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Multi-homing and Complementors**

Juan D. Carrillo and Guofu Tan  
University of Southern California

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# Platform Competition: The Role of Multi-homing and Complementors

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## Abstract

In this paper we present a model of platform competition in which two firms offer horizontally differentiated platforms and a group of complementors offers products that are complementary to each platform. Consumers can buy either or both platforms (single- or multi-homing) and complementors can produce for either or both platforms (single- or multi-production). We first characterize the pricing structure and find that, in equilibrium, consumers are more likely to multi-home as the differentiation of platforms decreases or as the number of complementors for either platform increases. We show that the platform and its complementors always benefit from an increase in the number of complementors in their own platform. When single-homing arises in equilibrium, the platform and its complementors suffer from an increase in the number of complementors in the rival platform. We also study the incentives of the platform to integrate with its complementors, to charge them a royalty or give a subsidy, and to sell its own complementary products to the rival platform.

*Keywords:* Platform competition, multi-homing, complementor, royalty and subsidy

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\*Carrillo: USC and CEPR, juandc@usc.edu. Tan: USC, guofutan@usc.edu. We thank seminar participants at Beijing University, University of Southern California, and University of Washington for helpful comments. We also thank Microsoft Corporation, the NET Institute, and USC for research support.

# 1 Introduction

In 2004 Sony and Toshiba announced their intention to release a new generation of non-compatible DVD players. Experts refer to this technological contest between Blu-Ray and HD-DVD as a modern version of the Betamax vs. VHS battle. Since the announcement, both players have devoted considerable effort into forming alliances with third parties (movie distributors, computer hardware providers, audio companies, etc.) that will enhance the attractiveness of their product.

Last November, Microsoft launched its latest video game platform together with 3 of the 18 games initially available for this console. Microsoft's business strategy has then consisted in attracting (through exclusive or non-exclusive agreements with game distributors) the most creative professionals to develop games for XBox 360.

The market structure of these two multi-billion Information Technology industries has three important similarities. First, there is platform competition between non-compatible differentiated products (Blu-Ray vs. HD-DVD or XBox vs. Playstation). Second, the platforms themselves provide little utility to consumers. In fact, a platform is mostly a means to enjoy some complementary goods (movies or games), making it only as valuable as the complementary products and services that can be accessed through it. As an immediate implication, platform providers are likely to produce also some complementors. More importantly, platforms will fiercely compete to attract the limited number of independent complementors (movie distributors or game developers) present in the market. Third, the profits of each platform and complementor depends on the prices of all the other market players, via their effect on the relative attractiveness of platforms.

This paper offers a model of platform competition that captures the main ingredients of the market structure described above. Our model allows us to address the following (interrelated) questions. What determines the consumers' decision to buy either or both platforms? What determines the complementors' decision to produce for either or both platforms? What are the optimal prices charged by platforms and complementors to consumers? How do they vary with the number of complementors accessible through each platform? What happens when platforms can charge royalties (or give subsidies) to complementors? What are the incentives of platform providers to vertically integrate some or all of the compatible complementors? What is the pricing structure under vertical integration? Do vertically integrated

platforms have incentives to make their complementors available for the rival platform? What are the incentives for platform standardization? What are the incentives for compatibility of platforms?

The basic elements of our model are the following. First, there are three groups of players: (i) two platforms which offer horizontally differentiated products; (ii) a continuum of consumers with different preferences over platforms; and (iii) an oligopolistic market of complementors. Consumers care exclusively about complementary products, but these can only be enjoyed through platforms. Second, there is a multi-party pricing structure: each platform charges positive or negative per-unit fees (royalties or subsidies) to the complementors in their group, and then both platforms and all the complementors set non-cooperatively prices to consumers. Third, consumers can buy either or both platforms (single- or multi-homing) and complementors can produce for either or both platforms (single- or multi-production).

Note that this setting combines some elements from two strands of research. First, as in the literature on virtual externalities (Church and Gandal (1992), Economides and Salop (1992)), players enjoy indirect network externalities. A platform becomes more attractive to consumers as the number of its complementors increases. In our model, however, platform competition results in a richer structure of interactions: the number of complementors in *each* platform affects pricing (and therefore profits) of both platforms and all complementors. Naturally, it also affects utility of all consumers. Second, as in the two-sided markets literature (Rochet and Tirole (2005), Armstrong (2005)), the value of a platform for one side of the market increases with the number of players in the other side of the market that adhere to it. Because of this externality, optimal price-setting by platforms requires cross-subsidization (higher prices in the side where demand is more inelastic). Oligopolistic competition in one side of the market and direct payments between players at both ends makes the nature of competition more complex in our setting. At the same time, it allows us to study new issues such as the incentives for vertical integration, standardization and compatibility.

Our model delivers a number of conclusions. We first characterize the optimal pricing structure in two different regimes depending on whether consumers single-home or multi-home. Naturally, the single- vs. multi-homing decision depends itself on prices. Solving simultaneously for firms' pricing and consumers' choices, we show that, in equilibrium, consumers are more likely to multi-home as the differentiation of platforms decreases and/or as the number of complementors for either platform increases. Under some

conditions and an intermediate level of differentiation, we also show the possibility of multiple equilibria due to a coordination problem: if each firm anticipates low (respective, high) prices by all other firms, then it has incentives to set low (respectively, high) prices inducing consumers to multi-home (respectively, single-home). We then argue that platforms and complementors always benefit from an increase in the number of complementors in their same platform. In the case of single-homing, they suffer from an increase in the number of complementors in the rival platform. This is simply because, under single-homing, a platform is more valuable to consumers the greater the number of its complementors *relative* to the complementors in the other platform. Higher value means possibility to charge higher prices while keeping consumers loyalty.

We further study the incentives of platforms to vertically integrate their complementors and show that an integrated group sets the price of each complementor equal to its marginal cost. The logic is similar to a two-part tariff: marginal cost pricing avoids distortions in the quantity of complementary goods sold. Profits are then recouped with the price of the (indispensable) platform. We last argue that an integrated group which can make its complementors available to the customers of the rival platform (Word for Apple or Microsoft games for Playstation) faces a trade-off: direct revenues generated by extra sales vs. loss of market share. We show that it is always optimal to put the complementors for sale, but at a price which exceeds the monopoly level.

The rest of the paper is organized as follows. In the next section, we introduce our model of platform competition and discuss a few examples that fit the general features of our model. In Section 3, we derive consumer demands for competing platform, characterize the pricing equilibrium, and discuss its properties. In Section 4, we study the incentives of platforms to integrate with their complementors and sell their complementary products to the rival platform. In Section 5, we allow platforms and their complementors to interact directly and determine sufficient conditions under which the platform subsidizes its complementors. Possible extensions are discussed in Section 6.

## 2 A Model of Platform Competition

### 2.1 Players

We consider a model of platform competition with virtual (indirect) network externalities. Our game has three different players: platforms, complementors and consumers.

- Platforms. There are two platforms,  $h \in \{A, B\}$ . Platforms provide access to complementary products and compete to attract consumers. Platforms are horizontally differentiated.

