# NET Institute* 

## www.NETinst.org

Working Paper \#06-24
November 2006

# Consumers' Dynamic Switching Decisions in the Cellular Service Industry 

Jiyoung Kim<br>University of Wisconsin-Madison

[^0]
# Consumers' Dynamic Switching Decisions in the Cellular Service Industry 

Jiyoung Kim *<br>University of Wisconsin-Madison

Nov. 2006


#### Abstract

This paper develops an empirical framework to analyze consumer's dynamic switching decision in the cellular service industry. It first incorporates the sequential problem of quantity, plan and firm subscription choice in the presence of switching costs into a dynamic structural model, which allows for fully heterogeneous consumers and multiple switching possibilities across networks. The model is estimated using the data set on the number of switching consumers and the evolution of observed plan/firm characteristics over time. Based on the BLP-style estimation methods, we combine a nested technique that uses parametric assumptions with the structural estimation algorithm. The magnitude of switching costs is estimated and it turns out that switching costs vary across networks. A dynamic model with restricted number of switching is likely to underestimate the switching costs. Lower switching costs encourage consumers to switch relatively early. Change in the variety of optional plans and plan characteristics also play a great role in the consumers' switching decision.


[^1]
## 1. Introduction

As the telecommunication technology has been developing, the size of the network ${ }^{1}$ industry has been increasing. Network service providers usually offer multiple tariffs competing in the menu of plans as well as the quality of service. The price schedule is non-linear in quantity. In the cellular phone service market, especially, firms provide optional calling plans with diverse combinations of both calling rate and non-price plan specifics. Such a pricing scheme aims to attract more subscribers and exploit the heterogeneity in consumer preferences. If multiple nonlinear tariffs require subscription before starting to use services, changing subscription status can generate switching costs for consumers. Consumer decision consists of two steps, which are a firm/plan choice and a usage choice. In the first step, the existing customer has three options at each period while using a cellular service: to stay with the current provider, to switch to another plan within the same provider, and to switch to a different provider. Then in the second step, the existing customer subsequently makes a usage decision conditional on the chosen plan.

When consumers decide whether to switch their service provider or not, they will try to figure out which provider offers the best matching plan for them. The best match would vary across heterogeneous consumers. Therefore tariff variety can be a crucial pricing strategy for firms. More variety is able to attract consumers from competitors as well as to retain its own customers by increasing the possibility of offering better matching options. The switching costs can also affect consumer's switching decision and firm strategies ${ }^{2}$. Consumers will decide to switch if the potential savings from switching exceed its costs. High switching costs prevent consumers from choosing a different network even though they would realize that there exists a plan with a better match which was either not chosen or not available previously. Note that the existence of switching costs makes the subscription demand of the consumer a dynamic problem.

Understanding the consumer switching behavior in the network industry has important implications for the service providers as well as the policy makers. First, the

[^2]cellular service industry has imposed switching costs on consumers, a unique characteristic of the network industry. They have been considered as anticompetitive elements of the market by generating lock-in effect. Policy makers have tried to reduce the switching costs to encourage market competition. This paper allows us to examine the role of switching costs in the consumer dynamics. Second, as the cellular phone service market has grown rapidly and the penetration rate has increased, the number of new subscribers cannot help shrinking. In the mature market with a shrinking base of potential subscribers, stealing competitors' customers and retaining its own subscribers becomes one of the most important marketing strategies for the firms. Therefore, firms get more interested in understanding what factors the consumer's switching decisions depend on.

This paper develops an empirical framework examining the demand for the network good with multiple non-linear pricing schemes. It constructs the model of the rational forward-looking consumer's switching decision. Firms provide differentiated network services, requiring subscription first over the temporal horizon. We incorporate the consumer's sequential choice problem into the dynamics. Consumers are fully heterogeneous and are allowed to repeat switching over time. Therefore, our model and estimation methods are applicable to other network industries such as wire phone service, internet service, cable TV service and etc. The dynamic structural model allows us to investigate important questions about the source of switching behavior.

The main estimation methods are based on the BLP. The analysis is in the framework of a discrete choice model with random idiosyncratic variations. Fixed point calculation is used to compute the predicted switching rate. Then distributional assumption on the relation between unobserved firm attributes and instruments allows us to construct a GMM estimator. While in the context of the BLP, however, we take some different approach for estimating the dynamic optimization process. We impose the parametric assumptions on a distribution of future products and endogenous switching probability, and then obtain related parameters in the structural estimation algorithm. We assume rational expectation which implies that consumer expectations match the actual realization of the future products distribution. And estimated reduced-form parameters of switching probability would be consistent with the dynamic optimization problems.

The rest of the paper is as follows. The next section contains literature reviews. Section 3 discusses the model. Two subsections of Section 3 will contain the model which allows multiple switching possibilities and the model which restricts the number of switching respectively. Section 4 presents estimation methods. Data are described in Section 5. Section 6 provides the empirical results and finally Section 7 concludes the paper.

## 2. Literature Review

Some empirical papers have studied the discrete choice model for the demand in the telecommunication industry. Train, McFadden, and Beb-Akiba (1987) estimate a nested logit model where plan choice and quantity choice are made simultaneously. Miravete (2002) develops a model which accounts for sequential characteristics of plan choice and quantity choice. He shows that the monopoly screens consumers with multiple optional calling plans. Utility specification for calling quantity in this model follows that of this paper. Narayanan, Chintagunta and Miravete (2005) analyze the demand for monopolistic local telephone service using micro-level data in the same context as Miravete (2002). Similarly, Iyengar (2004) studies the demand for wireless service by constructing a structural model incorporating sequential decisions of plan and quantity choice. Since consumer choices are limited to plan and volume decision within one firm in these studies, the demand for firm specific subscription has not been considered. Even though Iyengar (2004) considers the disconnecting rate in his paper, most papers ignore the consumer's switching decision, focusing instead on the plan and usage choice. Economides (2006) uses panel data at the subscriber-level to investigate the consumer decision for quantity choice and subscription choice over time. Economides, Seim and Viard (2005) develop a model that explains the consumer's switching decision for an internet service provider using a panel data set.

The dynamic structure and estimation methods in this paper are closely related to a group of recent empirical literature that studies the consumer demand for durable goods. Melnikov (2001) develops a model which analyzes the dynamics of consumer demand for a differentiated durable product, a computer printer market. The optimal timing of
consumers' purchases is formalized for the homogeneous consumers using a logit utility specification. Carranza (2004) allows for heterogeneous consumers in studying the dynamic demand for the digital camera market. They incorporate the endogeneity of price in their models, but restrict purchase to at most one time. Gordon (2006) analyzes the homogenous consumers' demand for computer processors, allowing for repeated purchases. Gowrisankaran and Rysman (2006) specify and estimate a dynamic model of demand for DVD players. They consider fully heterogeneous consumers, and also allow them to purchase the durable good multiple times. Carranza (2005) and Gowrisankaran and Rysman (2006) take the estimation approach that part of the dynamic choice problem is approximated parametrically. The former adopts a logit approximation for the endogenous participation probability, and the latter specifies a linear assumption on the distribution of future product quality. Besides these two papers, the estimation method of our model is based on Berry (1994) and BLP (1995) which develop the heterogeneous consumers' discrete choice model under the endogeneity of price and Rust (1987) where the optimal stopping decision is analyzed.

In terms of switching costs, empirical estimation and impact evaluation of switching costs have been explored in various industries. Carlsson and Lofgren (2004) estimate the switching costs of domestic flight routes in Sweden's airline industry. Chen and Hitt (2002) develop and implement an approach for measuring switching costs and brand loyalty for online service providers. They conclude that online brokerage firms have significant control over their switching costs through product and service design. None of them is derived from a dynamic model. Therefore the magnitude of switching costs has the possibility of overestimation or underestimation.

## 3. Model

### 3.1 Model with repeated switching

Consumers have fully heterogeneous preferences. We consider the existing consumers of operating providers, who have already subscribed to one of the cellular service firms and have been using the cellular service. Each firm announces a menu of
calling plans in the beginning of the month ${ }^{3}$. Denote $M$ as the number of firms in the market. Existing consumers have three options. They can renew contract with their current provider, or choose different provider given updated information on available calling plans. And they can also quit using the cellular service phone and leave the market by choosing the outside option. Firms and consumers have infinite horizons and a common discount factor $\beta$.

In the presence of multiple nonlinear tariffs, an optimization problem of the consumer who stays in the market consists of three steps. First, a consumer decides whether to quit using the cellular service. Second conditional on staying in the market, she chooses the best matching plan which maximizes her discounted value of future expected utility among available plans provided by all the firms, conditional on her information at time $t$. If she keeps the plan of the previous period or changes the plan within her initial provider, switching costs are supposed to be zero since she stays with the initial service provider. We assume that switching costs do not occur as long as a consumer remains in the same firm. If she chooses a different service provider, she has to suffer the switching costs. Then she has to make a usage decision, conditional on the chosen plan in the first step. Random usage shock $v_{i t}$ occurs to every consumer at this moment ${ }^{4}$. Given the shock, she decides the optimal quantity of calling minutes and subsequently the total amount to be paid is determined. Note that a consumer cannot predict the exact realization of her usage in the stage of plan choice because usage shock would not be realized until the plan choice is made.

