, with limited liability, also happen to be positively skewed. Buying a low priced, volatile stock is like a lottery ticket: There is some small chance of doubling or tripling in value, or much more, within a month, and there is a much larger probability that it will decline in value.

Representativeness. One way to explain representativeness is with a 1983 experiment. KT described a woman named Linda as “single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.” Then they asked: Which is more probable? A: Linda is a bank teller. Or B: Linda is a bank teller who is active in the women’s movement. The fact that many subjects choose B suggests that probability theory and the application of Bayes rule is not an ingrained skill. If Linda is a bank teller who is active in the women's movement (B is true), she *must* also be a bank teller (A is true). Suppose Linda is 70% likely to be active in the women's movement. Suppose also that Linda has a 1% chance of being a bank teller. B, at 0.7% = 1% x 70%, is less probable than A. The mistake arises, KT theorized, because the second job fits the description or is more "representative" of Linda. If we think of a bank teller who is active in the women's movement, a person like Linda comes to mind more readily.

What does this have to do with stocks and volatility? A similar problem arises when defining the characteristics of great investments. The layman and the quant take two different approaches. The layman tries to think of great investments – maybe buying Microsoft or Genzyme at their IPOs in 1986 – and draws the conclusion that small and speculative investments in new technologies – volatile stocks – are the path to investment success. Microsoft returned 70% per year in its first five years as a public company. The problem with this logic is similar to the Linda question. Subjects who answer incorrectly ignore the base rate probability of being a bank teller and its effect on the probability of B. The layman here largely ignores the large number of small and speculative investments that failed – and as a result is inclined to overpay of volatile stocks. The quant examines the full sample of stocks like Microsoft and Genzyme in an analysis like **Figure 1** and concludes that, without the knowledge to separate the Microsofts from the losers, this group of investments has fared poorly.

Overconfidence. Another bias underlying the preference for high volatility stocks is overconfidence. Experimenters ask subjects to estimate, for example, the population of Massachusetts, and to provide a 90% confidence interval around this answer. The confidence interval should be wide enough so that, for every 10 questions of this type, nine should contain the right answer. The experimental evidence shows that most people form confidence intervals that are far too narrow. In other words, they are overconfident in the accuracy of their knowledge. Moreover, the more obscure the question – if it is the population of Bhutan instead of Massachusetts – the more this calibration deteriorates.

Valuing stocks involves this same sort of forecasting. What will revenues be in 2014? Overconfident investors are likely to disagree. They will agree to disagree, sticking with the false precision of their estimates. The extent of disagreement is higher for more uncertain outcomes. For example, stocks that are either growing quickly or distressed – volatile stocks – will elicit a wider range of opinions.

One extra assumption is required to connect overconfidence, or more generally irreducible differences of opinion, to the demand for volatile stocks. We need pessimistic investors to act less aggressively than optimistic ones. In other words, we must have a general reluctance to short sell stocks. This extra assumption means that prices are set by optimists. So, stocks with a wider range of opinions sell for higher prices, and have lower future returns.