- Complementors. There are  $n$  firms in the market producing goods that complement platforms. Complementors can be of 3 types. Type- $A$  firms produce complementary products for platform  $A$ , type- $B$  firms produce complementary products for platform  $B$ , and type- $C$  firms produce complementary products for both platforms  $A$  and  $B$ . There are  $n_A$ ,  $n_B$  and  $n_C$  complementors of types  $A$ ,  $B$  and  $C$ , with  $n_A + n_B + n_C = n$ . For the time being, we assume that the number of complementors of each type is fixed. In a later section, we will endogenize the decision of complementors to produce for either or both platforms.

- Consumers. There is a continuum of consumers uniformly distributed in the Hotelling line. We index by  $x \in [0, 1]$  the location of consumer  $x$ . Platform differentiation is formalized by assuming that  $A$  is located at  $x = 0$  and  $B$  is located at  $x = 1$ . A consumer (he) incurs a linear transportation cost  $tz$  when the absolute distance between his location and the location of the platform he buys is  $z$ , where  $t$  captures the degree of differentiation. Buying a platform provides a fixed utility  $\bar{u}$  which, without loss of generality, will be normalized to 0. Most importantly, a platform provides access to its complementors. A consumer who buys platform  $A$  will also buy  $q_A^i$  units of the  $i^{\text{th}}$  type- $A$  complementary good ( $i \in \{1, \dots, n_A\}$ ) and  $q_{AC}^k$  units of the  $k^{\text{th}}$  type- $C$  complementary good ( $k \in \{1, \dots, n_C\}$ ) produced for platform  $A$ . Similarly, a consumer who buys platform  $B$  will buy  $q_B^j$  units of the  $j^{\text{th}}$  type- $B$  complementary good ( $j \in \{1, \dots, n_B\}$ ) and  $q_{BC}^k$  units of the  $k^{\text{th}}$  type- $C$  complementary good ( $k \in \{1, \dots, n_C\}$ ) produced for platform  $B$ . Consumers may choose to buy both platforms, which we call “multi-homing”, and therefore enjoy the products offered by all complementors.

## 2.2 Consumers' Utility, and Profits of Platforms and Complementors

Consumers enjoy a utility  $u(\tilde{q})$  from the purchase of  $\tilde{q}$  units of a complementary good, where  $u'(\tilde{q}) > 0$  and  $u''(\tilde{q}) < 0$  for all  $\tilde{q}$ . Therefore, if a complementor sets a per-unit price  $s$  for his product, consumers will demand a quantity  $q(s) = \arg \max_{\tilde{q}} u(\tilde{q}) - s\tilde{q}$ , yielding the familiar indirect utility function:

$$v(s) = u(q(s)) - sq(s)$$

where  $v'(s) = -q(s) < 0$  and  $v''(s) = -dq(s)/ds > 0$ .

Denote by  $P_h$  the price of platform  $h$ . Denote also by  $s_A^i$  and  $s_B^j$  the per-unit prices of the  $i^{\text{th}}$  type- $A$  and the  $j^{\text{th}}$  type- $B$  complementary goods. Last, denote by  $s_{AC}^k$  and  $s_{BC}^k$  the prices of the  $k^{\text{th}}$  type- $C$  complementary goods produced for platforms  $A$  and  $B$  respectively. Since a consumer who buys platform  $A$  cannot use the type- $C$  complementary goods produced for platform  $B$  (and vice versa), a type- $C$  complementor may choose to set different prices for the same good produced for different platforms ( $s_{AC}^k \geq s_{BC}^k$ ). Assume that the utility enjoyed by a consumer is additively separable in the number of complementary products. The payoff  $U_A(x)$  of a consumer located at  $x$  who purchases platform  $A$  and its complementary goods is given by:

$$U_A(x) = \sum_{i=1}^{n_A} v(s_A^i) + \sum_{k=1}^{n_C} v(s_{AC}^k) - P_A - tx \quad (1)$$

Similarly, his payoff of purchasing platform  $B$  and its complementary goods is:

$$U_B(x) = \sum_{j=1}^{n_B} v(s_B^j) + \sum_{k=1}^{n_C} v(s_{BC}^k) - P_B - t(1-x) \quad (2)$$

Additive separability in utility across complementary goods is assumed in the above specification to simplify our analysis. This means that the utility of a consumer is non-decreasing in the number of complementors associated with the platform he buys.<sup>1</sup> In the paper, however, we emphasize a second, indirect externality (which can be positive or negative): the utility of a consumer is affected by the number complementors associated to *each* platform via their effect on the prices of platforms, the prices of complementary goods

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<sup>1</sup>Most of our analysis can be carried through if we assume a general utility function of complementary goods.

in the same platform, and the prices of complementary goods in the rival platform.

When a consumer located at  $x$  multi-homes, his payoff  $U_{AB}(x)$  is given by:

$$U_{AB}(x) = \sum_{i=1}^{n_A} v(s_A^i) + \sum_{j=1}^{n_B} v(s_B^j) + \sum_{k=1}^{n_C} \max \{v(s_{AC}^k), v(s_{BC}^k)\} - P_A - P_B - t \quad (3)$$

This formalization of utility under multi-homing deserves some comments. First, since we have assumed differentiation on platforms, it seems natural to assume that multi-homers incur both transportation costs.<sup>2</sup> Second, since type- $C$  complementors offer essentially the same product for both platforms, multi-homers are unlikely to enjoy twice the benefits if they purchase the same complementary good for both platforms. We assume that a multi-homer purchases each type- $C$  complementary good only through the platform that yields highest utility. This extreme formalization of duplication of benefits is not essential. On the other hand, some duplication is important. Indeed, under no duplication  $U_{AB}(x) = U_A(x) + U_B(x)$ . As a result, consumer buys platform  $h$  if and only if  $U_h(x) \geq 0$  and therefore single-homing with full market coverage can never occur in equilibrium.

Given consumers' behavior, we can determine the profits of platforms and complementors. Assume for simplicity that both platform providers have zero marginal costs and all complementors have the same marginal cost  $c$  ( $> 0$ ). Denote by  $Q_h$  the fraction of consumers who buy platform  $h$  and its complementary products, and assume for the time being no direct pricing between platforms and complementors. The profit of platforms  $A$  and  $B$  are:

$$G_A = P_A Q_A \quad \text{and} \quad G_B = P_B Q_B \quad (4)$$

Similarly, the profit of the  $i^{\text{th}}$  type- $A$  and the  $j^{\text{th}}$  type- $B$  complementors are:

$$\Pi_A^i = \pi(s_A^i, c) Q_A \quad \text{and} \quad \Pi_B^j = \pi(s_B^j, c) Q_B \quad (5)$$

where  $\pi(s, c) \equiv (s - c)q(s)$  is the complementor's profit per consumer. Let  $s^M$  be the monopoly price that maximizes  $\pi(s, c)$ . The profit of type- $C$

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<sup>2</sup>The fact that  $U_{AB}$  is independent of  $x$  comes from the linearity of the transportation cost. It does not have a substantial impact in our analysis since the marginal cost of multi-homing (that is, the extra cost incurred by choosing to buy both platforms rather than just one of them) always depends on  $x$ .