Calling option $j_{m}$ of firm $m$ at time $t$ is characterized by observed plan-specific characteristics $x_{j_{j_{n}} t}$, free monthly allowance $A_{j_{m} t}$, per-minute calling rate $p_{j_{m_{t}} t}$, monthly fee $a_{j_{j_{m}}}$, and unobserved firm characteristics $\boldsymbol{\xi}_{m t}$. Total amount of payment for $i, T_{i j_{m} t}$ can be written as $T_{i j_{m} t}\left(q_{i j_{m} t}\right)=A_{j_{j_{n} t}}+p_{j_{m} t} \max \left\{q_{i j_{m} t}-a_{j_{m} t}, 0\right\}$, where $q_{i j_{m} t}$ represents the quantity of calling minutes of consumer $i$ for plan $j_{m}$. Observed plan-specific characteristics include the number of available text messages, the number of discounted lines, the possibility of forwarding

[^3]remained allowance, and the additional non-call related service options such as caller ID. Note that $x_{j_{m} t}$ and $\xi_{m t}$ can be different over periods since the firm can change the plan options and unobserved firm attributes can vary over time. Let $c_{\text {in }}$ denote the switching costs that occur when consumer $i$ switches from $m$ to a different service provider. Switching costs of each firm $\left\{c_{i m}\right\}_{m=1, \ldots, M}$ are assumed to be constant across periods.

The static utility that consumer $i$ obtains from using $q$ minutes under rate plan $j_{m}$ of firm $m$ at time $t$ is specified by

$$
\begin{aligned}
& u_{i j_{m} t} \\
& =\left(v_{i t}+\log q_{i j_{m} t}\right) q_{i j_{m} t}+\gamma_{i} x_{j_{m} t}+\xi_{m t}-\alpha_{i} T_{i j_{m} t}+\varepsilon_{i j_{m} t} \\
& =u\left(q_{i j_{m} t}: v_{i t}\right)-\alpha_{i} T_{i j_{m} t}+\gamma_{i} x_{j_{m} t}+\xi_{m t}+\varepsilon_{i j_{m} t}
\end{aligned}
$$

where $\varepsilon_{i j_{m} t}$ is an idiosyncratic preference shock of the plan $j_{m}$ which captures random variations and is independent across each other, consumers, plans and time. It is assumed to follow the extreme value distribution. The static net utility depends on the calling quantity $q_{i_{j_{m}} t}$, the consumer's heterogeneity $\alpha_{i}$ and $\gamma_{i}$, and unobserved firm attributes $\xi_{m t}$. In particular, the first part of the static utility function that contains calling volume $q$ follows that of Miravete (2002). This specific functional form helps the optimization problem of the calling quantity to be solved more easily. $\alpha_{i}$ and $\gamma_{i}$ are the random coefficients that weights the total payment and plan-specific characteristics, and do not vary across time for a given consumer. Let $\theta_{i}$ denote a group of parameters, $\left(\alpha_{i}, \gamma_{i},\left\{c_{i m}\right\}_{m \in M}\right) . \alpha_{i}, \gamma_{i}$ is assumed to be distributed normally with mean $\alpha, \gamma$ and $c_{\text {im }}$ follows a left-truncated normal distribution over a range of $c_{i m}>0$. The variance matrix $\Sigma$ will be estimated in the model. Finally, subscribed consumers can stop using the service any time and choose to take an outside option such as a wire telephone service or an internet phone service. We normalize the flow utility from those outside options to zero.

We solve the consumer's sequential decision process by backward induction. First, consider how each consumer makes the volume decision conditional on plan choice $j_{m}$. The usage choices satisfy the following maximization problem.

$$
q_{i j_{m} t}\left(\alpha_{i}, \gamma_{i}, v_{i t}\right)=\underset{q}{\arg \max } u\left(q_{i j_{m} t} t v_{i t}\right)-\alpha_{i} T_{i j_{m} t}\left(q_{i j_{n} t}\right)+\gamma_{i} x_{j_{m} t}+\xi_{m t}+\varepsilon_{i j_{n} t}
$$

The total payment is a non-linear function and has kinks when the actual calling quantity equals the monthly free allowance. The demand function with usage shock $v_{i t}$ is given by

$$
\begin{aligned}
\exp \left(v_{i t}-\alpha_{i} p_{j_{m} t}\right) & \text { if } \\
q_{i j_{m} t}\left(v_{i t}\right) & >\log A_{j_{m} t}+\alpha_{i} p_{j_{m} t} \\
A_{j_{m} t} & \text { if } \log A_{j_{m} t}<v_{i t}<\log \alpha_{i} p_{j_{m} t} \\
\exp \left(v_{i t}\right) & \text { if } \quad v_{i t}<\log A_{j_{m} t}
\end{aligned}
$$

Note that the optimal calling quantity depends on $\alpha_{i}$ which varies across consumers. The variation in consumer tastes generates a differentiated marginal utility, which results in different optimal usage choice. Since usage shock $v_{i t}$ is unknown to consumers before the actual plan choice is made, they have uncertainty in their usage level prior to the plan choice. Therefore consumers have to compare the expected utility of each plan for the plan/switching decision. Assuming $v_{i t}$ follows the normal distribution $N\left(0, v^{2}\right)$, we can obtain the closed form of the expected utility for each plan.

$$
E_{v}\left[u_{i j_{m} t}\right]=u_{i j_{m} t}^{e}+\varepsilon_{i j_{m} t}
$$

$u_{i j_{m} t}^{e}$ denotes the part of the expected utility from plan $j_{m}$ of firm $m$ at time $t$ which does not account for potential switching costs and idiosyncratic variations. We make two important assumptions on switching costs. First, we assume that switching costs occur only once at the period of switching. Second, they are supposed to depend on which firm consumers switch from, not switch to ${ }^{5}$. Switching costs in the telecommunication industries mainly consist of compatibility costs, transaction costs and search costs. Compatibility costs occur when the network operator forces its subscribers to use the exclusive handsets. Transaction costs are associated with changing the phone numbers, visiting the store to cut off the service or paying the termination fee. And search costs are the costs which consumers have to bear for

[^4]collecting information about the services of other network providers ${ }^{6}$. Most of costs are likely to occur near the time of switching and to depend on the initially subscribed firm. The second assumption might not capture the compatibility costs which depend on the firm consumers switch to. However, the compatibility of handsets between networks has kept decreasing recently.

Now consider the dynamic decision that each consumer faces every month. Note that whichever service provider a consumer was subscribing to, she would choose the best matching plan offered by the firm. Let $\Omega_{t}$ be the market characteristics at time $t$ including current plan characteristics, firm attributes and other market environments. We assume the Markov chain $P\left(\Omega_{t+1} \mid \Omega_{t}\right)$ where the evolution of $\Omega_{t+1}$ depends only on the current state $\Omega_{t}$. In this model, consumers are allowed to switch providers every period if they want. Even though a consumer signed a long-term contract in the past, she can close the contract only by paying high termination fee in reality. Suppose consumer $i$ has subscribed to firm $m$ at period $t-1$. At period $t$ she can either stay with $m$ or switch to any different service provider $k(\neq m)$. The Bellman equation is defined as

$$
V_{i m t}\left(\varepsilon_{i t}, \Omega_{t}\right)=\max \left\{\begin{array}{l}
\max _{j_{m} \in m}\left(u_{i j_{m} t}^{e}+\varepsilon_{i j_{m} t}+\beta E\left[V_{i m t+1}\left(\varepsilon_{i t+1}, \Omega_{t+1}\right) \mid \Omega_{t}\right]\right),  \tag{1.1}\\
\max _{j_{k} \in k}\left(u_{i j_{k} t}^{e}-c_{i m}+\varepsilon_{i_{j} t}+\beta E\left[V_{i k t+1}\left(\varepsilon_{i t+1}, \Omega_{t+1}\right) \mid \Omega_{t}\right]\right), \\
0
\end{array}\right\}
$$

for firm $k \in M-m . \varepsilon_{i t}$ is a vector of idiosyncratic shocks across all the firms in the market. Note that the maximum attainable utility from switching contains the term for switching costs. We assume that consumers observe realized values of the current idiosyncratic shock $\varepsilon$ but know only its distribution for the future value. Given the i.i.d. assumption on the distribution of $\varepsilon$, the expectation of the value function at any period $\tau>t$ can be integrated as

$$
E V_{i m \tau}\left(\Omega_{\tau}\right)=\int_{\varepsilon} V_{i m \tau}\left(\varepsilon_{i \tau}, \Omega_{\tau}\right) d f(\varepsilon)
$$

Consumer $i$ 's Bellman equation depends on which firm she subscribed to at the previous period, since where consumers depart from would determine the magnitude of

[^5]the switching costs which is the source of the dynamic feature in the decision process. From (1), the consumer can choose to hold off switching and keep her current service provider, or to switch to one of the available plans provided by different firms. Bellman equation of this model has a difference with that of other literatures studying the dynamic consumption behavior for common durable goods. Unlike the one-shot purchase of general durable goods industry, cellular service firms provide multiple tariff options for their subscribers. Given any firm, consumers will choose the plan which maximizes their expected utility among all the plans offered by the firm (second maximization problem). And eventually they will pick the firm which offers the plan which generates the highest maximum attainable utility (first maximization problem) as long as it outweighs the outside option. The Bellman equation above captures these two maximization problems.

