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Location Decisions of Competing Platforms

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Location Decisions of Competing Platforms*

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Abstract

There are examples of entry in two-sided markets, where first entrants occupy a 'central location' and serve agents with 'intermediate tastes', while later entrants are niche players. Why would the first entrant choose to become a 'general' platform, given that later entrants will not have enough room for differentiation, resulting in an intense price competition? This one-sided market logic may not apply in a two-sided market. A key difference in a two-sided market, stemming from the presence of cross-group network externalities, is stronger demand creation. We develop a model which can deliver the above mentioned empirical observation, when the network externalities are intermediate. On the other hand, when externalities are low, our model predicts that differentiation will be maximum, as it would be in a one-sided market. Finally, for strong externalities only one platform is active and locates at the center.

Keywords: Product Selection; Two-sided markets; Endogenous Locations; Cross-group Network Externalities.

JEL Classification Codes: D43, L13.

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1 Introduction

We examine 'product location' decisions of two competing platforms in a two-sided market.[1] It is well-known that in one-sided markets firms choose maximum differentiation, see d'Aspermont et. al. (1979), to mitigate the ensuing price competition. Most of the literature on two-sided markets has ignored the issue of product selection and usually assumes maximum horizontal differentiation when the market has features of spatial competition, e.g., Armstrong (2006).[2] This would be an innocuous assumption if maximum differentiation could extend in a two-sided market. However, as we argue, this may not be true.

We formulate a model with two competing platforms and two groups of agents. Each agent receives higher benefit when more members from the other group join the platform (cross-group network externality). We show that when the cross-group network externalities are weak, platforms differentiate maximally. On the other hand, strong externalities result in minimum differentiation, where only one platform is active. Finally, for intermediate externalities a pure strategy location equilibrium does not exist. Then, we assume that platforms make their location decisions sequentially. The first-mover locates in the middle and the follower locates at an extreme point (*asymmetric location equilibrium*). Both platforms have positive but asymmetric market shares, with the first mover having a higher market share.[3]

There are examples of sequential entry into a market, where the first mover occupies a 'central' location, i.e., the most attractive location, whereas the follower positions its product at a niche location. Consider, for example, the market for online video websites and in particular two important players in this market: YouTube and Hulu. YouTube, which was launched before Hulu, carries a huge number of diverse videos and clips, while Hulu serves those who watch commercial movies and TV shows. Our asymmetric location equilibrium can shed some light into these markets. One can view the platform that moves first, YouTube in this case, as a general video-sharing website that mostly caters to agents with intermediate preferences, while the follower, Hulu, can be viewed

[1] Two-sided (or multiple-sided) markets are markets that are organized around intermediaries or "platforms" with two (or multiple) sides who should join a platform in order for successful exchanges (trade) to take place, see Armstrong (2006), Caillaud and Jullien (2003) and Rochet and Tirole (2006). For example, videogame platforms (e.g., Nintendo, Sony, Microsoft) need to attract both gamers and game developers. Newspapers need to attract advertisers and readers. Credit cards need merchants and users. More formally, a two-sided market is defined as one where the volume of transactions between end-users depends on the structure of the fees and not only on the overall level of fees charged by platforms [Rochet and Tirole (2006)].

[2] Exceptions are the papers by Gabszewicz et. al. (2002), Peitz and Valletti (2008) and Kind et. al. (2007).

[3] We present our results in terms of the strength of the network externality, but, alternatively, we can vary the degree of differentiation (transportation cost) for any fixed positive externality. The two are inversely related, i.e., high externality is equivalent to low degree of differentiation and vice versa.

as a more specialized TV and commercial movie website. Consistent with our model predictions, YouTube has a higher market share than Hulu.[4]

Early entrants in the online dating market were mostly interested in attracting the 'average' man and woman (for example, Match.com which was launched in April of 1995, according to the Online Dating Magazine). Later entrants into this market entered as niche players aiming at signing up people from specific groups, e.g., online dating services for busy professionals, or for millionaires. Other examples of asymmetric location configurations include academic journals, where earlier entrants are typically general interest journals, while later entrants are usually specialized journals and search engines on the Internet, e.g., Google, Yahoo vs. Google Scholar.

The driving force behind the product selection decisions is demand creation. When network externalities are weak, price competition dominates demand creation. No platform wants to be a 'general' platform because if one platform locates at the most attractive (central) location product differentiation is reduced which creates stiff price competition. In contrast, when externalities are not weak, demand creation is important. A platform in this case benefits by being 'general' and attracting many agents from both sides of the market with intermediate preferences. The rival platform serves a niche market. The tension between demand creation and price competition is at the heart of most models with endogenous location decisions, regardless of whether the market exhibits one or two-sidedness. For instance, one can mitigate the intensity of price competition, and hence create a less than maximum differentiation, by changing the transportation cost functions, as in Economides (1986), or by allowing for multiple purchases, as in Kim and Serfes (2006), both in a one-sided framework. What we do in this paper is to identify a mechanism that can make demand creation stronger, and this is the cross-group externality in a two-sided market.[5]

Our duopoly model can also be used to understand the geographic locations of physical markets.[6] When externalities are low (or equivalently transportation cost is high), markets will locate at the

[4] According to "Disney's Hulu Deal Raises Questions About YouTube Model," Wall Street Journal, April 30, 2009, YouTube had 100 million viewers in March of 2009, while in the same time period Hulu had 41 million viewers.

[5] Our conjecture is that the stronger demand creation effect would also be present in a one-sided model with direct network externalities. For a model with direct network externalities and horizontal differentiation, but with fixed locations, see Griva and Vettas (2004). In that paper, when both firms have the same quality, (and consumer expectations are affected by prices, as it is the case in our model) either the market is shared equally, or one firm dominates the market, depending on the intensity of the network externality. With exogenously given asymmetric qualities, an asymmetric sharing equilibrium, i.e., where both firms have strictly positive but unequal market shares, can be obtained. One difference between Griva and Vettas (2004) and our model is that because we allow platforms to endogenously choose their spatial locations we obtain an asymmetric sharing equilibrium even with ex-ante symmetric firms.

[6] Jin and Rysman (2009) investigate the pricing decisions of sportcard conventions. These conventions are two-sided markets since they try to attract both consumers and dealers. An important decision of these conventions is where to locate, given the competition they face from rival conventions.

periphery, i.e., far apart from each other. For very strong externalities (or low transportation costs) only one market will be active and will locate at the center, while for intermediate externalities (transportation costs) there will be two active markets, one at the center and the other at the periphery. One implication of our model is that as the network externalities become stronger (or the transportation cost decreases) market shares will become more asymmetric and eventually one platform will dominate.

Gabszewicz et. al. (2002) develop a model of media (newspapers) competition that features readers who have horizontal preferences with respect to the political ideology of a newspaper and advertisers who are vertically heterogeneous. They show that when advertising revenue is important the two newspapers will choose the same political ideology (minimum differentiation). Without the advertising side the two newspapers differentiate maximally with respect to their political messages. An assumption that is made is that readers are indifferent to advertisements. Peitz and Valetti (2008) and Kind et. al. (2007) build on the Gabszewicz et. al. (2002) model by assuming that readers/viewers are not neutral about ads. They show that differentiation need not be maximal. We differ from the aforementioned papers in that: i) both groups of agents in our model have horizontal preferences and as a consequence ii) we allow platforms to choose locations with respect to both groups of agents and not only with respect to one group (readers/viewers in those two papers). A new prediction of our analysis is the asymmetric location equilibrium.

The rest of the paper is organized as follows. We present the model in the next Section. In Section 3, we solve the model and in Section 4 we solve the social planner's problem (first-best) and the problem of a multiproduct monopolist. In Section 5, we perform a robustness check where we allow platforms to choose locations only with respect to one group of agents. We conclude in Section 6. All proofs can be found in the Appendix.

2 The description of the benchmark model

There are two groups of agents $\ell = 1, 2$ and two horizontally differentiated platforms $k = A, B$.[7] We will denote the "other" group of agents by m. We capture platform differentiation as follows. There is a continuum of agents of group ℓ that is uniformly distributed on the $[0, 1]$ interval. Platform A is located at point a and platform B is located at point b, with $0 \leq a \leq b \leq 1$. We assume that transportation cost is quadratic in the distance d an agent has to 'travel' from his location to

[7] Our benchmark model follows closely the model in Armstrong (2006). We differ in that we endogenize the locations of the two platforms.

the location of the platform, td^2, where the parameter $t > 0$ measures the per-unit cost of travel. We assume that each agent joins only one platform (single-homing). Each member of a group who joins a given platform cares about the number of members from the other group who join the same platform. Denote by $n_{\ell k}$ the number of participants from group ℓ that platform k attracts. The maximum willingness to pay for a member of group ℓ if he joins platform k is given by $V + \alpha_\ell n_{mk}$, where V is a stand-alone benefit each agent receives independent of the number of participants from the other group on platform k. The parameter $\alpha_\ell > 0$ measures the cross-group network externality for group ℓ participants. For simplicity, we assume that $\alpha_1 = \alpha_2 = \alpha$. The indirect utility of an agent from group ℓ who is located at point $x \in [0, 1]$ is given by,

$$U_\ell = \begin{cases} V + \alpha n^e_{mA} - t\,(a - x_\ell)^2 - p_{\ell A}, & \text{if he joins platform } A \\ V + \alpha n^e_{mB} - t\,(b - x_\ell)^2 - p_{\ell B}, & \text{if he joins platform } B \end{cases} \tag{1}$$

where $p_{\ell k}$ is platform k's lump-sum charge to group ℓ participants and n^e_{mk} denotes the expectations agents from group ℓ have about how many agents from group m will join platform k. We assume that V is high enough which ensures that the market is covered. Prices cannot become negative and marginal cost is zero.[8] As it is usual in these models (e.g., Armstrong (2006)) we assume that horizontal differentiation is more important than the cross-group network externality, $t > \alpha$.

