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Abstract

This paper proposes an aggregative model of Total Factor Productivity (TFP) in the spirit of Houthakker (1955-1956). It considers a frictional labor market where production units are subject to idiosyncratic shocks and jobs are created and destroyed as in Mortensen and Pissarides (1994). An aggregate production function is derived by aggregating across production units in equilibrium. The level of TFP is explicitly shown to depend on the underlying distribution of shocks as well as on all the characteristics of the labor market as summarized by the job-destruction decision. The model is also used to study the effects of labor-market policies on the level of measured TFP.

1 Introduction

Hall and Jones (1999), Klenow and Rodríguez-Clare (1997), and Parente and Prescott (2000) have established that differences in Total Factor Productivity (TFP) account for a large fraction of the variation in output per worker across countries. Hall and Jones (1999) and Klenow and Rodríguez-Clare (1997) use data on output, labor input, average educational attainment and physical capital to decompose the differences in output per worker into differences in capital intensity, human capital per worker and TFP. Their levels accounting exercises reveal that differences in physical capital and educational attainment explain only a small amount of the differences in output per worker.[1] Parente and Prescott (2000) show that the standard growth model without TFP differences is not consistent with the observed income differences even when augmented to include a human capital sector. [2] So the message is that in order to understand income differences across countries,

[1] As a way of illustration, consider the following fact reported by Hall and Jones (1999). In 1988, output per worker in the five richest countries was on average 31.7 times that of the five poorest. Differences in capital intensities and educational attainments contributed factors of 1.8 and 2.2, respectively, to this difference. The remaining difference, a factor of 8.3 was accounted for by the TFP differential. Without this productivity difference, the average output per worker of the five richest countries would have only been about four times that of the five poorest. Hall and Jones (1999) and Klenow and Rodríguez-Clare (1997) also report somewhat striking productivity differences among OECD countries, with France and Italy having a higher and Germany a much lower level of TFP relative to the US.

[2] Specifically, they find that reasonable differences in saving rates cannot account for observed differences in steady-state income levels; and that the small diminishing returns to individuals investing in human capital that are needed to fit the empirical income differences imply that the time allocated to schooling is implausibly high. They also show that the factor difference in TFP needed to account for the income differences between the world's richest and poorest countries is between 2 and 3, not unreasonably high.

one first needs to understand what determines the level of TFP.

Hall and Jones (1999) conjecture that differences in observed TFP are driven by differences in the institutions and government policies they collectively refer to as "social infrastructure". Corrupt government officials, severe impediments to trade, poor contract enforcement and government interference in production are some of their examples of bad "social infrastructures" that could lead to low levels of TFP.[3]

Parente and Prescott (1994) propose that some countries have lower TFP than others because their process of technology adoption at the micro level is constrained by "barriers to riches". These barriers are essentially any institution or government policy that increases the cost of technology adoption. Parente and Prescott (1999) show that monopoly rights can act as a "barrier to riches".

This paper focuses on the theory underlying the aggregate production function to show how labor-market policies affect this function in general, and the level of measured TFP in particular. Specifically, I construct an aggregative model of TFP in the spirit of Houthakker (1955-1956): the basic idea is to derive an aggregate production function by aggregating across active production units. In equilibrium, the levels of output, inputs and

[3] Examples aside, the institutions and policies that Hall and Jones (1999) refer to as "social infrastructure" are defined by the two variables they use to proxy it in their regressions. The first is a measure of openness to trade; and the second, an index of government "anti-diversion" policies measuring (i) law and order, (ii) bureaucratic quality, (iii) corruption, (iv) risk of expropriation, and (v) government repudiation of contracts. In their empirical investigation, Acemoglu, Johnson and Robinson (2001) also highlight the role of institutions in determining different income levels across countries. Their focus is on the role property rights and checks against government power, and their definition of "institutions" is a risk-of-expropriation index.

TFP as well as the shape of the aggregate relationship between them depend on individual production decisions –such as which production units remain active in the face of idiosyncratic shocks– and these decisions are in turn affected by policies. So the model can be used to study the precise interaction between all these variables explicitly.

In the model proposed here, policy affects TFP because the latter is related to the average productivity of the units which are active, and policy induces changes in the productivity composition of active units. By distorting the way in which individual production units react to the economic environment, labor-market policies can make an economy exhibit a low level of TFP. As a result, two economies may exhibit different levels of TFP even if production units in both have access to the same technology and are subject to identical shocks. In this sense the determinants of TFP levels analyzed here are different from the barriers to technology adoption of Parente and Prescott (1999, 2000).[4]

At a theoretical level the paper also shows that, under some conditions,

[4] Although they focus on monopoly rights in their formal modelling, Parente and Prescott (2000) mention several labor-market policies as examples of "barriers to riches": "In India, for example, firms with more than 100 workers must obtain the government's permission to terminate any worker, and firms of all sizes are subject to state certification of changes in the tasks associated with a job." (pp. 107-108). "Another way the state protects the monopoly rights is by requiring large severance payments to laid-off workers." "Also in India, regulations require certain firms to award workers with lifetime employment and require firms with more that twenty-five workers to use official labor exchanges to fill any vacancy." (p. 108). "In Bangladesh, for example, private buyers of the state-owned jute mills were prohibited for one year from laying off any of the workforce they inherited. After one year, a worker could be laid off but not without a large severance payment." Parente and Prescott (2000) use these as instances of policies that can lower TFP by making technology adoption costly. But as shown here, these policies can also have a direct impact on level of TFP through composition effects.

a standard search model of the labor market –with its underlying meeting frictions and simple fixed-proportions micro-level production technologies– can generate an aggregate production function just like the one implied by the textbook neoclassical model of growth in which firms have access to a standard constant-returns Cobb-Douglas production technology. So in this sense, from the perspective of aggregate output, inputs and productivity, the neoclassical and the search paradigms can seem quite close.

The rest of the paper is organized as follows. Section 2 lays out the model. The equilibrium is characterized in Section 3, and the classical aggregation result of Houthakker (1955-1956) is extended to the dynamic equilibrium search setup in Section 4. This section also shows how, when aggregate inputs are correctly measured, the level of TFP depends on all the characteristics of the labor market summarized by the job-destruction decision. Section 5 introduces four policies: employment and hiring subsidies, firing taxes and unemployment benefits, and studies their effects on TFP. Section 6 extends the basic model to the case of serially-correlated shocks, generalizes the main aggregation result, and elaborates on how the observed level of TFP is affected by the different ways of measuring aggregate inputs that can be found in the literature. Section 7 concludes. All propositions are proved in the Appendix.

2 The Model

The labor market is modeled as in Mortensen and Pissarides (1994).[5] Time is continuous and the horizon infinite. There is a continuum of infinitely lived agents of two types: workers and firms. The size of the labor force is normalized to unity while the number of firms is determined endogenously by free entry. Both types are risk-neutral. Workers derive utility only from consumption. Each firm has a single job that can either be filled or vacant and searching. Similarly, workers can either be employed by a firm or unemployed and searching. No new offers arrive while an agent is in a relationship (i.e. there is no on-the-job search). I abstract from capital accumulation and assume labor-market participants take the aggregate stock of capital, K, as given.[6]

Assume meeting frictions can be represented by a function $m(u, v)$ that determines the instantaneous number of meetings as a function of the numbers of searchers on each side of the market; namely unemployed workers

[5] There are at least three reasons for carrying out the analysis in a search and matching framework. First, as will be discussed in Section 5, the labor-market policies considered will have testable implications not only for the level of TFP but also for the unemployment rate and the job-creation and destruction rates. Second, an explicit treatment of unemployment is relevant because –as will be shown in Section 6– the unemployment rate will affect empirical measures of TFP for the ways of measuring aggregate inputs that can be found in the literature. And finally, this framework has been used extensively to analyze the effects of similar policies on many other aggregate labor-market outcomes (see Ljungqvist and Sargent [2000], Pissarides [2000] and references therein).

[6] The model abstracts from saving and accumulation because the focus here is on isolating the effects of labor-market policies on the level of TFP. But even in the context of trying to explain income differences, Prescott (1998), and Parente and Prescott (2000) conclude that one cannot rely on policies that cause differences in saving rates, as they do not vary systematically with countries' incomes.

u and vacancies v. Suppose m exhibits constant returns to scale and is increasing in both arguments. Let $q(\theta)$ denote the (Poisson) rate with which a vacancy contacts an unemployed worker, where $\theta = v/u$.[7]

Each firm has access to a technology $f(x, n, k)$ that combines the hours supplied by the worker it employs, n, and capital, k, to produce a homogeneous consumption good. The match-specific level of technology is indexed by x. I assume that

$$f(x, n, k) = x \min(n, k) \tag{1}$$

and interpret k as the firm's "capacity". So output is linear in hours but is bounded above by the stock of capital the firm is operating with. The convention is that firm i has to choose and put in place k_i –its "scale of operation"– in order to engage in search and that this choice is irreversible.[8] This captures the idea that hours are a fully flexible factor while capital is relatively fixed. Firms rent capital from a competitive market at flow cost c.