Now we consider a dynamic decision of consumers. Consumer $i$ would choose to switch from firm $m$ to any firm $k(\neq m)$ only if

$$
\begin{align*}
& \max _{j_{k} \in k}\left(u_{i j_{k} t}^{e}-c_{i m t}+\varepsilon_{i j_{k} t}+\beta E\left[E V_{i k t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right)  \tag{1.2}\\
& >\max _{j_{m} \in m}\left(u_{i i_{m} t}^{e}+\varepsilon_{i i_{m} t}+\beta E\left[E V_{i m t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right)
\end{align*}
$$

and

$$
\max _{j_{k} \in k}\left(u_{i j_{k} t}^{e}-c_{i m t}+\varepsilon_{i_{j_{k} t}}+\beta E\left[E V_{i k+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right)>0
$$

Let $h_{i m t}$ denote the probability that consumer $i$ chooses to switch from $m$ to any firm.

$$
\begin{equation*}
h_{i m t}=\operatorname{Pr}\binom{\max _{j_{k} \in k}\left(u_{i j_{k} t}^{e}-c_{i m t}+\varepsilon_{i j_{k} t}+\beta E\left[E V_{i k t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right)}{>\max \left\{\max _{j_{m} \in m}\left(u_{i j_{m} t}^{e}+\varepsilon_{i j_{m} t}+\beta E\left[E V_{i m+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right), 0\right\}} \tag{1.3}
\end{equation*}
$$

for all $k \in M-m$. Let

$$
\varphi_{i m t}=\max _{j_{m} \in m}\left(u_{i j_{m} t}^{e}+\varepsilon_{i i_{m} t}+\beta E\left[E V_{i m t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right)
$$

be the part of the maximum expected utility obtained from firm $m \in M$ at time $t$ without switching costs $c$ where $\varphi_{i t}=\left\{\varphi_{i m t}\right\}_{m \in M}$, and let

$$
\delta_{i j_{m} t}=u_{i j_{n} t}^{e}+\beta E\left[E V_{i m t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]
$$

be the part of the expected utility obtained from specific plan $j_{m}$ which excludes the current idiosyncratic shock $\varepsilon$. Given the distributional assumption on $\varepsilon$, we can prove that $\varphi_{i n t}$ is distributed the extreme value with mode $\delta_{\text {intt }}$ (Appendix A):

$$
\begin{equation*}
\delta_{i m t}=\ln \left(\sum_{j_{m} \in m} \exp \left(\delta_{i_{m} t}\right)\right) \tag{1.4}
\end{equation*}
$$

The logit value represents the expected discounted utility from subscribing to the specific service provider at each period except for the utility decrease due to switching costs. Since the logit inclusive value $\delta_{\text {int }}$ is a sufficient statistic for the distribution of $\varphi_{i m t}$, the switching participation rate $h_{\text {imt }}$ can be written as a function of the logit inclusive values and switching costs (Appendix B).

$$
\begin{equation*}
h_{\text {int }}\left(\Omega_{t}\right)=h_{i m t}\left(\delta_{\text {int }}\left(\Omega_{t}\right), \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}\left(\Omega_{t}\right)-c_{i m}\left(\Omega_{t}\right)\right)\right) \tag{1.5}
\end{equation*}
$$

Now consider the probability that consumer $i$ of firm $m$ chooses the outside option. For all $k \in M-m$, the quitting probability is given by

$$
\begin{aligned}
& h_{i m t}^{q}=\operatorname{Pr}\left[\begin{array}{l}
\max _{j_{m} \in m}\left(u_{i j_{m} t}^{e}+\varepsilon_{i j_{m} t}+\beta E\left[E V_{i m t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right)<0, \\
\text { and } \left.\max _{j_{k} \in k}\left(u_{i k_{k} t}^{e}-c_{i m}+\varepsilon_{i k_{k} t}+\beta E\left[E V_{i k+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right)<0\right]
\end{array}\right] \\
& =\operatorname{Pr}\left[\varphi_{\text {int }}<0, \phi_{i M-m t}<0\right] \\
& =F_{\varphi_{p_{w+1}}}(0) \cdot F_{\phi_{\phi_{M-m}}}(0) \\
& =h_{i m t}^{q}\left(\delta_{i m t}\left(\Omega_{t}\right), \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}\left(\Omega_{t}\right)-c_{i m}\left(\Omega_{t}\right)\right)\right)
\end{aligned}
$$

where the cumulative distributions of $\varphi_{i n t}$ and $\phi_{i M-m t}, F_{\varphi_{i n t}}$ and $F_{\phi_{t}}$ are shown in the appendix A and B respectively.

Now in order to solve the dynamic optimization problem of consumers, we need to know consumer $i$ 's expectations about the future utility from using the service and the switching costs which are likely to be affected by the evolution of future market characteristics. Because of the large potential dimensionality of $\Omega$, it is hard to solve (2) directly. So we make assumptions on the process of market evolution over time. First, the evolution of the logit inclusive values is assumed to be Marcovian as in Hendel and

Nevo(2003) and Gowrisankaran and Rysman(2006) . Specifically, the expected logit inclusive value at time $t+1$ depends only on the logit inclusive values at time $t^{7}$. Then the Marcov process of $\delta_{i m t+1}, P\left(\delta_{i m t+1} \mid \delta_{i t}\right)$ is assumed to be

$$
\begin{equation*}
P\left(\delta_{i m t+1} \mid \delta_{i t}\right)=\pi_{m 1}+\pi_{m 2} \delta_{i m t}+\pi_{m 3} \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)+\mu_{i m t} \tag{1.6}
\end{equation*}
$$

where $\delta_{i t}=\left\{\delta_{i k t}\right\}_{k \in M}$ and $\mu_{i m t}$ follows the normal distribution of mean zero and variance $\sigma_{\mu}^{2}$. Note that we allow interaction between the firms in determining the future logit inclusive values. It would be more reasonable than when $\delta_{i m t+1}$ is a function of only $\delta_{i m t}$ since a firm's strategy is likely to be affected by its competitors' action and the inclusive value captures the realization of such strategies. We also allow the parameters of the product evolution to vary across firms. Overall, this assumption benefits the estimation by reducing the dimensionality of the state space for the expectation of the value function.

Now we can rewrite the expected value function, switching participation probability from $m$, and quitting probability from $m$ as

$$
\begin{align*}
& V_{i m t}\left(\varepsilon_{i t}, \delta_{i t}\right)= \max \left\{\begin{array}{l}
\max _{j_{k} \in k \in M-m}\left(u_{i j_{k} t}^{e}-c_{i m}+\varepsilon_{i j_{k} t}+\beta E\left[E V_{i k t+1}\left(\delta_{i t}\right)\right]\right), \\
\max _{j_{m} \in m}\left(u_{i j_{m} t}^{e}+\varepsilon_{i j_{m} t}+\beta E\left[E V_{i m t+1}\left(\delta_{i t}\right)\right]\right)
\end{array}\right\}  \tag{1.7}\\
& h_{i m t}=h_{i m t}\left(\delta_{i m t}, \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right) \\
& h_{i m t}^{q}=h_{i m t}^{q}\left(\delta_{i m t}, \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right) \tag{1.8}
\end{align*}
$$

respectively for $k \in M-m$. Then the probability that consumer $i$ who has subscribed to $m$ at time $t-1$ switches to a specific provider $n$ at time $t$ consists of two components; first, the probability that she decides to change her initial provider $m$ to any other provider; and second, the probability that she chooses a specific firm $n$ over the other operating firms in the market. The former is given by (8). The latter, the probability that

[^6]$n$ offers the highest maximum attainable utility among the firms conditional on deciding to switch is defined as:
$$
h_{i m \rightarrow n t}=\frac{\exp \left(\delta_{i n t}-c_{i m}\right)}{\sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)}
$$

Therefore consumer $i$ 's firm specific switching probability from $m$ to $n, s_{i m \rightarrow n t}$ can be written as (9).

$$
\begin{equation*}
s_{i m \rightarrow n t}=h_{i m t}\left(\delta_{i m t}, \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right) \frac{\exp \left(\delta_{i n t}-c_{i m}\right)}{\sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)} \tag{1.9}
\end{equation*}
$$