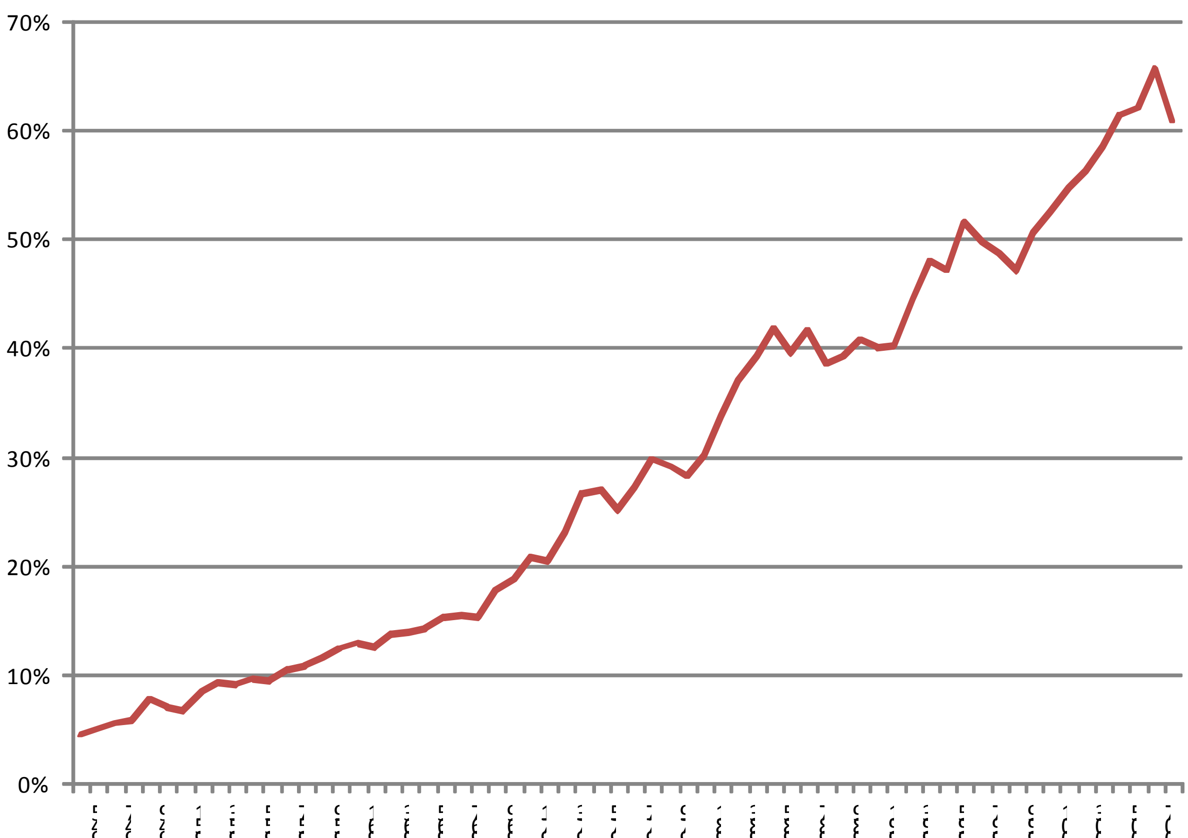
The Limits to Institutional Arbitrage in Low Volatility Stocks

A combination of preferences for lotteries and representativeness and overconfidence biases increases the demand for high volatility stocks among less than fully rational investors. The remaining and deeper puzzle is why sophisticated and well capitalized institutions do not offset this demand and capitalize on the return differences in **Figure 1**. What is especially puzzling is that this pattern has, if anything, gained force over a period where institutional management in the U.S. has doubled from 30% to over 60%, as the data from the Flow of Funds shows in **Figure 3**.

One part of the puzzle is why institutional investors do not short the very poor performing top volatility quintile. This has a simple answer. These are generally small stocks (in the full CRSP universe) and they are costly to trade in large quantity and difficult to short, because the volume of shares available to borrow is limited. The second and more interesting part of the puzzle is why institutional investors do not overweight the lower risk and higher performing low volatility quintile.

We think the limits to arbitrage here come from the typical contract in delegated investment management. The implicit or explicit mandate for the vast majority of institutional managers is to maximize the "information ratio" relative to a specific and fixed benchmark. For example, if the benchmark is the S&P 500, the information ratio is the average difference between the return earned by the investment manager and the return on the S&P 500, scaled by the volatility of the tracking error. Tracking error is the standard deviation of the return differences.

Figure 3. Institutional Ownership, 1968-2008.



Source: Institutional ownership from the Flow of Funds Table L.213. Assets managed by insurance companies (lines 12 and 13), public and private pension funds (lines 14, 15, and 16), open- and closed-end mutual funds (lines 17 and 18), and broker dealers (line 20). Assets under management are scaled by the market value of domestic corporations (line 21).

This contract has a number of advantages. It is arguably easier to understand the skill of an investment manager and the risks taken by examining the return relative to a benchmark. The ultimate investor cares more simply about risk, not tracking error. But, it comes at a cost. Roll (1992) analyzes the distortions that arise from a fixed benchmark mandate, and Brennan (1993) considers the effect on stock prices. In particular, a benchmark makes institutional investment managers less likely to exploit the patterns in **Figure 1**. We lay this out formally in an appendix for the mathematically inclined. But, the logic is simple.

In the Sharpe-Lintner CAPM, investors with common beliefs aim to maximize the expected return on their portfolios and minimize volatility. This leads to a simple relationship between risk and return. A stock's expected return is equal to the risk-free rate of return plus its beta times the market risk premium:

$$E(R) = R_f + \beta \times E(R_m - R_f) \quad (1)$$

Now, imagine that there is some extra, and less than fully rational demand for high volatility stocks that comes from a preference for lotteries and representativeness and overconfidence biases. This extra demand will push up the price of higher risk stocks and drive down their expected returns, and correspondingly push down the price of lower risk stocks and drive up their expected returns. Will an institution with a fixed benchmark exploit these mispricings? The short answer, illustrated with an example, is likely no.