Match productivity is stochastic and indexed by the random variable x. For an active match, the process that changes the productivity is Poisson with finite arrival rate λ. When a match of productivity x suffers a change, the new value x' is a draw from the fixed distribution $G(z)$. So the pro-

[7]Note that $q(\theta) = m(1/\theta, 1)$ and hence $q' < 0$. The probability a worker contacts a vacancy in a small time interval is $\theta q(\theta)$ and is increasing in θ. See Lagos (2000) for an environment in which a constant-returns aggregate matching function is explicitly derived from first principles.

[8]The idea is that in order to search, the firm must have borrowed some capital (e.g. to set up a plant). The firm is initially free to pick any size of plant k_i, but this choice is irreversible in the sense that once put in place, k_i cannot be changed. In a similar vein, technologies are assumed fixed and irreversible in Gilchrist and Williams (2000) and in Mortensen and Pissarides (1994).

ductivity process is persistent (since $\lambda < \infty$) but –conditional on change– it is independent of the firm's previous state.[9] The Poisson process and the productivity draws are *iid* across firms and there is no aggregate uncertainty. The focus will be on steady state outcomes.

In the next section I will show that there is a productivity level R such that active matches dissolve if productivity ever falls below R and new matches form only if their initial productivity is at least R.[10] Let $H_t(x)$ denote the cross-sectional distribution of productivities among active matches. That is, $H_t(x)$ is the fraction of matches producing at productivities x or lower at time t. The time path of $(1-u_t)H_t(x)$, namely of the *number* of matches producing at productivities x or lower at time t is given by[11]

$$\begin{aligned}\frac{d}{dt}\left[(1-u_t)H_t(x)\right] &= \lambda(1-u_t)\left[1-H_t(x)\right]\left[G(x)-G(R_t)\right] \\ &\quad +\theta q(\theta)u_t\left[G(x)-G(R_t)\right] \\ &\quad -\lambda(1-u_t)H_t(x)G(R_t) \\ &\quad -\delta(1-u_t)H_t(x) \\ &\quad -\lambda(1-u_t)H_t(x)\left[1-G(x)\right].\end{aligned}$$

[9] This is the process used by Mortensen and Pissarides (1994). For reasons that will be clear below, Section 6 generalizes the model by specifying that when a match of productivity x suffers a change, the new value x' is a draw from the fixed distribution $G(z|x)$. If $G(z|x_1) < G(z|x_0)$ when $x_0 < x_1$, then apart from persistent, idiosyncratic shocks are also positively correlated through time.

[10] Mortensen and Pissarides (1994) work with a bounded support and assume new matches start off with the highest productivity. I relax these assumptions and treat active and new matches symmetrically. In the model considered here, the initial productivity of a match is a non-degenerate random variable drawn from the same distribution as the innovations to active matches.

[11] The fact that active matches will form and continue only for productivities at least as large as R_t means that $H_t(R_t)=0$. So the derivation below focuses on $x \geq R_t$.

The first term accounts for the matches with productivities above x that get innovations below x but above R_t. The newly-formed matches that start off with productivities no larger than x are in the second term. The third term is the number of matches in the interval $[R_t, x]$ that get shocks below R_t and are destroyed. Let δ denote the parameter of an independent Poisson process that causes separations for unmodelled reasons. Then the fourth term accounts for matches in the interval $[R_t, x]$ that are destroyed for exogenous reasons. The last term accounts for the number of matches in the same interval that "move up" by virtue of having drawn productivities larger than x. Imposing steady states:

$$H(x) = \left[\frac{\lambda}{\delta+\lambda} + \frac{\theta q(\theta) u}{(\delta+\lambda)(1-u)}\right][G(x) - G(R)].$$

In addition, the steady-state unemployment rate is

$$u = \frac{\delta + \lambda G(R)}{\delta + \lambda G(R) + \theta q(\theta)[1 - G(R)]}. \tag{2}$$

Using this expression, the steady-state cross-sectional productivity distribution can be rewritten as

$$H(x) = \frac{G(x) - G(R)}{1 - G(R)}. \tag{3}$$

Firms can be either vacant and searching or filled. The problem of a searching firm is summarized by

$$rV = \max_k \left[-ck + q(\theta) \int \max[J(z) - V, 0]\, dG(z)\right], \tag{4}$$

where V is the asset value of a vacancy, $J(x)$ the asset value of a filled job and r the discount factor. There is entry of firms until all rents are exhausted, so $rV = 0$ in equilibrium. Letting $\pi(x)$ denote flow profit,

$$rJ(x) = \pi(x) + \lambda \int \max[J(z), V]\, dG(z) - \lambda J(x) - \delta[J(x) - V], \qquad (5)$$

where $\pi(x) = \max_n [x \min(n,k) - \phi n - ck - C(x,\phi)k - w(x)]$. Instantaneous profit is the residual output after the wage $w(x)$ and all other costs of production have been paid out. There are three such costs in this formulation: a fixed one, ck, which is the rental on capital; a variable cost, ϕn, that can be managed by varying hours; and a "maintenance cost" $C(x,\phi)$ per unit of capital. Given the wage-determination rule adopted below (Nash bargaining), the assumption that workers derive no utility from leisure implies that w is independent of n, so the profit-maximizing choice of hours is

$$n(x) = \begin{cases} k & \text{if } \phi < x \\ 0 & \text{if } x \leq \phi \end{cases} \qquad (6)$$

and hence flow profit is $\pi(x) = [\max(x - \phi, 0) - c - C(x,\phi)]k - w(x)$.

One can think of ϕ as the cost of electricity, for instance, with electricity usage being proportional to hours worked. This variable cost is introduced to allow for the possibility of "labor hoarding" and underutilization of capital, two pervasive features of the data. Specifically, for some parametrizations, it will be possible that at low productivity realizations the firm chooses to keep the worker employed despite requiring that she supplies zero hours. Below I show that this extreme variety of labor hoarding has interesting aggregate implications when it occurs in equilibrium. The maintenance cost is introduced in this section as a simple device to avoid a "flat spot" in flow profit which would otherwise carry over to the value functions. Since it is perhaps the only non-standard element of the model, I redo the whole analysis without this device in Section 6. So for now, I use a convenient specification for the maintenance cost, namely $C(x,\phi) = \max(\phi - x, 0)$.[12] With this specification, instantaneous profit becomes $\pi(x) = (x-\phi-c)k - w(x)$ for any x.

The value of employment and unemployment to a worker are denoted $W(x)$ and U respectively and solve

$$\begin{aligned} rU &= b + \theta q(\theta) \int \max[W(z) - U, 0]\, dG(z) && (7) \\ rW(x) &= w(x) + \lambda \int \max[W(z) - U, 0]\, dG(z) && (8) \\ &\quad - (\delta + \lambda)[W(x) - U], \end{aligned}$$

where $b \geq 0$ is a worker's flow income while unemployed.

3 Equilibrium

I follow the bulk of the labor search literature by letting $\beta \in [0,1)$ and assuming the instantaneous wage $w(x)$ solves

$$(1-\beta)[W(x) - U] = \beta J(x) \tag{9}$$

[12] One way to interpret this formulation is that machines require no maintainance if they are being operated by workers; so if $x \geq \phi$, the maintainance cost is zero and flow profit is just $(x-\phi-c)k - w(x)$. But when they stand idle, machines need to be *run,* even without a worker, in order to keep them operational. The time they need to be run depends on productivity. If $x = 0$, say, then the machine needs to be run at the cost of a full shift, ϕ, but if $x > 0$, then each machine needs to be run for less time, at cost $\phi - x$ per machine. So for $x < \phi$, output is zero, and flow profit is $-[(\phi - x + c)k + w(x)]$; the firm loses the maintainance cost, the cost of capital, and the wage payment to labor. Alternatively, one could interpret ϕ to be the depreciation rate of capital. In this case, $C(x,\phi)$ would be the (stochastic) depreciation rate suffered by the machine when it is not being used.

at all times. Letting $S(x) = J(x) + W(x) - U$ denote the surplus from a match, notice that (9) implies $J(x) = (1-\beta) S(x)$ and $W(x) - U = \beta S(x)$. These together with (5), (7) and (8) imply

$$(r + \delta + \lambda) S(x) = (x - \phi - c) k - rU + \lambda \int \max [S(z), 0] \, dG(z),$$

where

$$rU = b + \frac{\beta}{1-\beta} kc\theta. \tag{10}$$

Since $S'(x) = \frac{k}{r+\delta+\lambda} > 0$, there exists a unique R such that $S(R) > 0$ iff $x > R$. Hence matches separate whenever productivity falls below R.[13] Using this reservation strategy the surplus can be written as

$$(r + \delta + \lambda) S(x) = (x - \phi - c) k - rU + \lambda \int_R S(z) \, dG(z). \tag{11}$$

For completeness, (9) and the value functions can be manipulated to obtain expressions for instantaneous wages and profit:

$$w(x) = \beta (x - \phi - c) k + (1-\beta) rU \tag{12}$$

$$\pi(x) = (1-\beta) [(x - \phi - c) k - rU]. \tag{13}$$

Intuitively, the wage is a weighted average of output (net of the rental on capital and the variable and maintenance costs) and the worker's reservation wage.

