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SOCIAL NETWORKING AND INDIVIDUAL OUTCOMES: INDIVIDUAL DECISIONS AND MARKET CONTEXT[1]

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Abstract

This paper examines social interactions when social networking is endogenous. It employs a linear-quadratic model that accommodates contextual effects, and endogenous local interactions, that is where individuals react to the decisions of their neighbors, and endogenous global ones, where individuals react to the mean decision in the economy, both with a lag. Unlike the simple $VAR(1)$ structural model of individual interactions, the planner's problem here involves intertemporal optimization and leads to a system of linear difference equations with expectations. It highlights an asset-like property of socially optimal outcomes in every period which helps characterize the shadow values of connections among agents. Endogenous networking is easiest to characterize when individuals choose weights of social attachment to other agents. It highlights a simultaneity between decisions and patterns of social attachment. The paper also poses the inverse social interactions problem, asking whether it is possible to design a social network whose agents' decisions will obey an arbitrarily specified variance covariance matrix.

Keywords: Social Interactions, Social Networks, Neighborhood Effects, Endogenous Networking, Social Intermediation, Econometric Identification, Strong versus Weak Ties, Value of Social Connections.

JEL classification codes: D85, A14, J0.

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1 Introduction

Most of the empirical work on social interactions tries to analyze and identify econometrically interactions of a mean-field type. This literature typically assumes that individual agents are influenced by the average behavior of the social group to which they belong. Such reliance on a broad class of *mean field* models has shifted attention away from other features of social interactions, such as individuals' deliberate efforts to acquire desirable, or avoid undesirable, social connections. Ioannides and Zabel (2002) explicitly model group membership but the groups or neighborhoods they specify are non-overlapping and within each neighborhood, mean-field type interactions are assumed. The nascent experimental literature on network formation (see Kosfeld, 2004, for a survey) is a promising development. These studies focus, however, is on the evolution of networks in rather stylized small scale games.

The present paper differs from other approaches to endogenous group formation by its emphasis on macroeconomic aspects of network structure associated with the social groups which result from individuals' *social networking:* their initiatives to link with others. The paper investigates the link between social networking and social interactions when patterns of social interactions may be more complex than in the mean field case. If individuals in choosing their own actions, which we interpret as incomes, are affected by social interdependence but do not react strategically to it, individual income outcomes may be restricted with respect to the universe of social outcomes that may reached. Patterns of interdependence may also reflect self-selection associated with individuals' choice of groups.

The particular approach adopted by this paper is in part motivated by the desire to explore in greater depth the notion of social settings where some individuals may be more "influential" than others, and the observation that within groups, subgroups or cliques of agents exist with more intense mutual interaction. A natural question to pose pertains to the potential for econometric identification when observational data may be used to make inferences about interaction topology (also referred in the literature as network geography or morphology) along with individuals' behavioral parameters. A key consequence of social interactions in modelling social phenomena is that even if individuals are subject to random shocks that are independent, their individual reactions may end up being stochastically dependent, precisely because individual reactions reflect influences from others directly — one's own actions reflect those of others — or indirectly, because one's own actions reflect characteristics of others that influence behavior. Interdependence may also follow from the fact individuals take deliberate efforts to form social groups [Moffitt (2001)].

When association with others is valuable, it is also appropriate to wonder about existence of markets through which such "social networking" may be mediated. In fact, networking markets appear to be thriving sometimes and not so much at other times, at least according to the popular press; see, for example, Rivlin (2005a; 2005b). Creation of such markets is conceptually related to completing asset markets in an economy, an intuition which could be explored further in the paper.

The remainder of the paper is organized as follows. The paper first introduces a model of individual decisions that allows for local and global interactions. The model allows for the full range of social interactions with continuous outcomes, as they have been formalized by a number of researchers, including notably Bisin *et al.* (2004), Brock and Durlauf (2001), Glaeser, Sacerdote and Scheinkman (2003), Glaeser and Scheinkman (2001), Manski (1993; 2000), Moffitt (2001) and Weinberg (2004) in the economics literature. Our paper is most notably related to Weinberg (2004) who introduces utility from social interaction with others and emphasizes the role of the interaction structure on thresholds and of inputs, like own time, that an individual may associate with a given set of neighbors. Weinberg shows that endogeneity of association introduces additional structure that aids identification, thus relaxing the Manski "social reflection" problem.[2] Weinberg also provides empirical results with data from *Add Health* from the National Longitudinal Study of Adolescent Health. Unlike Weinberg, we are also exploring dynamic aspects of social networking Suitable specification of such interactions makes it possible to express a rich variety of patterns of social interdependence that may have important consequences in the context of the life cycle model as well, along the lines of earlier research by Binder and Pesaran (1998; 2001). Unlike Weinberg but similarly to Binder and Pesaran, we are also exploring dynamic aspects of social networking. We go beyond than those authors in obtaining precise analytical results when networking is endogenous. The paper is conceptually related to the problems addressed by Bala and Goyal (2004) and Goyal and Vega-Redondo (2004) but like Binder and Pesaran, *op. cit.*, and Weinberg, *op. cit.,* uses very different techniques in order to render results that are more amenable to empirical analysis.

The relationship of this paper to those earlier works will be clarified further below. We employ the model first in a static setting, which allows us to explore its implications for interdependence of individual outcomes in the long run. This development allows us to examine the "inverse social interactions" problem, that is whether or not an arbitrary social

[2]This is, in principle, the same effect as the one that endogeneity of neighborhood choice introduces into even a linear model [Brock and Durlauf (2001; Durlauf (2004)].

outcome, that is one that has been defined as a random vector with a given probability distribution, may be reached as a limit state of social interactions in the long run.

Next the paper turns to investigation of the range of social outcomes that may be reached when individuals choose deliberately how much they wish to be influenced by other individuals. Conceptually, this is related to choice of social groups or neighborhoods to belong to. We explore the literature in neighborhood choice and in formation of friendships in articulating different possibilities that may result in sorting in terms of observables or of unobservables. We explore similarities between the motivation that individuals have for social networking and the incentives to fully share risks.

2 Social Setting and Preferences

We employ the concept of social networking in the sense of acquiring access to benefits that emanate from activities of other individuals within a particular social setting, or, put differently, of acquiring access to desirable and avoiding undesirable social interactions.

We specify a social setting in terms of a social interactions topology. A social interactions structure, or topology, is defined in terms of the adjacency matrix $\mathbf{\Gamma}$ [Wasserman and Faust (1994)] of the graph $\mathcal{G}$ among $i = 1, \ldots, N$ individuals as follows:

$$\gamma_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are neighbors in } \mathcal{G}; \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

The adjacency matrix will be time-subscripted when appropriate. If social interactions are assumed to be undirected, the adjacency matrix (or sociomatrix) $\mathbf{\Gamma}$ is symmetric. We define an individual i's neighborhood as the set of other agents she is connected with in the sense of the adjacency matrix, $j \in \nu(i)$, $\forall j, \Gamma_{ij} = 1$. $\mathbf{\Gamma_i}$ denotes row i of the adjacency matrix, whose nonzero elements correspond to individual i's neighbors. Further below, it will be important to allow for a weighted adjacency matrix.

Social interactions may be embedded in a variety of settings, such as consumption activities, human capital accumulation, or more generally income-generating activities. We choose that last setting and assume that individual i derives utility U_{it} from an income generating activity, income for short, y_{it} in period t net of the (utility) cost of doing so. Utility U_{it} is assumed to be concave in y_{it}. It also depends on the incomes and the characteristics of i's neighbors in the sense of the social structure and possibly on the average income in the entire economy. Specification of the dependence on income and characteristics of others allows us to express a variety of social interactions, neighborhood and peer effects. We discuss these after we introduce notation.

Let

$\mathbf{Y}_t := (y_{1t}, y_{2t}, \dots, y_{Nt})'$ denote the (column) vector with elements all individuals' incomes at time t;

$\mathbf{x_i}$: a row $K-$vector with i's own characteristics;

$\mathbf{X}$: a $N \times K$ matrix of characteristics for all individuals;

$\mathbf{x}_{\nu(\mathbf{i})}$: a row $K-$vector with the mean characteristics of i's neighbors, $j \in \nu(i)$, $\mathbf{x}_{\nu(\mathbf{i})} = \frac{\mathbf{1}}{|\nu(\mathbf{i})|}\mathbf{\Gamma_i X}$;

ι : a row $N-$ vector of ones.

ϕ, θ, α and ω are column $K-$vectors of parameters.

We assume individual i at time t enjoys socializing with individuals with specific characteristics. This is expressed through taste over the mean *characteristics* of i's neighbors $j \in \nu(i)$, $\frac{1}{|\nu_t(i)|}\mathbf{\Gamma_{i,t} X}\phi$, a level effect. We allow for the marginal utility of own income to depend on neighbors' mean characteristics, which is expressed through the term $\frac{1}{|\nu_t(i)|}\mathbf{\Gamma_{i,t} X}\theta\mathbf{y_{it}}$, and separately, on her own characteristics, through a term $\mathbf{x}_i\alpha y_{it}$. The former may be referred to as a local *contextual* effect.