The timing of the game is as follows. In stage 1, the two platforms make their location decisions, either simultaneously or sequentially. In stage 2, the platforms make their pricing decisions simultaneously. Finally, in stage 3, the agents decide which platform to join.

One assumption of the main model is that platforms are forced to change their attributes with respect to both group of agents the same way. While this is certainly true when location is, for example, geographic, or the attribute affects both groups the same way, e.g., political ideology, it may not be an adequate representation in other cases. To better illustrate this, consider two rival online video websites which are differentiated with respect to the technical standards they have adopted. Furthermore, the degree of differentiation on the one side of the market may be different from the differentiation on the other side. That is, the way one side (viewers) downloads online videos may be different from the way the other side uploads them. The model becomes very messy if we allow platforms to change their locations in each group of agents separately, i.e., a_1, a_2 for platform A and b_1, b_2 for platform B. As a compromise, in Section 5, we allow platforms to choose their locations with respect to only one group of agents. In particular, we assume that the locations of the platforms for group 2 are fixed at 0 and 1. Platforms can only choose the locations for group 1. Our main results are robust.

[8] In most cases negative prices are unrealistic and create perverse incentives (see also Armstrong (2006) for a discussion on this issue).

3 Analysis

We look for a subgame perfect Nash equilibrium. We solve the game backwards.

3.1 Stage 3: Agent decisions and market shares

The marginal agent in group 1 can be found as follows.

$$V + \alpha n_{2A}^e - t\left(a - x_1\right)^2 - p_{1A} - \left(V + \alpha n_{2B}^e - t\left(b - x_1\right)^2 - p_{1B}\right) = 0$$

$$\Rightarrow \hat{x}_1 = \frac{p_{1B} - p_{1A} + t\left(b^2 - a^2\right) - \alpha\left(n_{2B}^e - n_{2A}^e\right)}{2t\left(b - a\right)}. \tag{2}$$

The fraction of agents from group 1 that joins platform A is $n_{1A} = \hat{x}_1$ and the fraction from group 1 that joins platform B is $n_{1B} = 1 - \hat{x}_1$. Similarly, we can find the marginal agent in group 2.

$$V + \alpha n_{1A}^e - t\left(a - x_2\right)^2 - p_{2A} - \left(V + \alpha n_{1B}^e - t\left(b - x_2\right)^2 - p_{2B}\right) = 0$$

$$\Rightarrow \hat{x}_2 = \frac{p_{2B} - p_{2A} + t\left(b^2 - a^2\right) - \alpha\left(n_{1B}^e - n_{1A}^e\right)}{2t\left(b - a\right)}. \tag{3}$$

The fraction of agents from group 2 that joins platform A is $n_{2A} = \hat{x}_2$ and the fraction from group 2 that joins platform B is $n_{2B} = 1 - \hat{x}_2$.

In equilibrium, it must be that expectations are confirmed, that is, $n_{1A} = n_{1A}^e$, $n_{1B} = n_{1B}^e$, $n_{2A} = n_{2A}^e$ and $n_{2B} = n_{2B}^e$. Using (2) and (3), this defines a system of four equations in four unknowns, n_{1A}, n_{1B}, n_{2A} and n_{2B}. By solving the system we obtain the market shares as a function of prices and parameters.

$$n_{1A} = \frac{t\left(b - a\right)\left(p_{1B} - p_{1A} - \alpha\right) + \alpha\left(p_{2B} - p_{2A}\right) + D}{2\left[t^2\left(b - a\right)^2 - \alpha^2\right]} \tag{4}$$

$$n_{1B} = \frac{t\left(b - a\right)\left(p_{1A} - p_{1B} + \alpha\right) + \alpha\left(p_{2A} - p_{2B}\right) - t^2\left(b^3 + a^3\right) + F}{2\left[t^2\left(b - a\right)^2 - \alpha^2\right]} \tag{5}$$

$$n_{2A} = \frac{t\left(b - a\right)\left(p_{2B} - p_{2A} - \alpha\right) + \alpha\left(p_{1B} - p_{1A}\right) + D}{2\left[t^2\left(b - a\right)^2 - \alpha^2\right]} \tag{6}$$

$$n_{2B} = \frac{t\left(b - a\right)\left(p_{2A} - p_{2B} + \alpha\right) + \alpha\left(p_{1A} - p_{1B}\right) - t^2\left(b^3 + a^3\right) + F}{2\left[t^2\left(b - a\right)^2 - \alpha^2\right]} \tag{7}$$

where $D \equiv t\alpha\left(b^2 - a^2\right) - \alpha^2 + t^2\left(b^3 + a^3\right) - t^2 ab\,(b+a)$ and $F \equiv -t\alpha\left(b^2 - a^2\right) + t^2 ab\,(b+a) + 2t^2\left(b^2 + a^2\right) - \alpha^2 - 4abt^2$.

3.2 Stage 2: Platforms' pricing decisions

Platform k chooses p_{1k} and p_{2k} to maximize its profits

$$\pi_k = p_{1k} n_{1k} + p_{2k} n_{2k},$$

where $n_{\ell k}$, $\ell = 1, 2$ and $k = A, B$, are given by (4), (5), (6) and (7). The profit functions are strictly concave in a platform's own prices if

$$\alpha < t\,(b - a)\,. \tag{8}$$

Alternatively, the above condition can be written as

$$a < b - \frac{\alpha}{t} \text{ or } b > a + \frac{\alpha}{t}.$$

If (8) is satisfied, then the first order conditions are also sufficient for profit maximization. The equilibrium prices then are given by

$$p_{1A} = p_{2A} = \frac{t\,(b-a)}{3}\,(2 + b + a) - \alpha \text{ and } p_{1B} = p_{2B} = \frac{t\,(b-a)}{3}\,(4 - b - a) - \alpha. \tag{9}$$

The equilibrium market shares, by substituting (9) into (4), (5), (6), and (7), are given by

$$n_{\ell A} = \frac{t\,(b-a)\,(2+b+a) - 3\alpha}{6\,(t\,(b-a) - \alpha)} \text{ and } n_{\ell B} = \frac{t\,(b-a)\,(4-b-a) - 3\alpha}{6\,(t\,(b-a) - \alpha)},\ \ell = 1, 2.$$

Note that if (8) is satisfied, then the denominators in the above expressions are positive. For an interior equilibrium we need the market shares to be in $(0, 1)$. It turns out that $n_{\ell k} \in (0, 1)$ if and only if

$$\alpha < \min\left\{\frac{t\,(b-a)}{3}\,(4-b-a)\,,\ \frac{t\,(b-a)}{3}\,(2+b+a)\right\} \tag{10}$$

or, equivalently, for $n_{\ell A}$ to be less than one (which implies that $n_{\ell B}$ is greater than zero) we must have

$$\alpha < \frac{t\,(b-a)}{3}\,(4-b-a) \Leftrightarrow a < \tilde{a} \equiv -\frac{1}{t}\left(-2t + \sqrt{3t\alpha + 4t^2 - 4bt^2 + b^2t^2}\right) \tag{11}$$

and for $n_{\ell A}$ to be greater than zero (which implies that $n_{\ell B}$ is less than one) we must have

$$\alpha < \frac{t\,(b-a)}{3}\,(2+b+a) \Leftrightarrow a < \hat{a} \equiv -\frac{1}{t}\left(t - \sqrt{-3t\alpha + t^2 + 2bt^2 + b^2t^2}\right). \tag{12}$$

For any given location b of platform B the market tips, either in favor of A or in favor of B, when platform A locates close enough to platform B, as the above thresholds indicate. The interior equilibrium profits as a function of the platforms' locations, after we substitute (9) into the profit functions, are[9]

$$\pi_A(a,b) = \frac{(t(b-a)(2+b+a)-3\alpha)^2}{9(t(b-a)-\alpha)} \text{ and } \pi_B(a,b) = \frac{(t(b-a)(4-b-a)-3\alpha)^2}{9(t(b-a)-\alpha)}. \tag{13}$$

It can be easily verified that when $\alpha = 0$ (no cross-group network externality) the equilibrium profits reduce to those in d'Aspermont et. al. (1979), where it is each platform's dominant strategy to locate at the extreme points (maximum differentiation). As we show next, this is not always the case when $\alpha > 0$.

3.3 Stage 1: Platforms' location decisions

Platforms choose their locations a and b to maximize profits as they are given by (13). We assume that they do so simultaneously. We make the assumption that when the market tips it is the platform that is closer to the middle point, $1/2$, that attracts all the agents. If they are equidistantly located from the middle, $a = 1-b$, then all agents join platform A.

Fix the location of platform B at a specific point b. Let's examine the profits of platform A when it moves from $a = 0$ to $a = b$. Two effects arise: i) price competition intensifies and ii) the platform attracts more agents from both groups. Initially, the intensified competition effect is stronger, but eventually demand creation dominates. The latter is due to the cross-group externality. The Lemma below summarizes the result.

