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Working Paper #05-23

October 2005

The Incentive To Participate In Open Source Projects: A Signaling Approach

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The incentive to participate in open source projects: a signalling approach*

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Current draft: October 30, 2005

Abstract

This paper examines the incentives of programmers to contribute to open source software projects on a voluntary basis. In particular, the paper looks at this incentive changes as (i) performance becomes more visible to the relevant audience, (ii) effort has a stronger impact on performance, and (iii) performance becomes more informative about talent. In all three cases, it is shown that whether we start from a stable interior equilibrium or an unstable interior equilibrium.

*The financial support of the NET insititute (http://www.NETinst.org) is gratefully acknowledged. For helpful discussions, I thank Jean Tirole.

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1 Introduction

Open source software (OSS) is a computer program whose source code - the instructions for the program, written in a human readable format - is distributed free of charge and can be modified, extended, adapted, and incorporated into other programs with relatively few restrictions. OSS is a rapidly expanding phenomenon: some OSS such as the Apache web server, dominate their product categories. In the personal computer market, the OSS such as the operating system Linux and the web browser Firefox gain rapid popularity. It is estimated that there are currently 29 million users of Linux worldwide and there were over 50 million downloads of Firefox.[1]

Apart from having millions of OSS users, there are also tens of thousands of participating programmers who contribute to various OSS projects, and there is also a growing number of firms who sell services, support, and documentation for OSS. The majority of the programmers who participate in OSS projects are unpaid volunteers. For example, Hars and Ou (2002) have surveyed 81 individuals involved in open source projects and found that only 16% received any direct monetary compensation for their contribution. This raises obvious questions about the incentives and motivations of the participating programmers who do not receive direct compensation for their efforts. There are three main, mostly complimentary, explanations for the willingness of programmers to contribute to OSS projects. The first two involve intrinsic motivations while the third involves extrinsic motivations.

The first explanation is that programmers simply like to be involved in open source projects, either because they simply enjoy being creative, or due to a sense of obligation or community related reasons. Indeed, a web-based survey conducted by Lakhani and Wolf (2003) reveals that the responding programmers were mainly driven by enjoyment-based intrinsic motivations.

The second explanation involves another type of intrinsic motivation. According to this explanation, system managers (e.g., users of Apache) who need improvements in software and are willing to make these improvements on their own. They then share these improvements with others in their community. A model along these lines is offered by Johnson (2001), who views

[1] See http://counter.li.org/estimate.php for the estimate on Linux and www.mozilla.org/products/firefox for the estimate on Firefox.

participation in OSS projects as a private provision of a public good (see Bessen, 2004, for a related model).

The third explanation, suggested by Lerner and Tirole (2002), is that programmers are willing to contribute to OSS projects in order to signal their ability to future employers, venture capitalists, or to peers and thereby boost their human capital or get ego gratification. Fershtman and Gandal (2004) examine a large data set on programmers' participation in OSS projects and argue that their findings are consistent with the hypothesis that programmers who contribute to OSS projects are driven by extrinsic motivations such as their desire to enhance their social status within the programmers' community or by their desire to signal their ability to potential employers. Hann et al (2004), examine a longitudinal data set of participant contributions made and accepted into three Apache open source projects for the period 1998 to 2002. They find that more contributions to the Apache open source projects do not result in wage increases for contributors. On the other hand, successful participation in the form of a higher status in the merit-based ranking within the Apache open source community is associated with a 13% - 27% increase in wages, depending on the rank attained. These findings are robust to various model specifications and remain true even after controlling for work and programming experience. Hann et al argue that their results are consistent with the notion that a high rank within the Apache Software Foundation is a credible signal of the productive capacity of a programmer.

Drawing on the "career concerns" literature (e.g., Holmström, 1999), Lerner and Tirole (2002) conjecture that the signalling incentive will become stronger as (i) performance becomes more visible to the relevant audience, (ii) effort has a stronger impact on performance, and (iii) performance becomes more informative about talent. The purpose of this paper is to examine these conjectures in the context of a formal model. The main finding in the paper is that the model always admit a no-effort equilibrium in which firms do not expect programmers to exert effort in order to contribute to OSS projects, and programmer in turn do not exert such effort. However, the model may also admit an even number of interior equilibria, half of which are stable and the other half is unstable. The analysis shows that the three conjectures are correct only if we start from a stable interior equilibrium but are incorrect if we start from an unstable interior equilibrium.

There are two closely related papers that also argue that programmers participate in

OSS projects in order signal their abilities to prospective employers. The two papers however differ from the current paper both in terms of their set up and in terms of their main focus. Lee, Moisa, and Weiss (2003) consider a model in which programmers need to choose between joining closed source software firms or OSS projects. If they join software firms, their wage reflects the expected productivity of all programmers who join software firms (talented ones and less talented ones). On the other hand, if they join OSS projects, they forgo current wages, but can signal their productivity to software firms and hence boost their future wages. The main focus of their analysis is on the relative sizes of the closed source system and the open-source system. In particular, their show that an open-source system will never exist alone in the market because mediocre programmers, who cannot benefit from signaling their talent, will always prefer to joint closed source software firms. On the other hand, a closed-source system can exist alone in the market, especially if the population of talented programmers is relatively small.

Leppämäki and Mustonen (2004) consider a model in which programmers signal their talent to software firms by choosing how many lines of code to contribute to an OSS project. As in the traditional Spence signalling model, talented programmers have a lower cost of writing lines of code. Consequently, in a separating equilibrium, only talented programmers contribute to the open source project and their contribution is chosen so as to deter untalented programmers from mimicking them. The model departs from the traditional Spence signalling model in that the freely available OSS project imposes either a positive or a negative externality on the commercial software offered by firms. The externality in turn affects the wages that software firms are willing to offer agents and hence the marginal benefit to signalling. Leppämäki and Mustonen focus on the effect of the externality on the incentive of talented agents to contribute to the OSS project. In particular, they show that if the OSS is a substitute (complement) for the commercial software then the contribution of talented programmers will end up being lower (higher) than in the case where OSS and the commercial software are independent of each.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 shows that the model can give rise to multiple equilibria and characterizes them. Section 4 study the comparative static properties of the model and in particular examines how the incentive to contribute to OSS projects is affected by the visibility of the contribution to prospective employers, by the sensitivity of performance to effort, and by how informative is the performance about talent. Finally, I examine the effect of intrinsic motivation to contribute to OSS projects in Section 5.