complementors depend on whether some consumers multi-home or not. Under multi-homing ( $Q_A + Q_B > 1$ ), consumers who buy both platforms will purchase the  $k^{\text{th}}$  type- $C$  complementary product only for the platform that yields highest indirect utility (for  $A$  if  $s_{AC}^k \leq s_{BC}^k$  and for  $B$  if  $s_{AC}^k > s_{BC}^k$ ). This choice is not available when all consumers single-home ( $Q_A + Q_B \leq 1$ ). Formally:

$$\Pi_C^k = \begin{cases} \pi(s_{AC}^k, c) Q_A + \pi(s_{BC}^k, c) Q_B & \text{if } Q_A + Q_B \leq 1 \\ \pi(s_{AC}^k, c) Q_A + \pi(s_{BC}^k, c) (1 - Q_A) & \text{if } Q_A + Q_B > 1 \text{ and } s_{AC}^k \leq s_{BC}^k \\ \pi(s_{AC}^k, c) (1 - Q_B) + \pi(s_{BC}^k, c) Q_B & \text{if } Q_A + Q_B > 1 \text{ and } s_{AC}^k > s_{BC}^k \end{cases} \quad (6)$$

Before proceeding, we briefly discuss the market relevance of our model.

### 2.3 Examples and Related Literature

There is a myriad of situations that can be roughly captured with our model. Typical examples of platform competition with complementors include IT industries: game consoles (Playstation and Xbox) and game developers, operating systems (MS Windows and Apple) and software applications, next generation of DVD players (HD-DVD and Blu-Ray) and movie distributors, etc. However, one can think of applications in many other traditional industries: shopping malls and retailers, golf or tennis clubs and facilities, etc.

For shopping malls and clubs, transportation costs may literally represent the distance traveled from the consumer's residence to the location of the facility. For game consoles and operating systems, they may represent the skills necessary to exploit all the advantages of the platform. In all cases, summing transportation costs seems a plausible first approximation to capture the costs of multi-homing.<sup>3</sup> Note also that, due to the platform specificity of products, type- $C$  firms can price discriminate: Adobe may charge different prices for the PC and Mac versions of its software, GAP may offer discounts in some shopping malls and not in others, EA can choose a different pricing policy for its Playstation and Xbox games, etc. By contrast, price discrimination would not be feasible for complementors that could interchangeably be used with either platform (e.g., a printer that can be connected to a PC or

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<sup>3</sup>Obviously, other specifications could be tailored to specific examples of multi-homing: a cost proportional to the relative usage of each platform, a cost per complementor rather than per platform, etc.

a Mac). Introducing this fourth type of complementors would hardly affect the analysis. Finally, in our examples, it also seems natural to assume that the benefits of purchasing the same GAP clothes in two different malls or the same video game for two consoles are limited.<sup>4</sup>

In the paper, we claim that in most markets with platform competition, consumers have the *option* to purchase multiple platforms (multi-homing) and firms have the *option* to produce for multiple platforms (multi-production).<sup>5</sup> Therefore, we think it is misguided to rule out these possibilities. First, because even in situations where single-homing and single-production emerge as equilibrium outcomes, the possibility to decide otherwise affects the strategic behavior of players. Second, because it is also important to determine which parameters of the market structure are responsible for these equilibrium choices.

Our model shares many features with the recent two-sided markets literature (Caillaud and Jullien (2003), Rochet and Tirole (2005), Armstrong (2005), ). However, two key differences are worth emphasizing. First, we consider oligopolistic competition in one side of the market (complementors). Second, we allow direct payments between the two sides.<sup>6</sup> As a result, our model is ill-suited for situations where both sides of the market are atomless (dating, real state or credit card markets) and/or end-users do not interact directly (viewers and advertisers). On the other hand, our setting fits better several IT industries such as personal computers, videogames and next generation DVDs.

Our study is also complementary to a recent paper by Economides and Katsamakas (2006) which studies the performance of proprietary platform and open source platform. Under the assumption of linear demand functions, the authors find that total profits for a vertically integrated proprietary industry structure are higher than that for vertically disintegrated proprietary and open source platforms.<sup>7</sup> Instead of assuming a linear demand system,

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<sup>4</sup>Once again, in some cases, duplication may not be an issue. For example, PC and Mac multi-homers may want to acquire basic softwares, such as antivirus and messaging service, for both operating systems.

<sup>5</sup>Naturally, there are prominent exceptions. Religious associations can be seen as clubs or platforms whose members are expected to single-home. Similarly, technological barriers may impede firms to join certain platforms.

<sup>6</sup>A reduced-form model with payments between end-users is discussed in Rochet and Tirole (2005).

<sup>7</sup>They also derive conditions under which a disintegrated proprietary platform charges per unit royalty or subsidy to its application software developers. When proprietary and

we derive the consumers' demands in a Hotelling setting and explicitly allow consumers to choose between multi-homing and single-homing. Our focus is on the effects of multi-homing and the number of complementors when two horizontally differentiated platforms compete.

Another recent paper by Doganoglu and Wright (2006) also studies the issue of multi-homing. Their focus is on whether multi-homing by consumers reduces the need to make products compatible in order for consumers to enjoy network benefits. They find that multi-homing weakens competition and makes compatibility less attractive to the firms, but increases the social desirability of compatibility. In their analysis, the network benefits are exogenously given; we explicitly model indirect network benefits by considering an oligopolistic structure of complementors.

### 3 The Pricing Game: Single-homing vs. Multi-homing

In this section, we determine the equilibrium of the pricing game. We first derive consumer demands for the two platforms and complementary products.

#### 3.1 Consumer Demands

Denote by  $x_A$  (respectively  $x_B$ ) the location of the consumer indifferent between buying platform  $B$  (respectively  $A$ ) and buying both platforms. Formally:

$$\begin{aligned}
 & U_B(x_A) = U_{AB}(x_A) \\
 \Leftrightarrow & \quad tx_A = \sum_i v(s_A^i) + \sum_k \max \{v(s_{AC}^k) - v(s_{BC}^k), 0\} - P_A \quad (7)
 \end{aligned}$$

$$\begin{aligned}
 & U_A(x_B) = U_{AB}(x_B) \\
 \Leftrightarrow & \quad t(1 - x_B) = \sum_j v(s_B^j) + \sum_k \max \{v(s_{BC}^k) - v(s_{AC}^k), 0\} - P_B \quad (8)
 \end{aligned}$$

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open source platforms compete, the authors provide sufficient conditions under which a vertically integrated proprietary platform dominates in terms of profitability and market shares.

