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MIXED SOURCE

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ABSTRACT. We study competitive interaction between profit-maximizing firms that sell software and complementary goods or services. In addition to tactical price competition, we allow firms to compete through business model reconfigurations. We consider three business models: the *proprietary* model (where all software modules offered by the firm are proprietary), the *open source* model (where all modules are open source), and the *mixed source* model (where a few modules are open). When a firm opens one of its modules, users can access and improve the source code. At the same time, however, opening a module sets up an open source (free) competitor. This hampers the firm's ability to capture value. We analyze three competitive situations: monopoly, commercial firm vs. non-profit open source project, and duopoly. We show that: (*i*) firms may become "more closed" in response to competition from an outside open source project; (*ii*) firms are more likely to open substitute, rather than complementary, modules to existing open source projects; (*iii*) when the products of two competing firms are similar in quality, firms differentiate through choosing different business models; and (*iv*) low-quality firms are generally more prone to opening some of their technologies than firms with high-quality products.

KEYWORDS: Open Source, User Innovation, Business Models, Complementarity, Vertical Differentiation, Value Creation, Value Capture (JEL O31, L17, D43).

1. INTRODUCTION

As is well understood by now, commercial firms may benefit from participating in open source (OS) software development by selling complementary goods or services. For example, IBM sells consulting services and proprietary software that is complementary to the OS software that it develops, Red Hat sells subscription services, and Sun sells complementary hardware such as servers. In particular, the combination of OS software and proprietary extensions has grown into an important phenomenon. In fact, the expressions *mixed source* and *hybrid source* have been used to refer to a business model whereby a software firm releases an open source version of its software and derives revenue from selling proprietary complementary code. Examples include: JasperSoft (business intelligence software), Zimbra

(server software for email and collaboration), SugarCRM (customer relationship management software), Hyperic (systems monitoring, server monitoring, and IT management software), xTuple (enterprise resource planning software), Zenoss (enterprise IT management software), Talend (data integration software), and Groundwork (IT management and network monitoring software).

Even fervent advocates for proprietary software have jumped on the bandwagon. Having stated in 2001 that "open-source is an intellectual property destroyer [...] I can't imagine something that could be worse than this for the software business," Microsoft has recently switched course to embrace the notion of mixed source.[1] Amongst other initiatives, it has partnered with Novell to put some of Microsoft's technologies on Linux and other open platforms. The *Mono* project, for example, consists of porting the .Net framework onto Linux and the *Moonlight* project is about providing an offer of Siverlight for Linux.[2] And in July 2009, Microsoft agreed to contribute some of its technology to Linux under a licensing agreement that allows developers outside Microsoft to modify the code.[3]

Open source has the potential to improve value creation because it benefits from the efforts of a large community of developers. Proprietary software, on the other hand, results in superior value capture because the intellectual property remains under the control of the original developer. Industry observers, however, point out that strict open source and proprietary approaches to software development "don't work in a world where innovators have to innovate, investors need a profit, employees need to eat, and customer needs must be met."[4] The reasoning is that proprietary development leads to little innovation and open source leads to little profits. According to Horacio Gutierrez (Microsoft's Deputy General Counsel for IP Licensing): "striking a balance between [embracing open-source software and brandishing patents] is one of the key things every commercial technology company must do in order to compete effectively."[5] Figure 1 illustrates the "popular case" for mixed source.

Oracle's April 2009, $7.4 million acquisition of Sun can be understood by reference to the figure. Sun's most important software products were the Java programming language and the Solaris operating system. On November 13, 2006, Sun released Java as open source under the

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[1]The quote is by James Allchin (Microsoft's executive in charge of Windows 2000 rollout). See Ben Charny, "Microsoft raps open-source approach," *CNET News.com*, May 3, 2001 and `http://www.infoworld.com/print/38795`.

[2]Silverlight is a Web-based digital video technology by Microsoft. It is a plug-in for delivering media and interactive applications for the Web.

[3]Microsoft announced the release of 20,000 lines of device driver code to the Linux community. See `http://www.marketwatch.com/story/microsoft-defends-nuanced-open-source-approach`.

[4]Greg Thomas, "Microsoft and Mixed Source Software." (October 22, 2008). See `http://www.infoworld.com/d/mobilize/microsoft-exec-touts-mixed-source-ventures-795`.

[5]See `http://www.marketwatch.com/story/microsoft-defends-nuanced-open-source-approach`.
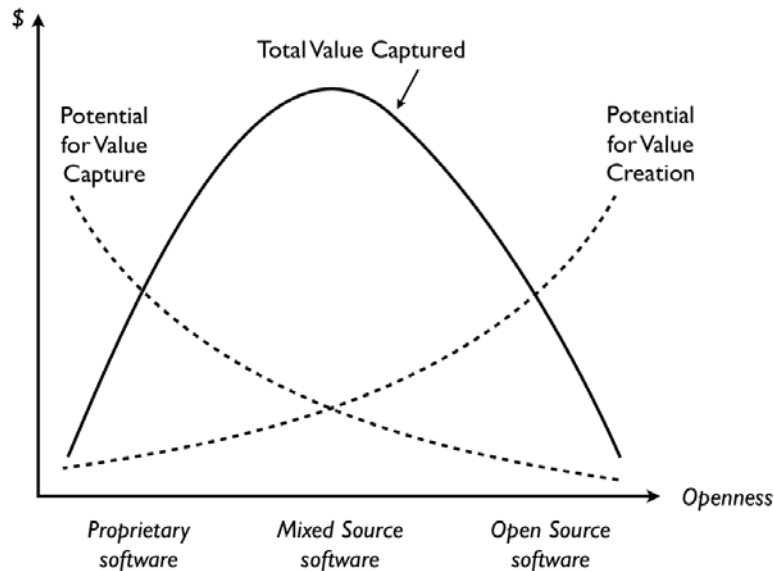
FIGURE 1. The case for mixed source development.

GNU General Public License. Likewise, a large part of Solaris's codebase was open sourced in 2005. Industry observers claimed that Sun's excessive openness prevented the company from capturing sufficient value. On the other hand, most of Oracle's products, such as the Oracle Database or the Oracle Fusion Middleware, were proprietary. The combination of Sun and Oracle created a mixed source corporation aiming a better balance between the potential for value creation and the potential for value capture as shown in Figure 1. The choice of Sun was quite natural because the Fusion Middleware was built on Java and Solaris was the leading platform for the Oracle Database. Both firms' main products were highly complementary.

While the straightforward rationale for mixed source presented in Figure 1 is appealing, the desirability (or lack thereof) of the mixed source business model is more nuanced. An implication of Figure 1 is that software firms *always* benefit from mixed source. However, we do not see every software firm opening (a portion of) its technologies. The question arises: under what circumstances should a profit-maximizing firm adopt a mixed source business model? And for firms considering the adoption of a mixed source business model, which technologies should be open and which should remain closed/proprietary? Should the core software be opened or is it better to open the extensions?

Likewise, the popular case for mixed source does not consider how business models of different industry players interact. While the adoption of mixed source may be optimal when the firm is alone in the market, competition may render this business model suboptimal. A

question that has not yet been addressed in the literature is: how should firms respond to competitors' adoption of mixed source business models?

Finally, the argument for mixed source of Figure 1 provides little advice on how firms that adopt that business model should price their products. What are the right pricing structures under mixed source compared to the proprietary business model? In this paper, we present a formal model that addresses these questions.

We set up a game-theoretical model where profit-maximizing firms that sell software and complementary goods or services (such as training or support services) must choose whether to open all or part of their technologies and the prices at which to sell their goods. The software that the firms offer is composed of two modules: a base (core) program and a set of extensions. The base program may be used without the extensions. The extensions, on the other hand, are valueless unless they are used in conjunction with a base program, *i.e.* the base program is an essential complement to the extensions (see Chen and Nalebuff 2006).[6] Firms may open the base, the extensions, or both.

Consistent with the terminology used in the software development community, we refer to the different degrees of openness as *business models*. Thus, we consider three business models. In the *proprietary* model, all the software offered by the firm is closed, and in the *open source* model, both modules are open. In the *mixed source* model, one module is open and the other one is closed. The mixed source model has two variants. In the *open core* model, the base program is open and the extensions closed, and in the *open extensions* model, the base program is closed and the extensions open.

Figure 2 shows examples of the different business models that we consider.

		Extensions	
		Open	Closed
Base	Open	**Open Source:** • Red Hat Linux • MySQL • OpenSolaris • Eclipse	**Open Core:** • JasperSoft • SugarCRM • Zimbra • Mac OS X
Base	Closed	**Open Extensions:** • MSFT .Net • Stata • Mathematica • Facebook	**Proprietary:** • MS Windows • MS Office • Oracle 11g • SAP

FIGURE 2. Examples of the different business models.

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[6]More generally, the extensions represent not only software modules, but all those technologies and protocols which have value only if used with the firm's core software.

The trade-off in our model builds on Figure 1. When a module is opened, users can access and improve the source code, which increases quality and value creation.[7] At the same time, however, the firm is setting up an open source competitor of its program which may cannibalize the revenues of the proprietary version. This hinders the firm's ability to capture value. We study how this trade off is affected by: (i) the value of complementary services that the firm may offer, such as support for software implementation or tailoring the software to customer specifications; (ii) the strength of user innovation; and (iii) competitors' responses, which include strategic reconfigurations in competitors' business models and tactical price adjustments.

We present our model and analysis in three parts, each building on the previous one. In Section 3, we analyze the case of an incumbent firm that enjoys a monopoly in its market segment (*i.e.* there are no open or commercial substitutes for the firm's software) and that sells its product (a combination of software and a complementary good or service) to a measure one of potential customers. We refer to the product sold by the monopolist as the *commercial product.* The incumbent monopolist chooses the business models through which it would like to operate. As long as the base module is kept proprietary, the firm remains a monopolist. The firm can benefit from user innovation by opening some (or all) of its software modules. However, when the base module is opened, a free, open source product emerges. We refer to the open source product set up by the monopolist when opening the base module as the *OS product.* The willingness to pay for the OS product is lower than that for the commercial product because it does not come bundled with the complementary good or service.

We find that when the value of the complementary good (z) and the strength of user innovation (σ) are low, the firm will open the extensions only. For intermediate values of z and σ, the firm prefers to open the base module instead of the extensions. For z and σ sufficiently large, the firm will open both modules. In the first two cases, the firm prefers to compete through a mixed source business model, and in the third case, the firm is using the pure open source business model. The intuition is simple. Large z increases the vertical differentiation between the commercial product and the OS product. Therefore, the business stealing effect is mild in this case. Likewise, when σ is large, the quality improvement that occurs when modules are opened, outweighs the negative competitive effect.

In Section 4, we consider the case in which the firm faces competition from an external open source project. This means that consumers may use a free open source product, even if the firm decides to compete through a proprietary business model. We allow the firm to replace any one of its modules with the available open source module/s if it desires to do so.

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[7]This effect is sometimes referred to as *demand-side learning.* See Casadesus-Masanell and Ghemawat (2006).

We find that the firm may become "more closed" in response to competition from an outside OS project: compared to the monopoly situation, the firm is more likely to use a proprietary business model. In particular, the firm will be wary of opening the set of extensions when the outside OS project has a base module. A combination of the outside base module and the firm's extensions result in a stronger free competitor.

When considering which mixed source business model to employ, the firm will always prefer the one that adds less value to the outside OS project. Thus, the firm is more likely to open substitute, rather than complementary, modules to the outside OS project.

Finally, we find that the firm will prefer to adopt the modules developed externally when these are of higher quality than the firm's own modules. When the outside OS project has modules of higher quality than those developed internally by the firm, the trade off between value creation and value capture of Figure 1 is broken: increased openness not only results in superior value creation but also in better value capture.

In Section 5 we study a profit-maximizing duopoly (firms H and L) which, in addition to prices, choose the business models through which they would like to compete. We assume that firm H has modules of higher quality than firm L. We analyze the following game: (i) in the first stage either H or L chooses business model, (ii) in the second stage, the other firm observes the first mover's business model and chooses its own, (iii) finally, both H and L simultaneously determine the prices of their commercial packages. This section studies *competition through business models*: firms best responding to each other's choices of business models, taking into account the equilibria of the subgames that different business models combinations set up.

We find that when the exogenous quality difference between H and L is low, one of the two firms competes through a proprietary business model and the other opens one module, generally the extensions. As the quality difference grows, cannibalization concerns lessen and both competitors elect to compete through the same mixed source business model. Likewise, when user innovation σ is low, only one of the two firms competes through a mixed source business model. The other firm is proprietary. And when σ is high, the equilibrium has both firms adopting a mixed source model.

We also find that both firms may prefer to compete through the proprietary business model when H is the first mover. This business model profile, however, never occurs when L is the leader. Thus, the low quality firm is more interested in competing through a mixed source business model than the high quality firm. Finally, we also show that quality leapfrogging may occur in equilibrium when L moves first but that it never happens when H leads.

Our theoretical model provides a formal base on which to build recommendations for management practice. In Section 6 we present simplified versions of our main results through

easily accessible matrices. In each case, we discuss what drives our propositions and lemmas and present policy recommendations that can be directly applied to the design of optimal business strategies. We also discuss the implications of our findings for the strategy literature, particularly as they relate to the subfield of value-based business strategy.

The rest of the paper is organized as follows. We first position our contribution in relation to the literature. In Section 2 we introduce the basic elements of our model. Section 3 presents the monopoly benchmark. In Section 4 we analyze competitive interaction between a profit-maximizing firm and a non-strategic OS competitor. The more challenging case where the business models of two strategic, for-profit companies are endogenously determined is presented in Section 5. Section 6 discusses the managerial implications and relates our results to a number of real-world examples. Section 7 concludes. The proofs of all propositions and lemmas are in Appendixes A and B. For completeness, in Appendix C we present the analysis of a few additional cases that, because of space constraints, we decided not to include in the main text.

1.1. **Related literature.** Our paper contributes to the literature on the economics of open source. For the most part, early papers on OS were concerned with explaining why individual developers contributed to OS projects, allegedly for free (see Lerner and Tirole (2005) and von Krogh and von Hippel (2006) for excellent surveys). The most common explanations were: altruism, personal gratification, peer recognition, and career concerns. Bagozzi and Dholakia (2006), for example, demonstrate that participation in OS development is partly explained by social and psychological factors and Roberts, Hann, and Slaughter (2006) find that status and career concern motivations significantly influence developers' levels of participation. Baldwin and Clark (2006) argue that the architecture of the code may affect the developers' incentives to contribute. Specifically, they show that a modular codebase mitigates free riding in OS development.

While the contributions of individual developers have played a crucial role in the growth of open source software, the same is true of contributions by commercial firms. In fact, in a carefully executed empirical piece, Bonaccorsi, Rossi, and Giannangeli (2006) show that fully proprietary and fully open software firms are rare. Instead, firms typically adopt a hybrid business models. The presence of complementarities has been documented by Fosfuri, Giarratana, and Luzzi (2008). The authors perform an econometric analysis and find that firms with a larger stock of hardware patents and trademarks are more likely to participate in OS. Shah (2006) investigates the effects of sponsorship of open source projects by commercial firms and finds that voluntary developers tend to contribute less, have different motivations

for contributing, and take on fewer code maintenance tasks than in the absence of such sponsorship.

On the theory front, a handful of papers have introduced profit maximizing OS firms (Henkel 2004, Bessen 2006, Schmidtke 2006, Haruvy, Sethi, and Zhou 2008). In these papers, commercial firms may profit from OS by selling complementary goods and services. However, these works consider OS firms in isolation, rather than in competition against proprietary firms. In fact, the interaction between open source and proprietary firms has been identified as an important topic for further research by Lerner and Tirole (2001, 2002, 2005).

The papers that study competition between the two development paradigms set up duopoly models of a profit-maximizing, proprietary firm and a community of not-for-profit/non-strategic OS developers that sell at zero price (Mustonen 2003, Bitzer 2004, Gaudeul 2005, Casadesus-Masanell and Ghemawat 2006, Economides and Katsamakas 2006, Lee and Mendelson 2008). In these papers, however, the business model choice by firms and the market structure are exogenously given. We endogeneize these constructs.

An important precedent for our paper is Llanes and de Elejalde (2009) which presents a model where profit-maximizing firms decide whether to be open source or proprietary. Firms profit from selling goods and services which are complementary to the software. The main trade-off is between the lower appropriability of the investment in R&D in OS and the duplication of effort in the proprietary model. These authors show that even when the appropriability of the investment in R&D is low, there are equilibria where both kinds of firms coexist in the same markets. These equilibria are characterized by an asymmetric market structure, with many small open source firms and a few large proprietary firms. Most importantly, this market structure arises endogenously, even though all firms are ex-ante symmetric.

Our paper builds upon this literature and presents a novel approach to the study of optimal business models and equilibrium market structure. Specifically, our modeling contributions are: (1) firms have multiple software modules and must decide which modules to open and which to keep proprietary, (2) firms choose not only between the two *pure* business models (open source and proprietary), but may also compete through a *mixed* source model, (3) we analyze the case of a monopolist, but also the cases in which a for-profit firm competes against a not-for-profit OS project, and against another for-profit firm. As a consequence of our modeling strategy, a variety of market structures may arise in equilibrium, combining proprietary, mixed source, and open source firms.

Our paper also contributes to an emerging literature in strategy that explores competitive interactions between organizations with different business models. While there are several formal models of asymmetric competition that exist in strategy (differences in costs, resource

endowments, or information, mainly), the asymmetries that this literature wrestles with are of a different nature: firms with fundamentally different objective functions, opposed approaches to competing, or different governance structures. The papers mentioned above on competition between open source and proprietary firms belong to this literature. In addition, Casadesus-Masanell and Yoffie (2007) study competitive interactions between two complementors, Microsoft and Intel, with asymmetries in their objectives functions stemming from technology—software *vs.* hardware. Casadesus-Masanell and Zhu (2009) study competitive interaction between a high-quality incumbent that faces a low-quality ad-sponsored competitor. Finally, Casadesus-Masanell and Hervas-Drane (2009) analyze competitive interactions between a free peer-to-peer file sharing network and a profit-maximizing firm that sells the same content at positive price and that distributes digital files through an efficient client-server architecture. For the most part, this literature has studied interactions between firms with exogenously given business models.[8] We contribute by endogeneizing the choice of business model by two profit-maximizing firms that compete to attract customers.

2. The model

2.1. **Preferences.** Preferences are based on a variant of Gabszewicz and Thisse's (1979) and Shaked and Sutton's (1982) models of vertical product differentiation. There is a continuum of consumers of mass 1, who differ in their valuations of the available products. Consumers are indexed by ρ, where $\rho \sim U[0,1]$. Consumer ρ's indirect utility from consuming good i is:

$$u_{\rho i} = \rho \, v_i - p_i,$$

where v_i and p_i are the quality and price of product i.

Given the list of qualities and prices for all products available, each consumer will choose the product that maximizes his indirect utility.

2.2. **Technology.** Consumers derive utility from consuming software modules and a complementary service z. There are two software modules: a base module and a set of extensions, denoted by a and b, respectively. With a slight abuse of notation, we use the same symbol to refer to a software module (or service) and to the value of that software module (or service). Let $a > 1$ and $b > 1$ be the value of modules a and b, respectively, and $z > 1$ be the value of service z. We assume zero marginal costs.