We allow, symmetrically, individual i at time t to be affected by her neighbors' income in two ways. One may be thought of as conformism: individual i suffers disutility from a gap between her own income and the mean income of her neighbors in the previous period, $y_{it} - \frac{1}{|\nu_{t-1}(i)|}\mathbf{\Gamma_{i,t} Y_{t-1}}$. A second way allows for the effect of a change in a neighbor's income to depend on i's own characteristics through a term $\frac{1}{|\nu_t(i)|}\mathbf{\Gamma_{i,t} Y_{t-1} x_i}\omega$. This allows the effect of the neighbors income to depend on individual background characteristics, e.g. younger people may experience a greater disutility from living in a high-income neighborhood. We also allow for a global conformist effect, through the divergence between an individual's current income and the mean income among all individuals in the previous period, $y_{it} - \bar{y}_{t-1}$. This implies *endogenous global interactions.*

We assume that each individual i incurs a (utility) cost for maintaining a connection with individual j at time t is a quadratic function of the "intensity" of the social attachment as measured by γ_{ij}, that is, $-c_1\mathbf{\Gamma_{i,t}}\iota - \mathbf{c_2 \Gamma_{i,t} \Gamma'_{i,t}}$, with $c_1, c_2 > 0$, where ι denotes an $N-$column vector of 1s.[3] This component of the utility function matters when we consider weighted adjacency matrices in section 6.

Finally, we allow for an individual's own marginal utility to include an additive stochastic shock, ε_{it}, which will be discussed in further detail below. Lagging the effect of neighbors' decisions, while keeping the contemporaneous neighborhood structure, is analytically convenient but not critical. We discuss below in more detail the consequences of including contemporaneous effects of neighbors' decisions.

All these effects are represented in a quadratic utility function, the simplest possible functional form that yields linear first order conditions:[4]

$$\begin{aligned} U_{it} = {} & \mathbf{\Gamma}_{i,t}\mathbf{X}\phi + (\alpha_0 + \mathbf{x}_i\alpha + \mathbf{\Gamma}_{i,t}\mathbf{X}\theta + \varepsilon_{i,t})\, y_{it} - \frac{1}{2}(1-\beta_g-\beta_\ell)y_{it}^2 \\ & - \frac{\beta_g}{2}\left(y_{it} - \bar{y}_{t-1}\right)^2 - \frac{\beta_\ell}{2}\left(y_{it} - \frac{1}{|\nu_t(i)|}\mathbf{\Gamma}_{i,t}\mathbf{Y}_{t-1}\right)^2 \\ & + \frac{1}{|\nu_t(i)|}\mathbf{\Gamma}_{i,t}\mathbf{Y}_{t-1}\mathbf{x}_i\omega - c_1\mathbf{\Gamma}_{\mathbf{i},\mathbf{t}}\iota - \mathbf{c_2}\mathbf{\Gamma}_{\mathbf{i},\mathbf{t}}\mathbf{\Gamma}'_{\mathbf{i},\mathbf{t}}. \end{aligned} \tag{2}$$

As we see below, some of these terms do not contribute to individuals' reaction functions, but do affect the utility an individual derives from networking with particular other individuals. This allows for social effects in the endogenous social networking process.

Our specification of the utility function nests a number of existing approaches in the literature, including Durlauf (2004), Glaeser, Sacerdote and Scheinkman (2003), Horst and Scheinkman (2004), and Weinberg (2004). Regarding the general formulation of the social interactions problem by Binder and Pesaran (1998; 2001), we note the following. First, Binder and Pesaran (2001) assume that the social weights are constant across individuals, whereas we posit an arbitrary adjacency matrix which may accommodate the full range between local and global interactions. Second, those authors do not examine deliberate social networking. Yet, they do provide an exhaustive analysis of solutions of rational expectations models for certain classes of social interactions. They also address the infinite regress problem (of forecasting what the average opinion expects the average opinion to be, etc.) by conditioning on public information only. Our utility function is analytically quite similar to that in

[3] When entries of the adjacency matrix are restricted to zero-one values, the total cost to individual i can be written simply as $-(c_1+c_2)\mathbf{\Gamma}_{\mathbf{i},\mathbf{t}}\iota$.

[4] An additional advantage of quadratic utility functions is that they lend themselves quite easily to formulation of decision problems as *robust control* problems, that is when agents make decisions without knowledge of the probability model generating the data. See Backus *et al.* (2004) for a simple presentation and Hansen and Sargent (2005) for a thorough development of robust control as a misspecification problem.

ibid. and to Weinberg (2004). By appropriate redefinition of the adjacency matrix it may accommodate habit persistence.[5] [6] It is possible that individuals may differ with respect to attitudes towards conformism or altruism, which would require a more general specification of preferences than (2).

3 Individual Decision Making

At the beginning of period t, individual i augments her information set Ψ_{it} by observing her own preference shock, ε_{it}, her neighbors' actual incomes in period $t-1$, $\mathbf{\Gamma}_{i,t}\mathbf{Y}_{t-1}$, and the mean lagged income for the entire economy, $\bar{y}_{t-1}$, and chooses an income plan $\{y_{it}, y_{it+1}, \ldots | \Psi_{it}\}$, so as to maximize expected lifetime utility conditional on Ψ_{it},

$$\mathcal{E}\left\{\sum_{s=t}\delta^{s-t}U_{i,s}\,|\Psi_{it}\right\}, \tag{3}$$

where $\delta,\ 0<\delta<1$, is the rate of time preference and individual utility is given by (2). We note that individual i in choosing her plan recognizes that y_{it} enters $U_{i,t+1}$ only if $\beta_g \neq 0$, that is only if endogenous global interactions are assumed. That is due to the fact that y_{it} enters period $t+1$ utility functions of all agents j other than agent i, $j \neq i$. That is, in our definition of the adjacency matrix, $\gamma_{ii}=0$, and the term $\mathbf{\Gamma}_{i,t+1}\mathbf{Y}_t$ does not contain y_{it}.[7]

The first order condition with respect to y_{it}, given the social structure, is:

$$y_{it} = \frac{\beta_\ell}{|\nu_t(i)|}\mathbf{\Gamma}_{i,t}\mathbf{Y}_{t-1} + \beta_g\bar{y}_{t-1} + \delta\frac{\beta_g}{N}\mathcal{E}\left\{(y_{i,t+1}-\bar{y}_t)\,|\Psi_{it}\right\} + \alpha_0 + \mathbf{x}_i\alpha + \mathbf{\Gamma}_{i,t}\mathbf{X}\theta + \varepsilon_{it}. \tag{4}$$

[5] That is, adopting their assumption would yield quadratic components of the form

$$\frac{1}{2}\left(y_{it}-\eta y_{i,t-1}\right)^2 - \frac{\beta}{2}\left(y_{it}-\eta y_{i,t-1} - \left(\frac{1}{|\nu(i)|}\mathbf{\Gamma}_i\mathbf{Y}_t - \eta\frac{1}{|\nu(i)|}\mathbf{\Gamma}_i\mathbf{Y}_{t-1}\right)\right)^2,$$

as in Binder and Pesaran (2001), Equ. (14)), with parameter $\beta>0$ measuring conformism. They model conformism by expressing a tradeoff between individual-specific growth in consumption, on one hand, and the gap between that growth and its counterpart among an individual's neighbors, on the other. They model altruism by expressing a tradeoff between the excess of one's own current income over target lagged income and its counterpart among one's neighbors.

[6] Following Equ. (15)) in *ibid.* would yield

$$-\frac{1}{2}\left(y_{it}-\eta y_{i,t-1} + \tau\left(\frac{1}{|\nu(i)|}\mathbf{\Gamma}_i\mathbf{Y}_t - \eta\frac{1}{|\nu(i)|}\mathbf{\Gamma}_i\mathbf{Y}_{t-1}\right)\right)^2,$$

with parameter $\tau>0$ reflecting altruism and $\tau<0$ reflecting jealousy.

[7] See Bisin, Horst and Özgür (2004) while assuming only one-sided interactions also allow for a global effect.

The interdependencies between individuals' decisions are clearer when the first order conditions for all individuals are put concisely in matrix form. We simplify by setting $\beta_g = 0$ and have:

$$\mathbf{Y_t} = \mathcal{A}_\mathbf{t} + \beta_\ell \mathbf{D_t} \mathbf{\Gamma_t} \mathbf{Y_{t-1}} + \varepsilon_\mathbf{t}, \tag{5}$$

where $\varepsilon_\mathbf{t} \equiv (\varepsilon_\mathbf{1t}, \varepsilon_\mathbf{2t}, \ldots, \varepsilon_\mathbf{Nt})'$, is the $N-$vector of shocks, $\mathbf{D_t}$ is an $N \times N$ diagonal matrix with elements the inverses of the size of each individual's neighborhood,

$$\mathbf{D}_{ii,t} = 1/|\nu_t(i)| = 1/(\mathbf{\Gamma_t}\mathbf{\Gamma_t})_{ii},$$

and

$$\mathcal{A}_t \equiv (\ldots, \alpha_0 + \mathbf{x}_i\alpha + \mathbf{\Gamma}_{i,t}\mathbf{X}\theta, \ldots)'$$

is a column $N-$vector of individual and contextual effects.

The evolution of the state of the economy, defined in terms of $\mathbf{Y_t}$, the vector of individuals' incomes, is fully described by (5), a VAR $(1,1)$ model, given the information set $\Psi_t = \bigcup_i \Psi_{i,t}$, and provided that the sequence of adjacency matrices $\mathbf{\Gamma}_t, t = 0, 1, \ldots,$ is specified. Intuitively, the economy evolves as a Nash system of social interactions that adapts to external shocks of two types, deterministic ones, as denoted by the evolution of the social structure $\mathbf{\Gamma_t}$, and stochastic ones, as denoted by the vectors of shocks ε_t, as $t = 1, \ldots, \infty$.

