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Market Penetration and Late Entry in Mobile Telephony

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Abstract

We consider some two dynamic models of entry in mobile telephony, with and without strategic pricing, and taking into account market penetration at entry, locked-in consumers and tariff-mediated network externalities. We show that on/off-net differentials may reduce the possibility of entry if incumbents are large, while they have no long-run effects if there are no locked-in consumers, or reduce the difference in subscriber numbers in their presence. Asymmetric fixed-to-mobile or mobile-to-mobile termination rates increase (decrease) market share and profit of the network with the higher (lower) rate. While the fixed-to-mobile waterbed effect is not full at the network level, it will be full in the aggregate.

Keywords: Mobile Telephony, Entry, Penetration, Mobile termination rates.

JEL codes: L13, L51, L96

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1 Introduction

An issue that presently is hotly debated between mobile telephony operators and national regulatory agencies is whether late entrants will be able to reach market shares comparable to those of previous entrants. If they were, there would be no need to maintain asymmetric regulation, such as higher mobile termination charges for small operators, over the long run. Indeed, the European Regulators' Group (ERG) and the European Commission are of the opinion that convergence in market shares will occur and that therefore regulation should become symmetric rather sooner than later.

Central arguments that point to long-run persistence of market share asymmetry are differences in dates of entry, market penetration of mobile telephony, and operators' pricing policies. In particular, small entrants argue that:

- A large difference in entry dates (up to 10 years in some cases) leaves incumbent networks with large numbers of locked-in customers;
- the larger the market penetration of mobile telephony at the date of entry of the late entrant, the smaller will be the number of customers that have not yet adhered to any mobile network, and thus the smaller will be the entrant's growth potential;
- the existence of on-/off-net differentials on large networks, i.e. differences between call prices within the same network and to other networks, make users prefer to be on large networks and therefore limit entrants' growth.

On the other hand, large incumbents and some regulators argue that entrants in mobile markets should be able to naturally achieve large market shares, and that failure to do so ultimately must be due to entrants' strategic mistakes. The European Regulators' Common Position on the symmetry of call termination rates (ERG 2008) is a case in point: The default is a presumption that convergence will occur, and support is offered in the form of numerical simulations (p.94). Yet, in these simulations convergence is driven directly by the assumption that consumers join all networks in equal numbers, while the entrants' problem in reality is precisely that they may attract fewer customers than large networks.

A paradigmatic case where (in a first phase) four operators entered at different points in time and came to hold roughly equal market shares is the United Kingdom. In most other countries, though, late entrants' market shares have remained small. Considering only entry dates, indeed the United

Kingdom is special in that the fourth operator entered within two years of the first adoption of digital technology, while in most other countries the difference in entry dates was much larger. Thus later entry seems to lead to a smaller long-run market share. This observation is reinforced by the evolution of the UK's fifth entrant, Hutchison, whose market share is still very small a few years after entry.

To our knowledge, no attempt has been made so far to rebuild the static theoretical models of telecommunications competition from a dynamic point of view. Necessary ingredients should be: development of mobile penetration over time, tariff-mediated network externalities and switching costs.

In this paper, will consider two models: First, a simple base-line model where firms' prices are identical and fixed, and networks grow in the presence of tariff-mediated network externalities and locked-in consumers. This model serves to highlight the dynamic issues related to penetration, network externalities and mobile call termination rates (MTRs).

In the second part of the paper we will consider how networks set prices strategically when they take into account future market penetration. We build and solve a differential game model of mobile network competition in a growing market. In this paper we will concentrate on the long-run outcome of this game, which will differ from the simple benchmark model because this outcome will have been reached through networks' choosing prices strategically based on their strengths.

In this second model we will also consider the effect of fixed-to-mobile (F2M) call termination rates and asymmetry in both termination rates. While mobile-to-mobile (M2M) termination rates affect on-/off-net differentials and thus tariff-mediated network effects, F2M termination rates affect the size of networks' "war chest". Thus their economic effects are different, and in our models we will consider them separately.

Our results are as follows: The simple non-strategic model without strategic pricing indicates that tariff-mediated network effects do matter, and in two respects: First, if the incumbent is sufficiently large at the entry date, then entry will fail. Second, if the market itself is very large in the long run then the entrant may get pushed out of the market by network effects.

Considering the complete differential where firms set prices responding to each other's subscriber numbers the above conclusion about the long run must be reconsidered. While the entrant cannot fully catch up with the incumbent if the latter starts out with locked-in consumers, tariff-mediated network effects due to positive (small and symmetric) mobile-to-mobile termination charges increase (rather than decrease) the entrant's long-run market share. Moreover, a larger total market size reduces (rather than increases) the difference.[1]

On the other hand, if there are no locked-in clients then both networks' long-run subscriber numbers will be equal, even if there are tariff-mediated network effects. The qualification in this case and the previous one is that network effects must not be too strong. If they were very strong then the entrant would not be sustainable in the long run.

A second set of results concerns differences in MTRs. While F2M and M2M rates work through different channels, their effects are similar: The network that is allowed to charge more for termination enjoys a higher market share and higher profits in the long run. We also show that while with symmetric fixed-to-mobile termination rates we have a full "waterbed effect", i.e. termination profits are fully handed on to mobile consumers, this is no longer true if F2M rates differ. In this latter case the network with the higher rate keeps part of the termination profits, while the other network actually spends more. On aggregate both effects cancel out, so that the waterbed effect is still full if we consider the whole market.

As an additional step we consider the importance of consumer expectations for price formation. If consumers' expectations are not consistent with their future behavior then networks will use this to their advantage by raising per-minute prices. On the other hand, if expectations are consistent then under two-part tariffs marginal cost pricing will prevail.

2 Literature Review

The equilibria and growth of network markets comprising single networks have been investigated since the seminal papers of Artle and Averous (1973) and Rohlfs (1974). The latter considered equilibrium configurations with network effects, while the former made a first attempt at deriving the logistic (S-shaped) growth curve from an out-of-equilibrium adjustment process. On the other hand, Cabral (1990) considered general continuous-time market grow processes under network effects, where the growth curve is drawn out by network equilibria which are shifted by an exogenous process affecting consumer preferences. Geroski (2000) considers different processes such as spread of information, differences between adopters and informational cascades which give rise to an S curve-like diffusion process.

[1] One caveat is in order: While in related work (Hoernig 2007) we have explicitly considered the interplay of call externalities and tariff-mediated network effects, in the present paper the former are absent – their inclusion would render the model intractable.

Gruber and Verboven (2001) consider an econometric model of the diffusion of mobile telephony in the European Union. They fit an S-curve and determine the effect of regulation and technology on diffusion speed and penetration. Koski and Kretschmer (2005) show that technical standardization accelerates diffusion and entry in mobile telephony. Doganoglu and Grzybowski (2007) consider diffusion and network effects in the German mobile market. They show that diffusion is affected strongly by the total number of mobile subscribers, but less so by their distribution over different networks.

Lieberman and Montgomery (1988) and Mueller (1997) survey several demand and supply side reasons of why a first-mover advantage can arise. Examples of supply side effects are learning curve effects, patents and preemption of scarce assets, while demand side effects are uncertainty about quality, switching costs, and network effects. In our paper we will concentrate exclusively on the latter two.