Software modules and service are complementary. The value of a package composed by software modules a and/or b and service z is the product of the value of its components. For

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[8]The exception is Casadesus-Masanell and Zhu (2009) who allow the incumbent (but not the entrant) to choose business model.

example, if a package is composed by modules a and b, its value is $v = a\,b$, and if it also includes service, its value becomes $v = a\,b\,z$.

Module a constitutes the base (core) of the program and module b is a set of extensions that run over module a. This means that module b has no value if it is not used with module a, so the value of a package which does not include module a is zero. Similarly, support has value only when used in conjunction with software. Although, for concreteness, we refer to b as software extensions, b represents more generally those technologies, protocols, documentation, and ideas that have value only if used in conjunction with the core software a. For instance, file format protocols are of little value if there is no core software with which to use them.

Modules may be opened or kept closed. The advantage of opening a module is that users can improve on it. We assume that when a module is opened, its value increases to $a_o = a\,(1 + \sigma\,q_o)$ or $b_o = b\,(1 + \sigma\,q_o)$ where σ measures the extent of user innovation and q_o is the measure of users of the open source module.[9]

Opening a module increases the value of all packages containing that module. For example, if a package includes both modules and service, and the base module a is opened, its value becomes $v = a_o\,b\,z$.

2.3. **Expectations.** When a module is opened, before users decide whether to adopt it or not, the firm and the users form expectations on the measure of users that will adopt it, q_o^e. Suppose, for example, that module a is opened. Before users decide whether to adopt it or not, they form expectations on the value of that module. This value depends on how many users are *expected* to use the module in equilibrium: $a_o^e = a\,(1 + \sigma\,q_o^e)$.

What is the relation between q_o^e and q_o? We follow Katz and Shapiro (1985) and assume fulfilled expectations. The expected number of users q_o^e is taken as given by consumers and firms when they make their decisions. What Katz and Shapiro's (1985) criterion requires is that those expectations be fulfilled in equilibrium: $q_o = q_o^e$. While the equilibrium q_o depends on the expectation q_o^e, the advantage of this approach is that there is no need to specify how expectations are formed.

2.4. **Business models.** We consider four business models. Two pure models and two mixed models. The pure models are the proprietary and the open source business models. These are pure models because all modules are either closed or open:

Proprietary (P): In the proprietary model both modules, a and b, are closed. This model has no user innovation.

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[9]The user innovation term is similar to a network externality. In contrast to network externalities, however, if the firm decides not to open the module, user innovation is absent.

Open source (O): In the open model both modules are open. User innovation is maximal in this case.

The two mixed models have one open module. The other module is closed. The two possibilities are:

Mixed source with open a (M_a): This business model has the base module open.[10] When M_a is adopted, an open source competitor that offers module a for free will emerge.

Mixed source with open b (M_b): In this model, only the extensions b are open. The implication is that no open source competitor emerges when this business model is adopted because the extensions cannot be used without a base module.

The most common definition of *business model* is "the logic of the firm, the way it operates to create and capture value for its stakeholders."[11] Thus, a firm's *real* business model includes a broad range of organizational and competitive elements such as products & markets, sources of revenue, incentive systems, hiring policies, information technologies, and so on. Detailed descriptions of business models are often too complex to be amenable to mathematical treatment. We follow Casadesus-Masanell and Zhu (2009) and represent business models through profit functions. Thus, in our formal development the choice of business model is represented through the choice of profit function.

Profit functions are reduced form representations of actual business models. As such, they do not capture the full richness of the entire business model. However, they allow the derivation of formal propositions, which is an important goal of our study. Representing business models through profit functions captures the idea that choosing a business model implies more than just determining the value of a particular variable or parameter: it entails the selection of the logic of the firm, the way it operates to create and capture value for its stakeholders.

2.5. **Cases.** We consider three alternative competitive scenarios. In Section 3, we analyze the case of a monopoly that must choose its business model and where to price its products. In Section 4, we examine a simple duopoly where a profit-maximizing firm faces an open source competitor that is nonstrategic. And in Section 5 we study the less tractable case of two for-profit firms that best-respond to each others' pricing and business model choices.

While the specifics of each scenario differ (we explain them in detail in the corresponding section), the generic sequence of decisions that we study is as follows:

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[10]This business model is sometimes called *open core*. See `http://www.linuxinsider.com/story/66807.html`, for example.

[11]See Baden-Fuller, MacMillan, Demil, and Lecocq (2008).

(1) Firms decide which business model to adopt.

(2) Expectations on q_o are formed and firms decide where to price their products.

(3) Given q_o^e, business models, and prices, consumers pick their preferred product.

We solve for the subgame perfect equilibrium with fulfilled expectations.

2.6. **Strategy vs. tactics.** Building on Ghemawat (1991) and Casadesus-Masanell and Ricart (2009), we refer to the choice of business model as *strategy*: the business model is a set of committed choices that lays the ground for the tactical interactions that will occur down the line. Thus, business model choices are *strategic* whereas pricing choices are *tactical*. This terminology reflects the fact that while the choice of business model often implies commitments which may be difficult to reverse, pricing choices are easy to change.[12]

2.7. **Modeling assumptions.** Before delving into the analysis, we discuss three important modeling assumptions.

Base vs. extensions. Our distinction between base and extensions reflects the fact that while some software modules can be used standalone, others require a base to function. This distinction captures important asymmetries between specific and general applications. Some software modules have general value, while others have been designed for highly specialized uses. The extent of module specificity affects its value to the open source community. In the case of general applications, a larger number of developers are likely to contribute and user creativity and innovation are likely to be enhanced. Although we assume that the extent of innovation *per user* is the same for both modules, endogenous adoption of the base is often larger than the extensions when they are opened. Therefore, consistent with the phenomenon, in our model there is more user innovation for the base than for the extensions.

User innovation. We have adopted a reduced-form approach to modeling user innovation: more users imply more user innovation and thus higher quality software. The strength of this relation is governed by the parameter σ. Our specification is simple and intuitive but has limitations. This modeling choice has been made for tractability.

An interesting direction for future research is to "open the black box" of user innovation, by endogenizing the users' decision to contribute and/or provide feedback. This would allow the study of many interesting issues arising from user innovation. For example, having more developers working on a particular software may increase its complexity, raising the communication and coordination requirements. This is well summarized by Brooks's Law:

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[12]For example, once the a module has been opened, there is no way back to being proprietary because the code is in the public domain.

"adding more programmers to a late project makes it later." Therefore, having more developers may imply higher coordination costs. The firm may end up bearing some of these costs. The works by Baldwin and Clark (2006) and MacCormack, Rusnak, and Baldwin (2006) suggest that open source projects have responded to this challenge by becoming more modular. Modularity is likely to have a positive impact on the quality of the firm's software and its development costs. In addition, modularity allows for a higher speed of debugging, which increases software stability. In fact, open source proponents claim that open source has effectively broken Brooks's Law.

While in our model all users value openness and user innovation, in reality there may be some users who disagree with the current direction of development. A more general model would allow users decide which elements of the code to adopt and which to discard. That is, a more complete model would allow users to choose the development path.

Compatibility. In Sections 4 and 5 we introduce a competing product for the firm. In one case, the competing product is provided by a non-profit open source project. In the other case, the competing product is provided by another commercial firm. In both cases, we assume full compatibility between the modules and service of the competing projects.[13]

We focus the analysis on the compatible case because, with compatibility, the interaction between the firm and any available outside open source projects is maximal. If the competing modules were incompatible with those of the firm, then the competitor could not take advantage of the modules opened by the firm, and the firm would be more likely to open its modules (this case would then be similar to the monopoly situation analyzed in Section 3). Likewise, the firm is less likely to adopt the modules of a competitor if they are incompatible with the complementary good or service that the firm is providing. A thorough analysis of partial compatibility and incompatibility presents an interesting direction for future research, but lies beyond the scope of the paper.

3. Monopoly benchmark

A monopolist facing a unit mass of consumers with utilities as described in Section 2.1 and a technology as given in Section 2.2 must decide to operate under one of four business models. Each business model gives rise to *one* product that the monopolist sells at positive price.[14] We refer to the product sold by the monopolist as the *commercial product.* There is

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[13]That is, when the firm opens a module, it can be combined with the complementary module of its competitor, and/or its support service. Likewise, if a competing module is opened, the firm can include it in its commercial product as it is fully compatible with its other module and/or service.

[14]The monopolist could consider selling several, horizontally differentiated, products. In Appendix B we allow the monopolist to produce and market as many products as she wishes, and we show that the monopolist will always prefer to offer only one product, which is the one that we are considering.

a one-to-one relationship between the four business models and the products offered by the monopolist. Letting v_c be the quality of the commercial product, we have:

P: $v_c = a\,b\,z$.
M_a: $v_c = a_o\,b\,z$.
M_b: $v_c = a\,b_o\,z$.
O: $v_c = a_o\,b_o\,z$.

When the monopolist opens module a, an open source competitor emerges. The reason is that module a can be used standalone and, thus, some users may prefer it to the monopolist's commercial offering (which is of higher quality but sold at positive price).[15] When b is opened, however, no open source competitor appears because module b cannot be used without the base a. Therefore, two of the available business models imply the emergence of competition (O and M_a) while the other two (P and M_b) ensure that the monopolist remains the only firm in the market.

Just as in the case of the commercial product, there is a one-to-one relationship between the four business models and the available open products. Letting v_o be the quality of the open source product, we have:

P: $v_o = 0$. No open source product available.
M_a: $v_o = a_o$.
M_b: $v_o = 0$. No open source product available.
O: $v_o = a_o\,b_o$.

The sequence of decisions is as follows:

(1) The firm chooses a business model.
(2) Expectations are formed on the share of the population that adopt the available open modules. Given these expectations, the monopolist sets the price of its commercial product.
(3) Consumers choose between the commercial product, the open source product (if available), and staying out of the market.

We now solve the monopolist's optimization problem. As usual, we proceed backwards. We first consider the consumers' purchase decision (given that one or two products are offered) and derive the demand functions. Then, we find the profit-maximizing price under each business model. Finally, we let the monopolist choose a business model. We impose that expectations be fulfilled in equilibrium.

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[15]In this section, we will refer to the commercial firm as the *monopolist* regardless of the fact that, when a is opened there is competition by an open source project.

3.1. **Consumer demands.** Suppose first that there is no open source product available. This means that consumers have two options: they can either buy the commercial product and obtain utility $u = \rho\, v_c - p$, or they can refrain from consuming and obtain $u = 0$. There is a cut-off point ρ_0 depending p such that only consumers with $\rho > \rho_0$ will buy the product of the firm at that price. The demand for the commercial product is:

$$q_c = 1 - \frac{p}{v_c}.$$

Suppose now that there is an open source alternative. Consumers can either buy the commercial product at price p, consume the open source good for free, or stay out of the market. Since the open source product gives utility $u = \rho\, v_o > 0$, consumers never choose to stay out and the relevant comparison is between the commercial and the open source products. Figure 3 shows the utility schedules for the commercial and open source products.
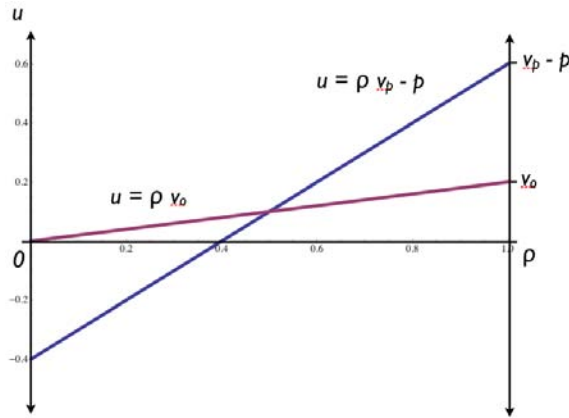

FIGURE 3. Consumer utility. Commercial and open source products.

Demand for the commercial product and the open source alternative are, respectively:

$$
\begin{aligned}
q_c &= 1 - \frac{p}{v_c - v_o} \\
q_o &= \frac{p}{v_c - v_o}.
\end{aligned}
$$

3.2. **Pricing decision.** In choosing prices, the firm is interested in maximizing profits:

$$\pi = p\, q_c.$$

When the commercial product is the only one available, the optimal price is $p = v_c/2$, which implies an equilibrium quantity of $1/2$ and profits of $\pi = v_c/4$. As is well-known, a monopolist that faces a linear demand function and has zero marginal cost of production

trades off volume and markup optimally at a price equal half the choke price and ends up serving half of the market.

When there is an OS alternative, the inverse demand function that the monopolist faces is $p = (v_c - v_o)(1 - q_c)$. This is a linear demand function. Thus, the optimal price is once again half the choke price $v_c - v_o$, half of the market is served, and profits are $\pi = (v_c - v_o)/4$.

Finally, the optimal price and profits depend on the precise functional forms of v_c and v_o which, as seen above, depend on the business model chosen by the firm. Specifically, the profits that each business model generate are as follows:

P: $\pi_p = a\,b\,z/4$.
M_a: $\pi_a = a_o\,(b\,z - 1)/4$.
M_b: $\pi_b = a\,b_o\,z/4$.
O: $\pi_{ab} = a_o\,b_o\,(z - 1)/4$.

3.3. **Business model choice.** The firm decides which module/s it should open, taking into account the expected number of users, its subsequent pricing decisions, the demand schedule, and that an open source competitor will emerge if module a is opened. The choice between business models depends on the comparison of profits which, in turn, depends on the strength of user innovation and the value of the modules and service (a, b, and z).

The first point to notice is that P is dominated by M_b. M_b is unambiguously superior to P because it takes advantage of user innovation to produce a higher quality product but it does not set up an open source competitor.[16]

Given that P is dominated by M_b, the three profit functions that must be compared are those for M_a, M_b, and O. The following thresholds are useful to state our first proposition which characterizes the monopolist's choice of business model.

$$\sigma_{a|b} := \frac{2}{b\,z - 2}, \quad \sigma_{a|ab} := \frac{b - 1}{b\,(z - 1)}, \quad \sigma_{b|ab} := \frac{1 + \sqrt{z\,(9\,z - 8)}}{4\,(z - 1)} - \frac{3}{4}.$$

Proposition 1. *Suppose that either $2 < b < 3$ and $z \leq \frac{2}{(3-b)b}$, or $b \geq 3$ hold. Then, if $\sigma \leq \sigma_{a|b}$, M_b is the optimal business model; if $\sigma_{a|b} < \sigma \leq \sigma_{a|ab}$, M_a is the optimal business model; and if $\sigma > \sigma_{a|ab}$, the optimal business model is O. Suppose, instead, that either $2 < b < 3$ and $z > \frac{2}{(3-b)b}$, or $1 \leq b \leq 2$ hold. Then, if $\sigma \leq \sigma_{b|ab}$, M_b is the optimal business model; and if $\sigma > \sigma_{b|ab}$, O is the optimal business model.*

Figure 4 shows the regions on which each business model is optimal in (σ, z) space.

---

[16]More generally, the firm can open all those technologies, protocols and ideas which have no value unless they are used with the base module, instead of remaining completely closed. For instance, if the firm's base software is necessary to read a particular format for saving data or to compile code written in a given
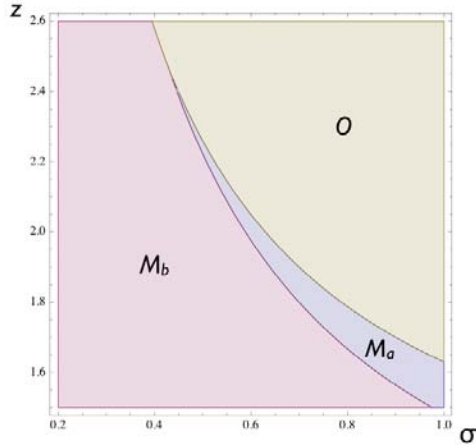
FIGURE 4. Optimal business models for a monopolist.

When σ and/or z are large, the optimal business model is O, regardless of the values of all other parameters. Large σ implies large user innovation when a and b are open. However, an open source competitor emerges, degrading the firm's ability to capture value. Notwithstanding, when σ is large the trade off is resolved in favor of opening the software to the maximal possible extent. When z is large, the vertical differentiation between the commercial product and the open source product that business model O sets up is large. As a consequence, the presence of the open source product does not harm much the ability of the firm to earn large profits. Thus, when z is large, the firm prefers to open both modules.

When σ is small the optimal business model is M_b as there is little to gain from user innovation and there is no point of being too open. As σ increases the preferred business model eventually becomes M_a. The difference between M_b and M_a is as follows. When b is opened, no open source competitor emerges. When a is opened, a new competitor surfaces which forces prices down. But M_b does not exploit user innovation to the same extent that M_a does: when M_b is chosen, $\frac{1}{2}$ of the market adopts the product[17] and when M_a is chosen, *everybody* adopts. The trade off is resolved in M_a's favor when σ is large.[18]

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programming language, the firm may open the file format or language without hampering its ability to capture vale. This is precisely what goes on in the cases of Microsoft's .Net framework, Stata, and Mathematica.

[17]Recall that the profit-maximizing price of the monopolist results in adoption by exactly half of the market.

[18]When $2 < b < 3$ and $z \geq \frac{2}{(3-b)b}$, whenever M_a is superior to M_b, it turns out that O is superior to M_a. This is why for $z \geq \frac{2}{(3-b)b}$, M_a is never chosen.

3.4. **Value creation and value capture.** The profit-maximizing choice of business model depends on the resolution of a trade-off between value creation and value capture which are defined as follows:[19]

> **Value creation (V):** Is the sum of the firm's profits and consumer surplus: $V = CS + \pi$.
>
> **Value capture (θ):** Is the proportion of the value created which is appropriated by the firm (in the form of profits): $\theta = \frac{\pi}{CS+\pi}$.

Using the above definitions, firm profits can be expressed as $\pi = \theta V$. It is straightforward to compute V and θ:

$$V = \frac{3}{8}\, v_c + \frac{1}{8}\, v_o, \qquad \theta = \frac{2\,(v_c - v_o)}{3\, v_c + v_o},$$

setting $v_o = 0$ when there is no open source alternative.

Figure 5A compares value creation and value capture for the different business models.[20] As discussed above, value capture under P coincides with that under M_b but value creation is larger under M_b. Given that profits are $\pi = \theta V$, we can easily plot the isoprofit curves. Obviously, curves farther away from the origin correspond to higher profits and the profit-maximizing business model is the one reaching the highest isoprofit curve. Figure 5B shows the isoprofit curves and business models. In this example, the optimal business model is M_b.

The figure shows that there is a trade off between value creation and value capture: M_b allows for maximal value capture but value creation is low, M_a erodes value capture but strengthens value creation, and O deteriorates value capture further but value creation is at its maximum.

A glance at V and θ reveals that regardless of the business model, value creation grows when a, b, z or σ increase. When a or σ increase, however, value capture remains constant. The reason is that value creation and profits grow at equal rates with a or σ regardless of whether there is an open source competitor or not. When z increases, value capture remains constant for P and M_b, and increases for M_a and O. And when b increases, value capture remains constant for P, M_b and O, and increases for M_a.