Lemma 1 *The profit function of platform A, for any fixed location b of platform B, exhibits a U-shape.*

The above Lemma suggests that platform A will either locate at 0 or right next to platform B. Symmetrically, the same is true for platform B. This property of the profit functions will be used in the proofs of the remaining Propositions.

To gain a better intuition about the properties of the profit functions as a function of the locations, let's look at Figures 4 and 5 where the profit of platform A is depicted as its location a varies for a fixed location b of platform B. The difference between the two figures is that in Figure 4 the location of platform B is farther away from the most attractive point (the center) than in

[9] We will deal with the tipping solution next when we will analyze the location game.

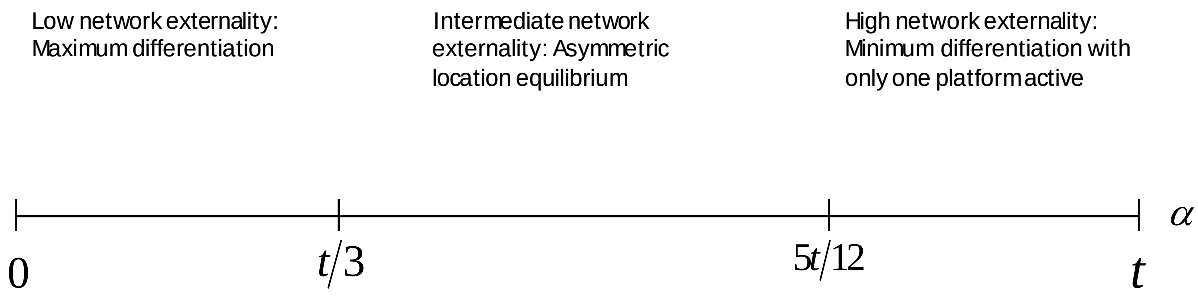

Figure 1: Equilibrium locations of the two platforms

Figure 5. When B is farther away from the center, as in Figure 4, prices and profits initially fall, but after a threshold, namely $\tilde{a}$, the market tips in favor of A and its profits rise. In contrast, when B is closer to the center, as in Figure 5, the market tips in favor of B after a certain threshold, namely $\hat{a}$. When A moves closer to the center, that is when it exceeds $1 - b$, the market tips again but now in favor of A. These properties are quite intuitive. For a high degree of horizontal differentiation the market is shared and any movement closer to the rival intensifies competition. After a certain point, however, tipping happens. The question then is: which is the platform that attracts all the agents? The answer is that it is the one closer to the most attractive location: the center.

Figure 1 summarizes the equilibrium locations as a function of the network externality α. These outcomes are presented formally in Propositions 2, 3 and 4.

Next, we examine whether an equilibrium with maximum differentiation, $a = 0$ and $b = 1$, exists. The next Proposition summarizes the result.

Proposition 2 ***(Maximum differentiation)*** *A maximum horizontal differentiation equilibrium, where $a = 0$ and $b = 1$, exists if and only if $\alpha \leq t/3$. The market is shared equally between the two platforms.*

When the cross-group network externalities are weak, a platform has no incentive to move close to the rival platform in order to become the dominant platform. The benefit from differentiation is minimized, when the two platforms are close to each other, and the dominant platform is only able to benefit from the larger market share. Since the externalities are weak, so is the benefit from the larger market share. Equilibrium prices are given by (9) after we set $a = 0$ and $b = 1$, i.e., prices are equal to $t - \alpha$.

The next Proposition presents the result when the cross-group network externality is high.

Proposition 3 ***(Minimum differentiation)*** *Suppose $\alpha \in [5t/12, t)$. The location equilibrium entails minimum horizontal differentiation and tipping in favor of one platform. In particular, $a = b = 1/2$ and platform A attracts all the agents. If $\alpha \in (t/3, 5t/12)$ a location equilibrium in pure strategies does not exist.*

The intuition behind the minimum differentiation result is as follows. When the cross-group network externalities are important, less than maximum differentiation makes tipping more likely and benefits the platform that is able to attract all agents. The dominant platform has an incentive to locate in the middle for two reasons: i) there is no room for the rival platform to differentiate itself enough by moving to an extreme and attract some agents and ii) the rival platform cannot move closer to the middle point and attract all agents. Indeed, the rival platform makes zero profits regardless of where it chooses to locate and for the equilibrium to exist we assume that it also locates at the middle point. So, the two platforms are not differentiated and in this case we assume that all agents coordinate and join one platform, platform A. Platform A charges prices equal to α and B's prices are zero. Recall that we do not allow for negative prices.

3.3.1 Platforms locate sequentially

When an equilibrium in pure strategies does not exist, we restore existence by assuming that platforms move sequentially, with platform A moving first. The next Proposition summarizes the subgame perfect Nash equilibrium.

Proposition 4 ***(Asymmetric location equilibrium)*** *Suppose $\alpha \in (t/3, 5t/12)$. Platform A moves first and locates at $a = 1/2$. Platform B moves second and locates at $b = 1$. Both platforms attract agents in equilibrium (sharing equilibrium). Platform A attracts more agents than platform B. The equilibrium profits are*

$$\pi_A \left(a = 1/2, b = 1\right) = \frac{\left(7t - 12\alpha\right)^2}{72\left(t - 2\alpha\right)} \text{ and } \pi_B \left(a = 1/2, b = 1\right) = \frac{\left(5t - 12\alpha\right)^2}{72\left(t - 2\alpha\right)}.$$

Platform A moves first and locates in the middle. Given that the externality is not as strong as in Proposition 3, platform B can attract some agents and make strictly positive profits when it locates at the extreme. Platform A has no incentive to move away from the middle point, because in this case B has a profitable deviation to locate closer to the middle point than A and leave A

with zero agents and profits.[10] The equilibrium prices, using (9), are given by

$$p_{1A} = p_{2A} = \frac{7t}{12} - \alpha \text{ and } p_{1B} = p_{2B} = \frac{5t}{12} - \alpha.$$

In the absence of network externalities, even if platforms move sequentially, the first mover will always locate at an extreme point to mitigate the ensuing price competition. Hence, the fact that the first mover in our model locates at the center, when the network externality is not weak, is a direct consequence of the network externalities. Furthermore, the equilibrium outcomes when platforms move simultaneously for $\alpha \notin (t/3, 5t/12)$, which is covered in Propositions 2 and 3, continue to hold even when platforms move sequentially (details are omitted and are available upon request).

One can draw an analogy between the present model and earlier literature on direct network externalities. One important issue in that literature was the trade-off between 'standardization' and variety, e.g., Farrell and Saloner (1986). Having only one platform in our model can be viewed as having one technical standard, at the expense of product variety. So, Proposition 4 predicts that for intermediate network externalities, there will be two 'standards' in equilibrium, with one being superior to the other. Note also that in our model the 'type' of the standard is not given but it is being determined endogenously. When externalities are low, as in Proposition 2, neither standard is superior to the other. Finally, when externalities are strong, as in Proposition 3, there will be only one standard in equilibrium.

4 Welfare analysis

A social planner chooses the locations a and b of the two platforms and the number of agents from each group that should join a platform to maximize the difference between aggregate network externality and aggregate transportation cost. We denote by x the number of agents from group 1 that join platform A and by y the number of agents from group 2 that join platform A. Total

[10] Tyagi (2000) examines location decisions of two firms, in a one-sided model, that enter sequentially and have different costs. If the second mover has lower cost, then it locates close to the most attractive location, while the first mover locates far away from the most attractive location. Our asymmetric location equilibrium result has a similar flavor, but it is the first mover in our model that locates at the most attractive location. Moreover, the underlying mechanisms between the two models are different and in our model firms are ex-ante symmetric.

welfare is given by

$$
\begin{aligned}
W &= \int_0^x \left(\alpha y - t\left(a-z\right)^2\right) dz + \int_x^1 \left(\alpha\left(1-y\right) - t\left(b-z\right)^2\right) dz + \\
&\quad \int_0^y \left(\alpha x - t\left(a-z\right)^2\right) dz + \int_y^1 \left(\alpha\left(1-x\right) - t\left(b-z\right)^2\right) dz \\
&= 4\alpha yx + tax^2 - ta^2x + tay^2 - ta^2y + 2\alpha - 2\alpha x - 2\alpha y \qquad (14) \\
&\quad -\frac{2t}{3} + 2tb - tbx^2 - 2tb^2 + tb^2x - tby^2 + tb^2y.
\end{aligned}
$$

The next Proposition summarizes the solution to the social planner's problem.

Proposition 5 ***(First-best)*** *For $\alpha \leq t/8$, the optimal locations are $a = 1/4$ and $b = 3/4$ and the agents are split equally between the two platforms. Total welfare is equal to $W = \alpha - t/24$. For $\alpha \geq t/8$, all agents from both groups join one platform which is located at the middle point $1/2$. Total welfare is equal to $W = 2\alpha - t/6$.*

The intuition is simple. Aggregate network externality is maximized when all agents join one platform. On the other hand, total transportation cost is minimized when the platforms are located at the first and third quartiles. When the externality is weak, transportation cost is relatively more important and the social planner splits the agents equally between the two platforms. For strong externalities one platform is chosen to dominate the market, since externalities are now relatively more important.