2 The model

Consider a competitive job market with a large number of agents, each of whom is either "talented" (i.e., has a high productivity) or "untalented" (i.e., has a low productivity). The marginal productivity of each talented agent if he is hired is w, while the marginal productivity of an untalented agent if he is hired is normalized to 0. Under full information, the wage of each agent is equal to his marginal product. Hence, the wage of talented agents is w while the wage of untalented agents is 0.

Under asymmetric information, it is common knowledge that the fraction of talented agents in the population is α, but firms cannot tell the agents' types apart before hiring them. To signal their types, agents can engage in some activity before they are hired by firms. Participation in an OSS project provides a good opportunity for talented agents to signal their ability due to the resulting exposure they get from peers. Specifically, I assume that when agents participate in an OSS project, they can either succeed (i.e., "solve a problem") or they can fail (i.e., "fail to come up with satisfactory results"). In particular, if an agent is talented and exerts effort e in the OSS project, his probability of success is $p(e, \gamma)$, where γ is a shift parameter. With probability $1 - p(e, \gamma)$ the agent fails. On the other hand, if the agent is untalented, his action succeeds with probability p_0 which is independent of his effort level. Since untalented agents cannot boost their probability of success, they do not exert any effort.

In and of itself, the activity does not benefit the firms nor the agents directly (for now I ignore the intrinsic motivation to participate in the OSS project). The only advantage of the activity from the firms and the agents' perspective is that it generates a signal on the agents' types. Firms cannot observe directly observe the efforts that the agents exert; rather they can only (imperfectly) observe whether the agent's activity has succeeded. In particular, firms observe a successful action with probability β. With probability $1-\beta$, as well as when the activity fails, firms observe nothing. Hence, β is a measure of the visibility of the agents' performance to potential employers. Whenever firms observe nothing, they cannot discern whether the agent has participated in the OSS project and did not succeed or whether he did not participate at all.

Using subscripts to denote partial derivatives, I make the following assumptions on the probability that a talented agent will succeed:

A1 $p_e(e,\gamma) > 0 > p_{ee}(e,\gamma)$

A2 $\lim_{e\to\infty} p_e(e,\gamma) = 0$

A3 $p(0,\gamma) = p_0 \geq 0, \quad \lim_{e\to\infty} p(e,\gamma) = 1$

A4 $p_\gamma(e,\gamma) > 0, \quad p_{e\gamma}(e,\gamma) > 0$

Assumption A1 says that effort raises the probability of success but does so at a decreasing rate. Assumption A2 implies that at the limit as e increases, the marginal effect of effort on the probability of success goes to 0. This assumption will ensure the existence of a solution to the maximization problem of agents. Assumption A3 says at one extreme, if talented agents do not exert effort, then their probability of success is equal to that of untalented agents, while on the other extreme, if their effort increases indefinitely, their probability of success approaches 1 in the limit. Assumption A4 implies that the shift parameter γ raises both the probability and the marginal probability that the activity will succeed. Hence, when γ increases, effort has a stronger impact on an agent's performance.

The payoff of each agents is increasing with his wage and decreasing with his effort level:

$$U = w - e.$$

3 Equilibrium

I now look for a Perfect Bayesian equilibrium in which talented agents exert effort, untalented agents do not exert effort, and the beliefs of firms are consistent with the agents' strategies. To characterize this equilibrium, suppose that firms believe that the effort of talented agents is $\widehat{e}$. Then, conditional on observing a successful action, firms believe that the agent is talented with probability

$$q(\widehat{e},\gamma \mid s) = \frac{\alpha p(\widehat{e},\gamma)}{\alpha p(\widehat{e},\gamma) + (1-\alpha)p_0}. \tag{1}$$

On the other hand, if firms do not observe a success, they cannot tell whether (i) the agent is talented, exerted effort, and failed, or (ii) the agent is talented, exerted effort and succeeded, but his success was unobserved, (iii) the agent is untalented and failed, or (iv) the agent is untalented, succeeded nonetheless, but his success was not observed. Hence, conditional on not observing a successful action, firms believe that the agent is talented with probability

$$\begin{aligned} q(\widehat{e},\gamma \quad | \quad n) &= \frac{\alpha\left((1-p(\widehat{e},\gamma)) + (1-\beta)p(\widehat{e},\gamma)\right)}{\alpha\left((1-p(\widehat{e},\gamma)) + (1-\beta)p(\widehat{e},\gamma)\right) + (1-\alpha)\left((1-p_0) + (1-\beta)p_0\right)} \\ &= \frac{\alpha(1-\beta p(\widehat{e},\gamma))}{1-\beta\left(\alpha p(\widehat{e},\gamma) + (1-\alpha)p_0\right)}. \end{aligned} \tag{2}$$

Note that given Assumption A3, $q(0,\gamma \mid s) = q(0,\gamma \mid n) = \alpha$: if firms expect talented agents to exert no effort, then success or failure is not an informative signal about the agent's talent. Moreover, note that $q(\widehat{e},\gamma \mid s)$ approaches 1 as p_0 approaches 0: if untalented agents cannot succeed then success is a sure sign that the agent is talented.

Next, we need to find the effort level that talented agents will exert. To this end, note that since the labor market is competitive, the wage of agents is $q(\widehat{e},\gamma \mid s)w$ following an observed success and $q(\widehat{e},\gamma \mid n)w$ otherwise. Hence, the expected payoff of talented agents given their effort level, e, and given the belief of firms, $\widehat{e}$, is

$$U(e) = \beta p(e,\gamma)q(\widehat{e},\gamma \mid s)w + \left((1-\beta)p(e,\gamma) + 1 - p(e,\gamma)\right)q(\widehat{e},\gamma \mid n)w - e. \tag{3}$$

The first term on the left-hand side reflects the idea that with probability $\beta p(e,\gamma)$, the talented agent's action succeeds and his success is observed by firms. The second term on the left-hand side reflects the idea that with probability $(1-\beta)p(e,\gamma)$, the successful action of a talented agent is not observed by firms and with probability $1-p(e,\gamma)$ it fails altogether. In both cases firms cannot tell whether the agent is talented or not and they pay him a wage $q(\widehat{e},\gamma \mid n)w$. The last term on the left-hand side of the equation is the agent's cost of effort.