I now do some analysis to characterize the job-creation and destruction decisions as summarized by θ and R respectively. Evaluating (11) at $x = R$,

$$\lambda \int_R S(z) \, dG(z) = rU - (R - \phi - c) k.$$

[13] Notice that separations are privately efficient. Moreover, they are also consensual in the sense that by (9), $J(x) > 0$ iff $W(x) - U > 0$; so the firm wants to destroy the match iff the worker wants to quit.

Notice that since the expected capital gain on the left-hand-side is positive, at $x = R$ net output is smaller than the worker's reservation wage. Thus (12) and (13) imply that $w(R) < rU$ and $\pi(R) < 0$: workers and firms sometimes tolerate instantaneous payoffs below those they could get by separating, in anticipation of a future productivity improvement.[14] Substituting this simpler expression for the expected capital gain term into (11) gives

$$S(x) = \frac{x - R}{r + \delta + \lambda} k. \tag{14}$$

Evaluating (11) at $x = R$ and using (14) to substitute $S(\cdot)$ yields the job-destruction condition:

$$R - \phi - c - \left(\frac{b}{k} + \frac{\beta}{1-\beta} c\theta \right) + \frac{\lambda}{r + \delta + \lambda} \int_R (x - R)\, dG(x) = 0. \tag{15}$$

As is standard, the destruction decision is independent of scale if b is. The natural interpretation of b is that it is unemployment insurance income. Along these lines, if one lets $b = \tau_b E_G [w(x) | x \geq R]$, where $\tau_b \in [0, 1)$ is the replacement rate, then $b = \hat{b}k$, with

$$\hat{b} = \frac{\tau_b \beta [\tilde{x}(R) - \phi - c + c\theta]}{1 - (1-\beta)\tau_b}$$

and $\tilde{x}(R) \equiv E_G [x | x \geq R] = [1 - G(R)]^{-1} \int_R x dG(x)$. Under this specification, b is linear in k and hence (15) is independent of k. In this case (15) becomes:

$$R - \frac{\tau_b \beta \tilde{x}(R)}{1-(1-\beta)\tau_b} - \frac{(1-\tau_b)(\phi + c)}{1-(1-\beta)\tau_b} - \frac{\beta c \theta}{(1-\beta)[1-(1-\beta)\tau_b]} + \frac{\lambda}{r+\delta+\lambda} \int_R (x - R)\, dG(x) = 0.$$

[14] This feature of the model is a consequence of the costly and time-consuming meeting process, as noted by Mortensen and Pissarides (1994).

In what follows I will always abstract from scale effects caused by unemployment income b by assuming it is a fraction of the average going wage. At times I may even resort to the especially tractable case with $\tau_b = b = 0$.

Substituting $rV = 0$ in (4) implies that at the optimal k,

$$(1-\beta)\int_R S(x)\,dG(x) = \frac{ck}{q(\theta)}.$$

That is, the expected profit from a filled job equals the expected hiring cost in an equilibrium with free entry. Using (14) to substitute $S(\cdot)$ out of this expression yields the job-creation condition:

$$\int_R (x-R)\,dG(x) = \frac{(r+\delta+\lambda)\,c}{(1-\beta)\,q(\theta)}. \tag{16}$$

The job-creation and destruction conditions jointly determine R and θ, and under the maintained assumptions they are independent of the choice of scale, k.[15] For given c and ϕ, an equilibrium is a vector $[\theta, R, H, U, w, u, k]$ such that (θ, R) jointly solve (15) and (16); and given (θ, R), H satisfies (3); U is given by (10); w by (12); and u by (2). In addition, the market for capital should clear, so k must satisfy $[1-(1-\theta)\,u]\,k = K$, where K is the aggregate supply of capital, which labor-market participants take as given.[16]

[15] For the case with $\tau_b = 0$, for instance, it is easy to show that there is a unique pair (θ, R) that satisfies (15) and (16). To see this notice that the slopes (in θ-R space) of the job-creation and destruction conditions are

$$\frac{-(r+\delta+\lambda)\,c\eta(\theta)}{(1-\beta)\,\theta q(\theta)\,[1-G(R)]} < 0 \text{ and } \frac{\beta c}{(1-\beta)\left\{1-\frac{\lambda[1-G(R)]}{r+\delta+\lambda}\right\}} > 0$$

respectively, with $\eta(\theta) \equiv -\theta q'(\theta)/q(\theta)$.

[16] Notice that using (14), (4) can be written as

$$rV = \max_k \left[-c + \frac{(1-\beta)\,q(\theta)}{r+\delta+\lambda}\int_R (x-R)\,dG(x)\right]k.$$

Note that if $R < \phi$, then the capital and workers in matches with realizations in $[R, \phi)$ remain employed but are not engaged in production. The firms in these states have excess capacity and hoard labor. The following section provides a sharper characterization of aggregate outcomes for a particular distribution of idiosyncratic shocks.

4 Aggregation

Let K_e denote the demand for capital from all firms with filled jobs. Since firms irreversibly choose the same amount of capital k upon entering the market,

$$K_e = \frac{1-u}{1-(1-\theta)u} K. \tag{17}$$

In general, aggregate output Y and the total number of hours worked, N, are given by $Y = (1-u)\int_\mu f[x, n(x), k]\, dH(x)$ and $N = (1-u)\int_\mu n(x)\, dH(x)$ respectively, with $\mu \equiv \max(R, \phi)$. Using (1) and (6), these expressions become

$$N = [1 - H(\mu)] K_e \tag{18}$$

$$Y(K_e, \mu) = [1 - H(\mu)] K_e E_H(x | x \geq \mu), \tag{19}$$

Since in equilibrium θ and R are independent of k, the objective is linear and the problem has a solution iff

$$\int_R (x - R)\, dG(x) \leq \frac{(r + \delta + \lambda) c}{(1-\beta) q(\theta)}.$$

But if the inequality is strict, then each firm i will choose $k_i = 0$ and the market is inactive. So a nontrivial equilibrium requires (16) to hold. Then any feasible k_i solves firm i's capacity problem: as for the standard neoclassical firm, individual size is indeterminate in equilibrium.

where $E_H(x|x \geq \mu) = [1 - H(\mu)]^{-1} \int_\mu x dH(x)$. Intuitively, since every firm-worker pair is setting hours either to zero or to full capacity k, the aggregate number of hours worked is just equal to the fraction of firm-worker pairs who engage in production times the total capital stock in filled jobs. Similarly, aggregate output equals the number of *active* units of capital, $[1 - H(\mu)] K_e$, times their average productivity.[17] Following Houthakker (1955-1956), one could imagine solving (18) for the aggregate "labor demand" by active firms $\mu(K_e, N)$ and then substituting it in (19) to obtain $Y[K_e, \mu(K_e, N)]$. Hereafter, I use $F(K_e, N)$ to denote $Y[K_e, \mu(K_e, N)]$ in order to simplify notation and stress the fact that this is the economy's "aggregate production function". Even for an arbitrary H, the aggregate production function is CRS. To see this, notice that $\mu(K_e, N)$ is homogeneous of degree zero and hence (19) indicates that for any $\zeta > 0$, $F(\zeta K_e, \zeta N) = \zeta F(K_e, N)$. Also, from (18) one sees that $-\mu_2(K_e, N) K_e dH(\mu) = 1$ and from (19) that $F_2(K_e, N) = -\mu_2(K_e, N) K_e \mu dH(\mu)$. Thus $F_2(K_e, N) = \mu$. So the mar-

[17] As mentioned previously, Mortensen and Pissarides (1994) assume that G has support $[0, 1]$ and that all new matches start off with productivity 1. So, with $\delta = 0$, aggregate output in their model evolves according to

$$\dot{Y} = k\theta q(\theta) u - \lambda Y + \lambda(1 - u) k \int_\mu^1 x dG(x).$$

Replacing $(1 - u) k$ with K_e, steady state output is

$$Y = \frac{\theta q(\theta) uk}{\lambda} + [1 - H(\mu)] K_e E_H(x|x \geq \mu),$$

which is essentially (19) except for the first term. Assuming that the initial productivity of a new match is a random draw from G –just as the innovations to the productivity of ongoing matches– allows for a density G with unbounded support. In addition, this alternative assumption smoothes aggregate output by getting rid of the "spike" $\theta q(\theta) uk\lambda^{-1}$.

ginal product of labor in the aggregate production function is equal to the marginal product of the least efficient unit of labor employed in production.[18]

Now suppose idiosyncratic shocks are draws from a Pareto distribution with parameters ε and α, namely

$$G(x) = \begin{cases} 0 & \text{if } x < \varepsilon \\ 1 - \left(\frac{\varepsilon}{x}\right)^{\alpha} & \text{if } \varepsilon \leq x \end{cases} \tag{20}$$

where $\varepsilon > 0$ and $\alpha > 2$. [19] Then, provided $R \geq \varepsilon$, $1 - G(R) = \left(\frac{\varepsilon}{R}\right)^{\alpha}$; and for any $x \geq R$,

$$G(x) - G(R) = \left(\frac{\varepsilon}{R}\right)^{\alpha}\left[1 - \left(\frac{R}{x}\right)^{\alpha}\right].$$

