Financial Markets and Firm Dynamics*

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September 13, 1999

Abstract

Recent studies have shown that the dynamics of firms (growth, job reallocation and exit) are negatively associated with firm's size. In this paper we analyze whether financial factors are important in generating this negative relation. We develop a model in which, at each point in time, firms are heterogeneous in the amount of equity, and the equity affects their financing decision. The production and investment behavior of small and large firms differs substantially, and the model replicates many of the key features of industry evolution: smaller firms experience faster growth, higher rates of job creation and destruction and lower survival rates.

Introduction

Recent studies of the relationship between firm size and growth have overturned the conclusion of Gibrat's Law which holds that firm size and growth are independent. Studies by Bronwyn Hall (1987) and David Evans (1987) show that the growth rate of manufacturing firms and their volatility is negatively associated with firm age and size. Firm size and age also play an important role in characterizing the dynamics of job reallocation. Davis, Haltiwanger, & Schuh (1996) show that the rates of job creation and job destruction in U.S. manufacturing firms are decreasing in firm age and size and that, conditional on the initial size, small firms grow faster than large firms. The empirical regularities of firm dynamics can be summarized as follows:

- Firm growth decreases with firm age and size.
- The variability of firm growth decreases with firm age and size.
- Job creation and destruction decrease with firm age and size.
- The probability of firm survival increases with firm age and size.

*We have received helpful comments and suggestions from Jeff Campbell, David Chapman, Hal Cole, Tom Cosimano, Joao Gomes, Hugo Hopenhayn, José-Víctor Ríos-Rull, and Harald Uhlig. This research is supported in part by NSF Grant SBR 9617396.

A number of authors have tried to explain these features of the economy as arising from persistent idiosyncratic shocks to firms production technology. Examples of this approach include Boyan Jovanovic (1982), Hugo Hopenhayn (1992) and Jeffrey Campbell (1998). In this paper we take a different (but complementary) approach. We ask whether this set of observations can be explained by the financial decisions that firms make. It seems natural to try to link patterns of firm growth with financial decisions because there are also important regularities in the financial characteristics of firms that are related to their size. Empirically, the financial behavior of firms is characterized by the following facts:[1]

- Small firms pay fewer dividends and invest more.
- Small firms take on more debt.
- Small firms have higher values of Tobin's q.
- The investment of small firm's is more sensitive to cash flows, even after controlling for their future profitability.
- All firms prefer to finance investment with internally generated funds.

In this paper we develop a model with heterogeneous, long-lived firms in which financing decisions are at the heart of the dynamic optimization problem they solve. Firms are heterogeneous because, in each period, they embody different amounts of equity. New firms are created with an initial amount of equity. Firms will then grow or fail as a consequence of their financial decisions and the realization of idiosyncratic shocks to their technology. The growth in firm equity is financed by retained earnings and the fact that profits are stochastic implies that at each point in time there will be a stationary distribution of heterogeneous firms. A key assumption of the model is that, once a firm has been created, the firm's equity can only be increased with reinvested profits. This assumption, which is made't/ keep the model tractable, is consistent with the empirical observation that most firm growth is financed with internal sources.

All firms have access to the same decreasing return-to-scale production technology which produces a single good with labor and capital inputs. The firm's production plan is financed in part with equity and in part with debt. The debt contract is a standard one-period debt contract signed with a financial intermediary: the financial intermediary lends funds at the end of the period and the firm commits to return the borrowed funds plus interest at the end of the next period. If at that time the firm is not able to repay the debt, even after liquidating its assets, the firm faces a bankruptcy problem and it is liquidated. The bank anticipates the possibility that the firm is not able to repay the debt and the interest rate charged depends on the probability of default. By pooling a large number of firms, the intermediary is able to obtain a non-stochastic return from the lending activity.

In deciding the optimal amount of debt, firms face a trade-off: on the one hand, more debt allows them to increase their expected profits by expanding the production scale; on the other hand, the expansion of the production scale implies a higher volatility of profits and a higher

[1] Figures 1a-1d illustrates the first three features of firm financial behavior based on the Compustat data used by Hall & Hall (1993). The fourth fact derives from the empirical findings of Fazzari, Hubbard, & Petersen (1988) and Gilchrist & Himmelberg (1994, 1998). The last fact is documented in Ross, Westerfield, & Jordan (1993).

probability of failure. Given that the firm's objective is a concave function of profits, this higher volatility (for a given expected value of profits) has a negative impact on the firm's value. As the equity of the firm increases, however, the firm becomes less concerned about the fluctuation of profits (in absolute value) and expands the production scale. Therefore, firms with less equity will choose smaller production plans.

The economic environment just described produces firm financial behavior and dynamics with many of the same characteristics that we observe in U.S. data. We show that smaller firms are more profitable, pay less dividends, take on more debt, invest more, have higher values of Tobin's q and their investment is more sensitive to cash flows. At the same time, smaller firms face a smaller probability of surviving, grow faster, and they experience greater variability of job creation and destruction.

The intuition for the firm dynamics that arise is fairly straight-forward. In equilibrium small firms grow faster because they earn higher rates of profits and they distribute, on average, smaller dividends as a percentage of equity. The reason why small firms distribute fewer dividends can be explained as follows. As explained above, a small firm in this economy is one with a small value of equity. Because the technology displays decreasing return to scale, smaller production plans imply higher marginal profits with respect to the scale of production. Because the scale of production depends on the equity of the firm, small firms have a greater incentive to reinvest their profits and increase their equity. Although small firms borrow less in absolute value, the higher marginal profit associated with smaller scales of production induces them to borrow more relative to their equity, and this allows those firms to have higher expected rates of profits (that is, the expected value of profits relative to the value of equity). The higher rate of profits and the reinvestment of these profits allow small firms to grow faster. At the same time, the fact that small firms take on more debt relative to their equity implies that they face higher probability of default. As a result, the probability of survival of smaller firms is lower. Similar considerations explain why the growth rate and the job reallocation of small firms is more volatile.

The alternative approach to explaining the characteristics of firm growth has been to assume that they arise because of technological differences. These differences can be characterized as arising because of persistent idiosyncratic shocks to the technology that firms use or because of learning about the technology. We explore the relative importance of technological differences for the dynamics of the firm by introducing persistent technology shocks. We then compare the growth of firms generated in two different environments. In the first environment there are no financial frictions and markets are complete. In the second environment markets are not complete because of financial frictions. We argue that differences in technology, if they are sufficiently persistent, are important in explaining the age dependence of firm growth, job reallocation, and failure. In the model with only financial frictions, size is the only key dimension of heterogeneity and once we condition on size, age becomes irrelevant to differentiate the dynamics of small and large firms. At the same time, however, the model with only technological differences is not fully capable of capturing the dependence of the firm dynamics on size. Therefore, we conclude that both, financial factors and technological differences, are important for the dynamics of the firms: while financial factors are more important for the size dependence, technological differences are more relevant for the dependence on age.

The paper proceeds as follows. In the next section we describe the basic model with financial frictions and we characterize the decision problem faced by firms. We then describe how the model is calibrated and analyze the characteristics of firm dynamics (growth, job reallocation and exit). After the analysis of the environment with financial frictions, we extend the model to allow for technological heterogeneity. A final section concludes.

1 Model Economy

We study an economy in which the production sector consists of a continuum of heterogeneous firms that maximize the expected discounted values of dividends, that is:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t d_t \right\} \tag{1}$$

where d_t is the dividend distributed at time t and β is the discount factor for the firm.

At each point in time, firms are heterogeneous in the amount of owned capital e. Henceforth, this capital is referred to as the firm *equity*. The amount of equity changes over time as firms reinvest profits. We assume that retained earnings is the only source of increased equity for the firm. We motivate this assumption by appealing to the empirical observation that firms mainly rely on internal sources of funds for finance.[2]

In each period, firms have access to the production technology $y = G(k, l, \varphi)$ where k is the input of capital that depreciates at rate δ, l is the input of labor, and φ is an idiosyncratic shock. The shock is observed after the inputs of labor and capital have been employed in production. We make the following assumptions about the function G and the shock φ.

Assumption 1.1 *The production function assumes the form* $G(k, l, \varphi) \equiv \varphi\, h(\Phi(k, l))$*, with* $\Phi : \mathbf{R}_+ \times \mathbf{R}_+ \rightarrow \mathbf{R}_+$ *continuously differentiable, strictly increasing and strictly concave in each argument, and homogeneous of degree 1; and* $h : \mathbf{R}_+ \rightarrow \mathbf{R}_+$ *strictly increasing, strictly concave and continuously differentiable.*

Assumption 1.2 *The technology shock* φ *is independently and identically distributed across firms and time, and it takes values in the set of real numbers* $\mathbf{R}$*. The density function* $f : \mathbf{R} \rightarrow [0, 1]$ *is continuous, differentiable and* $f(\varphi) > 0$, $\forall\, \varphi \in \mathbf{R}$.

The concavity of h along with the linear homogeneity of Φ implies that the production function displays decreasing returns-to-scale. At this stage we assume that the technology shocks are i.i.d. because we want to focus on how financial factors, rather than technological differences, affect the dynamics of small and large firms. The impact of technological differences will be analyzed in Section 4 where we extend the model to allow for persistent technology shocks.

The choice of k and l is made one period in advance, before observing the shock φ. The input of capital is financed in part with equity and in part with funds borrowed from a financial intermediary (firm's debt). Thus, $k = e + b$, where b is the firm's debt.

[2] Ross et al. (1993) and Smith (1977) document that firms raise more than 80 percent of equity from internal sources.

The contract signed between the firm and the intermediary is a standard one-period debt contract based on a non-contingent interest rate. In the next period, after the realization of the shock, if the firm is not able to repay the debt (inclusive of the interest) even after liquidating all its assets, the firm faces a bankruptcy problem and it is liquidated. In that case, the liquidation value of the firm's assets are confiscated by the intermediary in partial repayment of the debt. The intermediation sector is competitive and the interest rate charged by the intermediary, denoted $\tilde{r}$, is such that the expected return from the loan equals the risk-free market rate r.

Denote with $\underline{\varphi}$ the value of the shock below which the firm is not able to repay its debt. This is defined by the condition $(1-\delta)k - wl + G(k, l, \underline{\varphi}) = b(1+\tilde{r})$, where w is the wage rate paid on hired labor. Then the interest rate charged by the intermediary is implicitly defined by the following equation:

$$(1+r)b = (1+\tilde{r})b \int_{\underline{\varphi}}^{\infty} f(d\varphi) + \int_{-\infty}^{\underline{\varphi}} [(1-\delta)k - wl + G(k, l, \varphi)] f(d\varphi) \tag{2}$$

The amount of funds the firm can borrow is subject to an upper bound (borrowing limit). We assume that this limit depends on the value of the capital owned by the firm e and is given by the function $\bar{b}(e)$.

Assumption 1.3 *The function $\bar{b}(e)$ is continuous, non-decreasing and satisfies $\bar{b}(0) = 0$.*

The borrowing limit $\bar{b}(e)$ imposes the lower bound $e = 0$ to the size of the firm: a firm with zero equity cannot borrow, and therefore, it cannot produce. Consequently, its market value is also zero. Although the imposition of an exogenous borrowing limit may seem arbitrary, it can be justified by an enforceability argument as in Albuquerque & Hopenhayn (1997).[3] Given that it is impossible for the firm to get financing when $e = 0$, liquidation is the optimal action for the financial intermediary when the firm is not able to repay the debt. At the moment of choosing its debt, the firm also decides the distribution of dividends. Dividends cannot be negative, implying that the firm cannot raise funds by issuing new equity.

In the environment described so far, a firm exits the market only if it goes bankrupt. In reality, however, some firms exit the market for reasons that are not related to bankruptcy. To allow for this possibility, we assume that in each period the firm faces a probability η of becoming unproductive. This is equivalent to assuming that the shock φ assumes the value of zero forever. The firm observes the viability of its technology before deciding the production plan for the next period. In the case the firm ceases to be productive, the firm is liquidated and the liquidation value, given by the end-of-period resources after repaying its debt, is distributed as dividend.

Before analyzing the optimization problem faced by the firm, we make the following assumption on the lending rate r.

[3] For example, we can assume the following enforceability problem: after receiving the loan, a firm can distribute all the borrowed cash to the shareholders and then declare bankruptcy. In that case, if the liquidation value of the firm is smaller than the debt, the bank will realize a loss with probability one, independently of the interest rate charged in the contract. In order to prevent this moral hazard problem, the bank is willing to make loans to the firm only up to a limit. The incentive-compatible limit is equal to the value of the firm conditional on the firm implementing a non-deviating policy. Because the value of a firm is an increasing function of the equity, to simplify the analysis the liquidation value of the firm's capital is taken as proxy for the current value of the firm.