Comparing the first-best with the non-cooperative outcome, we can see that the two coincide only when network externalities are strong, i.e., $\alpha \geq 5t/12$, Proposition 3. For low externalities, horizontal differentiation is higher than in the first-best, i.e., 'too much' differentiation, as it is also the case in a one-sided Hoteling-type model. The two platforms differentiate maximally, while the social planner wants them at the first and third quartiles, as in a one-sided model. When externalities are intermediate, the degree of horizontal differentiation the platforms choose is higher than the first-best, Proposition 4.

4.1 Multiproduct monopolist

We examine when a multiproduct monopolist will find it profitable to switch from offering two products to offering only one. The price the monopolist can charge is one that leaves the marginal agent with zero surplus. We assume that V is high enough, so that the monopolist finds it optimal to serve all agents. The next Proposition, the proof of which is omitted, summarizes the monopolist's location choices.

Proposition 6 ***(Multiproduct monopolist)*** *When $\alpha \leq 3t/8$, the monopolist launches two platforms: one located at $a = 1/4$ and the other at $b = 3/4$. Total profits are $\Pi = 2V + \alpha - t/8$. The agents are split equally between the two platforms. When $a \geq 3t/8$, the monopolist launches one platform located at $1/2$. Total profits are $\Pi = 2V + 2\alpha - t/2$.*

The monopolist cannot extract all the surplus from the inframarginal agents, and that is why he has insufficient incentives, relative to the first-best, to launch only one platform. Only for relatively strong cross-group externality, i.e., greater then $3t/8$, the locations chosen by the monopolist coincide with the first-best locations. This can be seen by comparing the above Proposition with Proposition 5. This has interesting implications for merger policy. When $\alpha \geq t/8$, the first-best entails only one platform being active, locating at the center and attracting all agents. If $\alpha \leq 5t/12$, the non-cooperative equilibrium is inefficient since both platforms have strictly positive market shares, see Propositions 3 and 4. Therefore, for $\alpha \in [3t/8, 5t/12]$ a merger between the two platforms improves social welfare because the merged entity will launch only one platform instead of two, mimicking the first-best. Of course, this prediction should be viewed with caution because aggregate demand in our model is fixed.

5 Robustness check: Locations for one group are fixed

We assume that the locations of the platforms for group 2 are fixed at 0 and 1. Platforms can only choose the locations for group 1. The results do not change at all qualitatively.

The equilibrium prices when both platforms have strictly positive market shares are

$$p_{1A} = \frac{t\left(b-a\right)}{3}\left(2+b+a\right) - \alpha,\ p_{2A} = t - \alpha \text{ and } p_{1B} = \frac{t\left(b-a\right)}{3}\left(4-b-a\right) - \alpha,\ p_{2B} = t - \alpha.$$

The equilibrium profits functions $\pi_A(a,b)$ and $\pi_B(a,b)$ are lengthy and are omitted. The profit functions are strictly concave in a platform's own prices if

$$\alpha^2 < t^2\left(b-a\right). \tag{15}$$

By comparing (8) with (15), we can see that it is easier to satisfy the concavity condition when the locations on the one side are fixed. The equilibrium market shares are

$$n_{1A} = \frac{t^2\left(b-a\right)\left(2+b+a\right) - 3\alpha^2}{6\left(t^2\left(b-a\right)-\alpha^2\right)} \text{ and } n_{2A} = \frac{t\left(b-a\right)\left(3t-\left(1-a-b\right)\alpha\right) - 3\alpha^2}{6\left(t^2\left(b-a\right)-\alpha^2\right)} \tag{16}$$

$$n_{1B} = \frac{t^2\left(b-a\right)\left(4-b-a\right) - 3\alpha^2}{6\left(t^2\left(b-a\right)-\alpha^2\right)} \text{ and } n_{2B} = \frac{t\left(b-a\right)\left(3t+\left(1-a-b\right)\alpha\right) - 3\alpha^2}{6\left(t^2\left(b-a\right)-\alpha^2\right)}. \tag{17}$$

The market share of platform A in group 1, n_{1A}, is positive, which implies that n_{1B} is less than one, if and only if

$$a < \hat{a}_1 \equiv \frac{1}{t}\left(-t + \sqrt{t^2 + 2bt^2 + b^2t^2 - 3\alpha^2}\right). \tag{18}$$

The price p_{1A} is positive if and only if (12) holds. Observe that $\hat{a}_1 > \hat{a}$, suggesting that for $a \in (\hat{a}, \hat{a}_1)$, p_{1A} becomes negative, while n_{1A} is positive.

The market share of platform B in group 1, n_{1B}, is positive, which implies that n_{1A} is less than one, if and only if

$$a < \tilde{a}_1 \equiv \frac{1}{t}\left(2t - \sqrt{4t^2 - 4bt^2 + b^2t^2 + 3\alpha^2}\right). \tag{19}$$

The price p_{1B} is positive if and only if (11) holds. Observe that $\tilde{a}_1 > \tilde{a}$, suggesting that for $a \in (\tilde{a}, \tilde{a}_1)$, p_{1B} becomes negative, while n_{1B} is positive.

Since we do not allow for negative prices, we set the prices equal to zero when $a \in (\hat{a}, \hat{a}_1)$ or $a \in (\tilde{a}, \tilde{a}_1)$. In this case the threshold for a below which the market is shared decreases. For example, if we set $p_{1A} = 0$ when $a \geq \hat{a}$ the threshold $\hat{a}_1$, above which platform A has no market share in group 1, decreases. We will use this argument in the proofs of Propositions.

It turns out that $\hat{a}_1 \geq \tilde{a}_1$ if and only if $b \geq \bar{b}' \equiv 1/2 + \alpha^2/\left(2t^2\right)$. Note that $\bar{b}'$ is less than $\bar{b}$ from (22).

The market share of platform A in group 2, n_{2A}, is positive if and only if

$$a < \hat{a}_2 \equiv -\frac{1}{t\alpha}\left(-\frac{1}{2}t\alpha + \frac{3}{2}t^2 - \frac{1}{2}\sqrt{9t^4 - 12t\alpha^3 - 6t^3\alpha + 12bt^3\alpha + t^2\alpha^2 - 4bt^2\alpha^2 + 4b^2t^2\alpha^2}\right).$$

The market share of platform B in group 2, n_{2B}, is positive if and only if

$$a < \tilde{a}_2 \equiv -\frac{1}{t\alpha}\left(-\frac{1}{2}t\alpha - \frac{3}{2}t^2 + \frac{1}{2}\sqrt{9t^4 + 12t\alpha^3 + 6t^3\alpha - 12bt^3\alpha + t^2\alpha^2 - 4bt^2\alpha^2 + 4b^2t^2\alpha^2}\right).$$

It turns out that $\hat{a}_2 \geq \tilde{a}_2$ if and only if $b \geq \bar{b}' \equiv 1/2 + \alpha^2/\left(2t^2\right)$. Moreover, $\tilde{a}_1 \leq \tilde{a}_2$ and $\hat{a}_1 \geq \hat{a}_2$ if and only if $b \geq \bar{b}' \equiv 1/2 + \alpha^2/\left(2t^2\right)$. Therefore, we have two cases.

Case 1: $b \geq \bar{b}'$. The binding threshold is $\tilde{a}_1$. For any $a < \tilde{a}_1$ the equilibrium is interior. At $a = \tilde{a}_1$ the market for group 1 tips in favor of platform A. Given that $\tilde{a}_1 \leq \tilde{a}_2$, and our assumption that $\alpha < t$, the market for group 2 does not tip.

Case 2: $b \leq \bar{b}'$. The binding threshold is $\hat{a}_1$. For any $a < \hat{a}_1$ the equilibrium is interior. At $a = \hat{a}_1$ the market for group 1 tips in favor of platform B. Given that $\hat{a}_1 \leq \hat{a}_2$, and our assumption that $\alpha < t$, the market for group 2 does not tip.

Corner (tipping) solution. The market for group 1 has tipped in favor of platform A. Platform B's price in group 1 is zero. The indirect utility of the marginal consumer from group 1, who is located at 1 and joins platform A, is given by

$$U_{1A} = V - t\,(a-1)^2 + \alpha n_{2A} - p_{1A}.$$

Similarly, the indirect utility of the marginal consumer from group 1 who joins platform B is

$$U_{1B} = V - t\,(b-1)^2 + \alpha n_{2B}.$$

The market share of platform A in group 2 is

$$V - t\,(0-y)^2 + \alpha - p_{2A} = V - t\,(1-y)^2 - p_{2B} \Rightarrow y = n_{2A} = \frac{p_{B2} - p_{A2} + t + \alpha}{2t}$$

Platform A will set its price p_{1A}, using n_{2A}, so that the marginal agent in group 1 is indifferent between the two platforms, $U_{1A} = U_{1B}$. This yields

$$p_{1A} = \frac{-t^2a^2 + 2t^2a + \alpha^2 - \alpha p_{A2} + \alpha p_{B2} + t^2b^2 - 2t^2b}{t}.$$

The profit functions, using the above p_{1A}, are

$$\pi_A = p_{1A} + p_{2A}n_{2A} \text{ and } \pi_B = p_{2B}n_{2B}.$$

Taking the first order conditions with respect to p_{2A} and p_{2B} and solving we obtain

$$p_{2A} = p_{2B} = t - \alpha \text{ and } p_{1A} = \frac{-t^2a^2 + 2t^2a + \alpha^2 + t^2b^2 - 2t^2b}{t}. \tag{20}$$

The equilibrium profits are

$$\pi_A = \frac{-2t^2a^2 + 4t^2a + \alpha^2 + 2t^2b^2 - 4t^2b + t^2}{2t} \text{ and } \pi_B = \frac{(t-\alpha)^2}{2t}. \tag{21}$$

We can see that $\partial\pi_A/\partial a > 0$ for all $a < 1$, suggesting that when tipping occurs, platform A has an incentive to locate next to platform B. We also examined unilateral deviations from (20), using the interior market shares (16) and (17), to ensure that B has no incentive to lower its price in group 2 to attract more agents in group 1 and that A has no incentive to raise its price in group 1. The first order conditions of platform B, when the prices of A are fixed at (20), evaluated at $p_{1B} = 0$ and $p_{2B} = t - \alpha$, are given by

$$\frac{\partial\pi_B}{\partial p_{1B}} = -\frac{\alpha\,(t-\alpha)}{2\,(t^2\,(b-a) - \alpha^2)} < 0 \text{ and } \frac{\partial\pi_B}{\partial p_{2B}} = 0.$$

Platform B would like to lower its price in group 1, but negative prices are not allowed. The above first order conditions indicate that platform B does not find a local deviation profitable. Given that the profit function is strictly concave, see condition (15), a global deviation is unprofitable as well. Similarly, a deviation on part of platform A is unprofitable.

