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Abstract

Positive network externalities can arise when consumers benefit from the consumption of compatible products by other consumers (user-positive consumption externalities) or, alternatively, when they incur costs from the consumption of incompatible products by other consumers (nonuser-negative consumption externalities). But whereas user-positive externalities are typically mutually imposed and imply mutual benefit because they relate to interoperability, with nonuser-negative externalities the costs of incompatibility may be imposed unilaterally and borne asymmetrically. For example, increased risks of death and injury on the roads due to the co-existence of large and small vehicles are imposed exclusively by the owners of the large vehicles and borne exclusively by the occupants of the small vehicles. This paper compares the social optimality of incentives for compatibility under regimes involving user-positive and nonuser-negative externalities. Earlier work with respect to user-positive externalities (e.g., Katz and Shapiro, 1985) suggests that firms with relatively small networks or weak reputations tend to be biased in favor of compatibility, while individual firms' incentives for compatibility are suboptimal when their networks are closely matched in size. Meanwhile, intuition suggests that with nonuser-negative externalities incentives for incompatibility should always be excessive, reflecting the notion that activities involving unilaterally imposed negative externalities will always be overprovided by the market (in the absence of regulation or Coaseian mitigation). Using a "location" model of differentiated products, we find that, under *both* regimes, incentives for compatibility tend to be suboptimal when firms' networks are close in size, and excessive for the small firm when the networks differ greatly in size. Surprising public policy implications with respect to externalities are discussed.

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I. Introduction

It has long been recognized that, in various situations, the utility that an agent derives from consuming a good may depend upon the choices made by other agents with respect to consuming the same good, a compatible good, or competing goods. In their seminal paper, Katz and Shapiro (1985) noted that inter-user impacts on utility, or network externalities, exist where the number of users has a direct physical effect on the quality of the product (e.g., in the telephone network), or where the number of users has an indirect impact on the desirability of a product through its effect on the availability and quality of complementary goods (e.g., with respect to computer hardware and software, and service networks for durable goods). The cases they describe involve positive consumption externalities, whereby consumers benefit from the purchase or use of compatible products by other consumers. Following Katz and Shapiro, the network externalities literature has focused predominantly on positive consumption externalities; many discussions, like Katz and Shapiro's, have essentially equated the two.²

Network externalities are, however, not limited to positive consumption externalities. It is possible to classify four types of consumption externalities, based on the valence of the externality (positive or negative) and who the externality affects (product users or nonusers). Positive network externalities could theoretically arise from *two* of these: user-positive and nonuser-negative consumption externalities. In the latter case, a network externality arises because, when users of a product impose costs exclusively or to a greater extent on those who do not use the product, an incremental

² See, for example, Gilbert (1992), Katz and Shapiro (1992), and Economides (1996a, 1996b).

user increases the preferences of other consumers for using the product relative to not using it.

Nonuser-negative externalities are not merely an object of theoretical speculation. Examples abound. When a market offering competes with a public good (or, more generally, publicly-provided free good), such as bottled water with tap water, or private schools with public schools, a consumer who switches from the publicly-provided good to the market good reduces political support for maintaining the quality of the former.³ The consumer who switched is shielded from adverse effect, because she no longer relies on the public good. A similar situation is posed by the phenomenon of “white flight”: white people who flee urban areas that are becoming racially integrated impose “costs” (albeit as viewed from the racist perspective of preference for “whiter” communities) on the whites that remain behind. Meanwhile, they shield themselves, by fleeing, from the same costs.

Another example of the phenomenon is presented by what one might call “combatant goods,” which bundle greater imposition of external costs with greater protection against the same costs, relative to alternatives. For example, sport utility vehicles (SUVs) impose greater risks of injury and death on other motorists than do cars, while at the same time providing their occupants with increased protection against these same risks relative to cars. Visible car-theft deterrent devices (such as the Club) tend to push thieves to other cars, including those protected by invisible deterrent devices (such

³ Given the nonexcludability characteristic of public goods, when we talk about “switching away” from such goods we are talking about consumers who choose no longer to rely on the benefits they provide. For example, the consumer who places her house on stilts is still literally consuming the flood protection provided by a nearby levee, but she is choosing not to rely on it.

as Lojack); thus they redistribute crime rather than reducing it, while inoculating their owners against the effects of such redistribution (Ian Ayres and Steven D. Levitt 1998).

Yet a further example can be found in situations where a consumer's adoption of a noisome product reduces her displeasure from others' use of the product, either because personal use increases habituation to others' use, or because personal use changes attitudes through reaction to cognitive dissonance.⁴ For example, smokers tend to find secondhand smoke less objectionable than non-smokers (Beh 1989). Meanwhile, people who hire contractors to engage in construction work on their houses might be less likely to find neighbors' noisy construction projects objectionable. Similarly, the fact that one's neighbors are polluting a shared lake through pesticide runoff or by using motorboats may be viewed less unfavorably if one is doing these things oneself. Such "if you can't beat 'em, join 'em" influences might be relevant to a wide range of consumer choice situations, including not just decisions with respect to whether to use a product, as in the examples above, but also choices between competing products (e.g., loud gas-powered lawn mower versus quiet electric mower; McMansion versus modest house; etc.).

An important difference between the network externalities that typically arise from user-positive externalities and those typically arising from nonuser-negative externalities involves mutuality of effect. User-positive externalities typically relate to interoperability, that is, the ability of one user's product to work in connection with

⁴ Cognitive dissonance may be described as the internal conflict that results when an individual receives information that contradicts basic ego-supporting beliefs. For example, an individual tends to think of herself, "I am a nice person who would never do something to irritate or harm an undeserving person." So when the individual engages in an activity that she has found irritating in the past when others did it, she might avert the potential for internal stress by changing her attitude toward the activity so as to view it as less noisome. Evidence of such attitude changes in dissonance-provoking situations is provided by Leon Festinger and James M. Carlsmith (1959), Keith Davis and Edward Jones (1960), and David Glass (1964). For an economic discussion of cognitive dissonance, see George A. Akerlof and William T. Dickens (1982).

another user's product, or the ability of one user's product to make use of the complementary goods that another user's product employs. Thus the benefits from usership are mutual, that is, the consumers that enjoy the benefits of having other users on their network in turn benefit those other users. For example, when I purchase a computer with a Windows operating system, I benefit from the variety of software available for Windows due to the large number of other users; at the same time, as one of those users, I contribute to the software variety benefits that others receive. When it comes to compatibility, a key strategic question facing firms is whether, and to what extent, to extend the mutual benefits to competing products' consumers. Katz and Shapiro (1985) observe that firms with large networks are biased against compatibility, while firms with small networks are biased in favor of it. It would seem that mutual propagation of compatibility benefits is at the heart of this result: firms with small networks would seem to gain more when compatibility is extended to them because their consumers gain the benefits of interoperability with a larger mass of consumers, while what they offer in return is interoperability with a smaller mass of consumers.

But nonuser-negative externalities, arising in situations such as those depicted above, operate differently: they involve costs imposed unilaterally – *not* mutually – by agents who take an action on those who do not take the same action or a compatible action. In these situations, when the imposing action involves the use of a product, the strategic question facing the product manufacturer as relates to compatibility is how large to make the negative externality. That is, how incompatible should the product be with

competing products?⁵ Here, intuition suggests, because the external costs are imposed unilaterally, that private incentives for incompatibility are always excessive.

This paper compares incentives for compatibility under the regimes involving user-positive externalities and nonuser-negative externalities. We focus on incentives for unilateral action on compatibility (e.g., in the case of user-positive externalities, developing an adapter), rather than joint action (e.g., developing a standard). We analyze a “location” model of differentiated products. In this sense, the approach is similar to the analyses of network externalities offered by Farrell and Saloner (1992) and Matutes and Regibeau (1988, 1992), and different from the homogenous products model of Katz and Shapiro (1985). Our findings for user-positive externalities essentially replicate the results of Katz and Shapiro (1985) concerning the relationship of firm size to compatibility incentives. But the findings for nonuser-negative externalities do not bear out our intuition on excessive incentives. Instead, we find incentives for incompatibility that follow closely, though not exactly, Katz and Shapiro’s findings relating optimality of firms’ incentives to network size. Whereas firms that are close in size tend to have socially excessive incentives for incompatibility, an imposing firm has insufficient incentives for incompatibility if its “network” (customer base) is relatively very small or very large.