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[19]The expressions *value creation* and *value capture* became mainstream in strategy since the publication of Brandenburger and Stuart (1996) and Brandenburger and Nalebuff (1997). See also MacDonald and Ryall (2004). In Brandenburger and Stuart's (1996), all transactions with positive value take place. Therefore, value created equals potential value. However, Brandenburger and Stuart also recognize that there may exist 'frictions' which may introduce a wedge between potential and realized value. In our model, for example, the monopolist cannot price discriminate. Hence, there are transactions with positive potential value that do not occur in equilibrium (*i.e.* there is a welfare loss). Our definition of value creation corresponds with actual value created, which is more relevant for studying the choice of an optimal business model. For an insightful discussion of value creation and value capture and endogenous business models, see Salas-Fumas (2009).

[20]The parameter values are: $a = 1$, $b = 1.5$, $z = 1.2$ and $\sigma = 0.1$.

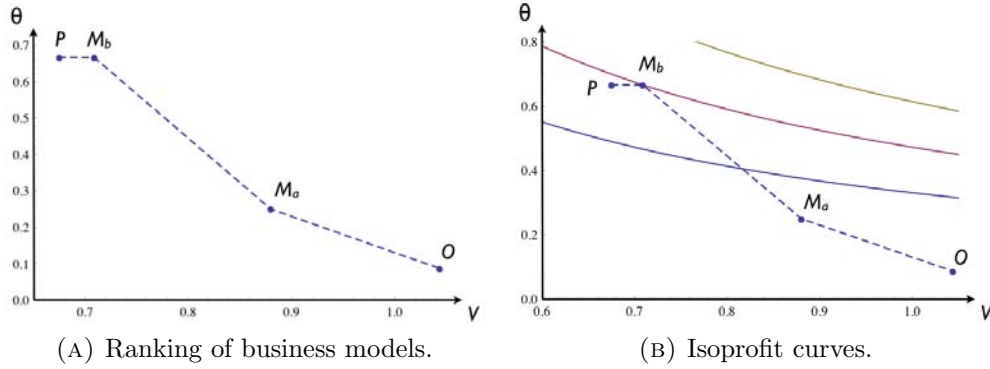(A) Ranking of business models. (B) Isoprofit curves.

FIGURE 5. Value creation & value capture in the monopoly case.

Note that our definition of value is equivalent to welfare. This means that Figure 5A also serves to rank the different business models in terms of welfare. The socially optimal model (in a second best sense) is O. As we have seen, the socially optimal business model is not always chosen by the monopolist.

4. COMPETITION FROM AN OUTSIDE OS PROJECT

In this section we analyze a situation where an incumbent firm (modeled like the monopolist of Section 3) faces a non-strategic competitor that enters the market with an independently developed open source product. Think, for example, of a non-profit open source project like Apache or Linux, competing against a for-profit firm like Microsoft or IBM. Because a substitute is available for the firm's product even if it decides not to open any of its modules, the open source project imposes competitive pressure on the incumbent.

Let $\hat{a} \geq 1$ and $\hat{b} \geq 1$ represent the external open source modules and their values. We will consider two cases: (1) the open source project has modules of *lower* quality than the firm, and (2) the open source project has modules of *higher* quality than the firm.[21]

As in Section 2.2, the value of a combination of modules and service is the product of their individual values, and a set of extensions (b or $\hat{b}$) has no value unless it is used with a base (a or $\hat{a}$). We assume full compatibility: either a or $\hat{a}$ may be combined with b or $\hat{b}$, and z. Therefore, the incumbent may embed an outside open source module in its commercial software if this leads to higher profit. Likewise, the open source project may combine one of its modules with one of the modules opened by the incumbent.[22]

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[21]We are assuming that the open source project provides substitutes for both modules. In Appendix C we analyze alternative scenarios in which the open source project provides only one of the two modules or provides one module with higher quality and another one with lower quality than those of the firm. The main conclusions of our analysis remain unaffected.

[22]See our discussion of compatibility in Section 2.7.

The incumbent must choose the business model through which it would like to compete. The choice set is the same as in the previous section: P, M_a, M_b, and O_{ab}. The only difference is that the firm may adopt the module/s available from the open source competitor. Therefore, there are two flavors to the business model with open a: M_a and $M_{\hat{a}}$. Likewise for M_b. Similarly, there are four flavors of business model O_{ab}.

Each business model gives rise to one commercial product which is sold at a positive price, but in this duopolistic setting there is always an open source product available to consumers, regardless of the business model choice made by the incumbent.

In addition to the commercial product, consumers may have a choice over two open source products, the outside OS project and the open product set up by the firm (if it decides to compete through a business model that involves opening at least one module). A slight complication that arises in this case is that there may be multiple equilibria. The reason is that the expected value of open products (and, thus, demand for them) depends on how many users are expected to use them. In some circumstances, more than one way to coordinate purchases satisfies Katz and Shapiro's (1985) criterion discussed in Section 2.3. To sidestep this issue, we assume that consumers coordinate to choose the open source product with highest (exogenous) quality, before taking into account the effect of user innovation.

The firm must choose its business model and price. The optimization problem is similar to that of Section 3. Given price and qualities, consumers choose their preferred product. The sequence of decisions is:

1. The firm chooses business model and whether or not to adopt any of the available modules from the outside open source project.
2. Expectations are formed on q_o and the firm sets price.
3. Given q_o^e, the choice of business model and price, consumers choose between the open source product and the commercial product.

Given v_o and v_c, the functional forms of demands and profits are the same as in the monopoly case (see Section 3.1). The only difference is that the open source products may now be based on the outside open source modules, the modules opened by the firm, or a combination of both.

4.1. The firm has modules of higher quality.

We start by analyzing what happens when $a > \hat{a}$ and $b > \hat{b}$. As discussed above, the possibility of adoption of the modules provided by the outside OS project mean that there are multiple implementations of the mixed and open source business models. The possibilities are: M_a, $M_{\hat{a}}$, M_b, $M_{\hat{b}}$, O_{ab}, $O_{\hat{a}b}$, $O_{a\hat{b}}$, and $O_{\hat{a}\hat{b}}$. In addition, the firm may also choose P.

We will show that: (1) the firm will never embed the external open source modules in its commercial offering, (2) when choosing between the different mixed source models, the firm will open the module that faces stronger competition from the open source project, and will keep closed the module which could potentially increase more the value of the open source product, and (3) the firm may choose to be P, M or O, so all possible business models may arise as an equilibrium outcome.

We begin by showing that whenever the firm is considering a mixed source or an open source business model, it will always choose to base its commercial product on its own modules (*i.e.* the firm will never adopt the modules supplied by the external open source project).

Lemma 1. *When the modules of the firm are of higher quality than those of a non-strategic open source competitor, the firm will never choose to embed those modules in its commercial product, i.e. the firm will never choose the $M_{\hat{a}}$, $M_{\hat{b}}$, $O_{a\hat{b}}$, $O_{\hat{a}b}$, or $O_{\hat{a}\hat{b}}$ business models.*

Lemma 1 is a consequence of complementarity. Suppose, for example, that the firm is considering a mixed source model with open base. The firm chooses between M_a and $M_{\hat{a}}$. The firm might be tempted to choose $M_{\hat{a}}$ because opening a implies that the resulting open source competitor will be of higher quality than if the firm chooses $M_{\hat{a}}$. However, complementarity with b and z implies that the quality of the firm's product increases relatively more than the quality of the open source product in a is opened. Therefore, the firm will prefer M_a to $M_{\hat{a}}$.

The above lemma implies that the firm will consider only P, M_a, M_b and O_{ab}. The values of v_c and v_o and profits are:

P: $v_c = a\,b\,z$, $v_o = \hat{a}_o\,\hat{b}_o$, and $\pi_p = (a\,b\,z - \hat{a}_o\,\hat{b}_o)/4$.

M_a: $v_c = a_o\,b\,z$, $v_o = a_o\hat{b}_o$, and $\pi_a = (a_o\,b\,z - a_o\,\hat{b}_o)/4$.

M_b: $v_c = a\,b_o\,z$, $v_o = \hat{a}\,b_o$, and $\pi_b = (a\,b_o\,z - \hat{a}_o\,b_o)/4$.

O_{ab}: $v_c = a_o\,b_o\,z$, $v_o = a_o\,b_o$, and $\pi_{ab} = a_o\,b_o\,(z - 1)/4$.

Next, we show that, for given a, b, $\hat{a}$, and $\hat{b}$, either M_a dominates M_b or vice versa (regardless of the values of z and σ). When $a/\hat{a} > b/\hat{b}$, the firm prefers to open b. Likewise, when $a/\hat{a} < b/\hat{b}$, the firm prefers to open a. There are two ways to interpret this result. On the one hand, the firm wants to open the module which faces greater competition from the OS project. On the other hand, the firm wants to keep proprietary the module which is more complementary to the software of the OS competitor.

Lemma 2. *M_a is preferred to M_b if and only if $a/\hat{a} < b/\hat{b}$, and vice versa.*

The intuition is that opening a or b produces the exact same benefit for the firm ($a_o\,b\,z = a\,b_o\,z$). The costs differ, however, because the quality of the open source competitor depends

on which module is opened. The quality of the competitor is $a_o \, \hat{b}_o$ if a is opened and $\hat{a}_o \, b_o$ if b is opened. In comparing M_a and M_b, the firm will prefer the business model that results in the lowest possible value for the outside OS project.

Finally, we proceed to characterize the optimal choice of business model for all possible values of σ and z. The above lemmas imply that there are only three candidates for optimal business model: P, O_{ab}, and either M_a or M_b.

Note that, contrary to the monopoly case, M_b does not dominate P for all parameter values. The reason is that when b is opened, users of the open source project can combine $\hat{a}$ with b to assemble a product of quality $\hat{a}_o \, b_o$. Having a stronger open source competitor may lead to decreased value capture by the for-profit firm.

The following thresholds are used in Proposition 2:

$$z_{a|p} := \frac{\hat{b}\,(2+\sigma)\,(2\,a\,(1+\sigma) - \hat{a}\,(2+\sigma))}{4\,a\,b\,\sigma}, \quad z_{b|p} := \frac{\hat{a}\,(2+\sigma)\,(2\,b\,(1+\sigma) - \hat{b}\,(2+\sigma))}{4\,a\,b\,\sigma},$$

$$z_{ab|a} := \frac{2\,b\,(1+\sigma) - \hat{b}\,(2+\sigma)}{2\,b\,\sigma}, \quad \text{and} \quad z_{ab|b} := \frac{2\,a\,(1+\sigma) - \hat{a}\,(2+\sigma)}{2\,a\,\sigma}.$$

Proposition 2 (Equilibrium regions)**.** *If $a/\hat{a} < b/\hat{b}$, then P is optimal when $z < z_{a|p}$, M_a is optimal when $z_{a|p} < z < z_{ab|a}$ and O_{ab} is optimal when $z > z_{ab|a}$. If $a/\hat{a} > b/\hat{b}$, then P is optimal when $z < z_{b|p}$, M_b is optimal when $z_{b|p} < z < z_{ab|b}$ and O_{ab} is optimal when $z > z_{ab|b}$.*

Figure 6 illustrates the optimal strategy for different parameter value combinations. For low values of σ and z, the firm chooses the proprietary business model (P), for intermediate values, the firm chooses a mixed source model (either M_a or M_b), and for high values of σ and z, the firm becomes completely open (O_{ab}).

The intuition is simple. When σ and z are low, there is little benefit from opening modules as user innovation is weak. Moreover, low z means that if the firm was to open any module, the outside OS project would become close in quality and competitive pressure would be strong. Therefore, the firm is better off with a proprietary business model in this case. As σ and z increase, so does the value creation implications of opening modules and the vertical differentiation between the OS project and the firm's product. When σ and z are sufficiently large, the additional value creation of opening one module outweighs the increased

(A) $a = 2$, $b = 3.5$, $\hat{a} = 1$, $\hat{b} = 2$ (B) $a = 2$, $b = 4.5$, $\hat{a} = 1$, $\hat{b} = 2$
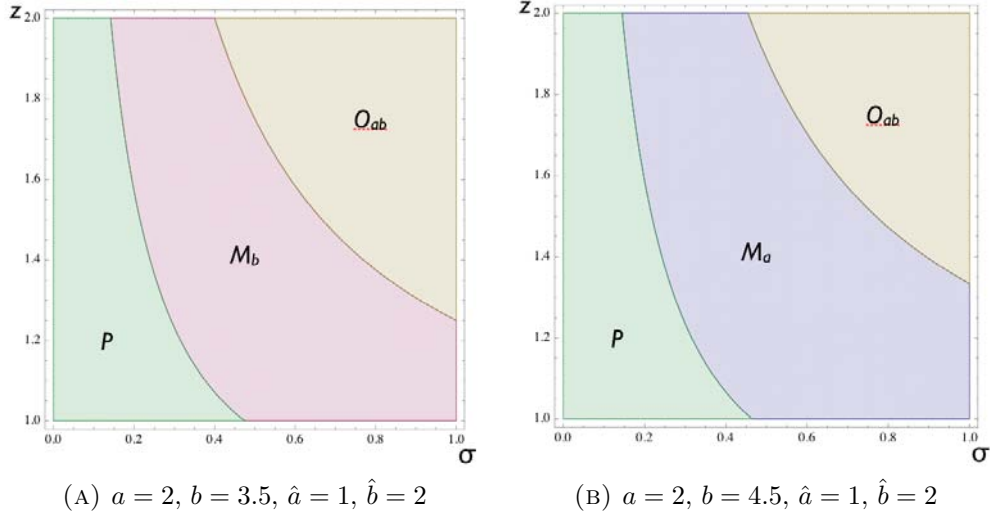
FIGURE 6. Equilibrium business models.

competitive pressure from the outside OS project. For the same reason, the firm is better off opening both modules when σ and z are very large.

An important implication of Proposition 2 is that the firm may prefer the proprietary (closed) business model when there is an outside OS competitor. In the monopoly case (Section 3), the firm never desired to be completely closed. This was because (i) opening the extensions was harmless if users did not have a base module with which to combine it and (ii) there was an innovation benefit when opening a module. When there is competition from an outside OS project, opening the extensions leads to increased competitive pressure, making P optimal in some opportunities.

One final issue that we need to address is the possibility of leapfrogging. Leapfrogging refers to a situation where the outside open source project ends up with quality above that of the commercial firm's product. This could happen if $\hat{a}\hat{b}$ was close to ab and z was close to 1. In this case leapfrogging could occur if the firm adopted a proprietary business model. It is easy to see, however, that leapfrogging never happens in equilibrium. If there was leapfrogging, the commercial firm would earn zero profits because the open source product would be of higher quality and given away for free. The commercial firm can easily prevent leapfrogging by opening a and/or b.

4.2. **Open source competitor has modules of higher quality.** In the previous section, we have shown that if $\hat{a} < a$ and $\hat{b} < b$, the firm will *never* include $\hat{a}$ nor $\hat{b}$ in its commercial product. The following lemma shows that when $\hat{a} > a$ and $\hat{b} > b$, the firm will *always* embed both modules in its commercial product.

Lemma 3. *When the modules of the non-strategic open source competitor are of higher quality than the modules of the firm, the firm will always choose to embed those modules in its commercial product, i.e. it will always choose the $O_{\hat{a}\hat{b}}$ business model.*

While the result may seem surprising, it is once again due to complementarity between the software modules and service. The firm may be tempted not to adopt the open source modules because doing so increases the quality of the open source product (the consumers of the firm engross the user base of the open source project, which increases its quality). However, complementarity implies that the quality of the firm's product increases relatively more than that of the open source project: the firm shares now the same user base as the open source project, the firm's product is based on higher quality modules, and its value is multiplied by $z > 1$.[23]

4.3. **Value creation and value capture.** We now compare the different business models in terms of value creation (V) and value capture (θ). The functional forms for V and θ are the same as in Section 3.3. What changes are the values of v_o and v_c that enter in these expressions.

4.3.1. *The firm has modules of higher quality.* Figure 7 compares value creation and value capture for the different business models for $a = 1.5$, $b = 1.5$, $z = 1.2$ and $\sigma = 0.1$. Figure 7A shows a case in which $a/\hat{a} > b/\hat{b}$ ($\hat{a} = 1$ and $\hat{b} = 1.2$), and Figure 7B shows a case in which $a/\hat{a} < b/\hat{b}$ ($\hat{a} = 1.2$ and $\hat{b} = 1$).

These plots have some features in common with the monopoly case. For example, P is still the business model with lowest value creation and O_{ab} the model with highest value creation. Also, O_{ab} is the model with lowest value capture. The mixed models (M_a and M_b) produce intermediate value creation and capture. However, there are important differences also. First, the line connecting P with M_b (or M_a) has negative slope indicating that the mixed models do not dominate P. Second, M_a may be to the north-west of M_b, as in Figure 7B. Clearly, the availability of outside OS modules affects the ranking of business models in terms of value creation and value capture.

4.3.2. *The open source competitor has modules of higher quality.* Figure 8 shows the ranking of business models when the outside OS project has modules of higher quality than those of the profit-maximizing firm, for $a = 2.5$, $\hat{a} = 3.5$, $b = 1.5$, $\hat{b} = 3$, $z = 4$, and $\sigma = 0.3$.

The plot shows clearly that the Open Source business model ($O_{\hat{a}\hat{b}}$) dominates all other business models. Thus, the tradeoff between value creation and value capture is completely

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[23]This result could be reversed if the open source modules were incompatible with the firm's complementary good or if the different software products were horizontally differentiated.

(A) $a/\hat{a} > b/\hat{b}$
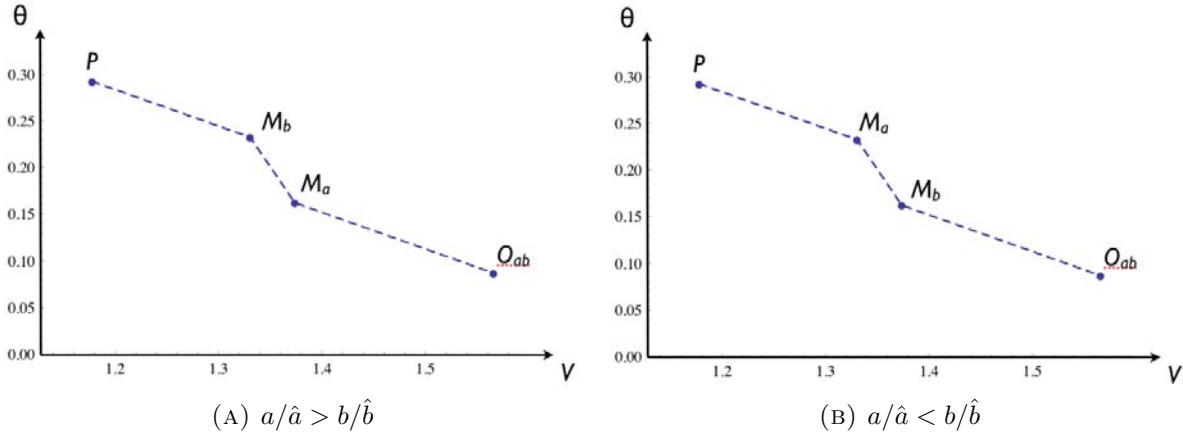
(B) $a/\hat{a} < b/\hat{b}$

FIGURE 7. Value ranking with competition from a low-quality OS project

broken when the OS competitor has modules of higher quality. Clearly, $O_{\hat{a}\hat{b}}$ reaches the highest isoprofit curve and therefore it is the best business model. In this case, private choice agrees with socially optimal behavior.
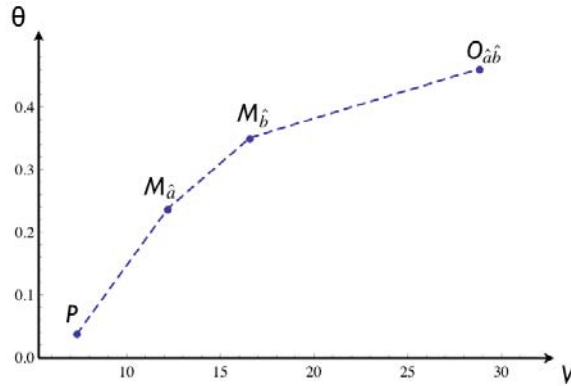

FIGURE 8. Value ranking with competition from a high-quality OS project

5. COMPETING THROUGH BUSINESS MODELS

In this section, we study competitive interaction between two firms that choose the business model through which they would like to compete. Specifically, we consider firms H and L endowed with modules a_i and b_i, $i \in \{H, L\}$. We assume that the modules offered by H are of higher quality than those offered by L: $a_H \geq a_L \geq 1$ and $b_H \geq b_L \geq 1$. The two-period model that we analyze is the natural generalization of that in Section 4. In the first period (strategic choices), firms choose business models. In the second period (tactical choices), firms choose the price at which their products are sold. We consider two alternative timings. In Subsection 5.1, H chooses business model first and in Subsection 5.2, L is the first mover.