Substituting these expressions in (3) one sees that the steady state productivity distribution of active matches is

$$H(x) = \begin{cases} 0 & \text{if } x < R \\ 1 - \left(\frac{R}{x}\right)^{\alpha} & \text{if } R \leq x. \end{cases} \tag{21}$$

This is the *cdf* of a Pareto distribution with parameters R and α. Using (21), $1 - H(\mu) = \left(\frac{R}{\mu}\right)^{\alpha}$ and $E_H(x|x \geq \mu) = \frac{\alpha}{\alpha-1}\mu$; the aggregates (18) and (19) specialize to

$$\begin{aligned} N &= \left(\frac{R}{\mu}\right)^{\alpha} K_e & (22) \\ Y(K_e, \mu) &= \frac{\alpha}{\alpha - 1} R^{\alpha} \mu^{1-\alpha} K_e. & (23) \end{aligned}$$

Inverting the former to get the aggregate labor demand $\mu = (K_e/N)^{1/\alpha} R$, and substituting it in the latter yields

$$F(K_e, N) = AK_e^{\gamma} N^{1-\gamma} \tag{24}$$

[18] I owe this argument to Erzo G. J. Luttmer.

[19] This distribution has mean $\bar{x} = \frac{\alpha}{\alpha-1}\varepsilon$ and variance equal to $\frac{\bar{x}}{(\alpha-2)(\alpha-1)}$. Assuming $\alpha > 2$ ensures both are finite.

where

$$A = \frac{R}{1-\gamma} \tag{25}$$

and $\gamma \equiv 1/\alpha$. This extends the classic aggregation result of Houthakker (1955-1956).[20] The factor A is what macroeconomists normally refer to as TFP. Its level depends on α, a parameter of the primitive distribution of productivity shocks, as well as on all the characteristics of the labor market as summarized by the destruction decision R. Notice that F expresses output as a function of the aggregate number of hours *worked*, N, and the total amount of capital hired by firms with filled jobs, K_e. One can also express output as a function of the aggregate capital stock, K, simply by substituting (17) in (24) to get $\hat{F}(K,N) = \hat{A}K^{\gamma}N^{1-\gamma}$, where $\hat{A} \equiv \left[\frac{1-u}{1-(1-\theta)u}\right]^{\gamma} A$.

The aggregate production function is Cobb-Douglas despite fixed proportions in the micro-level technologies. This results whenever utilization is imperfectly measured, namely when only a fraction of the capital stock included as an argument in the aggregate production function is actually being used in production.[21] Since having firm-worker pairs that sometimes choose

[20] Houthakker performed the aggregation over production units that employ two variable factors and face capacity constraints due to a fixed (unmodelled) factor. Here I have assumed each production unit employs a single variable factor (labor) as well as capital. Capital is chosen before engaging in search and then remains fixed, hence playing the role of the fixed factor constraining output at the time employment and production decisions are made. This formulation delivers an aggregate production function with constant returns to scale. In contrast, the setup used by Houthakker generates a function of the variable inputs only and it exhibits diminshing returns to scale. Another relevant difference is that the shift parameter in Houthakker's production function is solely a function of the parameters in the primitive productivity distribution. But here, decisions can shift the aggregate production function.

[21] To see this, notice that if there is no hoarding in equilibrium (i.e. if $\mu = R$) then $N = K_e$ and $F(K_e, N) = AK_e$. Similarly, if there is hoarding but utilization is perfectly measured, then aggregate output is again linear in the relevant capital stock. Explicitly, let

to be inactive affects the shape of the aggregates, I now establish under what conditions the equilibrium exhibits this property. For the remainder of the section let $\tau_b = 0$ to ease the algebra.

With G given by (20), (15) and (16) specialize to:

$$R - \phi - c - \frac{\beta}{1-\beta} c\theta + \frac{\lambda}{r+\delta+\lambda} \frac{\varepsilon^{\alpha} R^{1-\alpha}}{\alpha - 1} = 0 \tag{26}$$

$$\frac{\varepsilon^{\alpha} R^{1-\alpha}}{\alpha - 1} - \frac{(r+\delta+\lambda) c}{q(\theta)(1-\beta)} = 0. \tag{27}$$

By totally differentiating,

$$\frac{\partial R}{\partial \phi} = \frac{(r+\delta+\lambda)\eta(\theta)}{\beta\theta q(\theta)[1-G(R)] + (r+\delta+\lambda)\eta(\theta)\left\{1 - \frac{\lambda[1-G(R)]}{r+\delta+\lambda}\right\}} > 0 \tag{28}$$

and

$$\frac{\partial \theta}{\partial \phi} = \frac{-(1-\beta)\theta q(\theta)[1-G(R)]}{(r+\delta+\lambda)\eta(\theta) c} \frac{\partial R}{\partial \phi} < 0,$$

where $1 - G(R) = (\varepsilon/R)^{\alpha}$ and $\eta(\theta) \equiv -\theta q'(\theta)/q(\theta)$. An increase in ϕ has no direct effect on the job-creation condition, and it shifts the job-destruction condition up in θ-R space. This increases the equilibrium value of R and decreases the equilibrium value of θ. Combining (26) and (27), one sees that the sign of $\phi - R$ is the sign of $\lambda/q(\theta) - [1-(1-\theta)\beta]$. So at low productivity realizations, the firm is more likely to hoard labor than to break the match when λ is large (and hence the option value of keeping a match is large), and when q is small (and hence the expected cost of hiring a new worker is high). Market tightness θ enters the expression with an ambiguous sign

K_p denote the capital stock being used in production, that is $K_p = [1 - H(\mu)] K_e$. Then it follows from (19) that $Y = A'' K_p$, with $A'' \equiv E_H(x|x \geq \mu)$. So hoarding, together with imperfect measurement of utilization cause the aggregate to look Cobb-Douglas in capital and hours despite fixed proportions in the micro production functions.

because on the one hand a large θ makes hoarding more likely by increasing the expected recruiting cost; but on the other, through its effect on the worker's reservation wage, it also increases the value of her threat point in the wage bargain, which makes keeping an unproductive worker employed more costly and hoarding less likely. In fact, the latter effect disappears if the worker has no power in the wage bargain (i.e. if $\beta = 0$). Next, I provide a sufficient condition for $R < \phi$ to be possible in equilibrium under some parametrizations.

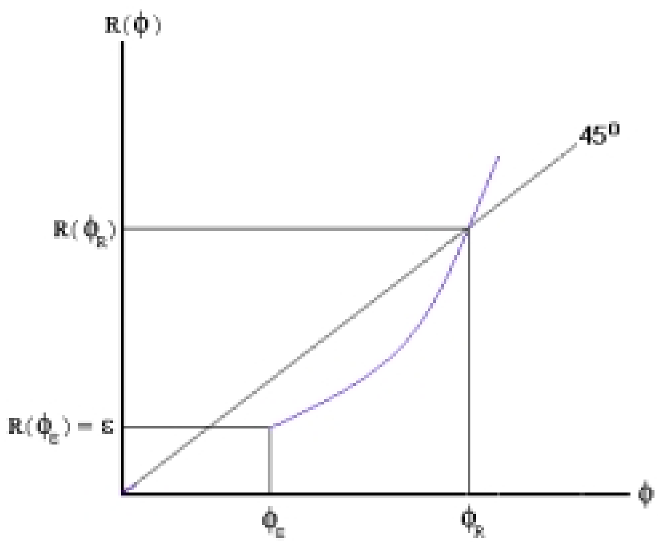

Figure 1: Destruction decision as a function of the variable cost.

Let θ_ε^* be defined by $q(\theta_\varepsilon^*) = \frac{(\alpha-1)(r+\delta+\lambda)c}{(1-\beta)\varepsilon}$ and $\phi_\varepsilon = \left[1 + \frac{\lambda}{(\alpha-1)(r+\delta+\lambda)}\right]\varepsilon - \left(1 + \frac{\beta}{1-\beta}\theta_\varepsilon^*\right)c$. Then if $\phi = \phi_\varepsilon$, (26) and (27) are solved by $\theta(\phi_\varepsilon) = \theta_\varepsilon^*$ and $R(\phi_\varepsilon) = \varepsilon$. Notice that if $R(\phi_\varepsilon) < \phi_\varepsilon$, then there is a nondegenerate interval $[\phi_\varepsilon, \phi_R)$ such that $R(\phi) < \phi$ iff $\phi \in [\phi_\varepsilon, \phi_R)$. An example of the function

$R(\phi)$ is illustrated in Figure 1.