The results below are very similar to their counterparts from the main analysis. There are only quantitative differences. More specifically, for low externalities, $\alpha \leq \left(\sqrt{2}-1\right)t$, platforms differentiate maximally; for intermediate externalities, $\alpha \in \left(\left(\sqrt{2}-1\right)t, \sqrt{15}t/6\right)$, we obtain an asymmetric location equilibrium; and for $\alpha \in \left[\sqrt{15}t/6, t\right)$, differentiation is minimum. One difference from our benchmark model is that the range of parameters for which maximum differentiation holds has expanded from $[0, t/3]$ to $\left[0, \left(\sqrt{2}-1\right)t\right]$. This is quite intuitive. When locations on the one side of the market are fixed, the demand creation effect is weaker. Hence, the incentives to locate at the center are weaker. Following this argument, tipping is easier when locations for both groups are changing: $\left[\sqrt{15}t/6, t\right)$ when one side is fixed to $[5t/12, t)$ when both sides are flexible.

Lemma 7 *The profit function of platform A for any fixed location b of platform B exhibits a U-shape.*

Proposition 8 ***(Maximum differentiation)*** *A maximum horizontal differentiation equilibrium, where $a = 0$ and $b = 1$, exists if and only if $\alpha \leq \left(\sqrt{2}-1\right)t$.*

Proposition 9 ***(Minimum differentiation)*** *Suppose $\alpha \in \left[\sqrt{15}t/6, t\right)$. The location equilibrium entails minimum horizontal differentiation and tipping in favor of one platform. In particular, $a = b = 1/2$ and platform A attracts all the agents. If $\alpha \in \left(\left(\sqrt{2}-1\right)t, \sqrt{15}t/6\right)$ a location equilibrium in pure strategies does not exist.*

Proposition 10 ***(Asymmetric location equilibrium)*** *Suppose $\alpha \in \left(\left(\sqrt{2}-1\right)t, \sqrt{15}t/6\right)$. Platform A moves first and locates at $a = 1/2$. Platform B moves second and locates at $b = 1$. Both platforms attract agents in equilibrium (sharing equilibrium). Platform A attracts more agents than platform B. The equilibrium profits are*

$$\begin{aligned}
\pi_A\left(a = 1/2, b = 1\right) &= \frac{-144\alpha t^2 + 121t^3 - 240t\alpha^2 + 288\alpha^3}{144\left(t^2 - 2\alpha^2\right)} \text{ and} \\
\pi_B\left(a = 1/2, b = 1\right) &= \frac{-144\alpha t^2 + 97t^3 - 192t\alpha^2 + 288\alpha^3}{144\left(t^2 - 2\alpha^2\right)}.
\end{aligned}$$

Based on the results from this extension of our benchmark model, we conjecture that our main predictions would not change much qualitatively even if we gave each platform the choice to change its locations for each group of agents separately. More precisely, for low cross-group externalities platforms would maximally differentiate, for intermediate externalities we would expect to observe the asymmetric location equilibrium and for high externalities minimum differentiation.

6 Conclusion

We examine location decisions of two horizontally differentiated competing platforms in a two-sided market. Our model can yield both symmetric and asymmetric location equilibria, depending on the strength of the cross-group network externality and the sequence of moves. There are two effects when a platform moves closer to the location of the rival: i) platforms become less differentiated and prices tend to decline and ii) its market share increases. Because of the positive externality each group of agents exerts on the other, the market share effect is stronger than in a one-sided market, hence there is a tendency for less than maximum differentiation. In particular, we show that when the cross-group externality is weak, the principle of maximum differentiation holds. However, for strong externalities the two platforms locate at the center and the market tips in favor of one platform. Finally, for intermediate externality, a pure strategy location equilibrium does not exist (when platforms move simultaneously). With sequential moves, we obtain an asymmetric location equilibrium, where the first mover locates at the center and the follower at an extreme location. Both platforms have strictly positive market shares. Our model offers an explanation for the coexistence (see the Introduction for examples) of 'general' platforms (first mover) that cater to agents with intermediate tastes with niche platforms (follower) that serve agents with extreme tastes. A merger between the two competing platforms can be social welfare enhancing.

Finally, we would like to highlight the role of 'product' selection. It is also true that even with fixed locations at the extremes the market tips when the externalities are strong enough (e.g., Armstrong (2006)). This is typical in models with cross-group network externalities. Nevertheless, when locations are fixed, we do not obtain asymmetric market structures where *both* platforms are active, which is the case when locations are endogenized (see Proposition 4). In that sense, a testable implication of our model is that market shares evolve 'more continuously' as the degree of network externalities (or the degree of differentiation) varies, starting from symmetric market structures when externalities are low, then moving to asymmetric market structures when externalities become stronger, and eventually, for very strong externalities, the market is dominated by one platform.

7 Appendix

7.1 Proof of Lemma 1

The thresholds (12) and (11) are very important at this stage. It turns out that $\hat{a} \geq \tilde{a}$ if and only if

$$b \geq \bar{b} \equiv \frac{1}{2} + \frac{\alpha}{2t}. \tag{22}$$

Symmetrically, we can define the thresholds for platform B for a fixed location a of platform A. When $\hat{a} \geq \tilde{a}$ the binding threshold is the $\tilde{a}$. In this case, as it will become evident below, the other threshold is irrelevant. The opposite is true when $\hat{a} \leq \tilde{a}$.

We make the assumption that when the market tips it is the platform that is closer to the middle point, 1/2, that attracts all the agents. If they are equidistantly located from the middle, $a = 1 - b$, then all agents join platform A. In what follows, we assume that $a \leq b$. In the proofs of the Propositions, we allow $a > b$, but in this case the roles of the platforms are reversed and the profit functions and the thresholds can be obtained via a simple relabeling.

When $b \geq \bar{b}$, as platform A increases a it attracts all agents when $a = \tilde{a} \geq 1 - b$ (market tips).[11] After this point, the market remains tipped in favor of A until $a = b$. Figure 2 depicts this case.

When $b \leq \bar{b}$, platform A's market share becomes zero when $a = \hat{a} \leq 1 - b$.[12] After this point, the market will tip in favor of A when $a = 1 - b$ (symmetric locations). The market will remain tipped in favor of A until $a = b$. Figure 3 depicts this case.

When $a > b$ the roles of the platforms are reversed (A becomes B and B becomes A). The analysis in this case will follow from the above two cases and it will be equivalent to fixing a and allowing b to vary. This in turn is equivalent to the above two cases after we set $b = 1 - a$.

The second order condition (8) is always satisfied when the market has not tipped, i.e., when $a < \min\{\tilde{a}, \hat{a}\}$.The derivative of platform A's profit function (13) with respect to a is

$$\frac{\partial \pi_A(a,b)}{\partial a} = \frac{\left(2at - 2bt + \alpha - 4abt + 4a\alpha + 3a^2t + b^2t\right)\left(2bt - 2at - 3\alpha - a^2t + b^2t\right)t}{9\left(bt - \alpha - at\right)^2}.$$

[11] It can be shown that $\tilde{a} \geq 1 - b$ if and only if $b \geq \bar{b}$.

[12] It can be shown that $\hat{a} \leq 1 - b$ if and only if $b \leq \bar{b}$. In addition, $\hat{a} = 0$, when

$$b \leq -1 + \frac{\sqrt{t^2 + 3t\alpha}}{t} < \bar{b}.$$

This suggests that when platform B moves closer to the middle after the above threshold platform A's market share will be zero even when $a = 0$.