Arthur (1989) studied competition between networks in a model of technology adoption. His emphasis is on the process that may tip the market to monopoly. De Bijl and Peitz (2002) consider competition and regulation in telecommunications markets in the presence of entrants. Their models are dynamic in the sense that the entrants' disadvantage disappears over time, but this process is exogenous and cannot be influenced by market players. In other words, whether it leads to equality in the long run depends on the researcher's assumptions and not on market forces.

Some empirical work relevant to our paper is the following: Huff and Robinson (1994) find that first entrants have sustainable market share advantages which third and later entrants are not able to erode even in the long run. Leo (2004) provides an overview of relative market entry dates of early and late entrants in European mobile telephony markets, and concludes that there is a clear first-mover advantage. Atiyas and Dogan (2007) show that, in the Turkish market, an incumbent duopoly lasting seven years significantly impaired the growth prospects of late entrants and advocate specific policy measures for fostering entrants' growth. Bijwaard *et al* (2008), in an econometric analysis, find that late entrants are at a clear disadvantage. Historical penetration levels are especially relevant, and it is more difficult to enter in markets with more existing players.

Tariff-mediated network externalities in telecommunications competition have until now between considered in a purely static context. In Laffont *et al* (1998), and most subsequent models, on-/off-net differentials are caused by above-cost termination charges. Hoernig (2007) showed that on-/off-net differentials arise also if networks take into account consumers' utility of receiving calls, and that in equilibrium larger networks choose larger differentials than small networks. Furthermore, small networks incur enduring access deficits towards larger networks. Peitz (2005) considered the effects of asymmetric termination charges in the presence of on-/off-net differentiation. ERG (2008), in position paper on the future regulation of mobile termination charges in the EU, argues that smaller networks should be allowed to charge higher termination charges, but only on a transitory basis. Reasons mentioned for letting them do so are exogenous cost differences and access deficits due to above-cost termination charges.

Gabrielsen and Vagstad (2008) consider the effects of switching costs on competition in mobile telephony. See also Farrell and Klemperer (2007) for a general treatment of switching costs and network effects.

3 A Simple Model of Pure Network Effects

3.1 Setup and First Results

Here we consider a simple model of market share dynamics, where for simplicity we assume networks' prices are constant and equal. Thus we do not consider a game between networks, in order to concentrate on the network effects created by date of entry, customer stock and market penetration. The second part of the paper will be dedicated to pricing issues and strategic behavior, modelled as a fully-fledged differential game.

We take the market growth process as such as exogenous, i.e. there is no feedback of entry on market growth. The advantage of this approach is that we can do without making specific assumptions about the market growth curve.[2] It also reduces the dimension of the state space, which reduces the complexity of the model. Thus assume that time $t \geq 0$ is continuous and that mobile market penetration $Z(t)$, i.e. the total number of mobile telephony subscribers, is an exogenous, increasing and continuously differentiable function of time. Let $Z(0) = Z_0 > 0$ and $Z_\infty = \lim_{t\to\infty} Z(t) < \infty$, that is, we assume that at time zero the market already exists, and that in the long run its growth levels off. There are two networks, where network 1 is the incumbent and network 2 is the entrant. Subscriber numbers are $s_1(t)$ and $s_2(t)$, respectively, with $s_1(t) + s_2(t) = Z(t)$ and $s_1(0) = Z(0)$, i.e. the second entry occurs at time zero and $s_2(0) = 0$. Market shares are $\alpha_i(t) = s_i(t)/Z(t)$. It turns out that it is more intuitive in the following to work with subscriber numbers than with market shares.

[2] In related research we consider the effect of mobile termination charges on market growth and fixed-to-mobile substitution. There the market growth process is endogenous.

Networks' prices are fixed, exogenous and symmetric. The price of a call minute from network i to network j is p_{ij}, with corresponding indirect utility per recipient of $v_{ij} = v(p_{ij}) = \max_q u(q) - p_{ij}q$, where v and u are consumers' indirect and direct utility functions over call minutes, respectively. For further reference, note that $-dv_{ij}/dp_{ij} = q_{ij}$ is the quantity of call minutes from network i to network j. On-/off-net differentials are created by prices $p_{ij} > p_{ii}$. A common example of different prices for on- and off-net calls arises under two-part tariffs, as we will assume here: The prices of calls are equal to their marginal cost $c + m$, where $c > 0$ is the cost of an on-net call. We have $m = 0$ for on-net calls, and $m > 0$ is the additional cost due to an above-cost MTR of off-net calls. If we define $p_m = c + m$ and $v_m = v(p_m)$ for $m \geq 0$, then $\delta = v_0 - v_m > 0$ measures the size of on-/off-net differentials in terms of consumer surplus from calling network i from either network i or j.

Networks $i = 1, 2$ also charge a fixed fee F_i, and for now we assume that $F_1 = F_2$. A consumer on network i therefore obtains the surplus at time t of

$$w_i(t) = s_i(t)\,v_0 + s_j(t)\,v_m - F_i + K, \tag{1}$$

where K is additional surplus unrelated to mobile subscriber numbers, for example the benefits of mobility or mobile-to-fixed calls. We assume that K is the same for both networks.

The two networks' services are differentiated in Hotelling fashion, with networks 1 and 2 occupying the left and right ends of a line of length 1, respectively. Consumers joining the market at time t are uniformly distributed along this line, with their mass given by $\dot{Z} = dZ/dt$. We assume that consumers are myopic and base their decision which network to join on present subscriber numbers.[3]

The share of new consumers joining network 2 is given by

$$y(t) = \frac{1}{2} + \sigma\left(w_2(t) - w_1(t)\right) \tag{2}$$

$$= \frac{1}{2} + \sigma\delta\left(s_2(t) - s_1(t)\right), \tag{3}$$

where $\sigma > 0$ measures the strength of horizontal preferences, i.e. competition is more intense if σ is large. Since we intend to model a market where the entrant has a non-zero market share, normally we assume that σ is small enough so that for all t we have $y(t) \in (0, 1)$.

[3] While we will consider forward-looking consumers in section 3.2, we believe that this is a plausible assumption, especially when consumers can switch networks.

Expression (3) shows that competition under equal prices conflates the competitiveness of the market (σ) with the strength of tariff-mediated network effects (δ), and that the difference in surplus offered to consumers is determined by the difference in subscriber numbers $s_2 - s_1$. Still, we will see below that long-run market shares depend on how many consumers are willing to switch networks.

For a start, we make the extreme assumption that switching costs are so high that consumers will never switch networks. The evolution of subscriber numbers of network 2 is thus given by $s_2(0) = 0$ and, since $s_1 = Z - s_2$,

$$\dot{s}_2 = \dot{Z}y = \dot{Z}\left[\frac{1}{2} + \sigma\delta\left(2s_2 - Z\right)\right], \tag{4}$$

while $y > 0$. If $y \leq 0$ then $\dot{s}_2 = 0$. Thus the market share path for $y > 0$ is described by a linear differential equation, which has solution

$$s_2(t) = \frac{Z(t)}{2} - \frac{Z_0}{2}e^{2\sigma\delta(Z(t)-Z_0)}. \tag{5}$$

This expression means that without network effects ($\delta = 0$) half of new consumers would join the entrant network, while the existence of network effects drives an ever greater number of consumers to the large network.