Now we can calculate the predicted switching rate between firms for at any period $t$. Since the static utility function contains random coefficients across heterogeneous consumers, we need to integrate (9) over the joint distribution of random coefficients $\theta_{i}$ to obtain the aggregated predicted switching rates. And finally, we need to specify the distributional assumptions on the joint distribution of the unobservable firm attributes and instrument variables which will be mentioned later. Those assumptions will allow us to identify the parameters of the model. As in standard BLP literatures, unobserved firm characteristics $\xi_{m t}$ at each period are assumed to be orthogonal to the firm specific attributes. Let $f(\theta)$ denote the joint density function of the random coefficients of the utility function. Then the aggregated switching rate and quitting rate can be obtained as

$$
\begin{align*}
& s_{m \rightarrow n t}(\xi, \theta, \Sigma, \beta)=\int h_{m t}\left(\delta_{i n t}, \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right) \frac{\exp \left(\delta_{i n t}-c_{i n}\right)}{\sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i n}\right)} d f(\theta)  \tag{1.10}\\
& h_{m t}^{q}(\xi, \theta, \Sigma, \beta)=\int h_{i m t}^{q}\left(\delta_{i n t}, \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right) d f(\theta)
\end{align*}
$$

### 3.2 Model with restricted switching

In this subsection, we consider the restricted model where consumers switch their service providers only once. Most of the discussion is similar with the earlier section where repeated switching is allowed. However, the value function (7) would be
different since the consumers have restricted options after switching. Once they experience switching to a new service provider, consumers can change plans only within the same firm. The Bellman equation for consumer $i$ who has switched to firm $m$ at time $\tau<t$ is defined as

$$
\begin{equation*}
B_{i m t}\left(\varepsilon_{i t}, \Omega_{t}\right)=\max _{j_{m} \in m}\left(u_{i j_{m} t}^{e}+\varepsilon_{i j_{m} t}+\beta E\left[E B_{i m t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right], 0\right) \tag{1.11}
\end{equation*}
$$

And for consumer $i$ who subscribed to $m$ initially and has never experienced switching her provider until time $t$, the Bellman equation is given by

$$
V_{i m t}^{b}\left(\varepsilon_{i t}, \Omega_{t}\right)=\max \left\{\begin{array}{l}
\max _{j_{k} \in k}\left(u_{i_{k} t}^{e}-c_{i m}+\varepsilon_{i_{j_{k}} t}+\beta E\left[E B_{i k+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right),  \tag{1.12}\\
\max _{j_{m} \in m}\left(u_{i j_{m} t}^{e}+\varepsilon_{i j_{m} t}+\beta E\left[E V_{i m t+1}^{b}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right), \\
0
\end{array}\right\}
$$

for $k \in M-m$. Let

$$
\begin{aligned}
\varphi_{i m t}^{b} & =\max _{j_{m} \in m}\left(u_{i j_{m} t}^{e}+\varepsilon_{i j_{m} t}+\beta E\left[E B_{i m t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right) \\
\varphi_{i m t}^{v} & =\max _{j_{m} \in m}\left(u_{i j_{m} t}^{e}+\varepsilon_{i j_{m} t}+\beta E\left[E V_{i m t+1}^{b}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right)
\end{aligned}
$$

be the part of the maximum attainable utility obtained from $m$ when consumer $i$ already switched to $m$ at $\tau<t$ and when she has never switched since the first period respectively. And denote the part of the expected utility from specific plan $j_{m}$ that is not due to idiosyncratic shock as

$$
\begin{aligned}
\delta_{i j_{m} t}^{b} & =u_{i j_{m} t}^{e}+\beta E\left[E B_{i m t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right] \\
\delta_{i j_{m} t}^{v} & =u_{i j_{m} t}^{e}+\beta E\left[E V_{i m t+1}^{b}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]
\end{aligned}
$$

Given the logit distribution of $\mathcal{E}$, we can show that $\varphi_{i m t}^{b}$ and $\varphi_{i m t}^{v}$ are distributed extreme value with modes (13) respectively.

$$
\begin{align*}
& \delta_{i m t}^{b}=\ln \left(\sum_{j_{m} \in m} \exp \left(\delta_{i j_{m} t}^{b}\right)\right)  \tag{1.13}\\
& \delta_{i m t}^{v}=\ln \left(\sum_{j_{m} \in m} \exp \left(\delta_{i j_{m} t}^{v}\right)\right)
\end{align*}
$$

As in the earlier section, we assume that the Marcov process of firm evolution follows a linear specification for the logit inclusive values. In order to solve the dynamic optimization problem, the future evolution of the logit inclusive values is required. We have a new form of inclusive values for a consumer who has experienced switching. In particular, we have two sets of logit inclusive values for each firm, which are for consumers who have already switched and for those who have never switched. We define similar specifications for the expectation of future inclusive values with a different set of parameters $\pi_{m}^{b}=\left(\pi_{m 1}^{b}, \pi_{m 2}^{b}, \pi_{m 3}^{b}\right)$ and $\pi_{m}^{v}=\left(\pi_{m 1}^{v}, \pi_{m 2}^{v}, \pi_{m 3}^{v}\right)$ for any $m$ with an i.i.d. drift $\mu_{\text {int }} \sim N\left(0, \sigma_{\mu}^{2}\right)$. We assume that the logit inclusive values would be affected by interaction with the other competitors' inclusive values. We specify Marcov process as

$$
\begin{align*}
& P\left(\delta_{i m t+1}^{b} \mid \delta_{i t}^{b}\right)=\pi_{m 1}^{b}+\pi_{m 2}^{b} \delta_{i m t}^{b}+\pi_{m 3}^{b} \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}^{b}\right)+\mu_{i m t}^{b}  \tag{1.14}\\
& P\left(\delta_{i m t+1}^{v} \mid \delta_{i t}^{v}\right)=\pi_{m 1}^{v}+\pi_{m 2}^{v} \delta_{i m t}^{v}+\pi_{m 3}^{v} \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}^{v}\right)+\mu_{i m t}^{v}
\end{align*}
$$

Then (1.12) can be written as

$$
V_{i m t}^{b}\left(\varepsilon_{i t}, \delta_{i t}^{b}, \delta_{i t}^{v}\right)=\max \left\{\begin{array}{l}
\max _{j_{k} \in k}\left(u_{i k_{k} t}^{e}-c_{i m t}+\varepsilon_{i j_{k} t}+\beta E\left[E B_{i k t+1}\left(\delta_{i t}^{b}\right)\right]\right),  \tag{1.15}\\
\max _{j_{m} \in m}\left(u_{i j_{m} t}^{e}+\varepsilon_{i j_{m} t}+\beta E\left[E V_{i m t+1}^{b}\left(\delta_{i t}^{v}\right)\right]\right), \\
0
\end{array}\right\}
$$

Now consider how to obtain the predicted switching rate. Note that consumers who have experienced switching at time $\tau<t$ cannot contribute to the switching rate since we restrict multiple switching in this subsection. The probability that consumer $i$ of firm $m$ decides to switch at time $t$ consists of two parts: the probability that she decides to switch from $m$ to any other firm, and the probability that she chooses a specific firm $n$ conditional on deciding to switch. They are defined as

$$
\begin{aligned}
& h_{i m t}^{b}=h_{i m t}^{b}\left(\delta_{i m t}^{v}, \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}^{b}-c_{i m}\right)\right) \\
& h_{i m \rightarrow t t}^{b}=\frac{\exp \left(\delta_{i n t}^{b}-c_{i m}\right)}{\sum_{k \in M-m} \exp \left(\delta_{i k t}^{b}-c_{i m}\right)}
\end{aligned}
$$

We can also obtain the function of quitting probability. For a consumer to choose the outside option, the maximum attainable utility from subscribing to any firm must be smaller than zero.

$$
\begin{aligned}
& h_{i m t}^{b q}=\operatorname{Pr}\left[\begin{array}{l}
\max _{j_{m} \in m}\left(u_{i j_{m} t}^{e}+\varepsilon_{i j_{m} t}+\beta E\left[E V_{i m t+1}^{b}\left(\delta_{i m t}^{v}\right)\right]\right)<0, \\
\text { and } \max _{j_{k} \in k}\left(u_{i j_{k} t}^{e}-c_{i m}+\varepsilon_{i j_{k} t}+\beta E\left[E B_{i k+1}\left(\delta_{i m t}^{b}\right)\right]\right)<0
\end{array}\right] \\
& =\operatorname{Pr}\left[\varphi_{i m t}^{v}<0, \phi_{i M-m t}^{b}<0\right] \\
& =F_{\phi_{m t}^{b}}(0) \cdot F_{\phi_{M-m t}^{b}}(0) \\
& =h_{i m t}^{b q}\left(\delta_{i m t}^{v}, \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}^{b}-c_{i m}\right)\right)
\end{aligned}
$$