To fix ideas, let us assume that the random vectors $\varepsilon_t, t = 1, \ldots,$ are i.i.d. over time and drawn from a normal distribution with mean $\mathbf{0}$ and variance covariance matrix $\mathbf{Q}$, $\varepsilon_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$. Then, by a standard derivation from stochastic systems theory [Kumar and Varaiya (1986), p. 27]of the $t-$step transition probability, the distribution for $\mathbf{Y_t}$, at time t, is characterized by the following proposition, whose proof is elementary that thus not repeated here.

Proposition 1.

Given the initial state of the economy, $\mathbf{Y}_0$, and under the assumption that the shocks ε_t are independent and identically distributed with a multivariate normal distribution $\mathcal{N}(\mathbf{0}, \mathbf{Q})$, then the distribution of the state of the system at time t, $\mathbf{Y}_t$, is normal,

$$\mathbf{Y}_t \sim \mathcal{N}\left(\prod_{s=1}^{t} \mathbf{A}_s \mathbf{Y}_0 + \sum_{j=0}^{t-1}\left(\prod_{s=j+1}^{t} \mathbf{A}_s\right)\mathcal{A}_j, \mathbf{\Sigma}_{t|0}\right), \tag{6}$$

where $\mathbf{A_t}$ is the $N \times N$ matrix defined as:

$$\mathbf{A_t} \equiv \beta_\ell \mathbf{D_t}\mathbf{\Gamma_t},$$

and the variance covariance matrix $\mathbf{\Sigma}_{t|0}$ *above is given from the time-varying linear difference equation:*

$$\mathbf{\Sigma}_{k+m|k} = \mathbf{A}_{k+m-1}\mathbf{\Sigma}_{k+m-1|k}\mathbf{A}'_{k+m-1} + \mathbf{Q},\ m > 1; \mathbf{\Sigma_{k|k}} = [\mathbf{0}]. \tag{7}$$

$\Sigma_{k+m|k}$ *is the matrix of mean-squared errors of the m-step predictor for* $\mathbf{Y_t}$*.*

Not surprisingly for a dynamic model, given a starting point $\mathbf{Y}_0$, the mean of vector of individuals' incomes after t periods reflect the full sequence of contextual effects, $\{\mathcal{A}_0, \ldots, \mathcal{A}_{t-1}\}$, weighted by the respective adjacency matrices. The dispersion of individual incomes, on the other hand, reflects the compound effect of weighted interactions as they augment the dispersion of the underlying shocks.

4 Properties of Social Outcomes at the Steady State

It is easier to see the properties of the state of the economy at the limit when $t \to \infty$. Then under the assumption of a time-invariant adjacency matrix, $\mathbf{A_t} = \mathbf{A}$, and vector of contextual effects, $\mathcal{A}_t = \mathcal{A}$ and the limit of the mean of $\mathbf{Y}_t$, $\mathbf{Y}^*$, is well defined and given by the unique solution to

$$\mathbf{Y}^* = \mathcal{A} + \mathbf{A}\mathbf{Y}^*. \tag{8}$$

A solution for the vector of mean individual incomes exists, provided that matrix $\mathbf{I} - \mathbf{A}$ is invertible. For this, it suffices that $\mathbf{A}$ be stable, namely that all its eigenvalues have magnitudes that are strictly smaller than one. Since $\mathbf{A}$ comes from an adjacency matrix, "normalized" by each row sum and scaled by β_ℓ, its stability is ensured by assuming $0 < \beta_\ell < 1$. So we have:

$$\mathbf{Y}^* = \left[\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \ldots\right]\mathcal{A}. \tag{9}$$

The asymptotic behavior of the variance covariance matrix of the vector of individuals' incomes, according to Equ. (5), is also easy to study provided that the adjacency matrix $\mathbf{\Gamma}_{i,t}$ is time-invariant. The limit $\mathbf{\Sigma}^*$ from Equ. (7) of the variance covariance matrix $\mathbf{\Sigma}_t$, as $t \to \infty$, exists. Theorem (3.4), in Kumar and Varaiya, *op. cit.*, or Proposition 4.1, in Bertsekas, *op. cit.*, as a special case, ensure that if matrix $\mathbf{A}$ is stable then the limit solution of (7) is a unique positive semidefinite matrix. We may state this in a slightly more general case by relaxing the assumption that the random vectors $\varepsilon_\mathbf{t}$ are independent draws from a given multivariate distribution and instead assume that they are defined in terms of factor loadings,

$$\varepsilon_{\mathbf{t}} = \mathbf{G}\mathbf{w}_{\mathbf{t}}, \tag{10}$$

where $\mathbf{G}$ is an $N \times M$ mixing matrix and $\mathbf{w}_t$ a random column $M-$vector, that is independently and identically distributed over time and obeys a normal distribution with mean $\mathbf{0}$ and variance covariance matrix $\mathbf{R}$, $\mathbf{w}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$. This case is of particular interest, because it allows us to express the contemporaneous stochastic shock in a factor-analytic form of contemporaneously interdependent components and therefore shocks to different individuals may be correlated. This approach also allows for a general form of serial correlation in the $\mathbf{w}_{\mathbf{t}}$s [*ibid.,* p. 27].[8]

Put succinctly in a proposition, whose proof readily follows from (7) and (10), we have:

Proposition 2.

The variance covariance matrix of vector of individuals' incomes when $t \to \infty$, *that is the limit of (7) when* ε_t *assumes the factor-analytic form according to (10), satisfies*

$$\mathbf{\Sigma}_\infty = \mathbf{A}\mathbf{\Sigma}_\infty\mathbf{A}' + \mathbf{G}\mathbf{R}\mathbf{G}'. \tag{11}$$

The matrix $\mathbf{\Sigma}_\infty$ in the equation above would not be a proper variance covariance matrix unless it is positive definite. This is ensured, provided the random error $\mathbf{w}_{\mathbf{t}}$ along with the matrix $\mathbf{A}$ "allow" the economy to reach all social outcomes in a well-defined sense. Clarifying this involves the concept of *controllability,* according to which a pair of matrices $\mathbf{A}, \mathbf{G}$ are *controllable*, if the matrix $[\mathbf{G},\ \mathbf{A}\mathbf{G},\ \dots\ ,\mathbf{A}^{\mathbf{N-1}}\mathbf{G}]$ with N rows and NM columns has rank N, that is all its rows are fully independent.

Proposition 3. *If the pair of matrices* $\mathbf{A}, \mathbf{G}$ *is controllable, then stability of matrix* $\mathbf{A}$ *is equivalent to existence of a unique solution to Equ. (11) that is positive definite and vice versa.*

Proof. Matrix $\mathbf{A}$ is indeed stable, as we discussed earlier above. Then, Theorem (3.9), in Kumar and Varaiya, *op. cit.*, applies. Alternatively, by applying Equ. (11) recursively we have that if $\lim_{t\to\infty} \mathbf{A}^t\mathbf{G}\mathbf{R}\mathbf{G}'(\mathbf{A}')^t = [\mathbf{0}]$, then

$$\mathbf{\Sigma}_\infty = \sum_{j=0}^{\infty} \mathbf{A}^j\mathbf{G}\mathbf{R}\mathbf{G}'(\mathbf{A}')^j.$$

[8] Such a factor-analytic approach links social interaction models with factor models in large cross-sections of time series that have been employed in generalizing the dynamic index models of business cycle research. See Reichlin (2002) for an excellent review of this recent literature.

It then follows that stability of matrix $\mathbf{A}$ is necessary and sufficient for the existence of $\mathbf{\Sigma}_\infty$. Q.E.D.

The interpretation of the condition for controllability is quite revealing in our context. If the $N \times NM$ matrix $[\mathbf{G}|\mathbf{AG}|\ldots|\mathbf{A^{N-1}G}]$ has rank N, then there exists a sequence of vectors $\widetilde{\mathbf{w}}_0, \ldots, \widetilde{\mathbf{w}}_{N-1}$, that take the economy from any arbitrary initial point to a designated final point in N steps. Let $\widetilde{\mathbf{Y}}_0$ and $\widetilde{\mathbf{Y}}_N$ be these points, respectively, defined as deviations from $\mathbf{Y}^*$, the unique solution of (8).

By applying Equ. (5), transformed in terms of deviations from $\mathbf{Y}^*$, $\widetilde{\mathbf{Y}}_t \equiv \mathbf{Y_t} - \mathbf{Y}^*$,

$$\widetilde{\mathbf{Y}}_t = \mathbf{A}\ \widetilde{\mathbf{Y}}_{t-1} + \mathbf{G}\mathbf{w}_t, \tag{12}$$

successively N times we stack the N equations so as to have $\widetilde{\mathbf{Y}}_N - \mathbf{A^N}\ \widetilde{\mathbf{Y}}_\mathbf{0}$ on the LHS and the column $NM-$vector obtained by stacking up the input vectors as the unknowns, $\widetilde{\mathbf{w}}_0, \ldots, \widetilde{\mathbf{w}}_N$. Controllability ensures that the solution for the input vectors, the $\widetilde{\mathbf{w}}_t$'s, is unique. Put differently, for the economy to move from anywhere to anywhere in N *steps*, as many input vectors are needed as there are members of the economy.

4.1 Extensions

We address next a number of possible extensions and generalizations. First, the stochastic shocks do not need to be Gaussian. As long as their second moments exist, the counterpart of (7) may be derived.