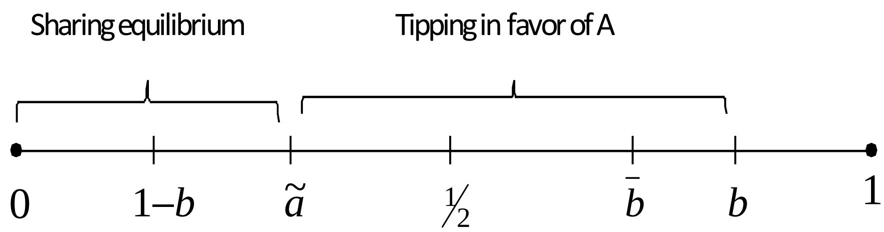

Figure 2: Type of equilibrium as the location a of platform A varies from 0 to b, with the location of platform B fixed at $b \geq \bar{b}$

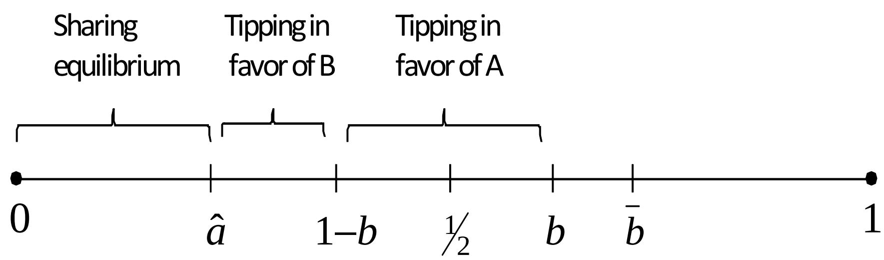

Figure 3: Type of equilibrium as the location a of platform A varies from 0 to b, with the location of platform B fixed at $b \leq \bar{b}$

Note that

$$\frac{\partial \pi_A (a = 0, b)}{\partial a} = \frac{\left(\alpha - 2bt + b^2 t\right)\left(2bt - 3\alpha + b^2 t\right)}{9 (bt - \alpha)^2} < 0$$

if $b > \left(-t + \sqrt{3t\alpha + t^2}\right)/t$.[13] So, the derivative of the profit function at $a = 0$ is always strictly negative when the market is shared.

[13] There are four roots when we solve

$$\frac{\partial \pi_A (a = 0, b)}{\partial a} = \frac{\left(\alpha - 2bt + b^2 t\right)\left(2bt - 3\alpha + b^2 t\right)}{9 (bt - \alpha)^2} = 0$$

with respect to b. One is negative and the other is greater than one, so we rule them out. The remaining two are

$$r_1 \equiv \frac{1}{t}\left(-t + \sqrt{3t\alpha + t^2}\right) \text{ and } r_2 \equiv \frac{1}{t}\left(t - \sqrt{-t\alpha + t^2}\right).$$

It can be shown that $r_1 \geq r_2$ if and only if $t \geq \alpha$, which we have assumed holds. It can be computed that $\hat{a}$ (from (12)) becomes zero when $b \leq r_1 \equiv \left(-t + \sqrt{3t\alpha + t^2}\right)/t$. Therefore, the other feasible root, root r_2, becomes irrelevant since when $b \leq r_1$ the market tips.

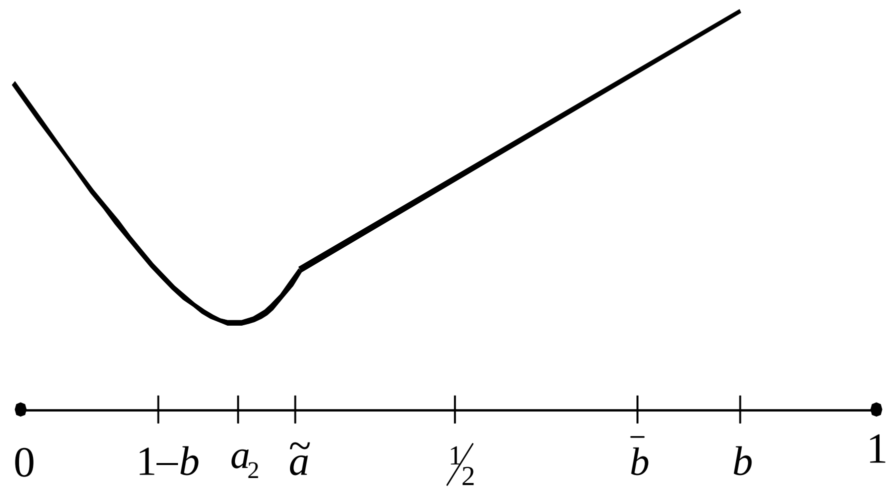

Figure 4: Profit function of platform A as its location a varies from 0 to b for a fixed location b of platform B at $b \geq \bar{b}$

For any a now, the derivative $\partial\pi_A(a,b)/\partial a$ becomes zero at

$$a = a_1 \equiv -\frac{1}{t}\left(t - \sqrt{-3t\alpha + t^2 + 2bt^2 + b^2t^2}\right) \tag{23}$$

$$a = a_2 \equiv \frac{1}{3t}\left(-t - 2\alpha + 2bt + \sqrt{t\alpha - 8bt\alpha + t^2 + 2bt^2 + 4\alpha^2 + b^2t^2}\right) \tag{24}$$

$$a = a_3 \equiv -\frac{1}{t}\left(t + \sqrt{-3t\alpha + t^2 + 2bt^2 + b^2t^2}\right)$$

$$a = a_4 \equiv \frac{1}{3t}\left(-t - 2\alpha + 2bt - \sqrt{t\alpha - 8bt\alpha + t^2 + 2bt^2 + 4\alpha^2 + b^2t^2}\right).$$

Roots a_3 and a_4 are negative, so we rule them out.[14] Also note that $a_1 = \hat{a}$, where $\hat{a}$ is given by (12). As we have mentioned before, we have $\hat{a} = a_1 \geq \tilde{a}$ if $b \geq \bar{b} \equiv 1/2 + \alpha/2t$ and $\tilde{a} \geq a_1 = \hat{a}$, if $b \leq \bar{b}$. This implies the following about the profit function of platform A for any fixed location b of platform B.

Case 1: $b \geq \bar{b} \equiv 1/2 + \alpha/(2t)$. Figure 4 depicts this case, where we have assumed that $a_2 \leq \tilde{a}$. Whether this holds or not depends, as we explain below, on the magnitude of α.

We have that $\hat{a} = a_1 \geq \tilde{a}$, so only root a_2 may be relevant. Given that it is the only relevant

[14] Root a_3 is clearly negative. Root a_4 is negative because at $\alpha = 0$, a_4 becomes

$$\frac{-t(2-b)}{3t} < 0$$

and the derivative of a_4 with respect to α is

$$\frac{1}{t}\left(-\frac{1}{6}\frac{t + 8t\alpha - 8bt}{\sqrt{t\alpha - 8bt\alpha + t^2 + 2bt^2 + 4\alpha^2 + b^2t^2}} - \frac{2}{3}\right)$$

which is always negative for $b \in [0,1]$.

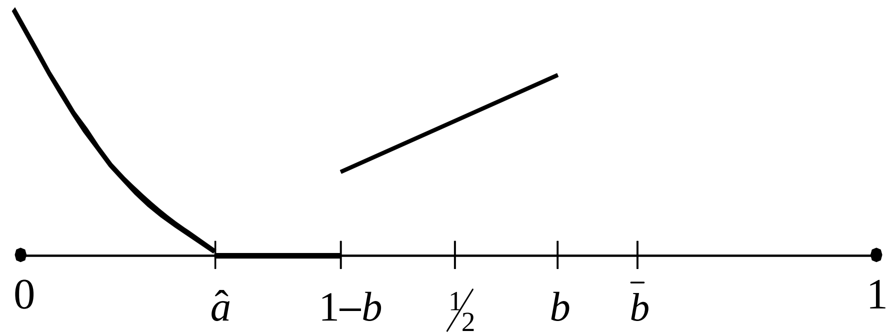

Figure 5: Profit function of platform A as its location a varies from 0 to b for a fixed location b of platform B at $b \leq \bar{b}$

root coupled with the fact that the profit function of platform A is strictly decreasing at $a = 0$ when sharing takes place, the profit function must attain a local minimum at $a = a_2$, if $a_2 \leq \tilde{a}$. The two platforms share the agents (no tipping) when $a < \tilde{a}$. As platform A moves closer to platform B its prices fall (see (9)) but the market shares after a certain point may increase. This may happen after $a = 1 - b$ where platform A is closer to the middle point $1/2$ than B. That is why a minimum may be attained at $a = a_2$ and after this point the profit function increases. This is not always true as a_2 may be greater than $\tilde{a}$, in which case the profit function of platform A is decreasing until $a = \tilde{a}$. Platform B is losing market share and at $a = \tilde{a}$ tipping occurs in favor of A. Profits increase for A as it moves closer to b because its distance to the marginal agents, who are located at 1, decreases. When $a > b$ the platforms reverse roles. Overall, the profit function of platform A as a function of its location a is U-shaped up to $a = b$ when $b \geq \bar{b}$.

Case 2: $b \leq \bar{b} \equiv 1/2 + \alpha/(2t)$. Figure 5 depicts this case.

Given that platform A's profit function is strictly decreasing at $a = 0$ when the market is shared and that $\hat{a} = a_1$ is now a relevant root, root a_2 becomes irrelevant. This is because at $a = \hat{a} = a_1$ the market tips in favor of B and the slope of A's profit function becomes zero. This implies that a_2 cannot be less than $\hat{a} = a_1$, since if that was the case there should be one more root in that range. Hence, in this case it must be that $a_2 > \hat{a} = a_1$ and therefore a_2 is irrelevant. The two platforms share the market when $a < \hat{a}$, unless $b \leq \left(-t + \sqrt{3t\alpha + t^2}\right)/t$, in which case platform A's market share is zero at $a = 0$. As platform A moves closer to b both prices and market shares fall (because now B is closer to the middle than in case 1 above) and at $a = \hat{a}$ the market tips in favor of B. Then, at $a = 1 - b$ the market tips in favor of A. As in case 1 above, the profit function is U-shaped up to $a = b$.