We can now state some results concerning network growth:

Proposition 1 *If networks charge symmetric prices and consumers never switch, then*

1. *If $Z_0 \geq Z^* = 1/2\sigma\delta$ the entrant gains zero market share.*
2. *If $Z_0 < Z^*$, the entrant network grows while $Z(t) < Z_0 - \frac{\ln(2\sigma\delta Z_0)}{2\sigma\delta}$. This upper limit and the entrant's speed of growth decline with initial market penetration Z_0.*
3. *The entrant's long-run subscriber number $s_2(\infty)$ decreases with initial market penetration Z_0 and the strength of termination-mediated network effects δ. There is a $\bar{Z} > 0$ such that $s_2(\infty)$ increases (decreases) with growth potential Z_∞ if $Z_\infty < (>)\ \bar{Z}$.*

Proof. We have that $\dot{s}_2(t) = \dot{Z}(t)\left(\frac{1}{2} - \sigma\delta Z_0 e^{2\sigma\delta(Z(t)-Z_0)}\right) > 0$, where the second term is decreasing in $Z(t)$. The first result is equivalent to $\dot{s}_2(0) \leq 0$, and the first part of the second is equivalent to $\dot{s}_2(t) \geq 0$. The rest follows because $d\dot{s}_2/dZ_0$ has the sign of $(2\sigma\delta Z_0 - 1)$. As for the third point: In the long run we have $s_2(\infty) = \left(Z_\infty - Z_0 e^{2\sigma\delta(Z_\infty - Z_0)}\right)/2$. Taking derivatives leads to the above result. ■

The difference in subscriber numbers between the incumbent network 1 and the entrant 2 is

$$s_1(t) - s_2(t) = Z_0 e^{2\sigma\delta(Z(t)-Z_0)},$$

which increases exponentially in $Z(t)$ if $\delta > 0$. Thus even if the entrant network grows, the incumbent network's size increases even faster due to network effects.

Now we reconsider network growth assuming that $(Z - Y_1)$ consumers are eventually able to switch networks, where $0 \leq Y_1 \leq s_1(0)$ is the number clients who are locked-in forever at the incumbent network. We assume that consumers $(Z - Y_1)$ are locked for some time until they become mobile at a rate of χ. These customers either rejoin their previous network or switch to the competing network, based on prices and subscriber numbers at time t.

New and switching consumers of the mobile group will join the entrant's network according to

$$y(t) = \frac{1}{2} + \sigma\delta(s_2 - s_1),$$

just as before. While $y > 0$, the entrant's growth in the first group of consumers is described by $s_2(0) = 0$ and

$$\dot{s}_2 = \left(\dot{Z} + \chi(Z - Y_1)\right)\left[\frac{1}{2} + \sigma\delta(2s_2 - Z)\right] - \chi s_2. \tag{6}$$

The interpretation of this expression is the following: At each point in time, χs_2 customers join the pool of customers that are selecting an operator, whose total size is $\left(\dot{Z} + \chi(Z - Y_1)\right)$. A share y of the latter then join the entrant's network. If $y \leq 0$ at some t then $\dot{s}_2 = -\chi s_2$ ever after, i.e. the entrant will shrink to zero.

We obtain the following results:

Proposition 2 *If the incumbent has Y_1 locked-in clients at $t \geq 0$, then:*

1. *If $Z_0 \geq Z^*$ the entrant gains zero market share.*
2. *If $Z_\infty < Z^*$ then the entrant's long-run number of subscribers is*

$$s_2(\infty) = \frac{1}{2}(Z_\infty - Y_1)\left(1 - \frac{Y_1}{Z^* - (Z_\infty - Y_1)}\right), \tag{7}$$

which is decreasing in σ, δ and Y_1. If $Z_\infty - Y_0 \geq Z^*$ then $s_2(\infty) = 0$.

Proof. The first statement follows from $\dot{s}_2(0) \leq 0$. The second statement follows from the steady state condition

$$0 = \chi (Z_\infty - Y_1) \left(\frac{1}{2} + \sigma\delta (2s_2 - Z_\infty) \right) - \chi s_2$$

and $s_2(\infty) > 0$. This value is consistent with $y(\infty) \geq 0$ if $Z_\infty \leq Z^*$. ■

Based on this simple model, and neglecting the strategic aspect of pricing, we have arrived at the following conclusions: A combination of high tariff-mediated network effects and a high number of subscribers at the incumbent make entry as such more difficult. In the long run, the entrant only survives if these network effects are not too strong, and if he does then an initial number of locked-in clients on the incumbent network translates into an even larger difference in subscribers in the long run.

The above analysis is limited by the assumption of equal and constant prices. It does not allow networks to set prices in a forward-looking manner, nor does it allow for a "fat cat"-effect due to locked-in consumers. As we will see, considering dynamic strategic pricing will make a difference.

3.2 Forward-looking Consumers

Contrary to what has been assumed in the previous section, we now consider consumers who are forward-looking and base the evaluation of benefits on their expectations of networks' long-run subscriber numbers. This assumption would makes sense if the market that develops fully over a very short period of time.

Let the expected final numbers of non-locked-in subscribers be s_i^∞, then

$$y = \frac{1}{2} + \sigma\delta (s_2^\infty - s_1^\infty),$$

if $y > 0$. Note that since y no longer depends on time, at each moment a constant proportion of consumers joins network 2.

We find the following result:

Proposition 3 *If consumers are forward-looking, then*

1. *If $Z_\infty \geq Z^*$ the entrant gains zero market share.*

2. *If $Z_\infty < Z^*$ then the entrant's long-run number of subscribers is*

$$s_2^\infty = \frac{1}{2} (Z_\infty - Y_1) \left(1 - \frac{Y_1}{Z^* - (Z_\infty - Y_1)} \right). \tag{8}$$

Proof. 1. No non-locked-in consumers will join network 2, i.e. if and only if they believe that $y \leq 0$, or $s_2^\infty \leq (Z_\infty - Z^*)/2$. This belief is consistent with the equilibrium outcome if and only if $s_2^\infty = 0$, or $Z_\infty \geq Z^*$.

2. On the other hand, if $y > 0$ we obtain the law of motion

$$\dot{s}_2 = \left(\dot{Z} + \chi (Z - Y_1)\right) \left[\frac{1}{2} + \sigma\delta (2s_2^\infty - Z_\infty)\right] - \chi s_2.$$

Consumers' expectations are correct if $s_2^\infty = s_2(\infty) > 0$. These are then determined by the condition $\dot{s}_2 = 0$, with the above solution for $(Z_\infty - Y_1) < Z^*$. This value is consistent with $y > 0$, or $s_2^\infty > (Z_\infty - Z^*)/2$, if $Z_\infty < Z^*$, which is a stricter condition. ■

The long-run market share is identical to the one we found above in (7) assuming myopic consumers. The reason why both coincide is that churn reshuffles consumers toward long-run market shares.

One difference between the two cases of myopic and forward-looking consumers here is that the possibility of entry in the segment of non-captive consumers depends on the value of Z_0 for the former, and Z_∞ for the latter. Thus with forward-looking consumers entry as such becomes more difficult, while the conditions for long-run survival are the same.

4 Endogenous Pricing

In this section we focus on networks' pricing decisions and how they will use pricing to steer growth. For simplicity, we will only consider consumers who may eventually be able to switch, i.e. the total number of mobile market customers at time t is $Z(t)$.[4]

4.1 Model Setup of Dynamic Network Competition

Suppose that networks set their two-part tariffs $T_i(t) = (p_{ii}(t), p_{ij}(t), F_i(t))$, $i \neq j$ in continuous time. Since competing mobile networks can observe each other's pricing decisions, we assume a "closed-loop information structure", i.e. networks observe past actions. Thus networks will react to each other's pricing, and the adequate equilibrium concept for competitive behavior is Markov-perfect equilibrium where networks set prices based on each others' subscriber numbers.