Second, we may generalize the model and interpret $\mathcal{A}_t$ as a vector of decision variables that contributes to the evolution of individual outcomes in the manner indicated by Equ. (5). That is, $\mathcal{A}_t$ may assume the form of closed-loop control and therefore could reflect the state of the economy as of time $t-1$, $\mathbf{Y_t} = \mathbf{B_t}\mathcal{A}_\mathbf{t} + \mathbf{A_t}\mathbf{Y_{t-1}} + \varepsilon_\mathbf{t}$, where $\mathbf{A_t}, \mathbf{B_t}$ are given matrices and the vector $\mathcal{A}_t$, the *control*, is optimally chosen so as to optimize an objective, which also includes a quadratic term in $\mathcal{A}_t$. Such a model would be very similar to a life cycle optimization of consumption decisions. This, of course, continues to be true in the presence of social interactions, as in the work by Binder and Pesaran (1998; 2001), discussed earlier. The optimal control, in this case, is defined through a matrix that satisfies an *algebraic Riccati equation* [Bertsekas (1995), 132–133].

Third, the properties that we have discussed earlier really pertain to a sequence of static problems that evolve over time. A number of straightforward generalizations that imply intertemporal tradeoffs can render the model genuinely dynamic. The simplest possible one, as we discussed above, would be to allow for a global effect, that is by allowing $\beta_g \neq 0$.

Fourth, an interesting extension would be to allow for the stochastic shocks to be decision variables. The simplest way to do so would be via endogenizing the factor loadings, that is by letting individuals set the elements of the mixing matrix in Equ. (10), $\mathbf{G}$, which could also be time-varying. This would then resemble a dynamic portfolio analysis problem, which we know to be quite tractable when individuals' utility functions are quadratic. We note the special interpretation that the choice of a portfolio of factors would confer: individuals choose factor loadings in order to offset shocks associated with the decisions of others that enter directly their own utility functions and therefore their own decisions as well.

Fifth, suppose that an arbitrarily chosen state of the economy may be specified via a random $N-$vector $\mathcal{Y}$, with a given distribution function, $\mathcal{Y} \sim \mathcal{N}(\mathbf{F}, \varsigma)$. That is, the incomes of the individual members of the economy are assumed to be stochastically interdependent and distributed in the above fashion. We define the *inverse social interactions problem* as whether such a state could be reached as an outcome of social interactions, that is as the limit vector of individuals' incomes, satisfying (5), when $t \to \infty$. That is, given $(\mathbf{F}, \varsigma)$ and the distributional characteristics of the random vector of factors $\mathbf{w_t}$, $\mathbf{w}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$, can we find a social interactions structure, defined by a vector of coefficients $\mathcal{A}$, an adjacency matrix $\mathbf{\Gamma}$, and a mixing matrix $\mathbf{G}$ of dimension $N \times M$, such that the random vector $\mathcal{Y}$ is the limit, when $t \to \infty$, of the vector of individual outcomes, $\mathbf{Y}_t$,

$$\mathbf{Y_t} = \mathcal{A} + \mathbf{A}\mathbf{Y_{t-1}} + \mathbf{G}\mathbf{w}_t. \tag{13}$$

The inverse social interactions problem is the counterpart here of problems posed by the business cycle literature, that is whether it might be possible to define vector autoregressive system that yields any desirable pattern of correlations among the shocks [Jovanovic (1987); Sargent (1987)]. We note that this problem is more general than examining the limit of the distribution of income in this economy.

First we note that given $\mathbf{F}$, (13) implies that $\mathbf{F} = [\mathbf{I} - \mathbf{A}]^{-1}\mathcal{A}$. If a mixing matrix $\mathbf{G}$ is given, then we may define the square root $\mathbf{S}$ of $\mathbf{GRG}'$, $\mathbf{SS}' = \mathbf{GRG}'$. The problem of finding a positive symmetric matrix of full rank N, $\mathbf{A}$, is overdetermined: there exist $\frac{1}{2}N(N+1)+N$ equations and $\frac{1}{2}N(N-1)$ unknowns. Therefore, there exist up to $2N$ degrees of freedom which may be used for the determination of the mixing matrix.

This result is readily interpretable in terms of weighted adjacency matrices. Imposing the restriction that the social interaction matrix takes the form of a matrix of zeros and ones, with $\mathbf{A} = \beta\mathbf{D}\mathbf{\Gamma}$, translates to the requirement that given ς, a matrix $\mathbf{\Gamma}$ and a social interactions coefficient β may be found that provide sufficient spanning for ς to be the limit distribution satisfying Equ. (11). Relaxing the requirement would be to allow for individuals' interaction coefficients to differ, $(\beta_1, \ldots, \beta_N)$, or alternatively, for the adjacency matrix to assume a weighted form with arbitrary positive entries, instead of just 0s and 1s. The case of a weighted adjacency matrix is taken up further below in section 6.

5 A Social Planner's Problem

Next we introduce a social planner who recognizes the interdependence of agents and also takes it into consideration in setting individuals' outcomes while respecting an existing topology of social interactions. This analysis helps assess the scope for social intermediation, as we shall see shortly below.

The planner seeks to choose individuals' incomes that maximize the sum of individuals' expected lifetimes utilities.[9] The planner is forward-looking and in recognizing individuals' interdependence she conditions her decisions on knowledge of all incomes in the previous periods, $\{\mathbf{Y}_0, \mathbf{Y}_1, \ldots, \mathbf{Y}_{t-1}\}$, and of all individual preference information available as of the beginning of every time period, Ψ_t. At the beginning of each period t, the planner chooses a plan $\{\mathbf{Y_t}, \mathbf{Y_{t+1}}, \ldots,\}$ so as to maximize the expectation of individuals' lifetime utility, according to

$$\sum_{i=1}^{N} \left[U_{it} + \mathcal{E}\left\{ \sum_{s=t+1} \delta^{s-t} U_{i,s} | \Psi_t \right\} \right]. \tag{14}$$

The planner's maximization assumes actual outcomes as of time period t and the exogenous evolution of the adjacency matrix $\mathbf{\Gamma}_t$, and where we have also simplified utility function (2) by assuming that $\beta_g = 0, \omega = \mathbf{0}$.

Differentiating objective function (14) with respect to $y_{it}, i = 1, \ldots, N$, yields the first order conditions:

$$y_{it} + \delta\beta_\ell \sum_{j \neq i} \frac{\gamma_{ji,t+1}}{|\nu_{t+1}(j)|} \frac{1}{|\nu_{t+1}(j)|} \mathbf{\Gamma}_{j,t+1} \mathbf{Y}_t$$

$$= \frac{\beta_\ell}{|\nu_{t+1}(i)|} \mathbf{\Gamma}_{it} \mathbf{Y}_{t-1} + \delta\beta_\ell \sum_{j \neq i} \frac{\gamma_{ji,t+1}}{|\nu_{t+1}(j)|} \mathcal{E}\left\{ y_{j,t+1} | t \right\} + \mathcal{A}_{it} + \mathbf{G}_i \mathbf{w}_t. \tag{15}$$

[9] We eschew introduction of arbitrary positive weights $\lambda_1, \ldots, \lambda_N$, because they are not really necessary for expressing the planner's problem. There are sufficient degrees of freedom elsewhere in the individuals' utility functions.

We may rewrite these conditions for all i's in matrix form as follows:

$$\begin{aligned} & \left[\mathbf{I} + \delta\beta_\ell \mathbf{\Gamma}'_{t+1}\mathbf{D}_{t+1}\mathbf{D}_{t+1}\mathbf{\Gamma}_{t+1}\right]\mathbf{Y}_t \\ = \;& \mathcal{A}_t + \beta_\ell \mathbf{D_{t+1}}\mathbf{\Gamma}_t\mathbf{Y}_{t-1} + \delta\beta_\ell\mathbf{\Gamma}'_{t+1}\mathbf{D}_{t+1}\,\mathcal{E}\left\{\mathbf{Y}_{t+1}|\Psi_t\right\} + \mathbf{G}\mathbf{w}_t. \end{aligned} \tag{16}$$

By working in the standard fashion for linear-quadratic rational expectations models, we solve for $\mathbf{Y}_t$ as a function of $\mathbf{Y}_{t-1}$ and of $\mathcal{E}\left\{\mathbf{Y}_{t+1}|t\right\}$. Specifically, we simplify by assuming that the adjacency matrix and the matrix of contextual effects are time invariant, and define:

$$\mathbf{H} = \mathbf{I} + \delta\beta_\ell\mathbf{\Gamma}'\mathbf{D}\mathbf{D}\mathbf{\Gamma}$$

Then solving Equ. (16) yields:

$$\mathbf{Y}_t = \mathbf{H}^{-1}\left[\mathcal{A} + \beta_\ell\mathbf{D}\mathbf{\Gamma}\mathbf{Y}_{t-1} + \delta\beta_\ell\mathbf{\Gamma}'\mathbf{D}\,\mathcal{E}\left\{\mathbf{Y}_{t+1}|\Psi_t\right\} + \mathbf{G}\mathbf{w}_t\right]. \tag{17}$$

By advancing the time subscript, taking expectations and using the law of iterated expectations, we may express $\mathcal{E}\left\{\mathbf{Y}_{t+1}|\Psi_t\right\}$ in terms of $\mathbf{Y}_t$ and $\mathcal{E}\left\{\mathbf{Y}_{t+2}|\Psi_t\right\}$, and so on. That is:

$$\mathcal{E}\left\{\mathbf{Y}_{t+1}|t\right\} = \mathbf{H}^{-1}\left[\mathcal{A} + \beta_\ell\mathbf{D}\mathbf{\Gamma}\mathbf{Y}_t + \delta\beta_\ell\mathbf{\Gamma}'\mathbf{D}\,\mathcal{E}\left\{\mathbf{Y}_{t+2}|t\right\}\right].$$

By iterating forwards from time $t+1$, we obtain the standard form of the forward solution for systems of linear rational expectations models. Put concisely in the form of a proposition we have:

Proposition 4.