Assumption 1.4 *The interest rate r is such that $1/\beta - 1 > r > 0$.*

This condition imposes that the firm discounts dividends at a higher rate than the lending rate. The imposition of this condition is necessary for the firm's problem to be well defined. Moreover, as we will see in the next section, it implies that the equity size of the firm is bounded.[4]

1.1 The Firm's Problem

In this section we describe the optimization problem solved by the firm using a recursive formulation. For analytical convenience, we describe the firm's problem at the end of the period, after the payment of dividends. Although the choice of b, k, l takes place at the same time in which the firm chooses the dividends, and therefore, e', it will be convenient to analyze these two choices as if they were sequential. Define $\Omega(e)$ to be the value of the firm at the end of the period after the payment of dividends and before choosing the production scale. The firm's problem can then be written as:

$$\Omega(e) = \max_{b \leq \bar{b}(e),k,l} \left\{ \beta \int_{\underline{\varphi}(e,b,k,l)}^{\infty} \left[\max_{e'} \left\{ e + \pi - e' + \Omega(e') \right\} (1-\eta) + (e+\pi)\eta \right] f(d\varphi) \right\} \tag{3}$$

subject to

$$k = e + b \tag{4}$$

$$0 \leq e' \leq e + \pi \tag{5}$$

$$\pi(e, b, k, l, \varphi) = G(k, l, \varphi) - wl - \tilde{r}b - \delta k \tag{6}$$

$$(1+r)b = (1+\tilde{r})b \int_{\underline{\varphi}}^{\infty} f(d\varphi) + \int_{-\infty}^{\underline{\varphi}} [(1-\delta)k - wl + G(k,l,\varphi)] f(d\varphi) \tag{7}$$

In this formulation, the firm's choice occurs in two stages: at the end of the period it chooses the production scale, that is, the variables b, k, l. Then in the next period, after the realization of the shock, if the firm is able to repay the debt ($\varphi > \underline{\varphi}$) and continues to be productive (which happens with probability $1 - \eta$), it chooses the next period equity e'. In the event a firm is not able to repay the debt ($\varphi < \underline{\varphi}$) or ceases to be productive (which happens with probability η) it will be liquidated. In the event of failure the liquidation value of the firm is confiscated by the financial intermediary in partial repayment of the debt (in the case of negative value, the intermediary covers the losses); in the event in which the firm ceases to be productive, the liquidation value of the firm is equal to end-of-period resources $e + \pi$, that are distributed as

[4] There are different ways to motivate this assumption. For example, we can assume that the remuneration of managers is proportional to the firm's equity. Alternatively, we could assume that in the event a firm becomes unproductive, its liquidation involves a cost that is also proportional to the initial equity of the firm.

dividends. Notice that the whole expression is discounted because dividends are next period dividends.

Equation (5) imposes an upper and a lower bound to the next period equity of the firm: the next period equity cannot be smaller than zero and greater than the end-of-period resources. This simply says that the firm cannot raise equity with external sources. Equation (6) defines the firm's net profit and equation (7), derived in (2), implicitly defines the interest rate charged by the financial intermediary.[5]

We now state three lemmas that will be used below in the proof of the existence and uniqueness of the firm's optimal policy. The first two lemmas provide a partial characterization of the firm's policy. The second lemma characterizes the properties of the firm's profit function.

Lemma 1.1 *The optimal capital-labor ratio is independent of the scale of production and it only depends on the price ratio $w/(r+\delta)$.*

Proof 1.1 *Direct derivation of the first order conditions with respect to k and l gives:*

$$\frac{w}{r+\delta} = \frac{\partial\Phi(k,l)/\partial l}{\partial\Phi(k,l)/\partial k} \tag{8}$$

Given the properties of Φ defined in assumption 1.1, the term on the right-hand side of equation (8) only depends on the ratio k/l, and it is monotonically increasing in this ratio. Therefore, there exists only one value of k/l that satisfies (8).

Q.E.D.

The importance of this lemma is that, combined with the equation (4), we can study the choice of the production scale as the choice of only one variable, that is, b. This, in turn, allows us to write the production function as $\varphi h(A\cdot(e+b)) = \varphi F(e+b)$, where A depends on r, w and δ. Given the strict concavity of h, the function F is strictly concave in e and b. In addition, we can also express the labor cost as $wl = \phi(e+b)$, where ϕ is also a function of r, w and δ. Notice that the chosen value of b must be in the compact and convex set $B(e) \equiv [\underline{b}(e), \bar{b}(e)]$. The lower bound is $\underline{b}(e) = -e$ and derives from the fact that the input of capital k cannot be negative.

Lemma 1.2 *Under assumption 1.4 there exists $\bar{e} < \infty$ such that $e' \leq \bar{e}$*

This lemma can be easily understood once we take into consideration the decreasing return-to-scale property of the production function. This implies that after a certain scale of production, the firm gets a higher return by lending its funds to the intermediary at the market rate r, rather than expanding its scale of production. Consequently, in the limit the marginal return from accumulating equity converges to r. Because the firm discounts dividends at a higher rate, the accumulation of equity must be bounded. On the other hand, if the discount rate is smaller than

[5] There are different ways to formulate the firm's problem recursively. For example, we could choose the end-of-period resources $e+\pi$ as the state variable for the firm's value. Although this alternative formulation may simplify the presentation of the firm's problem, at the same time it complicates the technical analysis because the value of the firm would not be bounded.

the lending rate, the firm will have an incentive to postpone dividends forever and its problem is not well defined.

For later use, define $q(e, b, \varphi)$ to be the firm's end-of-period resource function. This function is given by:

$$q(e,b,\varphi) \equiv \begin{cases} (1-\delta-\phi)(e+b)+\varphi F(e+b)-(1+\tilde{r})b & \text{if} \quad \varphi > \underline{\varphi} \\ \\ 0 & \text{if} \quad \varphi \leq \underline{\varphi} \end{cases} \tag{9}$$

The following lemma defines the properties of this function.

Lemma 1.3 *Let (e_1, b_1) and (e_2, b_2) be two arbitrary points in the feasible space, let (e_α, b_α) be a convex combination of these two points, and define the function $\psi(\varphi)$ as:*

$$\psi(\varphi) = q(e_\alpha, b_\alpha, \varphi) - \alpha q(e_1, b_1, \varphi) - (1-\alpha) q(e_2, b_2, \varphi) \tag{10}$$

Then there exists $\hat{\varphi} < \infty$ such that $\psi(\varphi) \leq 0$ if $\varphi \leq \hat{\varphi}$, and $\psi(\varphi) > 0$ if $\varphi > \hat{\varphi}$. Moreover, $\int \psi(\varphi) f(d\varphi) > 0$.

Proof 1.2 *Appendix.*

The lemma states that the end-of-period resource function q, is strictly concave in (e, b) (for a given φ) only for large values of the shock. When we take the expected value, however, the concave points dominate the non-concave ones and the expected value of q is strictly concave. The fact that the function q is not concave for all values of the shock, makes it difficult to prove the uniqueness of the firm's choice. In order to assure that the firm's policy is unique, we have to impose some restrictions on the stochastic process for the technology shock φ.

Define $\underline{\mathbf{S}}$ to be the set that contains $\underline{\varphi}(e, b)$ for all $b \in B(e)$ and for all $e \in [0, \bar{e}]$. Formally, $\underline{\mathbf{S}} \equiv \{\varphi \mid \varphi = \underline{\varphi}(e,b), \forall b \in B(e), \forall e \in [0, \bar{e}]\}$. By restricting the values that the probability density function f can take over the set $\underline{\mathbf{S}}$, we are able to prove the concavity of the firm's value function and the uniqueness of the firm's optimal choice.

Proposition 1.1 *Let ϵ be a strictly positive scalar. If for ϵ small enough $f(\varphi) \leq \epsilon$, $\forall \varphi \in \underline{\mathbf{S}}$, the firm's problem is well defined and there exists a unique strictly increasing and concave function $\Omega(e)$ that satisfies the functional equation (3). In addition the firm's solution is unique and the policy rule $b(e)$ is continuous in e.*

Proof 1.1 *Appendix.*

The proof of the above proposition implicitly characterizes the optimal dividend policy of the firm which is given by:

$$d = \begin{cases} 0, & \text{if} \quad e + \pi(e,b,\varphi) \leq \bar{e} \\ \\ e + \pi(e,b,\varphi) - \bar{e}, & \text{if} \quad e + \pi(e,b,\varphi) > \bar{e} \end{cases} \tag{11}$$

Therefore, a firm distributes dividends only after its equity has reached $\bar{e}$. This result derives from the fact that the financial frictions assumed in the model create a strict link between the equity of the firm and its production scale: in order for the firm to expand its production scale, the firm needs to increase its equity. Given the assumption of decreasing returns to scale, when $e < \bar{e}$, the marginal profit with respect to the production scale is high and the firm would like to expand the scale of production. The way to make this possible (optimally or by constrain) is by retaining profits and increasing the value of equity.[6]

1.2 New entry and invariant distribution of firms

Each period there will be entry by newly created firms. There is a cost to create a new firm which is composed of two components: a fixed cost κ and a variable cost λ proportional to the amount of the initial equity (initial size of the firm). The presence of the fixed component is necessary because, absent κ, the optimal size of a new firm in terms of equity would be zero in the limit, and the return from creating new firms would tend to infinity. The variable component is introduced to control for the optimal size of new entrant firms. As we will see, when $\lambda = 0$, the optimal size of new entrants would be the maximum size $\bar{e}$, while empirically the size of new firms is generally small.

If $\Omega(e)$ is the end-of-period (after dividend) value of a firm with e units of equity, the surplus generated by creating a new firm is $\Omega(e) - \kappa - (1+\lambda)e$. The optimal size of a new firm, denoted by e_0, maximizes this surplus, that is:

$$e_0 = \arg\max_{e \geq 0} \Big\{ \Omega(e) - \kappa - (1+\lambda)e \Big\} \tag{12}$$

Figure 1 illustrates how the optimal size of new firms is determined. The figure reports the value of the firm $\Omega(e)$. The optimal size of new firms is given by the value of equity e for which the line with slope $1+\lambda$ is tangent to $\Omega(e)$. In this partial equilibrium analysis with fixed prices, a stationary equilibrium is possible only if the surplus from creating a new firm is zero, that is, $\Omega(e) = \kappa + (1+\lambda)e$. Therefore, in what follows we assume that the fix cost κ is such that this arbitrage condition is satisfied. In this case the intercept of the tangent line is $-\kappa$.[7]

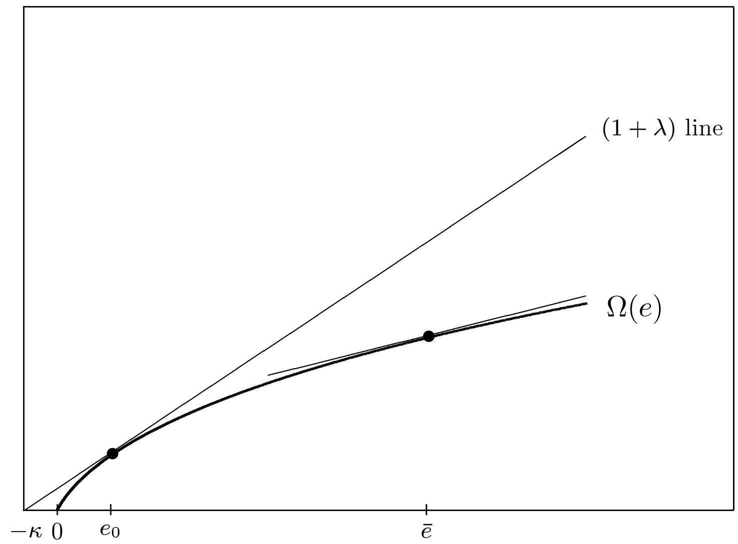
The properties of the value function stated in proposition 1.1 guarantee that there exists a unique value of the fix cost κ for which the arbitrage condition is satisfied. This value of κ also implies that the size of new entrants e_0 is unique.

Figure 1 also shows how the maximum size of the firm is determined. This is denoted by $\bar{e}$ and it is the value of equity for which the slope of the firm's value curve is equal to 1. Once the firm has reached this size, there is no further advantage in increasing the size of the firm, and the optimal dividend policy consists of retaining profits until the equity of the firm reaches $\bar{e}$. Notice that if $\lambda = 0$, the optimal size of new entrants is $\bar{e}$. When $\lambda > 0$, the optimal size of new entrants is smaller than $\bar{e}$, given that the slope of the firm's value curve at this point is only 1.

[6] It is important here not to confuse the marginal (static) profit from the total marginal return to scale, which takes into consideration also the changes in the next period value of the firm. If $e < \bar{e}$, the marginal profit is positive. However, if the borrowing limit is not binding, the total marginal return to scale is necessarily zero (otherwise the firm would borrow more).

[7] In a general equilibrium analysis, the entrance of new firms would induce changes in the prices and in the value function of the firm Ω until there are no gains from creating a new firm.

Figure 1: Current value of new and old firms as a function of equity.



Given the system of prices, this framework generates a complex dynamics of firms. Does the sequence of distributions of firms generated by the model converge to an invariant distribution? The answer to this question depends on the properties of the transition function generated by the optimal decision rule of the firm $b(e)$. Let $Q(e, \mathbf{A}) : [0, \bar{e}] \times \mathcal{A} \to [0, 1]$ be the transition function, where $\mathcal{A}$ is the collection of all Borel sets that are subsets of $[0, \bar{e}]$, and $\mathbf{A}$ is one of its elements. Note that we can define the transition function over the measurable space $([0, \bar{e}], \mathcal{A})$ because the equity size of the firm cannot be smaller than zero, and the optimal firm policy is such that $e' \leq \bar{e}$. The function Q delivers the following distribution function for the next period equity e', given the current value of e:

$$\int_0^x Q(e, de') = \begin{cases} (1-\eta)\displaystyle\int_{\underline{\varphi}}^{\underline{\varphi}+\frac{x}{F(e+b)}} f(d\varphi) & \text{if} \quad x < e_0 \\ (1-\eta)\displaystyle\int_{\underline{\varphi}}^{\underline{\varphi}+\frac{x}{F(e+b)}} f(d\varphi) + 1 - (1-\eta)\displaystyle\int_{\underline{\varphi}}^{\infty} f(d\varphi) & \text{if} \quad e_0 \leq x < \bar{e} \\ 1 & \text{if} \quad x \geq \bar{e} \end{cases} \tag{13}$$

In the specification of the transition function we assume that exiting firms re-enter with the size e_0. Given a current probability measure of firms μ_t, the function Q delivers a new probability measure μ_{t+1} through the mapping $\Psi : M([0, \bar{e}], \mathcal{A}) \to M([0, \bar{e}], \mathcal{A})$, where $M([0, \bar{e}], \mathcal{A})$ is the space of probability measures on $([0, \bar{e}], \mathcal{A})$. The mapping Ψ is defined as:

$$\mu_{t+1}(\mathbf{A}) = \Psi(\mu_t)(\mathbf{A}) = \int_e Q(e, \mathbf{A}) d\mu \tag{14}$$

An invariant probability measure μ^* is the fixed point of Ψ, *i.e.*, $\mu^* = \Psi(\mu^*)$. In the following lemma we prove that Q has the Feller property. This property turns out to be useful in the subsequent proof that an invariant probability measure of firms exists.