[4] Considering locked-in consumers as above adds a second state variable, which makes the model much more difficult to solve, unless one assumes that these consumers do not react to price differences.

At each point in time t, new and churning consumers choose one of the two available contracts, with tariffs $T_i(t)$. These tariffs are valid for the duration of the contract, and contracts terminate at rate $\chi > 0$. Similar to what we assumed above, at time $t = 0$ there is a group of clients $0 < Y_1 \leq s_1(0)$ on the incumbent network who will never switch. For consistency of notation let $Y_2 = 0$ and $Y = Y_1 + Y_2$. Thus the size of the cohort selecting networks at time t is $\dot{Z} + \chi(Z - Y)$. These consumers choose networks myopically, and thus a fraction $y_i(t)$ adheres to network i:

$$\begin{aligned} y_i(t) &= \frac{1}{2} + \sigma\left[s_i(t)(v_{ii} - v_{ji}) - s_j(t)(v_{jj} - v_{ij}) - F_i + F_j\right] \\ &= \frac{1}{2} + \sigma(2\delta s_i - \delta_j Z - F_i + F_j), \end{aligned} \tag{9}$$

where $\delta_i = (v_{ii} - v_{ji})$ is the difference in surplus of calling network i from either network i or j, and $\delta = (\delta_i + \delta_j)/2$. If networks set the same per-minute prices we have $\delta_i = \delta$.

We also assume that consumers who join at time t only make calls to other consumers they know are already on either one of the networks. This assumption maintains consistency with consumer expectations and simplifies flow profits. We will analyze the implications of different assumptions about consumer expectations and calling patterns below in Section 4.4.1.

Flow profits at time $u \geq t$, per consumer joining network i at time t, are

$$\begin{aligned} P_i(t) &= F_i(t) + s_i(t)(p_{ii}(t) - c)\, q_{ii}(t) \\ &\quad + s_j(t)(p_{ij}(t) - c - m)\, q_{ij}(t), \end{aligned} \tag{10}$$

where c and $m = a - c_t$ are the on-net cost of calls and the margin between mobile-to-mobile termination charge and termination cost, respectively. The discounted profits from a consumer joining at time t are

$$\tilde{P}_i(t) = \int_t^\infty P_i(t)\, e^{-(\chi+r)(u-t)} du = \frac{1}{r+\chi} P_i(t).$$

Thus network i's discounted profits over all cohorts of consumers joining at $t \geq 0$ are[5]

$$\Pi_i = \int_0^\infty \left[y_i(t)\left(\dot{Z}(t) + \chi Z(t)\right)\tilde{P}_i(t) + s_i(t)\, s_j(t)\, Q_i + s_i(t)\, R_i\right] e^{-rt} dt - M_i, \tag{11}$$

[5] The incumbent network will continue to make profits on clients that have joined before time 0, but these profits are not relevant for future pricing decisions.

where $Q_i = m_i q_{ji}$ and $R_i = (b_i - c_t) q_{fi}$ are profits from mobile-to-mobile and fixed-to-mobile termination. Both Q_i and R_i are assumed to be piecewise constant. In particular, we want to be able to analyze the effect of transitory termination rate asymmetries $Q_2 > Q_1$ and / or $R_2 > R_1$. Let $Q = (Q_1 + Q_2)/2$ and $R = (R_1 + R_2)/2$. The term M_0 refers to discounted mobile termination profits of clients who are already present at time 0, which for simplicity we have counted in the first expression,

$$\begin{aligned} M_i &= \int_0^\infty \left(Y_i e^{-rt} + (s_i(0) - Y_i) e^{-(r+\chi)t}\right)(R_i + Q_i s_j(0))\, dt \\ &= (R_i + Q_i s_j(0)) \left(\frac{Y_i}{r} + \frac{s_i(0) - Y_i}{r+\chi}\right), \end{aligned}$$

where the latter identity holds if R_i is constant over time. Evidently $M_1 > 0$ and $M_2 = 0$.

First we show that under two-part tariffs networks charge prices per minute equal to marginal cost.

Conjecture 4 *At each $t \geq 0$ we have $p_{ii}(t) = c$ and $p_{ij}(t) = c + m$, for $i = 1, 2$ and $j \neq i$.*

Proof. . Consider a given path of subscriber shares $y(t)$, $t \geq 0$, and therefore also of subscriber numbers s_i and s_j. In order to maximize profits Π_i over p_{ii} and p_{ij} given the path y, network i maximizes every $\tilde{P}_i(t)$, substituting F_i from (9).

$$F_1 = \frac{1}{\sigma}\left(y - \frac{1}{2}\right) - s_2(t)(v_{22} - v_{12}) + s_1(t)(v_{11} - v_{21}) + F_2.$$

Thus network i solves:

$$\begin{aligned} &\max_{p_{ii}} \frac{1}{r+\chi}\left[s_i(t) v_{ii}(t) + s_i(t)(p_{ii}(t) - c) q_{ii}(t)\right], \\ &\max_{p_{ij}} \frac{1}{r+\chi}\left[s_j(t) v_{ij}(t) + s_j(t)(p_{ij}(t) - c - m) q_{ij}(t)\right]. \end{aligned}$$

The corresponding first-order conditions are

$$\begin{aligned} \frac{s_i(t)}{r+\chi}\left[-q_{ii} + q_{ii} + (p_{ii} - c) q'_{ii}\right] &= 0, \\ \frac{s_j(t)}{r+\chi}\left[-q_{ij} + q_{ij} + (p_{ij} - c - m) q'_{ij}\right] &= 0, \end{aligned}$$

which lead to the above result. ■

Because calls are priced as cost we obtain $P_i(t) = F_i(t)$. Defining $\zeta(t) = \dot{Z}(t) + \chi(Z(t) - Y)$, flow profits and the law of motion become:

$$\begin{aligned} \Pi_i &= \int_0^\infty \left[\frac{\zeta y_i F_i}{r+\chi} + s_i (Z - s_i) Q_i + s_i R_i \right] e^{-rt} dt \\ \dot{s}_i &= \zeta y_i - \chi (s_i - Y_i). \end{aligned}$$

Together with (9), we have a linear-quadratic differential game with time-varying coefficients, for which Markov-perfect equilibria exist and can be characterized (see e.g. Dockner *et al* 2000).