⁵ Enlarging external costs might be accomplished by dint of product design. For example, external costs imposed by the Club might be manipulated by altering its size, color, and shape to make it more or less obvious to prospective thieves. (For further discussion of visible car-theft deterrents, see Ayres and Levitt 1998.) Similarly, the external costs imposed by SUVs might be enlarged by building these vehicles with stiffer and higher front ends, thereby increasing the damage imposed on the vehicles with which they collide (Keith Bradsher 2002; Howard Latin and Bobby Kasolas 2002). Enlarging external costs might also be accomplished through marketing messages. For example, under the practice known as “blockbusting,” unscrupulous real estate agents “warn” white residents in a neighborhood to sell quickly and avoid losses on their houses as blacks move in. More subtle practices include planting large “For Sale” or “Sold” signs on neighborhood properties (Georgetown Law Journal 1970). Such actions call attention to, and thereby exacerbate perceptions of, the negative effect that the exodus of home owners has on those who choose to stay put.

The next section lays out the general model. Section 3 derives welfare results for the user-positive externalities case. Section 4 derives welfare results for the nonuser-negative externalities case. Section 5 offers a public policy discussion and concludes.

II. A Model of Differentiated Product Duopoly with Network Externalities

Consider a market for two products, A and B, sold at prices p_A and p_B , respectively. Consumers are distributed uniformly on a unit segment based on their preferences for A versus B, with the total number of consumers normalized to 1. Consumers choose whether to purchase A or B; each consumer will choose at most one unit of one of the two products. We consider a general framework of network effects as given by the following utility functions, representing the utility that the consumer located at a point j ($1 \geq j \geq 0$) obtains from purchasing a unit of product A or B, respectively:

$$U_A(j) = v + \theta - t(1 - j) + \sigma_A \lambda Q_A + \sigma_{BX} \lambda Q_B - p_A \quad (1)$$

$$U_B(j) = v - \theta - tj + \sigma_B \lambda Q_B + \sigma_{AX} \lambda Q_A - p_B \quad (2)$$

Here, v represents the demand for all products; θ , which may be positive or negative, parameterizes the demand for A relative to B; t represents the intensity of consumers' relative preferences for A or B ($t > 0$); Q_i is the number of consumers who purchase product i ($i = A, B$); λ parameterizes the overall size of the network effect ($\lambda \geq 0$); and σ_i sizes and signs an *own* component of the network effect ($\sigma_i \in [-1, 1]$), while σ_{ix} similarly sizes and signs a *cross* component of the network effect ($\sigma_{ix} \in [-1, 1]$). A

consumer who chooses neither A nor B receives utility of zero. Each consumer makes the choice that maximizes her utility.

Now, consider two cases: (I) $\sigma_A = \sigma_B = 1$, $\lambda > 0$ are given, and firm A sets $\sigma_X \equiv \sigma_{AX} = \sigma_{BX} \in [0,1]$; and (II) $\sigma_{AX} = -1$, $\sigma_A = \sigma_B = \sigma_{BX} = 0$ are given, and firm A sets λ . The first case is the classic case of a user-positive externality: incremental users of A and B provide a benefit, λ , to other users of the same product. The decision that firm A faces is whether, and to what extent, to include firm B's consumers in the network. Does firm A make B's consumers fully compatible with its own consumers, or partially compatible, or not at all? Note that the decision to make firm B's consumers compatible also means that firm A's consumers are compatible with firm B's, so that B's consumers receive increased network benefits as well; thus the benefits are mutual. Further, note that we assume, in Katz and Shapiro's (1985) parlance, that the compatibility technology is an "adaptor," hence A and B are compatible if either A or B decide to undertake the expense to make them compatible. Since our purpose is to examine whether the level of compatibility chosen by a firm of a given network size is too high or too low, we assume without loss of generality that only A makes the decision of whether to make the products compatible.

The second case involves a nonuser-negative externality: firm A considers the possibility of imposing a negative externality that only affects the users of product B. We shall show that the effect of doing this is also to create a network externality: when $\lambda > 0$, the reservation price of users of A increases with the number of users of A, all else

equal.⁶ But A's decision to make B's users more *incompatible* with product A does not have a mutual effect: A's users are not reciprocally harmed by users of B. That is, B is made more incompatible with A, but A is not made more incompatible with B.

It might seem that we already understand the relative private and social incentives for compatibility in this case. As discussed in the introduction, since the incompatibility decision involves a unilaterally imposed negative externality, the incompatibility incentives of firm A would seem always to be excessive, unlike in the case of user-positive externalities. The model considers whether that expectation is correct.

III. Equilibrium with User-Positive Externalities

Setting parameters to the values proposed for case (I) above, (1) and (2) become:

$$U_A(j) = v + \theta - t(1-j) + \lambda Q_A + \sigma_X \lambda Q_B - p_A \quad (3)$$

$$U_B(j) = v - \theta - tj + \lambda Q_B + \sigma_X \lambda Q_A - p_B \quad (4)$$

Let us assume v is large enough that all consumers choose A or B at equilibrium prices, implying $Q_A = 1 - Q_B$.⁷

Combining (3) and (4) reveals that the consumer at j prefers A over B if

$$2\theta - t(1-2j) + (1 - \sigma_X)\lambda(Q_A - Q_B) + p_B > p_A. \text{ We can therefore think of}$$

⁶ Nagler (2008) examines a more general framework allowing $\sigma_A \in [-1, 0]$, so that the degree of “selectivity” of the negative externality is a parameter in the analysis. The extreme case $\sigma_A = -1$ represents a pure negative externality due to the use of product A, with no consequent network externality; while varying values of $\sigma_A \in (-1, 0]$ varies both the degree of selectivity and size of the network externality.

⁷ Consider $\bar{v} \equiv \theta + t + \varepsilon$ for arbitrary $\varepsilon > 0$. Then, for all $\lambda, \sigma_X > 0$, there exists $p_B > 0$ such that $U_B > 0$. Thus \bar{v} satisfies the requirement.

$$\Psi_j = 2\theta - t(1 - 2j) + (1 - \sigma_x)\lambda(Q_A - Q_B) + p_B \quad (5)$$

as the consumer's reservation price for A, relative to B. It is interesting also to note that the relative quantity of A versus B matters more to the relative willingness-to-pay the less compatible the two products are.

Following Katz and Shapiro (1985), we assume that firm A incurs a fixed cost of compatibility, $C(\sigma_x \lambda)$. This is assumed to depend upon the size of the compatibility benefit received by its users from each incremental user of B, and which B's users receive in turn from the incremental user of A. For simplicity, assume $C(\sigma_x \lambda) = k\sigma_x \lambda$ for $k > 0$. Firm A sets p_A and σ_x to maximize

$$\Pi_A = p_A Q_A - k\sigma_x \lambda \quad (6)$$

while firm B sets p_B to maximize

$$\Pi_B = p_B Q_B \quad (7)$$

We restrict attention to $\lambda < t$, which is required for a stable interior solution; otherwise a small exogenous shift of consumers between products results, through the network effect, in all consumers shifting.