Lemma 1.4 *The transition function Q has the Feller property.*

Proof 1.3 *Appendix.*

We then have the following proposition.

Proposition 1.2 *An invariant probability measure of firms μ^* exists. If in addition*
(a) $k(e) = e + b(e)$ is not decreasing in e;
(b) for $\epsilon > 0$ sufficiently small, $f'(\varphi) < \epsilon$;
then μ^* is unique and the sequence of probability measures $\{\Psi^n(\mu_0)\}_{n=0}^{\infty}$ converges weakly to μ^* from any arbitrary $\mu_0 \in M([0, \bar{e}], \mathcal{A})$.

Proof 1.2 *Appendix.*

The first part of the proposition is proved by applying Theorem 12.10 in Stokey, Lucas, & Prescott (1989). To apply this theorem we need to show that Q has the Feller property which is proved in lemma 1.4 (we also need the compactness of the state space $[0, \bar{e}]$ which is obvious). To prove the second part of the proposition we need two extra assumptions: first, that Q is monotone and, second, that Q satisfies a mixing condition. The two conditions (a) and (b) in the above proposition guarantee that Q is monotone, while the mixing condition derives from assuming that the density function of the shock φ is defined over the real line and for any $\varphi \in \mathbf{R}$, there exists $\epsilon > 0$ such that $f(\varphi) \geq \epsilon$. The proof is then based on the application of Theorem 12.12 in Stokey et al. (1989).

Under the conditions of proposition 1.1, the non-decreasing property of k (condition (a) of the above proposition) can be easily established. Therefore, if the density function satisfies condition (b), the invariant distribution is unique.

2 Calibration

We calibrate the model assuming that a period is a year and we set the risk-free lending rate r equal to 4 percent. The depreciation rate is assumed to be 0.07.

The production technology is characterized by the functions h and Φ, and by the stochastic properties of the technology shock φ. The functional forms for these two functions are specified as $h(x) = x^{\nu}$ and $\Phi(k, l) = k^{\theta} l^{1-\theta}$, and the technology shock is assumed to be normally distributed with mean m and standard deviation σ. The parameter ν determines the degree of returns-to-scale. Studies of the manufacturing sector as in Basu & Fernald (1997), find that this parameter is close to 1. We assign a value of 0.95. Then, given the value of ν, we choose $\theta = 0.30$ which implies a capital income share of approximately 36 percent.

In the sample of firms analyzed by Evans (1987), the average probability of exit is about 5 percent. This is the average exit rate we want the model to replicate. However, only a fraction of this exit is due to bankruptcy. Using the estimates of Dun and Bradstreet Corporation for the period 1984-92, we assume that the bankruptcy rate is 1 percent. Consequently, we set $\eta = 0.04$ which is the rate of exogenous exit. The borrowing limit is assumed to be $\bar{b}(e) = 2e$. As we will see, only very small firms borrow up to the limit, so the results of the paper are not sensitive to a relaxation of this limit.

The proportional cost to create a new firm, that is, the parameter λ, is important in determining the size of new entrants: the larger the value of λ , the smaller the size of new entrants. By fixing the value of this parameter to 0.60, we keep the size of new entrants relatively small as in the data. The fixed cost κ is such that the arbitrage condition for the creation of new firms—that is, the condition $\Omega(e_0) = \kappa + (1 + \lambda)e_0$—is satisfied.

There are still three parameters to be pinned down. Those are the parameters m and σ and β. The calibration of these parameters is obtained by imposing the following three conditions: (a) The largest firm employs 2,000 workers; (b) The average probability of default is 1 percent; (c) The value of debt of the largest firms is 25 percent the value of its assets.

With the largest firm employing 2,000 workers, the model generates enough heterogeneity covering the range of heterogeneity studied, for example, in Evans (1987). Condition (c) derives from balance sheet evidence of large firms. For example, in the sample of firms analyzed by Hall & Hall (1993), the ratio between debt and total assets is about 0.25. The full set of parameter values is reported in table 1.

Table 1: Calibration values for the model parameters.

Lending rate	r	0.040
Intertemporal discount rate	β	0.956
Mean idiosyncratic shock	m	0.065
Standard deviation shock	σ	0.090
Production technology parameter	θ	0.300
Return to scale parameter	ν	0.950
Depreciation rate	δ	0.070
Probability of exogenous exit	η	0.040
Fix set-up cost	κ	2.412
Variable set-up cost	λ	0.600

3 Properties of the calibrated economy

3.1 The financial behavior of firms

Figure 2a plots the value of debt which is an increasing function of equity. It is important to point out that this increasing relationship between equity and debt is not a direct consequence of the borrowing limit. In fact, as is shown in the figure, the borrowing limit is never binding except for the smallest firms. We would have the same shape if we had imposed the borrowing limit only for the smallest firms.[8]

To understand why firms do not borrow up to the limit, we have to consider the trade-off that they face in deciding the optimal amount of debt. On the one hand, more debt allows them to expand the production scale and increase their expected profits; on the other, the expansion of the production scale implies a higher volatility of profits and a higher probability of failure. Given that a large fraction of profits is reinvested, and the firm's future value is a concave function of equity (see figure 2e), the firm's objective is a concave function of profits. This implies that the higher volatility of profits (for a given expected value) has a negative impact on the firm's value. Therefore, in deciding whether to expand the scale of production by borrowing more, the firm compares the marginal increase in the expected profits with the marginal increase in its volatility (and therefore, in the volatility of next period equity). Due to the decreasing return-to-scale property of the production function, as the firm increases its equity and implements larger production plans, the marginal expected profits from further increasing the production scale decreases. This implies that the firm becomes more concern about the volatility of profits and borrows less in proportion to its equity. Therefore, as the firm grows, the composition of the sources of finance changes in favor of internal sources.

Figures 2c plots the expected rate of profits, which is decreasing in the size of the firm. This derives from the combination of the financing policy outlined above and the decreasing return-to-scale property of the production function. The higher profitability of smaller firms implies that they have a greater incentive to reinvest profits. In fact, as shown by figure 2d, the expected rate of dividend is increasing in the size of the firm. The higher profitability of small firms, associated with their lower rate of distributed dividends, implies that small firms invest more. Also notice that, in this economy, firms with higher profits invest more independently of their future profitability.[9] Therefore, the model is consistent with the empirical findings of Fazzari et al. (1988) and Gilchrist & Himmelberg (1994, 1998) for which cash flows have a significant impact on firms' investment, even after controlling for their future profitability.

Figures 2e and 2f plot the firms' value and the value of Tobin's q. As can be seen from these pictures, the firm's value is an increasing and concave function of equity and Tobin's q is decreasing in firm size. As a consequence of higher borrowing, small firms face a higher probability of default as shown in figure 2g. This in turn implies that small firms have lower survival rates.

The steady state distribution of firms over equity is plotted in Figure 2h. If we exclude the largest size, the shape of this distribution presents a degree of skewness toward small firms which is also an empirical regularity of the data. There is a concentration of firms at the bottom of the distribution because the optimal size of new entrants is small. The concentration of firms in the largest class, instead, follows from the existence, in the model, of an upper bound to the firm's size. In the data we have firms that employ many more workers than the largest firms in the model. Although the number of these firms is relatively small, they account for a large fraction of aggregate production. Accordingly, the largest firms in the model must be interpreted as representing the production of firms employing more than 2,000 workers: the large share in

[8] Of course, the borrowing limit affects indirectly this relation. Because more debt increases the probability of falling in the constrained range, the firm do not borrow up to the limit in order to reduce this probability.

[9] Remember that shocks are i.i.d.

production of these big firms is accounted for in the model by an increase in the number of firms rather than their size.

The joint distribution of firms over equity (size) and age is reported in Figure 3. As can be seen from this figure a large number of firms are small and young.

3.2 Industry Dynamics

In this section we describe the industry dynamics generated by the calibrated model and, in particular, we show how the growth rate of the firm, its volatility, the reallocation of jobs and the survival rate change with the firm's size and age.

The growth rate of the firm in terms of equity, denoted by g, is a function of its equity e, its borrowing decision b, and the idiosyncratic shock φ. It is described by the following function:

$$g(e,b,\varphi)=\begin{cases} -1 & \text{if} \quad \varphi \leq \underline{\varphi}(e,b) \\ \frac{(\varphi-\underline{\varphi}(e,b))F(e+b)-e}{e} & \text{if} \quad \underline{\varphi}(e,b)<\varphi<\bar{\varphi}(e,b) \\ \frac{\bar{e}-e}{e} & \text{if} \quad \varphi \geq \bar{\varphi}(e,b) \end{cases} \tag{15}$$

where $\bar{\varphi}(e,b)$ is the value of the shock for which the end-of-period resources of the firm is equal to $\bar{e}$. We are interested in the expected value of the growth rate given the initial size of the firm e and its policy b, that is, $\int g(e,b,\varphi)f(d\varphi)$. The volatility of growth is measured by its standard deviation computed as $[\int(g(e,b,\varphi)-E(g))^2 f(d\varphi)]^{1/2}$.

Figure 4a reports the average growth rate as a function of the initial size of the firm in terms of equity and Figure 4b its standard deviation. As can be seen from these graphs, the growth rate and its volatility is decreasing in the firm's size. The higher growth rate of small firms derives from the higher rate of profits of these firms, and from their lower dividend payments (See Figures 2c and 2d).

Figure 4c plots the rates of job creation and job destruction as functions of the firm size. Following Davis et al. (1996), job creation is defined as the sum of the employment gains of expanding firms, and job destruction is defined as the sum of the employment losses of contracting firms. As can be seen from the figure, both creation and destruction are decreasing in the firm's size.

Figures 4e plots the probability of exit, that is, the probability of failure plus the exogenous probability η, which is decreasing in the firm's size.

Figures 4e-4h plot the same variables of figures 4a-4d but as a function of the firm's age. These variables are computed by averaging them according to the size distribution of each age class of firms. As can be seen from these figures, the growth rate, its volatility, the rates of job reallocation and the exit rate are all decreasing in firm's age. The dependence of these variables on the age of the firm is explained by the same factors that explain their dependence on the size of the firm: younger firms experience faster growth, higher rates of job reallocation and lower survival rates because they are smaller.

4 An economy with persistent shocks

The analysis conducted in the previous section shows that a model driven the financial decisions of firms captures many of the salient qualitative features of industry dynamics. In particular, we observe that the model generates the higher rates of job reallocation and lower survival rates of small firms. At the same time it replicates decreasing unconditional rates of job reallocation and exit rates as a function of the firm's age. However, if we control for size, age becomes irrelevant in differentiating the dynamics of small and large firms: the dependence on age shown in Figures 4e-4h derives only from the fact that young firms are on average smaller. Although this is also an empirical regularity of the data, the results of Evans (1987), Hall (1987) and Davis et al. (1996) also show that the dynamics of young firms differs from the dynamics of older firms for each size class of firms. This further feature of the data is not captured by the model economy developed in the previous sections.

The fact that age is irrelevant for the firm's dynamics, derives from the assumption that technology shocks are independently and identically distributed. This implies that once we fix the size, firms are all alike independently of their previous history. This modeling strategy has the advantage of isolating the impact of financial factors on the dynamics of firms. However, it necessarily misses other factors that may be important in shaping the dynamics of firms, in addition to financial factors. For that reason we view our modeling strategy as complementary, rather than alternative, to models in which the dynamics of firms are driven by technological differences.

To explore the importance of technological differences in shaping the heterogeneous dynamics of firms, in this section we extend the previous model by introducing persistent technology shocks. As in models developed in Hopenhayn & Rogerson (1993), Campbell & Fisher (1997), and Gomes (1997), among others, we assume that the shock follows an exogenous Markov process. The functional form of the production function is the same as in Section 1, that is, $G(k, l, \varsigma) = \varsigma h(\Phi(k, l))$. The technology shock is now denoted by ς and it is the sum of two components, $\varsigma = z + \varphi$. The variable z takes values in the finite set $\mathbf{Z} = \{z_1, .., z_N\}$ and follows a first order Markov process with transition probability $\Gamma(z'/z)$. The second component of the shock, φ, is independently and identically distributed and satisfies the same properties assumed in section 1. This economy differs from the previous one in that the mean value of the shock follows a Markov process rather than being constant.

We assume that z is observed at the end of the period, and therefore, it is known at the moment in which the firm chooses the dividend policy and production plan (still unknown however is the value of φ). The analysis of the existence and uniqueness of the firm policy conducted in Section 1.1 can be easily extended to this new version of the model. In the recursive formulation, the firm's value is now also a function of the shock z, that is, $\Omega(z, e)$.