Because the predicted switching probability can be applied only to the consumers who have never switched their service providers, the firm specific switching probability is derived from the Bellman equation (15). Therefore, $h_{\text {int }}^{b}$ depends on its own inclusive value $\delta_{i m t}^{v}$ which includes the full option value from possible future switching and its competitors' inclusive values $\delta_{i k t}^{b}$ for $k \in M-m$ which contains the restricted option value from no future switching. Following the same step as in the earlier section, the probability that consumer $i$ who subscribed to $m$ at time $t-1$ decides to switch from $m$ to $n$ is

$$
\begin{equation*}
s_{i m \rightarrow n t}^{b}=h_{i m t}^{b}\left(\delta_{i n t}^{v}, \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}^{b}-c_{i m}\right)\right) \frac{\exp \left(\delta_{i n t}^{b}-c_{i m}\right)}{\sum_{k \in M-m} \exp \left(\delta_{i k t}^{b}-c_{i m}\right)} \tag{1.16}
\end{equation*}
$$

Then we can obtain the predicted aggregate switching rates and quitting rates by integrating (16) over the joint density distribution of $\theta$.

$$
\begin{align*}
& s_{m \rightarrow n t}^{b}(\xi, \theta, \Sigma, \beta)=\int \Gamma_{m t} h_{i n t}^{b}\left(\delta_{i m t}^{v}, \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}^{b}-c_{i m}\right)\right) \frac{\exp \left(\delta_{i n t}^{b}-c_{i m}\right)}{\sum_{k \in M-m} \exp \left(\delta_{i k t}^{b}-c_{i n}\right)} d f(\theta) \\
& h_{m t}^{b q}(\xi, \theta, \Sigma, \beta)=\int \Gamma_{m t} h_{i m t}^{b q}\left(\delta_{i n t}^{v}, \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}^{b}-c_{i m}\right)\right) d f(\theta) \tag{1.17}
\end{align*}
$$

Since the consumers who have experienced switching are assumed not to switch repeatedly, the ratio of potential switchers is included in the aggregate switching rates.

$$
\Gamma_{m t}=\frac{\Pi_{m t-1}\left(1-h_{m t-1}-h_{m t-1}^{q}\right)+\Pi_{m t-1}^{N}}{Q_{m t}}
$$

for $t>1$ and $\Gamma_{m 1}=1$ where $\Pi_{m t}=\Pi_{m t-1}\left(1-h_{m t-1}-h_{m t-1}^{q}\right)+\Pi_{m t-1}^{N}$ for $t>1$ and $\Pi_{m 1}=Q_{m 1}$. $Q_{m t}$ is the number of total subscribers at the beginning of period $t$. Finally, $\Pi_{m t}^{N}$ denotes the number of total subscribers and the number of new subscribers for firm $m$ during period $t$.

## 4. Estimation

Estimation of the model is based on the BLP technique. First, the fixed point algorithm allows us to compute the vector of the mean implied utilities with which predicted and observed aggregate switching rates become identical. Then next, the preference parameters and reduced-form parameters of the switching participation can be estimated by interacting the vector of unobserved firm attributes with instruments. The distributional assumption on orthogonality between $\xi$ and instrument variables $z$ is required. Finally, the transition matrix which is based on the linear regression (6) can be obtained by computing the dynamic programming problem conditional on a $\xi$ vector and parameters for every simulated consumer.

Estimated parameters $(\theta, \Sigma)$ include the mean switching costs across firms, the mean consumer tastes for the price and the plan-specific characteristics, and the variance of the parameters. We assume that consumers have the common discount factor as the monthly level of 0.99 , instead of estimating $\beta$. We focus on the estimation of the first model with multiple switching possibilities. To begin with, the predicted aggregate
switching rate from $n$ to $m$ (10) was obtained by integrating the individual's predicted switching probability over the joint distribution of the random coefficients.
Given the joint distribution of $f(\theta), D$ random draws from $f(\theta)$ can be simulated to compute a consistent estimator of the integral. We assume that the parameters $\theta_{i}=\left(\alpha_{i}, \gamma_{i},\left\{c_{i m}\right\}_{m \in M}\right)$ follow the normal distribution $\theta_{i} \sim N(\theta, \Sigma)$. Then (10) can be computed using $D$ random draws $\theta_{d}$ as

$$
\begin{align*}
& \hat{s}_{m \rightarrow n t}(\theta)=\frac{1}{D} \sum_{d=1}^{D} h_{d m t}\left(\theta_{d}\right) \frac{\exp \left(\delta_{d n t}\left(\theta_{d}\right)-c_{d m}\right)}{\sum_{k \in M-m} \exp \left(\delta_{d k t}\left(\theta_{d}\right)-c_{d m}\right)}  \tag{1.18}\\
& \hat{h}_{m t}^{q}(\theta)=\frac{1}{D} \sum_{d=1}^{D} h_{d m t}^{q}\left(\delta_{d m t}\left(\theta_{d}\right), \ln \sum_{k \in M-m} \exp \left(\delta_{d k t}\left(\theta_{d}\right)-c_{d m}\right)\right)
\end{align*}
$$

As discussed in the previous section, $h_{i m t}$ is a switching participation probability that consumer $i$ decides to switch from $m$ to any other service provider, and a function of $\left(\delta_{i n t}, \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right)$. Since the value function of consumer $i$, (8) has two maximization problems in its own maximization function, $h_{i m t}$ is a complicated exponential function of logit inclusive values. We adopt a parametric assumption on $h_{i m t}$ and estimate its parameters in the estimation process. Specifically, we impose a logistic approximation for the functional form of $h_{\text {imt }}$. Let $\lambda=\left(\lambda_{0}, \lambda_{1}, \lambda_{2}\right)$ be a set of reduced-form parameters.

$$
\begin{equation*}
\hat{h}_{i n t}=\frac{1}{1+\exp \left(\lambda_{0}+\lambda_{1} \delta_{i n t}+\lambda_{2} \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right)} \tag{1.19}
\end{equation*}
$$

We now mention two things about the parametric assumption. First, (19) is a reduced-form of the switching participation probability whose underlying parameters are $\left(\theta_{i}, \Sigma_{i}\right)$ of the model. This probability is the result of the consumer's dynamic optimization decision, so it may not have any closed-form solution. Second, the estimated parameters $\lambda$ cannot be used to simulate counterfactual scenarios since they are non-structural. But we can still derive the counterfactual equilibria using the
estimated structural parameters. For each simulated draw $\theta_{i}$, we can compute (3) directly and obtain the true switching participation rate of simulated consumer $i$.

Parameters $(\theta, \Sigma, \lambda)$ are estimated using the distributional assumption between the implied mean utilities with the instrument variables. To compute the implied mean utility, we utilize the fixed point algorithm as in Berry (1994) and BLP (1995). Define $\delta_{n t}^{c m}=u_{n t}^{e}-c_{m}$ for $n \in M-m$ and $\delta_{m t}^{f}=u_{m t}^{e}$. Then we perform the following fixed point calculation on $\boldsymbol{\delta}_{n t}^{c m}$ and $\boldsymbol{\delta}_{m t}^{f}$, where $\hat{\boldsymbol{S}}_{m \rightarrow n t}\left(\boldsymbol{\delta}^{c m}, \boldsymbol{\delta}_{m}^{f}\right)$ and $\hat{h}_{m t}^{q}\left(\boldsymbol{\delta}^{c m}, \boldsymbol{\delta}_{m}^{f}\right)$ is the predicted switching probability from $m$ to $n$ and the predicted quitting probability from $m$ respectively, which can be calculated from (18).

$$
\begin{align*}
& \delta_{n t}^{c m}=\delta_{n t}^{c m}+\left\{\ln \left(s_{m \rightarrow n t}\right)-\ln \left(\hat{s}_{m \rightarrow n t}\left(\delta^{c m}, \delta_{m}^{f}\right)\right)\right\} \\
& \delta_{m t}^{f}=\delta_{m t}^{f}+\left\{\ln \left(h_{m t}^{q}\right)-\ln \left(\hat{h}_{m t}^{q}\left(\delta^{c m}, \delta_{m}^{f}\right)\right)\right\} \tag{1.20}
\end{align*}
$$

After obtaining a vector of the mean utilities, the moment condition can be formed by interacting the unobservable firm attributes $\xi$ with a set of relevant instruments $z$. Instrument variables may be correlated with the prices but supposed to be exogenous to the unobserved firm attributes. The instruments $z$ include the following variables: the monthly ARPU (Average Revenue per User), the advertising costs and the number of plans and plan families of other firms. The pricing and marketing structures of competing firms affect the firm's own pricing strategy. However, the unobserved firm attributes are likely to be uncorrelated with them. Then GMM can be used to estimate the parameters from the predicted moments ${ }^{8}$.

Finally, we solve the individual dynamic optimization problem, conditional on a vector of $\left(\boldsymbol{\delta}^{c m}, \delta_{m}^{f}\right)$ and the estimated parameters. We consider fully heterogeneous consumers in the model. Therefore, the expected Bellman equation, the logit inclusive values and the transition of industry evolution are different across every consumer. In order to obtain these values we draw $\theta_{i}$ from its distribution. And for each draw, we iteratively update the logit inclusive values (4), the expected evolution of logit inclusive values from (6), and the value function (7) until convergence.