Assuming that there is a large number of talented agents, each will ignore the effect of his own effort level on $\widehat{e}$. Since Assumption A1 ensures that $U''(e) < 0$, the effort level that each talented agent will choose given the firms' beliefs, $\widehat{e}$, is defined implicitly by the following first order condition:

$$U'(e) = \beta p_e(e,\gamma)\Delta(\widehat{e},\gamma)w - 1 \leq 0, \qquad eU'(e) = 0, \tag{4}$$

where

$$\begin{aligned} \Delta(\widehat{e},\gamma) &\equiv q(\widehat{e},\gamma \mid s) - q(\widehat{e},\gamma \mid n) \\ &= \frac{\alpha p(\widehat{e},\gamma)}{\alpha p(\widehat{e},\gamma) + (1-\alpha)p_0} - \frac{\alpha(1-\beta p(\widehat{e},\gamma))}{1-\beta\left(\alpha p(\widehat{e},\gamma)+(1-\alpha)p_0\right)} \\ &= \frac{\alpha\left(1-\alpha\right)\left(p(\widehat{e},\gamma)-p_0\right)}{\left(\alpha p(\widehat{e},\gamma)+(1-\alpha)p_0\right)\left(1-\beta\left(\alpha p(\widehat{e},\gamma)+(1-\alpha)p_0\right)\right)}, \end{aligned} \tag{5}$$

is the increase in the probability that firms assign to an agent being talented following an observed success. The expression $\beta p_e(e,\gamma)\Delta(\widehat{e},\gamma)w$ represents the marginal benefit from effort which is equal to the marginal effect of effort on the probability that a successful action will be observed, $\beta p_e(e,\gamma)$, times the extra wage that an agents gets in this event, $\Delta(\widehat{e},\gamma)w$. At an interior optimum, this marginal benefit must be equal to the marginal cost of effort, which is 1. But, if $\beta p_e(e,\gamma)\Delta(\widehat{e},\gamma)w$ is smaller than 1 for all positive effort levels, then the talented agent will not exert any effort.

Before proceeding, it is worth noting that

$$\Delta_{\widehat{e}}(\widehat{e},\gamma) = \frac{\alpha\left(1-\alpha\right)p_e(\widehat{e},\gamma)\left[\beta\alpha^2\left(p(\widehat{e},\gamma)-p_0\right)^2 + p_0(1-\beta p_0)\right]}{\left(\alpha p(\widehat{e},\gamma)+(1-\alpha)p_0\right)^2\left(1-\beta\left(\alpha p(\widehat{e},\gamma)+(1-\alpha)p_0\right)\right)^2} > 0. \tag{6}$$

That is, if firms believe that talented agents exert more effort, then observed success leads to a larger increase in the probability that firms assign to an agent being talented. Recalling that $\Delta(\widehat{e},\gamma)w$ is the extra expected wage that an agent receives following an observed success, this implies that as firms believe that talented agents exert more effort, they are willing to pay higher wages to agents who were observed to be successful. Moreover, since by Assumption A3, $q(0,\gamma \mid s) = q(0,\gamma \mid n) = \alpha$, then $\Delta(0,\gamma) = 0$. Hence, if firms believe that talented agents do not exert effort, then observed success does not increase their assessment that the agent is talented.

Let $BR(\widehat{e})$ denote the solution of (4). This function is the best-response of each talented agent against the firms' beliefs about his effort level. In equilibrium, the firms' beliefs must be consistent with the true efforts of the talented agents. Hence, the equilibrium effort level, e^*, is defined implicitly by the equation

$$e^* = BR(e^*). \tag{7}$$

In other words, the equilibrium is defined by the intersection of the best response function, $BR(\widehat{e})$, with the 45^o line in the $(e, \widehat{e})$ space. Given its central role in what follows, I now study the properties of $BR(\widehat{e})$ in the next lemma. To establish this lemma, I first make the following assumption on the marginal productivity of a talented agent if he is hired by a firm:

A5 The marginal productivity of a talented agent is such that

$$w > \overline{w} \equiv \frac{(\alpha + (1-\alpha)p_0)\,(1 - \beta\,(\alpha + (1-\alpha)p_0))}{\beta\alpha\,(1-\alpha)\,(1-p_0)\,p_0}. \tag{8}$$

Lemma 1: *Suppose that Assumption A5 holds. Then, the best response of talented agents against the firms' beliefs about their effort levels, $BR(\widehat{e})$ has the following properties:*

(i) *Suppose that $BR(\widehat{e}) = 0$ for all $0 < e \leq \widehat{e}_1$ and $BR(\widehat{e}) > 0$ for all $e > \widehat{e}_1$, where $\widehat{e}_1$ is implicitly defined by the equation $\beta p_0 \Delta(\widehat{e}_1, \gamma) w = 1$.*

(ii) *$BR'(\widehat{e}) > 0$ for all $e > \widehat{e}_1$ and $\lim_{\widehat{e}\to\infty} BR'(\widehat{e}) = 0$.*

Proof: (i) First, note that since $\Delta(0, \gamma) = 0$, $U'(e) = -1$ when $\widehat{e} = 0$, so $BR(0) = 0$. Otherwise, if $\widehat{e} > 0$, then $\Delta(\widehat{e}, \gamma) > 0$. Since $p_{ee}(e, \gamma) < 0$, $U'(e)$ is a strictly decreasing function of e for all $\widehat{e} > 0$. Assumption A2 implies that as e goes to infinity, $U'(e)$ goes to -1. Hence, $U'(e) = 0$ attains a unique interior solution if and only if

$$U'(0) = \beta p_0 \Delta(\widehat{e}, \gamma) w - 1 > 0, \tag{9}$$

where the equality follows because by Assumption A3, $p(0,\gamma) = p_0$.