Some new assumptions are needed in describing the entrance of new firms. Depending on the level of technology with which new firms enter the market, we will have a different age dependence of the dynamics of the firm. One possibility is to assume that new entrants are initially low productivity types, that is, $z = z_1$. This assumption would be consistent with a learning (by doing) interpretation of the technology shock z. Alternatively, we could assume that the new entrants have high productivity, that is, $z = z_N$. This is consistent with the view that new entrants possess better technologies and perform better than incumbent firms as in Greenwood & Jovanovic (1999). We explain later how this second assumption has some advantages in generating the right age dependence of firm dynamics. Accordingly, in what follows, we will assume that the level of technology of new entrants is $z = z_N$.

In each period there are many draws of new projects, where each project is characterized by the technology level z. As in the previous section, the implementation of a new project requires an initial fixed and variable cost. Given a draw z, the value of the new project is $\Omega(z, e_0(z))$, where $e_0(z)$ is the initial optimal equity when the initial shock is z. The initial equity is determined as described in Section 1.2. A new project will be implemented only if $\kappa+(1+\lambda)e_0(z) \geq \Omega(z, e_0(z))$. We assume that this condition is satisfied with equality for $z = z_N$, and therefore, only projects with the highest initial level of technology are implemented.[10]

In the next two sections we describe the properties of this model in two different environments. In the first environment we assume complete markets, and therefore, an absence of financial frictions. In the second environment, we assume the same financial frictions assumed in Section 1. By comparing the performance of the model in these two different environments, we are able to evaluate the importance of financial frictions for industry dynamics and isolate the dynamics induced by the persistence of shocks.

4.1 A complete markets economy

In the frictionless economy we assume that there are no restrictions on the use of equity and debt in the financing of the firm. In addition, we assume that $r = 1/\beta - 1$. This can be interpreted as a general equilibrium condition. These assumptions have two consequences. First, there is no bankruptcy in the economy as the firm can always issue new equity to repay its debt. Second, the Modigliani-Miller theorem holds and there is indeterminacy in the use of equity and debt to finance investment.[11]

In this environment the size of the firm (measured in terms of capital input k or labor input l) is simply determined by maximizing current expected profits. This implies that there is a unique correspondence between z and the size of the firm and, once we condition on size, the age of the firm becomes irrelevant for its dynamics. This is formally stated in the following proposition.

Proposition 4.1 *In the frictionless economy, firms of the same size experience the same dynamics independently of their age.*

[10] Although in this paper we do not conduct a general equilibrium analysis, the condition $\kappa + (1 + \lambda)e_0(z_N) \geq \Omega(z_N, e_0(z_N))$ can be interpreted as a general equilibrium condition. Because in the economy a large number of projects are drawn in each period, if this condition is not satisfied, many projects (at least those with $z = z_N$) will be implemented. The increase in the number of firms raises the demand for loans and labor which in turn increases the prices r and w. The increase in these prices will reduce the expected profits and the value of the firm, until the arbitrage condition for the creation of new projects with technology z_N is satisfied.

[11] Notice that, in order to have a frictionless economy, it is enough to eliminate only one of the two forms of rigidity assumed in the model: the inability to issue new shares and the borrowing limit. If the firm cannot issue new shares, but it can borrow without limit, then the firm can implement the desired scale of production by borrowing more, whatever the value of its equity. On the other hand, the borrowing limit becomes irrelevant if the firm can increase its equity by issuing new shares.

The statement of this proposition is quite intuitive and easy to understand, and therefore, we omit the formal proof. Thus, the model with correlated shocks and no financial frictions shares with the previous model (independent shocks and financial frictions) the property that age is irrelevant for the dynamics of the firm, once we control for its size. What about the dependence of the firm's dynamics on size? To simplify the analysis, let's consider the case in which the shock takes two values and the transition probability matrix is symmetric. We then have the following proposition.

Proposition 4.2 *Assume that $z \in \{z_1, z_2\}$ and $\Gamma(z'/z)$ is symmetric. Then small firms growth faster than large firms and the rate of job reallocation is independent of the firm size.*

The proof of the proposition is trivial. In this economy there are only two types of firms: small firms with current shock z_1 and large firms with current shock z_2. Neglecting the possibility of exit, small firms will only grow while large firms will only shrink. Moreover, small firms will never destroy jobs (except in the case of exogenous exit) while large firms will never create jobs. Job reallocation is defined as the sum of job creation and job destruction. Because the number of jobs destroyed by a large firm in the event in which z switches from z_2 to z_1 is equal to the number of job created by a small firm when z switches from z_1 to z_2, we have that the volume (and rate) of job reallocation is the same between small and large firms. Looking at the individual components of job reallocation, this model is consistent with the fact that job creation is decreasing in the firm's size, but it inconsistent with the empirical fact that job destruction is also decreasing in the firm's size.

The above proposition also holds when the shock z takes more than two values and the transition probability matrix is symmetric with decreasing probability of changing the current z to farther values. Without restriction on the transition matrix it is not possible to derive a general pattern of industry dynamics and it is possible to have a non-monotone relation between growth and size and between job reallocation and size. Therefore, while the model with financial frictions is able to generate the decreasing pattern of job reallocation observed in the data, a frictionless economy in which the firm dynamics is only driven by technology shocks cannot.

4.2 Incomplete markets economy

We now analyze the economy with both financial frictions and correlated shocks. As in the previous section, we assume that the shock z takes only two values with symmetric transition probability matrix. Although the calibration of the stochastic process for z does not have a large impact on the dependence of the firm dynamics on its size, the dependence on age crucially depends on the properties of this process. We assume that the shock z is highly persistent with $\Gamma(z = z_1/z = z_1) = \Gamma(z = z_2/z = z_2) = 0.99$. The two values of the shock, are chosen so that the employment of the largest firm with current $z = z_1$ is 50 percent of the employment of the largest firm with current $z = z_2$. The choice of these parameters and how they affect the dynamic properties of the model are discussed after the presentation of the results.

Figure 5a plots the value of debt for each type of firm $z \in \{z_1, z_2\}$, as a function of its equity. For each equity size, high productivity firms borrow more and implement larger production scales in order to take advantage of their higher productivity. As discussed for the model with independent shocks, in deciding the scale of production, firms face a trade-off. On the one hand, a larger production scale allows higher expected profits. On the other, a larger production scale implies higher volatility of profits, to which the firm is averse (because the firm's value is a concave function of profits). When the productivity level z increases, the marginal expected profit increases and the firm is willing to face higher risk by borrowing more and expands the scale of production in order to take advantage of the higher return. As shown in Figure 5b, firms with high value of z enjoy higher rate of profits. The higher profits allow these firms to grow faster (see Figure 5c). The increase in the volatility of profits associated with more debt, however, also implies that their growth rate is more volatile. As a consequence of this, high productivity firms experience higher rates of job reallocation (see Figures 5e and 5f) and higher failure rates (see Figure 5d).

This heterogeneous behavior of firms of different types, introduces an age dependence in the dynamics of firms. Figures 6a-6h plot the average growth rate, its standard deviation, the rates of job reallocation (creation and destruction) and the failure rate, as a function of the firm's size and age. In order to separate the size effect from the age effect, these variables are plotted for different age classes of firms (left graphs) and for different size classes (right graphs). These variables are all decreasing in the size of the firm and in its age, even after controlling, respectively, for age and size. Quantitatively, the age dependence is small, but this may depend on the simple structure chosen for the stochastic process of z. Also notice that the age effect is more important among the class of small firms and it almost disappear for very large firms.

Regarding the size dependence, this is driven by the same factors that generate this dependence in the model with independent shocks. The age dependence, instead, derives from the technological composition of firms of different age. As shown in Figure 5h, the proportion of young firms with low z is smaller than old firms. This derives from the assumption that new entrants are of the high technology type. Now because firms with $z = z_2$ experience higher rates of growth, job reallocation and failure, we also have that for each size class, younger firms grow faster and face higher rates of job reallocation and failure. Thus, in an economy with persistent shocks to technology, in order to have a significant dependence of firm dynamics on age, there must be a heterogeneous composition of firm types in each age class of firms. In this respect the degree of persistence plays an important role. If the shock is not highly persistent, the heterogeneous composition of firm types becomes insignificant after a few periods. With a very persistent shock, the heterogeneity vanishes slowly and the age dependence is maintained for a large range of ages. The initial technology levels of firms also play an important role. If we make the alternative assumption that new entrants are of the low productivity type, then the age dependence would be of the wrong sign, with old firms experiencing higher rates of growth, job reallocation and exit. Finally, the calibration of z_1 and z_2 is important in differentiating the behaviors of firms of different types: the more different the behavior of firms of different types is, the larger the dependence of the firm dynamics on age.

Before closing, we outline an alternative model in which persistence is introduced through a learning mechanism similar to Jovanovic (1982). In this framework the variable z is constant for each firm, but different firms have different values of z. The crucial assumption is that z is not known by the firm, and the firm decision is based on its belief about this parameter. As the firm operates, it updates its belief using Bayes rule (learning). At the moment of entrance, firms get a signal of their type z and only those firms that receive a signal above a certain threshold will enter. The distribution of beliefs of new firms is then skewed toward the high types, more than the true distribution of new entrants. However, as learning takes place, the distribution of beliefs converges to the true distribution. Now, because the firms with high beliefs implement larger production scales, they experience higher rates of job reallocation and failure. Therefore, we will have the same age effect described above and deriving from the age composition of firm type. In this model the distribution of beliefs play the same role played by the observed technology level assumed above. In contrast to the previous model, however, the technological level of young firms is not different from old firms. What changes with age is only the distribution of beliefs.

Without financial frictions, this model is able to generate the age dependence of job reallocation. However, the growth rate of the firm would be increasing in age. This is because the distribution of beliefs of young firms has a larger mass in high values of z, and on average they implement larger production plans. As they learn, they reduce on average the production scale. In the limit, with full learning, the average growth rate is zero (once we control for exit). Therefore, young firms have a negative growth rate, while old firms have a growth rate close to zero. This also implies another counterfactual property of this model, that is, the fact that young firms are larger than old firms. This problem could be eliminated by making the assumptions necessary for the beliefs of new entrants to be more skewed toward low types. With this assumption, the model would capture the age dependence of the firm dynamics. However, size is not necessarily an important variable: depending on the properties of the stochastic process of the shock, once we condition on age, size may be irrelevant for the properties of job reallocation.

We conclude by observing that the introduction of financial frictions in a model with learning as outlined above, would successfully capture many of the features of firm dynamics observed in the data: The learning mechanism would be primarily important for capturing the age effect; the financial mechanism, would be primarily important for the size effect.

5 Conclusion

This paper has developed a model with firm heterogeneity in which long-lived firms solve a dynamic intertemporal optimization problem. Financial factors are crucial in differentiating the production and investment decisions of firms of different size and this generates dynamics of entry, exit and growth that mimic many of the key empirical observations about industry evolution. An important finding of this paper is that financial factors play an important role in affecting the dynamics of firms (growth, job reallocation and exit). The analysis of the paper also provides some insight on the way financial factors may differentiate the investment behavior of small and large firms. In contrast to other models of industry dynamics, the heterogeneity here arises from i.i.d. shocks to technology. It is the financial decisions of the firms that generates persistence in the patterns of industry evolution.

A Appendix

A.1 Proof of lemma 1.3.

The variable $\underline{\varphi}$ satisfies the condition

$$(1-\delta-\phi)(e+b)+\underline{\varphi}F(e+b)-(1+\tilde{r})b=0 \tag{16}$$

Using this condition to eliminate $(1-\delta-\phi)(e+b)-(1+\tilde{r}_l)b$ in the function $q(e,b,\varphi)$, we get:

$$q(e,b,\varphi)=(\varphi-\underline{\varphi})F(e+b) \tag{17}$$

Therefore, q is a linear function of φ with slope $F(e+b)$ and intercept $-\underline{\varphi}F(e+b)$. Because F is strictly concave, we have that $F(e_\alpha+b_\alpha)>\alpha F(e_1+b_1)+(1-\alpha)F(e_2+b_2)$. This implies that the slope of $q(e_\alpha,b_\alpha,\varphi)$ is greater than the slope of $\alpha q(e_1,b_1,\varphi)+(1-\alpha)q(e_2,b_2,\varphi)$. Therefore, if for small value of φ the function $q(e_\alpha,b_\alpha,\varphi)$ is smaller than $\alpha q(e_1,b_1,\varphi)+(1-\alpha)q(e_2,b_2,\varphi)$, then as we increase φ the value of the first function get closer to the values of the second function until it crosses it. In the case in which the function $q(e_\alpha,b_\alpha,\varphi)$ is greater than $\alpha q(e_1,b_1,\varphi)+(1-\alpha)q(e_2,b_2,\varphi)$, even for small values of φ, then the first function will be greater than the second for all value of φ.

Let's prove now the second part of the lemma. Simple algebra gives us:

$$E(q(e,b,\varphi)) = \int_{\underline{\varphi}}^{\infty} q(e,b,\varphi)f(d\varphi) = (e+b)(1-\delta-\phi)-(1+r)b+E(\varphi)F(e+b) \tag{18}$$

Because F is strictly concave, the expected value of q is strictly concave, and therefore, $\int\psi(\varphi)f(d\varphi)=E(q(e_\alpha,b_\alpha,\varphi))-\alpha E(q(e_1,b_1,\varphi))-(1-\alpha)E(q(e_2,b_2,\varphi))>0$.

Q.E.D.

A.2 Proof of proposition 1.1.