So a sufficient condition for hoarding to occur in equilibrium (for at least some range of ϕ) is that $\phi_\varepsilon - \varepsilon > 0$, or equivalently, that $T(\lambda, \zeta) > 0$, where

$$T(\lambda, \zeta) \equiv \frac{\lambda \varepsilon}{(\alpha - 1)(r + \delta + \lambda)} - \left(1 + \frac{\beta}{1-\beta}\theta_\varepsilon^*\right) c.$$

The parameter ζ summarizes the efficiency of matching, with the property that $\partial m(u,v)/\partial\zeta > 0$ and hence that $\partial q(\theta)/\partial\zeta > 0$ for all θ. Figure 2 plots the boundary $T(\lambda, \zeta) = 0$ in λ-ζ space.

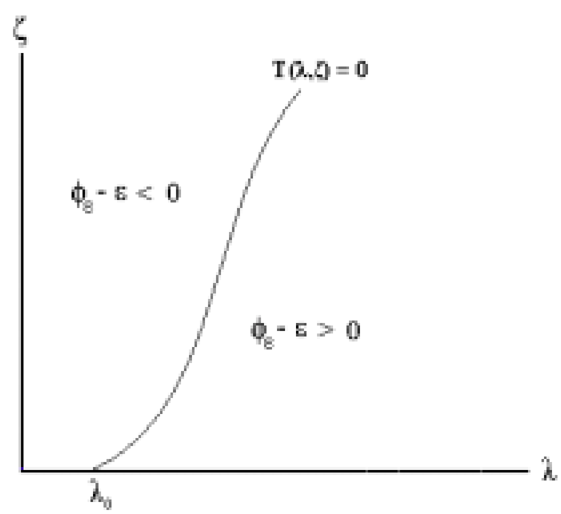

Figure 2: Range of parameters for which there is hoarding.

The condition $\phi_\varepsilon - \varepsilon > 0$ is satisfied for the values of the parameters λ and ζ that lie below boundary.[22] Intuitively, the parameter restriction that makes

[22] The equilibrium may or may not exhibit hoarding for parametrizations that lie above the boundary. Note that θ_ε^* goes to zero as ζ goes to zero. So $T(\lambda, 0) = 0$ iff $\lambda = \lambda_0$, where $\lambda_0 \equiv \frac{c(\alpha-1)(r+\delta)}{\varepsilon - c(a-1)}$ is the point at which the boundary intercepts the horizontal axis

hoarding possible holds for relatively large λ (i.e. when bad shocks are very transitory) and relatively low ζ (i.e. when the search process needed to replace the worker is very costly). Having characterized the relevant properties of the equilibrium, the following section studies the effects of labor-market policies on the level of TFP.

5 Labor-Market Policies and the Level of TFP

This section considers the effects of four policies: employment and hiring subsidies, firing taxes and unemployment benefits. I follow Pissarides (2000) and model the subsidies as transfers from the government to the firm and the firing tax as a payment from the firm to the government.[23] The value function $W(x)$ is still given by (8), while (4), (5) and (7) generalize to

$$\begin{aligned} rV &= \max_k \left[-ck + q(\theta) \int \max\left[J_o(z) + \tau_h k - V, 0\right] dG(z) \right], \\ rJ(x) &= \pi(x) + \tau_e k + \lambda \int \max\left[J(z), V - \tau_f k\right] dG(z) - \lambda J(x) \\ &\quad -\delta \left[J(x) + \tau_f k - V\right], \\ rU &= b + \theta q(\theta) \int \max\left[W_o(z) - U, 0\right] dG(z). \end{aligned}$$

in Figure 2. Formally, this boundary is upward-sloping because

$$\frac{\partial T}{\partial \zeta} = -\frac{\beta \theta^*_\varepsilon}{1-\beta} \frac{\partial \theta^*_\varepsilon}{\partial \zeta} < 0 \text{ and } \frac{\partial T}{\partial \lambda} = \frac{\varepsilon(r+\delta)}{(\alpha-1)(r+\delta+\lambda)^2} - \frac{bc^2(\alpha-1)}{(1-\beta)\varepsilon(1-\beta)q'(\theta^*_\varepsilon)} > 0.$$

[23] I assume that upon separation the firm must pay the firing tax to the government because in the present setup, firing taxes would be completely neutral under the alternative scheme where the firm compensates the fired worker directly. (The effects of such a policy would be completely undone by the wage bargain.) To keep the analysis simple, the government's financing constraints will be ignored. A natural extension would be requiring the government to run a balanced budget. An example of a scheme which is self-financing in the steady state is $\tau_f = \tau_h$ and $\tau_b = \tau_e = 0$.

The policy variables are τ_h (hiring subsidy), τ_e (employment subsidy), τ_f (firing tax) and b (unemployment benefit). Note that all payments are assumed to be proportional to the firm's size, as measured by k.[24] There are two reasons why the bargaining situation faced by a firm and worker when they first meet and are still considering whether to form a match is different from the one they face every instant after having agreed to form a match. The first is that in the initial bargain there is a one-time hiring subsidy at stake. The second, is that at that point the firm is not yet "locked in" by the firing tax. I use $w_o(x)$ to denote the wage that solves the initial bargain and $w(x)$ to denote the subsequent one. So $W_o(x) - W(x) = J(x) - J_o(x) = w_o(x) - w(x)$, and hence

$$J_o(x) + W_o(x) = J(x) + W(x). \tag{29}$$

The wages $w_o(x)$ and $w(x)$ are respectively characterized by

$$\begin{aligned} \beta\left[J_o(x) + \tau_h k\right] &= (1-\beta)\left[W_o(x) - U\right] \\ \beta\left[J(x) + \tau_f k\right] &= (1-\beta)\left[W(x) - U\right]. \end{aligned}$$

Letting $S_o(x) = J_o(x) + W_o(x) + \tau_h k - U$ and $S(x) = J(x) + W(x) + \tau_f k - U$ be the initial and the subsequent surplus respectively, the first-order conditions imply that $W_o(x) - U = \beta S_o(x)$, $W(x) - U = \beta S(x)$, $J_o(x) + \tau_h k = (1-\beta) S_o(x)$ and $J(x) + \tau_f k = (1-\beta) S(x)$. Combining these with the value functions gives

$$(r+\delta+\lambda) S(x) = (x - \phi - c) k + \tau_e k + r\tau_f k - rU + \lambda \int \max\left[S(z), 0\right] dG(z),$$

[24] This assumption is useful because it ensures that the policies introduce no scale effects into the job-creation and destruction decisions.

with rU as in (10). Since $S'(x) > 0$, there is a unique R such that $S(x) \geq 0$ iff $x \geq R$. Using this reservation property, the surplus of an ongoing match can be written as

$$(r+\delta+\lambda) S(x) = (x-\phi-c) k + \tau_e k + r\tau_f k - rU + \lambda \int_R S(z)\, dG(z), \quad (30)$$

a natural generalization of (11). One can work with the value functions and the first order conditions of the Nash problem to derive expressions for wages and profit. The key observations are that $w_o(x)$ is decreasing in the firing tax but increasing in the hiring and employment subsidies, while $w(x)$ is increasing in the employment subsidy and the firing tax and independent of the hiring subsidy.[25] Evaluating (30) at $x = R$,

$$\lambda \int_R S(z)\, dG(z) = rU - [(R-\phi-c) k + \tau_e k + r\tau_f k],$$

and substituting this back into (30) yields (14). Using (14) to substitute $S(z)$ out of (30), evaluating at $x = R$ and using (10) produces the job-destruction condition that generalizes (15):

$$R - \phi - c + \tau_e + r\tau_f - \left(\tau_b + \frac{\beta}{1-\beta} c\theta\right) + \frac{\lambda}{r+\delta+\lambda} \int_R (x-R)\, dG(x) = 0.$$

[25] The wages and profit agreed upon in an ongoing match are:

$$\begin{aligned} w(x) &= \beta [(x-\phi-c) k + \tau_e k + r\tau_f k] + (1-\beta) rU \\ \pi(x) &= (1-\beta) [(x-\phi-c) k - rU] - \beta [\tau_e k + r\tau_f k], \end{aligned}$$

while those in an initial match are:

$$\begin{aligned} w_o(x) &= \beta [(x-\phi-c) k + \tau_e k + (r+\delta+\lambda) \tau_h k - (\delta+\lambda) \tau_f k] + (1-\beta) rU \\ \pi_o(x) &= (1-\beta) [(x-\phi-c) k - rU] - \beta [\tau_e k + (r+\delta+\lambda) \tau_h k - (\delta+\lambda) \tau_f k]. \end{aligned}$$

For simplicity, I have specified $b = \tau_b k$ where $\tau_b \in [0, 1)$ is akin to a replacement rate.[26] Increases in the employment subsidy and the firing tax reduce R for given θ. In other words, an increase in τ_e or τ_f shifts the job-destruction condition down in θ-R space. Conversely, an increase in τ_b raises the worker's outside option and hence increases R for given θ.