The above two cases will be used in the proofs of Propositions 2 and 3. These cases illustrate that the profit function will also be U-shaped when $a > b$. This follows because the case of $a > b$ is a simple relabeling of the above two cases.

7.2 Proof of Proposition 2

We set $b = 1$ and examine how the profit function of platform A behaves as a ranges from 0 to 1. Because $b \geq \bar{b}$, only the $\tilde{a}$ threshold is relevant. As we proved in Lemma 1, the profit function of A exhibits a U-shape pattern with respect to a (see also Figure 4). This implies that any interior location $a \in (0,1)$ yields lower profits than the extreme locations. For an interior pricing equilibrium we need (11), $a < \tilde{a}$, to be satisfied. When $a = 0$, we have that $\partial \pi_A (a = 0, b = 1) / \partial a = -t/3 < 0$, which implies that platform A has no incentive to locally deviate from $a = 0$, when platform B is at $b = 1$. (This is the standard result from one-sided markets). But A can deviate globally and induce tipping in its favor.

Next, we examine the tipping case. This case arises when $a \geq \tilde{a}$. When $b = 1$, this amounts to

$$a \geq \tilde{a} \equiv -\frac{1}{t}\left(\sqrt{3t\alpha + t^2} - 2t\right).$$

When $a \geq \tilde{a}$, platform B's market shares and prices are zero. The marginal agents from both groups are located at 1. Platform A has attracted all the agents. The indirect utility of the marginal agent from group ℓ if he joins platform B is V, while if he joins platform A instead his indirect utility becomes $V + \alpha - t(1-a)^2 - p_{\ell A}$. So, platform A will set its price to keep the marginal agent indifferent between the two platforms. This yields

$$p_{\ell A} = \alpha - t(1-a)^2.$$

Prices and profits for platform A increase in a. Platform A then locates at $a = 1$, charges $p_{\ell A} = \alpha$ and dominates the market. Platform A's profits are

$$\pi_A (a = 1, b = 1) = 2\alpha.$$

On the other hand when $a = 0$ the market is shared. Hence, from (13), platform A's profits are

$$\pi_A (a = 0, b = 1) = t - \alpha.$$

Platform A has no incentive to deviate from $a = 0$ if and only if $t - \alpha \geq 2\alpha \Rightarrow \alpha \leq t/3$.

If $\alpha > t/3$, then platform A has an incentive to locate at $a = 1$ and dominate the market. Platform B in this case earns zero profits. Hence, a maximum horizontal differentiation equilibrium does not exist.

7.3 Proof of Proposition 3

We look for a location equilibrium when $\alpha > t/3$. In Proposition 2, we proved that platform A has an incentive to locate at $a = 1$, when B is located at $b = 1$ and $\alpha > t/3$. But this location configuration cannot be an equilibrium. Platform B's profits are zero, and if platform B deviates to $b = 0$, and assumes the role of platform A, its profits will become strictly positive. This is because when differentiation is maximum the equilibrium is always sharing, given our assumption that $t > \alpha$. Hence, $b = 1$ cannot be part of an equilibrium when $\alpha > t/3$.

To this end, we fix $b < 1$ and we examine the properties of $\pi_A(a, b < 1)$. We have the following two cases.

Case 1: $b \geq \bar{b}$. Figure 4 in Lemma 1 depicts this case.

Case 2: $b \leq \bar{b}$. Figure 5 in Lemma 1 depicts this case.

For any fixed b we either have sharing or tipping. The above two cases suggest that if the equilibrium involves sharing the optimal location of A is either at $a = 0$ or at $a = 1$, as the platform wants to move away from the rival. Without loss of generality let's assume that $b \geq 1/2$. Hence, platform A will either locate at zero (given that $b \geq 1/2$, $a = 0$ is optimal when the equilibrium involves sharing) or it will induce tipping in its favor and locate at $a = b$. Note that if $a > b$ the market will tip in favor of B. If $a = 0$, platform B, as we showed in Proposition 2, will have an incentive to also locate at zero. If it is $a = b > 1/2$, platform B makes zero profits and can move to a location that is closer to the middle in order to attract all the agents and enjoy strictly positive profits. If it is $a = 1/2 < b$, A will have an incentive to move to $a = b$. Hence, the only possible equilibrium is $a = b = 1/2$, with platform A attracting all agents. To be an equilibrium, platform B must not be able to secure strictly positive profits even when it locates at the extreme. We had showed that $\hat{a} = 0$ (from (12)), when

$$b \leq -1 + \frac{\sqrt{t^2 + 3t\alpha}}{t} < \bar{b}.$$

If

$$-1 + \frac{\sqrt{t^2 + 3t\alpha}}{t} \geq \frac{1}{2}$$

then platform A cannot obtain positive profits regardless of where it moves if B is located at $1/2$. The above inequality holds if and only if $\alpha \geq 5t/12$. Here, the roles of A and B are reversed: A is located at $1/2$ and B chooses where to locate in order to make strictly positive profits. With a simple relabeling, the above discussion implies that platform B cannot obtain strictly positive profits regardless of where it locates.

If $\alpha < 5t/12$ a location equilibrium in pure strategies does not exist. Platform B will have an incentive to move to $b = 1$, but then platform A will have an incentive to locate at $a = 1$ and so on.

7.4 Proof of Proposition 4

Fix $a \leq 1/2$ and look at the optimal location of platform B. We showed in Lemma 1 that platform A's profit function is U-shaped with respect to a for any $b \geq a$. Via a simple relabeling the same holds for platform B's profit function with respect to b for any fixed $a \leq b$. Hence, if $a < 1/2$, platform B's profit is decreasing starting from $b = 1$ until $b = 1 - a$. After that point, the market tips in favor of B and its profits tend to 2α as b tends to a, independent of where a is. These profits are higher than the profits platform B obtains if it stays at $b = 1$. This can be seen as follows. When $a = 0$, 2α is higher than the profits of platform B when $b = 1$ (which are $t - \alpha$) if and only if $\alpha > t/3$. Moreover, as a increases both the (interior) prices of platform B and its market shares decrease. Therefore, moving away from $b = 1$ and locating arbitrarily close to a will yield higher profits for B. This suggests that a location configuration with $a < 1/2$ and $b = 1$ cannot be an equilibrium. Platform B has a profitable deviation and platform A makes zero profits. The subgame perfect equilibrium is for platform A to locate at $a = 1/2$ and B to locate at $b = 1$. Given $a = 1/2$, platform B has no profitable deviation. In addition, following from the proof of Proposition 3, platform B attracts agents (sharing equilibrium). Platform A has no incentive to locate to $a < 1/2$ because in that case platform B can locate closer to the middle point and attract all the agents, leaving A with zero profits.

The profits of platform A are

$$\pi_A (a = 1/2, b = 1) = \frac{(7t - 12\alpha)^2}{72 (t - 2\alpha)}$$

and of platform B are

$$\pi_B (a = 1/2, b = 1) = \frac{(5t - 12\alpha)^2}{72 (t - 2\alpha)}.$$

7.5 Proof of Proposition 5

We differentiate (14) with respect to a, b, x and y. There is only one interior solution to the system of first order conditions

$$x = y = \frac{1}{2} \text{ and } a = \frac{1}{4}, b = \frac{3}{4}.$$

The interior solution yields welfare equal to $W = \alpha - t/24$. The corner solution is the one where the social planner has only one platform serving all agents (tipping). If we set $x = y = 1$, for example, then it is easy to verify that welfare is maximized at $a = 1/2$ and is equal to $W = 2\alpha - t/6$. Finally, it can be easily verified that the interior solution dominates the corner solution if and only if $\alpha \leq t/8$.

7.6 Proof of Lemma 7

The derivative of platform A's profit function with respect to a is

$$\frac{\partial \pi_A(a,b)}{\partial a} = -\frac{t\left(3t^2a^2 + 2t^2a - 4t^2ab - 2t^2b + t^2b^2 + \alpha^2 + 4a\alpha^2\right)\left(t^2a^2 + 2t^2a - 2t^2b - t^2b^2 + 3\alpha^2\right)}{18\left(t^2(b-a) - \alpha^2\right)^2}.$$

Note that

$$\frac{\partial \pi_A(a=0,b)}{\partial a} = -\frac{t\left(-2t^2b + b^2t^2 + \alpha^2\right)\left(-2bt^2 + 3\alpha^2 - b^2t^2\right)}{18\left(bt^2 - \alpha^2\right)^2} < 0$$

if $b > \left(-t + \sqrt{3\alpha^2 + t^2}\right)/t$.[15] So, the derivative of the profit function at $a = 0$ is always strictly negative when the market is shared.