For an interior solution, $\Psi_{j^*} = p_A$, where j^* represents the threshold consumer (i.e., $Q_A = 1 - j^*$). Assuming such a solution, using (5), and making appropriate substitutions we obtain

$$Q_A = \frac{t - p_A + p_B - (1 - \sigma_x)\lambda + 2\theta}{2[t - (1 - \sigma_x)\lambda]} \quad (8)$$

and

$$Q_B = 1 - Q_A = \frac{t - (1 - \sigma_X) \lambda + p_A - p_B - 2\theta}{2[t - (1 - \sigma_X) \lambda]} \quad (9)$$

The first-order conditions for firm A's profit maximization with respect to p_A and σ_X , respectively, are given by

$$\frac{t - (1 - \sigma_X^*) \lambda - 2p_A^* + p_B^* + 2\theta}{2[t - (1 - \sigma_X^*) \lambda]} = 0 \quad (10)$$

$$\frac{p_A^* \left(\frac{1}{2} - Q_A\right)}{[t - (1 - \sigma_X^*) \lambda]} = k \quad (11)$$

The first-order condition for firm B's problem is

$$\frac{t - (1 - \sigma_X^*) \lambda + p_A^* - 2p_B^* - 2\theta}{2[t - (1 - \sigma_X^*) \lambda]} = 0 \quad (12)$$

It is immediately clear from (11) that a corner solution is the only equilibrium when $Q_A \geq \frac{1}{2}$: firm A would like to set $\sigma_X < 0$ because the marginal benefit of compatibility at any positive level of compatibility is negative when firm A has more than half the market. Meanwhile, for $Q_A < \frac{1}{2}$, the smaller Q_A , the greater firm A wishes to set σ_X . Thus, the smaller the market share of a firm on this range, the greater its incentives for compatibility.

Solving (10) and (12) together yields

$$p_A^* = t - (1 - \sigma_X^*) \lambda + \frac{2}{3} \theta \quad (13)$$

and

$$p_B^* = t - (1 - \sigma_X^*) \lambda - \frac{2}{3} \theta \quad (14)$$

This yields the result, consistent with Farrell and Saloner (1992), that compatibility in the presence of network externalities under a differentiated product duopoly implies higher prices. Incentives to cut price to achieve greater sales through enlargement of the own-product-specific network effect are diminished the more compatible the products are. Substituting (13) and (14) into (8) provides a useful partial-reduced-form for Q_A ,

$$Q_A = \frac{t + \frac{2}{3}\theta - (1 - \sigma_X^*)\lambda}{2[t - (1 - \sigma_X^*)\lambda]} \quad (15)$$

Solving (11) explicitly for $(1 - \sigma_X^*)\lambda$ yields two roots:

$$(1 - \sigma_X^*)\lambda = t + \frac{1}{6k}\theta \left[1 \pm \sqrt{1 - 8k} \right] \quad (16)$$

As we demonstrate in the appendix, the values of σ_X^* that correspond to both roots are maxima. It is not necessary to our welfare results to determine which value of σ_X^* is preferred by firm A; we are able to proceed with (16). Substituting (16) into (13) and (14) yields the following corresponding equilibrium prices and quantities:

$$(p_A^*, p_B^*) = \left(\frac{2}{3}\theta - \frac{1}{6k}\theta \left[1 \pm \sqrt{1 - 8k} \right], -\frac{2}{3}\theta - \frac{1}{6k}\theta \left[1 \pm \sqrt{1 - 8k} \right] \right) \quad (17)$$

$$(Q_A^*, Q_B^*) = \left(\frac{1}{2} - \frac{2k}{1 \pm \sqrt{1 - 8k}}, \frac{1}{2} + \frac{2k}{1 \pm \sqrt{1 - 8k}} \right) \quad (18)$$

The equilibrium offers an interesting finding on how firm A uses compatibility over the range of interior solution: it sets σ_X as a “buffer” to keep Q_A at an optimizing level that is independent of θ . A lower level of demand will cause A to set σ_X and p_A higher (hence, p_B will be higher as well – recall that prices rise with compatibility), keeping Q_A steady at the level given in (18). Meanwhile, when demand is high enough

or low enough to correspond to a corner solution with respect to compatibility, firm A does not buffer its output. Equation (16) shows that $\sigma_X > 0$ requires $\theta < \frac{6k(\lambda-t)}{1\pm\sqrt{1-8k}} < 0$. At higher levels of demand, as inspection of (15) indicates, firm A sets $\sigma_X = 0$ and allows Q_A to vary positively with p_A . Meanwhile, $\sigma_X < 1$ requires $\theta > \frac{-6kt}{1\pm\sqrt{1-8k}}$.⁸ When demand is below this lower threshold, firm A favors full compatibility, sets $\sigma_X = 1$, and again allows Q_A to vary positively with p_A .

We now turn to the question of how the level of compatibility chosen by firm A relates to the social optimum. Define welfare as

$$W \equiv \Pi_A + \Pi_B + \int_{j^*}^1 U_A(j) dj + \int_0^{j^*} U_B(j) dj \quad (19)$$

Substituting and integrating, we obtain

$$W = \Pi_A + [2(1-\sigma_X)\lambda - t]Q_A^2 + (2\theta + t + 2\sigma_X\lambda - p_A - 2\lambda)Q_A + v - \theta + \lambda - \frac{t}{2} \quad (20)$$

Differentiating this expression with respect to σ_X , we obtain the following results:

PROPOSITION 1: Unless the costs of compatibility are very large, when the firms are the same or close to the same size, the unilateral private incentives for each firm with respect to compatibility are too low. When the firms are not close in size, the smaller firm has socially excessive incentives to seek compatibility unilaterally.

COROLLARY 1: When the firm's network is small enough for it to choose partial or full compatibility, it always overinvests in compatibility.

⁸ Note that $\frac{6k(\lambda-t)}{1\pm\sqrt{1-8k}} > \frac{-6kt}{1\pm\sqrt{1-8k}}$.

The results are essentially consistent with the findings of Katz and Shapiro (1985) that firms with large networks or good reputations are biased against compatibility, whereas those with small networks or weak reputations are biased in favor of it. In fact, we find in our simple model that any firm with a small enough network to seek some degree of compatibility is biased in favor of compatibility. This finding, which has a “Catch 22” flavor to it, has to do with a horizontal externality implicit in the use of compatibility in the “zero-sum” context of the model. A firm who seeks compatibility does so as a method of increasing the relative reservation price of consumers with respect to its product. Because increases in firm A’s sales come at the expense of firm B over the range of parameter values considered in the model, compatibility is providing a private benefit to the firm that exceeds its social benefit.

IV. Equilibrium with Nonuser-Negative Externalities

Now let us set parameters to the values proposed for case (II). (1) and (2) become:

$$U_A(j) = v + \theta - t(1-j) - p_A \quad (21)$$

$$U_B(j) = v - \theta - tj - \lambda Q_A - p_B \quad (22)$$

Again assume v large enough that all consumers choose A or B at equilibrium prices.⁹

Combining (21) and (22) we obtain

⁹ As shall be shown, $Q_A + Q_B = 1$ implies v does not appear in the first-order conditions for A’s profit maximization. This means $\lambda^* \Big|_{Q_A + Q_B = 1}$, A’s profit-maximizing choice of λ subject to all consumers

$$\Psi_j = 2\theta + \lambda Q_A - t(1-2j) + p_B \quad (23)$$

as the consumer's reservation price for A, relative to B.

Note that, if $\lambda > 0$, the consumer's relative reservation price for A increases with Q_A . Therefore, a network externality exists for A if $\lambda > 0$. In other words, a negative externality that selectively affects nonusers fosters a network externality.

Firm A incurs a fixed cost of *incompatibility*, $C(\lambda)$, which depends upon the size of the incompatibility cost imposed on product B's users by each incremental user of A.

For simplicity, assume $C(\lambda) = k\lambda$ for $k, \lambda > 0$.¹⁰

Firm A therefore sets p_A and λ to maximize

$$\Pi_A = p_A Q_A - k\lambda \quad (24)$$

while, as in the previous case, firm B sets p_B to maximize (7).

For an interior solution, $\Psi_{j^*} = p_A$, where j^* represents the threshold consumer (i.e., $Q_A = 1 - j^*$). Assuming such a solution, using (23), and making appropriate substitutions we obtain

$$Q_A = \frac{t - p_A + p_B + 2\theta}{2t - \lambda} \quad (25)$$

and

$$Q_B = 1 - Q_A = \frac{t - \lambda + p_A - p_B - 2\theta}{2t - \lambda} \quad (26)$$

choosing to purchase A or B, is a function of exogenous parameters other than v . Accordingly,

$\bar{v} \equiv \theta + t + \lambda \Big|_{Q_A + Q_B = 1} + \varepsilon$ satisfies the requirement for arbitrarily small $\varepsilon > 0$.