By free entry, $rV = 0$, and

$$(1-\beta) \int_R S_o(x)\, dG(x) = \frac{ck}{q(\theta)}. \tag{31}$$

Finally, using (29), $S_o(x) = S(x) + (\tau_h - \tau_f)\, k$, which combined with (14) can be used to substitute $S_o(x)$ from (31) to obtain the job-creation condition:

$$\frac{1}{r+\delta+\lambda} \int_R (x-R)\, dG(x) + [1 - G(R)]\,(\tau_h - \tau_f) = \frac{c}{(1-\beta)\, q(\theta)}.$$

For given R, the hiring subsidy increases and the firing tax decreases job-creation. The other policy instruments have no direct effect on the entry decision. Finally, assuming G is as in (20), the job-destruction and creation conditions specialize to:

$$\begin{aligned} R - \phi - c + \tau_e + r\tau_f - \left(\tau_b + \tfrac{\beta}{1-\beta} c\theta\right) + \tfrac{\lambda \varepsilon^\alpha R^{1-\alpha}}{(\alpha-1)(r+\delta+\lambda)} &= 0 && (32) \\ \tfrac{\varepsilon^\alpha R^{1-\alpha}}{(\alpha-1)(r+\delta+\lambda)} + \left(\frac{\varepsilon}{R}\right)^\alpha (\tau_h - \tau_f) - \tfrac{c}{q(\theta)(1-\beta)} &= 0. && (33) \end{aligned}$$

The main properties of the equilibrium are summarized in the following proposition.

[26] This formulation of the unemployment compensation is a clean way to ensure the job-destruction equation is independent k. Another –perhaps more realistic– way to obtain the same result would be to adopt the specification outlined before, where $b = \tau_b E_x [w(x) | x \geq R]$. The formulation in the text yields the same qualitative results, but it is simpler because b remains independent of R.

Proposition 1. Let θ_ϵ^* be defined by

$$q\left(\theta_\epsilon^*\right) = \frac{(\alpha-1)(r+\delta+\lambda)c}{(1-\beta)\left[\varepsilon+(\alpha-1)(r+\delta+\lambda)\left(\tau_h-\tau_f\right)\right]}, \text{ and let}$$

$$\phi_\epsilon = \left[1+\frac{\lambda}{(\alpha-1)(r+\delta+\lambda)}\right]\varepsilon - \left(1+\frac{\beta}{1-\beta}\theta_\epsilon^*\right)c + \tau_h + r\tau_f - \tau_b > 0.$$

If $\varepsilon + (\alpha-1)(r+\delta+\lambda)(\tau_h - \tau_f) > 0$, then for any $\phi > \phi_\epsilon$: (a) there exists a unique equilibrium; (b) $R > \varepsilon$; (c) $\partial R/\partial\phi > 0$ and (d) $\partial\theta/\partial\phi < 0$. If in addition, $\phi_\epsilon - \varepsilon > 0$, then: (e) there is a nondegenerate interval $(\phi_\epsilon, \widetilde{\phi})$ such that $R(\phi) < \phi$ for all $\phi \in (\phi_\epsilon, \widetilde{\phi})$.

Proof. See the Appendix.

Aggregate output is still given by (24); the aggregate stock of capital demanded by filled jobs, K_e, is still given by (17); and the aggregate number of hours worked, N, is still as in (22). In addition, if the measure of capital used to construct aggregate output is K_e, then the level of TFP is still given by (25). The following proposition, which holds under the assumptions stated in Proposition 1, summarizes the effects that labor market policies have on A, the level of TFP.

Proposition 2. Employment subsidies and firing restrictions reduce A. Hiring subsidies and unemployment benefits increase A.

Proof. See the Appendix.

Since A is proportional to R, policy instruments have the same qualitative effect on TFP as on the destruction rate. Proposition 2 is illustrated in Figure 3.

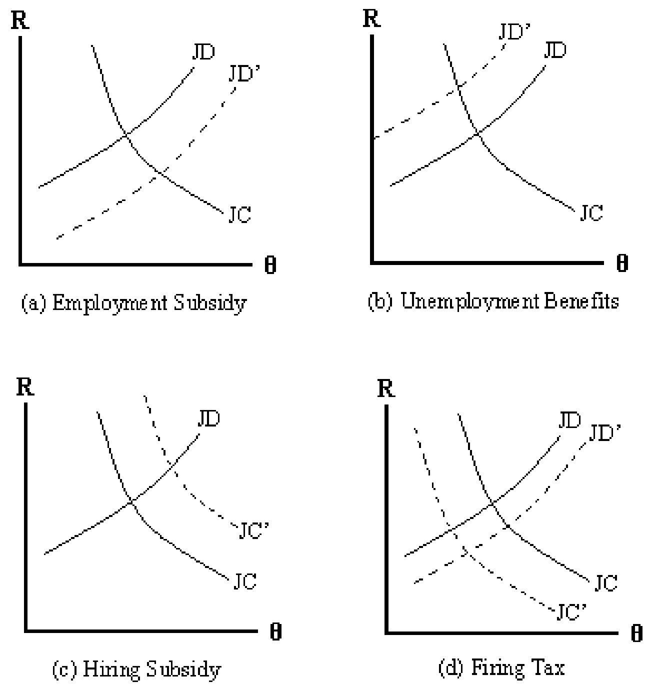

Figure 3: Equilibrium effects of various policies.

Employment subsidies make firms more tolerant of low productivity realizations, and hence lower the average productivity of active firms. All else equal, an economy with relatively high subsidies to continued employment will exhibit a low job-destruction rate, a high job-creation rate, and hence low levels of unemployment and measured TFP. Firing taxes have a similar qualitative effect on job-destruction, but that mechanism is reinforced by a relatively low rate of job-creation (which reduces the reservation wage and hence makes firms even more tolerant of low productivity realizations). So firing restrictions will reduce measured TFP, as well as the job-creation and destruction rates. Hiring subsidies have no direct effect on the destruction decision, but they stimulate job-creation. This increases market tightness which in turn increases the workers' outside option and raises measured TFP, job-creation and destruction. Unemployment benefits also cause R to rise through an increase in the worker's reservation wage. Consequently, economies with relatively high unemployment benefits will tend to exhibit relatively high levels of TFP and unemployment.

6 Extensions

This section extends the basic model to the case of serially-correlated shocks, provides a generalization of the main aggregation result, and shows how the observed level of TFP is affected by the different ways of measuring aggregate inputs that can be found in the literature.

6.1 Correlated Shocks

The maintenance cost $C(x, \phi)$ was introduced in Section 2 as a simple device to avoid "flat spots" in the value functions.[27] Here I show that by extending the model in a natural way, one can drop the maintenance cost without affecting the main results. To this end, I generalize the productivity process by allowing for serially correlated shocks: when match of productivity x suffers a change, the new value x' is a draw from the fixed distribution $G(x'|x)$.

[27] If $\pi(x) = [\max(x - \phi, 0) - c]\, k - w(x)$, then $\pi(x)$ is flat up to ϕ and then rises with slope k. It is easy to show that in this case $J(x)$ is also flat up to ϕ and then rises with slope $\frac{k}{r+\delta+\lambda}$. Note that since R is defined by $J(R) = 0$, this implies that generically the equilibrium will have $\phi < R$ (except for the knife-edge case in which R is indeterminate). Ruling out this type of flat spots in J allows for the possibility that $R < \phi$ in equilibrium.

Assuming $G(x|x_1) < G(x|x_0)$ if $x_0 < x_1$, allows idiosyncratic shocks to be positively correlated through time. For this case, the cross-section of productivities evolves according to

$$\begin{aligned}\frac{d}{dt}\left[(1-u_t)H_t(x)\right] &= \lambda(1-u_t)\int_x^{\infty}\left[G(x|s)-G(R_t|s)\right]dH_t(s)\\ &\quad +\theta q(\theta)u_t\int_{-\infty}^{\infty}\left[G(x|s)-G(R_t|s)\right]dH_t(s)\\ &\quad -\lambda(1-u_t)\int_{-\infty}^{x}G(R_t|s)\,dH_t(s)\\ &\quad -\int_{-\infty}^{x}\left[1-G(x|s)\right]dH_t(s)\\ &\quad -\delta(1-u_t)H_t(x)\lambda(1-u_t).\end{aligned}$$

The first term accounts for the matches with productivities above x that get innovations below x but above R_t. The newly-formed matches that start off with productivities no larger than x are in the second term. Notice the assumption that upon contact, the worker and firm draw their productivity level from the density corresponding to the average productivity among active matches.[28] The third term is the number of matches in the interval $[R_t, x]$ that get shocks below R_t and are destroyed. The fourth term accounts for the number of matches in the same interval that "move up" by virtue of having drawn productivities larger than x. The last term accounts for matches in

[28] When shocks were *iid*, I could just specify that new matches drew z from $G(z)$ just as active matches did when forced to update their shock. However, with correlated shocks active matches with state z draw the new shock z' from $G(z'|z)$. Since vacancies and unemployed workers have no productivity attached to them, I assume their initial draw z' is from the average density $\int G(z'|z)\,dH(z)$. As a way of motivating this, imagine –as Mortensen and Pissarides (1994) do– that firms must irreversibly adopt a "technology" to engage in production. The present specification then means that they pick their technology at random from all those active at the time the match is created.

the interval $[R_t, x]$ that are destroyed for exogenous reasons. Imposing steady states and re-arranging:

$$H(x) = \left[\frac{\lambda}{\delta+\lambda} + \frac{\theta q(\theta) u}{(\delta+\lambda)(1-u)}\right] \int [G(x|s) - G(R|s)]\, dH(s).$$

The steady-state unemployment rate is

$$u = \frac{\delta + \lambda \int G(R|s)\, dH(s)}{\delta + \lambda \int G(R|s)\, dH(s) + \theta q(\theta) \int [1 - G(R|s)]\, dH(s)}. \tag{34}$$

Using this expression, the steady-state cross-sectional productivity distribution can be rewritten as

$$H(x) = \frac{\int [G(x|s) - G(R|s)]\, dH(s)}{\int [1 - G(R|s)]\, dH(s)}, \tag{35}$$

a natural generalization of (3).