Similarly, denote by  $\bar{x}$  the location of the consumer indifferent between buying platform  $A$  and buying platform  $B$ :

$$U_A(\bar{x}) = U_B(\bar{x}) \Leftrightarrow 2t\bar{x} = t + P_B - P_A + \sum_i v(s_A^i) - \sum_j v(s_B^j) + \sum_k (v(s_{AC}^k) - v(s_{BC}^k)) \quad (9)$$

Using (7) and (8), there is Multi-Homing (MH) in equilibrium if and only if:

$$\begin{aligned} x_B < x_A \Leftrightarrow P_A + P_B + t < \sum_i v(s_A^i) + \sum_j v(s_B^j) \\ + \sum_k \max \{v(s_{AC}^k) - v(s_{BC}^k), v(s_{BC}^k) - v(s_{AC}^k)\} \end{aligned}$$

Naturally, if MH occurs, consumers located in  $[0, x_B]$  buy only platform  $A$ , consumers in  $[x_B, x_A]$  buy both platforms and consumers in  $[x_A, 1]$  buy only platform  $B$ . Conversely, there is Single-Homing (SH) in equilibrium if and only if

$$\begin{aligned} x_B \geq x_A \Leftrightarrow P_A + P_B + t \geq \sum_i v(s_A^i) + \sum_j v(s_B^j) \\ + \sum_k \max \{v(s_{AC}^k) - v(s_{BC}^k), v(s_{BC}^k) - v(s_{AC}^k)\} \end{aligned}$$

If SH with full market coverage occurs, consumers located in  $[0, \bar{x}]$  buy platform  $A$  and consumers located in  $[\bar{x}, 1]$  buy platform  $B$ . Overall, using (7), (8),  $(C_{MH})$  and  $(C_{SH})$  and given the uniform distribution of consumers, the demand for platform  $A$  is

$$Q_A = \begin{cases} 1 & \text{if } C_{MH} \text{ and } P_A < \sum_i v(s_A^i) + \sum_k \max \{v(s_{AC}^k) - v(s_{BC}^k), 0\} - t \\ x_A & \text{if } C_{MH} \text{ and } P_A \geq \sum_i v(s_A^i) + \sum_k \max \{v(s_{AC}^k) - v(s_{BC}^k), 0\} - t \\ \bar{x} & \text{if } C_{SH} \end{cases} \quad (10)$$

Note that the demand  $Q_A$  is continuous, downward-sloping, and piecewise linear in  $P_A$ . It has two kinks and it is decreasing in prices of the products that are complementary to platform  $A$ . Moreover, the demand for  $A$  is increasing in  $P_B$  in the single-homing region and independent of  $P_B$  in the multi-homing region.

Similarly, the demand for platform  $B$  is:

$$Q_B = \begin{cases} 1 & \text{if } C_{MH} \text{ and } P_B < \sum_j v(s_B^j) + \sum_k \max \{v(s_{BC}^k) - v(s_{AC}^k), 0\} - t \\ 1 - x_B & \text{if } C_{MH} \text{ and } P_B \geq \sum_j v(s_B^j) + \sum_k \max \{v(s_{BC}^k) - v(s_{AC}^k), 0\} - t \\ 1 - \bar{x} & \text{if } C_{SH} \end{cases} \quad (11)$$

The demand for a complementary product associated with platform  $h$  is simply  $q(s_h)Q_h$ , where  $s_h$  is the price of the complementary product.

We shall derive pricing equilibria by considering two cases: SH and MH.

### 3.2 Single-homing Equilibrium

Suppose given a profile of prices all consumers choose SH. Then platform A solves

$$\max_{P_A} P_A \left( t + P_B - P_A + \sum_i v(s_A^i) - \sum_j v(s_B^j) + \sum_k \max \{v(s_{AC}^k) - v(s_{BC}^k), 0\} \right) / (2t).$$

The necessary condition for an interior solution yields

$$t + P_B - 2P_A + \sum v(s_A^i) - \sum v(s_B^j) + \sum_k \max \{v(s_{AC}^k) - v(s_{BC}^k), 0\} = 0.$$

Similarly, the necessary condition for platform B is given by

$$t + P_A - 2P_B - \sum v(s_A^i) + \sum v(s_B^j) + \sum_k \max \{v(s_{BC}^k) - v(s_{AC}^k), 0\} = 0.$$

Note that the best reply for each platform is increasing in the price of the rival platform, decreasing in the prices of its own complementary goods, and increasing in the prices of its rival complementary goods. Therefore, prices of platforms are strategic complements; prices of platform and of rival complementors are also strategic complements; and prices of platform and of its own complementors are strategic substitutes.

The decision for a type-C complementor is much simpler. It solves the following optimization problem

$$\max_{s_{AC}^k, s_{BC}^k} \Pi_C^k = \pi(s_{AC}^k, c)\bar{x} + \pi(s_{BC}^k, c)(1 - \bar{x}).$$

It is straightforward to see that, at the optimal solution, each type-C complementor charges a monopoly price  $s_{AC}^k = s_{BC}^k = s^M$  and receives monopoly profit  $\pi^M$ .

We now examine the decision of a type-A complementor. Complementor  $i$  in group A solves

$$\max_{s_A^i} \pi(s_A^i, c) \left( t + P_B - P_A + \sum v(s_A^i) - \sum v(s_B^j) \right) / (2t).$$

The necessary condition for an interior solution is given by

$$t + P_B - P_A + \sum v(s_A^i) - \sum v(s_B^j) - \phi(s_A^i) = 0,$$

where

$$\phi(s) \equiv \frac{q(s)\pi(s, c)}{\pi_1(s, c)}$$

and  $\pi_1(s, c)$  is the partial derivative of  $\pi$  with respect to  $s$ . Similarly, the necessary condition for complementors in group B is given by

$$t + P_A - P_B - \sum v(s_A^i) + \sum v(s_B^j) - \phi(s_B^j) = 0.$$

Imposing symmetry on prices within each group of complementors yields the following necessary conditions for a SH equilibrium to arise

$$\begin{aligned} \phi(s_A) &= t + \frac{1}{3} (n_A v(s_A) - n_B v(s_B)), \\ \phi(s_B) &= t - \frac{1}{3} (n_A v(s_A) - n_B v(s_B)), \\ P_A &= \phi(s_A), \\ P_B &= \phi(s_B). \end{aligned}$$

In the rest of the paper, we make the following assumption:  $\phi(s)$  is strictly increasing for  $s \in [c, s^M]$ . In Section 3.5, we present an example of constant elasticity demands in which  $\phi$  satisfies this assumption. Moreover, define

$$\bar{n}_A = (3t + n_B v(c)) \max\{1, 1/v(\phi^{-1}(2t))\}.$$

The following lemma establishes existence and uniqueness of a solution to the above necessary conditions for a SH equilibrium, which we denote by  $(P_A^*, P_B^*, s_A^*, s_B^*)$ .

**Lemma 1:** *When  $n_A \leq \bar{n}_A$ , then there exists a unique price profile that satisfies the above necessary conditions. When  $n_A > \bar{n}_A$ , the equilibrium will be at the corner.*

At the interior solution, the utility of the marginal consumer at location  $\bar{x}$  is

$$U_A(\bar{x}) = U_B(\bar{x}) = \frac{1}{2}(n_A v(s_A^*) + n_B v(s_B^*)) + n_C v(s^M) - \frac{3}{2}t,$$

so that the full market coverage requires  $U_A(\bar{x}) = U_B(\bar{x}) \geq 0$ , or

$$t \leq \frac{1}{3}(n_A v(s_A^*) + n_B v(s_B^*)) + \frac{2}{3}n_C v(s^M).$$

Moreover, the utility of consumer choosing multi-homing is

$$U_{AB}(\bar{x}) = n_A v(s_A^*) + n_B v(s_B^*) + n_C v(s^M) - 3t,$$

SH equilibrium requires  $U_{AB}(\bar{x}) < U_h(\bar{x})$  which is equivalent to

$$-t \leq \frac{1}{3}(n_A v(s_A^*) + n_B v(s_B^*)) \leq t.$$

Finally, for  $(P_A^*, P_B^*, s_A^*, s_B^*)$  to be an equilibrium, we further verify that no firm has incentive to deviate unilaterally to the MH region. This imposes an extra restriction on the parameters of the model. We therefore have the following result.