Since $\Delta(0,\gamma) = 0$, condition (9) clearly fails when $\widehat{e} = 0$, and by continuity, it also fails for sufficiently small values of $\widehat{e}$. On the other hand, since $\Delta_{\widehat{e}}(\widehat{e},\gamma) > 0$, an increase in $\widehat{e}$ raises $U'(0)$. Recalling from Assumption A3 that $\lim_{e\to\infty} p(e,\gamma) = 1$, it follows that in the limit, as $\widehat{e}$ increases,

$$\lim_{\widehat{e}\to\infty} \Delta(\widehat{e},\gamma) = \frac{\alpha(1-\alpha)(1-p_0)}{(\alpha+(1-\alpha)p_0)(1-\beta(\alpha+(1-\alpha)p_0))}.$$

This implies in turn that (9) can be satisfied for a large enough $\widehat{e}$ if and only if

$$\lim_{\widehat{e}\to\infty} U'(0) = \frac{\beta\alpha(1-\alpha)(1-p_0)p_0 w}{(\alpha+(1-\alpha)p_0)(1-\beta(\alpha+(1-\alpha)p_0))} - 1 > 0. \tag{10}$$

A sufficient condition for $\lim_{\widehat{e}\to\infty} U'(0) > 0$ is that $w > \overline{w}$, where $\overline{w}$ is defined by (8).

Therefore, whenever $w > \overline{w}$, there exists a unique value of $\widehat{e}$, denoted $\widehat{e}_1$, such that $U'(0) > 0$ for all $\widehat{e} > \widehat{e}_1$ and $U'(0) < 0$ otherwise, where $\widehat{e}_1$ is implicitly defined by the equation $U'(0) = \beta p_0 \Delta(\widehat{e},\gamma) w - 1 = 0$.

This implies in turn that for all $\widehat{e} \leq \widehat{e}_1$, $U'(e) < 1$ for all e so $BR(\widehat{e}) = 0$. On the other hand, for all $\widehat{e} > \widehat{e}_1$, $U'(e) > 0$ for sufficiently small values of e. Since $U'(e)$ is a strictly decreasing function of e and since $U'(e)$ goes to -1 as e goes to infinity, it follows that whenever $\widehat{e} > \widehat{e}_1$, there exists a unique value of e that solves the equation $U'(e) = 0$. Hence, $BR(\widehat{e}) > 0$ for all $\widehat{e} > \widehat{e}_1$.

(ii) As part (i) shows, $BR(\widehat{e}) > 0$ for all $\widehat{e} > \widehat{e}_1$ and it is defined implicitly by the equation $U'(e) = 0$. That is, $U'(BR(\widehat{e})) = 0$. Fully differentiating this equation with respect to $\widehat{e}$ and rearranging terms, yields

$$BR'(\widehat{e}) = -\frac{p_e(e,\gamma)\Delta_{\widehat{e}}(\widehat{e},\gamma)}{p_{ee}(e,\gamma)\Delta(\widehat{e},\gamma)} > 0, \tag{11}$$

where the inequality follows because $p_{ee}(e,\gamma) < 0$ and because $\Delta_{\widehat{e}}(\widehat{e},\gamma) > 0$. To complete the proof, note that as $\widehat{e}$ increases so does e. However, Assumption A2 shows that $\lim_{e\to\infty} p_e(e,\gamma) = 0$. Hence, $BR'(\widehat{e})$ goes to 0 as $\widehat{e}$ goes to infinity. ■

Using Lemma 1, I can now characterize the equilibrium effort level of talented agents. To this end, recall from (7) that the equilibrium condition is given by $e^* = BR(e^*)$. Since $BR(\widehat{e})$ passes through the origin, $e^* = 0$ is a solution to the equilibrium condition. Hence, there always exists a no-effort equilibrium in which talented agents are not expected to exert effort and in fact do not exert effort. The question is whether there are additional solutions to the equilibrium condition $e^* = BR(e^*)$?

To address this question, I present $BR(\widehat{e})$ in Figure 1, using Lemma 1. The figure shows $BR(\widehat{e})$ in the $(e, \widehat{e})$ space. As the figure shows, $BR(\widehat{e})$ coincides with the vertical axis for sufficiently small values of $\widehat{e}$. As $\widehat{e}$ increases above $\widehat{e}_1$, $BR(\widehat{e})$ increases with $\widehat{e}$. Since $BR'(\widehat{e})$ goes to 0 as $\widehat{e}$ goes to infinity, $BR(\widehat{e})$ eventually becomes very steep.[2] Figure 1 also shows the 45^0 line. The equilibrium effort level of talented agents is determined by the intersection of $BR(\widehat{e})$ with the 45^0 line. As the figure shows, there are in general two possibilities depending on the shape of $BR(\widehat{e})$.

The first possibility, illustrated in Figure 1a, arises when $BR(\widehat{e})$ intersects the 45^0 line only at $e = 0$. In this case, the model does not admit interior equilibria in which $e^* > 0$. A sufficient (though not necessary) condition for case (i) is that $BR'(\widehat{e}) < 1$ for all $\widehat{e} > \widehat{e}_1$. The second possibility, illustrated in Figure 1b, arises when $BR(\widehat{e})$ intersects the 45^0 line at least once from above at some $\widehat{e} > \widehat{e}_1$. In this case, we do have interior equilibria in which $e^* > 0$. But, since $BR'(\widehat{e})$ goes to 0 as $\widehat{e}$ goes to infinity, $BR(\widehat{e})$ must intersect the 45^0 line at least one more time but from below. Hence, if there are interior equilibria in which $e^* > 0$, then their number must be even. A necessary condition for the model to admit only two interior equilibria (apart from the no-effort equilibrium) is that $BR''(\widehat{e}) < 0$. Using (11), it follows that this is the case whenever

$$BR''(\widehat{e}) = -\frac{p_e(e,\gamma)}{p_{ee}(e,\gamma)} \frac{d}{d\widehat{e}} \left[\frac{\Delta_{\widehat{e}}(\widehat{e},\gamma)}{\Delta(\widehat{e},\gamma)} \right].$$

Since $p_{ee}(e,\gamma) < 0$, it follows that $BR''(\widehat{e}) < 0$ if and only if $\frac{d}{d\widehat{e}} \left[\frac{\Delta_{\widehat{e}}(\widehat{e},\gamma)}{\Delta(\widehat{e},\gamma)} \right] < 0$.