The firm's problem upon entering the market is now summarized by

$$rV = \max_k \left[-ck + q(\theta) \int \int \max[J(z) - V, 0]\, dG(z|x)\, dH(x)\right]. \tag{36}$$

Again, there is entry of firms until all rents are exhausted, so $rV = 0$ in equilibrium. The value of a filled job with productivity x is

$$rJ(x) = \pi(x) + \lambda \int \max[J(z), V]\, dG(z|x) - \lambda J(x) - \delta[J(x) - V], \tag{37}$$

where $\pi(x) = \max_n [x \min(n, k) - \phi n - ck - w(x)]$. Flow profit $\pi(x)$ is the residual remaining after the wage $w(x)$ and all other costs of production have been paid out. There are only two such costs in this formulation: the fixed cost, ck, and the variable one, ϕn. The wage w is independent of n, so the profit-maximizing choice of hours is still given by (6), and hence

$\pi(x) = y(x) - w(x)$, where $y(x) \equiv [\max(x - \phi, 0) - c]\, k$ is output net of the variable cost and the rental on capital.

The values of unemployment and employment to a worker are:

$$
\begin{aligned}
rU &= b + \theta q(\theta) \int \int \max[W(z) - U, 0]\, dG(z|x)\, dH(x) \qquad (38) \\
rW(x) &= w(x) + \lambda \int \max[W(z) - U, 0]\, dG(z|x) \qquad (39) \\
&\quad - (\delta + \lambda)[W(x) - U],
\end{aligned}
$$

where $w(x)$ it is still characterized by (9). Letting $S(x) = J(x) + W(x) - U$ denote the surplus from a match, notice that (9) implies that $J(x) = (1 - \beta) S(x)$ and $W(x) - U = \beta S(x)$. These together with (37), (38) and (39) imply

$$
(r + \delta + \lambda) S(x) = y(x) - rU + \lambda \int \max[S(z), 0]\, dG(z|x),
$$

where rU is given by (10). The fact that $S'(x) > 0$ implies that there exists a unique R such that $S(R) > 0$ iff $x > R$. Hence matches separate whenever productivity falls below R. For completeness, (9) and the value functions can be manipulated to obtain expressions for instantaneous wages and profit:

$$
\begin{aligned}
w(x) &= \beta y(x) + (1 - \beta) rU \qquad (40) \\
\pi(x) &= (1 - \beta)[y(x) - rU]. \qquad (41)
\end{aligned}
$$

It turns out that one can get a much sharper characterization of the equilibrium by putting some structure on the conditional distribution $G(s|x)$. In what follows, I assume that $dG(s|x) = \xi(x)\, \hat{g}(s)$ where $\xi'(x) > 0$.[29] The

[29] The Pareto distribution considered below is an example of a density that satisfies this condition.

surplus from a match x is now

$$(r+\delta+\lambda) S(x) = y(x) - rU + \lambda\xi(x) \int_R S(z)\,\hat{g}(z)\,dz, \tag{42}$$

and evaluating it at $x = R$ yields

$$\lambda \int_R S(z)\,\hat{g}(z)\,dz = \frac{rU - y(R)}{\xi(R)}.$$

Since the expected capital gain on the left-hand-side is positive, at $x = R$ net output is smaller than the worker's reservation wage. From (40) and (41) one again verifies that $w(R) < rU$ and $\pi(R) < 0$. Substituting the simpler expression for the expected capital gain term into (42) yields[30]

$$(r+\delta+\lambda) S(x) = y(x) - rU + \frac{\xi(x)}{\xi(R)}[rU - y(R)]. \tag{43}$$

Note that $y'(x) \geq 0$, and that the expected capital gain from the next draw (the second term) is increasing in current productivity because $\xi'(x) > 0$ (i.e. a higher shock today means the next innovation will be drawn from a better distribution). Thus $S'(x) = y'(x) + [\xi'(x)/\xi(R)][rU - y(R)] > 0$. Just as

[30] A word of caution is in order here. In general, the lower bound of the support of the density $\xi(x)\,\hat{g}(s)$ could itself be a function of x. For instance, assume the support is $[\varepsilon(x), \infty)$. Then, formally, the capital gain term in (42) should be written as $\lambda\xi(x)\int_{\max[R,\varepsilon(x)]} S(z)\,\hat{g}(z)\,dz$, and then evaluating the surplus at $x = R$ would yield

$$\lambda \int_{\max[R,\varepsilon(R)]} S(z)\,\hat{g}(z)\,dz = \frac{rU - y(R)}{\xi(R)}.$$

Thus

$$\lambda \int_{\max[R,\varepsilon(x)]} S(z)\,\hat{g}(z)\,dz = \frac{rU - y(R)}{\xi(R)}$$

and (43) follows iff $R > \varepsilon(x)$ for all x. A restriction that one can assume –and must later verify– to be satisfied in equilibrium.

before, the equilibrium can have $\phi < R$ or $R < \phi$. Evaluating (42) at $x = R$ and using (43) to substitute $S(z)$ yields the job-destruction condition:

$$y(R) - rU + \frac{\lambda}{r+\delta+\lambda}\int_R \left\{ y(z) - rU + \frac{\xi(z)}{\xi(R)}\left[rU - y(R)\right] \right\} dG(z|R) = 0.$$

Equation (36) and $rV = 0$ imply that at the optimal k,

$$(1-\beta)\int\int_R S(z)\, dG(z|x)\, dH(x) = \frac{ck}{q(\theta)},$$

namely the expected profit from a filled job equals the expected recruiting cost. Using (43) to substitute $S(z)$ out of this expression yields the job-creation condition:

$$\int\int_R \left\{ y(z) - rU + \frac{\xi(z)}{\xi(R)}\left[rU - y(R)\right] \right\} dG(z|x)\, dH(x) = \frac{(r+\delta+\lambda)\, ck}{(1-\beta)\, q(\theta)}.$$

After some manipulations, the job-destruction and creation conditions respectively simplify to:

$$\tfrac{-[rU-y(R)]}{k} + \tfrac{\lambda}{r+\delta+\lambda}\left\{ \varphi(\mu|R) + \tfrac{rU-y(R)}{\xi(R)k}\int_R \left[\xi(x) - \xi(R)\right] dG(x|R) \right\} = 0$$

$$\int \varphi(\mu|z)\, dH(z) + \tfrac{rU-y(R)}{\xi(R)k}\int\int_R \left[\xi(x) - \xi(R)\right] dG(x|z)\, dH(z) = \tfrac{(r+\delta+\lambda)c}{(1-\beta)q(\theta)},$$

where $\varphi(\mu|R) \equiv \int_\mu [1 - G(x|R)]\, dx$ and $\mu \equiv \max(\phi, R)$. Given the cross-sectional productivity distribution H, the job-creation and destruction conditions jointly determine R and θ. Observe that these conditions are analogous to those in Mortensen and Pissarides (1994) when $\phi = 0$ and $\xi(x) = \xi$ for all x. More formally, for given c and ϕ, an equilibrium is a list $[R, \theta, H, U, w, u, k]$ such that R, θ and H jointly solve (35) and the job-creation and the job-destruction conditions; rU is given by (10); w by (40);

and u satisfies (34). In addition, the market for capital should clear, so k must satisfy $[1-(1-\theta)u]k = K$.