**Result 1:** *There exist  $t^*$  and  $\bar{t}$ , with  $t^* < \bar{t}$ , such that an equilibrium with single-homing and full coverage of the market occurs whenever  $t^* \leq t \leq \bar{t}$ .*

When  $t > \bar{t}$ , there are two possible cases. In one case, the equilibrium is such that the market is not fully covered and two platforms are local monopolies. In the other case, the market is fully covered and there are multiple equilibria in prices.

The above equilibrium thresholds can also be stated in terms of parameters  $n_A$  or  $n_B$ . In particular, SH equilibrium exists when  $n_A \leq n_A^*$  or  $n_B \leq n_B^*$ .

### 3.3 Multi-homing Equilibrium

Suppose given a price profile, some of the consumers choose MH. Then platform A solves

$$\max_{P_A} P_A x_A.$$

The necessary condition for an interior solution yields

$$\sum v(s_A^i) - 2P_A = 0.$$

Thus, the solution is

$$P_A = \sum v(s_A^i)/2$$

if

$$t \geq \sum v(s_A^i)/2$$

and

$$P_A = \sum v(s_A^i) - t$$

otherwise. Thus, the solution is interior if and only if  $2t \geq \sum v(s_A^i)$ .

Similarly,

$$\sum v(s_B^j) - 2P_B = 0.$$

The solution is interior iff  $2t \geq \sum v(s_B^j)$ . Otherwise,  $P_B = \sum v(s_B^j)$ .

Note that given the possibility of MH, the best reply for each platform is independent of the rival platform's price and of the prices of the rival complementors, and is decreasing in the prices of its own complementors' prices.

A type-C complementor chooses prices  $(s_{AC}^k, s_{BC}^k)$  to maximize  $\pi(s_{AC}^k, c) Q_A + \pi(s_{BC}^k, c) (1 - Q_A)$  if  $s_{AC}^k \leq s_{BC}^k$ , and  $\pi(s_{AC}^k, c) (1 - Q_B) + \pi(s_{BC}^k, c) Q_B$  if  $s_{AC}^k > s_{BC}^k$ . It is easy to see that the solution is  $s_{AC}^k = s_{BC}^k = s^M$ .

Complementor  $i$  in group A solves

$$\max_{s_A^i} \pi(s_A^i, c) x_A.$$

The necessary condition for an interior solution is given by

$$\phi(s_A^i) = \sum v(s_A^i) - P_A.$$

Similarly,

$$\phi(s_B^j) = \sum v(s_B^j) - P_B.$$

Imposing symmetry on the prices of complementors within each platform yields

$$P_A = \phi(s_A) = n_A v(s_A)/2,$$

$$P_B = \phi(s_B) = n_B v(s_B)/2.$$



**Lemma 2:** *There exists a unique price profile that satisfy the above necessary conditions.*

Lemma 2 follows directly from the fact that  $\phi(s)/v(s)$  is strictly increasing. We denote by  $(P_A^{**}, P_B^{**}, s_A^{**}, s_B^{**})$  the unique solution. Note that  $s_A^{**}$  is increasing in  $n_A$ , but independent of  $n_B$  and  $t$ , and  $s_A^{**} < s^M$ .

Now, also note that

$$tx_A = \phi(s_A^{**}), \quad t(1 - x_B) = \phi(s_B^{**}).$$

Thus, an interior solution requires  $t \geq \phi(s_A^{**})$  and  $t \geq \phi(s_B^{**})$ . Moreover, multi-homing requires  $x_B < x_A$ , which occurs if and only if

$$t < \phi(s_A^{**}) + \phi(s_B^{**}).$$

When  $t < \phi(s_A^{**})$ , in equilibrium  $x_A = 1$ ,  $s_A^{**} = s^M$ ,  $P_A^{**} = n_A v(s^M) - t$ . Similarly,  $t < \phi(s_B^{**})$ , in equilibrium  $s_B^{**} = s^M$  and  $P_B^{**} = n_B v(s^M) - t$ . Thus, if  $t < \phi(\min\{s_A^{**}, s_B^{**}\})$ , then in equilibrium all types of consumers choose MH.

We also need to verify that the players do not have incentives to deviate from MH to the SH region. This discussion leads us to the following result.

**Result 2:** *There exists a value  $t^{**}$  such that an equilibrium with multi-homing arises whenever  $t < t^{**}$ .*

This equilibrium result can also be stated in terms of parameters  $n_A$  and  $n_B$ . In particular, MH equilibrium exists when  $n_A > n_A^{**}$  or  $n_B > n_B^{**}$ .

We conjecture that  $t^* < t^{**}$ . This conjecture implies that there is an interval of  $t$  where two equilibria coexist, one with single-homing and the other with multi-homing. There is a coordination issue here: If a player expects other players to choose low prices, then he will also choose a low price. This is the multi-homing equilibrium. On the other hand, if a player expects others to choose high prices, he will choose a high price, leading to a single-homing equilibrium.

### 3.4 Properties of the Price Equilibria

There are two types of comparative static properties of the price equilibria. The first concerns a change in the number of complementors,  $n_A$  and  $n_B$ , and the second is on the platform differentiation parameter,  $t$ .

**Proposition 1:** *As  $n_A$  increases, the following holds:*

(i) *SH equilibrium initially arises and then MH equilibrium occurs; (ii) the price of platform A, the price of A-type complementors, the market share for A as well as the profits for A-type firms all increase; and (iii) the price of platform B, the price of B-type complementors, the market share for B as well as the profits for B-type firms all decrease initially (SH equilibrium) and then stay constant (MH equilibrium).*

The first implication of the above set of comparative statics points to the importance of the number of complementors to the market share of the platform and the profits of the platform and its complementors. All players associated with the same platform benefit from an increase in the number of complementors. This is essentially an indirect network effect. However, how such an indirect network externality may have an impact on the rival platform and its complementors depends on the size of the indirect network effect (as the equilibrium shifts from SH to MH). It initially has negative impact on the rival group. As the indirect network effect becomes large, the two groups become independent.

As  $t$  increases, at the MH equilibrium all the prices stay constant and the market share and profit for each platform fall. At the SH equilibrium the prices for each platform and complementary products increase with  $t$ . In this case, it is not clear how the profits change with respect to  $t$ .

### 3.5 An Example: Constant Elasticity of Demand

We now consider an example to illustrate the properties of the  $\phi$  function discussed above. Suppose

$$u(q) = \frac{q^{1-\alpha}}{1-\alpha},$$

where  $0 < \alpha < 1$ . It follows that

$$\begin{aligned} q(s) &= s^{-1/\alpha} \\ v(s) &= \frac{\alpha}{1-\alpha} s^{1-1/\alpha}, \end{aligned}$$

and the monopoly price is

$$s^M = \frac{c}{1-\alpha}.$$

Then

$$\phi(s) = \frac{\alpha}{1-\alpha} \frac{s - (1-\alpha)s^M}{s^M - s} s^{1-1/\alpha}$$

for  $s \in [c, s^M]$ . It can be easily verified that for  $s \in [c, s^M]$ ,  $\phi$  is strictly increasing and convex, with  $\phi(c) = 0$  and  $\lim_{s \rightarrow s^M} \phi(s) = +\infty$ .