4.2 Pricing Equilibrium

4.2.1 Equilibrium Conditions

Since we will determine a Markov-perfect equilibrium, we will use the Hamilton-Jacobi-Bellman equation describing the value function $V^i = \Pi_i + M_i$ and pricing policy F_i corresponding to each player's maximization problem. Letting π_i be flow profits, the HJB equation is

$$rV^i(s_i, t) - \frac{\partial V^i}{\partial t}(x, t) = \max_{F_i} \left\{ \pi_i + \frac{\partial V^i}{\partial s_i}(s_i, t)\, \dot{s}_i \right\} \tag{12}$$

Since the game is linear-quadratic, policy functions will be linear in the state,

$$F_i(t) = g_1^i(t)\, s_i(t) + g_0^i(t),$$

and value functions quadratic,

$$V^i(s_i, t) = V_2^i(t)\, s_i^2(t) + V_1^i(t)\, s_i(t) + V_0^i(t),$$

where $V_2^i(t) < 0$. We find the following necessary conditions (The proofs of this and the next proposition can be found in the appendix):

Proposition 5 *1. The equilibrium value function parameters must obey the following differential equations: $W_2 = V_2^i + V_2^j$ is defined by the Riccati differential equation*

$$\dot{W}_2 = 2Q + (r + 2\chi) W_2 - \frac{8\sigma}{9} \frac{\zeta}{r+\chi} (\delta + (r+\chi) W_2)^2, \tag{13}$$

and the parameters V_k^i then follow the linear differential equations

$$\dot{V}_2^i = Q_i + (r + 2\chi) V_2^i - \frac{4\sigma}{9} \frac{\zeta}{r+\chi} (\delta + (r+\chi) W_2)^2, \quad (14)$$

$$\dot{V}_1^i = \frac{4\sigma\zeta}{9} (\delta + (r+\chi) W_2) \left(\frac{\delta_j Z - \frac{3}{2\sigma}}{r+\chi} + V_1^j - V_1^i + 2ZV_2^j \right) + (r+\chi) V_1^i - 2\chi Y_i V_2^i - ZQ_i - R_i, \quad (15)$$

$$\dot{V}_0^i = rV_0^i - \chi Y_i V_1^i - \frac{\sigma (r+\chi) \zeta}{9} \left(\frac{\delta_j Z - \frac{3}{2\sigma}}{r+\chi} + V_1^j - V_1^i + 2ZV_2^j \right)^2. \quad (16)$$

The boundary conditions are $\lim_{t\to\infty} W_2 = \bar{W}_2$ *and* $\lim_{t\to\infty} V_k^i \to \bar{V}_k^i$, *which are the long-run equilibrium values derived in the next section.*

2. The equilibrium policy function parameters are given by

$$g_1^i = \frac{2}{3}\delta + \frac{2}{3} (r+\chi) \left(V_2^j - 2V_2^i \right), \quad (17)$$

$$g_0^i = \frac{1}{2\sigma} - \frac{1}{3}\delta_j Z - \frac{r+\chi}{3} \left(V_1^j + 2V_1^i + 2ZV_2^j \right). \quad (18)$$

As is typical for linear-quadratic differential games, the fundamental equation describing the equilibrium is a Riccati equation, which normally is solved using numerical methods.

We would like to highlight one case where the model can be solved analytically. This is the case where $\dot{Z} + \chi (Z - Y) = \chi (Z_\infty - Y)$ for all t, or $Z(t) = Z_\infty - (Z_\infty - Z_0) e^{-\chi t}$, which can thus be seen as an approximation to the upper branch of a logistic growth curve with convergence rate χ. For this specific case, condition (13) has an unstable negative steady state and a stable positive steady state for the parameter W_2.[6] The unstable steady state is equal to the long-run equilibrium value $\bar{W}_2$, and being an unstable steady state, there is no other solution candidate that converges to it. Thus $W_2(t) = \bar{W}_2$ for all t is the only solution candidate. Using this result, it is easy to see that all value function and policy function parameters will be constant and equal to their long-run values.

4.3 The Long-Run Equilibrium

4.3.1 Steady State Conditions

In the long run market size will converge to Z_∞ while $\dot{Z} \to 0$, i.e. $\zeta \to \chi (Z_\infty - Y)$. The parameters of the value function V_k^i and g_l^i also converge to limit values $\bar{V}_k^i$ and $\bar{g}_l^i$, and subscriber numbers converge to s_i^∞.

[6] We develop the corresponding proofs in the next section. For similar arguments about boundary conditions see the book by Dockner et al..

Proposition 6 *The long-run value function is characterized as follows:*

1. If m_1 and m_2 are small enough there is a unique $\bar{W}_2 < 0$, with

$$\bar{W}_2 = -\frac{\delta}{r+\chi} + 3\frac{3(r+2\chi) - \sqrt{9(r+2\chi)^2 + 32\sigma\zeta(2Q(r+\chi) - \delta(r+2\chi))}}{16\sigma(r+\chi)\zeta}$$

2. The value function parameters are given by

$$\begin{aligned}\bar{V}_2^i &= \frac{Q_j - Q_i}{2(r+2\chi)} + \frac{\bar{W}_2}{2}, \\ \bar{V}_0^i &= \frac{\chi}{r} Y_i \bar{V}_1^i + \frac{\sigma(r+\chi)\zeta}{9}\left(\frac{\delta_j Z - \frac{3}{2\sigma}}{(r+\chi)} + \bar{V}_1^j - \bar{V}_1^i + 2Z\bar{V}_2^j\right)^2,\end{aligned}$$

and we state the expression for $\bar{V}_1^i$ in the proof.

With symmetric termination, i.e. $Q_1 = Q_2$ and $R_1 = R_2$, we obtain

$$\bar{V}_2^i = \frac{\bar{W}_2}{2},\ \bar{V}_1^i - \bar{V}_1^j = -\frac{A\chi\bar{W}_2}{(r+\chi)}(Y_i - Y_j),$$

where $A = \left(\frac{8\zeta\sigma}{9(r+\chi)}(\delta + (r+\chi)X_{11}) - 1\right)^{-1}$, and the policy function parameters

$$\begin{aligned}g_1 &= g_1^i = g_1^j = \frac{2}{3}\delta - \frac{1}{3}(r+\chi)\bar{W}_2, \\ g_0^i &= \frac{1}{2\sigma} - \frac{1}{3}\delta Z - \frac{r+\chi}{3}\left(V_1^j + 2V_1^i + Z\bar{W}_2\right).\end{aligned}$$

Thus in the long run networks' pricing in response to a group of locked-in customers of the incumbent network differs only in the constant part of the policy function, with

$$g_0^i - g_0^j = \frac{r+\chi}{3}\left(\bar{V}_1^j - \bar{V}_1^i\right) = \frac{A\chi\bar{W}_2}{3}(Y_i - Y_j).$$

Since we assumed $Y_1 > 0$ and $Y_2 = 0$, it is the incumbent network that sets a higher fixed fee in the long run even if subscriber numbers we to equalize. On the other hand, differences in locked-in consumers do not affect how networks set their fixed fee as a function of size, i.e. g_1 is unaffected.

Since one can show that $g_1 > 0$ if m is small, the larger network then sets a higher fixed fee for two reasons: 1. Its larger base of initial locked-in customers, 2. a larger long-run market share.

4.3.2 The Determinants of Long-Run Subscriber Numbers

Our next step is to consider this distribution of subscribers in the long run when both networks remain in the market. Setting $\dot{s}_i = \zeta y_i - \chi (s_i - Y_i) = 0$, substituting the policy functions and solving for $s_i^\infty = s_i(\infty)$, we obtain

$$s_i^\infty = \frac{1}{2} \frac{Z_\infty + Y_i - Y_j - 2\sigma (Z_\infty - Y) \left(\left(\delta_j - g_1^j \right) Z_\infty + g_0^i - g_0^j \right)}{1 + \sigma (Z_\infty - Y) \left(g_1^i + g_1^j - 2\delta \right)}. \qquad (19)$$

This long-run steady state is stable if $1 + \sigma (Z_\infty - Y) \left(g_1^i + g_1^j - 2\delta \right) > 0$, which is true for small m_i.