Example. For simplicity, assume an institutional manager where the benchmark is the market portfolio, and suppose the expected return on the market is 5% over the risk-free rate and the volatility of the market is 20%. Take a stock with a β of 0.8, and imagine that it is undervalued, with an expected return greater than the CAPM benchmark in Equation (1) by an amount α. Overweighting the stock by a small amount, say 0.1%, will increase the expected active return by 0.1% x { α - 0.25 x $E(R_m-R_f)$ } = 0.1% x { α -

2.5% }. The extra variance of the portfolio is at least { 0.1% }^{2} x { σ_m^2 x $(1 - \beta)^2$ } = { 0.1% }^{2} x 0.0016, or the component of tracking error that comes from having a portfolio β that is not equal to 1.0. This investment manager would not start overweighting the stock until its α exceeds 2.5% per year.

Another way to see this is to consider the information ratio of the low volatility strategy in **Figure 1**. **Table 1** shows that the Sharpe ratio is high at 0.38. But, the information ratio, or the ratio of the excess return over the fixed benchmark to the tracking error, is much less impressive at 0.15.

Table 1. Returns by Volatility Quintile, 1968.01-2008.12.

	Low	**2**	**3**	**4**	**High**
Panel A: β Sorts					
Geometric Average $R_p - R_f$	4.16%	3.24%	3.37%	1.28%	-0.59%
Average $R_p - R_f$	4.85%	4.16%	4.64%	3.35%	3.18%
SD	12.28%	13.85%	16.10%	20.17%	27.12%
Sharpe Ratio	0.39	0.30	0.29	0.17	0.12
Average $R_p - R_m$	0.84%	0.15%	0.63%	-0.66%	-0.83%
Tracking Error	9.20%	6.49%	4.91%	6.30%	13.70%
Information Ratio	0.09	0.02	0.13	-0.11	-0.06
β	0.63	0.79	0.96	1.22	1.58
α	2.32%	0.97%	0.78%	-1.55%	-3.14%
$t(\alpha)$	2.10	1.11	1.02	-1.89	-1.97
Panel B: σ Sorts					
Geometric Average $R_p - R_f$	4.24%	3.27%	2.67%	0.60%	-6.69%
Average $R_p - R_f$	5.03%	4.66%	4.99%	4.32%	-1.64%
SD	13.15%	16.77%	21.40%	26.95%	31.99%
Sharpe Ratio	0.38	0.28	0.23	0.16	-0.05
Average $R_p - R_m$	1.02%	0.65%	0.98%	0.31%	-5.65%
Tracking Error	6.75%	4.59%	7.87%	14.24%	20.36%
Information Ratio	0.15	0.14	0.12	0.02	-0.28
β	0.75	1.01	1.28	1.53	1.70
α	2.02%	0.60%	-0.14%	-1.81%	-8.44%
$t(\alpha)$	2.37	0.84	-0.14	-1.01	-3.16