I summarize this discussion in the following Proposition:

[2] Note that since Figure 1 shows $BR(\widehat{e})$ in the $(e, \widehat{e})$ space, a steep curve is associated with small value of $RB'(\widehat{e})$.

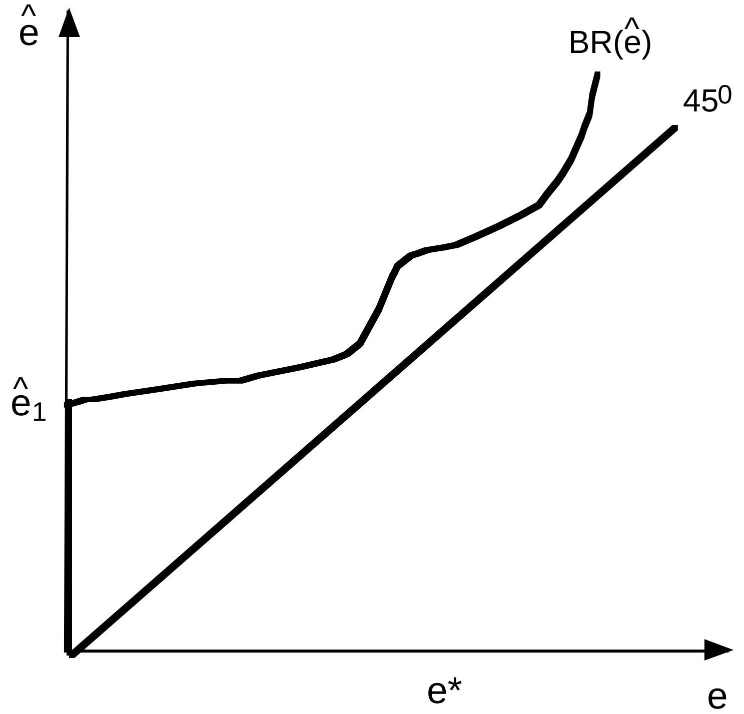

Figure 1a: No interior equilibria

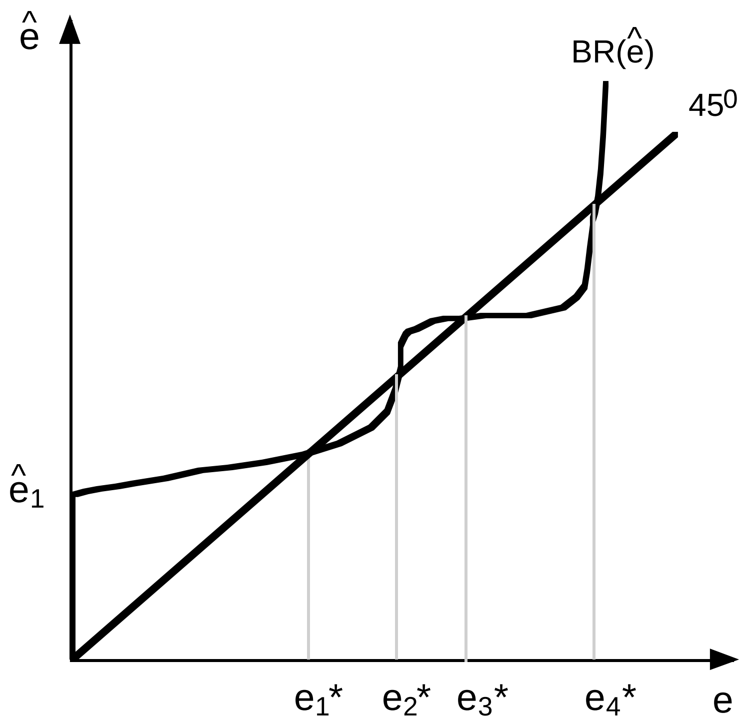

Figure 1b: Four interior equilibria

Proposition 1: *A sufficient condition for the no-effort equilibrium to be unique is that* $BR'(\widehat{e}) < 1$ *for all* $\widehat{e} > \widehat{e}_1$*. If however the model admits interior equilibria in which* $e^* > 0$*, then their number must be even.*

Next, suppose that there exist interior equilibria in which $e^* > 0$. Recalling that in equilibrium the firms' beliefs must be consistent with the true efforts of the talented agents, i.e., $\widehat{e} = e^*$, and substituting this equality into equation (4), the equilibrium effort level, e^*, is implicitly defined by

$$\beta G(e^*, \gamma)w = 1, \qquad G(e^*, \gamma) \equiv p_e(e^*, \gamma)\Delta(e^*, \gamma). \tag{12}$$

It should be noted that the left-hand side of equation (12) differs from the left-hand side of equation (4) because in the latter, the beliefs of firms about the efforts of talented agents are arbitrary, while in the former they are consistent with the true efforts of talented agents. Hence, $\beta G(e^*, \gamma)w$ can be interpreted as the marginal benefit of effort from an agents' point of view in equilibrium (i.e., given that firms hold correct beliefs about the agent's effort).

In the next section, I will study the comparative statics properties of e^*. Since the function $G(e^*, \gamma)$ plays a key role in that analysis, I now establish an important property of $G(e^*, \gamma)$.

Lemma 2: $G_e(e^*, \gamma) \geq (<) 0$ *as* $-\frac{p_{ee}(e^*,\gamma)}{p_e(e^*,\gamma)} \leq (>) \frac{\alpha}{w(1-\alpha)} \left(1 + \frac{p_0(1-\beta p_0)}{(p(e^*,\gamma)-p_0)^2}\right)$.