Part A. *Maximization of expected social welfare, defined as the sum of expected life time utilities of all agents requires that the vector of outcomes* $\mathbf{Y}_t$ *satisfies the system of linear difference equations (16) with expectations.*

Part B. *The expectation of the vector of optimal outcomes at the steady state, when all covariates are time-invariant,* $\tilde{\mathbf{Y}}^*$, *satisfies*

$$\mathcal{E}\left\{\tilde{\mathbf{Y}}^*\right\} = \mathcal{A} + \beta_\ell\mathbf{D}\mathbf{\Gamma}\mathcal{E}\left\{\tilde{\mathbf{Y}}^*\right\} + \delta\beta_\ell\mathbf{\Gamma}'\mathbf{D}\left[\mathbf{I} - \mathbf{D}\mathbf{\Gamma}\right]\mathcal{E}\left\{\tilde{\mathbf{Y}}^*\right\}, \tag{18}$$

and is well defined and unique.

The economic interpretation of Equ. (18) is straightforward. At the steady state, the vector of expected outcomes is equal to the vector of contextual effects, of the own lagged feedbacks, and of the net "conformist" effect on one's neighbors in the following period. Precise results regarding the properties of the solution to the above model may be obtained by applying Proposition 3, Binder and Pesaran (1998). We plan to return to this issue in a later version of the paper. We do note that a richer behavioral model than the one in Section 3, while possible and attractive, would have complicated considerably the use we make of our behavioral model later on in the paper.

A number of remarks are in order. *First*, systems of linear difference equations with expectations, like (16), have been studied extensively and are amenable to solution in the standard fashion for such models [Sargent (1987); Binder and Pesaran, *op. cit.*]. Expectations enter because the planner recognizes that setting individual i's period t outcome affects the expectation of the utilities of her neighbors in the following period. In contrast, individuals' optimization considerations are backward looking, and therefore the first-order conditions yield a simple vector autoregression of order 1.

Second, by employing the methods used earlier and summarized in Proposition 1 above, it is possible to extend Proposition 4 so as to clarify the evolution of the variance-covariance matrix for the socially optimal outcomes vector.

Third, iterating Equ. (17) forwards demonstrates an *asset-like property* of socially optimal period t outcomes, $\mathbf{Y}_t$. That is, the value from setting an individual's outcome in this period depends upon others' outcomes next period, and so on. This confers an element of asset to agents' outcomes and leads to *an asset theory* of social interactions. At the social optimum, individual outcomes at time t reflect the present value of the future stream of contextual effects of all current neighbors, adjusted by discount factors that are functions of the rate of time preference and multiplied by the social interactions coefficient and weighted by the directness of the connections between agents. It is not surprising that the planner is forward looking in assessing the effects that different individuals' actions have on their own outcomes and those of their neighbors. The one period forward dependence is, of course, translated into the standard infinitely forward dependence familiar from rational expectations models.

Fourth, we note that there is another aspect of the social value of social interactions which the effect discussed above does not capture. That is, the value for an agent of having versus not having a particular connection with another agent. Associated with every solution path is an expected value of the social welfare function, the planner's objective function. Therefore, in principle, we may assess the impact of a change in the adjacency matrix on the socially optimal solution and thus on the expected value of the planner's objective function. That is, adding a new link changes the solution path in a particular way and produces an increment in the expected value of the planner's objective function. In general, this yields a shadow value for the corresponding link which is a benchmark by means of which we may measure the social value of intermediation. This calculation is straightforward conceptually. In practice, it is quite tedious to compute the social value of an additional connection.

Since the individually optimal outcomes are inefficient, the social optimum may be implemented, at least conceptually, through an appropriately chosen set of prices, $\pi_{it}, i = 1, \ldots, N$. By adapting individual i's utility so as to include a term $-\pi_{it} y_{it}$, maximization of $U_{it} - \pi_{it} y_{it}$ brings about the social optimum, provided that the prices are set equal to the respective marginal social effects, which in view of quadratic utility functions are equal to the difference between the socially optimal and individually optimal outcomes. For completeness, we also need to impose conditions of incentive compatibility and allow for transfers that leave individuals at least indifferent while budget balance is satisfied.

So far we have assumed that the network of social connections is given. We next turn to the situation where this network is endogenous. In that case, implementing individual prices π_{it} in general is insufficient as a tool for the social planner to implement the social optimum of outcomes and network connections.

6 Endogenous Networking

How could social network connections come about? This section explores the notion that they are initiated by means of individuals' initiatives, when individuals stand to benefit by doing so. We examine endogenous networking by assuming that individuals choose weights which they associate with their connections with other individuals. This approach assuages the inherent difficulty of dealing with discrete endogenous variables while taking advantage of a natural interpretation of weighted social connections. Individuals' social attachments do differ: they vary from close friendships to mere acquaintances. And, naturally, weighted adjacency matrices are used in the social science literature. A formulation of endogenous networking in a discrete setting of the model of social connections we have been working with so far is relegated to an appendix in a working paper version of the paper. It demonstrates that characterization of individual networking initiatives is quite awkward, and so is the associated welfare analysis.

Analysis of endogenous networking is facilitated by assuming a finite horizon, T, version of the typical individual's problem and considering the networking decision prior to setting the period T outcome.[10] The impact on utility in period T is easier to obtain when we work with the indirect utility function, given a network topology. Setting y_{iT} so as to maximize the expectation of period T utility,

$$U_{iT} = \mathbf{\Gamma}_{i,T}\mathbf{X}\phi + (\mathcal{A}_{iT} + \varepsilon_{iT})\, y_{iT} - \frac{1}{2}(1-\beta_\ell)y_{iT}^2$$

$$-\frac{\beta_\ell}{2}\left(y_{iT} - \frac{1}{|\nu_T(i)|}\mathbf{\Gamma}_{i,T}\mathbf{Y}_{T-1}\right)^2 + \frac{1}{|\nu_T(i)|}\mathbf{\Gamma}_{i,T}\mathbf{Y}_{T-1}\mathbf{x}_i\omega - c_1\mathbf{\Gamma_{i,T}}\iota - \mathbf{c_2}\mathbf{\Gamma_{i,T}}\mathbf{\Gamma'_{i,T}}, \qquad (19)$$

yields an indirect utility function for agent i :

$$\tilde{U}_{iT}\left(\mathbf{\Gamma}_T;\mathbf{Y}_{T-1}\right) \equiv \mathcal{E}_{\varepsilon_t}\left[\max_{y_{iT}} : U_{iT} \mid \Psi_{i,T-1}\right]$$

$$\begin{aligned} =\; & \mathbf{\Gamma}_{i,T}\mathbf{X}\phi + \frac{1}{2}\mathcal{A}_{iT}^2 + (\beta_\ell\mathcal{A}_{iT} + \mathbf{x}_i\omega)\,\frac{1}{|\nu_T(i)|}\mathbf{\Gamma}_{i,T}\mathbf{Y}_{T-1} \\ -\; & \frac{1}{2}\beta_\ell(1-\beta_\ell)\left(\frac{1}{|\nu_T(i)|}\mathbf{\Gamma}_{i,T}\mathbf{Y}_{T-1}\right)^2 - c_1\mathbf{\Gamma_{i,T}}\iota - \mathbf{c_2}\mathbf{\Gamma_{i,T}}\mathbf{\Gamma'_{i,T}} + \frac{\mathbf{1}}{\mathbf{2}}\sigma_\varepsilon^{\mathbf{2}}. \end{aligned} \qquad (20)$$

A network of connections among N individuals is defined by means of a $N \times N$ *weighted* adjacency matrix $\mathbf{\Gamma}_t$, of intensities of social attachment:

$$\gamma_{ij,t}\begin{cases} \neq 0, & \text{if } i \text{ is influenced by } j \text{ in } \mathcal{G}; \\ = 0, & \text{otherwise.} \end{cases} \qquad (21)$$

This formulation combines the notion of an adjacency matrix in graphs with the notion of varying intensities of social contacts[11] and at the same time allows for the network to be directed. For $c_2 > 0$ the marginal cost of a connection are increasing with intensity. This has an interpretation in terms of opportunity cost: short and superficial encounters e.g. at parties bear a lower opportunity cost than spending quality time together. We return below to an interpretation of individuals' choice of intensities of social attachment as neighborhood choice.

Individuals are assumed to choose by which other individuals they are influenced under the assumption that they optimize their own outcomes for any given social structure and take

[10]The analytics employed and intuition gained by the finite horizon approach may be used to extend the model to an infinite horizon setting. See Bisin, *et al., op. cit.* for an extension with one-sided local but global social interactions, as well.