## 4 Integration between Platform and its Complementors

In this section we analyze the incentives of platform integrating with its complementors. If platform A integrates with its complementors, its total profit is given by

$$\Pi_A = \left( P_A + \sum_i \pi_A^i \right) Q_A.$$

It follows that

$$\frac{\partial \Pi_A}{\partial P_A} = Q_A - \left( P_A + \sum \pi_A^i \right) \delta$$

where  $\delta = 1/t$  if MH and  $1/(2t)$  if SH. Moreover,

$$\begin{aligned} \frac{\partial \Pi_A}{\partial s_A^{i'}} &= \left( q(s_A^{i'}) + (s_A^{i'} - c_A^{i'}) q'(s_A^{i'}) \right) Q_A - \left( P_A + \sum \pi_A^i \right) q(s_A^{i'}) \delta \\ &= \frac{\partial \Pi_A}{\partial P_A} q(s_A^{i'}) + (s_A^{i'} - c_A^{i'}) q'(s_A^{i'}) Q_A. \end{aligned}$$

Thus, we have

**Proposition 2:** *Independently of whether group B is integrated or not, an integrated group A would set prices of its complementary products equal to marginal costs.*

This result is similar to the idea of two-part tariff. Pricing at the marginal cost avoids quantity distortion and profits can be recouped with a fixed fee (i.e., the price of the platform).

It follows that the best reply of  $P_A$  is determined by

$$Q_A = P_A \delta.$$

Thus, the profit for integrated A group is

$$(P_A)^2 \delta.$$

To compute group A's profit, we need to consider whether group B is integrated or not.

Suppose both groups are integrated. Group B would also choose complementary prices equal to marginal costs. The FOC for platform price is

$$Q_B = P_B \delta.$$

If MH arises in equilibrium, then

$$P_A = \frac{1}{2}n_A v(c), \quad P_B = \frac{1}{2}n_B v(c).$$

If SH arises in equilibrium, then

$$P_A = t + \frac{1}{3}(n_A - n_B)v(c), \quad P_B = t - \frac{1}{3}(n_A - n_B)v(c).$$

It follows that the equilibrium profits for the integrated A are

$$\left(\frac{1}{2}n_A v(c)\right)^2 / t$$

in the case of MH and

$$\left(t + \frac{1}{3}(n_A - n_B)v(c)\right)^2 / (2t)$$

in the case of SH.

Clearly, in the case of MH, each platform makes higher profit under *2-side* integration than no integration. However, since the total profits under non-integration are equal to

$$\begin{aligned} & (\phi(s_A) + n_A \pi(s_A, c)) \phi(s_A) / t \\ &= \left(\frac{1}{2}n_A\right)^2 ((v(s_A))^2 + 2(s_A - c)q(s_A)v(s_A)) / t \end{aligned}$$

which exceeds the profit under *2-side* integration, since

$$(v(s))^2 + 2(s - c)q(s)v(s) - (v(c))^2$$

decreases with  $s$  for  $s > c$  and is equal to 0 when  $s = c$ . Thus, we have

**Proposition 3:** *Suppose MH arises as an equilibrium outcome in both cases of no integration and 2-side integration. Then*

- (i) each platform makes higher profits under 2-side integration than no integration; and  
(ii) the total profits for each group under no integration are higher than that under 2-side integration.

Proposition 3 implies that platforms have incentives to use a fixed royalty fee to extract rents from independent complementors.

What happens when SH arises as an equilibrium? Suppose first  $n_A = n_B$ . Then the profit for the platform under 2-side integration is  $t/2$ , which is identical to the profit under no integration. This implies that the total profits under no integration are higher than that under 2-side integration. Suppose now that  $n_A > n_B$ . We can show that full integration benefits platform A and hurts B. To summarize, we have

**Proposition 4:** *Suppose SH arises as an equilibrium outcome in both cases of no integration and 2-sided integration. Then*

- (i) when  $n_A = n_B$ , the profit for the platform under no integration is identical to that under 2-sided integration, but the total profits for each group under no integration are higher than that under 2-sided integration;  
(ii) when  $n_A > n_B$ , 2-sided integration benefits platform A, but hurts B.

Next, we consider a situation in which platform A sells one of its complementary products to consumers who purchase platform B.

Let  $s$  be its price. We consider the case of SH. Then

$$Q_A = \left( t + P_B - P_A + \sum v(s_A^i) - \sum v(s_B^j) - v(s) \right) / (2t).$$

Platform A has the following payoff

$$\begin{aligned} \Pi_A &= \left( P_A + \sum \pi_A^i \right) Q_A + \pi(s, c) Q_B \\ &= \left( P_A + \sum \pi_A^i - \pi(s, c) \right) Q_A + \pi(s, c). \end{aligned}$$

Note that

$$\frac{\partial \Pi_A}{\partial P_A} = Q_A - \left( P_A + \sum \pi_A^i - \pi(s, c) \right) / (2t)$$

and

$$\begin{aligned} \frac{\partial \Pi_A}{\partial s_A^i} &= \left( q(s_A^i) + (s_A^i - c)q'(s_A^i) \right) Q_A - \left( P_A + \sum \pi_A^i - \pi(s, c) \right) q(s_A^i) / (2t) \\ &= \frac{\partial \Pi_A}{\partial P_A} q(s_A^i) + (s_A^i - c)q'(s_A^i) Q_A. \end{aligned}$$

This implies that  $s_A^i = c$  for all  $i$ . Moreover,

$$\begin{aligned}
\frac{\partial \Pi_A}{\partial s} &= \left( P_A + \sum \pi_A^i - \pi(s, c) \right) \frac{\partial Q_A}{\partial s} + \pi_1(s, c)(1 - Q_A) \\
&= \left( P_A + \sum \pi_A^i - \pi(s, c) \right) q(s)/(2t) + \pi_1(s, c)(1 - Q_A) \\
&= \left( Q_A - \frac{\partial \Pi_A}{\partial P_A} \right) q(s) + \pi_1(s, c)(1 - Q_A) \\
&= Q_A(q(s) - \pi_1(s, c)) - \frac{\partial \Pi_A}{\partial P_A} q(s) + \pi_1(s, c) \\
&= -(s - c)q'(s)Q_A - \frac{\partial \Pi_A}{\partial P_A} q(s) + \pi_1(s, c).
\end{aligned}$$

Thus, at an interior solution,  $\pi_1(s, c) < 0$  and

$$\frac{s - c}{s} = \frac{1}{(1 - Q_A)\epsilon}$$

where  $\epsilon = -sq'(s)/q(s)$  is the elasticity of demand for each complementary good conditional on purchasing the corresponding platform. The familiar price-cost markup formula is thus modified by market share. We can also examine the optimal price in the case of MH. The results are summarized as follows:

**Proposition 5:** *Selling complementary products to the consumers of rival platform leads no loss of market share under MH and potential loss of market share under SH.*

## 5 Direct Interaction between Platform and its Complementors: Royalties vs. Subsidies

Consider the case in which there is single-homing. One of the platforms, say  $A$ , can charge a royalty  $r_A$  to each complementor per unit of output  $q_A^i$  sold to consumers. What will be the optimal royalty?