[^7]The estimation of the one-time switching model needs only a slight extension of a multiple- switching model. The only difference is that we have two groups of individuals, which are consumers who already switched and consumers who have never switched. So it is required to compute the optimization problems separately for each group. We use the simulation technique to obtain the predicted firm specific switching probability (9). A parametric assumption is still imposed on switching participation rate $h_{\text {int }}^{b}$ with parameters $\lambda^{b}$. Then fixed point calculation allows us to compute the groups of mean implied utilities. GMM is applied as in the multiple-switching model. In the final step, we update (13),(14) and (15) to complete solving for the optimization problems of consumers who have never switched.

## 5. Data

We apply the model which was addressed earlier to the Korean cellular service market. It is one of the most developed telecommunication markets in the world. It has exhibited remarkable growth since 1990s. As a result of dramatic diffusion, cellular service penetration reached up to 80 percentage of the population in 2005. The market has begun to show evidence that it has reached a fully mature stage of development. The growth rate of new subscription has slowed down, and call related qualities of networks have been converging due to technological improvement. It makes maintaining current customers and stealing consumers from their competitors more important for the firms. We have observed that each firm keeps offering new calling options and improving customer benefits periodically. Table 1 presents the average features of plan across providers.

As number portability ${ }^{9}$ has been introduced in the cellular service market, the Korean government collects the number of switching consumers between networks every month. Three firms have been operating in the market during the sample period, which are SK, KT and LG in order of market share. Timing of the policy enforcement was sequential

[^8]for the firms. It started from January 2004 for SK which is the first leading firm, July 2004 for KT which is the second leading firm and January 2005 for LG whose market share is the lowest. Figure 1, 2 and 3 show the switching rates of each firm after the enforcement of number portability. Switching rate of a firm is calculated as the ratio of the number of switchers from given firm at each month to the number of total consumers of the firm at the beginning of the month. .

We observe that firms have adjusted their menu of optional plans more actively by reducing prices, including more benefits (for example, more text messages or more free allowance) to the existing plans, or introducing new plans to the market during the sample period. Furthermore, considerable number of consumers switched their provider. In particular, we observe an interesting point in the evolution of switching rate. Switching rate tends to start at a high level, and drop fast. Then it again increases after experiencing a huge decrease.

Besides the monthly number of switchers, we utilize the following firm-level data; the monthly number of total subscribers, the monthly number of terminating consumers, the monthly number of new subscribers, the monthly ARPU and the quarterly advertising costs. Firms announce the updated information on their menu of optional plans whenever there are some changes, so the evolution of the menu of tariffs has been tracked.

## 6. Results

### 6.1 Estimated Parameters

The estimation results are shown in Table 1. The first column contains the parameter estimates from the consumer dynamic model which allows multiple-switching in Section 3.1. The second column provides the parameter estimates from the dynamic model where consumers are restricted to switch at most one time, which is addressed in Section 3.2.

First, consider the estimates from the dynamic model with repeated switching possibility. All of the mean plan specific characteristics have positive signs. More text messages and a higher level of customer benefits (the number of discounted lines and
the possibility of forwarding remained allowance to the next month) increase the utility of consumers. The mean coefficient of the total monthly payment is negative. In particular, the standard deviation level reveals that almost every consumer would be negatively affected by higher prices. As seen in Section 3.1, price coefficient $\alpha$ also determines the optimal calling quantity in the usage decision given optional plan. A consumer with higher $\alpha$ is likely to have smaller usage. There are considerable amounts of variation in consumer evaluation. But the utilities from calling service and the plan specifics turn out to be positive for most consumers. Overall, the estimated parameters show a reasonable sign and magnitude. The mean switching costs vary across firms and the differences are statistically significant. It implies that firms have different abilities to lock in their customers.

Reduced-form estimates of switching probability are also presented in the table. The probability that a consumer decides to switch from a specific firm is positively correlated with the inclusive values of its competitors including switching costs and is negatively correlated with its own inclusive value, as the signs of the coefficients imply. Higher inclusive value represents either more variety of plans or lower price. As competitors provide better menus of plans, it is likely for a firm to lose more consumers. Similarly, a firm can protect its customers from competitors by offering better plans or increasing product quality. Higher switching costs would discourage consumers from deciding to switch by decreasing the potential benefit from changing service providers. Figure 4 shows the switching probabilities of the mean consumer at the first period. Consumers with lower switching costs are more likely to switch their service provider than consumers with higher switching costs. The results imply that increasing its own switching costs can be one of a firm's strategies to retain customers.

Next, column 2 presents the estimates from the dynamic model with restricted switching. Signs of the plan specific characteristics vary. Some plan-specific characteristics have negative signs, but they are not statistically significant. The magnitudes of the estimates are generally similar to those of the dynamic model with multiple switching. No consistent change can be observed. The mean price coefficient is still negative and significant. Noticeable changes the multiple-switching model can be found in the estimates of the switching costs. The mean switching costs for every firm are lower than those for multiple-switching. Furthermore, they turn out to be
significantly different from the estimates in column 1 . When the number of switching is restricted, consumers have to stay with the same provider after experiencing switch unless they stop using the cellular service and choose an outside option. It decreases the variety of choice for consumers, which would consequently reduce the option values. Then the switching costs estimates may get smaller in order to match the observed switching probabilities conditional on less option values. Therefore, restricting consumer switching to at most one time is likely to underestimate the switching costs.

### 6.2 Additional results and Implications

We use the estimated parameters to investigate the dynamic features of consumer switching behavior and make predictions for switching decisions under different assumptions. As mentioned above, we cannot directly adopt the reduced-form parameters of switching participation rate for obtaining counterfactual equilibria. Instead, we solve the dynamic optimization problem for every simulated consumer using the estimated structural parameters. In this section, we present the application results for the SK which has the longest sample period.

First, we investigate variations in the switching rates under a counterfactual assumption on the fixed product evolution ${ }^{10}$. Figure 5 illustrates the evolution of switching rates when the variety of optional plans and plan characteristics including prices remain constant and consumers. We use the estimate results from the model where repeated switching is allowed. The dashed line is the path of the actual switching rates during our sample period that we observed ${ }^{11}$. The solid line shows the path of the simulated switching rate under the assumption on the same level of logit inclusive values for the firms would remain in the future. This figure shows that changes in the menu of plans and plan specific characteristics play an important role in switching decisions. The simulated switching rates would be higher than the observed rate at the beginning of our sample period. However, switching rate would decrease remarkably fast and appear to converge to zero by the end of our sample period. Variation in the

[^9]future inclusive values is the source of the option value from waiting. A stable future reduces the option values, so it accelerates early switching.

In order to understand the pattern of switching rates, we examine the average switching costs of switching consumers every period, and the difference in valuation of the firm which they switch from and the firm which they switch to. The results are illustrated in Figure 6 and Figure 7. Consumers who switch in earlier periods (first six months) have relatively lower switching costs. The average switching costs tend to increase. Furthermore, the difference in switching consumers' valuation for the firms is lower for the early switchers and increasing as the switching timing is delayed. Figure 8 illustrates the time path of the logit values for all the firms in the market. We can observe the increase in consumers' valuations. Remind that the number portability which can reduce switching costs has been enforced in the beginning of our sample period. These figures may suggest that remarkably high switching rates during the earlier periods are attributed to the consumers with relatively lower switching costs and that gradually increasing switching rates after those periods are due to the improvement of cellular services such as the introduction of more attractive plans or increase in the service quality.

Finally, we evaluate the ratio of repeated switching among the total switching rates. Figure 9 shows the simulation results of repeated switching ratio (dashed line) among the total switching rates (solid line). We find that repeated switching explains a very small fraction of the total switching rates. It implies that the duration of subscription tend to be more than 2 years for most consumers. Even though the multiple switching rates are relatively low, it is noticeable that the ratio appears to increase near the end of the sample period.

### 6.3. Fit of the model

In the process of our estimation, we impose some parametric specifications on the expected inclusive values and the switching probability for each firm. Consumer $i$ 's logit inclusive value of firm $m$ at time $t+1, \delta_{i m t+1}$ is assumed to depend on a vector of the logit inclusive values at time $t, \delta_{i t}$. The logit approximation is applied to
consumer $i$ 's switching probability from $m$ at time $t, h_{i m t}$ that is determined by its own switching costs and the inclusive values $\delta_{i t}$. The validity of these assumptions is examined in the following figures.

We use estimation results from the first dynamic model where repeated switching is allowed. First, Figure 10 provides the observed and the $95 \%$ interval of the predicted logit inclusive values for the mean consumer over time. We perform the same process for the switching probability of the mean consumer and the results are given in Figure 11. The model fits quite well for both cases.

## 7. Conclusion

This paper investigates the dynamics of consumer's optimal timing decision to switch providers across differentiated networks in the cellular service industry. It develops a structural model that accounts for consumer heterogeneity, the sequential choices resulting from the characteristics of pricing scheme, and the existence of switching costs. Consumers are allowed to switch repeatedly upon their prediction for the future as the market features change. We use a monthly data set on the number of consumers who switch their provider, firm attributes and plan specific characteristics. Number portability has been enforced during the sample period.