Proof: Straightforward differentiation reveals that

$$\begin{aligned} G_e(e^*, \gamma) &= p_{ee}(e^*, \gamma)\Delta(e^*, \gamma) + p_e(e^*, \gamma)\Delta_e(e^*, \gamma) \\ &= \left[\frac{p_{ee}(e^*, \gamma)}{p_e(e^*, \gamma)} + \frac{\Delta_e(e^*, \gamma)}{\Delta(e^*, \gamma)}\right] G(e^*, \gamma). \end{aligned} \tag{13}$$

But, using equations (5), (6), and (12), yields

$$\begin{aligned} \frac{\Delta_e(e^*, \gamma)}{\Delta(e^*, \gamma)} &= \frac{p_e(e^*, \gamma)\left[\beta\alpha^2\left(p(e^*, \gamma) - p_0\right)^2 + p_0(1 - \beta p_0)\right]}{\left(\alpha p(e^*, \gamma) + (1-\alpha)p_0\right)\left(1 - \beta\left(\alpha p(e^*, \gamma) + (1-\alpha)p_0\right)\right)\left(p(e^*, \gamma) - p_0\right)} \\ &= \frac{\alpha}{(1-\alpha)w}\left(1 + \frac{p_0\left(1 - \beta p_0\right)}{\left(p(e^*, \gamma) - p_0\right)^2}\right) \end{aligned} \tag{14}$$

Substituting from (14) into (13),

$$G_e(e^*,\gamma) = \left[\frac{p_{ee}(e^*,\gamma)}{p_e(e^*,\gamma)} + \frac{\alpha}{w\,(1-\alpha)}\left(1+\frac{p_0\,(1-\beta p_0)}{(p(e^*,\gamma)-p_0)^2}\right)\right] G(e^*,\gamma). \qquad (15)$$

The result follows by noting that the sign of $G_e(e^*,\gamma)$ depends on the sign of the square bracketed expression. This expression can be either negative or positive since $\frac{p_{ee}(e^*,\gamma)}{p_e(e^*,\gamma)}$ is negative by Assumption A1, while $\frac{\alpha}{w(1-\alpha)}\left(1+\frac{p_0(1-\beta p_0)}{(p(e^*,\gamma)-p_0)^2}\right)$ is positive. ■

Lemma 2 shows that $G(e^*,\gamma)$ may either increase or decrease with e. Intuitively, holding the belief of firms, $\widehat{e}$, constant, the marginal benefit of effort from an agent's point of view is decreasing with effort because effort raises the likelihood of success at a decreasing rate. Hence at first glance it would seem that $G(e^*,\gamma)$ should be decreasing with e. However, when talented agents exert more effort (and this is anticipated by firms), their extra wage following a success increases. This effect raises the marginal benefit of effort, which in turn implies that $G(e^*,\gamma)$ should be increasing with e. The first, negative, effect is more likely to dominate the second, positive, effect when α is small (there are few talented agents in the population) and when w is large (the productivity of talented agents is large); in both cases, the wage gap between success and failure is particularly large. Hence, $G_e(e^*,\gamma)$ is likely to be negative when α is small and when w is large and positive when α is large and w is small.

At a more technical level, note from (13) and (11) that

$$\begin{aligned} G_e(e^*,\gamma) &= \left[1+\frac{p_e(e^*,\gamma)\Delta_e(e^*,\gamma)}{p_{ee}(e^*,\gamma)\Delta(e^*,\gamma)}\right] p_{ee}(e^*,\gamma)\Delta(e^*,\gamma) \\ &= \left[1-BR'(e^*)\right] p_{ee}(e^*,\gamma)\Delta(e^*,\gamma). \end{aligned}$$

Since $p_{ee}(e^*,\gamma)\Delta(e^*,\gamma)$, it follows that $G_e(e^*,\gamma) \geq 0$ if $BR'(e^*) > 1$ and $G_e(e^*,\gamma) < 0$ if $BR'(e^*) < 1$. To interpret these conditions, note that $BR'(e^*)$ is just the slope of the best response function of talented agents against the beliefs of firms evaluated at the equilibrium effort level. When $BR'(e^*) > 1$, the best response at the equilibrium point, $BR(e^*)$, is steeper than the 45^0 line and hence cuts it from below. On the other hand, when $BR'(e^*) < 1$, $BR(e^*)$, is flatter than the 45^0 line and hence cuts it from above.

Notice that when $BR(e^*)$ cuts the 45^0 line from below (e.g., the equilibria e_2^* and e_4^* in

Figure 1b), the resulting equilibrium is stable in the sense that a Cournot tatônnement process will lead to a convergence to the equilibrium point starting from any close neighborhood of the equilibrium point. On the other hand, when $BR(e^*)$ cuts the 45^0 line from above (e.g., the equilibria e_1^* and e_3^* in Figure 1b), the resulting equilibrium is unstable. Hence,

Proposition 2: *Suppose that the model admits interior equilibria in which $e^* > 0$. Then, a given interior equilibrium is stable if $G_e(e^*, \gamma) < 0$ and unstable if $G_e(e^*, \gamma) \geq 0$.*

Since Lemma 2 indicates that $G_e(e^*, \gamma) < 0$ is more likely when α is small and w is large while $G_e(e^*, \gamma) \geq 0$ is more likely when α is large and w is small, one can conclude that stable equilibria are more likely when α is small and w is large while unstable equilibria are more likely when the reverse is true.

4 Comparative statics

Given Lemma 2, I now examine the conjectures of Lerner and Tirole (2002) that the signalling incentive of agents is stronger:

(i) the more visible the performance to the relevant audience,

(ii) the higher the impact of effort on performance, and

(iii) the more informative the performance about talent.

4.1 The effect of the visibility of performance on effort

To examine conjecture (i), recall that β is a measure of the visibility of the agents' performance to firms. Hence, I examine conjecture (i) by looking at the effect of an increase in β on e^*:

Proposition 3: *An increase in β which measures the visibility of the agents' performance to firms, increases the effort level that talented agents exert in stable interior equilibria but lowers it in unstable interior equilibria.*

Proof: Differentiating equation (12) with respect to e^* and β and rearranging terms,

$$\frac{\partial e^*}{\partial \beta} = -\frac{G(e^*,\gamma) + \beta \frac{\partial G(e^*,\gamma)}{\partial \beta}}{\beta G_e(e^*,\gamma)},$$

where

$$\begin{aligned}
\frac{\partial G(e^*,\gamma)}{\partial \beta} &= p_e(e^*,\gamma)\frac{\partial \Delta(e^*,\gamma)}{\partial \beta} \\
&= p_e(e^*,\gamma)\frac{\alpha(1-\alpha)(p(\widehat{e},\gamma)-p_0)}{(\alpha p(\widehat{e},\gamma)+(1-\alpha)p_0)}\frac{\alpha p(\widehat{e},\gamma)+(1-\alpha)p_0}{(1-\beta(\alpha p(\widehat{e},\gamma)+(1-\alpha)p_0))^2} \\
&= p_e(e^*,\gamma)\Delta(e^*,\gamma)\frac{\alpha p(\widehat{e},\gamma)+(1-\alpha)p_0}{(1-\beta(\alpha p(\widehat{e},\gamma)+(1-\alpha)p_0))} > 0.
\end{aligned}$$