Consider the following timing. First, platform  $A$  sets the royalty. Then, platforms and complementors simultaneously choose the prices to charge to consumers. Note that for a complementor, a royalty  $r_A$  is like an increase in

the marginal cost of production from  $c$  to  $c + r_A$ . Let's denote  $\pi(s_A, r_A) = (s_A - c - r_A)q(s_A)$  the profit of a complementor and

$$\phi(s, r) \equiv \frac{\pi(s, c + r)q(s)}{\pi_1(s, c + r)}.$$

From the equilibrium conditions derived before, we find that

$$\frac{\partial s_A}{\partial r_A} > 0 \quad \text{and} \quad \frac{\partial s_B}{\partial r_A} > 0.$$

Let's move now to stage 1. Platform  $A$  chooses the royalty  $r_A$  that solves:

$$\max_{r_A} \left[ n_A r_A q(s_A) + \frac{[\phi(s_A, r_A)]^2}{2t} \right]$$

where the first term represents the direct revenue generated by the sale of royalties and the second term represents the indirect revenue generated by the sale of platforms.

Let us focus on the second term, the indirect effect. We have

$$\frac{d\phi(s_A, r_A)}{dr_A} = \phi_2(s_A, r_A) + \phi_1(s_A, r_A) \frac{\partial s_A}{\partial r_A}$$

The first term captures the fact that as  $r_A$  increases, complementary products are more costly to produce. The second term describes the fact that complementors increase prices as their marginal cost increases, which affects positively the profit of the platform provider. Putting both terms together and given the expression for  $\frac{\partial s_A}{\partial r_A}$  yields

$$\frac{d\phi(s_A, r_A)}{dr_A} < 0.$$

Not surprisingly, the first effect dominates the second. As royalties increase, the market share and profit from sales of platforms decrease. Royalties increase since the prices of platforms decrease.

It turns out that the equilibrium profit for the platform is given by

$$\Pi = \frac{(n(v(s_A) + r_A q(s_A)) - d)^2}{4}$$

where  $d$  is the marginal cost of the platform. The following proposition provides a sufficient condition for subsidy to be optimal. This condition

holds for the class of constant elasticity demands, but is violated for linear demands.

**Proposition 6:** *Platforms subsidize their complementors if the price elasticity of complementary demands,  $-q'(s)s/q(s)$ , is non-decreasing.*

It would be interesting to examine whether it is an anti-competitive practice that platform subsidizes its complementors. It certainly hurts the rival platform and its complementors.

## 6 Discussion and Future Research

In this paper, we have provided a model to examine the role of complementors and consumer multi-homing in platform competition. One innovative feature of our model is the explicit consideration of the option of consumers to adopt multiple, competing platforms in the context of a Hotelling framework. This introduces an extra complexity in characterizing equilibrium prices since demand curves have several kinds.

The other innovation in our paper is to explicitly model the role of complementors in platform competition. This feature is particularly significant in the current war of expectations management on next generation DVD formats (Blu-Ray and HD-DVD). Players (platforms and complementors) form coalitions in setting standards.

Our analysis offers some insights on the incentives of the competing platforms in setting prices to consumers and complementors, in integrating vertically some or all of the compatible complementors, and in making their own complementary products available to the rival platform. Many issues remain open.

### 6.1 Complementor's Choice: Single-Production vs. Multi-Production

One direction for our future research is to endogenize the complementors' choice to produce for one or both platforms. We may consider horizontal differentiation in the firms' marginal cost of adapting their technology to each platform. We expect that tipping may occur when the technological differentiation parameter is small, as the cost of adaptation is offset by the increasing returns in the number of complementors in the same platform. By



contrast, for higher levels of technological differentiation, there is a symmetric equilibrium where each platform attracts an equal number of complementors.

We now briefly outline our approach to modeling complementors' choices to produce for one or both platforms. For each complementor, there exists a fixed cost of adopting its technology to work with a platform and there is a horizontal differentiation between the two platforms in terms of complementors' adoption/development costs. We assume that firms are located uniformly on a line segment between 0 and  $n$ . For a firm located at  $y$ , its cost of adopting is technology to platform A is  $\theta y$ , to B is  $\theta(n - y)$ , and to both (type-C firm) is  $\theta y + \theta(n - y) = \theta n$ .

Each firm chooses to become A, B or C type before the pricing game begins. If it chooses C type, it will make a monopoly profit  $\pi^M - \theta n$ . Its net profit from being A type is  $\Pi_A(n_A, n_B) - \theta y$ , where  $\Pi_A(n_A, n_B)$  is the profit of A-type complementor derived in the last section and it depends on the number of complementors in each group. The net profit for B-type firm can be written analogously.

A full characterization of the equilibrium at this stage depends on the properties of  $\Pi_A(n_A, n_B)$ ,  $\Pi_B(n_A, n_B)$  and the size of  $\theta$ . We expect that there will be a negative relationship between the degree of multi-homing of consumers and multi-production of complementors.

## 6.2 Compatibility and Standardization

Another direction for our future research is to apply our framework to study players' decision to standardize their platforms or make their platforms not compatible to each other. In our setting, we view a *complete standardization* as a situation in which platform providers agree to choose a single platform (a single location in the Hotelling setting) and share the revenues from royalties and from selling the platform. The current generation of CD products is a good example. On the other hand, platform providers may choose differentiated platforms (different locations in the Hotelling setting) but agree to make all complementary products fully compatible with both platforms. This is an outcome of *complete compatibility without standardization*. Platform providers may engage in maximal differentiation in platforms at the same time induce complementors to make their products not compatible with rival platforms. In this situation, depending on the size of direct and indirect networks effects as well as on the heterogeneity of consumers' tastes, *multiple non-compatible platforms* may coexist, or only *one single platform*

survives in the market. The latter is the same outcome as the one with complete standardization, but arises *ex post* from market competition rather than from an *ex ante* agreement among platform providers. It would be interesting to derive conditions for each of the above four cases to arise as an equilibrium outcome. However, endogenous determination of platform locations (standardization or not) is a challenging task.

As a final remark, we expect that our framework applies well beyond IT industries. Examples include golf or tennis clubs offering valuable complementary services (tennis lessons, pool, gym, restaurant, etc.) to its members, and shopping malls competing to offer the best possible retailers to shoppers.

## 7 Appendix

*Proof of Lemma 1:*

We need to show that the following two equations

$$\phi(s_A) = t + \frac{1}{3} (n_A v(s_A) - n_B v(s_B)),$$

$$\phi(s_B) = t - \frac{1}{3} (n_A v(s_A) - n_B v(s_B)),$$

determine a unique solution  $(s_A^*, s_B^*)$ . Each equation determines a curve in the  $(s_A, s_B)$  space, which we denote by A-curve and B-curve respectively. We proceed in three steps.