In order to consider the effects of the initial number of locked-in customers, assume again symmetric termination charges. In this case we have

$$s_i^\infty = \frac{1}{2} (Z_\infty - Y) + Y_i - \theta (Y_i - Y_j) \qquad (20)$$

where

$$\theta = \frac{\sigma (Z_\infty - Y) \left(\frac{\chi A \bar{W}_2}{3} + g_1^i - \delta \right)}{1 + 2\sigma (Z_\infty - Y) (g_1^i - \delta)},$$

That is, subscriber numbers in the long run result from an equal distribution over both networks of the new and non-locked-in subscribers, the original number of locked-in subscribers, and an additional portion whose distribution depends on the initial difference in locked-in customers.

We have the following result:

Proposition 7 *If termination is symmetric and above cost (m is positive but small) then long-run difference in subscriber numbers is smaller than the incumbent's number of locked-in clients.*

Proof. A Taylor expansions around $m = 0$ shows that $\theta \approx \frac{\sigma}{3} (Z_\infty - Y) q(c) m > 0$, which together with $Y_1 > 0$ and $Y_2 = 0$ yields the result. ■

Thus a positive but sufficiently small on-/off-net differential may actually increase the size of the entrant in the long run. The reason for this is that the incumbent will charge a higher fixed fee, and therefore the entrant can partially (but never fully) catch up.

A second question is: If there is no initial group of locked-in customers with the incumbent, do we need asymmetric termination charges, either fixed or mobile, in order to achieve equal long-run subscriber numbers? The answer is clear:

Proposition 8 *If termination is symmetric and there are no locked-in customers at the incumbent, in the long-run subscriber numbers will be equal.*

Proof. Immediate by setting $Y_1 = Y_2 = 0$ in (20). ■

A third issue that we want to address is the effect of the long-run size of the market Z_∞ on the imbalance in subscriber numbers. Will a larger market in the long run swamp initial differences or amplify them? The answer is the following:

Proposition 9 *If termination is symmetric, a larger long-run market size reduces the difference in subscriber numbers due to differences in locked-in clients if m is positive (but small).*

Proof. As was shown in the proof of Proposition 7, the multiplier θ is increasing in $(Z_\infty - Y)$. ■

4.3.3 Mobile Termination Rates

As a final point, we consider the effects of termination charges. First we show an important result concerning fixed-to-mobile termination:

Proposition 10 *If fixed-to-mobile termination is symmetric there is a full waterbed effect, i.e. termination profits are fully passed on to consumers on both networks. If fixed-to-mobile termination rates differ (while $m_1 = m_2$ and sufficiently small) then the network with the higher (lower) rate has a less (more) than full waterbed, which translates into an increase (decrease) in market share and profits. Yet on aggregate the waterbed effect is still full.*

Proof. With $R_1 = R_2 = R$, one can show that $g_0^i = -R + const$, where the latter does not depend on R. Since g_1^i does not depend on R, this implies that fixed fees F_i are reduced by the full amount of fixed-to-mobile termination profits.

If $R_1 \neq R_2$, then $g_0^i = \frac{1}{2\sigma} - \frac{2}{3}R_i - \frac{1}{3}R_j + O(m)$ but $g_0^1 + g_0^2 = \frac{1}{\sigma} - R_i - R_j + O(m)$. Furthermore, we find that

$$\begin{aligned}
\Pi_i &= V_2^i (s_1^\infty)^2 + V_1^i s_1^\infty + V_0^i - (R_i + Q_i s_j)\left(\frac{Y_i}{r} + \frac{s_i - Y_i}{r+\chi}\right) \\
&= \frac{1}{2\sigma}\frac{\chi}{r+\chi}\frac{Z_\infty - Y}{2}\left(1 + \frac{2}{3}\sigma(R_i - R_j)\right)^2 + O(m),
\end{aligned}$$

and

$$s_i^{\infty} = Y_1 + \frac{Z_{\infty} - Y}{2}\left(1 + \frac{2}{3}\sigma\left(R_1 - R_2\right)\right) + O\left(m\right). \ \blacksquare$$

Thus as is common in static models of network competition in two-part tariffs, we obtain a full waterbed effect for fixed-to-mobile termination rates in the symmetric case that is usually considered. But an important qualification arises: Differences in termination rates matter. The network with the higher fixed-to-mobile termination rates will indeed have an advantage, and passes less of its higher termination profits to consumers. On the other hand, the network with the lower charges passes on more than it receives to consumers. These two effects compensate so that in this model the waterbed effect is still full when considering the whole market rather than single firms.

As concerns mobile-to-mobile termination rates, similar results hold. Assume now that fixed-to-mobile termination rates are identical ($R_1 = R_2$), and that the incumbent's mobile-to-mobile termination is cost-based, i.e. $\delta_1 = 0$ and $Q_1 = 0$.

Proposition 11 *If the incumbent's mobile-to-mobile termination is cost-based, the entrant's (incumbent's) market and profits increase (decrease) with a positive (but small) termination margin m_2.*

Proof. For small m_2, we have $s_1^{\infty} \approx Y_1 + \frac{Z_{\infty}-Y_1}{2}\left(1 - \sigma\eta q\left(c\right)m_2\right)$ and $s_2^{\infty} \approx \frac{Z_{\infty}-Y_1}{2}\left(1 + \sigma\eta q\left(c\right)m_2\right)$ where $\eta > 0$. Furthermore,

$$\begin{aligned}\Pi_1 &\approx \frac{Z_{\infty} - Y_1}{2}\left(\frac{1}{2\sigma}\frac{\chi}{r+\chi} - q\left(c\right)\gamma_1 m_2\right), \\ \Pi_2 &\approx \frac{Z_{\infty} - Y_2}{2}\left(\frac{1}{2\sigma}\frac{\chi}{r+\chi} + q\left(c\right)\gamma_2 m_2\right),\end{aligned}$$

where $\gamma_i > 0$. $\blacksquare$

Thus both an asymmetry in fixed-to-mobile and mobile-to-mobile termination rates can help the entrant grow. Which of the two is better in terms of welfare, i.e. fewer distortions, cannot be established without specifying the effect on the surplus of consumers on the fixed network.

4.4 Extensions

4.4.1 The Importance of Consistent Consumer Expectations

Above we have assumed that networks' clients only make calls to those customers that they know of when they join their respective networks, which is consistent with the assumption of myopic consumers. We have seen that under this assumption the standard pricing formulas under two-part tariffs remain valid.

Now we will consider the implications of clients making calls even to others who join later. This implies that consumers' myopic expectations and their actual calling behavior are inconsistent, which networks can exploit.