As it will become apparent in what follows, the model with endogenous business models for both players is complex. To make progress, we simplify by assuming that $z = 1$ so that no additional willingness to pay is derived from service. This assumption reduces the attractiveness of opening modules. The reason is that the vertical differentiation between the commercial products and the open source competitors that are set up when base modules are opened is not as large as it would be if $z > 1$. Therefore, if we find that it is desirable to open modules when $z = 1$, it should be even more desirable if z was greater than 1.

We are able to obtain analytical expressions of profits for each combination of business models of the two players. However, these expressions are long and intricate. This forces us to solve the first period of the game (the choice of busines model) numerically. Analytical solutions are intractable. As we will see, several interesting results can be derived from the numerical analysis.[24]

5.1. **H moves first.** The extensive form game that we study has two periods. Strategic choices take place in the first period and tactical interaction occurs in the second period. The first period has two stages. In the first stage, H chooses its business model and in the second stage L chooses its business model.

As the first mover, H has four business models at its disposal: P, M_{a_H}, M_{b_H}, and $O_{a_H b_H}$. Because L moves second, there are cases in which it has more than four business models to choose from. For example, if H chose M_{a_H}, then in addition to P, M_{a_L}, M_{b_L}, and $O_{a_L b_L}$, L can also choose to compete through M_{a_H} or $O_{a_H b_L}$. However, whenever the H and L versions of a given module are open, all consumers will use the H version (because it is free, compatible, and of higher quality). Therefore, if H competes through model M_{a_H} and L competes through business model M_{a_L}, L's effective model will be M_{a_H} as even those consumers buying L's product will combine b_L with a_H.

Therefore, we will assume that whenever both firms open the same module, there is a *de facto* adoption of the higher quality one by L's customers.[25] The implication is that, regardless of the business model choice made by H in the first stage, L effectively has four business models to choose from.[26]

The timing of the game is:

(1) H chooses business model.
(2) L chooses business model.

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[24]The *Mathematica* code is available from the authors.

[25]Clearly, this is possible because we assume that the modules of different firms are compatible. See Section 2.7 for details.

[26]More precisely, when H opens module i_H ($i \in \{a, b\}$), from the point of view of L, opening i_L or adopting i_H are equivalent strategies. For ease of exposition, we collapse both strategies into one: adoption by L.

(3) Expectations are formed over the market shares of the different products. Given these expectations, H and L simultaneously choose prices, p_H and p_L.[27]

(4) Consumers choose their preferred product.

Although we allow firms to be completely open, our assumption that $z = 1$ implies that if a firm opened everything, then it would not be able to sell its commercial product at a positive price because the open source product would be as good as the commercial version. Therefore, opening both modules is a strictly dominated strategy for both firms and it can be ignored when solving for the subgame perfect equilibrium of the game.

Figure 9 shows a partial description of the game tree.[28] Depending on H and L's decisions on what modules to open, there may be two or three products available in the market. Three products are available when either firm opens module a because an open source competitor emerges. Otherwise, only two products are available.

Note that in this game H can never push L off the market by opening a and setting up a high-quality open source competitor. The reason is that L can always adopt the opened module and have quality at least as high as that of the free open source product. For the same reason, if H opens a module, L will never choose to open the complementary module. As a consequence, there are two branches of the game in Figure 9 that will never be part of an equilibrium: $\{M_{a_H}, M_{b_L}\}$ and $\{M_{b_H}, M_{a_L}\}$.

We solve for the subgame perfect equilibria recursively. We start by determining consumer choices and solving the pricing subgame given the business models, and later solve for the equilibrium business model choices.

5.1.1. *Consumer demands and equilibrium prices with two products.* If neither firm opens a base module, consumers choose between two commercial products. Let v_i and p_i be the quality and price of the product offered by $i \in \{H, L\}$. Demands are

$$q_H = 1 - \frac{p_H - p_L}{v_H - v_L} \quad \text{and} \quad q_L = \frac{p_H - p_L}{v_H - v_L} - \frac{p_L}{v_L}.$$

Finding the equilibrium prices and substituting in, we obtain

$$q_H = \frac{2v_H}{4\,v_H - v_L} \quad \text{and} \quad q_L = \frac{v_H}{4\,v_H - v_L}.$$

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[27]As in previous sections, the market shares of the different products are taken as given by consumers and firms when they make their choices. Fulfilled expectations imply that in equilibrium, those expectations are correct.

[28]The game tree is *partial* because we have not written down explicitly the pricing subgame nor the payoff functions. Moreover, we have not included the strictly dominated strategy of being completely open.
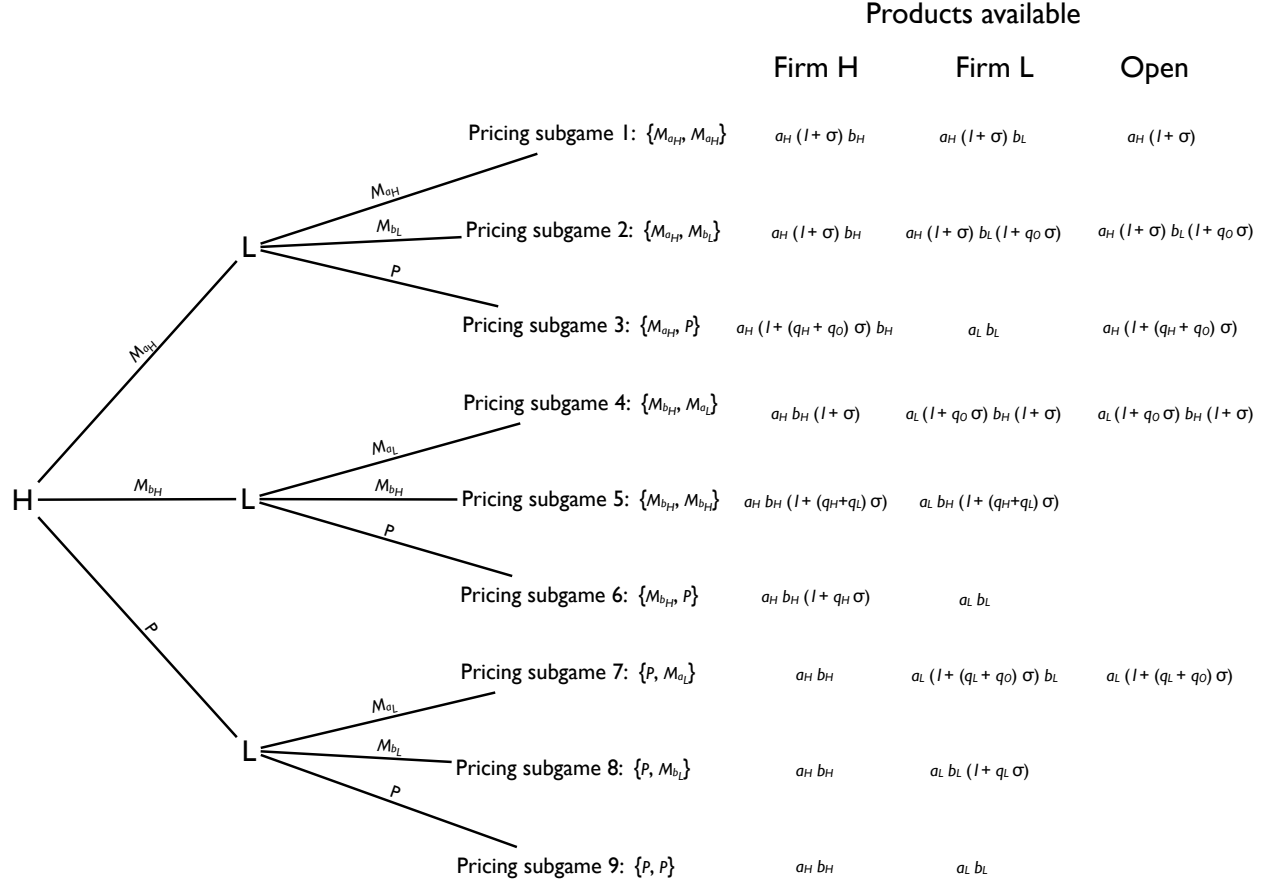
	Products available		
	Firm H	Firm L	Open
Pricing subgame 1: $\{M_{a_H}, M_{a_H}\}$	$a_H(1+\sigma)b_H$	$a_H(1+\sigma)b_L$	$a_H(1+\sigma)$
Pricing subgame 2: $\{M_{a_H}, M_{b_L}\}$	$a_H(1+\sigma)b_H$	$a_H(1+\sigma)b_L(1+q_O\sigma)$	$a_H(1+\sigma)b_L(1+q_O\sigma)$
Pricing subgame 3: $\{M_{a_H}, P\}$	$a_H(1+(q_H+q_O)\sigma)b_H$	$a_L b_L$	$a_H(1+(q_H+q_O)\sigma)$
Pricing subgame 4: $\{M_{b_H}, M_{a_L}\}$	$a_H b_H(1+\sigma)$	$a_L(1+q_O\sigma)b_H(1+\sigma)$	$a_L(1+q_O\sigma)b_H(1+\sigma)$
Pricing subgame 5: $\{M_{b_H}, M_{b_H}\}$	$a_H b_H(1+(q_H+q_L)\sigma)$	$a_L b_H(1+(q_H+q_L)\sigma)$	
Pricing subgame 6: $\{M_{b_H}, P\}$	$a_H b_H(1+q_H\sigma)$	$a_L b_L$	
Pricing subgame 7: $\{P, M_{a_L}\}$	$a_H b_H$	$a_L(1+(q_L+q_O)\sigma)b_L$	$a_L(1+(q_L+q_O)\sigma)$
Pricing subgame 8: $\{P, M_{b_L}\}$	$a_H b_H$	$a_L b_L(1+q_L\sigma)$	
Pricing subgame 9: $\{P, P\}$	$a_H b_H$	$a_L b_L$	

(Game tree: H moves first with branches M_{a_H}, M_{b_H}, P; each leading to an L node with branches leading to the pricing subgames above.)

FIGURE 9. The game after elimination of dominated strategies. H moves first.

Equilibrium profits are

$$\pi_H = \frac{4\,v_H^2\,(v_H - v_L)}{(4\,v_H - v_L)^2} \quad \text{and} \quad \pi_L = \frac{v_H\,v_L\,(v_H - v_L)}{(4\,v_H - v_L)^2}.$$

The particular profits in each subgame are obtained by substituting in the corresponding expressions for product quality, v_H and v_L, given in Figure 9 and imposing that the firms' and consumers' expectations be fulfilled in equilibrium.

Consider, for example, the subgame where H has opened b_H and L has adopted b_H. This is the subgame $\{M_{b_H},\, M_{b_H}\}$. In this case, $v_H = a_H\, b_H\,(1 + \sigma\,(q_H^e + q_L^e))$ and $v_L = a_L\, b_H\,(1 + \sigma\,(q_H^e + q_L^e))$, where q_i^e $(i \in \{H, L\})$ is the expected demand for i. Substituting v_H and v_L in the equilibrium demands we have

$$q_H = \frac{2a_H}{4\,a_H - a_L} \quad \text{and} \quad q_L = \frac{a_H}{4\,a_H - a_L}.$$

Fulfilled expectations require $q_H + q_L = q_H^e + q_L^e$. Hence,

$$q_H^e + q_L^e = \frac{3a_H}{4\,a_H - a_L}.$$

Substituting in the profit functions we obtain:

$$\pi_H = \frac{4\,a_H^2\,b_H(a_H - a_L)(a_H(4 + 3\,\sigma) - a_L)}{(4\,a_H - a_L)^3},$$

and

$$\pi_L = \frac{a_H\,a_L\,b_H(a_H - a_L)(a_H(4 + 3\,\sigma) - a_L)}{(4\,a_H - a_L)^3}.$$

Other cases are solved similarly. The equilibrium expression for $q_H^e + q_L^e$ is particularly simple in this example because $q_H + q_L$ does not depend on q_H^e or q_L^e. More generally, $q_H + q_L$ will depend on q^e and the resulting expressions for q^e and profits are convoluted. This is why analytical solutions for the first period become intractable and we have to resort to a numerical analysis.

Regarding leapfrogging, the only subgame where it may occur is when H keeps both modules closed and L opens b_L. Leapfrogging occurs in this subgame when σ is large. Let

$$\sigma_{NLF} := \frac{3\,(a_H\,b_H - a_L\,b_L)}{a_L\,b_L} \quad \text{and} \quad \sigma_{LF} := \frac{3\,(a_H\,b_H - a_L\,b_L)}{2\,(a_L\,b_L)}.$$

Clearly, $0 < \sigma_{LF} < \sigma_{NLF}$. It is not hard to see that when $\sigma > \sigma_{LF}$ there is an equilibrium with leapfrogging. Likewise, when $\sigma < \sigma_{NLF}$ there is an equilibrium without leapfrogging. Therefore, when $\sigma_{LF} < \sigma < \sigma_{NLF}$ there are two equilibria. As in previous sections, we assume that whenever there are multiple equilibria, consumers coordinate to choose the equilibrium with higher (exogenous) quality. Therefore, when $\sigma < \sigma_{NLF}$ the equilibrium that we consider has no leapfrogging.

Allowing for the possibility of leapfrogging is important because H has to determine what is L's best response to each strategy it may take. In the game where H moves first, however, leapfrogging does not occur in equilibrium because H can always prevent it by opening b_H and be at least as well off.

5.1.2. *Consumer demands and equilibrium prices with three products.* If at least one of the firms opens its base module a, there may be three products in equilibrium.[29] Let v_i and p_i,

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[29]There are two subgames where even with one of the base modules open, there are only one or two active products in equilibrium. The subgames are $\{M_{a_H}, M_{b_L}\}$ and $\{M_{a_L}, M_{b_H}\}$. In these two subgames, L's product is of equal quality than the open source product. Therefore, L gets no demand if it sets a positive price. Moreover, depending on the parameter values, the quality of the open source product may be higher than that of H's product. When this happens, there is only one product that obtains positive demand in equilibrium. As discussed above, these subgames are never played.

$i \in \{H, L, O\}$ be the quality and price of the different products, where O refers to the open source product, and $p_O = 0$ (by definition of open source). Demands are

$$q_H = 1 - \frac{p_H - p_L}{v_H - v_L}, \qquad q_L = \frac{p_H - p_L}{v_H - v_L} - \frac{p_L}{v_L - v_O}, \quad \text{and} \quad q_O = \frac{p_L}{v_L - v_O}.$$

After substituting in the equilibrium prices, demands become

$$q_H = \frac{2 (v_H - v_O)}{4 v_H - v_L - 3 v_O}, \qquad q_L = \frac{v_H - v_O}{4 v_H - v_L - 3 v_O}, \quad \text{and} \quad q_O = \frac{v_H - v_L}{4 v_H - v_L - 3 v_O}.$$

Equilibrium profits are

$$\pi_H \;=\; \frac{4 (v_H - v_O)^2 (v_H - v_L)}{(4 v_H - v_L - 3 v_O)^2}$$

and

$$\pi_L \;=\; \frac{(v_H - v_O) (v_L - v_O) (v_H - v_L)}{(4 v_H - v_L - 3 v_O)^2}.$$

The particular profits in each subgame are obtained by substituting in the corresponding expressions for product quality, v_H, v_L, and v_O, as given in Figure 9 and imposing that the firms' and consumers' expectations be fulfilled in equilibrium.

As in the previous case, there is one subgame where leapfrogging may occur: $\{P, M_{a_L}\}$. As before, there are parameters for which the subgame has equilibria with and without leapfrogging, in which case we select the equilibrium without leapfrogging. While it is important to consider the possibility of leapfrogging to compute the equilibrium of the complete game, leapfrogging does not occur in the equilibrium.

Having obtained the continuation profits for each of the nine possible subgames we now move to studying the first period. The first period has two stages. In the first stage, H chooses business model. In the second stage, L chooses business model. Thus, in choosing its business model, H takes into account how L will best respond through its choice of business models.

5.1.3. *Business model choice.* Figure 10 shows the equilibrium business model configurations for different parameter values. To produce the figure, we have set $a_L = b_L$, $a_H = \lambda a_L$, and $b_H = \lambda b_L$, where $\lambda > 1$. The larger is λ, the larger is the exogenous quality difference between the products of firms H and L. The plots have λ on the vertical axis, and the strength of user innovation σ on the horizontal axis.