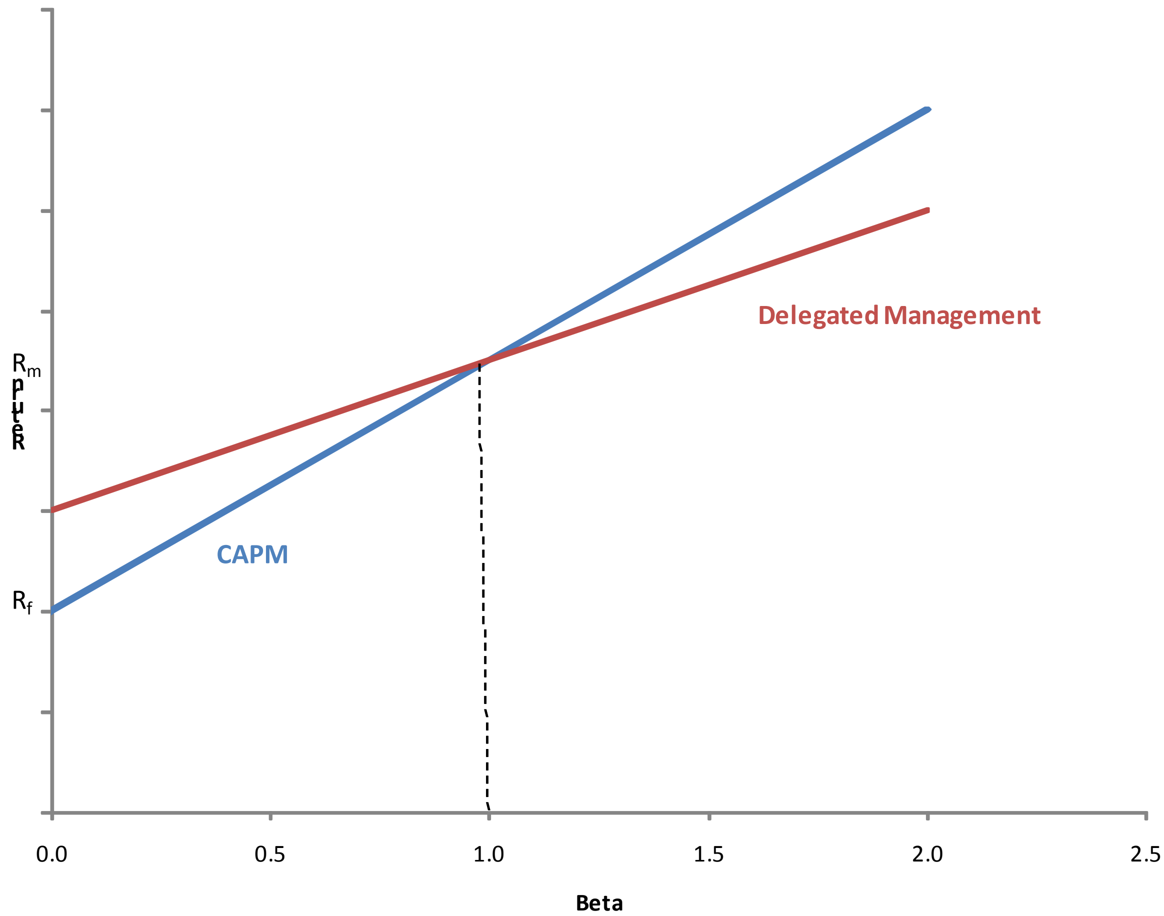
Source: We form portfolios by dividing the CRSP universe into five equal sized quintiles according to trailing beta in Panel A and trailing volatility in Panel B. Betas and standard deviations are measured using up to five years of monthly returns. The return on the market R_m and the risk free rate R_f are taken from Ken French's website. Average returns are monthly averages multiplied by 12. Standard deviation and tracking error are monthly standard deviations multiplied by the square root of 12.

To recap, an investment manager with a fixed benchmark is well suited to exploit mispricings that involve stocks with approximately market risk. A stock with a β around 1 and a positive α will be overweighted, and a stock with a β around 1 and a negative α will be underweighted. However, they will do little to correct the undervaluation of low β stocks and the overvaluation of high β stocks. In fact, not only will traditional asset managers with a fixed benchmark fail to compensate for noise trader demand for high β stocks, they are part of the problem. By the same logic in the example, an investment manager will overweight a stock with a β of 1.2, if its α is zero or modestly negative. In a simple equilibrium described in the appendix along the lines of Brennan (1993), with no irrational investors in the system, the presence of delegated investment management with a fixed benchmark will cause the CAPM relationship to fail. In particular, it will be too flat as shown in **Figure 4**:

$$E(R) = \{R_f + c\} + \beta \times \{E(R_m - R_f) - c\} \quad (2)$$

The constant c depends on aggregate risk aversion, the tracking error mandate of the investment manager, and the fraction of asset management that is delegated. Requiring investment managers to target a β of 1 will help, but it will not solve the problem of limited arbitrage. Such a manager will still be reluctant to overweight an underpriced stock with a β less than 1. A simple check of this logic is to look at the β for mutual fund managers. Over the past ten years, mutual fund βs have averaged 1.10.

Figure 4. Delegated investment management with a fixed benchmark flattens the CAPM relationship.



Putting the Pieces Together

The combination of irrational investor demand, stemming from a preference for lotteries and representativeness and overconfidence biases, and delegated investment management with fixed benchmarks causes the relationship between risk and return to breakdown. High risk stocks, whether measured by σ or β, do not earn a commensurate return. And, low risk stocks outperform. Despite this pattern, sophisticated investors are largely on the sideline because the mandate of maximizing return subject to tracking error makes arbitraging these mispricings unattractive.