¹⁰ Nagler (2008) assumes a convex cost of incompatibility, with a linear, increasing marginal cost to enlarging the negative externality. The structure used here simplifies the equilibrium solution, but does not have a significant impact on the main results.

The first-order conditions for A's and B's profit maximization, respectively, are given by

$$\frac{t - 2p_A^* + p_B^* + 2\theta}{2t - \lambda^*} = 0 \quad (27)$$

$$\frac{p_A^* Q_A}{2t - \lambda^*} = k \quad (28)$$

and

$$\frac{t - \lambda^* + p_A^* - 2p_B^* - 2\theta}{2t - \lambda^*} = 0 \quad (29)$$

Solving (27) and (29) together yields

$$p_A^* = t + \frac{2}{3}\theta - \frac{1}{3}\lambda^* \quad (30)$$

$$p_B^* = t - \frac{2}{3}\theta - \frac{2}{3}\lambda^* \quad (31)$$

Comparing (30) and (31) to (13) and (14), one is struck by the similarity of the equations. With σ_x set to zero, the equations are identical, but for the coefficients on λ^* . Thus, in the current case, we obtain a pricing result that is the precise flipside to the result in the previous case: *incompatibility implies lower prices*. In both the case of user-positive externalities and nonuser-negative externalities, the price effect is proportional to the size of the network effect.

However, if one compares the price differential in the current case with the differential in the previous case, an important difference emerges. With user-positive externalities, the price differential between the products is independent of the network effect. This follows naturally from the mutuality of the network effect. But with nonuser-negative externalities, the price premium for product A *increases* with the network effect. Because this case involves a negative externality imposed unilaterally, the “victimized” product, B, is in effect degraded relative to the imposing product, A.

This will have important implications for the social optimality of the privately chosen level of incompatibility.

We now determine the equilibrium value of λ and corresponding equilibrium prices and quantities. Substituting (30) and (31) into (25) obtains

$$Q_A = \frac{t + \frac{2}{3}\theta - \frac{1}{3}\lambda^*}{2t - \lambda^*} \quad (32)$$

and substituting this into (28) yields

$$\lambda^* = 2t - \frac{\frac{1}{3}(t + 2\theta)}{\sqrt{k} - \frac{1}{3}} \quad (33)$$

Note that an interior solution in quantities requires $\lambda < 2t$, hence $k < \frac{1}{9}$ for $\theta < -\frac{t}{2}$, and

$k > \frac{1}{9}$ for $\theta > -\frac{t}{2}$. Moreover, $\lambda > 0$ requires $\frac{t + 2\theta}{\sqrt{k} - \frac{1}{3}} < 6t$. Thus, observing what

happens as k approaches $\frac{1}{9}$ in (33), it becomes evident that $k > \frac{1}{9}$ implies a corner solution of $\lambda = 0$ for all $\theta < -\frac{t}{2}$, and $k < \frac{1}{9}$ implies $\lambda = 0$ for all $\theta > -\frac{t}{2}$.

Does λ^* given in (33) represent a maximum? Using (30) and (32), we may re-write the first derivative of A's profit function with respect to λ as

$$\frac{\partial \Pi_A}{\partial \lambda} = Q_A^2 - k \quad (34)$$

The second derivative is therefore

$$\frac{\partial^2 \Pi_A}{\partial \lambda^2} = 2Q_A \frac{\partial Q_A}{\partial \lambda} \quad (35)$$

where, using (32),

$$\frac{\partial Q_A}{\partial \lambda} = \frac{\frac{1}{3}(t + 2\theta)}{(2t - \lambda^*)^2} \quad (36)$$

Successive substitution of (33) into (36) and then into (35) shows that the second derivative is positive when $\theta > -\frac{t}{2}$, and negative otherwise. When demand for product A is relatively large, the marginal revenue product of λ increases in λ , while marginal cost of λ is constant. A corner solution equilibrium is the result: Firm A's profits are maximized by setting λ large enough to achieve $Q_A = 1$ (if the cost of increasing λ to this value is small enough relative to the benefit of taking the entire market) or else setting $\lambda = 0$ (if raising λ is prohibitively costly). However, when demand for product A is relatively small, the marginal revenue product of λ decreases in λ , while marginal cost is constant. Consequently, the first-order condition for profit maximization yields a maximum.

For $\theta < -\frac{t}{2}$, the profit-maximum represented by (33) corresponds to the following prices and quantities:

$$(p_A^*, p_B^*) = \left(\frac{\frac{\sqrt{k}}{3}(t+2\theta)}{\sqrt{k}-\frac{1}{3}}, \frac{\frac{1}{3}(t+2\theta)[1-\sqrt{k}]}{\sqrt{k}-\frac{1}{3}} \right) \quad (37)$$

$$(Q_A^*, Q_B^*) = (\sqrt{k}, 1-\sqrt{k}) \quad (38)$$

As in the case of user-positive externalities, we find a “buffering” result, that is, Firm A sets λ as a buffer to keep Q_A at an optimizing level that is independent of θ . A lower level of θ causes A to set λ lower and p_A higher. Firm B raises p_B as well – recall that prices *fall* with *incompatibility* – and the price differential $p_B - p_A$, which is positive in this region of low relative demand for A, increases as λ falls. Thus, Q_A remains steady at the level given in (38).

When demand is low enough to correspond to $\lambda = 0$ in (33), to wit, when $\theta < 3t\sqrt{k} - \frac{3}{2}t$, equilibrium prices and quantities are given, respectively, by

$$(p_A^*, p_B^*) = (t + \frac{2}{3}\theta; t - \frac{2}{3}\theta) \quad (39)$$

$$(Q_A, Q_B) = (\frac{1}{2} + \frac{\theta}{3t}; \frac{1}{2} - \frac{\theta}{3t}) \quad (40)$$

In this region, λ is not available for buffering output, so firm A sets its price to allow Q_A to decline with θ .

We now turn to the question of how the level of incompatibility chosen by firm A relates to the social optimum. Define welfare as above in (19). Substituting and integrating, we obtain

$$W = \Pi_A + [\lambda - t]Q_A^2 + [2\theta - p_A + t - \lambda]Q_A - \frac{t}{2} + v - \theta \quad (41)$$

Differentiating with respect to λ , we obtain the following result:

PROPOSITION 2: When the imposing firm is small relative to its competitor, or when it is relatively large and the costs of incompatibility are large but not prohibitive, its incentives for incompatibility may be too low. When the imposing firm and its competitor are close in size, its incentives for incompatibility are too high, except when the costs of incompatibility are relatively large, in which case social and private incentives conform for zero incompatibility (i.e., perfect compatibility).

Table 1 summarizes more specifically the social optimality outcomes with respect to firm A's incompatibility decision in terms of the incompatibility cost parameter, k , and

relative demand parameter, θ . As with Proposition 2, these results are derived in the appendix.

*** INSERT TABLE 1 APPROXIMATELY HERE ***

The intuition of the results for nonuser-negative externalities can be seen from the car and sport-utility vehicle case example. When demands for cars and SUVs are relatively close in size, the SUV manufacturer's incentives for incompatibility may be excessive. Making SUVs more hazardous to car drivers provides maximum benefit to the SUV manufacturer when the network sizes for the two vehicle types are near equal because the effect on SUV sales at the margin is greatest. However, the social cost of vehicle incompatibility is also highest in this situation, since the probability of deadly car versus SUV accidents is greatest when cars and SUVs coexist on the road in near equal numbers (White, 2004).