The labels inside the regions indicate the equilibrium business model configurations. The first element corresponds to the business model of the first mover (H in this case), and the second element corresponds to the second mover. For example, $\{M_{b_H}, P\}$ means that H has chosen to open b_H (mixed source business model with open b) and L has chosen to stay
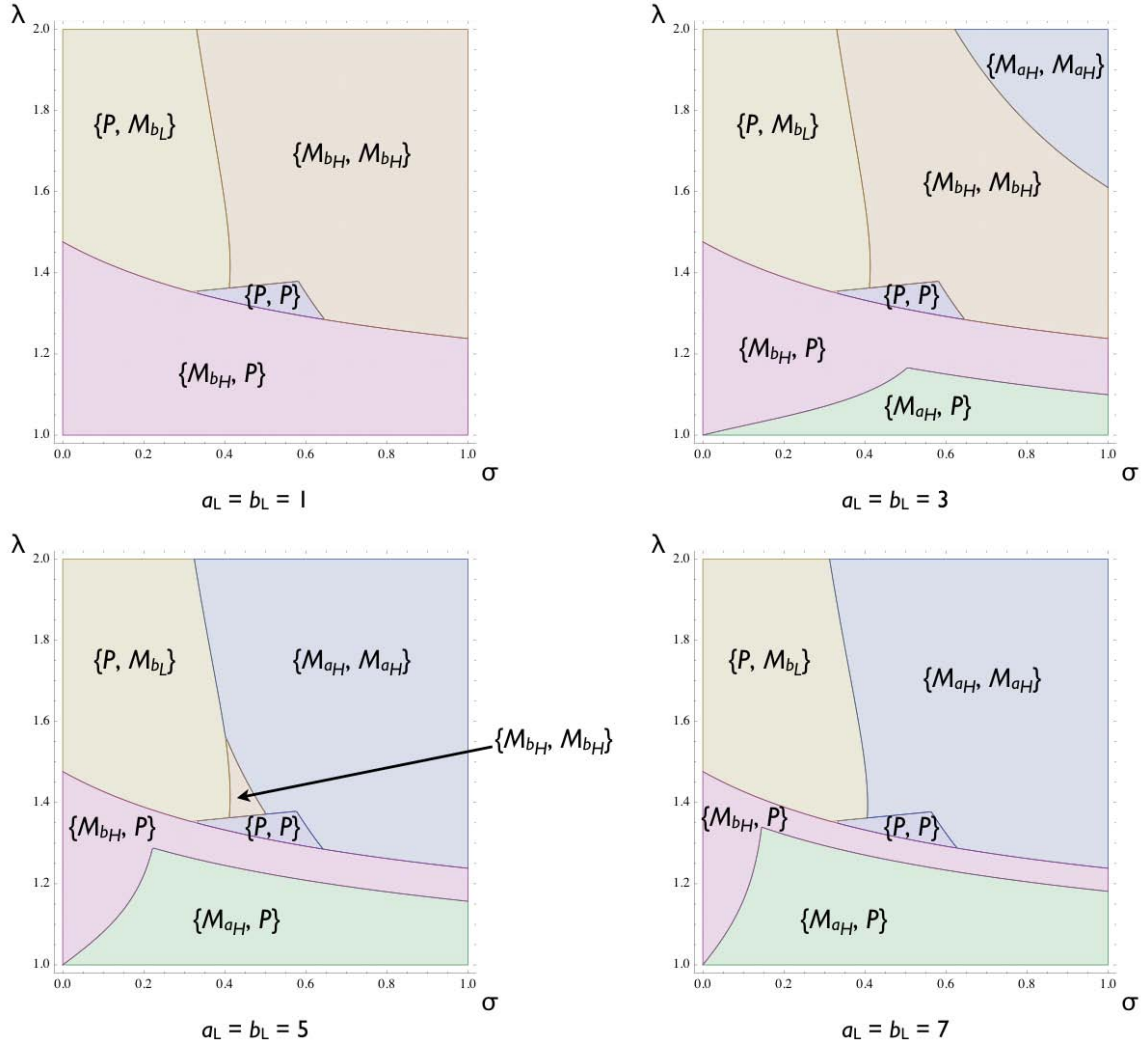
FIGURE 10. Equilibrium regions when H moves first.

closed (proprietary model). Likewise, $\{M_{b_H}, M_{b_H}\}$ means that H has opened b_H and L has adopted b_H.

Several features of these plots are worth highlighting:

(1) When λ is low, H opens one of its modules and L does not adopt. The reason is that when λ is low, the modules of H and L are very similar in quality. Therefore, L prefers to increase its vertical differentiation with H. Thus, H knows that if it opens

a module, it will not be adopted by L.[30] Therefore, when the products of H and L
are similar, the firms prefer to compete with different business models.

(2) When λ is high but σ is low, H stays closed and L opens b_L. H knows that if it
opens a module, it will be adopted by L. The negative competitive effect of L getting
closer in quality is larger than the positive effect of winding up with higher quality
due to increased user innovation. This happens because user innovation is weak and
initial quality differences are large in that region.

(3) There is a small region of intermediate values of λ and σ for which both firms choose
P, the closed business model. This is because the strength of user innovation and
initial differences in quality are such that L would adopt any module opened by H.
Still, L prefers to remain closed when H chooses P because by opening a module in
that case, L becomes too close in quality to H.

(4) For relatively large values of λ and σ, both firms choose a mixed source business
model. This is because user innovation is strong and H prefers to open a module even
if L is expected to adopt, and L prefers to adopt because initial quality differences
are sufficiently large.

(5) When b_L is low, firms have more incentives to open b than to open a. As b_L increases,
it is more likely that firms will open a. This is because if an open source product (O)
was to arise when b_L is low, it would have quality close to that of L's product. This,
in turn, would imply low prices for L, which would also affect H. When b_L is large
the region corresponding to $\{M_{b_H}, M_{b_H}\}$ is completely replaced by $\{M_{a_H}, M_{a_H}\}$.

(6) There are three business model configurations which never arise in equilibrium:
$\{M_{a_H}, M_{b_L}\}$, $\{M_{b_H}, M_{a_L}\}$ and $\{P, M_{a_L}\}$.

5.1.4. *Value creation and value capture.* Strategic interaction affects the creation and capture
of value associated to the different business models. The trade-off between value creation
and capture depends on the equilibrium under analysis, and several configurations can be
analyzed depending on parameter values. Nevertheless, there are regularities that can be
easily described by studying a particular example. In this section, we focus on H's actions,
taking into account L's best response for each possible business model choice.

Figure 11 shows value creation and value capture for both firms for the case of $a_L = b_L = 5$,
$\sigma = 0.5$, and different values of λ. For each case, value creation and capture for H is
represented with small circles. L is represented with squares.

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[30]This is a well known feature of Shaked and Sutton's (1982) model of vertical product differentiation. If
quality differences are small, price competition will be strong and will lead to low profits. In this case, both
firms will benefit if the quality differential between their products increases. However, if initial differences
in quality are large, L will benefit from getting (somewhat) closer to H.

For all values of the parameters, L captures less value than H. We can see that the nature of the trade-off for H changes as λ increases. For $\lambda = 1.1$, there is really no trade-off. M_{a_H} provides higher capture and creation than M_{b_H} and P, and even though O gives more creation than M_{a_H}, value capture is 0 under O.
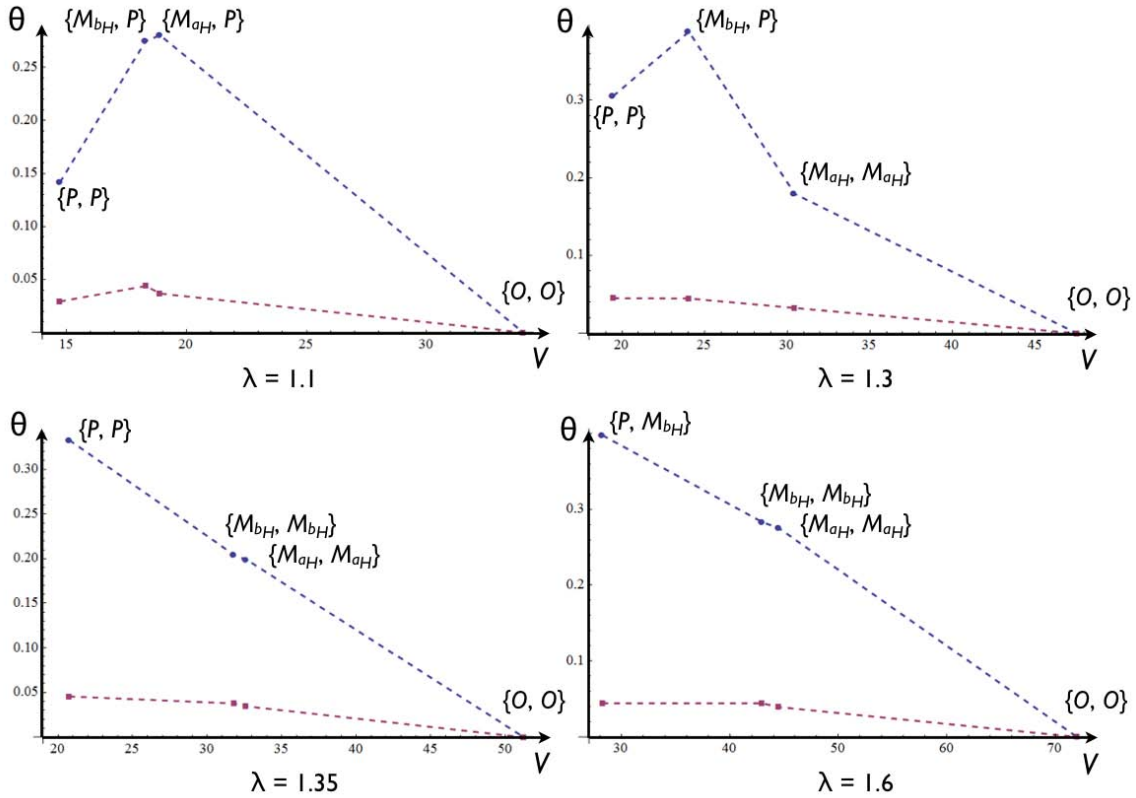

Figure 11. Value capture and value creation.

As λ increases, the trade-off becomes more pronounced. For $\lambda = 1.3$, M_{b_H} gives more capture than M_{a_H}, and for $\lambda = 1.35$ and $\lambda = 1.6$, P gives more capture than M_{a_H} and M_{b_H}. This means that the trade-off becomes more noticeable as the quality differential between the firms increases.

5.2. **L moves first.** We now study the game where L chooses its business model first. The timing is as follows:

(1) L chooses business model.
(2) H chooses business model.
(3) Expectations are formed over the market shares of the different products. Given these expectations, H and L simultaneously choose prices, p_H and p_L.

(4) Consumers choose their preferred product.

As in the previous case, opening both modules is a strictly dominated strategy, so we ignore that choice. In addition and as discussed above, if both firms open the same module, L's consumers will replace the lower quality module by the higher quality one (because it is open and freely available) and there will be a *de facto* adoption of H's module by L. Figure 12 displays the partial representation of the game tree. Note that we have labeled the subgame where L chooses M_{a_L} and H chooses M_{a_H} as $\{M_{a_H}, M_{a_H}\}$. This reflects the fact that although L has chosen to open a_L, those individuals that purchase L's product, will combine b_L with the best base module that is freely available. And that is a_H. Therefore, the business model profile $\{M_{a_L}, M_{a_H}\}$ is in fact $\{M_{a_H}, M_{a_H}\}$.[31]
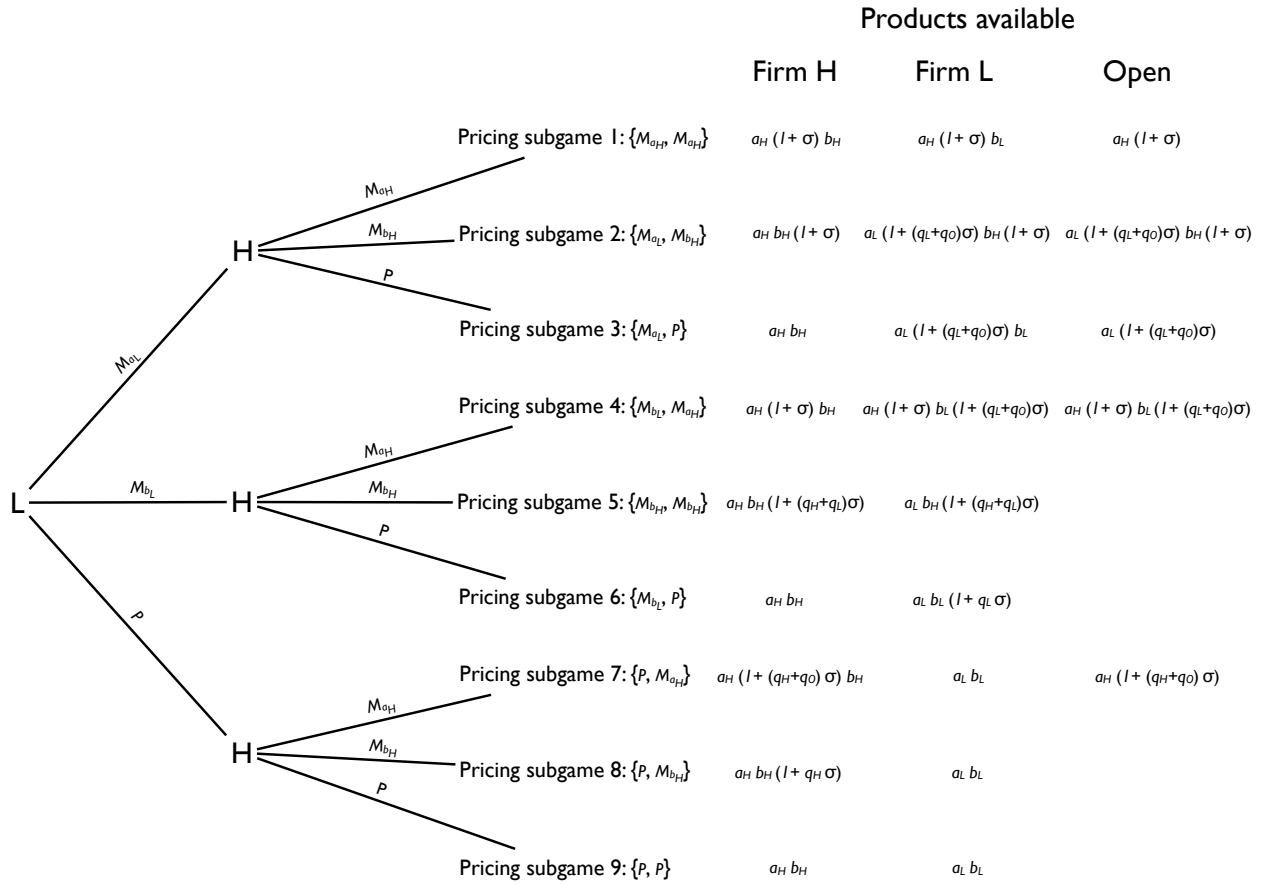

Products available

	Firm H	Firm L	Open
Pricing subgame 1: $\{M_{a_H}, M_{a_H}\}$	$a_H(1+\sigma)b_H$	$a_H(1+\sigma)b_L$	$a_H(1+\sigma)$
Pricing subgame 2: $\{M_{a_L}, M_{b_H}\}$	$a_H b_H(1+\sigma)$	$a_L(1+(q_L+q_0)\sigma)b_H(1+\sigma)$	$a_L(1+(q_L+q_0)\sigma)b_H(1+\sigma)$
Pricing subgame 3: $\{M_{a_L}, P\}$	$a_H b_H$	$a_L(1+(q_L+q_0)\sigma)b_L$	$a_L(1+(q_L+q_0)\sigma)$
Pricing subgame 4: $\{M_{b_L}, M_{a_H}\}$	$a_H(1+\sigma)b_H$	$a_H(1+\sigma)b_L(1+(q_L+q_0)\sigma)$	$a_H(1+\sigma)b_L(1+(q_L+q_0)\sigma)$
Pricing subgame 5: $\{M_{b_H}, M_{b_H}\}$	$a_H b_H(1+(q_H+q_L)\sigma)$	$a_L b_H(1+(q_H+q_L)\sigma)$	
Pricing subgame 6: $\{M_{b_L}, P\}$	$a_H b_H$	$a_L b_L(1+q_L\sigma)$	
Pricing subgame 7: $\{P, M_{a_H}\}$	$a_H(1+(q_H+q_0)\sigma)b_H$	$a_L b_L$	$a_H(1+(q_H+q_0)\sigma)$
Pricing subgame 8: $\{P, M_{b_H}\}$	$a_H b_H(1+q_H\sigma)$	$a_L b_L$	
Pricing subgame 9: $\{P, P\}$	$a_H b_H$	$a_L b_L$	

FIGURE 12. The game where L moves first after elimination of dominated strategies.

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[31]The same applies to the game where L chooses M_{b_L} and H chooses M_{b_H}.

As in the model where H chooses first, depending on L and H's business model choices, two or three products will compete to attract customers in the second period.[32] In contrast to the game where H chooses first, however, in this game even if three products are available in the second stage, in equilibrium it may be the case that only one or two of these products winds up receiving positive demand.

For example, consider the subgame where L has opened a_L and H has opened b_H. In principle, there are three products available for customers to choose: the (commercial) products of firms L and H and the open source product that combines the opened a_L and b_H. In this case, however, if $p_L > 0$, L's product gets no demand because it has the same quality as the open product which is given away for free. Also, if σ is large, then the open source product will be of higher quality than the product offered by H.

In Figure 12, we have enumerated the different subgames. Before solving for the equilibrium, we should point out the following features of the game:

(1) O leapfrogs L: L is pushed out of the market only when the quality of the open source product is higher than that of L's product. In subgame 7, O *may* leapfrog L.
(2) O leapfrogs H: H is pushed out of the market only when the quality of the open source product is higher than that of H's product. In subgames 2, 3 and 4, O may leapfrog H.
(3) L leapfrogs H: In subgames 2, 3, 4 and 6, L may leapfrog H. When L leapfrogs H, but O does not, H is not pushed out the market.
(4) No leapfrogging: Leapfrogging (either L over H, O over L, or O over H) may occur in subgames 2, 3, 4, 6, and 7. In subgames 1, 5, 8, and 9 there is no leapfrogging.

As usual, we solve the game recursively. Because there are so many subgames with potential leapfrogging and so many possible equilibrium product configurations, this game is harder to solve than the one of Section 5.1.

First, we solve the second period. As we just discussed, the number of products in equilibrium depends on each business model configuration. There are four possibilities: (i) one open product, (ii) one open and one commercial product, (iii) two commercial products, and (iv) one open and two commercial products (three products). In case (i), both firms have zero-profits. We have analyzed case (ii) in Sections 3.1 and 3.2. Finally, cases (iii) and (iv) correspond to our analysis in Section 5.1.

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[32]In some subgames, there are parameter values for which there are multiple equilibria. In these cases, we have selected (a) the equilibrium with the largest number of products and (b) the equilibrium in which H gets more demand.

Once we have the continuation profits for the nine subgames, we proceed to solve numerically for the first period outcomes (choice of business model). We present the equilibrium regions for different parameter values in the following section.