The estimation is basically in the range of the BLP-style technique. But the difference in our model is that we adopt a nested estimation method. Reduced-form specifications are imposed on the expectation of the market evolution process, and the switching participation rate which is a result of individual dynamic optimization problem. These parameters are estimated within the structural estimation algorithm. We examine the two dynamic models, the one which allows multiple-switching and the other which allows one-time switching only. Two dynamic models generate sensible estimates. However, the dynamic model with restricted number of switching is likely to underestimate the magnitude of the switching costs. We find that consumers differ substantially in their preferences and switching costs. Consumers with lower switching costs are likely to decide to switch their service provider earlier. Switching decision is
affected by improvement in product quality and price as well as the switching costs. Increase in consumers' valuation can encourage consumers to decide to switch by making it more likely for benefits from switching to exceed switching costs.

This paper focuses on the demand side, taking the plan changes and firm behaviors as exogenous. A more comprehensive work in the future will involve the research in the supply side which endogenizes firms' pricing and product entry decisions. Since most network industries provide non-linear pricing schemes, the supply side of the cellular service industry would have important implications.

## References

Berry, S. (1994). Estimating Discrete Choice Models of Product Differentiation, RAND Journal of Economics, 25(2), 242-262.

Berry, S., Levinsohn, J., and A. Pakes (1995). Automobile Prices in Market Equilibrium, Econometrica, 63(4), 841-890.

Carlsson, F. and A. Lofgren (2004). Airline Choice, Switching costs and Frequent Flyer Programs, Working papers in Econoimis no. 123.

Carranza, J.E. (2004). Empirical Analysis of the Evolution of Quality in Durable Goods' Markets, PhD Dissertation, Yale University, New Haven, CT.

Carranza, J.E. (2005). Demand for Durable Goods and the Dynamics of Quality, Manuscript, University of Wisconsin.

Chen, P. and L.M. Hitt (2002). Measuring Switching costs and Their Determinants in Internet Enabled Business: A Study of the Online Brokerage Industry, Information Systems Research, Fall 2002, forthcoming.

Economides, N., Seim, K. and B.V. Viard (2005). Quantifying the Benefits of Entry into Local Phone Service. Manuscript, New York University and Stanford University.

Gordon, B. (2006). Estimating a Dynamic Model of Demand for Durable Goods. Manuscript, Carnegie Mellon University.

Gowrisankaran, G. and M. Rysman (2006) Dynamics of Consumer Demand for New Durable Goods, Unpublished Manuscript, Washington University of St. Louis and Boston University

Hendel, I. and A. Nevo (2003). Measuring the Implications of Sales and Consumer Stockpiling, Working Paper, Northwestern University.

Inceoglu, F. and M. Park (2004). Diffusion of New Products under Network Effects, Manuscript, Sabanci University, Turkey.

Iyengar, R. (2004). A Structural Demand Analysis for Wireless Services under Nonlinear Pricing Schemes, Manuscript, University of Pennsylvania.

Klemperer, P. (1987). Competition When Consumers Have Switching Costs: An Overview with Applications to Industrial Organization, Macroeconomics, and International Trade, The Review of Economic Studies, 62, 515-539.

Melnikov, O. (2001). Demand for Differentiated Products: The Case of the U.S. Computer Printer Market. Manuscript, Yale University

Miravete, E.J. (2002). Estimating Demand for Local Telephone Service with Asymmetric Information and Optional Calling Plans, Review of Economic Studies, 69, 943-971.

Narayanan, S., P.K. Chintagunta, and E.J. Miravete (2005). The Role of Self Selection and Usage Uncertainty in the Demand for Local Telephone Service, Manuscript,

University of Chicago and University of Pennsylvania.
Nevo, A. (2000). A Practitioner's Guide to Estimation of Random Coefficients Logit Model of Demand. Journal of Economics and Management Strategy, 9, 513-548.

Nevo, A. (2001). Measuring Market Power in the Ready-to-eat Breakfast Cereal Industry, Econometrica, 69, 307-342.

Park, S. (2004). Quantitative Analysis of Network Externalities in Competing
Technologies: The VCR Case, Review of Economics and Statistics, 86, 937-945.
Rust, J. (1987). Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher, Econometrica, 55, 999-1033.

Song, I. and P. Chintagunta (2003). A Micromodel of New Product Adoption with Heterogenous and Forward-looking Consumers: An Application to the Digital Camera Category, Quantitative Marketing and Econometrics, 1, 371-407.

Train, K.E., D.L. McFadden, and M. Beb-Akiba (1987). The Demand for Local Telephone Service: A Fully Discrete Model of Residential Calling Patterns and Service Choices, RAND Journal of Economics, 18, 109-123.

## Appendix A: The distribution of $\varphi_{i n t}$

The same proof is contained in Melnikov (2001) and Carranza (2005).
Let $\varphi_{i m t}=\max _{j_{m} \in m}\left(u_{i j_{m} t}^{e}+\varepsilon_{i j_{m} t}+\beta E\left[E V_{i m t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right)$ is the maximum attainable utility that consumer $i$ get from firm $m$ at time $t$.

Let $v_{i j_{m} t}=u_{i m_{m} t}^{e}+\varepsilon_{i m_{m} t}+\beta E\left[E V_{i m t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]$

The cumulative distribution of $\varphi_{i m t}$ is given by,

$$
F_{\varphi_{i n t}}(z)=\operatorname{Pr}\left[\varepsilon_{i l t}<z-v_{i l t}, \cdots, \varepsilon_{i J_{m} t}<z-v_{i J_{m^{t}}}\right]
$$

Given the i.i.d. logit assumption on $\mathcal{\varepsilon}$,

$$
\begin{aligned}
& F_{\varphi_{p_{m t}}}(z)=\prod_{j_{m} \in J_{m}} \exp \left(-\exp \left(-z+v_{i_{j_{m}} t}\right)\right) \\
& =\exp \left(-\sum_{j_{m} \in J_{m}} \exp \left(-z+v_{i j_{m} t}\right)\right) \\
& =\exp \left(-\exp (-z) \sum_{j_{m} \in J_{m}} \exp \left(v_{i j_{m} t}\right)\right) \\
& =\exp \left(-\exp \left((-z)+\ln \sum_{j_{m} \in J_{m}} \exp \left(v_{i_{j_{m} t}}\right)\right)\right) \\
& =\exp \left(-\exp \left((-z)+\delta_{i m t}\right)\right)
\end{aligned}
$$

where $\delta_{i m t}=\ln \sum_{j_{m} \in J_{m}} \exp \left(v_{i j_{m} t}\right)$. Therefore, $\varphi_{i m t}$ is distributed Type I extreme value with the mode $\delta_{i n t}$.

## Appendix B: The switching participation rate $h_{i v t}$

Using the result from the appendix A, we can derive a functional form of the switching participation rate $h_{i n t}$ that consumer $i$ decides to switch from firm $m$ to any other provider at time $t$. For $k \in M-m$

$$
\begin{aligned}
& h_{i m t}=\operatorname{Pr}\left[\begin{array}{l}
\max _{j_{k} \in k}\left(u_{i j_{k} t}^{e}-c_{i m}+\varepsilon_{i j_{k} t}+\beta E\left[E V_{i m t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right) \\
>\max \left\{\max _{j_{m} \in m}\left(u_{i j_{m} t}^{e}+\varepsilon_{i j_{m} t}+\beta E\left[E V_{i m t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right), 0\right\}
\end{array}\right] \\
& =\operatorname{Pr}\left[\begin{array}{l}
\max _{j_{k} \in k}\left(u_{i j_{k} t}^{e}-c_{i m}+\varepsilon_{i j_{k} t}+\beta E\left[E V_{i m t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right) \\
>\max _{j_{m} \in m}\left(u_{i j_{m} t}^{e}+\varepsilon_{i_{i_{m} t}}+\beta E\left[E V_{i m t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right), \\
\text { and } \\
\max _{j_{k} \in k}\left(u_{i j_{k} t}^{e}-c_{i m}+\varepsilon_{i j_{k} t}+\beta E\left[E V_{i m t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right)>0
\end{array}\right] \\
& =\operatorname{Pr}\left[\begin{array}{l}
\max _{j_{k} \in k}\left(\delta_{i j_{k} t}+\varepsilon_{i j_{k} t}\right)-c_{i m}>\max _{j_{m} \in m}\left(\delta_{i_{i_{m} t}}+\varepsilon_{i j_{m} t}\right) \\
\text { and } \max _{j_{k} \in k}\left(\delta_{i j_{k} t}+\varepsilon_{i j_{k} t}\right)-c_{i m}>0
\end{array}\right]
\end{aligned}
$$