Given lemma 1.2, we start the proof of the proposition by assuming that the optimal dividend policy is such that the firm will distribute all profits only after reaching the size $\bar{e}$. Conditional on this policy, we prove the existence of a solution to the firm's choice of b, and we characterize the firm's value function defined over the interval $[0,\bar{e}]$. The properties of the firm's value function, then, allow us to prove the optimality of the assumed dividend policy by showing that there is no incentive to deviate from this policy in the current period when the firm takes as given the implementation of that policy in future periods.

As observed before, the borrowing choice b is restricted to be in the compact and convex set $B(e)=[-e,\bar{b}(e)]$. The firm's problem can then be written as:

$$\begin{aligned}\Omega(e) = \beta\max_{b\in B(e)}\Big\{&\Big\{E[q(e,b,\varphi)-\bar{e}\,|\,\mathbf{S}_2]P(\mathbf{S}_2)+E[\Omega(q(e,b,\varphi))\,|\,\mathbf{S}_1]P(\mathbf{S}_1)+\Omega(\bar{e})P(\mathbf{S}_2)\Big\}(1-\eta) \\ &+E[q(e,b,\varphi)\,|\,\mathbf{S}_1\cup\mathbf{S}_2]P(\mathbf{S}_1\cup\mathbf{S}_2)\eta\Big\}\end{aligned} \tag{19}$$

where $\mathbf{S}_1$ is the subset of shocks for which $0\le q(e,b,\varphi)\le\bar{e}$ and $\mathbf{S}_2$ is the subset of shocks for which $q(e,b,\varphi)>\bar{e}$, and P stands for the relative probability. Defining $u(e,b)=\beta E[q(e,b,\varphi)-\bar{e}\,|\,\mathbf{S}_2]P(\mathbf{S}_2)(1-\eta)+\beta E[q(e,b,\varphi)\,|\,\mathbf{S}_1\cup\mathbf{S}_2]P(\mathbf{S}_1\cup\mathbf{S}_2)\eta$, the problem can be rewritten as:

$$\Omega(e)=\max_{b\in B(e)}\Big\{u(e,b)+\beta\{E[\Omega(q(e,b,\varphi))\,|\,\mathbf{S}_1]P(\mathbf{S}_1)+\Omega(\bar{e})P(\mathbf{S}_2)\}(1-\eta)\Big\} \tag{20}$$

The function $u(e,b)$ is continuous and bounded $\forall\, e \in [0,\bar{e}]$, and $\forall\, b \in B(e)$. Moreover, if Ω is a continuous and bounded function, the term $E[\Omega(q(e,b,\varphi))\,|\,\mathbf{S}_1]P(\mathbf{S}_1) = \int_{\underline{\varphi}}^{\bar{\varphi}} \Omega(q(e,b,\varphi))f(d\varphi)$ is also bounded and continuous. Therefore, the mapping $T(\Omega)$ defined as:

$$T(\Omega) = \max_{b\in B(e)} \Big\{u(e,b) + \beta\{E[\Omega(q(e,b,\varphi))\,|\,\mathbf{S}_1]P(\mathbf{S}_1) + \Omega(\bar{e})P(\mathbf{S}_2)\}(1-\eta)\Big\} \tag{21}$$

maps the set of bounded and continuous functions into itself and there is a unique continuous and bounded function Ω^* that satisfies the functional equation $\Omega^* = T(\Omega^*)$.

The fact that T maps the set of continuous and bounded functions into itself can be proved by verifying the conditions for the theorem of the maximum. If Ω is a continuous and bounded function, then the boundedness and continuity of $u(e,b)$ and $\Omega(e)$ implies that the objective function is continuous and bounded. Because the correspondence $B(e)$ is continuous, compact and convex valued, the maximum exists and the function $T(\Omega)(e)$ is continuous and bounded.

The unique solution to the functional equation $\Omega = T(\Omega)$ can be proved by showing that T is a contraction and by applying the contraction mapping theorem. Blackwell's conditions of monotonicity and discounting allow us to verify that T is a contraction.

- (**Monotonicity**) We want to show that if Ω^1 and Ω^2 are bounded and continuous functions satisfying $\Omega^1(e) \le \Omega^2(e)$, $\forall\ e \in [0,\bar{e}]$, then $T(\Omega^1) \le T(\Omega^2)$. Define b_1 to be the solution to the optimization problem in the mapping $T(\Omega^1)$. After defining $\Omega^2(e) = \Omega^1(e) + \omega(e)$, where $\omega(e) \ge 0$ $\forall\ e \in [0,\bar{e}]$, we have the following relations:

$$\begin{aligned}
T(\Omega^1)(e) &= u(e,b_1) + \beta\{E[\Omega^1(q(e,b_1,\varphi))\,|\,\mathbf{S}_1]P(\mathbf{S}_1) + \Omega^1(\bar{e})P(\mathbf{S}_2)\}(1-\eta)\\
&= u(e,b_1) + \beta\{E[\Omega^1(q(e,b_1,\varphi)) + \omega(q(e,b_1,\varphi))\,|\,\mathbf{S}_1]P(\mathbf{S}_1)[\Omega^1(\bar{e}) + \omega(\bar{e})]P(\mathbf{S}_2) -\\
&\qquad\qquad E[\omega(q(e,b_1,\varphi))\,|\,\mathbf{S}_1]P(\mathbf{S}_1) + \omega(\bar{e})P(\mathbf{S}_2)\}(1-\eta)\\
&\le u(e,b_1) + \beta\{E[\Omega^1(q(e,b_1,\varphi)) + \omega(q(e,b_1,\varphi))\,|\,\mathbf{S}_1]P(\mathbf{S}_1) +\\
&\qquad\qquad [\Omega^1(\bar{e}) + \omega(\bar{e})]P(\mathbf{S}_2)\}(1-\eta)\\
&\le \max_b \Big\{u(e,b) + \beta\{E[\Omega^2(q(e,b,\varphi))\,|\,\mathbf{S}_1]P(\mathbf{S}_1) + \Omega^2(\bar{e})P(\mathbf{S}_2)\}(1-\eta)\Big\}\\
&= T(\Omega^2)(e)
\end{aligned}$$

- (**Discounting**) We want to show that $T(\Omega + a)(e) \le T(\Omega)(e) + \beta a$.

$$\begin{aligned}
T(\Omega + a)(e) &= \max_b \Big\{u(e,b) + \beta\{E[\Omega(q(e,b,\varphi))\,|\,\mathbf{S}_1]P(\mathbf{S}_1) + \Omega(\bar{e})P(\mathbf{S}_2)\}(1-\eta) +\\
&\qquad\qquad \beta a\{P(\mathbf{S}_1) + P(\mathbf{S}_2)\}(1-\eta)\Big\}\\
&\le \max_b \Big\{u(e,b) + \beta\{E[\Omega(q(e,b,\varphi))\,|\,\mathbf{S}_1]P(\mathbf{S}_1) + \Omega(\bar{e})P(\mathbf{S}_2)\}(1-\eta)\Big\} + \beta a\\
&= T(\Omega)(e) + \beta a
\end{aligned}$$

We want to show now that the firm's solution is unique. Define $\tilde{\Omega}(x)$ to be the objective in the functional equation (21) expressed as a function of the end-of-period resources $x = q(e,b,\varphi)$. The functional equation (21) can be written as:

$$T(\Omega) = \beta \max_b \Big\{\tilde{\Omega}(q(e,b,\varphi))\Big\} \tag{22}$$

Observe first that, if the assumed dividend policy is optimal, then the slope of $\tilde{\Omega(x)}$ in $x \in [0,\bar{e}]$ must be not smaller than 1 and equal to 1 for $x = \bar{e}$. If we assume that Ω is strictly concave, then $\tilde{\Omega}(x)$ is strictly increasing, concave and with slope not smaller than 1. If, in addition, the function $q(e,b,\varphi)$ is concave in (e,b), for all φ, then the concavity and slope of $\tilde{\Omega}$ implies that T maps concave functions into concave functions. Unfortunately, lemma 1.3 shows that q is not concave for all φ.

Take two points for equity e_1, e_2 and two points for debt b_1, b_2 and define e_α, b_α to be a convex combination of these two points with parameter $0 < \alpha < 1$. Then, by lemma 1.3, there exists $\hat{\varphi}$ such that the term:

$$\psi_\alpha(\varphi) = q(e_\alpha, b_\alpha, \varphi) - \alpha q(e_1, b_1, \varphi) - (1-\alpha) q(e_2, b_2, \varphi) \tag{23}$$

is greater than zero if $\varphi > \hat{\varphi}$, and non-positive if $\varphi \leq \hat{\varphi}$. The concavity of $\int q(e,b,\varphi) f(d\varphi)$, however, implies that the concave points dominates the convex points and, as stated in lemma 1.3, we have that $\int \psi_\alpha(\varphi) f(d\varphi) > 0$

Now consider the term

$$\tilde{\Omega}(q(e_\alpha, b_\alpha, \varphi)) - \alpha \tilde{\Omega}(q(e_1, b_1, \varphi)) - (1-\alpha)\tilde{\Omega}(q(e_2, b_2, \varphi)) \tag{24}$$

By the concavity and slope of $\tilde{\Omega}$, we have that this term is not smaller than $\psi_\alpha(\varphi)$ if $\psi_\alpha(\varphi) \geq 0$, and is smaller than $\psi_\alpha(\varphi)$ if $\psi_\alpha(\varphi) < 0$. Therefore, the concavity of the expected value of q does not necessarily implies the concavity of the expected value of $\tilde{\Omega}(q)$. In order to obtain the concavity of $E(\tilde{\Omega})$, we need to restrict the quantitative importance of the shocks φ for which the function q is not concave. One way to do this is by assuming that the probability density function f assumes small values in the neighbor of the shocks $\underline{\varphi}$. In that way we have that:

$$\begin{aligned} \int [\tilde{\Omega}(q(e_\alpha, b_\alpha, \varphi)) - \alpha \tilde{\Omega}(q(e_1, b_1, \varphi)) - (1-\alpha)\tilde{\Omega}(q(e_2, b_2, \varphi))] f(\varphi) d\varphi &\geq \\ \int [q(e_\alpha, b_\alpha, \varphi) - \alpha q(e_1, b_1, \varphi) - (1-\alpha) q(e_2, b_2, \varphi)] f(\varphi) d\varphi &> 0 \end{aligned}$$

The first inequality comes from the imposed restriction on f, while the second inequality comes from the strict concavity of the expected value of q.

Given this result, it is easy to prove that the mapping T maps concave functions into strictly concave functions. Define b_1 and b_2 to be the optimal debts when the firm has respectively e_1 and e_2 units of equity, and define b_α and e_α to be a convex combination of (b_1, b_2) and (e_1, e_2) respectively. Then we have:

$$\begin{aligned} \Omega(e_\alpha) &= \beta \max_b \int \tilde{\Omega}(q(e_\alpha, b, \varphi)) f(d\varphi)(1-\eta) \\ &\geq \beta \int \tilde{\Omega}(q(e_\alpha, b_\alpha, \varphi)) f(d\varphi)(1-\eta) \\ &> \beta\alpha \int \tilde{\Omega}(q(e_1, b_1, \varphi)) f(d\varphi)(1-\eta) + \beta(1-\alpha) \int \tilde{\Omega}(q(e_2, b_2, \varphi)) f(d\varphi)(1-\eta) \\ &= \beta\alpha \max_b \int \tilde{\Omega}(q(e_1, b, \varphi)) f(d\varphi)(1-\eta) + \beta(1-\alpha) \max_b \int \tilde{\Omega}(q(e_2, b, \varphi)) f(d\varphi)(1-\eta) \\ &= \alpha T(\Omega)(e_1) + (1-\alpha) T(\Omega)(e_2) \\ &= \alpha \Omega(e_1) + (1-\alpha)\Omega(e_2) \end{aligned}$$

Therefore, the fixed point Ω^* is strictly concave. In addition, because the objective function is strictly concave in b, then the optimization problem consists of maximizing a strictly concave function over a compact and convex set and the solution is unique. The theorem of the maximum also guarantees that the solution is continuous in e.

The next step is to show that the assumed dividend policy is optimal. Of course, this follows from the properties of Ω which is strictly concave in $[0, \bar{e}]$ and satisfies $\Omega(0) = 0$. Consequently, if the slope of

Ω is 1 at $\bar{e}$, it must be greater than 1 for $e < \bar{e}$ and smaller than 1 for $e > \bar{e}$. Therefore, it is optimal to reinvest profits until the firm has reached the size $\bar{e}$ and to distribute dividends after the firm has reached this size.

Q.E.D.

A.3 Proof of lemma 1.4.

The transition function Q has the Feller property if the function $T(Q)(e)$ defined as:

$$T(Q)(e) = \int_0^{\bar{e}} v(e')Q(e, de') \tag{25}$$

is continuous for any continuous and bounded function v. Conditional on being productive (which happens with probability $1 - \eta$), the next period equity is given by:

$$e' = \begin{cases} e_0 & \text{if} \quad \varphi \leq \underline{\varphi} \\ (\varphi - \underline{\varphi})F(e+b) & \text{if} \quad \underline{\varphi} < \varphi < \bar{\varphi} \\ \bar{e} & \text{if} \quad \varphi \geq \bar{\varphi} \end{cases} \tag{26}$$

Therefore, the function $T(Q)(e)$ can be written as:

$$T(Q)(e) = \eta v(e_0) + (1-\eta)\left[v(e_0)\int_{-\infty}^{0} f\left(\underline{\varphi}(e,b) + \frac{e'}{F(e+b)}\right)F(e+b)de' + \right. \tag{27}$$
$$\left. \int_0^{\bar{e}} v(e')f\left(\underline{\varphi}(e,b) + \frac{e'}{F(e+b)}\right)F(e+b)de' + v(\bar{e})\int_{\bar{e}}^{\infty} f\left(\underline{\varphi}(e,b) + \frac{e'}{F(e+b)}\right)F(e+b)de'\right]$$

Notice that in the construction of the above functional, the entrance of new firms is included by assuming that exiting firms re-enter with size e_0. Because $b(e)$ is continuous in e and F is a continuous function of e and b, then $F(e+b(e))$ is continuous in e. If in addition $\underline{\varphi}(e, b(e))$ is continuous in e, then the continuity of the density function f implies that $T(Q)$ is continuous. So we only need to prove that $\underline{\varphi}$ is continuous.