Meanwhile, when SUVs significantly outnumber cars, the manufacturer's incentives for incompatibility may be too low. This is because manufacturers fail to account for the social benefit that SUV-imposed external costs have of increasing homogeneity of the product mix, so that the incidence of car versus SUV accidents is reduced. Similarly, SUV firms' incentives for incompatibility are too low when cars significantly outnumber SUVs. In this situation, the increase in the price differential between SUVs and cars has a negative effect on SUV sales that outstrips the positive network effect. So, though SUVs are made more dangerous, the number of SUVs declines sufficiently to increase welfare overall. In both cases of lopsided network size, the manufacturer considers mainly the marginal effect of incompatibility on his sales, and this is smaller the more lopsided the network sizes are.

Though not exact, there is a strong correspondence between the results we obtained with respect to user-positive externalities and those that arise under nonuser-negative externalities. The clearest correspondence exists for firms with relatively low demand (i.e., small networks). We observe under nonuser-negative externalities that such firms have suboptimal incentives for incompatibility from a social welfare perspective, just as firms with small networks had excessive incentives for compatibility under user-positive externalities. When the two firms are close in size, the results also conform in most cases. When $k < \frac{2\sqrt{7}+1}{27}$ and $\theta \in \left(\frac{9tk}{2}, t\left[\frac{-1+2\sqrt{7+27k}}{6}\right]\right)$, firm A sets λ too high. Thus, under nonuser-negative externalities, a firm's incentives for incompatibility may be excessive for moderate levels of relative demand, so long as the costs of incompatibility are not too large. This corresponds to the case of moderate demand under user-positive externalities, in which private incentives for compatibility are too low.

Interestingly, we find a departure in our results on nonuser-negative externalities with respect to Katz and Shapiro's (1985) findings regarding firms with large networks. While Katz and Shapiro find that firms with large networks or good reputations tend to be biased against compatibility, we find that they might be biased against *in*compatibility. Specifically, for $k \in \left(\frac{2\sqrt{7}+1}{27}, \frac{1}{3}\right)$, when $\theta \in \left(t\left[\frac{-1+2\sqrt{7+27k}}{6}\right], \frac{9tk}{2}\right)$, firm A sets λ too low. The same thing happens for $k \in \left(\frac{1}{3}, \frac{2}{3}\right)$ when $\theta \in \left(t\left[\frac{-1+2\sqrt{7+27k}}{6}\right], \frac{3t}{2}\right)$.

V. Conclusion

Previous analyses of incentives for compatibility in the context of network externalities have focused on the case of user-positive externalities. The results of these studies have suggested that firms undervalue the utility that inframarginal customers gain from having a product that is compatible with products used by others. They focus excessively on compatibility as a tool to win over marginal customers. User-positive externalities are conferred mutually by consumers on each other in pairings of compatible products. For this reason, it would appear that the tendency to overweight marginal customers' utility is what causes firms with large or medium-sized networks to choose too little compatibility while firms with small networks choose too much. Small firms seem to gain more from the mutual exchange of benefits.

This paper has suggested that mutuality is not the reason, or not the only reason, that firms' compatibility incentives are what they are. Our simple model indicates that compatibility is still favored excessively by smaller firms when network sizes are lopsided, and sought insufficiently by firms when they are close in size, even when network externalities arise from unilateral, rather than mutual, external impacts.

The general implication is that public policy has a role in encouraging compatibility when competing products have near-equal network sizes. This is true not only when user-positive externalities exist, but also when external costs are imposed by users on non-users. Conversely, policy makers may need to dampen unilateral private incentives for compatibility when network sizes are lopsided. The surprising thing is that this may actually mean encouraging firms to impose larger external costs that selectively affect rivals' products! For example, if SUVs represented a small enough share of the motor vehicle market, it might actually improve welfare to make them more hazardous to

car drivers, because the price effects of doing would further curtail sales of SUVs. If instead the overwhelming majority of vehicles were SUVs, making them more hazardous would improve welfare by reducing further the number of car drivers that incur incompatibility losses due to SUVs. In both cases, increased incompatibility at a per-unit level improves welfare by increasing standardization and thereby reducing the adverse effects of incompatibility at an aggregate level.

Beyond pure compatibility considerations, the broader implications of our results for public policy are perhaps equally surprising. The wisdom that external costs are provided excessively in the market and should be reduced is called into question when one considers that, in many cases, such costs have implications for the competitive equilibrium in markets.¹¹ Situations involving user-imposed externalities should be scrutinized to consider whether the externalities selectively, or asymmetrically, affect non-users (i.e., are nonuser-negative). The desirability of certain policy prescriptions, such as the use of Pigouvian taxes, might be affected by such asymmetries.

Appendix

A1. Second Order Conditions – Positive Consumption Externalities Case

The Hessian in this case is given by

$$|H| = \begin{vmatrix} \frac{\partial^2 \Pi_A}{\partial p_A^2} & \frac{\partial^2 \Pi_A}{\partial p_A \partial \sigma_X} \\ \frac{\partial^2 \Pi_A}{\partial p_A \partial \sigma_X} & \frac{\partial^2 \Pi_A}{\partial \sigma_X^2} \end{vmatrix} \quad (42)$$

¹¹ This issue is explored directly by Nagler (2008).

where, using (8), (13), (14), and the first-order condition $Q_A + p_A \frac{\partial Q_A}{\partial p_A} = 0$, the components are given by

$$\begin{aligned} \frac{\partial^2 \Pi_A}{\partial \sigma_X^2} &= p_A \frac{\partial^2 Q_A}{\partial \sigma_X^2} = p_A \frac{\partial}{\partial \sigma_X} \frac{2[t-(1-\sigma_X)\lambda]\lambda - 2\lambda[t-p_A+p_B-(1-\sigma_X)\lambda+2\theta]}{4[t-(1-\sigma_X)\lambda]^2} \\ &= p_A \frac{2\lambda^2 \left\{ \frac{2}{3}\theta \right\}}{2[t-(1-\sigma_X)\lambda]^3} \end{aligned} \quad (43)$$

$$\frac{\partial^2 \Pi_A}{\partial p_A^2} = 2 \frac{\partial Q_A}{\partial p_A} + p \frac{\partial^2 Q_A}{\partial p_A^2} = 2 \left[\frac{-1}{2[t-(1-\sigma_X)\lambda]} \right] + p[0] = \frac{-1}{[t-(1-\sigma_X)\lambda]} \quad (44)$$

$$\frac{\partial^2 \Pi_A}{\partial p_A \partial \sigma_X} = \frac{\partial Q_A}{\partial \sigma_X} + p_A \frac{\partial^2 Q_A}{\partial p_A \partial \sigma_X} = \frac{\lambda \left(\frac{1}{2} - Q_A \right)}{[t-(1-\sigma_X^*)\lambda]} + p_A \frac{-\lambda \frac{\partial Q_A}{\partial p_A}}{[t-(1-\sigma_X^*)\lambda]} = \frac{\lambda}{2[t-(1-\sigma_X^*)\lambda]} \quad (45)$$

Substitution into (42) yields

$$\begin{aligned} |H| &= \begin{vmatrix} \frac{-1}{[t-(1-\sigma_X)\lambda]} & \frac{\lambda}{2[t-(1-\sigma_X^*)\lambda]} \\ \frac{\lambda}{2[t-(1-\sigma_X^*)\lambda]} & p_A \frac{2\lambda^2 \left\{ \frac{2}{3}\theta \right\}}{2[t-(1-\sigma_X)\lambda]^3} \end{vmatrix} \\ &= \frac{-\lambda^2 \left\{ \frac{4}{3}\theta + t - (1-\sigma_X^*)\lambda \right\}^2}{4[t-(1-\sigma_X)\lambda]^4} < 0 \end{aligned} \quad (46)$$

So all solutions to the first-order conditions are maxima.