[11]We thank Alan Kirman and Bruce Weinberg who emphasized this interpretation.

all others' decisions as given.[12] All previous derivations in the paper so far readily apply to the case of a weighted adjacency matrix, except that it is no longer necessary to normalize by the size of neighborhood. Formally, individual i seeks to maximize expected period T utility, given by (20), with respect to

$$\mathbf{\Gamma}_{i,T} = (\gamma_{i1,T}, \ldots, \gamma_{ij,T}, \ldots, \gamma_{iN,T}), \text{ with } \gamma_{ij,T} \geq 0, \; j = 1, \ldots, N,$$

and conditional on her information set Ψ_{it}. The results are summarized in the following proposition.

Proposition 5.

The indirect utility function (20) for individual i is concave with respect to $\mathbf{\Gamma}_{i,T}$, *individual i's vector of social weights, provided that* $\mathbf{X} = 0$. *It admits a unique maximum with respect to* $\mathbf{\Gamma}_{i,T}$ *in that case.*

Proof. By differentiating (20) twice with respect to $\mathbf{\Gamma}_{i,T}$ yields the Hessian,

$$\mathbf{H} = [\mathbf{X}\theta + \beta_\ell \mathbf{Y_{T-1}}] (\mathbf{X}\theta)' + \beta_\ell [\mathbf{X}\theta - (\mathbf{1} - \beta_\ell)\mathbf{Y_{T-1}}] \mathbf{Y'_{T-1}} - \mathbf{2c_2 I}$$

$$= \mathbf{X}\theta(\mathbf{X}\theta)' + \beta_\ell [\mathbf{X}\theta\mathbf{Y_{T-1}} + (\mathbf{X}\theta\mathbf{Y'_{T-1}})'] - \beta_\ell(\mathbf{1} - \beta_\ell)\mathbf{Y_{T_1}}\mathbf{Y'_{T-1}} - \mathbf{2c_2 I}.$$

To ensure concavity, $\mathbf{H}$ must be negative semidefinite. In the simple case in which $\mathbf{X} = \mathbf{0}$, $\mathbf{H}$ reduces to $\mathbf{H} = -\beta_\ell(\mathbf{1} - \beta_\ell)\mathbf{Y_{T_1}}\mathbf{Y'_{T-1}} - \mathbf{2c_2 I}$, which is negative semidefinite as long as $\beta_\ell < 1$, which ensures that the only terms in equation (20) that are quadratic in $\mathbf{\Gamma_{i,T}}$ receive a negative sign. For $\mathbf{X} \neq \mathbf{0}$, interpretation of the condition is more involved. Since the term $\mathbf{X}\theta(\mathbf{X}\theta)'$ is positive definite and the term $\mathbf{X}\theta\mathbf{Y_{T-1}} + (\mathbf{X}\theta\mathbf{Y'_{T-1}})'$ is ambiguous in sign, a sufficiently large coefficient of the marginal connection cost would be required to make the utility function concave with respect to the weighted adjacency matrix. In case $\mathbf{H}$ is negative semidefinite, the first order conditions,

$$\mathbf{\Gamma'}_{i,T} = -\left\{[\mathbf{X}\theta + \beta_\ell \mathbf{Y_{T-1}}] (\mathbf{X}\theta)' + \beta_\ell [\mathbf{X}\theta - (\mathbf{1} - \beta_\ell)\mathbf{Y_{T-1}}] \mathbf{Y'_{T-1}} - \mathbf{2c_2 I}\right\}^{-1} \times$$
$$[\mathbf{X}\phi + \mathcal{A}_{iT}\mathbf{X}\theta + (\beta_\ell \mathcal{A}_{iT} + \mathbf{x}_i\omega)\mathbf{Y}_{T-1} - c_1\iota], \tag{22}$$

characterize the unique maximum. We note that (22) is in the form of a linear system in terms of $\mathbf{\Gamma'}_{i,T}$.

Q.E.D.

[12] In the terminology of DeMarzo *et al.* (2003), if $\gamma_{ij} > 0$, then i "listens" to j. A weighted adjacency matrix may come about even when connections are indicated by either 0 or 1. One such possibility would be when agents update their beliefs using Bayes theorem. See DeMarzo *et al.* (2003).

A number of remarks are in order. First, for certain applications, it may be important to require the entries of the adjacency matrix to be positive and normalized. Restricting $\mathbf{\Gamma}_{i,T}$ so that it lies in the positive orthant of R^N, $\gamma_{ij,T} \geq 0,\ j = 1, \ldots, N$, a convex set, is straightforward but quite stringent. That is, that would translate to restrictions on values of parameters and of variables. While some of the variables are constant, others depend on the actual state of the economy as of the preceding period, $\mathbf{Y}_{T-1}$. These values accumulate past values of stochastic shocks and are therefore random. Consequently, endogenous weighted networking implies that agents in setting their period $T-1$ decisions must take into consideration that $\mathbf{\Gamma}_T$, the social adjacency matrix in period T, is an outcome of individuals' uncoordinated decisions and stochastic. We note that positivity of the $\gamma_{ij,T}$s is quite critical for a meaningful interpretation of endogenous networking as being akin to neighborhood choice. If the adjacency matrix were to be restricted to be positive, then particular patterns in $\mathbf{\Gamma}$, like a block-diagonal structure with many small blocks is reminiscent of so-called "small worlds," emerging endogenously.

Second, endogenous determination of the social adjacency matrix introduces intertemporal linkages in the individual's decision making for two reasons: first, period $T-1$ decisions affect the period T adjacency matrix $\mathbf{\Gamma}_T$; and second, unlike when the adjacency matrix was taken as given and the assumption was made that all diagonal terms were assumed to be equal to zero, $\gamma_{ii,t} = 0$, now they may well be nonzero, thereby strengthening habit formation. From a psychological perspective, one can argue whether positive $\gamma_{ii,t}$'s should be interpreted as the result of willful acts or are expressing addiction. This intertemporal linkage in turn leads to a clarification of the asset value of a connection between agents. Because of our functional specification for individual utility according to Equ. (2), additional information, in the form of total and marginal contextual effects, is brought to bear upon the problem of determining the social structure and also enters the asset value of social connections. In particular, additional information is included, over and above information included in the reaction function, such as the term $\mathbf{X}\phi$. Third, once the connection weights are endogenized, weights are not necessarily equal, that is the adjacency matrix is not symmetric, let alone constant over time.

We refrain from pursuing further these issues in the present paper. However, in an important sense, networking, once it has been endogenized no longer implies that social interactions assume the form of the mean field case. Furthermore, variable intensities of attachment subsume the issue of strong versus weak ties [Grannovetter (1994)] as a special case.

6.1 Endogenous Networking and Individual Decisions

In obtaining the first-order conditions for individual i's decision in period $T-1$ we need to recognize that $\gamma_{ii,T}$, which is optimally chosen and satisfies (22) above, need not be equal to zero. Therefore $y_{i,T-1}$ enters period T indirect utility $\tilde{U}_{iT}$ via the term $\mathbf{\Gamma}_{i,T}\mathbf{Y}_{T-1}$.[13] Therefore, we now have for all $y_{i,T-1}$ in matrix form:

$$[\delta\beta_\ell(1-\beta_\ell)[\text{Diag}\mathbf{\Gamma_T}]\mathbf{\Gamma}_T + \mathbf{I}]\,\mathbf{Y}_{T-1} = \delta[\text{Diag}\mathbf{\Gamma_T}][\beta_\ell\mathcal{A}_T + \mathbf{X}\omega] + \beta_\ell\mathbf{\Gamma}_{T-1}\mathbf{Y}_{T-2} + \mathcal{A}_{T-1} + \epsilon_{T-1}. \tag{23}$$

where $[\text{Diag}\mathbf{\Gamma_T}]$ denotes the diagonal matrix made up of the diagonal terms of $\mathbf{\Gamma_T}$.

Not surprisingly, the strength of the effect depends on $\delta\gamma_{ii,T}$. This weighs the own period T individual and contextual effects $\mathcal{A}_{i,T}$ and the own endogenous local interaction effect on period T utility, as they are added to the period $T-1$ individual and contextual effects, $\mathcal{A}_{i,T-1}$. It also weighs the own conformist effect of the current decision $y_{i,T-1}$ on itself through its presence in period T utility.

Finally, we turn to the joint evolution of individual decisions and the social network. We note that the setting of Y_{T-1} and Γ_T are based on the same information. Therefore, (23) and (22) may be treated as a simultaneous system of equations: period $T-1$ individual decisions and period T weights of social attachment are jointly determined. These equations may be rewritten schematically as:

$$\begin{aligned} \mathbf{\Gamma}_T &= \mathcal{G}\left(\mathbf{\Gamma}_T, \mathbf{Y}_{T-1}|\mathbf{X}\right), \\ \mathbf{Y}_{T-1} &= \mathcal{Y}\left(\mathbf{\Gamma}_T, \mathbf{Y}_{T-1}, \mathbf{Y}_{T-2}, \varepsilon_{T-1}|\mathbf{X}\right). \end{aligned} \tag{24}$$

The above system of equations is autonomous with only a contemporaneous stochastic shock, the vector ε_{T-1}. It exhibits complicated dynamic dependence: it is second-order with respect to $\mathbf{Y}_{T-1}$ and first-order with respect to $\mathbf{\Gamma}_T$. It is amenable to the usual treatment for dynamical systems, but the fact that $\mathbf{\Gamma}_T$ is a matrix makes matters cumbersome. One especially has to ensure that the system is dynamically stable, perhaps by imposing a kind of transversality condition that restricts the absolute value of entries in the $\mathbf{\Gamma}$ matrix. We plan to pursue further the properties of this system in future research.