Let $\quad v_{i j_{k} t}=u_{i j_{k} t}^{e}+\varepsilon_{i j_{k} t}+\beta E\left[E V_{i k+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]$ for all $k \in M-m$.If we denote $\phi_{i M-m t}=\max _{j_{k} \in k \in M-m}\left(u_{i k_{k} t}^{e}-c_{i m}+\varepsilon_{i j_{k} t}+\beta E\left[E V_{i k t+1}\left(\Omega_{t+1}\right) \mid \Omega_{t}\right]\right)$, the cumulative distribution of $\phi_{i M-m t}$ is given by,

$$
\begin{aligned}
& F_{\phi_{\phi_{k}}}(z)=\prod_{k \in M-m} \prod_{j_{k} \in k} \exp \left(-\exp \left(-z+v_{i j_{k} t}-c_{i m}\right)\right) \\
& =\prod_{k \in M-m} \exp \left(-\exp (-z) \sum_{j_{k} \in k} \exp \left(v_{i j_{k} t}-c_{i m}\right)\right) \\
& =\prod_{k \in M-m} \exp \left(-\exp \left((-z)+\ln \sum_{j_{k} \in k} \exp \left(v_{i j_{k} t}-c_{i m}\right)\right)\right) \\
& =\prod_{k \in M-m} \exp \left(-\exp \left((-z)+\left(\delta_{i k t}-c_{i m}\right)\right)\right) \\
& =\exp \left(-\exp \left(-z+\ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right)\right)
\end{aligned}
$$

Now, the switching participation rate is,

$$
\begin{aligned}
& h_{i n t}=\operatorname{Pr}\left[\begin{array}{l}
\max _{j_{k} \in k}\left(\delta_{i_{j_{k} t}}+\varepsilon_{i j_{k} t}\right)-c_{i m}>\max _{j_{m} \in m}\left(\delta_{i j_{m} t}+\varepsilon_{i j_{m} t}\right) \\
\text { and } \max _{j_{k} \in k}\left(\delta_{i_{j_{k} t}}+\varepsilon_{i_{j_{k} t}}\right)-c_{i m}>0
\end{array}\right] \\
& =\operatorname{Pr}\left[\begin{array}{l}
\max _{j_{k} \in k}\left(\delta_{i j_{k} t}-c_{i m}+\varepsilon_{i j_{k} t}\right)>\max _{j_{m} \in m}\left(\delta_{i j_{m} t}+\varepsilon_{i j_{m} t}\right) \\
\text { and } \max _{j_{k} \in k}\left(\delta_{i j_{k} t}-c_{i m}+\varepsilon_{i j_{k} t}\right)>0
\end{array}\right] \\
& =\left[1-\exp \left(-\exp \left(-\max _{j_{m} \in m}\left(\delta_{i j_{m} t}+\varepsilon_{i j_{m} t}\right)+\ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right)\right)\right] \\
& \cdot\left[1-\exp \left(-\exp \left(\ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right)\right)\right] \\
& =\int_{\varphi_{i m t}}\left\{\begin{array}{l}
{\left[1-\exp \left(-\exp \left(-\max _{j_{m} \in m}\left(\delta_{i j_{m} t}+\varepsilon_{i j_{m} t}\right)+\ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right)\right)\right]} \\
\cdot\left[1-\exp \left(-\exp \left(\ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right)\right)\right] d f\left(\varphi_{i m t}\right) .
\end{array}\right. \\
& =\int_{\varphi_{\text {imt }}}\left\{\begin{array}{l}
{\left[1-\exp \left(-\exp \left(-\varphi_{i m t}+\ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right)\right)\right]} \\
\cdot\left[1-\exp \left(-\exp \left(\ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right)\right)\right] d f\left(\varphi_{\text {imt }}\right)
\end{array}\right\} \\
& =h_{i m t}\left(\delta_{i m t}, \ln \sum_{k \in M-m} \exp \left(\delta_{i k t}-c_{i m}\right)\right)
\end{aligned}
$$

The density function of $\varphi_{\text {imt }}, f\left(\varphi_{\text {intt }}\right)$ is a function of $m$ 's logit inclusive value $\delta_{i m t}$ as shown in Appendix A. Therefore, consumeri's switching participation rate from $m$ at time $t$ depends on the logit inclusive value of all the firms, and finally its own switching costs $c_{i m}$.

Table1. Average plan features across service providers

|  | November 2003 | May 2006 |
| :--- | :---: | :---: |
| Number of plan families | 4.33 | 7 |
| Number of plans | 32 | 44.33 |
| Monthly fee of basic plan | 15.58 (2006 US \$)* | 13.12 (2006 US \$) |
| Calling fee of basic plan | $0.022(2006$ US \$) | 0.019 (2006 US \$) |

* Won, Currency unit of Korea is converted to the US dollar after adjusting the price change.

Figure1. Switching rates of SK


Figure2. Switching rates of KT


Figure3. Switching rates of LG


Table2. Estimation results

| Parameter | Dynamic model with <br> Multiple-switching | Dynamic model with <br> Restricted switching |
| :--- | :---: | :---: |
| $\alpha$ | $-0.035(0.0062)^{*}$ | $-0.022(0.0093)^{*}$ |
| $\gamma$-number of txt | $0.196(0.0034)^{*}$ | $-0.025(0.011)^{*}$ |
| $\gamma$-forward allowance | $0.359(0.021)^{*}$ | $0.507(0.095)$ |
| $\gamma$-number of discounted lines | $0.151(0.083)$ | $0.162(0.027)^{*}$ |
| $c_{S K}$ | $4.029(0.712)^{*}$ | $1.171(1.015)^{*}$ |
| $c_{K T}$ | $3.291(0.463)^{*}$ | $1.625(0.163)^{*}$ |
| $c_{L G}$ | $2.778(1.408)$ | $1.008(4.808)$ |
| $\lambda_{0}$ | $5.443(2.005)^{*}$ | $3.003(2.150)$ |
| $\lambda_{1}$ | $0.208(0.078)^{*}$ | $0.420(0.065)^{*}$ |
| $\lambda_{2}$ | $-0.338(0.076)^{*}$ | $-0.915(0.115)^{*}$ |
| $\sigma_{\alpha}$ | $0.0108(0.0024)^{*}$ | $0.0123(0.0057)^{*}$ |
| $\sigma_{\gamma}$ | $0.0672(0.0105)^{*}$ | $0.0831(0.0559)$ |
| $\sigma_{c}$ | $1.487(0.393)^{*}$ | $0.623(0.0554)^{*}$ |

1) Standard errors in parenthesis; 2) * significant at $5 \%$ level

Figure4. Variation in switching probability


Figure5. Switching rates under the fixed product evolution


Figure6. Average switching costs of switching customers


Figure7. Difference in valuation for firms of switching consumers (average)


Figure8. Evolution of logit inclusive value


Figure9. Share of repeated switching


Figure10. 95\% interval of logit inclusive value


Figure11.95\% interval of switching participation probability



[^0]:    * The Networks, Electronic Commerce, and Telecommunications ("NET") Institute, http://www.NETinst.org, is a non-profit institution devoted to research on network industries, electronic commerce, telecommunications, the Internet, "virtual networks" comprised of computers that share the same technical standard or operating system, and on network issues in general.

[^1]:    * Department of Economics, University of Wisconsin-Madison, 1180 Observatory Drive, Madison, WI 53706. E-mail: jiyoungkim@wisc.edu. I would like to thank Juan Esteban Carranza, Jack Porter and Jean-Francois Houde for helpful comments. Financial supports from NET Institute, www.NETinst.org is gratefully acknowledged.

[^2]:    1 Network is composed of complementary nodes and links. A service delivered over a network requires the use of two or more network components.
    ${ }^{2}$ Klemperer (1987), Farrell and Shapiro (1988), Beggs and Klemperer (1992)

[^3]:    ${ }^{3}$ Consumers can obtain information on the pricing schemes, customer benefits and other related new through the firm webpage, advertisement or visit to the local agency.
    ${ }^{4}$ Random usage shock captures the unexpected factors or situations which can affect individual's consumption behavior. It is time-specific and is supposed to last no longer than a month. For example, sudden illness or business trip are likely to change consumer's normal usage pattern.

[^4]:    ${ }^{5}$ The model can be extended to have different switching costs across the firms which consumers switch to.

[^5]:    ${ }^{6}$ Klemperer (1995)

[^6]:    ${ }^{7}$ Since we deal with the inclusive values $\delta_{i m t}$ not individual $\delta_{i j_{m} t}$, high inclusive value can indicate both two cases, which are 1) firm $m$ is offering the large number of optional plans with relatively high prices 2) firm $m$ is offering the small number of optional plans with relatively low prices.

[^7]:    ${ }^{8}$ Nevo (2000)

[^8]:    ${ }^{9}$ Number portability is supposed to reduce switching costs by allowing consumers to keep their current phone number even after changing their service provider. Also the process of switching providers has been simplified. Many countries including Hong Kong, UK and USA have enforced number portability for their telecommunication market.

[^9]:    ${ }^{10}$ The evolution of products indicates the change in the plan characteristics and variety.
    ${ }^{11}$ Observed switching rates are consistent with the switching rates which are generated by the estimated model since the model matches the estimated rate with the actual rate by the structure.