The function $\underline{\varphi}(e, b)$ is implicitly defined by $(1 - \delta - \phi)(e + b) + \underline{\varphi}F(e + b) - (1 + \tilde{r})b = 0$. Using equation (2) to eliminate $(1 + \tilde{r})b$ and rearranging we get:

$$(1 + r)b = F(e + b)\psi(\underline{\varphi}) + (1 - \delta - \phi)(e + b) \tag{28}$$

where $\psi(\underline{\varphi}) = \int_{-\infty}^{\underline{\varphi}} \varphi f(\varphi)d\varphi + \underline{\varphi}\int_{\underline{\varphi}}^{\infty} f(\varphi)d\varphi$, is strictly increasing and continuous in $\underline{\varphi}$ under assumption 1.2. This implies that ψ is invertible and the inverse function is continuous. Then the continuity $F(e+b)$ implies that $\underline{\varphi}(e, b)$ is continuous (singleton) function of e and b.

Q.E.D.

A.4 Proof of proposition 1.2.

Because the probability measure of firms has support in the compact set $[0, \bar{e}]$ and, as proved in lemma 1.4 the transition function Q has the Feller property, then Theorem 12.10 in Stokey et al. (1989) shows that there exist an invariant distribution μ^*. In order to prove that μ^* is unique we need extra conditions. As shown in theorem 12.12 in Stokey et al. (1989), if Q is monotone and it satisfies a mixing condition, then the invariant probability measure μ^* is unique. We want to show then that under conditions (a) and (b) of the proposition, Q is monotone and it satisfies the mixing condition.

- (**Monotonicity**) To prove that Q is monotone, we have to show that $Q(e_1, .)$ dominates $Q(e_2, .)$ $\forall e_1, e_2 \in [0, \bar{e}]$, with $e_1 \geq e_2$. The dominance means that for all bounded increasing functions v, $\int v(e')Q(e_1, de') \geq \int v(e')Q(e_2, de')$. When the state space is defined in $\mathbf{R}^1$, then $Q(e_1, .)$ dominates $Q(e_2, .)$ if and only if $\int_0^x Q(e_1, de') \leq \int_0^x Q(e_2, de')$, $\forall x \in [0, \bar{e}]$. The distribution function is defined in (13). Under conditions (a) and (b) of the proposition, it is easy to show that the distribution function is monotone. In fact, under the assumption that k is not decreasing in e, F is increasing in e. Consequently, the distribution function for smaller e results from the integration of the density function f over a larger set. If in addition the density function f changes in a sufficiently smooth way (assumption (b)), then we have the result.

- (**Mixing condition**) We have to prove that there exists $\hat{e} \in [0, \bar{e}]$, $\epsilon > 0$, and $N \geq 1$ such that $\Psi^N(0, [\hat{e}, \bar{e}]) \geq \epsilon$ and $\Psi^N(\bar{e}, [0, \hat{e}]) \geq \epsilon$. Because we are assuming that $f(\varphi) > 0 \; \forall \varphi \in \mathbf{R}$, then the mixing condition is obviously satisfied.

Q.E.D.

B Appendix: Computational procedure

The computational procedure consists of the following steps.

1. Guess the upper bound $\bar{e}$ and choose a discrete grid in the space of firms' equity, *i.e.*, $e \in \mathbf{E} \equiv \{e_1, ..., e_n\}$. In this grid, $e_1 = 0$ and $e_n = \bar{e}$.
2. Guess initial steady state values of debt b_i^*, $i \in \{1, .., n\}$.
3. Guess initial values for Ω_i, $i \in \{1, .., n\}$.
4. Approximate with a second order Taylor expansion the function $\tilde{\Omega}_i$ around the guessed points for the steady state values b_i^*. This function is defined as:

$$\begin{aligned} \tilde{\Omega}_i(b) &= \beta \sum_{j=1}^{n-1} \int_{e_j}^{e_{j+1}} \left[\Omega_j + \left(\frac{\Omega_{j+1} - \Omega_j}{e_{j+1} - e_j} \right) (x - e_j) \right] \Phi_i(b, dx)(1 - \eta) \qquad (29) \\ &+ \beta \int_{e_n}^{\infty} (x - e_n + \Omega_n)\Phi_i(b, dx)(1 - \eta) + \beta \int_0^{\infty} x\Phi_i(b, dx)\eta \end{aligned}$$

   where $\Phi_i(b, x)$ is the density function for the end-of-period resources for a firm with equity e_i and debt b. The value function Ω is approximated with peace-wise linear functions joining the grid points in which the value function is computed. The definition of $\tilde{\Omega}_i$ takes as given the dividend policy of the firm consisting in retaining all profits until the firm reaches the size e_n.
5. Solve for the firm's policy b_i by differentiating the function $\tilde{\Omega}_i(b)$ with respect to b.
6. Eliminate b from $\tilde{\Omega}_i$ using the policy rules found in the previous step. The found values are the new guesses for Ω_i. The procedure then restarts from step 4 until all firms' value functions have converged.
7. After value function convergence, check whether the firm policies found in step 5 reproduce the guesses for the steady state values of debt b_i^*. If not, update this guesses and restart the procedure from step 3 until convergence.

8. Check the optimality of the upper bounds $\bar{e}$, that is:

$$\beta \frac{\partial \Omega(e)}{\partial e}\bigg|_{e = e_n} = 1 \tag{30}$$

In order to check for this condition, we compute the numerical derivative of $\Omega(e)$ at e_n, taking as given the value of b_n^* found before. If the condition is not satisfied, update the initial guesses for $\bar{e}$ and restart the procedure from step 1 until convergence.

9. Find the optimal size of new firms e_0 and the set-up cost κ using the conditions:[12]

$$\Omega(e_0) = \kappa + (1 + \lambda)e_0 \tag{31}$$

$$\frac{\partial \Omega(e)}{\partial e}\bigg|_{e = e_0} = 1 + \lambda \tag{32}$$

10. Given the size of new entrants, iterate on the measure of firms until convergence. In each iteration, it is assumed that the measure of new entrants is equal to the measure of exiting firms.

[12] If we were able to determine the function $\Omega(e)$ exactly, then the point e_0 would be the tangent point to the line with slope $1 + \lambda$ and departing from $-\kappa$. Because in the computational procedure the function Ω is approximated with peace-wise linear functions, then, although unlikely, it is possible that a full linear segment of the function coincides with the line with slope $1 + \lambda$. In that case we assume that there is an equal probability for new firms to be of the two sizes.

(Chapter head:)*
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Figure 1a – Dividends–asset ratio