[13] It also enters indirectly through the dependence of the endogenous $\mathbf{\Gamma}_{i,T}$ on Y_{T-1}, but by the envelope theorem, those terms cancel out.

6.2 Comparison with Exogenous Networking

As a contrast to exogenous networking, consider a stylized exogenous networking setting where individuals engage in social networking by progressively connecting with the acquaintances of their acquaintances, and so on. Tracking the evolving acquaintance sets is quite straightforward [Brueckner and Smyrnov (2004)]. Starting with an adjacency matrix $\mathbf{\Gamma_0}$, it is straightforward to see that *all* individuals acquire connections with their second neighbors, that is the acquaintances of their acquaintances, then the adjacency matrix becomes $\mathbf{\Gamma_1} = \mathbf{\Gamma_0} + \mathbf{\Gamma_0^2}$. Extending to third neighbors (graph-distance three), the adjacency matrix becomes $\mathbf{\Gamma_2} = \mathbf{\Gamma_1} + (\mathbf{\Gamma_1})^{\mathbf{2}} = \mathbf{\Gamma_0} + \mathbf{2\Gamma_0^2} + \mathbf{2\Gamma_0^3} + \mathbf{\Gamma_0^4}$. Endogenous networking as examined earlier would lead only by chance to such a description of the evolution of networking at the aggregate. Brueckner and Smyrnov (2004) derive a number of results for the case in which $\mathbf{\Gamma_0}$ is irreducible, that is, in networks in which every agent is directly or indirectly connected to every other agent. Notwithstanding the tractability of the network evolution process, one drawback of this approach is that it is hard to come up with an economic justification of this particular evolution process.

7 Conclusions

We conclude by first providing a brief summary. This paper examines social interactions when social networking may be endogenous. It employs a linear-quadratic model that accommodates contextual effects and endogenous interactions, that is local ones where individuals react to the decisions of their neighbors, and global ones, where individuals react to the mean decision in the economy, both with a lag. Unlike the simple $VAR(1)$ form of the structural model, the planner's problem involves intertemporal optimization and leads to a system of linear difference equations with expectations. It also highlights an asset-like property of socially optimal outcomes in every period which helps characterize the shadow values of connections among agents. Endogenous networking is easiest to characterize when individuals choose weights of social attachment to other agents. It is much harder to do so when networking is discrete. The paper also poses the inverse social interactions problem, that is whether it is possible to design a social network whose agents' decisions will obey an arbitrarily specified variance covariance matrix.

We would like to think of this paper as addressing problems that pursue further the logic of one of Alan Kirman's greatest contributions. That is, to further study economies with interacting agents in ways that help bridge an apparent gap with mainstream economics [Kirman (1997; 2003)]. Although the paper does not deal with another of Kirman's key areas of interest, aggregation, it hopes to contribute indirectly in the following fashion. First, aggregation that ignores the presence of social interactions is a tricky venture [c.f. Kirman (1992)]. Second, social interactions give rise to a *social multiplier*, construed here in the sense of Glaeser, Sacerdote and Scheinkman (2003). That is, *social multiplier* is defined as the ratio of the coefficient of a contextual effect obtained from a regression with aggregate data to that obtained from a regression with individual data. As a consequence of aggregation, the social multiplier overstates individual effects, in varying degrees that depend on precise patterns of interdependence and sorting in the data. The tools developed in this paper allows us to explore the following hypothesis. Might the social multiplier be due to the exogeneity of social interactions as assumed by earlier papers? We know from *ibid.* that sorting across groups on the basis of observables reduces the social multiplier, but sorting on unbservables has an ambiguous effect. Our model involves both effects, by positing what is essentially an optimal "social" portfolio problem. Endogenous social networking makes the case forcefully that non-market interactions need not be of the mean field type.

In a mechanical sense, our model represents endogeneity of the infrastructure of trade. Dependence on neighbors' income could be a reduced form for the benefits derived from trading with others. In our model, agents are vying to choose a desirable set of others to be influenced from. We would like to think of this as a precursor to opening up routes for trading. That is a much harder problem, but one that promises attractive payoffs. When markets are incomplete, opening up new markets is a tricky business from the viewpoint of social welfare [Newbery and Stiglitz (1984)]. An alternative interpretation of the model is in terms of endogenous group formation, and could be pursued further. Therefore, for several reasons the type of phenomena that Alan Kirman has chosen to emphasize throughout his work are central to making economics more relevant and a better tool for understanding how societies function.

8 Bibliography

Backus, David, Bryan Routledge, and Stanley Zin (2004), "Exotic Preferences for Macroeconomists," *NBER Macroeconomics Annual,* MIT Press, Cambridge.

Bala, Venkatesh, and Sanjeev Goyal (2000), "A Noncooperative Model of Network Formation," *Econometrica,* 68, 5, 1181–1229.

Bertsekas, Dimitri P. (1972), "Infinite-Time Reachability of State-Space Regions by Using Feeedback Control," *IEEE Transactions in Automatic Control*, AC–17, 5, 604–613.

Bertsekas, Dimitri P. (1995), *Dynamic Programming and Stochastic Control, Volume 1,* Athena Scientific, Belmont, MA.

Binder, Michael, and M. Hashem Pesaran (1998), "Decision Making in the Presence of Heterogeneous Information and Social Interactions," *International Economic Review,* 39, 4, 1027–1052.

Binder, Michael, and M. Hashem Pesaran (2001), "Life-cycle Consumption under Social Interactions," *Journal of Economic Dynamics and Control,* 25, 35–83.

Bisin, Alberto, Ulrich Horst, and Onur Özgür *et al.* (2004), "Rational Expectations Equilibria of Economies with Local Interactions," *Journal of Economic Theory,* forthcoming.

Brock, William A., and Steven N. Durlauf (2001) "Interactions-Based Models," 3297–3380, in Heckman, James J., and Edward Leamer, eds., *Handbook of Econometrics,* Volume 5, North-Holland, Amsterdam.

Brueckner, Jan K. and Oleg Smyrnov (2004), "Workings of the Melting Pot: Social Networks and the Evolution of Population Attributes," CESifo working paper No. 1320, October.

Calvó-Armengol, Antoni, and Matthew O. Jackson (2004), "The Effects of Social Networks on Employment and Inequality," *American Economic Review*, 94, 3, 426–454.

DeMarzo, Peter M., Dimitri Vayanos, and Jeffrey Zwiebel (2003), "Persuasion Bias, Social Influence, and Unidimensional Opinions, *Quarterly Journal of Economics,* CXVIII, 3, 909–968.

Durlauf, Steven N. (2004), "Neighborhood Effects," Ch. 50, in J. Vernon Henderson and Jacques-Francois Thisse, eds., *Handbook of Regional and Urban Economics: Vol. 4, Cities and Geography,* North-Holland, 2004, 2173–2234.

Glaeser, Edward L., Bruce I. Sacerdote and Jose A. Scheinkman (2003), "The Social Multiplier," *Journal of the European Economic Association,* 1, 2–3, 345–353.

Glaeser, Edward L., and Jose A. Scheinkman (2001), "Non-market Interactions," working paper; presented at the World Congress of the Econometric Society, Seattle, 2000.

Goyal, Sanjeev, and Fernando Vega-Redondo (2004), "Structural Holes in Social Networks," working paper, University of Essex, December.

Gronovetter, Mark (1994), *Getting a Job: A Study of Contacts and Careers,* 2nd edition, Harvard university Press, Cambridge, MA.

Hansen, Lars P., and Thomas Sargent (2005), *Misspecification in Recursive Macroeconomic Theory,* University of Chicago and the Hoover Institution, manuscript.

Horst, Ulrich, and Jose A. Scheinkman (2004), "Equilibria in Systems of Social Interactions," *Journal of Economic Theory,* forthcoming.

Ioannides, Yannis M. (2004), "Random Graphs and Social Networks: an Economics Perspective," presented at the IUI Conference on Business and Social Networks, Vaxholm, Sweden, June.

Ioannides, Yannis M., and Linda Datcher Loury (2004), "Job Information Networks, Neighborhood Effects, and Inequality," *Journal of Economic Literature,* XLII, December, 1056–1093.

Ioannides, Yannis M., and Jeffrey E. Zabel (2002), "Interactions, Neighborhood Selection and Housing Demand," Tufts working paper 2002-08; revised March 2004.

Jackson, Matthew O., and Asher Wolinsky (1996), "A Strategic Model of Social and Economic Networks," *Journal of Economic Theory,* 71, 44–74.

Jovanovic, Boyan (1987), "Micro Shocks and Aggregate Risk," *The Quarterly Journal of Economics,* 102, 2, 395–410.

Kirman, Alan P. (1992), "Whom or What Does the Representative Individual Represent?" *Journal of Economic Perspectives,* 6, 2, 117-136.

Kirman, Alan P. (1997), "The Economy as an Interactive System," 491–531, in Arthur, W. Brian, Steven N. Durlauf and David A. lane, eds., *The Economy as an Evolving Complex System II,* Addison-Wesley, Reading, MA.

Kirman, Alan P. (2003), "Economic Networks," 273–294, in Bornholdt, S., and H. G. Schuster, eds., *Handbook of Graphs and Networks: From the Genome to the Internet,* Wiley - VCH, Weinheim.

Kosfeld, Michael (2004), "Economic Networks in the Laboratory: A Survey," working paper, Institute for Empirical Research in Economics, University of Zurich.

Kumar, P. R., and P. Varaiya (1986) *Stochastic Systems: Estimation, Identification and Adaptive Control*, Prentice Hall, Englewood Cliffs, N. J.