The implication is clear. There is a solid investment thesis for low volatility strategies. As long as fixed benchmark contracts are the dominant form of implicit or explicit contracts between investors and investment management firms, the anomaly in **Figure 1** is unlikely to go away.

Table 2. Risk and return across asset classes.

			Sharpe Ratio			*CAPM Performance*		
	Average Return	**Excess Return**	**SD**	**Sharpe**	β	**t(β)**	α	**t(α)**
Short Term Government	5.70%	0.00%	0.02%					
Intermediate Term	7.88%	2.18%	5.58%	0.39	0.04	2.72	2.00%	2.31
Long Term Government	8.90%	3.20%	10.54%	0.30	0.13	4.44	2.67%	1.65
Corporate Bonds	8.48%	2.77%	9.58%	0.29	0.18	6.77	2.05%	1.43
Large Company Stocks	9.86%	4.16%	15.33%	0.27	0.95	131.77	0.26%	0.65
Small Stocks	13.04%	7.34%	21.79%	0.34	1.14	33.50	2.65%	1.41

Source: We compute the average return and beta by asset class, using data from Ibbotson Associates. The return on the market Rm and the risk free rate Rf are taken from Ken French's website. Average returns are monthly averages multiplied by 12.

There are some additional, more subtle predictions. Investment managers with fixed benchmarks will be unlikely to exploit mispricings where stocks of different risks have similar returns *within a particular mandate*. But, risk and return are more likely to line up *across mandates*: for example, between intermediate and long-term bonds; between government and corporate bonds; between stocks and bonds; and, between large and small company stocks. The CAPM works in the right direction at the macro level in the data from Ibbotson Associates presented below in **Figure 5**, though the returns lower risk investments in government and investment grade bonds are higher than the CAPM would predict.

Compounding and Transaction Costs

The last ingredient in **Figure 1** is compounding. The average monthly returns are certainly different. The top quintile has an average return of 41.9 basis points per month, while the bottom quintile has an average return of -13.7 basis points. But, the compound monthly differences are even larger at 35.3 basis points and -55.7 basis points. The extra 35 basis point difference in cumulative returns comes from compounding a lower volatility monthly series.

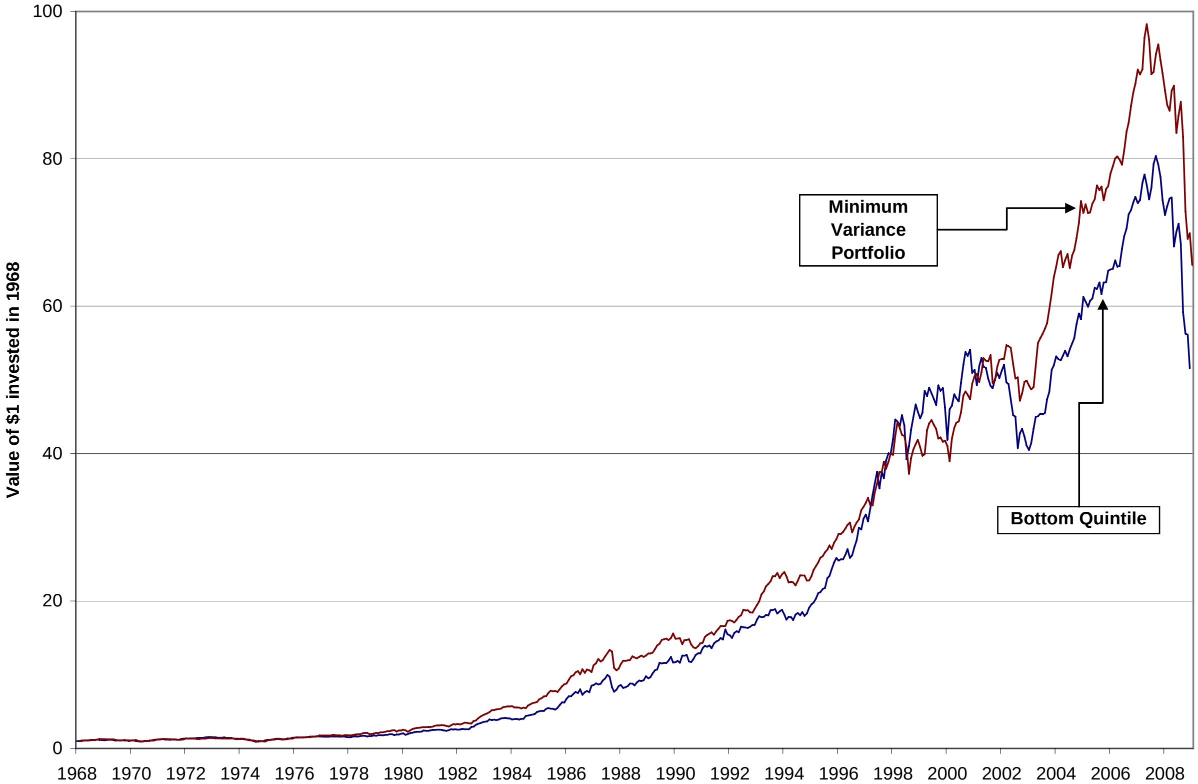
Which is the right measure of performance, the arithmetic monthly average or the compound returns in **Figure 1**? There is some ambiguity. By rebalancing every month – say back to 50% in cash and 50% in a stock portfolio – an investor can compound gross returns at the arithmetic average rate, multiplied by 50% of course. But, this strategy involves considerable transaction costs. And, delaying rebalancing even for a few months erodes the returns.