5.2.1. *Business model choice.* Figure 13 shows the equilibrium business model configurations for different parameter values. As in the previous section, we have set $a_L = b_L$, $a_H = \lambda\, a_L$, and $b_H = \lambda\, b_L$, where $\lambda > 1$.
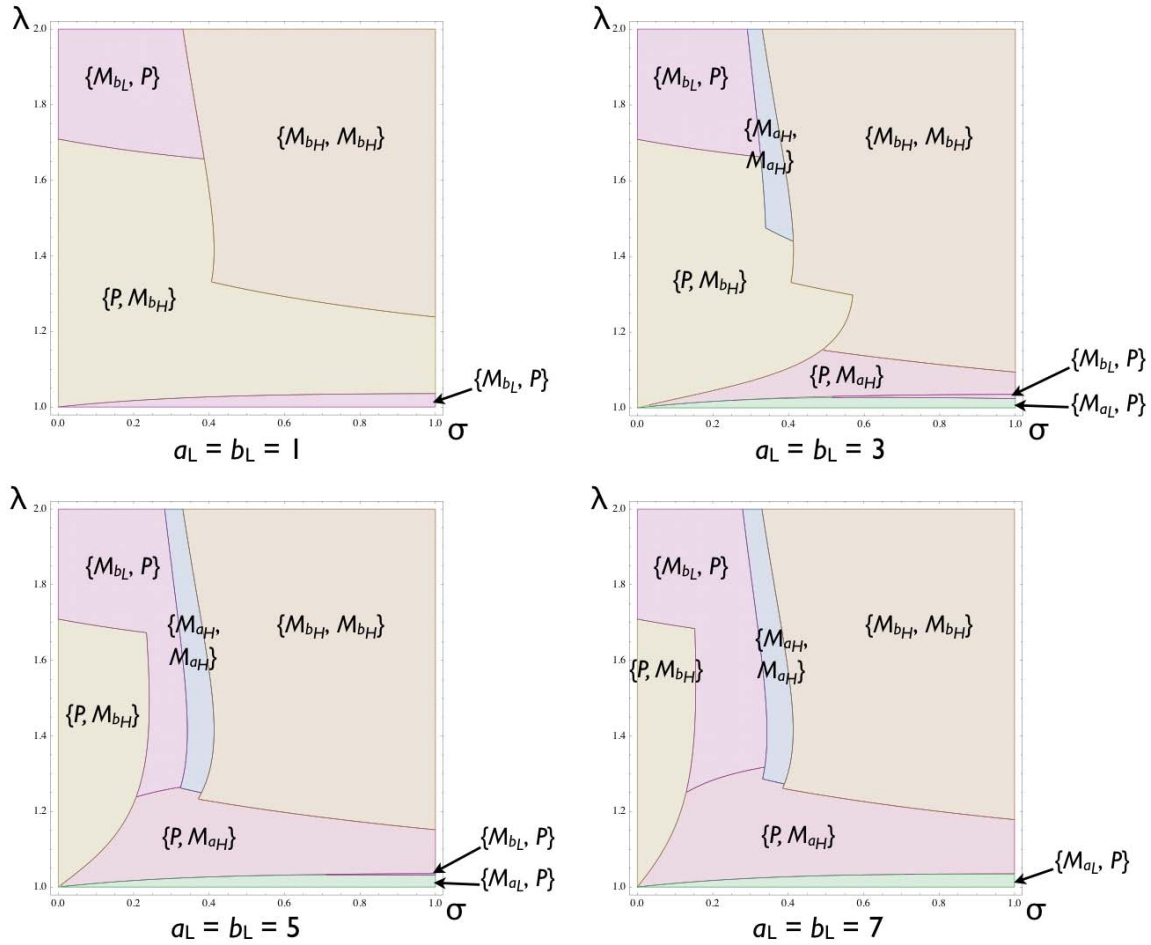

FIGURE 13. Equilibrium regions when L moves first

The labels inside the regions indicate the equilibrium business model configuration. The first element in the label corresponds to L, the first mover. For example, $\{M_{b_L}, P\}$ means that L has chosen to open b_L and H has chosen to stay closed. Therefore, $\{M_{b_L}, P\}$ when L moves first is equivalent to $\{P, M_{b_L}\}$ when H moves first (given the same parameters, both of them generate the same equilibrium demands and payoffs).

Several features of these plots are worth underlining:

(1) When λ is low and/or σ is low the equilibrium has one mixed source firm and one proprietary firm. When λ is low, firms do not want to get too close in quality because of the harsh price competition that ensues when products are not too differentiated. This means that only one firm can take advantage of user innovation when λ is low. The intuition why when σ is low and λ is large there is one mixed source firm and one proprietary firm is that if H opened a module, it would be adopted by L and the products would become insufficiently differentiated (from H's viewpoint).[33]

(2) There are two cases in which L will be the unique mixed source firm: when λ is large, but σ is small, and when λ is very small (close to 1). In the first case, L opens b_L, but H does not want to open b_H because the effect of L getting closer in quality is larger than the effect of higher quality due to user innovation. In the second case, L opens some module, and leapfrogs H, but H prefers to stay as the lower quality firm, with a larger quality differential, rather than overtaking L and having very similar quality.

(3) When λ and σ are relatively high, both firms choose to be mixed source. The initial quality differential is very large, so L wants to get closer in quality to H. On the other hand, user innovation is significant and both firms want to take advantage of the increase in quality from opening some module.

(4) When a_L and b_L are small, firms prefer to open b rather than a. As a_L and b_L increase, it is more likely that firms will open a. The reason is that when firms open a an open source product arises, and if b_L is small this open source product would have a similar quality to L's product. This, in turn, would imply low prices for L, which would also affect H.

5.2.2. *Value creation and value capture.* As in the previous case, the trade-off between value creation and value capture is affected by the strategic interaction between the two firms. In this section, we focus on the effects of L's actions, taking into account H's best response for each possible action.

Figure 14 shows value creation and value capture for both firms for the case of $a_L = b_L = 3$, $\sigma = 0.5$, and different values of λ. For each case, value capture for H is represented with small circles, and value capture for L is represented with squares.

A first result is that when L plays first, it may capture more value than H. This will happen when λ is very close to 1, so that both firms are very similar. In this case, if L

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[33]Having understood that when λ is low the equilibrium has one mixed source firm and one proprietary firm regardless of the value of σ, we need only consider low σ when λ is high.

decides to open some module first, H will decide to remain closed, and as a result L will leapfrog H.

An interesting result is that when λ is small, there is room for only one mixed source firm in the market. We can see that if L decides to remain closed, H will open a_H, but if L opens some module, H will react by remaining completely closed.

We can also see that for small λ, opening a_L gives higher value capture and value creation than opening b_L, so there is no trade-off. For larger values of λ, opening b_L gives higher value capture, so a trade-off between the two mixed source business models arises.
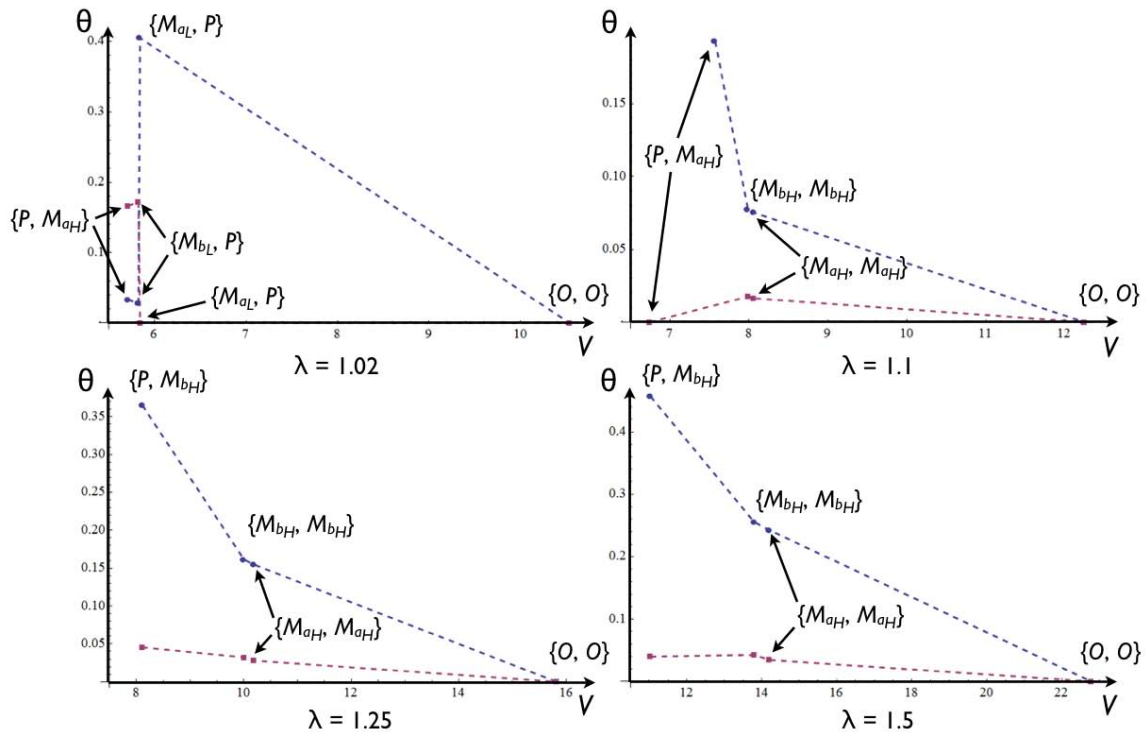

FIGURE 14. Value capture and value creation.

5.3. **Comparison of the two timings.** The two games analyzed above have important differences. First, leapfrogging may occur in equilibrium when L plays first. Second, when L plays first, the area in which both firms choose the proprietary business model P disappears. Therefore, when L plays first, it is more likely that we have at least one firm competing through a mixed source business model. Third, the area with open a is smaller than when H plays first. This is because L prefers a world with open b rather than open a. Finally, the equilibrium region where both firms open b is larger when L moves first.

6. Managerial implications

We have set up and analyzed a model to better understand the optimal choice of business model in industries where technological openness and user innovation are important. Our theoretical development provides a formal base on which to build recommendations for management practice. Our propositions, however, may be difficult to interpret by managers because of the inherent complexity of the formal model. We now present simplified versions of our main results through easily accessible matrices. In each case, we discuss what drives our propositions and lemmas and present policy recommendations that can be directly applied to the design of optimal business strategies.

6.1. **Monopoly.** The firm will in general choose to be open: when there are no competing products in the market, there is little harm in opening those technologies which have no standalone value. Thus, the firm will prefer to open the extensions even if the value of service and user innovation are low. Still, there are situations where the firm may find it optimal to open the core or to become completely open. This will happen when user innovation and the value of the complementary good are high. In this case the firm will want to build a strong user base and profit from selling the complementary good.

Figure 15 (a stylized version of Figure 4) shows the business policy recommendation for different degrees of the value of service and user innovation. The firm should choose to be open source when service and user innovation are high, and should choose a mixed source business model when either service or user innovation, or both, are low.

Figure 15. Optimal business model in the monopoly case.

Our findings show the relevance of the mixed source *open extensions* business model, which is an inexpensive way to become open. Microsoft's .Net framework and Stata are two examples. In the first case, Microsoft is committed to opening the languages that can be compiled with .Net (Visual Basic, C#, J#, *et cetera*), even to the point of promoting open standards. Users of those languages, however, need .Net (which is kept proprietary)

to compile the code that they develop. Likewise, StataCorp has opened hundreds of ado files, which are programs that implement econometric techniques used to perform specific tasks (such as Maximum Likelihood estimations for particular econometric models). While the ado files are open, users need to use Stata (which is kept proprietary) to compile those programs.

6.2. **Competing against a non-profit Open Source project.** The presence of an open source alternative affects the optimal decisions of the firm. Our first finding is that the firm should embed the modules of the open source project when they have higher quality. The result is due to complementarity between the software modules and service.

An example is IBM's support of Linux. IBM had several competing operating systems (like Z/OS), but it began supporting Linux because it was of higher quality, had a growing user base, and it could profit from selling Linux related support and consultancy services. Currently, IBM provides support for over 500 software products running on Linux, and has more than 15,000 Linux-related customers worldwide.[34]

When the outside OS project is of lower quality than the firm's product, the optimal course of action depends on the extent of user innovation and the value of the complementary good/service. Figure 16 shows a stylized version of Figures 6A and 6B. As in the monopoly case, the firm should choose the open source model when user innovation and value of service are high. The firm should compete with a mixed source model when user innovation is intermediate or high. In contrast to monopoly, however, the firm should remain proprietary when user innovation is low.

Therefore, the firm will sometimes react to competition from a non-profit open source competitor by becoming more closed. This result questions the conventional wisdom that firms will always react to open source by becoming more open.

FIGURE 16. Competition with a non-profit Open Source project.

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[34]See http://www.ibm.com/linux/ (accessed May 12, 2009).

Which module should the firm open when choosing the mixed source business model? The prescription is unambiguous: always open the module that faces stronger competition from the outside OS project.

6.3. **Competition between for-profit, commercial firms.** In this case, the optimal strategy depends on whether the firm has the higher or the lower quality product and on whether the firm is the leader or the follower in choosing its business model. Figures 17A and 17B show the optimal business model for both firms (these are stylized versions of Figures 10 and 13). In every cell, the business model at the top corresponds to the firm moving first and the one at the bottom to the firm moving second. For example, when the low-quality firm moves first, it should choose the proprietary business model and the high-quality firm the mixed source model when user innovation is low and the vertical product differentiation is intermediate.

(A) High-quality firm moves first   (B) Low-quality firm moves first
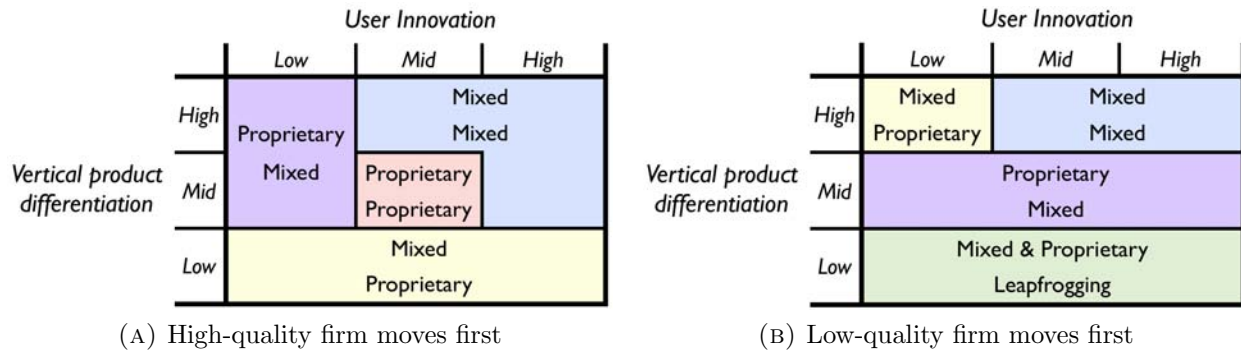
FIGURE 17. Optimal business model in the strategic duopoly case.

As we can see in both figures, when the initial quality difference or user innovation are low, one firm should compete with the proprietary business model and the other will be mixed source. When the quality difference and user innovation are high, however, both firms should compete with the same mixed business model.

While Figures 17A and 17B are similar, there is an important difference: if the firm with lower quality chooses business model first, it may end up leapfrogging the quality of the other firm's product. This happens when the initial quality difference is low. In this case, both firms prefer to move first and the market outcome depends on which firm is more agile to respond to the opportunity presented by open source. Moreover, when the low quality firm moves first, the equilibrium area in which both firms choose the proprietary business model disappears. The market tends to be more open when firms with lower quality products are more agile than firms with high quality products.

The duopoly case provides support for the observation that firms react to changes in their competitors business models by adapting their own business models. A relevant example is the case of IBM and JBoss.[35] Both firms were competing in the middleware segment.[36] Initially, IBM was proprietary, but JBoss started gaining market share using an open source model and selling complementing proprietary software and support services. To respond, IBM bought a small firm called Gluecode that was developing proprietary extensions over the open source middleware project Apache Geronimo. In a move aimed at reducing the user base of JBoss, IBM completely opened Gluecode's software. Interestingly, IBM maintained more advanced middleware programs proprietary. The example clearly illustrates the notion of competition through business models introduced in Section 5.

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[35]See `http://www.forbes.com/2005/06/15/jboss-ibm-linux_cz_dl_0615jboss.html` (accessed September 11, 2009).

[36]Middleware is software which is used as a bridge to connect other programs, and is one of the most important market segments in the software industry.

7. Concluding remarks

We have analyzed a model where firms with modular software must decide which modules to open and which to keep proprietary. We have allowed firms to choose between two "pure" business models (open source and proprietary) and two "mixed" models (open core and open extensions). We have considered three different industry structures: a monopoly benchmark, competition between a for-profit firm against a non-profit OS project, and a strategic duopoly. The analysis has delivered several novel findings which can be directly applied to the design of optimal business strategies. Given the recent outburst of attention on open business models, the topic deserves further study. The obvious next step is relaxing two convenient, though limiting, important assumptions: software compatibility and the absence of horizontal differentiation.

The paper contributes to the recent literature in strategy that analyzes competitive interactions between organizations with different business models. Interest in this line of research has increased in the past decade as new technologies, regulatory changes, and new customer demands have allowed firms to implement new approaches to competing in a wide range of industries spanning from airlines (e.g., Ryanair) to furniture (e.g., IKEA) and from the circus (e.g., Cirque du Soleil) to betting (e.g., Betfair). In fact, many of the fastest-growing firms in the recent past appear to have taken advantage of opportunities sparked by globalization, deregulation, or technological change to "compete differently" and to innovate in their business models (see Kim and Mauborgne (2005) and Markides (2008) for additional examples).

Where do the business models that we observe come from? Our contention is that, to a large extent, the configuration of business models in an industry is the equilibrium outcome of a search process for higher profits. We have proposed and illustrated a methodology for the study of endogenous business models; a two-period game where in the first period (the *strategy* period) business models are chosen and in the second period (the *tactics* period) firms interact by making tactical choices as allowed by their models. By choosing particular business models, firms are committing to the tactical subgames that will ultimately determine the profits that they will earn. To the best of our knowledge, ours is the first paper in the strategy literature where firms interacting in the market place, in addition to best responding to each other through tactical choices (such as prices), compete by reconfiguring their business models.[37]

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[37]Casadesus-Masanell and Zhu (2009) consider business model reconfigurations by one firm only. In the present paper, *both* competitors business models are endogenous.

Our approach to modeling competition through business models has similarities with the biform games introduced by Brandenburger and Stuart (2007). A biform game is a two-period game in which players first make strategic choices that determine the second period cooperative game that ensues. The first period is the *strategy* period where the bounds of the *tactical* subgame are set. The difference between our approach and biform games is that our second period is a noncooperative game. Therefore, the size of the final "pie of value" in our setting depends on the first period business model choices *and* on the equilibrium tactical interaction. In the biform game formulation, the size of the pie is determined by the first period choices and it remains fixed in the second period.

One significant implication of our approach to modeling the second period is the following. In Brandenburger and Stuart (2007), the larger the value created in the strategy period, the better off all firms end up because every unit of value created winds up being shared between all industry participants in the second period (in exogenously given proportions). In our formulation, however, larger value created in the strategy period does *not* imply improved value capture for all participants because (*i*) the equilibrium of the tactical game is generally not efficient; and (*ii*) the bargaining power of the different firms in the second period depends on the business models chosen. As a consequence, and as pointed out by Salas-Fumas (2009), the advice "choose your business model to *create* and *capture* as much value as possible" does not apply generally. As our results show, value creation and value capture need not be positively correlated. Perhaps better advice for profit-maximizing firms competing through business models is: "choose your business model to *capture* as much value as possible, taking into account the likely strategic and tactical reactions of other firms with which you will interact."

The generic two-period game that we have presented can be applied to all sorts of competitive settings such as strategies to fight ad-sponsored rivals, strategies to fight low cost entrants, strategies to fight platform players, and the like. We hope that our analysis of mixed source is helpful to researchers and practitioners willing to consider competition beyond tactics and to have provided a solid first step towards a general framework for the study of competition through business models.