Manski, Charles F. (1993), "Identification of Endogenous Social Effects: The Reflection Problem," *Review of Economic Studies,* 60, 531–542.

Moffitt, Robert A. (2001) "Policy Interventions, Low-Level Equilibria and Social Interactions," in Durlauf, Steven N., and H. Peyton Young, eds., *Social Dynamics,* MIT Press, 45–82.

Newman, Mark E. J. (2003), "The Structure and Function of Networks," *SIAM Review*, 45, 167–256.

Newbery, David M. G., and Joseph E. Stiglitz (1984), "Pareto Inferior Trade," *Review of Economic Studies,* 51, 1, 1–12.

Reichlin, Lucrezia (2002), "Factor Models in Large Cross-Sections of Time Series," presented at World Congress of the Econometric Society, Seattle, 2000; working paper, ECARES, Universite Libre de Bruxelles.

Rivlin, Gary (2005a), "Friendster, Love and Money," *New York Times*, January 24.

Rivlin, Gary (2005b), "Skeptics Take Another Look at Social Sites," *New York Times*, May 9.

Sargent, Thomas J. (1987), *Macroeconomic Theory, 2nd edition* Academic Press.

Wasserman, Stanley, and Katherine Faust (1994), *Social Network Analysis: Methods and Applications,* Cambridge University Press, Cambridge.

Weinberg, Bruce A. (2004), "Social Interactions and Endogenous Associations," Department of Economics, Ohio State University working paper.

9 APPENDIX: Endogenous Discrete Networking

We consider next the problem of individual i at time $t = T - 1$, who is contemplating a networking initiative, denoted by the matrix $[\Delta\mathbf{\Gamma}_T]$, prior to setting the period T decision. The resulting network topology is given by:

$$\mathbf{\Gamma}_T = \mathbf{\Gamma}_{T-1} + \Delta\mathbf{\Gamma}_T. \tag{25}$$

We simplify derivation of the impact of networking by considering first its effect in the last period of a finite horizon version of the problem of individuals' lifetime utility maximization. This does not impose any undue restrictions because individually optimal outcomes satisfy conditions which are backward looking.

Since networking is defined in terms of the entries of the adjacency matrix, which are discrete variables, the contribution of different networking initiatives to optimum utility may now be defined in terms marginal but discrete changes, $[\Delta\Gamma_{ij}]$ to the adjacency matrix.

Specifically, given $\mathbf{\Gamma}_{T-1}$ and $\mathbf{Y}_{T-1}$, marginal changes in the network topology may be ranked in terms of their contributions to expected utility. The admissible changes in the network topology, $[\Delta\gamma_{ij,T}]$, may be defined as follows. If a connection does not exist in period $T-1$ between agents i and k, $\gamma_{ik,T-1} = 0$, then it may be created or not created; if such a connection does exist, $\gamma_{ij,T-1} = 1$, then it may be severed or not severed. That is:

$$\begin{array}{lllll} \text{if } \gamma_{ik,T-1} = 0, & \text{and } \mathcal{D}_{ik,T} \geq 0, & \text{then:} & \Delta\gamma_{ik} = 1, & \gamma_{ik,T} = 1; \\ \text{if } \gamma_{ik,T-1} = 0, & \text{and } \mathcal{D}_{ik,T} < 0, & \text{then:} & \Delta\gamma_{ik} = 0, & \gamma_{ik,T} = 0; \\ \text{if } \gamma_{ik,T-1} = 1, & \text{and } \mathcal{D}_{ik,T} < 0, & \text{then:} & \Delta\gamma_{ik,T} = -1, & \gamma_{ik,T} = 0; \\ \text{if } \gamma_{ik,T-1} = 1, & \text{and } \mathcal{D}_{ik,T} \geq 0, & \text{then:} & \Delta\gamma_{ik,T} = 0, & \gamma_{ik,T} = 1, \end{array} \tag{26}$$

where the quantity $\mathcal{D}_{ik,T}$ is defined as follows:

$$\mathcal{D}_{ik,T} = \mathbf{X}_k\phi + [\beta_\ell(2-\beta_\ell)\mathcal{A}_{iT} + \mathbf{x}_i\omega]\frac{1}{|\nu_{T-1}(i)|+1}y_{k,T-1} - \frac{1}{2}\beta_\ell[1-\beta_\ell]$$

$$\left[\frac{1}{|\nu_{T-1}(i)|^2}\left[y_{k,T-1}^2 + \sum_{j\neq i}\gamma_{ij,T-1}\gamma_{ik,T-1}y_{j,T-1}y_{k,T-1}\right] + \frac{(|\nu_{T-1}(i)|+1)^2 - |\nu_{T-1}(i)|^2}{|\nu_{T-1}(i)|^2(|\nu_{T-1}(i)|+1)^2}\left[\sum_{j\neq k}\gamma_{ij,T-1}y_{j,T-1}\right]^2\right]. \tag{27}$$

These definitions may be applied symmetrically for the networking decisions contemplated by individual k. Consequently,

$$\mathcal{D}_{ki,T} = \mathbf{X}_i\phi + [\beta_\ell(2-\beta_\ell)\mathcal{A}_{kT} + \omega]\frac{1}{|\nu_{T-1}(k)|+1}y_{k,T-1} - \frac{1}{2}\beta_\ell[1-\beta_\ell]$$

$$\left[\frac{1}{|\nu_{T-1}(k)|^2}\left[y_{i,T-1}^2+\sum_{j\neq k}\gamma_{kj,T-1}\gamma_{ki,T-1}y_{j,T-1}y_{i,T-1}\right]+\frac{(|\nu_{T-1}(k)|+1)^2-|\nu_{T-1}(k)|^2}{|\nu_{T-1}(k)|^2(|\nu_{T-1}(k)|+1)^2}\left[\sum_{j\neq i}\gamma_{kj,T-1}y_{j,T-1}\right]^2\right]. \tag{28}$$

These definitions along with the transition functions (26), suitably adapted for all agents, define convex sets in R^N which describe fully the motion of the system at any point in time. So, we summarize the networking decisions of all agents by stating that:

$$\Delta\mathbf{\Gamma}_T = \mathcal{D}_{T-1}\left(\mathbf{Y}_{T-1}, \mathbf{\Gamma}_{T-1}\right). \tag{29}$$

with $\mathcal{D}(\cdot)$ an $N \times N$ matrix.

Working backward in the fashion of dynamic programming, we have that agent i at the beginning of period $T-1$ chooses $y_{i,T-1}$, given $\mathbf{\Gamma}_{i,T-1}$ so as to maximize the expectation of remaining lifetime utility. However, because under the assumptions of the basic model $\tilde{U}_{iT}\left(\mathbf{\Gamma}_{i,T}; \mathbf{Y}_{T-1}\right)$ depends on the components of $\mathbf{Y}_{T-1}$ other than y_{iT-1}, its optimal value is given by equation, Equ. (5), as above. By adapting the definition of indirect utility in Equ. (20), we have that remaining expected lifetime utility becomes:

$$\tilde{U}_{iT-1}\left(\mathbf{\Gamma}_{i,T-1}; \mathbf{Y}_{T-2}\right) + \delta\tilde{U}_{iT}\left(\mathbf{\Gamma}_{i,T}; \mathbf{Y}_{T-1}\right). \tag{30}$$

Individual i chooses her period $T-1$ networking, $\Delta\mathbf{\Gamma}_{i,T-1}$, given $\mathbf{\Gamma}_{i,T-2}$ and prior to the realization of that period's shocks, and plans to choose $\Delta\mathbf{\Gamma}_{i,T}$, given $\mathbf{\Gamma}_{i,T-1}$ so as to maximize the optimum value of remaining expected utility. Although individual i ignores the effects that her own actions have on the utilities of others, she still needs to form expectations about the effects of networking by others on the entire pattern of social interactions and therefore on her own utility. These considerations suggest that the economy's adjacency matrix in the following period is to be regarded as random. However, the dynamic programming formulation may still be used, but in the context of the classic optimal control literature, the system matrix becomes stochastic [Bertsekas (1995), p. 141].

We have considered so far the consequences of only a single networking initiative by each agent. Since the indirect utility function, defined in (20) is quadratic in $\mathbf{\Gamma}$, the impacts of deliberate networking with various other agents, which is initiated by individual i and described according to (27), may be ranked. With the assumption of utility costs of forming new connections, which additional connections each agent will undertake is fully determined, and so is the law of motion for the evolution of the patterns of social interactions in the economy. We summarize by stating that

$$\mathbf{\Gamma}_t = \mathcal{L}\left(\mathbf{Y}_{t-1}, \mathbf{\Gamma}_{t-1}\right), \tag{31}$$

where the function $\mathcal{L}(\cdot)$ summarizes the above results.

A number of remarks are in order. First, we note that although the law of motion (31) is stated in terms of quantities that are known as of time t, the evolution of the state of the economy over time depends on realizations of shocks in every period. Second, although the contribution to agent i's utility from opening up a connection with agent k is defined symmetrically to the contribution to agent k's utility from opening up a connection with agent i, those two different contributions need not both have the same sign. This raises the possibility that opening and severing of connections would cycle, unless we impose the condition that directly affected agents must be in mutual agreement. However, presence of costs does introduce friction, which mitigates some of these issues. Third, an extension of the model would be to allow individual the option of initiating contacts with others according to a *Poisson clock* in the style of Blume (1993).