Flow profits at time $u \geq t$, per consumer joining network i at time t, become

$$\begin{aligned} P_i(t,u) &= F_i(t) + s_i(u)(p_{ii}(t) - c)\, q_{ii}(t) \\ &\quad + s_j(u)(p_{ij}(t) - c - m)\, q_{ij}(t). \end{aligned} \tag{21}$$

Letting

$$\tilde{s}_i(t) = (r+\chi)\int_t^\infty s_i(u)\, e^{-(\chi+r)(u-t)}du,$$

we have $\tilde{s}_i(t) > s_i(t)$ while network i grows. The discounted profits from a consumer joining at time t are

$$\begin{aligned} \tilde{P}_i(t) &= \int_t^\infty P_i(t,u)\, e^{-(\chi+r)(u-t)}du \\ &= \frac{1}{r+\chi}\left(F_i(t) + \tilde{s}_i(u)(p_{ii}(t) - c)\, q_{ii}(t)\right. \\ &\quad \left. + \tilde{s}_j(u)(p_{ij}(t) - c - m)\, q_{ij}(t)\right). \end{aligned}$$

Profits Π_i are defined as before. Let $\eta = -pq'/q$ be the price elasticity of demand. We then obtain the following result:

Proposition 12 *If networks' clients make calls to other clients who join later, but do not take this into account when selecting networks, then networks set per-minute prices above cost. More precisely,*

$$\begin{aligned} \frac{p_{ii}(t) - c}{p_{ii}(t)} &= \frac{\tilde{s}_i(t) - s_i(t)}{\tilde{s}_i(t)}\frac{1}{\eta}, \\ \frac{p_{ij}(t) - (c+m)}{p_{ij}(t)} &= \frac{\tilde{s}_j(t) - s_j(t)}{\tilde{s}_j(t)}\frac{1}{\eta}. \end{aligned}$$

Proof. . Consider a given path of subscriber shares $y(t)$, $t \geq 0$, and therefore also of subscriber numbers s_i and s_j. In order to maximize profits Π_i over p_{ii} and p_{ij} given the path y, network i maximizes every $\tilde{P}_i(t)$, substituting F_i from (9). Thus network i solves:

$$\begin{aligned} &\max_{p_{ii}} \left\{ s_i v_{ii} + \tilde{s}_i (p_{ii} - c)\, q_{ii} \right\}, \\ &\max_{p_{ij}} \left\{ s_j v_{ij} + \tilde{s}_j (p_{ij} - c - m)\, q_{ij} \right\} \end{aligned}$$

As for the on-net price, the first-order condition is

$$(\tilde{s}_i - s_i)\, q_{ii} + \tilde{s}_i\, (p_{ii} - c)\, q'_{ii} = 0,$$

which leads to the above result. As concerns the off-net price, the first-order condition is similar since the only substantial difference stems from the higher off-net cost:

$$(\tilde{s}_j - s_j)\, q_{ij} + \tilde{s}_j\, (p_{ij} - c - m)\, q'_{ij} = 0. \;\blacksquare$$

Thus we have shown the maybe counter-intuitive result that networks would charge higher per-minutes prices while they are growing if consumers are myopic. Prices then level off downwards towards marginal cost when the market reaches maturity. The reason for these above-cost prices per minute is that networks know that consumers will make more calls once on the network than what they considered when choosing networks.

Actually, given that networks compete in two-part tariffs, the higher per-minute prices will be compensated by lower fixed fees. That is, while we can have "penetration pricing" implemented through fixed fees (including handset subsidies), networks charge prices above cost to cash in on network growth.

Let us now assume that consumers foresee perfectly how networks' market shares develop, and also take into account that they will be able to switch networks in the future. We will maintain the assumption that prices are fixed for the duration of the contract. Now their expectations about network growth and actual calls made will be aligned.

This assumption leads to a redefinition of the subscriber market share y in (9). More precisely, the discounted surplus that every consumer expects to receive when joining network i at time t is

$$\tilde{w}_i\,(t) = \frac{1}{r+\chi}\,(\tilde{s}_i\,(t)\, v_0 + \tilde{s}_j\,(t)\, v_m - F_i) + K,$$

and subscriber shares become:

$$y_i\,(t) = \frac{1}{2} + \frac{\sigma}{r+\chi}\,[\tilde{s}_i\,(t)\,(v_{ii} - v_{ji}) - \tilde{s}_j\,(t)\,(v_{jj} - v_{ij}) - F_i + F_j]\,.$$

Aligning consumers' expectations of who they will call with the calls they will actually make returns per-minutes prices to marginal cost:

Proposition 13 *If clients have rational expectations about network sizes and their own permanence on networks, per-minute prices will be at cost.*

Proof. Following the same logic as in the previous Proposition, network i solves $\tilde{s}_i \max_{p_{ii}} \{v_{ii} + (p_{ii} - c)\, q_{ii}\}$ and $\tilde{s}_j \max_{p_{ij}} \{v_{ij} + (p_{ij} - c - m)\, q_{ij}\}$, both of which lead to marginal-cost pricing. ■

Thus as long as consumer expectations are consistent with their effect on profits, marginal-cost pricing will prevail.

5 Conclusions

In this paper we have presented a first go at a full dynamic model of competition between telecommunications networks. In particular, we have shown that some "obvious" conclusions concerning network effects need no longer hold when firms pricing decisions are taken into account. Such a conclusion is that tariff-mediated network effects are always to the detriment of the entrant. We have shown two significant results in this respect: These network effects matter for long-run market shares if and only if there are locked-in clients at the incumbent at the entry date.[7] On the other hand, (small) tariff-mediated network effects in the presence of locked-in clients at the incumbent imply that the long-run difference in subscribers is smaller than the initial number of locked-in clients, i.e. the entrant can partially catch up.

We have also shown that asymmetric termination rates, be it mobile-to-mobile or fixed-to-mobile, are to the advantages of the network with the higher rates, both in terms of market share and profits.

While in this paper we have concentrated on the effects in the long run, future research will consider short-run growth through numerical simulations.

[7] ERG (2008) contains a simple simulation model which "demonstrates" that entrants' and incumbents' market shares quickly converge, but this model assumes that there are no locked-in consumers.

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Appendix: Proofs

Proof of Proposition 5:

Proof. 1. Value function: Substitute $F_j = g_1^j s_j + g_0^j = g_1^j Z + g_0^j - g_1^j s_i$ into y_i and define the parameters $z = \zeta / (r + \chi)$ and

$$\begin{aligned} P_{11}^i &= -Q_i, \; P_{22} = -\sigma z, \; P_{12}^i = \sigma z \left(2\delta - g_1^j\right), \\ P_1^i &= (ZQ_i + R_i), \; P_2^i = z \left(\frac{1}{2} - \sigma \delta_j Z + \sigma \left(g_0^j + g_1^j Z \right) \right). \end{aligned}$$

Also, let

$$f_0^i = (r + \chi) P_2 + \chi Y_i, \; f_1^i = (r + \chi) P_{12} - \chi, \; f_2 = (r + \chi) P_{22},$$

then

$$\begin{aligned} \pi_i &= P_{11}^i s_i^2 + P_{22} F_i^2 + P_{12}^i s_i F_i + P_1^i s_i + P_2^i F_i, \\ \dot{s}_i &= f_0^i + f_1^i s_i + f_2 F_i. \end{aligned}$$

Substituting these into the HJB equation results in

$$\begin{aligned} & r \left(V_2^i s^2 + V_1^i s + V_0^i \right) - \left(\dot{V}_2^i s^2 + \dot{V}_1^i s + \dot{V}_0^i \right) = \\ & \max_F \left\{ P_2^i F + P_{12}^i s F + P_{22} F^2 + P_{11}^i x^2 + P_1^i s \right. \\ & \left. + \left(2V_2^i s + V_1^i \right) \left(f_0^i + f_1^i s + f_2 F \right) \right\} \end{aligned}$$

2. Policy function: The maximization in the HJB equation produces the first-order condition

$$P_2^i + P_{12}^i s + 2P_{22} F + \left(2V_2^i s + V_1^i \right) f_2 = 0,$$

with second-order condition $2P_{22} = -\frac{2\sigma\zeta}{r+\chi} < 0$. Solving for F leads to

$$F = -\frac{(2V_2^i f_2 + P_{12}^i)}{2P_{22}} s - \frac{(V_1^i f_2 + P_2^i)}{2P_{22}},$$

or

$$\begin{aligned} g_1^i &= -\frac{2V_2^i f_2 + P_{12}^i}{2P_{22}} = \delta - \frac{g_1^j}{2} - (r + \chi) V_2^i \\ g_0^i &= -\frac{V_1^i f_2 + P_2^i}{2P_{22}} = \frac{1}{4\sigma} + \frac{g_1^j Z + g_0^j - \delta_j Z}{2} - \frac{r + \chi}{2} V_1^i \end{aligned}$$