Searching for Lower Volatility

Given the power of compounding low volatility returns and the outperformance of low volatility stocks, a natural question is whether we can do even better than the low volatility quintile in **Figure 1** by taking more efficient advantage of the benefits of diversification. Leaving returns aside, we can do better, if we have a good estimate of not only individual firm volatility, but also the correlations among stocks. A portfolio of two *correlated* stocks each with low volatility can be more volatile than a portfolio of two *uncorrelated* but slightly more individually volatile stocks.

With this in mind, we construct a minimum variance portfolio that takes advantage of forecast correlation as well as volatility. Following the method of Clarke, de Silva and Thorley (2006), we use only the top 1000 stocks in the CRSP universe – a realistic and implementable strategy – and we compare the returns on the low volatility portfolio to the performance of the lowest quintile (200 stocks now, not the bottom quintile of the whole CRSP universe as in **Figure 1**) sorted by volatility. The minimum variance portfolio has lower volatility at 11.5%, versus 12.8% for the bottom 200 stocks. The compound annual return is correspondingly higher at 10.7%, versus 10.1%. These patterns are visually apparent in the cumulative returns plotted in **Figure 5** below.

Figure 5. A low volatility portfolio versus a portfolio of low volatility stocks



Source: The solid line is a minimum variance portfolio of the top 1000 stocks by market capitalization in the CRSP universe using six methods. The covariance matrix is estimated using the method of Clarke, de Silva and Thorley (2006). We limit the individual stock weights to fall between zero and three percent. The dashed line is the portfolio of the lowest quintile by trailing volatility. Volatility is measured as the standard deviation of up to 60 months of trailing returns.

The Best of Both Worlds

Most stock market anomalies can be thought of as "different returns, similar risks" anomalies. Value and momentum strategies, for example, are of this sort; the return differences are emphasized, not risk differences. Institutional investment managers are well positioned to take advantage of such anomalies because they can generate high excess return while maintaining average risks, matching their benchmark's risk and controlling tracking error.

But the low volatility anomaly is of a quite different character. Exploiting it involves holding stocks with more or less similar returns, which does not help the investment manager's excess returns, and different risks, which only increases their tracking error. So, even though irrational investors are happy to overpay for high risk while shunning low risk, the investment manager is generally not incentivized to exploit it. And thus, the anomaly persists. Indeed, it has grown stronger over time, in step with the increase in institutional investment management.

In summary, the behavioral finance explanation for The Greatest Anomaly is that irrational investors prefer total risk while at the same time institutional investors are incentivized to manage risk against their benchmarks. The theoretical diagnosis also implies the practical prescription. Investors who want to maximize return subject to total risk must incentivize their managers to do just that, by focusing on the benchmark-free Sharpe ratio, not the information ratio.

In recent years, more and more managers and their sophisticated clients have noticed the appealing risk-return profiles of Low Volatility and Maximum Sharpe Ratio strategies. For them, our behavioral finance insights are good news, because they suggests that as long as most of the investing world sticks with benchmarks, the advantage is theirs.

References

Ang, Andrew, R. Hodrick, Y. Xing, X. Zhang. "The Cross-Section of Volatility and Expected Returns." Journal of Finance, 61 (2006), pp. 259-299.