A2. Proof of Proposition 1 and Corollary 1

Differentiate (20) with respect to σ_X , use (13) and (15), and assume an interior solution:

$$\begin{aligned}
\frac{\partial W}{\partial \sigma_X} &= \frac{\partial \Pi_A}{\partial \sigma_X} + \frac{\partial}{\partial [(1-\sigma_X)\lambda]} \left\{ [2(1-\sigma_X)\lambda - t] Q_A^2 \right. \\
&\quad \left. + (2\theta - p_A - [2(1-\sigma_X)\lambda - t]) Q_A \right\} \cdot \frac{\partial [(1-\sigma_X)\lambda]}{\partial \sigma_X} \\
&= 0 + \left\{ 2Q_A^2 + 2[2(1-\sigma_X)\lambda - t] Q_A \frac{\partial Q_A}{\partial [(1-\sigma_X)\lambda]} \right. \\
&\quad \left. - \left[2 + \frac{\partial p_A}{\partial [(1-\sigma_X)\lambda]} \right] Q_A + (2\theta - p_A - [2(1-\sigma_X)\lambda - t]) \frac{\partial Q_A}{\partial [(1-\sigma_X)\lambda]} \right\} \cdot -\lambda \\
&= \left\{ 2Q_A^2 + 2[2(1-\sigma_X)\lambda - t] Q_A \left[\frac{-2[t-(1-\sigma_X)\lambda] + 2p_A}{4[t-(1-\sigma_X)\lambda]^2} \right] \right. \\
&\quad \left. - [2 + -1] Q_A + (2\theta - p_A - [2(1-\sigma_X)\lambda - t]) \left[\frac{-2[t-(1-\sigma_X)\lambda] + 2p_A}{4[t-(1-\sigma_X)\lambda]^2} \right] \right\} \cdot -\lambda \\
&= \left\{ 2Q_A^2 - Q_A + \{(2Q_A - 1)[2(1-\sigma_X)\lambda - t] + 2\theta - p_A\} \left[\frac{-1}{2[t-(1-\sigma_X)\lambda]} + \frac{p_A}{2[t-(1-\sigma_X)\lambda]^2} \right] \right\} \cdot -\lambda \\
&= \left\{ 2Q_A(Q_A - \frac{1}{2}) + \frac{\{(2Q_A - 1)[2(1-\sigma_X)\lambda - t] + 2\theta - p_A\} (Q_A - \frac{1}{2})}{[t - (1-\sigma_X)\lambda]} \right\} \cdot -\lambda \\
&= -\lambda(Q_A - \frac{1}{2}) \left\{ 2Q_A + \frac{2Q_A[2(1-\sigma_X)\lambda - t]}{t - (1-\sigma_X)\lambda} + \frac{2\theta - p_A - 2(1-\sigma_X)\lambda + t}{t - (1-\sigma_X)\lambda} \right\} \\
&= -\lambda(Q_A - \frac{1}{2}) \left\{ \frac{2Q_A(1-\sigma_X)\lambda}{t - (1-\sigma_X)\lambda} + \frac{2\theta - p_A - 2(1-\sigma_X)\lambda + t}{t - (1-\sigma_X)\lambda} \right\} \\
&= -\lambda(Q_A - \frac{1}{2}) \left\{ \frac{2 \left\{ \frac{t + \frac{2}{3}\theta - (1-\sigma_X^*)\lambda}{2[t-(1-\sigma_X^*)\lambda]} \right\} (1-\sigma_X^*)\lambda}{t - (1-\sigma_X^*)\lambda} + \frac{2\theta - [t + \frac{2}{3}\theta - (1-\sigma_X^*)\lambda] - 2(1-\sigma_X^*)\lambda + t}{t - (1-\sigma_X^*)\lambda} \right\} \\
&= -\lambda(Q_A - \frac{1}{2}) \left\{ \frac{2(1-\sigma_X^*)\lambda \{t + \frac{2}{3}\theta - (1-\sigma_X^*)\lambda\}}{2[t - (1-\sigma_X^*)\lambda]^2} + \frac{\frac{4}{3}\theta - (1-\sigma_X^*)\lambda}{t - (1-\sigma_X^*)\lambda} \right\} \\
&= -\lambda(Q_A - \frac{1}{2}) \left\{ \frac{\frac{4}{3}\theta(1-\sigma_X^*)\lambda}{2[t - (1-\sigma_X^*)\lambda]^2} + \frac{\frac{4}{3}\theta}{[t - (1-\sigma_X^*)\lambda]} \right\} \\
&= -\frac{\frac{4}{3}\theta\lambda(Q_A - \frac{1}{2})}{2[t - (1-\sigma_X^*)\lambda]^2} [2t - (1-\sigma_X^*)\lambda] \tag{47}
\end{aligned}$$

Since an interior solution requires $\theta < 0$ and $Q_A < \frac{1}{2}$, it follows that $\frac{\partial W}{\partial \sigma_X} < 0$ for all interior solutions. Thus, whenever firm A's network size is small enough that it chooses at least partial compatibility, it overinvests in compatibility.

Now we consider the corner solution corresponding to $\sigma_X = 0$. We begin by noting that $\frac{\partial \Pi_A}{\partial \sigma_X} \neq 0$ at $\sigma_X = 0$; therefore we may substitute (47) in for $\frac{\partial W}{\partial \sigma_X}$, but we must add $\frac{\partial \Pi_A}{\partial \sigma_X}$ back in. We do so and evaluate the resulting expression at $\sigma_X = 0$:

$$\begin{aligned}
\frac{\partial W}{\partial \sigma_X} &= p_A \frac{\partial Q_A}{\partial \sigma_X} + Q_A \frac{\partial p_A}{\partial \sigma_X} - k\lambda - \frac{\frac{4}{3}\theta\lambda(Q_A - \frac{1}{2})}{2[t - (1 - \sigma_X^*)\lambda]^2} [2t - (1 - \sigma_X^*)\lambda] \\
&= -\lambda p_A \frac{-2[t - (1 - \sigma_X)\lambda] + 2p_A}{4[t - (1 - \sigma_X)\lambda]^2} + Q_A \lambda - k\lambda - \frac{\frac{4}{3}\theta\lambda(Q_A - \frac{1}{2})}{2[t - (1 - \sigma_X^*)\lambda]^2} [2t - (1 - \sigma_X^*)\lambda] \\
&= -\lambda p_A \frac{p_A - [t - \lambda]}{2[t - \lambda]^2} + \frac{p_A}{2(t - \lambda)} \lambda - k\lambda - \frac{\frac{4}{3}\theta\lambda(\frac{p_A}{2(t - \lambda)} - \frac{1}{2})}{2[t - \lambda]^2} [2t - \lambda] \\
&= \frac{1}{2(t - \lambda)^2} \left[\lambda(t - \lambda + \frac{2}{3}\theta)(t - \lambda - \frac{2}{3}\theta) - 2k\lambda(t - \lambda)^2 - \frac{\frac{4}{9}\theta^2\lambda(2t - \lambda)}{(t - \lambda)} \right] \\
&= \frac{\lambda}{2(t - \lambda)^2} \left[(t - \lambda)^2 - \frac{4}{9}\theta^2 - 2k(t - \lambda)^2 - \frac{\frac{4}{9}\theta^2(2t - \lambda)}{(t - \lambda)} \right] \\
&= \frac{\lambda}{2(t - \lambda)^3} \left[-\frac{4}{9}\theta^2(t - \lambda) + (1 - 2k)(t - \lambda)^3 - \frac{4}{9}\theta^2(2t - \lambda) \right] \\
&= \frac{\lambda}{2(t - \lambda)^3} \left[(1 - 2k)(t - \lambda)^3 - \frac{4}{9}\theta^2(3t - 2\lambda) \right]
\end{aligned} \tag{48}$$

So long as $k \leq \frac{1}{2}$, $\frac{\partial W}{\partial \sigma_X} > 0$ for θ close to zero.