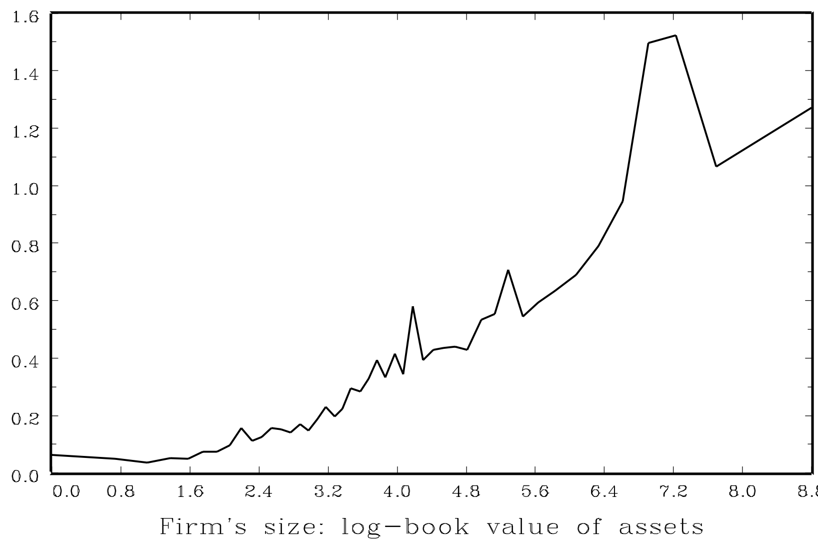


Figure 1b – Investment–asset ratio

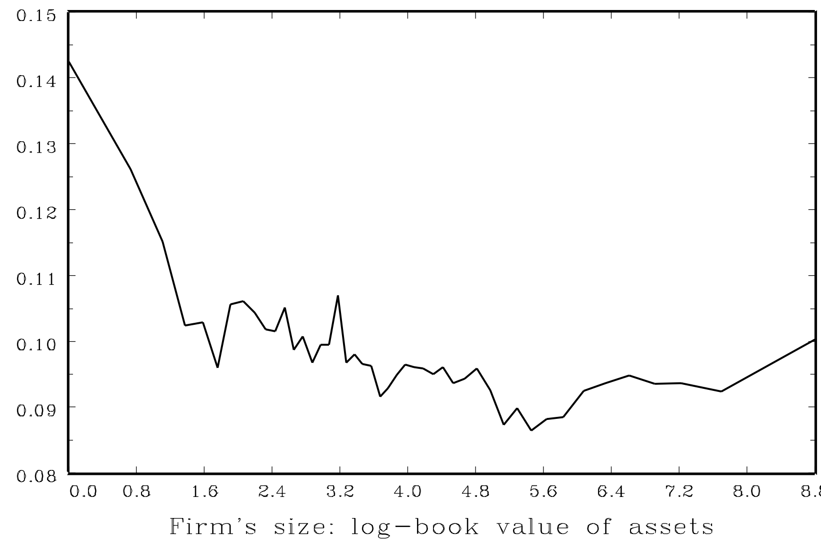


Figure 1c – Long term debt–asset ratio

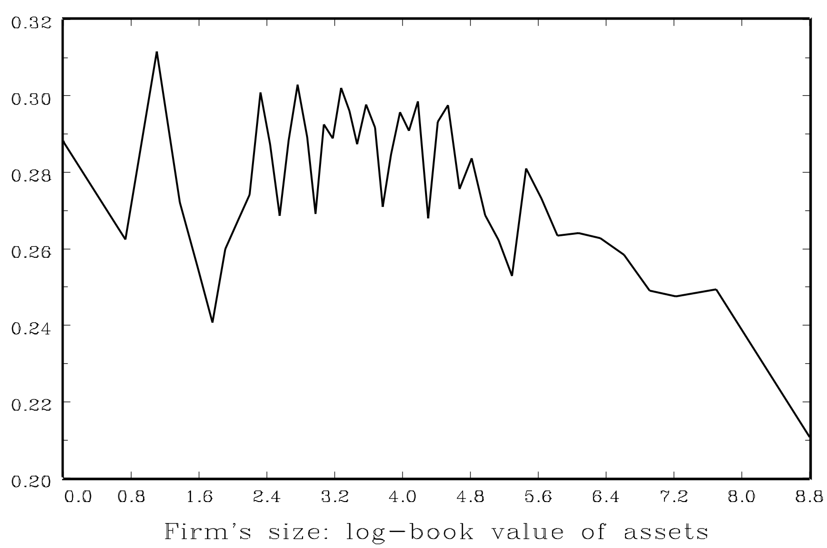


Figure 1d – Tobin's q

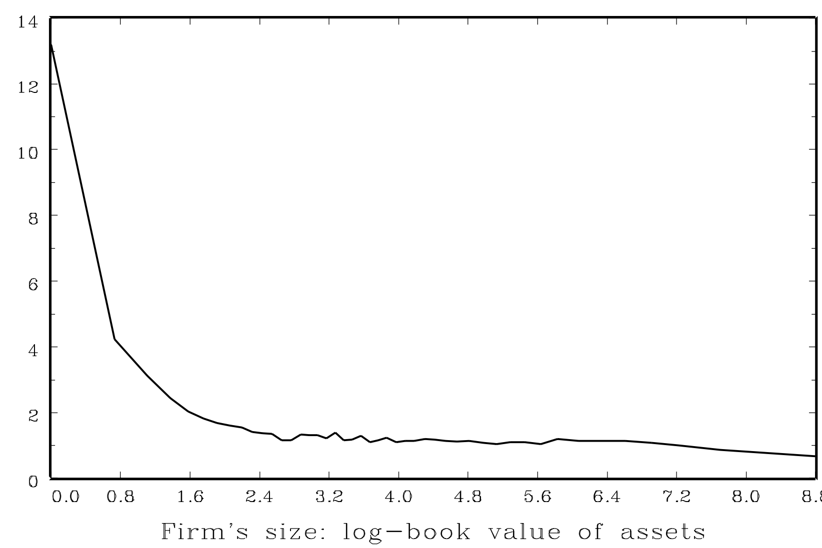


Figure 2a – Short-term debt

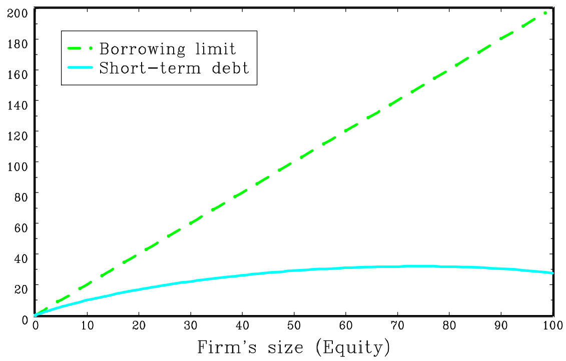


Figure 2b – Debt as fraction of equity

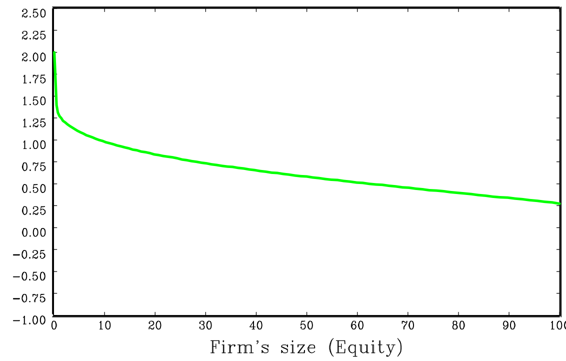


Figure 2c – Expected profit rate

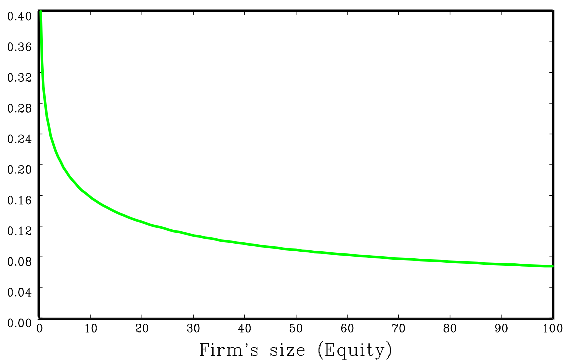


Figure 2d – Expected rate of dividend

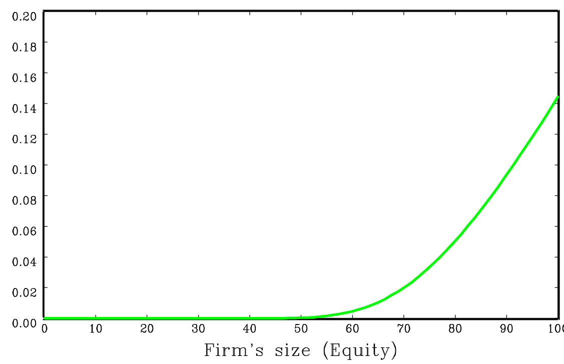


Figure 2e – Firm's value

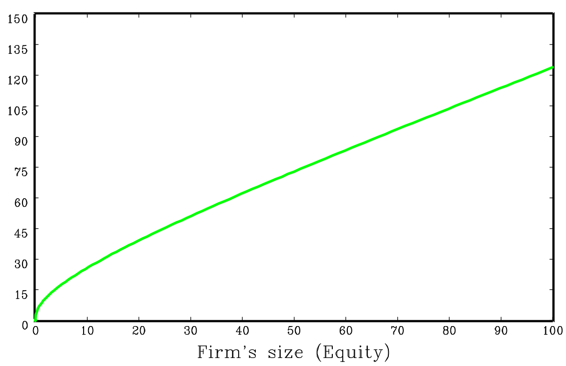


Figure 1f – Tobin's q

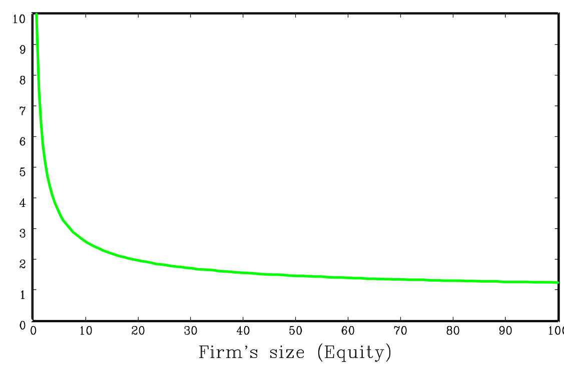


Figure 2g – Failure rate

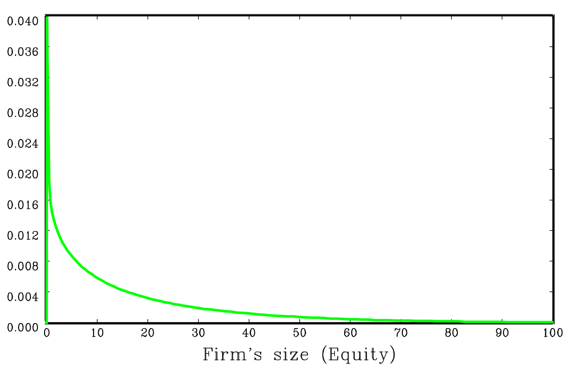


Figure 2h – Stationary distribution of firms

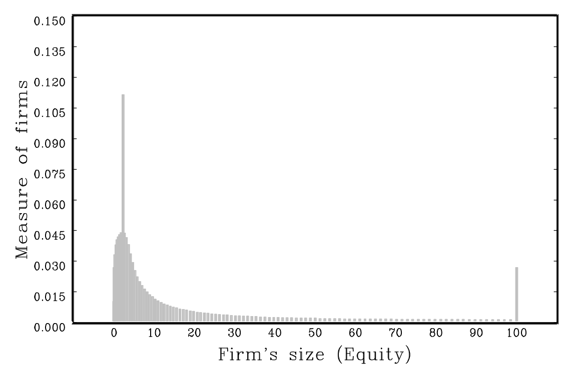


Figure 3 – Age and size distribution of firms

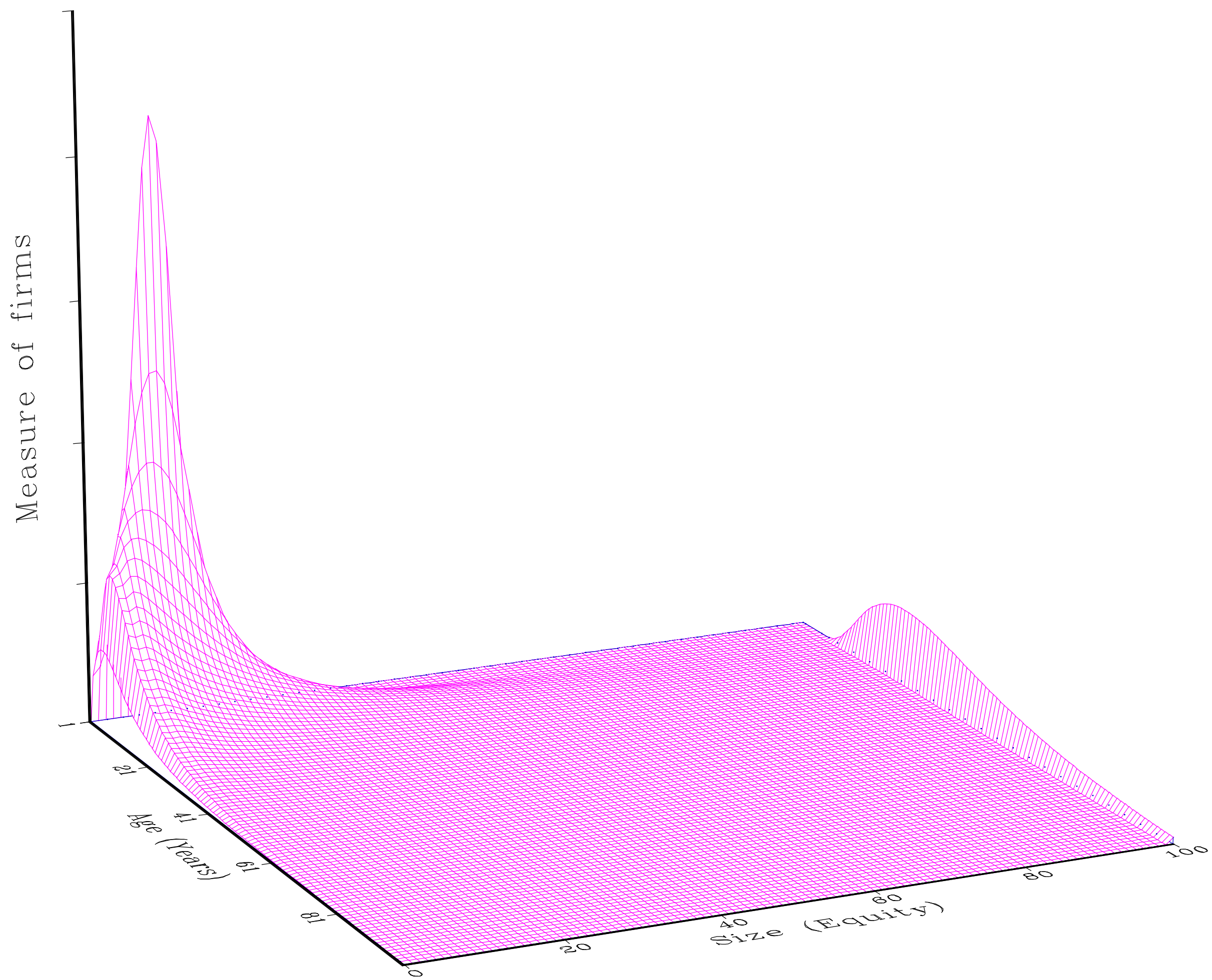


Figure 4a – Growth rate

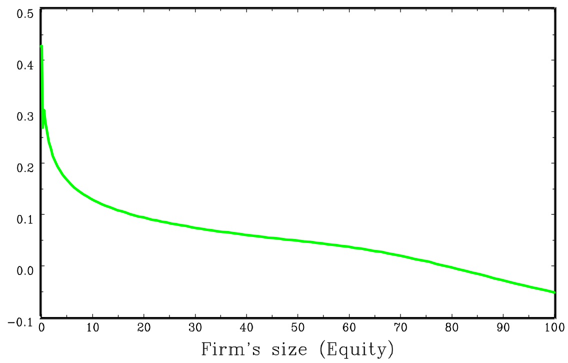


Figure 4b – Standard deviation of growth

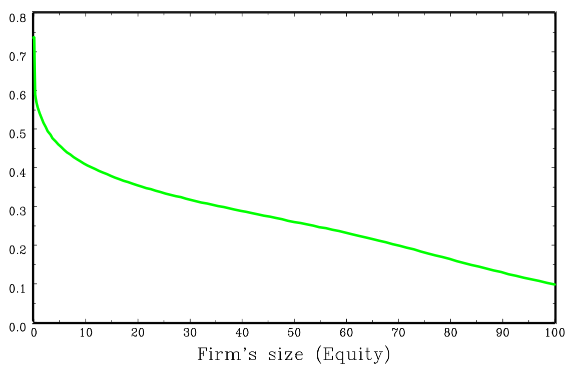


Figure 4c – Job reallocation

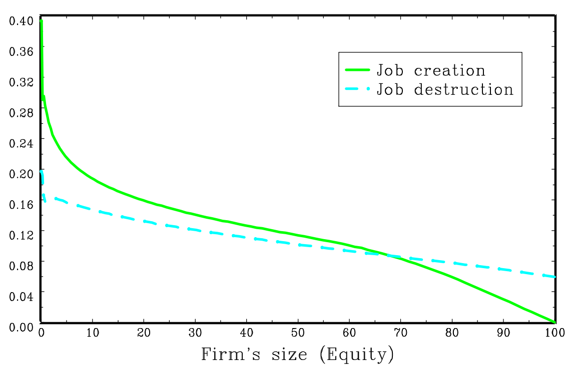


Figure 4d – Exit rate

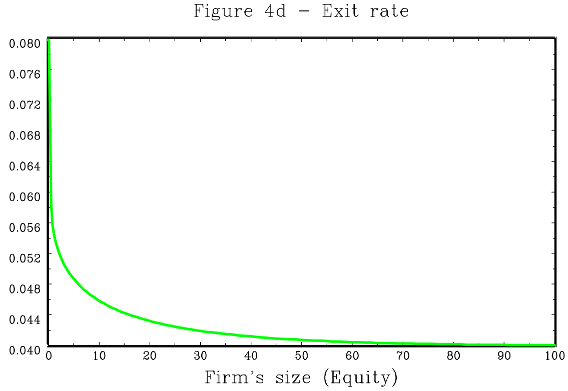


Figure 4e – Growth rate

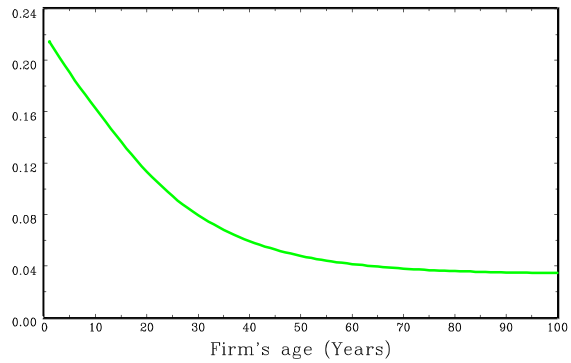


Figure 4f – Standard deviation of growth

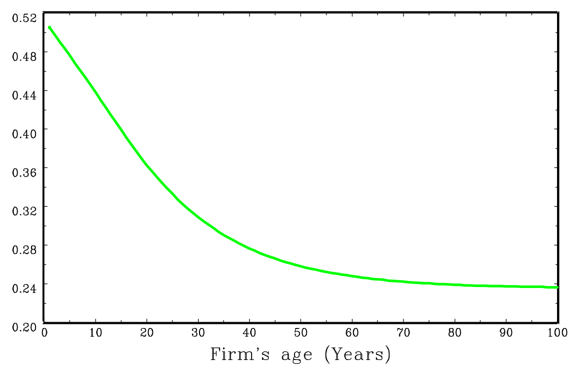