-. "High Idiosyncratic Volatility and Low Returns: International and Further U.S. Evidence." Journal of Financial Economics, 91 (2009), pp. 1-23.

Baker, Malcolm, J. Wurgler. "Investor Sentiment and the Cross-Section of Stock Returns." Journal of Finance, 61 (2006), pp. 1645-1680.

-. "Investor Sentiment in the Stock Market." Journal of Economic Perspectives, Vol. 21, No. 2 (2007), pp. 129-152.

Barberis, Nicholas, M. Huang. "Stocks as Lotteries: The Implications of Probability Weighting for Security Prices." American Economic Review 98 (2008), pp. 2066-2100.

Barberis, Nicholas, R. Thaler. "A Survey of Behavioral Finance." In G. Constantinides, M. Harris, R. Stulz, eds., Handbook of the Economics of Finance, North-Holland.

Black, Fischer. "Capital Market Equilibrium with Restricted Borrowing." Journal of Business, Vol. 45, No. 3 (1972), pp. 444-455.

Black, Fischer, M. C. Jensen, M. Scholes. "The Capital Asset Pricing Model: Some Empirical Tests." In M. C. Jensen, ed., Studies in the Theory of Capital Markets. New York: Praeger, 1972, pp. 79-121.

Brennan, Michael J. "Agency and Asset Pricing." Working paper, University of California, Los Angeles, 1993.

"Minimum-Variance Portfolios in the U.S. Equity Clarke, Roger G., H. de Silva, S. Thorley. Market." The Journal of Portfolio Management, Vol. 33, No. 1 (2006), pp. 10-24.

Fama, Eugene F., K. R. French. "The Cross-Section of Expected Stock Returns." Journal of Finance, 47 (1992), pp. 427-465.

Miller, Edward M. "Risk, Uncertainty, and Divergence of Opinion." Journal of Finance, 32 (1977), pp. 1151-1168.

Mitton, Todd, K. Vorkink. "Equilibrium Underdiversification and the Preference for Skewness." Review of Financial Studies, Vol. 20, No. 4 (2007), pp. 1255-1288.

Roll, Richard. "A Mean/Variance Analysis of Tracking Error." Journal of Portfolio Management, Vol. 18, No. 4 (1992), pp. 13-22.

Shleifer, A. Inefficient Markets: An Introduction to Behavioral Finance. Oxford, UK: Oxford UP, 2000.

Appendix. Delegated portfolio management and the CAPM

This is a short derivation is similar in setup to Brennan (1993) and shows that delegated portfolio management with a fixed benchmark will tend to flatten the CAPM relationship and make low volatility and low beta stocks and low volatility portfolios an attractive investment strategy. To show this somewhat more formally, we need to make a few assumptions.

1. **Stocks and bonds.** There are stocks i = 1 to N that pay a per share dividend of d_i per share next period. **d** has mean **1** and covariance Σ. Each has 1/N shares outstanding. For simplicity, assume that $\Sigma = \beta\beta'\sigma_m^2$ with $\beta'\mathbf{1} = N$. To make Σ invertible, we need e.g. $\beta\beta'\sigma_m^2 + \sigma^2\,\mathbf{I}$. The second term drops out for large N. There is a risk-free bond that returns r_f. Define dollar expected return with some abuse of notation as $R_i - R_f \approx 1 - p_i\,(1+r_f)$ and dollar market return $R_m - R_f \approx \{1/N\}\,\{\mathbf{1}'(\mathbf{1}-\mathbf{p}(1+r_f)\}$.

2. **Investors.** There are two representative investors j = 1, 2, who are mean-variance utility maximizers, each with capital k_j.

3. **Investment strategies.** Each investor makes a scalar asset allocation decision a_j between stocks and bonds, and a vector portfolio choice decision $\mathbf{n_j}$. Both investors are CARA, mean-variance utility maximizers (in dollars) with a risk aversion parameter of α.

    a. Investor 1 delegates his portfolio choice. Investor 1 allocates a fraction a_1 of his capital to an intermediary who chooses a portfolio $\mathbf{n_1}$ on investor 1's behalf.

    b. Investor 2 chooses his own portfolio. Investor 1 allocates a fraction a_2 of his capital to stocks and he chooses a portfolio $\mathbf{n_2}$. This can be collapsed to a single choice variable $\mathbf{n_2}$, with $a_2 = 1$.