First, note that both curves are strictly increasing for  $(s_A, s_B) \in (c, s^M)^2$ , since  $\phi'(s_h) > 0$ ,  $h = A, B$ , and

$$\begin{aligned} \frac{ds_B}{ds_A} \Big|_A &= \frac{3\phi'(s_A) + n_A q(s_A)}{n_B q(s_B)} > 0, \\ \frac{ds_B}{ds_A} \Big|_B &= \frac{n_A q(s_A)}{3\phi'(s_B) + n_B q(s_B)} > 0. \end{aligned}$$

Second, if the two curves intersect at  $(s_A, s_B)$ , then

$$\frac{ds_B}{ds_A} \Big|_A > \frac{ds_B}{ds_A} \Big|_B$$

and therefore they intersect uniquely.

Third, we show that the two curves indeed intersect. Without loss of generality, suppose  $n_A \geq n_B$ . Consider the behavior of the A-curve in the  $(s_A, s_B)$  space: It begins below the 45 degrees line since  $s_A > c$  when  $s_B = c$ ; it is upward-sloping; and it is above the 45 degrees line since  $s_A < s^M$  when  $s_B = s^M$ . If  $3t \geq (n_A - n_B)v(c)$ , then the B-curve begins above the 45 degrees line, increases, and becomes below the 45 degrees line. Therefore, by the continuity of the curves and the Intermediate Value Theorem, they must intersect. If  $3t < (n_A - n_B)v(c)$ , then the claim holds as long as

$$\phi^{-1}(2t) \geq v^{-1}\left(\frac{3t + n_B v(c)}{n_A}\right)$$

or

$$n_A \leq \frac{3t + n_B v(c)}{v(\phi^{-1}(2t))}.$$

The claim follows. QED.

*Proof of Proposition 4:*

Suppose first  $n_A = n_B$ . Then the profit for the platform under *2-side* integration is  $t/2$ , which is identical to the profit under no integration. This implies that the total profits under no integration are higher than that under *2-side* integration.

Suppose now that  $n_A > n_B$ . Note that A benefits if and only if

$$t + \frac{1}{3}(n_A - n_B)v(c) > t + \frac{1}{3}(n_A v(s_A^*) - n_B v(s_B^*))$$

or equivalently,

$$n_A(v(c) - v(s_A^*)) > n_B(v(c) - v(s_B^*)).$$

Define

$$\begin{aligned} \psi_1 &= n_A(v(c) - v(s_A)) - n_B(v(c) - v(s_B)), \\ \psi_2 &= -n_A v(s_A) + n_B v(s_B). \end{aligned}$$

It follows that  $\psi_1 = 0$  and  $\psi_2 = 0$  determine two monotonically increasing curves in the  $(s_A, s_B)$  plane. Note that  $s_A > s_B$  implies  $\psi_1 > 0$  and that  $s_A < s_B$  implies  $\psi_2 < 0$ . It follows that the curve  $\psi_1 = 0$  is above the 45 degree's line and  $\psi_2 = 0$  is below the 45 degree's line. Since the equilibrium under no integration,  $(s_A^*, s_B^*)$ , is below the 45 degree's line, it follows that

$$\psi_1(s_A^*, s_B^*) = n_A(v(c) - v(s_A^*)) - n_B(v(c) - v(s_B^*)) > 0.$$

Therefore, when  $n_A > n_B$ , *2-side* integration benefits platform A. In this case, we can also show that *2-side* integration hurts B. QED.

*Proof of Proposition 6:*

Given  $r = r_A$ , complementor  $i$  solves

$$\max_{s_i} \log \pi(s_i, r + c) + \log \bar{x}.$$

The necessary condition for an interior solution is given by

$$\phi(s_i, r + c) = \sum v(s_j) - P.$$

Platform solves

$$\max_P \left( P - d + r \sum q(s_i) \right) \bar{x}(s, P).$$

The solution is

$$2P = d + \sum v(s_i) - r \sum q(s_i).$$

Imposing symmetry among complementors, the equilibrium is given by

$$2P = d + nv(s) - rnq(s)$$

$$2\phi(s, r + c) = n(v(s) + rq(s)) - d.$$

It follows that

$$\begin{aligned} \bar{x} &= P - d + r \sum q(s_i) \\ &= \frac{n(v(s) + rq(s)) - d}{2} \end{aligned}$$

and the profit of platform is given by

$$\Pi = \frac{(n(v(s) + rq(s)) - d)^2}{4}.$$

Thus, platform chooses  $r$  to maximize

$$\psi \equiv v(s) + rq(s)$$

subject to

$$2\phi(s, r + c) = n(v(s) + rq(s)) - d.$$

Note that

$$\begin{aligned}\frac{d\psi}{dr} &= q(s) + (rq'(s) - q(s))\frac{ds}{dr} \\ &= q(s) \left[ 1 + \left( \frac{rq'(s)}{q(s)} - 1 \right) \frac{ds}{dr} \right]\end{aligned}$$

where

$$\frac{ds}{dr} = \frac{nq - 2\phi_2}{nq + 2\phi_1 - nrq'} > 0.$$

It follows that

$$\frac{d\psi}{dr} \frac{nq + 2\phi_1 - nrq'}{q(s)} = 2\phi_1 + 2\phi_2 \left( 1 - \frac{rq'(s)}{q(s)} \right).$$

Let

$$\Delta = \frac{\phi_1}{\phi} + \frac{\phi_2}{\phi} \left( 1 - \frac{rq'(s)}{q(s)} \right).$$

Note that  $\log \phi = \log q + \log \pi - \log \pi_1$ , implying that

$$\begin{aligned}\frac{\phi_1}{\phi} &= \frac{q'(s)}{q(s)} + \frac{\pi_1}{\pi} - \frac{\pi_{11}}{\pi_1} \\ \frac{\phi_2}{\phi} &= -\frac{q}{\pi} + \frac{q'}{\pi_1}.\end{aligned}$$

It follows that

$$\Delta = \frac{q'}{q} + \frac{\pi_1 - q + rq'}{\pi} + \frac{-\pi_{11} + q' - r(q')^2/q}{\pi_1}$$

and

$$\begin{aligned}\pi\pi_1\Delta &= \frac{q'}{q}\pi\pi_1 + \pi_1(\pi_1 - q + rq') + \pi(-\pi_{11} + q') - \pi rq' \frac{q'}{q} \\ &= (s - r - c)^2(2(q')^2 - qq'') + (s - c)qq'.\end{aligned}$$

Note that

$$-\epsilon(s)q = q's$$

and

$$-\epsilon'q^2 = (qq'' - (q')^2)s + qq'.$$

It follows that

$$\begin{aligned}\pi\pi_1\Delta &= \pi^2\epsilon'/s + (s-r-c)^2((q')^2 + qq'/s) + (s-c)qq' \\ &= \pi^2\epsilon'/s + q'[(s-r-c)(\pi/s + \pi_1) + rq].\end{aligned}$$

If the price elasticity is non-increasing, then at any non-negative royalty rate,  $\Delta < 0$  since in equilibrium  $\pi_1 > 0$ . The claim follows. QED.

## 8 References

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