Figure 4g – Job reallocation

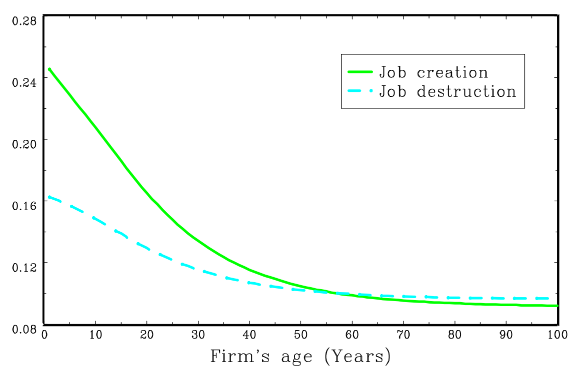


Figure 4h – Exit rate

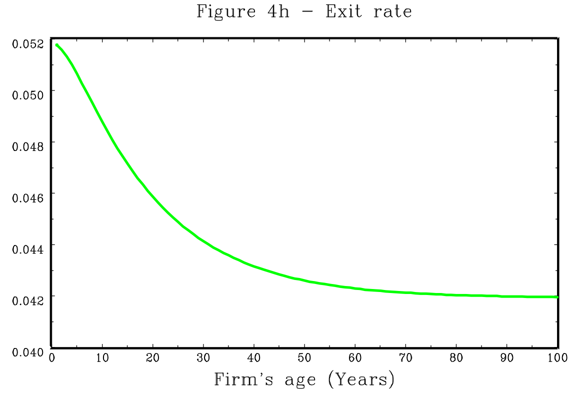


Figure 5a – Debt as fraction of equity

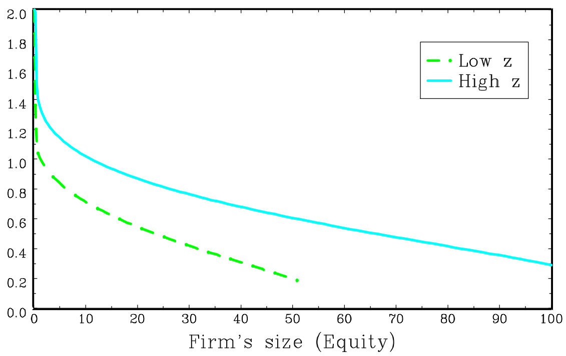


Figure 5b – Profit rate

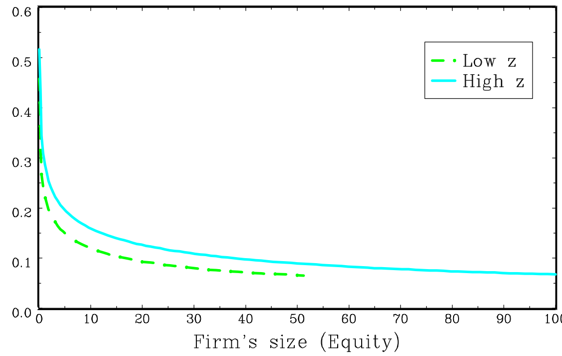


Figure 5c – Growth rate

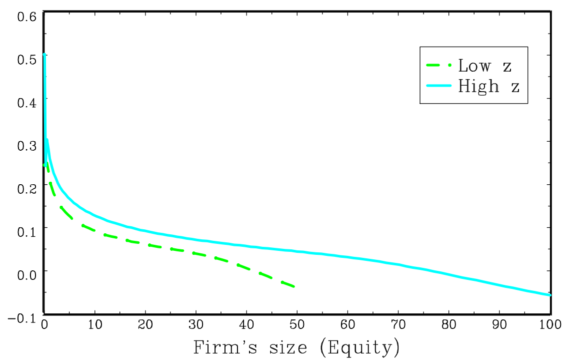


Figure 5d – Failure rate

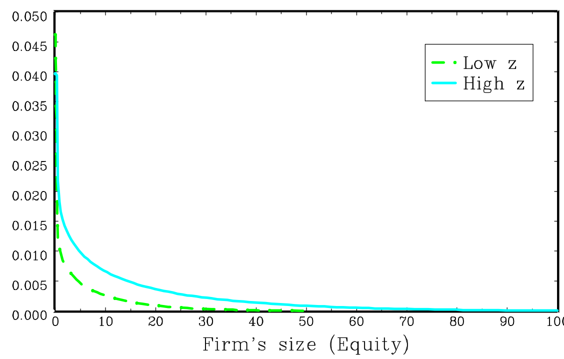


Figure 5e – Job creation

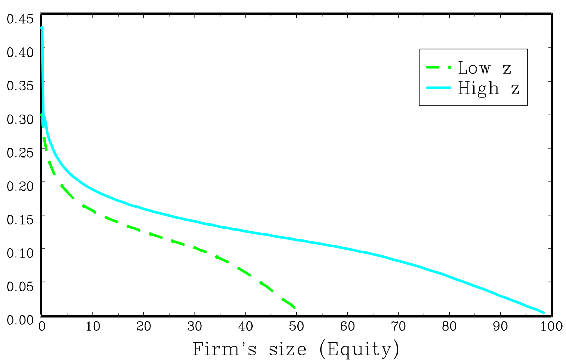


Figure 5f – Job destruction

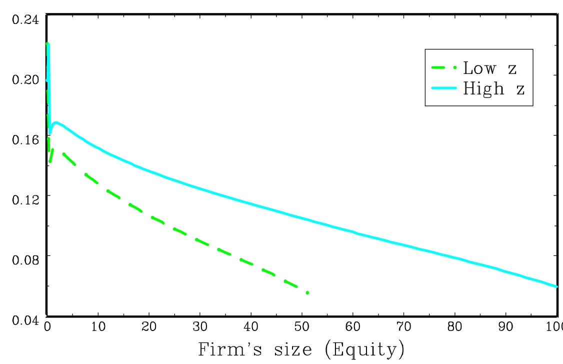


Figure 5g – Fraction of firms with low z

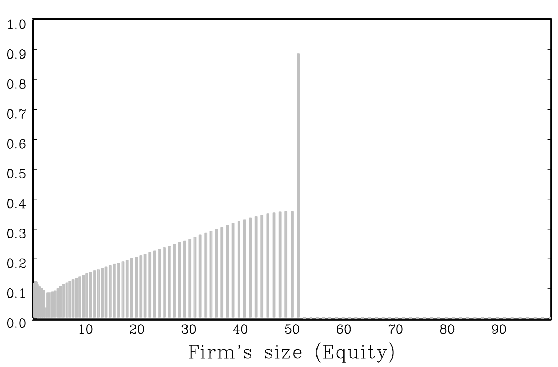
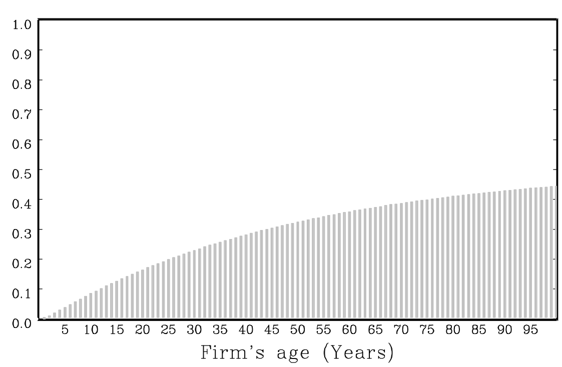


Figure 5h – Fraction of firms with low z



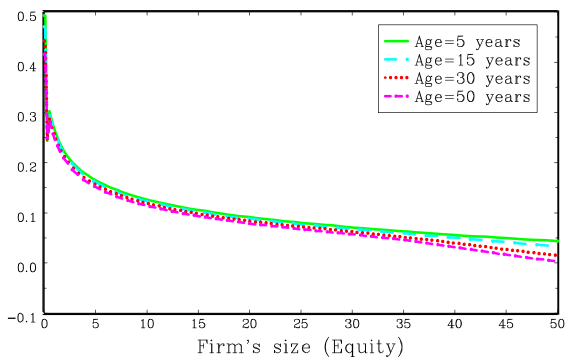


Figure 6a − Size profile: growth rate

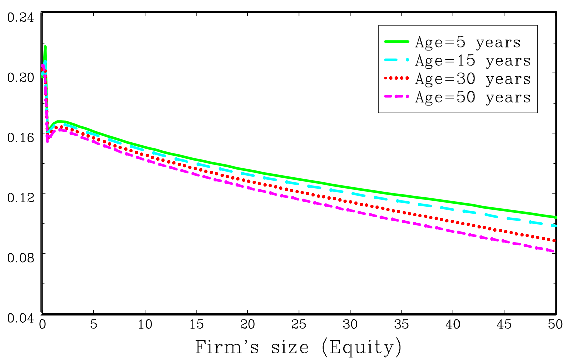


Figure 6b − Size profile: job destruction

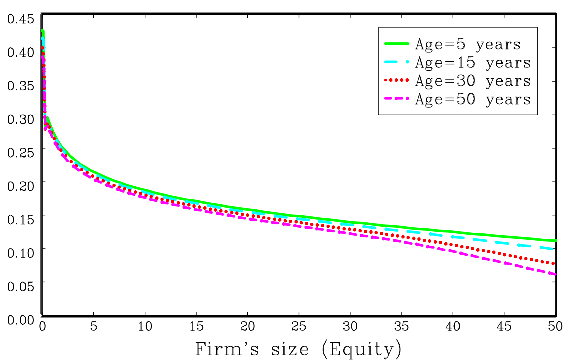


Figure 6c − Size profile: job creation

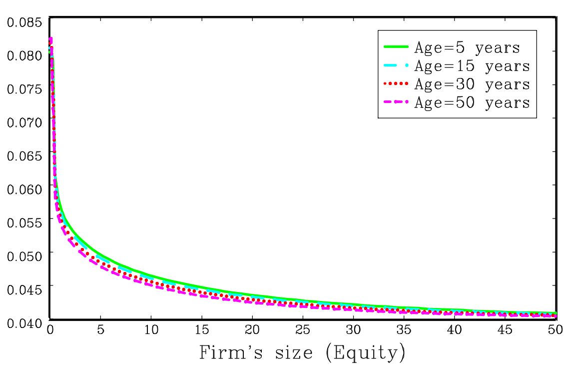


Figure 6d − Size profile: exit rate

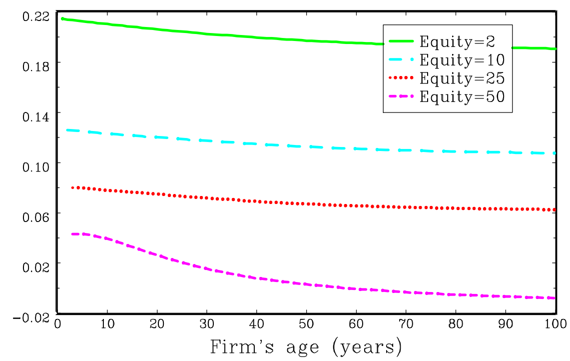


Figure 6e − Age profile: growth rate

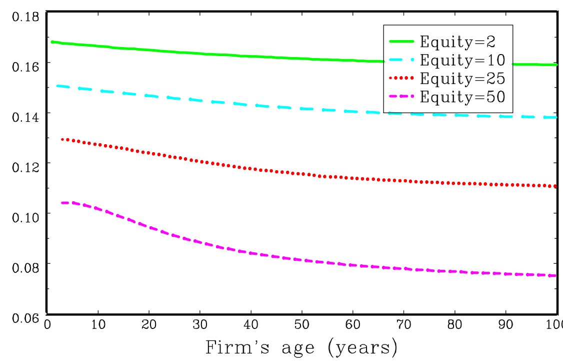


Figure 6f − Age profile: job destruction

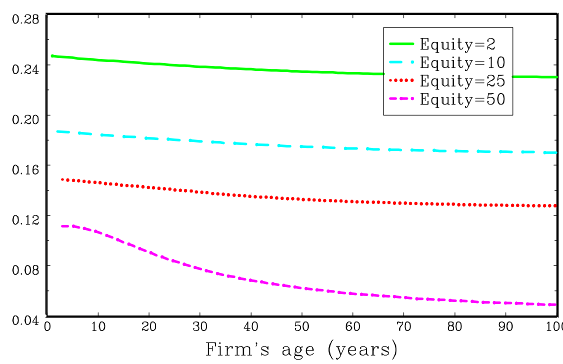


Figure 6g − Age profile: job creation

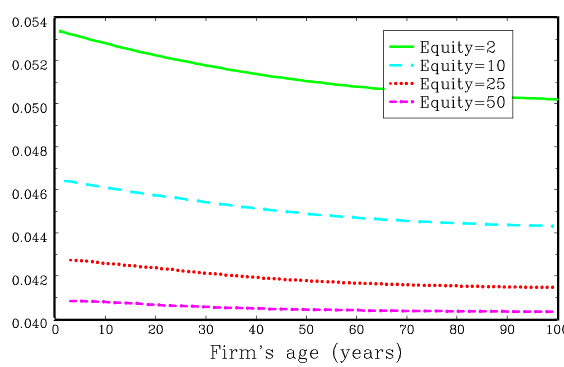


Figure 6h − Age profile: exit rate