A3. Proof of Proposition 2 and Derivation of Table 1

To begin, let us differentiate (41) with respect to λ , assume an interior solution (i.e., $\theta \in (-\frac{3t}{2}, -\frac{t}{2})$ and $k < \frac{1}{9}$),

$$\begin{aligned}
W &= \Pi_A + [\lambda - t]Q_A^2 + [2\theta - p_A + t - \lambda]Q_A - \frac{t}{2} + v - \theta \\
\Rightarrow \frac{\partial W}{\partial \lambda} &= 0 + 2[\lambda - t]Q_A \frac{\partial Q_A}{\partial \lambda} + Q_A^2 - \left(\frac{\partial p_A}{\partial \lambda} + 1\right)Q_A + [2\theta - p_A + t - \lambda] \frac{\partial Q_A}{\partial \lambda}
\end{aligned} \tag{49}$$

Using (30) and (32), we find

$$\frac{\partial p_A}{\partial \lambda} = -\frac{1}{3}; \frac{\partial Q_A}{\partial \lambda} = \frac{Q_A - \frac{1}{3}}{2t - \lambda} \quad (50)$$

Substituting (30), (32), and (50) into (49) and factoring yields

$$\frac{\partial W}{\partial \lambda} = \frac{-\frac{1}{3}(2t - \lambda)^3 - \frac{4}{3}t(2\theta + t)(2t - \lambda) + \frac{1}{3}(4t - \lambda)(2\theta + t)^2}{3(2t - \lambda)^3} \quad (51)$$

Since $\lambda < 2t$ on $0 < Q_A < 1$, hence on c , it follows that $\frac{\partial W}{\partial \lambda} < 0$ at $\theta = -\frac{t}{2}$. If we can show that $\frac{\partial W}{\partial \lambda} > 0$ at $\theta = -\frac{3t}{2}$, then we will have proven that there exists $\underline{\theta}(k) \in (-\frac{3t}{2}, -\frac{t}{2})$ such that, for $\theta < \underline{\theta}$, firm A sets λ too low. In the neighborhood of $\theta = -\frac{3t}{2}$, $\frac{\partial \Pi_A}{\partial \lambda}$ approaches $-k$. So, using (51), and substituting in $\theta = -\frac{3t}{2}$ and $\lambda = 0$, we obtain:

$$\begin{aligned} \frac{\partial W}{\partial \lambda} &= \frac{\partial \Pi_A}{\partial \lambda} + \frac{-\frac{1}{3}(2t - \lambda)^3 - \frac{4}{3}t(2\theta + t)(2t - \lambda) + \frac{1}{3}(4t - \lambda)(2\theta + t)^2}{3(2t - \lambda)^3} \\ &= -k + \frac{-\frac{1}{3}(2t - \lambda)^3 - \frac{4}{3}t(2\theta + t)(2t - \lambda) + \frac{1}{3}(4t - \lambda)(2\theta + t)^2}{3(2t - \lambda)^3} \\ &= -k + \frac{-\frac{8}{3}t^3 - \frac{4}{3}t(-2t)(2t) + \frac{1}{3}(4t)(-2t)^2}{3(2t)^3} \\ &= -k + \frac{-\frac{8}{3}t^3 + \frac{16}{3}t^3 + \frac{16}{3}t^3}{24t^3} = -k + \frac{24}{24t^3}t^3 = -k + \frac{1}{3} \end{aligned} \quad (52)$$

So, $\frac{\partial W}{\partial \lambda} > 0$ if and only if $k < \frac{1}{3}$. This satisfies the interior solution requirement of $k < \frac{1}{9}$,

so we have proven the first part for this case.

Now consider $\theta \in (-\frac{3t}{2}, -\frac{t}{2})$ with $k \in (\frac{1}{9}, \frac{1}{3})$. In this case, k is sufficiently large that a corner solution of $\lambda = 0$ holds for all $\theta < -\frac{t}{2}$. Note that $\frac{\partial \Pi_A}{\partial \lambda} \neq 0$ at $\lambda = 0$; therefore we may substitute (53) in for $\frac{\partial W}{\partial \lambda}$, but we must add $\frac{\partial \Pi_A}{\partial \sigma_x}$ back in. We do so and evaluate the resulting expression at $\lambda = 0$, simplifying:

$$\begin{aligned}
\frac{\partial W}{\partial \lambda} &= p_A \frac{\partial Q_A}{\partial \lambda} + Q_A \frac{\partial p_A}{\partial \lambda} - k + \frac{-\frac{1}{3}(2t-\lambda)^3 - \frac{4}{3}t(2\theta+t)(2t-\lambda) + \frac{1}{3}(4t-\lambda)(2\theta+t)^2}{3(2t-\lambda)^3} \\
&= p_A \left(\frac{Q_A - \frac{1}{3}}{2t-\lambda} \right) - \frac{1}{3}Q_A - k + \frac{-\frac{1}{3}(2t-\lambda)^3 - \frac{4}{3}t(2\theta+t)(2t-\lambda) + \frac{1}{3}(4t-\lambda)(2\theta+t)^2}{3(2t-\lambda)^3} \\
&= \left(\frac{t + \frac{2}{3}\theta - \frac{1}{3}\lambda}{2t-\lambda} \right)^2 - \frac{2}{3} \left(\frac{t + \frac{2}{3}\theta - \frac{1}{3}\lambda}{2t-\lambda} \right) - k \\
&\quad + \frac{-\frac{1}{3}(2t-\lambda)^3 - \frac{4}{3}t(2\theta+t)(2t-\lambda) + \frac{1}{3}(4t-\lambda)(2\theta+t)^2}{3(2t-\lambda)^3} \\
&= \left(\frac{t + \frac{2}{3}\theta}{2t} \right)^2 - \frac{2}{3} \left(\frac{t + \frac{2}{3}\theta}{2t} \right) - k + \frac{-\frac{1}{3}(2t)^3 - \frac{4}{3}t(2\theta+t)(2t) + \frac{1}{3}(4t)(2\theta+t)^2}{3(2t)^3} \\
&= \frac{6t(t + \frac{2}{3}\theta)^2}{3(2t)^3} - \frac{8t^2(t + \frac{2}{3}\theta)}{3(2t)^3} + \frac{-3k(2t)^3 - \frac{1}{3}(2t)^3 - \frac{4}{3}t(2\theta+t)(2t) + \frac{1}{3}(4t)(2\theta+t)^2}{3(2t)^3} \\
&= \frac{\frac{8}{3}\theta t - 24kt^2 - 6t^2 + 8\theta^2}{24t^2} \tag{54}
\end{aligned}$$

Differentiating this expression with respect to θ reveals that $\frac{\partial W}{\partial \lambda}$ is monotone in θ on

$$\theta \in \left(-\frac{3t}{2}, -\frac{t}{2} \right):$$

$$\frac{\partial^2 W}{\partial \lambda \partial \theta} = \frac{\frac{8}{3}t + 16\theta}{24t^2} = 0 \Rightarrow \theta = -\frac{t}{6} > -\frac{t}{2} \tag{55}$$

It remains to check the sign of $\frac{\partial W}{\partial \lambda}$ at the endpoints of the interval. At $\theta = -\frac{3t}{2}$,

$$\frac{\partial W}{\partial \lambda} = \frac{1}{3} - k, \text{ which is positive for } k < \frac{1}{3}. \text{ At } \theta = -\frac{t}{2}, \text{ it can shown using (54) that } \frac{\partial W}{\partial \lambda} < 0.$$

Therefore, for $k \in \left(\frac{1}{9}, \frac{1}{3} \right)$, there exists $\underline{\theta}(k) \in \left(-\frac{3t}{2}, -\frac{t}{2} \right)$ such that, for $\theta < \underline{\theta}$, firm A sets

λ too low. (It may be observed in passing that $k > \frac{1}{3}$ implies $\frac{\partial W}{\partial \lambda} < 0$ everywhere on

$\theta \in \left(-\frac{3t}{2}, -\frac{t}{2} \right)$, so private and social incentives conform for setting $\lambda = 0$ when

$\theta \in \left(-\frac{3t}{2}, -\frac{t}{2} \right)$ and $k > \frac{1}{3}$.)

Now consider $\theta \in \left(-\frac{t}{2}, \frac{3t}{2}\right)$. As noted in the text, this range corresponds to a corner solution in λ : firm A sets λ to achieve $Q_A = 1$ when the cost of increasing λ is small enough, and it sets $\lambda = 0$ otherwise. Using (32), we find that $\lambda = \frac{3}{2}t - \theta$ corresponds to $Q_A = 1$. Using (24) and comparing firm A's profits at $\lambda = 0$ with its profits when $Q_A = 1$ and $\lambda = \frac{3}{2}t - \theta$, we find that firm A will opt for $Q_A = 1$ when $k < \frac{2\theta}{9t}$, or, rearranging, when $\theta > \frac{9}{2}tk$. Thus, we are able to recast firm A's threshold in terms of a level of demand large enough to make increasing λ worthwhile.