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APPENDIX A: PROOFS OF PROPOSITIONS AND LEMMAS IN TEXT

Proposition 1. *Suppose that either $2 < b < 3$ and $z \leq \frac{2}{(3-b)b}$, or $b \geq 3$ hold. Then, if $\sigma \leq \sigma_{a|b}$, M_b is the optimal business model; if $\sigma_{a|b} < \sigma \leq \sigma_{a|ab}$, M_a is the optimal business model; and if $\sigma > \sigma_{a|ab}$, the optimal business model is O. Suppose, instead, that either $2 < b < 3$ and $z > \frac{2}{(3-b)b}$, or $1 \leq b \leq 2$ hold. Then, if $\sigma \leq \sigma_{b|ab}$, M_b is the optimal business model; and if $\sigma > \sigma_{b|ab}$, O is the optimal business model.*

Proof. The thresholds

$$\sigma_{a|b} := \frac{2}{bz - 2}, \quad \sigma_{a|ab} := \frac{b-1}{b(z-1)}, \quad \sigma_{b|ab} := \frac{1 + \sqrt{z(9z - 8)}}{4(z-1)} - \frac{3}{4}$$

are such that:

$$\begin{aligned}
\pi_a &> \pi_b & \text{if and only if} & \quad \sigma > \sigma_{a|b}, \\
\pi_a &> \pi_{ab} & \text{if and only if} & \quad \sigma < \sigma_{a|ab}, \\
\pi_b &> \pi_{ab} & \text{if and only if} & \quad \sigma < \sigma_{b|ab}.
\end{aligned}$$

We study the six possible orders between the thresholds, taking into account that $\sigma > 0$, $b > 1$, and $z > 1$.

A little algebra shows that the only two possibilities are: $\sigma_{a|ab} < \sigma_{b|ab} < \sigma_{a|b}$ and $\sigma_{a|b} < \sigma_{b|ab} < \sigma_{a|ab}$. The other four rankings are impossible, given our assumptions on the values that the model parameters may take. Let's consider the two possible orders separately.

Case 1: $\sigma_{a|ab} < \sigma_{b|ab} < \sigma_{a|b}$. This happens when $1 < b \leq 2$ and $z > \frac{2}{b}$ or $2 < b < 3$ and $z > \frac{2}{3b - b^2}$. Figure 18 illustrates this situation.
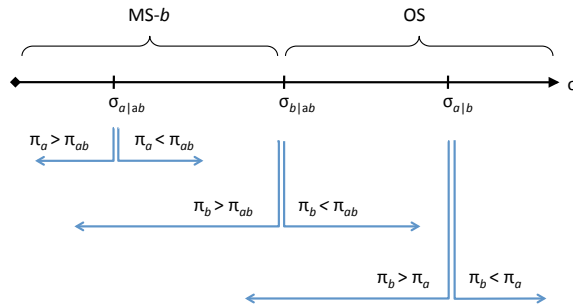

FIGURE 18. $\sigma_{a|ab} < \sigma_{b|ab} < \sigma_{a|b}$.

Case 2: $\sigma_{a|b} < \sigma_{b|ab} < \sigma_{a|ab}$. This happens when $2 < b < 3$ and $1 < z < \frac{2}{3b - b^2}$ or when $b \geq 3$. Figure 19 illustrates this situation.
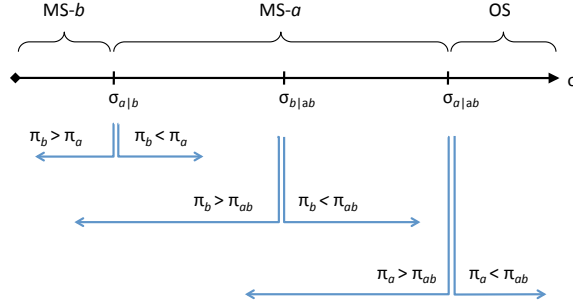
∎

FIGURE 19. $\sigma_{a|b} < \sigma_{b|ab} < \sigma_{a|ab}$.

Lemma 1. *When the modules of the firm are of higher quality than those of a non-strategic open source competitor, the firm will never choose to embed those modules in its commercial product, i.e. the firm will never choose the $M_{\hat{a}}$, $M_{\hat{b}}$, $O_{a\hat{b}}$, $O_{\hat{a}b}$, or $O_{\hat{a}\hat{b}}$ business models.*

Proof. We have to prove that $M_{\hat{a}}$, $M_{\hat{b}}$ and $O_{\hat{a}\hat{b}}$ are always dominated by other model. It will suffice to compare $M_{\hat{a}}$ with M_a, $M_{\hat{b}}$ with M_b, and $O_{\hat{a}\hat{b}}$ with O_{ab}.

We will now prove that $M_{\hat{a}}$ is dominated by M_a for all parameter values. In any of the two business models, in equilibrium $1/2$ of the users consume the commercial product, and $1/2$ of the users choose the open source product. With the first business model, the firm's product has quality $v_c = \hat{a}(1 + \sigma) b z$ and the open source product has quality $v_o = \hat{a}(1 + \sigma) \hat{b}(1 + \sigma/2)$. Profits are $\hat{a}(1 + \sigma)(b z - \hat{b}(1 + \sigma/2))$. With the second business model, the firm's product has quality $v_c = a(1 + \sigma) b z$ and the open source product has quality $v_o = a(1 + \sigma) \hat{b}(1 + \sigma/2)$. Profits become $a(1 + \sigma)(b z - \hat{b}(1 + \sigma/2))$. Clearly profits are higher under M_a because $a > \hat{a}$. Even though the open source product increases its quality when the firm open its product, in comparison with the case in which it adopts the inferior OS module, the quality of the commercial product increases even more, because of its complementarity with b and z.

The proofs for $M_{\hat{b}}$ and $O_{\hat{a}\hat{b}}$ follow the same steps, appropriately redefining the firm profits in each case.   ∎

Lemma 2. *M_a is preferred to M_b if and only if $a/\hat{a} < b/\hat{b}$, and vice versa.*

Proof. To see this, note that profits under M_a and M_b are, respectively, $\pi_a = (a_o b z - a_o \hat{b}_o)/4$ and $\pi_b = (a b_o z - \hat{a}_o b_o)/4$. The result follows directly from observing that $a_o b z := a b_o z$. Hence, the comparison is between $a_o \hat{b}_o$ and $\hat{a} b_o$ or, more simply, between $a \hat{b}$ and $\hat{a} b$.   ∎

Proposition 2 (Equilibrium regions). *If $a/\hat{a} < b/\hat{b}$, then P is optimal when $z < z_{a|p}$, M_a is optimal when $z_{a|p} < z < z_{ab|a}$ and O_{ab} is optimal when $z > z_{ab|a}$. If $a/\hat{a} > b/\hat{b}$, then P is optimal when $z < z_{b|p}$, M_b is optimal when $z_{b|p} < z < z_{ab|b}$ and O_{ab} is optimal when $z > z_{ab|b}$.*

Proof. It is easy to see that:

$$\pi_a > \pi_p \quad \text{if and only if} \quad z > \frac{\hat{b}(2+\sigma)(2a(1+\sigma)-\hat{a}(2+\sigma))}{4\,a\,b\,\sigma} := z_{a|p},$$

$$\pi_b > \pi_p \quad \text{if and only if} \quad z > \frac{\hat{a}(2+\sigma)(2b(1+\sigma)-\hat{b}(2+\sigma))}{4\,a\,b\,\sigma} := z_{b|p},$$

$$\pi_{ab} > \pi_a \quad \text{if and only if} \quad z > \frac{2b(1+\sigma)-\hat{b}(2+\sigma)}{2\,b\,\sigma} := z_{ab|a},$$

$$\pi_{ab} > \pi_b \quad \text{if and only if} \quad z > \frac{2a(1+\sigma)-\hat{a}(2+\sigma)}{2\,a\,\sigma} := z_{ab|b}.$$

Given this, the proposition follows from Lemma 2 and a straightforward comparison of the thresholds presented above. ∎

Lemma 3. *When the modules of the non-strategic open source competitor are of higher quality than the modules of the firm, the firm will always choose to embed those modules in its commercial product, i.e. it will always choose the $O_{\hat{a}\hat{b}}$ business model.*

Proof. With any business model, if the commercial product ends up with lower quality than the open source product in equilibrium, it has zero demand and thus firm's profit become zero. The firm will then adopt some or all of the OS modules. Therefore, we can concentrate on the case in which z is large enough, so that the commercial product would have higher quality than the open source product in equilibrium if the firm were to choose a business model other than $O_{\hat{a}\hat{b}}$. In equilibrium, then, the commercial product and the open source product have each $1/2$ of the market. This means that if the firm adopts an OS module, the user innovation term of any module will be $(1+\sigma)$, whereas if the firm does not adopt the OS module the user innovation term is $(1+\sigma/2)$. Therefore, when the firm adopts the module, there are two effects: (1) the commercial product increases quality because it changes modules from a or b to $\hat{a}$ or $\hat{b}$, which have higher quality, (2) the commercial product and the open source product increase quality because the user innovation term for each module goes from $(1+\sigma/2)$ to $(1+\sigma)$. However, the second effect is higher for the firm, because it gets multiplied by $z > 1$. Clearly, the firm will benefit from adopting both OS modules. ∎

Appendix B: Multiproduct Monopolist

In our analysis of the monopolist, we have imposed that the firm offers one commercial product only. Moreover, we have assumed that the product that the firm commercializes is composed of the base module, the extensions, and service. Because the independent modules and service are available to the monopolist for separate commercialization, one question that arises is whether the monopolist prefers to offer more than one commercial product.

The answer turns out to be negative: even if we allowed the monopolist to sell more than one product, she will choose to offer only the product that we have considered: a base with extensions and service.

Lemma 4. *If we let the monopolist choose how many commercial products to sell, she will choose to offer one product only. Moreover, the chosen product is the one that we have considered in Section 3.*

Proof. We consider all possible product combinations available to the monopolist.

Case 1: Monopoly with four products. The monopoly offers the base module at price p_1, the base module together with the set of extensions at price p_2, the base module with service at price p_3, and the complete package (base module, extensions, and service) at price p_4.

Let the qualities of the three products be v_1, v_2, v_3, and v_4. Clearly, $0 < v_1 < v_2 < v_3 < v_4$. The demand functions for each of the products are:

$$q_1 = \frac{p_2 - p_1}{v_2 - v_1} - \frac{p_1}{v_1}$$

$$q_2 = \frac{p_3 - p_2}{v_3 - v_2} - \frac{p_2 - p_1}{v_2 - v_1}$$

$$q_3 = \frac{p_4 - p_3}{v_4 - v_3} - \frac{p_3 - p_2}{v_3 - v_2}$$

$$q_4 = 1 - \frac{p_4 - p_3}{v_4 - v_3}$$

Profits are $\pi_{1,2,3,4} = q_1 \, p_1 + q_2 \, p_2 + q_3 \, p_3 + q_4 \, p_4$. Solving the system of three first-order conditions yields:

$$p_1 = \frac{v_1}{2}, \ p_2 = \frac{v_2}{2}, \ p_3 = \frac{v_3}{2}, \ \text{and} \ p_4 = \frac{v_4}{2}.$$

Substituting in the demand functions, we obtain: $q_1 = 0$, $q_2 = 0$, $q_3 = 0$, and $q_4 = \frac{1}{2}$. Therefore, the profit-maximizing monopolist prefers to offer the complete product only.

Case 2: Monopoly with three products. The monopoly offers the base module at price p_1, the base module together with the set of extensions or the base module with service at price p_2, and the complete package (base module, extensions, and service) at price p_3.

Let the qualities of the three products be v_1, v_2, and v_3. Clearly, $0 < v_1 < v_2 < v_3$. The demand functions for each of the products are:

$$q_1 = \frac{p_2 - p_1}{v_2 - v_1} - \frac{p_1}{v_1}$$

$$q_2 = \frac{p_3 - p_2}{v_3 - v_2} - \frac{p_2 - p_1}{v_2 - v_1}$$

$$q_3 = 1 - \frac{p_3 - p_2}{v_3 - v_2}$$

Profits are $\pi_{1,2,3} = q_1\,p_1 + q_2\,p_2 + q_3\,p_3$. Solving the system of three first-order conditions yields:

$$p_1 = \frac{v_1}{2}, \ p_2 = \frac{v_2}{2}, \text{ and } p_3 = \frac{v_3}{2}.$$

Substituting in the demand functions, we obtain: $q_1 = 0$, $q_2 = 0$, and $q_3 = \frac{1}{2}$. Therefore, the profit-maximizing monopolist prefers to offer the complete product only.

Case 3: Monopoly with two products. There are several combinations of two products that the monopolist may offer. In the analysis that we are now conducting, they are all equivalent. For example, the firm may offer the base module at price p_1 and the base module together with the set of extensions at price p_2. Or it may offer the base module at price p_1 and the base module together with the set of extensions and service at price p_2.

Let the qualities of the two products be v_1 and v_2. Clearly, $0 < v_1 < v_2$. The demand functions for each of the products are:

$$q_1 = \frac{p_2 - p_1}{v_2 - v_1} - \frac{p_1}{v_1}$$

$$q_2 = 1 - \frac{p_2 - p_1}{v_2 - v_1}$$

Profits are $\pi_{1,2} = q_1\,p_1 + q_2\,p_2$. The profit-maximizing solution has: $q_1 = 0$ and $q_2 = \frac{1}{2}$. Once again, the profit-maximizing monopolist prefers to offer the complete product only.

Case 4: Monopoly with two products when there is an open source alternative. This case corresponds to the monopolist opening module a, which sets up an OS competitor, and fighting the open source alternative with two, rather than one commercial product. The two products are: the base module with the set of extensions sold at p_1, and the complete product (base module, extensions, and service) sold at p_2.

Let the quality of the open source product be v_o and those of the commercial products be v_1 and v_2. Clearly, $0 < v_o < v_1 < v_2$. The demand functions for each of the commercial products are:

$$q_1 = \frac{p_2 - p_1}{v_2 - v_1} - \frac{p_1}{v_1 - v_0}$$

$$q_2 = 1 - \frac{p_2 - p_1}{v_2 - v_1}$$

Profits are $\pi_{1,2} = q_1\,p_1 + q_2\,p_2$. The profit-maximizing solution has: $q_1 = 0$ and $q_2 = \frac{1}{2}$. Once again, the profit-maximizing monopolist prefers to offer the complete product only. The free open sorce product obtains a market share of $\frac{1}{2}$.

Case 5: Monopoly with three products when there is an open source alternative. This last case corresponds to the monopolist opening module a, which sets up an OS competitor, and fighting the open source alternative with three commercial products. The three products are: the base module with the set of extensions sold at p_1, the base module with service sold at p_2, and the complete product (base module, extensions, and service) sold at p_3. Following the same steps as in the above cases, it is easy to see that the profit-maximizing monopolist prefers to offer the complete product only.

<div align="right">■</div>

Appendix C: Alternative Competitive Scenarios

In this appendix, we analyze alternative scenarios when there is a non-profit open source competitor for the products of the firm.

7.1. Preliminaries: Relevant business models.
The possibility of adoption of the modules provided by the outside OS project means that there are multiple implementations of the mixed and open source business models. The possibilities are: M_a, $M_{\hat{a}}$, M_b, $M_{\hat{b}}$, O_{ab}, $O_{\hat{a}b}$, $O_{a\hat{b}}$, and $O_{\hat{a}\hat{b}}$. The following lemma says that given the values of a, $\hat{a}$, b, and $\hat{b}$, only one implementation of each business model is relevant.

Lemma 5. *When choosing between M_i and O_{ij} ($i, j \in \{a, \hat{a}, b, \hat{b}\}$, $i \neq j$) business models, the profit-maximizing firm will never choose models embedding modules of lower quality. If $\hat{a} > a$ ($\hat{a} \leq a$), the firm will never choose an M_i or O_{ij} business model that uses a ($\hat{a}$). Likewise, if $\hat{b} > b$ ($\hat{b} \leq b$), the firm will never choose an M_i or O_{ij} business model that uses b ($\hat{b}$).*

Proof. There are many cases depending on which modules are provided by the OS project. The continuation profit for the incumbent is always $(v_c - v_o)/4$, but the specific values of v_c and v_o depend on the case under analysis.

For ease of exposition, we will decompose quality into two parts: $v_c = m_c\,k_c$ and $v_o = m_o\,k_o$. m_c and m_o depend on whether the OS module has been adopted or not. $k_c > 1$ and $k_o \geq 1$ are constants that do not change regardless of adoption of the OS module. For example, suppose that $\hat{a} > a$ and that the firm is opening b, with $b \leq \hat{b}$. We are interested in comparing the payoffs of adopting $\hat{a}$ with those of opening a. If the firm opens a, then $v_c = a(1 + \sigma/2)b(1 + \sigma)z$, and $v_o = \hat{a}(1+\sigma/2)b(1+\sigma)$. If the firm adopts $\hat{a}$, then $v_c = \hat{a}(1+\sigma)b(1+\sigma)z$, and $v_o = \hat{a}(1+\sigma)b(1+\sigma)$. In this case, $k_c = b(1 + \sigma)z$ and $k_o = b(1 + \sigma)$.

Recall that the user innovation term depends on the expectations of the measure of consumers using an open module. When the commercial product C and the outside open source product OS

are based on different open modules, the user innovation term is $(1+\sigma/2)$ for both products. When C and OS share the same open module, the user innovation term becomes $(1+\sigma)$.

The decomposition $v_c = m_c\,k_c$ and $v_o = m_o\,k_o$ allows us to classify every possible case into one of three possibilities:

(1) $\hat{a} > a$ or $\hat{a} \leq a$, for any value of b and $\hat{b}$.
(2) $\hat{b} > b$ or $\hat{b} \leq b$, when $\hat{a}$ exists or a is open.
(3) $\hat{b} > b$ or $\hat{b} \leq b$, when $\hat{a}$ does not exist and a is not open.

We analyze each case separately.

Case 1: $\hat{a} > a$ or $\hat{a} \leq a$, for any value of b and $\hat{b}$.

Suppose $\hat{a} > a$. We do not impose any restrictions on b and $\hat{b}$: b may be open or closed, there may be a $\hat{b}$ or not, and $\hat{b}$ may be larger or smaller than b if the former exists. If the firm opens a, $v_c = a(1+\sigma/2)k_c$ and $v_o = \hat{a}(1+\sigma/2)k_o$. If the firm adopts $\hat{a}$, $v_c = \hat{a}(1+\sigma)k_c$ and $v_o = \hat{a}(1+\sigma)k_o$. Clearly, the firm will always adopt $\hat{a}$. Suppose now that $\hat{a} \leq a$. If the firm opens a, $v_c = a(1+\sigma)k_c$ and $v_o = a(1+\sigma)k_o$. If the firm adopts $\hat{a}$, $v_c = \hat{a}(1+\sigma)k_c$ and $v_o = \hat{a}(1+\sigma)k_c$. In this case, the firm will open a. Therefore, if the firm is considering a MS or OS model with open a, it will include the higher quality module in its commercial version.

Case 2: $\hat{b} > b$ or $\hat{b} \leq b$, when $\hat{a}$ exists or a is open.

The proof is essentially the same as that of case 1.

Case 3: $\hat{b} > b$ or $\hat{b} \leq b$, when $\hat{a}$ does not exist and a is not open.

In this case, if the firm opens b, then $v_c = b(1+\sigma/2)k_c$ and $v_o = 0$. If the firm adopts $\hat{b}$, then $v_c = \hat{b}(1+\sigma)k_c$ and $v_o = 0$. Therefore, if the firm is considering a MS or OS model with either b or $\hat{b}$ open, it will choose the model with $\hat{b}$. The opposite result is obtained when $\hat{b} < b$. This concludes the proof. ∎

Lemma 5 offers a powerful simplification: the firm will only consider four business models. For example, if $\hat{a} > a$ and $\hat{b} \leq b$ or $\hat{b}$ does not exist, the firm will only consider P, $M_{\hat{a}}$, M_b and $O_{\hat{a}b}$; if $\hat{b} > b$ and $\hat{a} \leq a$ or $\hat{a}$ does not exist, the firm will only consider P, M_a, $M_{\hat{b}}$ and $O_{a\hat{b}}$; and so on.