The sign of $\frac{\partial e^*}{\partial \beta}$ is equal to the sign of $-G_e(e^*,\gamma)$ which by Proposition 2 is positive in stable interior equilibria and negative in unstable interior equilibria. ■

Proposition 3 is illustrated in Figure 2. The equilibrium effort level, e^*, is attained at the point where $\beta G(e^*,\gamma)w$, which is the equilibrium marginal benefit of effort, cuts the horizontal line whose height is 1 and which represents the marginal cost of effort. An increase in β shifts the equilibrium marginal benefit of effort upward. Whether this leads to an increase or a decrease in e^* depends on whether $G(e^*,\gamma)$ is upward or downward sloping. When $G(e^*,\gamma)$ is downward sloping, which as Lemma 2 shows is likely to occur when α is small and w is large, an increase in β leads to an increase in e^*. On the other hand, when α is large and w is small, $G(e,\gamma)$ is likely to upward sloping so an increase in β lead to a decrease in e^*.

Proposition 3 shows that Lerner and Tirole's (2002) conjecture that the signalling incentive of agents will become stronger as their performance becomes more visible to the relevant audience is true only if the model admits interior equilibria and then only in interior equilibria that are stable. This a likely to be the case when there are few talented agents around (α is small) and when talented agents earn a high wage (w is high). Otherwise, this conjecture is incorrect: the signalling incentive of agents will become stronger as the agents' effort becomes less visible.

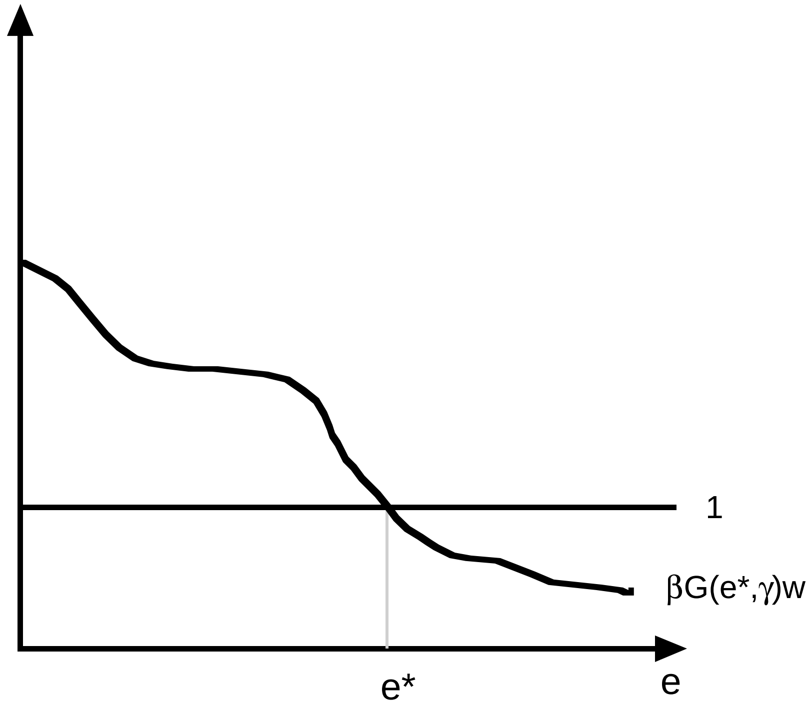

Figure 2a: α is small and w is large - G(e*,γ) is downward sloping

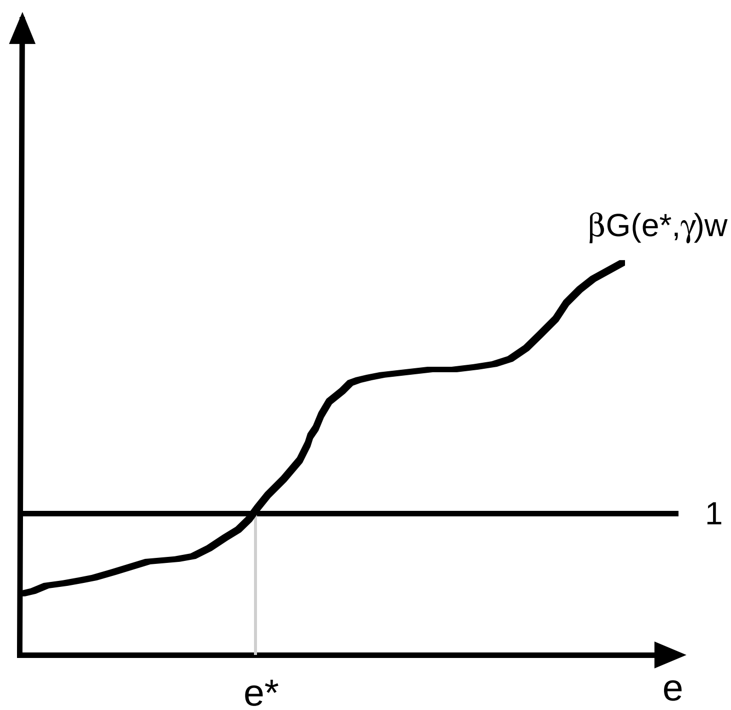

Figure 2b: α is large and w is small - G(e*,γ) is upward sloping

4.2 The effect of the sensitivity of performance to effort on effort

Next, I examine conjecture (ii) that the signalling incentive of agents will become stronger as effort has a stronger impact on performance. Recalling that effort has a larger impact on the probability of success when γ increases, it is obvious that in order to examine this conjecture I need to study the effect of an increase in γ on e^*:

Proposition 4 : *An increase in γ which ensures that effort has a larger impact on the probability that the action will succeed increases the effort level that talented agents exert in the activity in stable interior equilibria and decreases it in unstable interior equilibria.*

.