The corresponding social threshold in θ for raising λ to set $Q_A = 1$ is derived by substituting $\lambda = 0$ into $\frac{\partial W}{\partial \lambda} = 0$ and solving for θ . Setting (54) equal to zero and solving for θ yields (using quadratic formula) $\theta = t \left[\frac{-1 \pm 2\sqrt{7+27k}}{6} \right]$. Here, only the positive-signed root corresponds to $\theta \in \left(-\frac{t}{2}, \frac{3t}{2}\right)$, so that is the relevant one. One can check that $\theta > t \left[\frac{-1+2\sqrt{7+27k}}{6} \right]$ corresponds to $\frac{\partial W}{\partial \lambda} > 0$ in (54).

Equating the social and private thresholds and solving for k :

$$t \left[\frac{-1+2\sqrt{7+27k}}{6} \right] = \frac{9tk}{2} \Rightarrow k = \frac{1+2\sqrt{7}}{27} \quad (56)$$

where only the positive-signed root corresponds to $k > 0$ and is therefore relevant. Thus, for $k > \frac{1+2\sqrt{7}}{27}$, the private threshold level of θ exceeds the social threshold, so that for $\theta \in \left(t \left[\frac{-1+2\sqrt{7+27k}}{6} \right], \frac{3t}{2}\right)$, firm A sets λ too low, while for $\theta \in \left(-\frac{t}{2}, t \left[\frac{-1+2\sqrt{7+27k}}{6} \right]\right)$, firm A's incentives conform with social incentives for setting $\lambda = 0$. Meanwhile, for $k < \frac{1+2\sqrt{7}}{27}$, the social threshold exceeds the private threshold, so that for $\theta \in \left(\frac{9tk}{2}, t \left[\frac{-1+2\sqrt{7+27k}}{6} \right]\right)$, firm

A sets λ too high. When $\theta \in \left(-\frac{t}{2}, \frac{9tk}{2}\right)$, $k < \frac{1+2\sqrt{7}}{27}$ corresponds to firm A's incentives conforming with social incentives for setting $\lambda = 0$. Finally, $\theta \in \left(t \left[\frac{-1+2\sqrt{7+27k}}{6} \right], \frac{3t}{2}\right)$ and $k < \frac{1+2\sqrt{7}}{27}$ imply that private and social incentives conform for setting $\lambda = \frac{3}{2}t - \theta$ and $Q_A = 1$.

References

- Akerlof, G.A., Dickens, W.T., 1982. The economic consequences of cognitive dissonance. *American Economic Review*, 72, 307-319.
- Ayres, I., Levitt., S.D., 1998. Measuring positive externalities from unobservable victim precaution. *The Quarterly Journal of Economics*, 113:1, 43-77.
- Beh, H., 1989. The effect of passive smoking on vigilance performance. *Ergonomics*, 32:10, 1227-1236.
- Bradsher, K., 2002. High and mighty: SUVs – the world's most dangerous vehicles and how they got that way. New York: Public Affairs.
- Davis, K.E., Jones, E.E., 1960. Changes in interpersonal perception as a means of reducing cognitive dissonance. *Journal of Abnormal and Social Psychology*, 61:3, 402-410.
- Economides, N., 1996a. Network externalities, complementarities, and invitations to enter. *European Journal of Political Economy*, 12, 211-232.
- Economides, N., 1996b. The economics of networks. *International Journal of Industrial Organization*, 14, 673-699.
- Farrell, J., Saloner, G., 1986. Installed base and compatibility: innovation, product preannouncements, and predation. *American Economic Review*, 76:5, 940-955.
- Farrell, J., Saloner, G., 1992. Converters, compatibility, and control of interfaces. *Journal of Industrial Economics*, 40:1, 9-35.
- Festinger, L., Carlsmith, J. M., 1959. Cognitive consequences of forced compliance. *Journal of Abnormal and Social Psychology*, 58, 203-210.

- Georgetown Law Journal, 1970. Blockbusting. 59, 170-189.
- Gilbert, R.J., 1992. Symposium on compatibility: incentives and market structure. *Journal of Industrial Economics*, 40, 1-8.
- Glass, D., 1964. Changes in liking as a means of reducing cognitive discrepancies between self-esteem and aggression. *Journal of Personality*, 32, 531-549.
- Katz, M.L., Shapiro, C., 1985. Network externalities, competition, and compatibility. *American Economic Review*, 75, 424-440.
- Katz, M.L., Shapiro, C., 1992. Product introduction with network externalities. *Journal of Industrial Economics*, 40:1, 55-83.
- Latin, H., Kasolas, B., 2002. Bad designs, lethal products: the duty to protect other motorists against SUV collision risks. *Boston University Law Review*, 82, 1161-1229.
- Matutes, C., Regibeau, P., 1988. Mix and match: product compatibility without externalities. *RAND Journal of Economics*, 19, 221-234.
- Matutes, C., Regibeau, P., 1992. Compatibility and bundling of complementary goods in a duopoly. *Journal of Industrial Economics*, 40, 37-54.
- Nagler, M.G., 2008. If you can't beat 'em: negative externalities, consumer choice, and welfare. Working paper, The City College of New York.
- White, M.J., 2004. The "arms race" on American roads: the effect of sport utility vehicles and pickup trucks on traffic safety. *Journal of Law and Economics*, 47, 333-355.

Table 1. Summary of social optimality outcomes for firm A's incompatibility decision.

	$\theta < -\frac{3t}{2}$	$\theta \in \left(-\frac{3t}{2}, -\frac{t}{2}\right)$	$\theta \in \left(-\frac{t}{2}, \frac{3t}{2}\right)$	$\theta > \frac{3t}{2}$
$k \in \left(0, \frac{1+2\sqrt{7}}{27}\right)$	Private and social incentives conform for $\lambda = 0$.	There exists $\underline{\theta}(k) \in \left(-\frac{3t}{2}, -\frac{t}{2}\right)$ such that for $\theta < \underline{\theta}$ firm A sets λ too low .	$\theta \in \left(-\frac{t}{2}, \frac{9tk}{2}\right)$: private and social incentives conform for $\lambda = 0$. $\theta \in \left(\frac{9tk}{2}, t \left[\frac{-1+2\sqrt{7+27k}}{6} \right] \right)$: firm A sets λ too high . $\theta \in \left(t \left[\frac{-1+2\sqrt{7+27k}}{6} \right], \frac{3t}{2} \right)$: private and social incentives conform for $Q_A = 1$.	Private and social incentives conform for $\lambda = 0$. ($Q_A = 1$ regardless.)
$k \in \left(\frac{1+2\sqrt{7}}{27}, \frac{1}{3}\right)$			$\theta \in \left(-\frac{t}{2}, t \left[\frac{-1+2\sqrt{7+27k}}{6} \right] \right)$: private and social incentives conform for $\lambda = 0$. $\theta \in \left(t \left[\frac{-1+2\sqrt{7+27k}}{6} \right], \frac{9tk}{2} \right)$: firm A sets λ too low . $\theta \in \left(\frac{9tk}{2}, \frac{3t}{2}\right)$: private and social incentives conform for $Q_A = 1$.	
$k \in \left(\frac{1}{3}, \frac{2}{3}\right)$		Private and social incentives conform for $\lambda = 0$.	$\theta \in \left(-\frac{t}{2}, t \left[\frac{-1+2\sqrt{7+27k}}{6} \right] \right)$: private and social incentives conform for $\lambda = 0$. $\theta \in \left(t \left[\frac{-1+2\sqrt{7+27k}}{6} \right], \frac{3t}{2} \right)$: firm A sets λ too low .	
$k > \frac{2}{3}$			Private and social incentives conform for $\lambda = 0$.	