4. **Intermediation.** There is a single intermediary chooses a portfolio $\mathbf{n_1}$ to maximize E { $(\mathbf{n_1}-\mathbf{n_b})'\mathbf{d}$ } - $\gamma(\mathbf{n_1}-\mathbf{n_b})'\Sigma(\mathbf{n_1}-\mathbf{n_b})$, where $\mathbf{n_b} = \{1/N\}\mathbf{1}$ is the market portfolio and $(\mathbf{n_1}-\mathbf{n_b})'\mathbf{1} = 0$.

If there are only investors of type 2, then the CAPM holds. Investor 2 chooses **n** to maximize E { $\mathbf{n}'\mathbf{d}$ } - $\alpha\mathbf{n}'\Sigma\mathbf{n}$ } + $(k-\mathbf{n}'\mathbf{p})(1+r_f)$. The first order condition is $\mathbf{n} = \{1/(2\alpha)\}\ \Sigma^{-1}\{\mathbf{1}-\mathbf{p}(1+r_f)\}$ and the market clearing condition is $\mathbf{n} = \{1/N\}\mathbf{1}$. Putting these two together, and rearranging terms leads to the following CAPM relationship between risk and return: (3)

$$\mathbf{R} - \mathbf{1}R_f \approx \mathbf{1}-\mathbf{p}(1+r_f) = \{1/N\}\ \{\mathbf{1}'(\mathbf{1}-\mathbf{p}(1+r_f)\}\boldsymbol{\beta} \approx (R_m-R_f)\boldsymbol{\beta}, \text{ or} \tag{3}$$

$$R_i \approx R_f + (R_m - R_f)\beta_i$$

If there are investors of type 1 and type 2, things are somewhat more complicated. Investor 2 chooses $\mathbf{n_2}$ as before. The intermediary chooses $\mathbf{n_1}$ to max E { $(\mathbf{n_1}- \mathbf{n_b})'\mathbf{d}$ } - $\gamma(\mathbf{n_1}- \mathbf{n_b})'\Sigma(\mathbf{n_1}- \mathbf{n_b})$ s.t. $(\mathbf{n_1}- \mathbf{n_b})'\ \mathbf{1} = 0$. The first order condition is $\mathbf{n_1} = \mathbf{n_b} + \{1/(2\gamma)\}\ \Sigma^{-1}\{\mathbf{1}-\mathbf{p}(1+r_f) - \lambda\ \mathbf{1}\}\ \lambda = \mathbf{1}'\Sigma^{-1}\{\mathbf{1}-\mathbf{p}(1+r_f)\}\ /\ \mathbf{1}'\Sigma^{-1}\mathbf{1}$. Investor 1 chooses a_1 to max a_1 { E { $\mathbf{n_1}'\mathbf{d}$ } - $\alpha a_1^2\mathbf{n_1}(\gamma)'\Sigma\mathbf{n_1}$ } + $(k_1-a_1\mathbf{n_1}'\mathbf{p})\ (1+r_f)$. The first order condition here is $a_1 = \{1/(2\alpha\ \mathbf{n_1}'\Sigma\mathbf{n_1})\}\ \{\mathbf{n_1}'(\mathbf{1}-\mathbf{p}(1+r_f)\}$. Combining these three choices $\mathbf{n}_2$, $\mathbf{n}_1$, and a_1 with the market clearing condition of $a_1\mathbf{n_1}+ \mathbf{n_2} = \{1/N\}\mathbf{1}$ and rearranging leads to the following relationship between risk and return:

$$\mathbf{R} - \mathbf{1}R_f \approx \{\mathbf{1}-\mathbf{p}(1+r_f)\} = \{1/N\}\ \{\mathbf{1}'(\mathbf{1}-\mathbf{p}(1+r_f)\}\ \boldsymbol{\beta} + (C\lambda/A)(\mathbf{1}-\boldsymbol{\beta}) \approx (R_m-R_f)\boldsymbol{\beta} \tag{4}$$

where $A = \{1/(2\alpha)+a_1/(2\gamma)\ \}$; $B = (1-a_1)$; $C = \{\ a_1/(2\gamma)\}$, or

$$R_i \approx (R_f + c)+ (R_m - R_f - c)\beta_I$$

where $c = (C\lambda/A)$

The important thing to notice is that Equation (4) is a flattened version of Equation (3). Returns are increasing in β, but not quite at a rate equal to R_m - R_f. The second term is positive for stocks with a b less than 1 and negative for stocks that have a β greater than 1. That is to say, relative to the CAPM, low β stocks are undervalued and high β stocks are overvalued.