Proof: Differentiating equation (12) with respect to e^* and γ and rearranging terms,

$$\frac{\partial e^*}{\partial \gamma} = \frac{G_\gamma(e^*,\gamma)}{-G_e(e^*,\gamma)} = \frac{p_{e\gamma}(e^*,\gamma)\Delta(e^*,\gamma) + p_e(e^*,\gamma)\Delta_\gamma(e^*,\gamma)}{-G_e(e^*,\gamma)},$$

where $p_{e\gamma}(e^*,\gamma) > 0$ by Assumption A3, and

$$\Delta_\gamma(e^*,\gamma) = \frac{\alpha(1-\alpha)\, p_\gamma(\widehat{e},\gamma)\left[\beta\alpha^2\left(p(\widehat{e},\gamma)-p_0\right)^2 + p_0(1-\beta p_0)\right]}{\left(\alpha p(\widehat{e},\gamma)+(1-\alpha)p_0\right)^2\left(1-\beta\left(\alpha p(\widehat{e},\gamma)+(1-\alpha)p_0\right)\right)^2} > 0.$$

Hence, the sign of $\frac{\partial e^*}{\partial \gamma}$ is equal to the sign of $-G_e(e^*,\gamma)$ which is positive in stable interior equilibria and negative in unstable interior equilibria. ■

As in the case of an increase in β, an increase in γ shifts $G(e,\gamma)$ upward. Hence, an increase in γ will also lead to more effort by talented agents if $G(e,\gamma)$ is decreasing with e and to less effort if $G(e,\gamma)$ is increasing with e. Again, Lemma 2 shows that $G(e,\gamma)$ is decreasing with e if α is small and w is large, but increasing with e if α is large and w is small. Hence, Lerner and Tirole's (2002) conjecture that the signalling incentive will become stronger as effort has a stronger impact on performance is true only when initially, there are few talented agents around (α is small) and when talented agents earn a high wage (w is high). Otherwise, this conjecture is incorrect: the signalling incentive of agents will become stronger as the agents' effort has a weaker impact on their performance.

4.3 The effect of the informativeness of performance about talent on effort

Conjecture (iii) of Lerner and Tirole states that the signalling incentive of agents will become stronger as performance becomes more informative about talent. This conjecture can be examined by studying the effect of a change in the parameter p_0 on the equilibrium effort level of talented agents, e^*. This is because a decrease in p_0 implies that a successful agent is more likely to be talented; that is, when p_0 decreases towards 0, $q(\widehat{e}, \gamma \mid s)$, which is the probability that a successful agent is talented, increases towards 1.

Proposition 5 : *Suppose that* $\Delta_{p_0}(e^*, \gamma) > 0$*. Then a decrease in* p_0 *which ensures that performance is more informative about talent, increases the effort level that talented agents exert in the activity in stable interior equilibria but decreases it in unstable interior equilibria. If* $\Delta_{p_0}(e^*, \gamma) < 0$ *then the reverse is true.*

Proof: Differentiating equation (12) with respect to e^* and γ and rearranging terms,

$$\frac{\partial e^*}{\partial p_0} = \frac{G_{p_0}(e^*, \gamma)}{-G_e(e^*, \gamma)} = \frac{p_e(e^*, \gamma)\Delta_{p_0}(e^*, \gamma)}{-G_e(e^*, \gamma)},$$

where using the notation $x \equiv \alpha p(\widehat{e}, \gamma) + (1 - \alpha)p_0$,

$$\Delta_{p_0}(e^*, \gamma) = \frac{dq(\widehat{e}, \gamma \mid s)}{dp_0} - \frac{dq(\widehat{e}, \gamma \mid n)}{dp_0} < 0,$$

where the inequality follows because (1) implies that $\frac{dq(\widehat{e},\gamma|s)}{dp_0} < 0$ while (2) implies that $\frac{dq(\widehat{e},\gamma|n)}{dp_0} > 0$. Hence, the sign of $\frac{\partial e^*}{\partial p_0}$ is equal to the sign of $G_e(e^*, \gamma)$, which by Proposition 2 is negative in stable interior equilibria and positive in unstable interior equilibria. Hence, a decrease in p_0 raises e^* in stable interior equilibria and lowers e^* in unstable interior equilibria. ■

Like Propositions 3 and 4, a decrease in p_0 shifts $G(e, \gamma)$ upward. When $G(e, \gamma)$ is decreasing with e, which is the case in stable interior equilibria, this shifts induces more effort. On the other hand, in unstable interior equilibria, $G(e, \gamma)$ is increasing with e so the decrease in p_0 induces less effort. As before, whether the equilibrium is stable or not depends, among other things, on whether α is small and w is large or conversely. In any event, once again the conjecture is true only in stable interior equilibria but not true otherwise.

5 Intrinsic motivation for participation in OSS projects

Up to now I only considered extrinsic motivation for participation in OSS projects. Talented agents took place in these projects in order to try and generate positive signals about their talent and hence boost their prospects in the labor market. However, this view is obviously too narrow given that many participants contribute to OSS projects for other reasons like their sense of creativity, or the desire to solve problems that they face in performing daily tasks (like system managers). The question is how such intrinsic motivations are going to effect matters.

To address this issue, suppose that apart from their ability to boost their prospects in the labor market, agents also draw a positive utility v from successful contributions to OSS projects. Given v, the utility of talented agents becomes

$$U(e) = p(e,\gamma)\left[v + \beta q(\widehat{e},\gamma \mid s)w\right] + \left((1-\beta)p(e,\gamma) + 1 - p(e,\gamma)\right) q(\widehat{e},\gamma \mid n)w - e.$$

The effort level that each talented agent will choose given the firms' beliefs, $\widehat{e}$, is now defined implicitly by the following first order condition:

$$U'(e) = p_e(e,\gamma)\left[v + \beta\Delta(\widehat{e},\gamma)w\right] - 1 \leq 0, \qquad eU'(e) = 0.$$

As one can see, v raises the marginal benefit from effort and hence, other things being equal, it expands the set of parameters for which the model attains an interior equilibrium. Moreover, v shifts the best response function of talented agents outward in the sense that holding the belief of firms, $\widehat{e}$, constant, an increase in v leads to an increase in $BR(\widehat{e})$. Consequently, it is clear that an increase in v will lead to more effort in stable interior equilibria but less effort in unstable equilibria.

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