Of course, Lemma 1 (in the main text) is a particular case of the more general Lemma 5. The intuition is the same as before, complementarity implies that the firm always benefits more than the open source project from adopting superior modules.

Having established this helpful result, we move on to analyzing the cases that may arise. We first study the case of competition by a low-quality outside OS project. We consider two cases: (1) the entrant competes with a base module only, and (2) the entrant competes with the set of extensions only. Recall that we have analyzed the case in which the entrant competes with both modules in the main text.

7.2. Outside OS project competes with base module $\hat{a} \leq a$.
We begin studying the case where the outside OS project consists only of a base module $\hat{a}$ which is of lower quality than the

firm's corresponding module a. Regardless of the business model chosen by the firm, consumers have the option of consuming an OS product with value $v_o > 0$. However, the firm affects v_o through its choice of business model. For example, if the firm decides not to open any modules, then $v_o = \hat{a}_o$. If the firm opens module a, then $v_o = a_o$ because $\hat{a} \leq a$ and all consumers prefer the firm's open module. If the firm opens module b only, then $v_o = \hat{a}_o \, b_o$ because full compatibility implies that consumers may combine modules from different developers. Finally, if the firm opens both a and b, then $v_o = a_o \, b_o$.

The firm's profits are $(v_c - v_o)/4$, where v_c is the value of the commercial product. Lemma 5 states that the firm will never adopt $\hat{a}$. Therefore the four business models that the firm will consider, the corresponding values v_c and v_o, and profits are:

P: $v_c = a \, b \, z$, $v_o = \hat{a}_o$, and $\pi_p = (a \, b \, z - \hat{a}_o)/4$.
M_a: $v_c = a_o \, b \, z$, $v_o = a_o$, and $\pi_a = a_o \, (b \, z - 1)/4$.
M_b: $v_c = a \, b_o \, z$, $v_o = \hat{a}_o \, b_o$, and $\pi_b = (a \, b_o \, z - \hat{a}_o \, b_o)/4$.
O_{ab}: $v_c = a_o \, b_o \, z$, $v_o = a_o \, b_o$, and $\pi_{ab} = a_o \, b_o \, (z - 1)/4$.

Contrary to the monopoly case, M_b does not dominate P. The reason is that when b is opened, users of the open source project can combine $\hat{a}$ with b to assemble a product of quality $\hat{a}_o \, b_o$. Having a stronger open source competitor may lead to decreased value capture by the for-profit firm. We begin by comparing M_a and M_b. The following lemma states that there is a thresold $\sigma_{a|b}$ independent of z such that M_a is preferred to M_b if and only if $\sigma > \sigma_{a|b}$.

Lemma 6. *Let $\sigma_{a|b} := 2 \left(\frac{a}{\hat{a} \, b} - 1 \right)$. When $\sigma > \sigma_{a|b}$, then M_a is preferred to M_b, and if the opposite happens, M_b is preferred to M_a. In particular, if $b > a/\hat{a}$, then M_b is never optimal.*

Proof. M_a is preferred to M_b when $\pi_a > \pi_b$, that is, when $a_o \, (b \, z - 1)/4 > (a \, b_o \, z - \hat{a}_o \, b_o)/4$. Rearranging this inequality we obtain the desired result. The opposite result is obtained similarly.
∎

The intuition behind this result is similar to that behind Lemma 2. The firm prefers to open the module which adds less value to the outside OS product. An implication of Lemma 6 is that when $\sigma > \sigma_{a|b}$, the firm considers (at most) P, M_a, and O_{ab} as candidates to an optimal business model. And when $\sigma < \sigma_{a|b}$, it considers (at most) P, M_b, and O_{ab}.

We now fully characterize the firm's choice of business model. The following are helpful for Proposition 3:

$$z_{a|p} := \frac{2 \, a \, (1 + \sigma) - \hat{a} \, (2 + \sigma)}{2 \, a \, b \, \sigma}, \quad z_{b|p} := \frac{\hat{a} \, (2 + \sigma) \, (b \, (1 + \sigma) - 1)}{2 \, a \, b \, \sigma},$$

$$z_{ab|a} := 1 + \frac{b - 1}{b \, \sigma}, \quad \text{and} \quad z_{ab|b} := \frac{2 \, a \, (1 + \sigma) - \hat{a} \, (2 + \sigma)}{2 \, a \, \sigma}.$$

Proposition 3 (Equilibrium regions). *If $\sigma > \sigma_{a|b}$, then P is optimal when $z < z_{a|p}$, M_a is optimal when $z_{a|p} < z < z_{ab|a}$ and O_{ab} is optimal when $z > z_{ab|a}$. If $\sigma < \sigma_{a|b}$, P is optimal when $z < z_{b|p}$, M_b is optimal when $z_{b|p} < z < z_{ab|b}$ and O is optimal when $z > z_{ab|b}$.*

Proof. Noticing that

$$\pi_a > \pi_p \quad \text{if and only if} \quad z > \frac{2a(1+\sigma) - \hat{a}(2+\sigma)}{2\,a\,b\,\sigma} := z_{a|p},$$

$$\pi_b > \pi_p \quad \text{if and only if} \quad z > \frac{\hat{a}(2+\sigma)(b(1+\sigma) - 1)}{2\,a\,b\,\sigma} := z_{b|p},$$

$$\pi_{ab} > \pi_a \quad \text{if and only if} \quad z > 1 + \frac{b-1}{b\,\sigma} := z_{ab|a},$$

$$\pi_{ab} > \pi_b \quad \text{if and only if} \quad z > \frac{2a(1+\sigma) - \hat{a}(2+\sigma)}{2\,a\,\sigma} := \sigma_{ab|b}.$$

this Proposition follows from Lemma 6 and a straightforward profit comparison. ∎

Proposition 3 describes how the choice of business model varies with σ and z. For low values of σ and z, the firm chooses the proprietary business model (P), for intermediate values, the firm chooses a mixed source model (either M_a or M_b), and for high values of σ and z, the firm becomes completely open (O_{ab}). Figure 20 shows the regions for different parameter values.

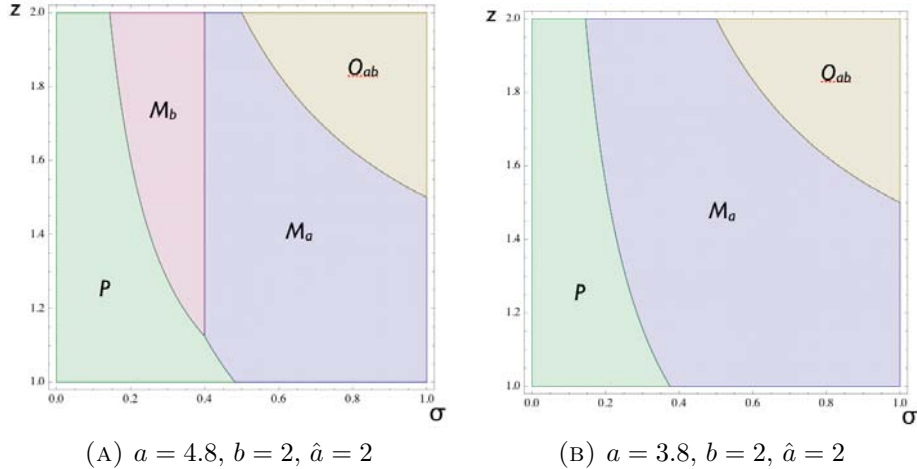(A) $a = 4.8$, $b = 2$, $\hat{a} = 2$ (B) $a = 3.8$, $b = 2$, $\hat{a} = 2$

FIGURE 20. Equilibrium business models.

It is interesting to note the similarity of this case with the case in which the firm competes with both modules. What is really important behind these two cases is that there is an outside base module which can be combined with the extensions of the firm, if it decides to open them.

As a final remark, note that leapfrogging by the open source project will not happen in equilibrium. The commercial firm can easily prevent leapfrogging by opening a.

7.3. **Outside OS project competes with add-on module $\hat{b} \leq b$.** We now consider the case in which the outside OS project consists only of extensions which are of lower quality than those of the profit-maximizing firm.

Because the outside open source project consists of $\hat{b}$ only, it cannot be used alone; it needs a base module with which to combine. The implication is that this case is similar to the monopoly of Section 3 in that if the firm decides not to open the base module, the competitor open source product has no value and no effect in the market.

The values of v_c and v_o and profits under each business model in this case are:

P: $v_c = a\, b\, z$, $v_o = 0$, and $\pi_p = a\, b\, z / 4$.
M_a: $v_c = a_o\, b\, z$, $v_o = a_o \hat{b}_o$, and $\pi_a = (a_o\, b\, z - a_o\, \hat{b}_o)/4$.
M_b: $v_c = a\, b_o\, z$, $v_o = 0$, and $\pi_b = (a\, b_o\, z)/4$.
O_{ab}: $v_c = a_o\, b_o\, z$, $v_o = a_o\, b_o$, and $\pi_{ab} = a_o\, b_o\, (z - 1)/4$.

As in the monopoly case (and for the same reason), P is dominated by M_b. This, together with Lemma 5, implies that the firm needs only consider M_a, M_b, O_{ab}. The following are helpful to characterize the firm's choice of business model:

$$z_{a|b} := \frac{\hat{b}\,(1 + \sigma)\,(2 + \sigma)}{b\,\sigma}, \quad z_{ab|a} := \frac{2\,b\,(1 + \sigma) - \hat{b}\,(2 + \sigma)}{2\,b\,\sigma}, \quad \text{and} \quad z_{ab|b} := \frac{2\,(1 + \sigma)^2}{\sigma\,(3 + 2\sigma)}.$$

Proposition 4. *If $\frac{b}{\hat{b}} > \frac{(2+\sigma)(3+2\sigma)}{2(1+\sigma)}$, then M_b is optimal when $z < z_{a|b}$, M_a is optimal when $z_{a|b} < z < z_{a|ab}$, and O_{ab} is optimal when $z > z_{a|ab}$. If $\frac{b}{\hat{b}} < \frac{(2+\sigma)(3+2\sigma)}{2(1+\sigma)}$, then M_b is optimal when $z < z_{b|ab}$ and O_{ab} is optimal when $z > z_{b|ab}$. M_a is never optimal when $\frac{b}{\hat{b}} < \frac{(2+\sigma)(3+2\sigma)}{2(1+\sigma)}$. Notice that a sufficient condition is $b < 3\hat{b}$.*

Proof. As in the monopoly case (and for the same reason), P is dominated by M_b. This, together with Lemma 5, implies that the firm needs only consider M_a, M_b, O_{ab}. Thus, only three thresholds are relevant, $z_{a|b}$, $z_{ab|a}$, and $z_{ab|b}$:

$$\pi_a > \pi_b \quad \text{if and only if} \quad z > \frac{\hat{b}(1+\sigma)(2+\sigma)}{b\,\sigma} := z_{a|b},$$

$$\pi_{ab} > \pi_a \quad \text{if and only if} \quad z > \frac{2b(1+\sigma) - \hat{b}(2+\sigma)}{2\,b\,\sigma} := z_{ab|a},$$

$$\pi_{ab} > \pi_b \quad \text{if and only if} \quad z > \frac{2(1+\sigma)^2}{\sigma(3+2\sigma)} := z_{ab|b}.$$

If $z_{a|b} > z_{ab|a}$, M_a is always dominated by M_b or OS. Rearranging this inequality, we get $\frac{b}{\hat{b}} < \frac{(2+\sigma)(3+2\sigma)}{2(1+\sigma)}$. Therefore, if $\frac{b}{\hat{b}} < \frac{(2+\sigma)(3+2\sigma)}{2(1+\sigma)}$, then M_a is never optimal, and the firm will choose between M_b and O_{ab}. When $\frac{b}{\hat{b}} > \frac{(2+\sigma)(3+2\sigma)}{2(1+\sigma)}$, the firm will choose between M_a, M_b and O_{ab}. The optimal choice is found by comparing the different thresholds. ∎

The proposition states that depending on the values of σ and z, there may be two or three optimal business models in equilibrium. For low values of σ and z, the firm opens b. For high values of σ and z, the firm opens both modules. And for intermediate values of σ, the firm may consider opening a. Opening a happens only when b is large (more precisely, only when $b/\hat{b}$ is large). In this case, although opening a creates a more formidable outside open source competitor, there is a large increase in the value of the commercial package because of the complementarity between a and b. If b is low, on the other hand, the firm will prefer to keep a closed to prevent having to remain a monopoly. Figure 21 illustrates the two situations that may emerge.
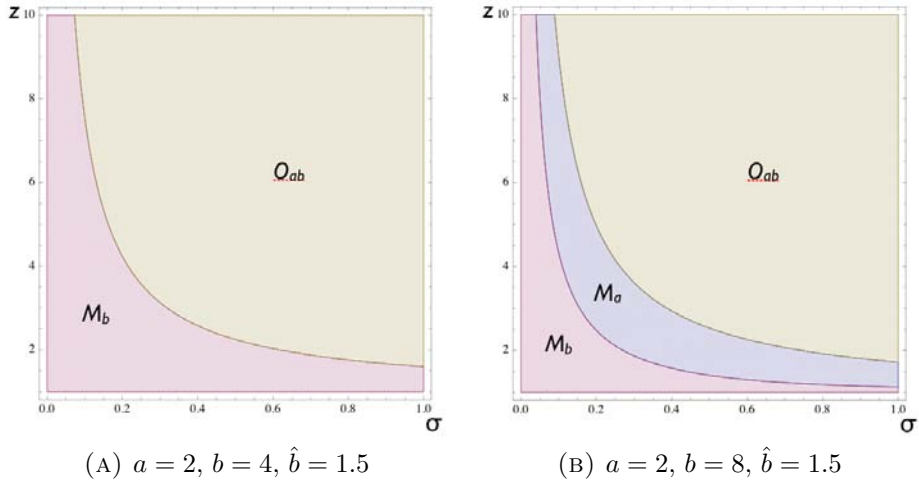

(A) $a = 2, b = 4, \hat{b} = 1.5$ (B) $a = 2, b = 8, \hat{b} = 1.5$

FIGURE 21. Equilibrium business models.

7.4. **Outside OS project competes with either a base module $\hat{a} > a$ or an add-on module $\hat{b} > b$.** In this subsection we consider the simple case in which one of the modules available through the outside OS project has higher quality than that of the firm.[38] Remember that we have already analyzed the case in which the open source project provides both modules, and both modules have higher quality than those of the firm in the main text.

The following lemma says that when $\hat{a} > a$ or $\hat{b} > b$, only two business models may arise: one of the two mixed models ($M_{\hat{a}}$ or $M_{\hat{b}}$) and the open source model ($O_{\hat{a}\hat{b}}$).

Lemma 7. *If $\hat{a} > a$ and $\hat{b} \leq b$ or $\hat{b}$ does not exist, the firm will choose $M_{\hat{a}}$ or $O_{\hat{a}\hat{b}}$. If $\hat{b} > b$ and $\hat{a} \leq a$ or $\hat{a}$ does not exist, the firm will choose $M_{\hat{b}}$ or $O_{a\hat{b}}$.*

Proof. There are many cases, depending on the modules offered by the OS project. Remember that profits are always $(v_c - v_o)/4$, but the exact form of v_c and v_o change with the case under analysis.

Suppose first that $\hat{a} > a$. We assume nothing regarding b: b may be open or closed, $\hat{b}$ may exist or not, and if it exists $\hat{b}$ may be larger than b or not. If the firm does not adopt $\hat{a}$, $v_c = a\,k_c$ and

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[38]The other module may not be offered by the outside OS project or it may be of lower quality.

$v_o = \hat{a}(1 + \sigma/2)k_o$, where $k_c > 1$ and $k_o \geq 1$ are constants which do not depend on the decision to adopt $\hat{a}$ (for example, if $\hat{b} < b$ and b is not open, $k_c = b\,z$ and $k_o = \hat{b}(1 + \sigma/2)$). If the firm adopts $\hat{a}$, then $v_c = \hat{a}(1 + \sigma)k_c$ and $v_o = \hat{a}(1 + \sigma)k_o$. Clearly, the firm will always adopt $\hat{a}$ whenever $\hat{a} > a$.

Suppose now that $\hat{b} > b$, and that $\hat{a}$ does not exist and a is not open. If the firm does not adopt $\hat{b}$, then $v_c = b\,k_c$ and $v_o = 0$. If the firm adopts $\hat{b}$, $v_c = \hat{b}(1 + \sigma/2)k_c$ and $v_o = 0$. Therefore, the firm will adopt $\hat{b}$.

Finally, suppose that $\hat{b} > b$, and that $\hat{a}$ exists or a is open. $\hat{a}$ may be larger than a or not. If the firm does not adopt $\hat{b}$, then $v_c = b\,k_c$ and $v_o = \hat{b}(1 + \sigma/2)k_o$. If the firm adopts $\hat{b}$, $v_c = \hat{b}(1 + \sigma)k_c$ and $v_o = \hat{b}(1 + \sigma)k_o$. Therefore, the firm will always adopt $\hat{b}$ if $\hat{b} > b$. ∎

Lemma 7 extends the results of Lemma 3. The intuition is the same as before. The firm compares $a\,b\,z$ to $\hat{a}_o\,b\,z$, $a\,\hat{b}_o\,z$, or $\hat{a}_o\,\hat{b}_o\,z$. Incorporating the available open modules has an obvious positive effect: because $\hat{a} > a$ and $\hat{b} > b$ the resulting product will be of higher quality than $a\,b\,z$. Moreover, because $\hat{a}$ and $\hat{b}$ are open modules, there is also a benefit from user innovation. The drawback of adopting $\hat{a}$ and/or $\hat{b}$ is that the quality of the outside OS project also increases (because more users end up adopting the module/s and there is more user innovation). As a result, competitive pressure intensifies. But because the software modules and service z are complementary, the increase in quality in the commercial product is always larger than that of the outside OS product and, thus, the positive effect always dominates the negative effect.

The following lemma says that when the outside OS project has one high-quality module only, the firm will choose to compete through an open business model if user innovation σ is strong or the value of service z is high.

Lemma 8. *When the OS project has only one module with higher quality than the corresponding firm's module, there is a function $\underline{\sigma}(z) > 0$, with $\underline{\sigma}'(z) < 0$ such that firms choose $M_{\hat{a}}$ or $M_{\hat{b}}$ for $\sigma < \underline{\sigma}(z)$ and $O_{\hat{a}\hat{b}}$ for $\sigma > \underline{\sigma}(z)$.*

Proof. We are examining cases in which one of the OS modules is better than that of the firm, and the other module is worse. We know that the firm will always adopt the better OS module. What we have to determine is in which situation it will choose to open its own module. There are two effects of opening a module. On the one hand, user innovation increases the quality of C and OS products. This effect is always positive (v_c always increases more than v_o), and will be larger when σ or z are larger. On the other hand, the quality of the OS product increases because now it has access to a better module. This effect is always negative. Therefore, for OS to be preferred to MS, σ and z have to be large enough. Finally, a lower z implies that the corresponding σ necessary for OS to be optimal has to be larger. ∎

The intuition is the same as that given in the earlier lemmas for why the open business model dominates when σ and/or z are high.

In conclusion, we can see that the analysis of these cases confirms our previous findings.