Solving the two conditions for g_1^i and g_1^j leads to

$$g_1^i = \frac{2}{3}\delta + \frac{2}{3}(r+\chi)\left(V_2^j - 2V_2^i\right),$$

and the two corresponding conditions for g_0^i and g_0^j imply

$$g_0^i = \frac{1}{2\sigma} - \frac{1}{3}\delta_j Z - \frac{r+\chi}{3}\left(V_1^j + 2V_1^i + 2ZV_2^j\right).$$

3. Value function parameters: Substituting $F_i = g_1^i s + g_0^i$ as maximizer into the value function, and grouping coefficients associated to the same power of s, we obtain the three conditions

$$\begin{aligned}
\dot{V}_2^i &= -P_{12}^i g_1 - P_{22} g_1^2 - P_{11}^i + \left(r - 2\left(f_1^i + f_2 g_1^i\right)\right) V_2^i, && (22)\\
\dot{V}_1^i &= \left(r - f_1^i - f_2 g_1^i\right) V_1^i - 2V_2^i\left(f_0^i + f_2 g_0^i\right) && (23)\\
&\quad -P_2^i g_1^i - P_{12}^i g_0^i - 2P_{22} g_0^i g_1^i - P_1^i, &&\\
\dot{V}_0^i &= rV_0^i - V_1^i\left(f_0^i + f_2 g_0^i\right) - P_2^i g_0^i - P_{22}\left(g_0^i\right)^2. && (24)
\end{aligned}$$

3a. Parameters V_2^i and V_2^j: Substituting the original parameter values into (22), we obtain

$$\dot{V}_2^i = Q_i + (r+2\chi) V_2^i - \frac{4}{9} z\sigma\left(\delta + (r+\chi)\left(V_2^i + V_2^j\right)\right)^2$$

Defining $W_2 = V_2^i + V_2^j$, $Q = (Q_i + Q_j)/2$ and summing (22) over both i and j, yields

$$\dot{W}_2 = 2Q + (r+2\chi) W_2 - \frac{8\sigma}{9}\frac{\dot{Z} + \chi Z}{r+\chi}\left(\delta + (r+\chi) W_2\right)^2, \qquad (25)$$

which is a non-autonomous Riccati differential equation for W_2 which has a unique solution given the boundary condition that we will derive using the long-run equilibrium. Given a solution W_2, V_2^i can be determined by the linear differential equation

$$\dot{V}_2^i = Q_i + (r+2\chi) V_2^i - \frac{4\sigma}{9}\frac{\dot{Z} + \chi Z}{r+\chi}\left(\delta + (r+\chi) W_2\right)^2, \qquad (26)$$

with a corresponding long-run boundary condition.

3b. Parameters V_1^i and V_1^j: Using (23), we find the coupled linear differential equations

$$\begin{aligned}\dot{V}_1^i &= \frac{4\sigma\zeta}{9}\left(\delta+(r+\chi)W_2\right)\left(\frac{\delta_j Z-\frac{3}{2\sigma}}{r+\chi}+V_1^j-V_1^i+2ZV_2^j\right) &(27)\\ &+(r+\chi)V_1^i-2\chi Y_i V_2^i-ZQ_i-R_i,\\ \dot{V}_1^j &= \frac{4\sigma\zeta}{9}\left(\delta+(r+\chi)W_2\right)\left(\frac{\delta_i Z-\frac{3}{2\sigma}}{r+\chi}+V_1^i-V_1^j+2ZV_2^i\right) &(28)\\ &+(r+\chi)V_1^j-2\chi Y_j V_2^j-ZQ_j-R_j.\end{aligned}$$

3c. Parameters V_0^i and V_0^j: Using (24), we find the linear differential equations

$$\dot{V}_0^i = rV_0^i-\chi Y_i V_1^i-\frac{\sigma(r+\chi)\zeta}{9}\left(\frac{\delta_j Z-\frac{3}{2\sigma}}{r+\chi}+V_1^j-V_1^i+2ZV_2^j\right)^2. \blacksquare \quad (29)$$

Proof of Proposition 6: Proof. 1. The expression for $\bar{W}_2$ is found by solving (13) with $\dot{W}_2=0$. For symmetric MTRs, i.e. $Q_i=Q$ and $\delta_i=\delta$, we have

$$\begin{aligned}Q &= mq(c+m)=q(c)m+q'(c)m^2+O\left(m^3\right)\\ \delta &= v(c)-v(c+m)=q(c)m+\frac{1}{2}q'(c)m^2+O\left(m^3\right),\end{aligned}$$

thus $Q\approx\delta$ for $m\approx 0$. The term under the root is certainly positive for m small enough, because then also δ is small. It can be shown that $\bar{W}_2<0$ if either $\delta\geq\frac{9(r+2\chi)}{16\sigma\psi}$ (which we rule out) or if $\delta<\frac{9(r+2\chi)}{16\sigma\psi}$ and $9Q(r+\chi)-4\delta^2\psi\sigma>0$. Joining the two leads to the sufficient conditions $Q(r+\chi)-\frac{\delta}{4}(r+2\chi)>(r+\chi)\left(Q-\frac{\delta}{2}\right)>0$ since $Q\approx\delta$ for small m. This result continues to hold if the m_i are small but different. On the other hand, the larger root $\bar{W}_2'$ of $\dot{W}_2=0$ cannot be negative because there would be a contradiction with the term under the root sign having to be non-negative. This latter result does not depend on the m_i being small.

2. As for $\bar{V}_2^i$, solve (14) with $\dot{V}_2^i=0$ using the long-run condition for W_2 to substitute out the quadratic term, to obtain

$$\bar{V}_2^i=\frac{Q_j-Q_i}{2(r+2\chi)}+\frac{\bar{W}_2}{2}.$$

The expression for $\bar{V}_1^i$ is more complicated,

$$\bar{V}_1^i=\frac{2\chi Y_i\bar{V}_2^i+ZQ_i+R_i}{r+\chi}-\frac{2\zeta\left(\delta+(r+\chi)\bar{W}_2\right)\left(2\sigma\delta_j Z-3+2\sigma(r+\chi)\left(2Z\bar{V}_2^j-\left(\bar{V}_1^i-\bar{V}_1^j\right)\right)\right)}{9(r+\chi)^2}$$

with

$$\left(\bar{V}_1^i - \bar{V}_1^j\right) = \left(Z - \frac{A\zeta}{(r+\chi)}\right)\frac{Q_i - Q_j}{r+2\chi} - \frac{(1+A)\,Z}{2\,(r+\chi)}\,(\delta_1 - \delta_2)$$
$$-A\frac{\chi X_{11}\,(Y_i - Y_j) + (R_1 - R_2)}{(r+\chi)}$$

and $A = \left(\frac{8\sigma\zeta}{9(r+\chi)}\left(\delta + (r+\chi)\,\bar{W}_2\right) - 1\right)^{-1}$. The last term, $\bar{V}_0^i$, again follows directly from $\dot{V}_0^i = 0$. ■