For any a now, the derivative $\partial \pi_A(a,b)/\partial a$ becomes zero at

$$a = a_1 \equiv \frac{1}{t}\left(-t + \sqrt{t^2 + 2bt^2 + b^2t^2 - 3\alpha^2}\right) \quad (25)$$

$$a = a_2 \equiv \frac{1}{3t^2}\left(-t^2 + 2bt^2 - 2\alpha^2 + \sqrt{t^4 + 2bt^4 + 4\alpha^4 + b^2t^4 + t^2\alpha^2 - 8bt^2\alpha^2}\right) \quad (26)$$

$$a = a_3 \equiv \frac{1}{t^2}\left(-t^2 - \sqrt{t^4 + 2bt^4 + b^2t^4 - 3t^2\alpha^2}\right)$$

$$a = a_4 \equiv \frac{1}{3t^2}\left(-t^2 + 2bt^2 - 2\alpha^2 - \sqrt{t^4 + 2bt^4 + 4\alpha^4 + b^2t^4 + t^2\alpha^2 - 8bt^2\alpha^2}\right).$$

Roots a_3 and a_4 are negative, so we rule them out.[16] Also note that $a_1 = \hat{a}_1$, where $\hat{a}_1$ is given by (18). As we have mentioned before, we have $\hat{a}_1 = a_1 \geq \tilde{a}_1$ if $b \geq \bar{b}' \equiv 1/2 + \alpha^2/2t^2$ and

[15] There are four roots when we solve

$$\frac{\partial \pi_A(a=0,b)}{\partial a} = -\frac{t\left(-2t^2b + b^2t^2 + \alpha^2\right)\left(-2bt^2 + 3\alpha^2 - b^2t^2\right)}{18\left(bt^2 - \alpha^2\right)^2} = 0$$

with respect to b. One is negative and the other is greater than one, so we rule them out. The remaining two are

$$r_1 \equiv \frac{1}{t}\left(-t + \sqrt{3\alpha^2 + t^2}\right) \text{ and } r_2 \equiv \frac{1}{t}\left(t - \sqrt{-\alpha^2 + t^2}\right).$$

It can be shown that $r_1 \geq r_2$ if and only if $t \geq \alpha$, which we have assumed holds. It can be computed that $\hat{a}_1$ (from (18)) becomes zero when $b \leq r_1 \equiv \left(-t + \sqrt{3\alpha^2 + t^2}\right)/t$. Therefore, the other feasible root, root r_2, becomes irrelevant since when $b \leq r_1$ the market tips.

[16] Root a_3 is clearly negative. Root a_4 is negative because at $\alpha = 0$, a_4 becomes

$$\frac{-(2-b)}{3} < 0$$

$\tilde{a}_1 \geq a_1 = \hat{a}_1$, if $b \leq \bar{b}'$. Following the analysis from Lemma 1 we can conclude that the profit function of platform A for any fixed location b of platform B is U-shaped.

7.7 Proof of Proposition 8

We set $b = 1$ and examine how the profit function of platform A behaves as a ranges from 0 to 1. Because $b \geq \bar{b}'$, only the $\tilde{a}_1$ threshold is relevant. Overall, the profit function of A exhibits a U-shape pattern with respect to a. This implies that any interior location $a \in (0,1)$ yields lower profits than the extreme locations. When $a = 0$, we have that $\partial \pi_A (a = 0, b = 1) / \partial a = -t/6 < 0$, which implies that platform A has no incentive to locally deviate from $a = 0$, when platform B is at $b = 1$. (This is the standard result from one-sided markets). But A can deviate globally and induce tipping in its favor.

Next, we examine the tipping case. The profits of platform A are given by (21). Platform A then locates at $a = 1$ and dominates the market. Platform A's profits are

$$\pi_A (a = 1, b = 1) = \frac{t^2 + \alpha^2}{2t}.$$

On the other hand, when $a = 0$ the market is shared. Hence, from (13), platform A's profits are

$$\pi_A (a = 0, b = 1) = t - \alpha.$$

Platform A has no incentive to deviate from $a = 0$ if and only if $t - \alpha \geq \left(t^2 + \alpha^2\right) / 2t \Rightarrow \alpha \leq \left(\sqrt{2} - 1\right) t$. If $\alpha > \left(\sqrt{2} - 1\right) t$, then platform A has an incentive to locate at $a = 1$ and dominate the market. Platform B in this case earns zero profits. Hence, a maximum horizontal differentiation equilibrium does not exist.

7.8 Proof of Proposition 9

We now investigate the case where $\alpha > \left(\sqrt{2} - 1\right) t$. In Proposition 8, we proved that platform A has an incentive to locate at $a = 1$, when B is located at $b = 1$ and $\alpha > \left(\sqrt{2} - 1\right) t$. But this location configuration cannot be an equilibrium. Platform B's profits are zero, and if platform B deviates to $b = 0$, and assumes the role of platform A, its profits will become strictly positive. This is because when differentiation is maximum the equilibrium is always sharing, given our assumption

and the derivative of a_4 with respect to α is

$$\frac{1}{t^2} \left(-\frac{4}{3}\alpha - \frac{1}{6} \frac{16\alpha^3 + 2t^2\alpha - 16bt^2\alpha}{\sqrt{t^4 + 2bt^4 + 4\alpha^4 + b^2t^4 + t^2\alpha^2 - 8bt^2\alpha^2}} \right)$$

which is always negative for $b \in [0,1]$.

that $t > \alpha$. Thus, $b = 1$ cannot be part of an equilibrium when $\alpha > \left(\sqrt{2} - 1\right) t$. To this end, we fix $b < 1$ and we examine the properties of $\pi_A(a, b < 1)$. We have the following two cases.

Case 1: $b \geq \bar{b}'$. Figure 4 depicts this case.

Case 2: $b \leq \bar{b}'$. Figure 5 depicts this case.

For any fixed b we either have sharing or tipping. The above two cases suggest that if the equilibrium involves sharing the optimal location of A is either at $a = 0$ or at $a = 1$, as the platform wants to move away from the rival. Without loss of generality let's assume that $b \geq 1/2$. Hence, platform A will either locate at zero (given that $b \geq 1/2$, $a = 0$ is optimal when the equilibrium involves sharing) or it will induce tipping in its favor and locate at $a = b$. Note that if $a > b$ the market will tip in favor of B. If $a = 0$, platform B, as we showed in Proposition 8, will have an incentive to also locate at zero. If it is $a = b > 1/2$, platform B makes zero profits and can move to a location that is closer to the middle in order to attract all the agents and enjoy strictly positive profits. If it is $a = 1/2 < b$, A will have an incentive to move to $a = b$. Hence, the only possible equilibrium is $a = b = 1/2$, with platform A attracting all agents. To be an equilibrium, platform B must not be able to secure strictly positive profits even when it locates at the extreme. We had showed that $\hat{a}_1 = 0$ (from (18)), when

$$b \leq -1 + \frac{\sqrt{t^2 + 3\alpha^2}}{t} < \bar{b}'.$$

If

$$-1 + \frac{\sqrt{t^2 + 3\alpha^2}}{t} \geq \frac{1}{2}$$

then platform A cannot obtain positive profits regardless of where it moves if B is located at $1/2$. The above inequality holds if and only if $\alpha \geq \sqrt{15}t/6$. Here, the roles of A and B are reversed: A is located at $1/2$ and B chooses where to locate in order to make strictly positive profits. With a simple relabeling, the above discussion implies that platform B cannot obtain strictly positive profits regardless of where it locates. If $\alpha < \sqrt{15}t/6$ a location equilibrium in pure strategies does not exist. Platform B will have an incentive to move to $b = 1$, but then platform A will have an incentive to locate at $a = 1$ and so on.

7.9 Proof of Proposition 10

Fix $a \leq 1/2$ and look at the optimal location of platform B. Platform A's profit function is U-shaped with respect to a for any $b \geq a$. We showed in Lemma 7 that platform A's profit function is U-shaped with respect to a for any $b \geq a$. Via a simple relabeling the same holds for platform

B's profit function with respect to b for any fixed $a \leq b$. Hence, if $a < 1/2$, platform B's profit is decreasing starting from $b = 1$ until $b = 1 - a$. After that point, the market tips in favor of B and its profits tend to 2α as b tends to a, independent of where a is. These profits are higher than the profits platform B obtains if it stays at $b = 1$. This can be seen as follows. When $a = 0$, 2α is higher than the profits of platform B when $b = 1$ (which are $t - \alpha$) if and only if $\alpha > \left(\sqrt{2} - 1\right) t$. Moreover, as a increases both the (interior) prices of platform B and its market shares decrease. Therefore, moving away from $b = 1$ and locating arbitrarily close to a will yield higher profits for B. This suggests that a location configuration with $a < 1/2$ and $b = 1$ cannot be an equilibrium. Platform B has a profitable deviation and platform A makes zero profits. The subgame perfect equilibrium is for platform A to locate at $a = 1/2$ and B to locate at $b = 1$. Given $a = 1/2$, platform B has no profitable deviation. In addition, following from the proof of Proposition 9, platform B attracts agents (sharing equilibrium). Platform A has no incentive to locate to $a < 1/2$ because in that case platform B can locate closer to the middle point and attract all the agents, leaving A with zero profits.

The profits of platform A are

$$\pi_A \left(a = 1/2, b = 1\right) = \frac{-144\alpha t^2 + 121t^3 - 240t\alpha^2 + 288\alpha^3}{144\left(t^2 - 2\alpha^2\right)}$$

and of platform B are

$$\pi_B \left(a = 1/2, b = 1\right) = \frac{-144\alpha t^2 + 97t^3 - 192t\alpha^2 + 288\alpha^3}{144\left(t^2 - 2\alpha^2\right)}.$$

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