Now suppose idiosyncratic shocks are draws from

$$G(x|s) = \begin{cases} 0 & \text{if } x < \varepsilon(s) \\ 1 - \left[\frac{\varepsilon(s)}{x}\right]^{\alpha} & \text{if } \varepsilon(s) \leq x \end{cases}$$

where $\varepsilon(\cdot)$ is a continuously differentiable function and $\alpha > 2$. I introduce positively correlated shocks by assuming that $\varepsilon' > 0$. (The special case of *iid* shocks corresponds to $\varepsilon' = 0$.) In addition, suppose there is an $\underline{\varepsilon} > 0$ such that $\varepsilon(\underline{\varepsilon}) = \underline{\varepsilon}$ and $\varepsilon(s) = 0$ if $s < \underline{\varepsilon}$, and that $\lim_{s\to\infty} \varepsilon(s) = 1 + \underline{\varepsilon} \equiv \overline{\varepsilon}$.[31]

Then for $R \geq \varepsilon(s)$, $1 - G(R|s) = \left[\frac{\varepsilon(s)}{R}\right]^{\alpha}$; and hence for any $x \geq R$,

$$G(x|s) - G(R|s) = \left[\frac{\varepsilon(s)}{R}\right]^{\alpha}\left[1 - \left(\frac{R}{x}\right)^{\alpha}\right].$$

After substituting these expressions in (35) it becomes clear that the steady state productivity cross-section is still given by (21). So for this case, the job-creation and destruction conditions, respectively, specialize to

$$\frac{\mu^{1-\alpha}R^{\alpha}}{\alpha-1} - \frac{\alpha[y(R)-rU]}{k}\left[\frac{R}{\varepsilon(R)}\right]^{\alpha}\int_R \frac{\varepsilon(x)^{\alpha}-\varepsilon(R)^{\alpha}}{x^{1+\alpha}}dx = \frac{(r+\delta+\lambda)c}{(1-\beta)q(\theta)}\left[\alpha\int_R \frac{\varepsilon(x)^{\alpha}}{x^{1+\alpha}}dx\right]^{-1}$$

$$\left[1 - \frac{\lambda\alpha}{r+\delta+\lambda}\int_R \frac{\varepsilon(x)^{\alpha}-\varepsilon(R)^{\alpha}}{x^{1+\alpha}}dx\right]\frac{y(R)-rU}{k} + \frac{\lambda}{r+\delta+\lambda}\frac{\varepsilon(R)^{\alpha}\mu^{1-\alpha}}{\alpha-1} = 0.$$

Under relatively mild conditions, it can be shown that the job-creation condition slopes down and the destruction condition up in θ-R space, implying a unique (θ, R) pair. A parameter restriction analogous to the one depicted in Figure 2 guaranteeing that there is a range of values for ϕ such that

[31] An example of an $\varepsilon(\cdot)$ satisfying all these conditions is $\varepsilon(s) = 1 + \underline{\varepsilon} - e^{\underline{\varepsilon}-s}$, for any $\underline{\varepsilon} > 0$.

$R < \phi$ can still be derived.[32] Following the procedure used in Section 4, it is straightforward to verify that output still aggregates to (24).

6.2 More on Aggregation

Section 4 established that when the idiosyncratic shocks are drawn from a Pareto distribution, aggregate output always looks like a Cobb-Douglas function of the aggregate labor and capital inputs. This subsection generalizes the previous result by characterizing the distribution of shocks that gives rise to an aggregate CES production function.

Suppose the primitive distribution of shocks, G, is given by

$$G(x) = \begin{cases} 0 & \text{if } x < \varepsilon \\ 1 - \left[\frac{1}{\sigma}\left(\frac{x}{\varepsilon}\right)^{\frac{\rho}{1-\rho}} - \frac{1-\sigma}{\sigma}\right]^{-1/\rho} & \text{if } \varepsilon \leq x \end{cases} \tag{44}$$

with $\varepsilon > 0$ and $\rho, \sigma \in (0,1)$.[33] Substituting (44) into (3) one sees that for any $R \geq \varepsilon$, the steady state productivity distribution of active matches is

$$H(x) = 1 - \kappa \left[\frac{1}{\sigma}\left(\frac{x}{\varepsilon}\right)^{\frac{\rho}{1-\rho}} - \frac{1-\sigma}{\sigma}\right]^{-1/\rho} \tag{45}$$

if $R \leq x$; and $H(x) = 0$ if $x < R$, with $\kappa \equiv [1 - G(R)]^{-1}$. Using the method proposed by Levhari (1968), one can show the following result.

[32] In addition, one should always make sure that the equilibrium satisfies $R > \overline{\varepsilon}$. Recall that the derivation of (43) implicitly assumes that $R > \varepsilon(s)$ for all s. This condition is satisfied if $R > \overline{\varepsilon}$. Showing that equilibria with $R < \phi$ are possible for some parametrizations is now rather tedious, so the basic idea is only outlined here. Let $\phi_{\overline{\varepsilon}}$ be the value of ϕ such that $\theta^*_{\overline{\varepsilon}}$ and $R(\phi_{\overline{\varepsilon}}) = \overline{\varepsilon}$ solve the job-creation and destruction conditions. Then if $\phi_{\overline{\varepsilon}} - \overline{\varepsilon} > 0$, there will be an interval $(\phi_{\overline{\varepsilon}}, \widehat{\phi})$ such that $R(\phi) < \phi$ iff $\phi \in (\phi_{\overline{\varepsilon}}, \widehat{\phi})$. If, in addition, $\partial R(\phi)/\partial\phi > 0$, then $\phi_{\overline{\varepsilon}} - \overline{\varepsilon} > 0$ also implies $\overline{\varepsilon} < R(\phi)$ for all $\phi \in (\phi_{\overline{\varepsilon}}, \widehat{\phi})$. Finally, notice that $R > \overline{\varepsilon}$ also implies that *every* match faces a positive probability of being destroyed for endogenous reasons. To see why, suppose $R = \varpi < \overline{\varepsilon}$; then any match that reaches a state $s > \varepsilon^{-1}(\varpi)$ will never be destroyed endogenously.

[33] Under these conditions $G'(x) \geq 0$ and $\lim_{x\to\infty} G(x) = 1$, so G is a proper *cdf*. Hereafter we return to the case with *iid* shocks.

Proposition 3. If the primitive distribution of the idiosyncratic shocks is given by (44), then in equilibrium, the aggregates Y, K_e and N satisfy $Y = B\left[\sigma \bar{A} K_e^{\rho} + (1-\sigma) N^{\rho}\right]^{1/\rho}$, with $B = \frac{\varepsilon}{1-\sigma}$, and $\bar{A} = \left[\frac{1}{\sigma}\left(\frac{R}{\varepsilon}\right)^{\frac{\rho}{1-\rho}} - \frac{1-\sigma}{\sigma}\right]$.

Proof. See the Appendix.

In this case, all the characteristics of the labor market as summarized by R affect the measured productivity of inputs asymmetrically.[34] Notice that as $\rho \to 0$, (44) approaches the Pareto distribution in (20) with parameters ε and $\alpha = 1/\sigma$. So in this sense, the CES aggregate in Proposition 3 approaches the Cobb-Douglas aggregate in (24) as the elasticity of substitution $1/(1-\rho)$ approaches unity.[35]

6.3 Measurement

I conclude this section by showing how the observed level of TFP is affected by the different ways of measuring aggregate inputs that can be found in the literature. The measure of capital input used by Hall and Jones (1999) did not adjust for utilization. This means that K instead of K_e was used in the production function, which would imply $\hat{F}(K,N) = \hat{A} K^{\gamma} N^{1-\gamma}$, with $\hat{A} = \left[\frac{1-u}{1-(1-\theta)u}\right]^{\gamma} A$, as mentioned in Section 4. But in addition, Hall and Jones

[34] If the aggregation were performed using (44) instead of its truncation, then the aggregate would instead be $Y = \frac{R}{1-\sigma}\left[\sigma K_e^{\rho} + (1-\sigma) N^{\rho}\right]^{1/\rho}$. However, there is no primitive density that has (44) as its truncation.

[35] Notice, however, that the truncation of (44) does not approach (21) as $\rho \to 0$. That is, even though the primitive distribution approaches a Pareto, its truncation does not limit a truncated Pareto. This is because the density in (44) is not "closed" under truncations (as for example the Pareto and the exponential distributions are). This "discontinuity" introduced by the truncation is the reason why if we take the limit on the truncated *cdf* or on the CES aggregate directly, we don't obtain exactly (24).

(1999) report they did not have data on hours per worker for all countries in their sample, so they used the number of employed workers instead of hours worked as a measure labor input. Letting $E = 1 - u$ denote employment and using (17) and (22), the number of hours worked is $N = (R/\mu)^{1/\gamma} \frac{KE}{1-(1-\theta)u}$, so their measurements of inputs imply that the aggregate relationship between inputs, output and TFP that they observed was $\tilde{F}(K, E) = \tilde{A} K^{\gamma} E^{1-\gamma}$, with $\tilde{A} = \left[\frac{(R/\mu)^{1/\gamma} K}{1-(1-\theta)u}\right]^{1-\gamma} \hat{A}$.

7 Concluding Remarks

This paper presented a theory of aggregate TFP differences based on the interaction between institutions and the microeconomics underlying the aggregate production function. It focused on a precise type of institutions, namely labor-market policies as measured by the magnitudes of hiring and employment subsidies, unemployment benefits and firing taxes. In the model, firm-level technologies are subject to idiosyncratic shocks that induce a cross-sectional distribution of productivities. Labor-market policies affect the productivity composition of active firms through their effects on the job-creation and destruction decisions.

Policies that make firing difficult make firms less willing to give up relatively unproductive opportunities to search for better ones, lowering the average productivity among active matches, and aggregate TFP. Employment subsidies also make firms more tolerant of low productivity realizations and hence they also decrease TFP. Unemployment benefits have the opposite effect. Hiring subsidies stimulate job creation and cause more competition among firms. As a result, firms become more selective and only pursue very productive ventures. The cross-sectional distribution of productivities shifts to the right, and aggregate TFP rises.

The model could serve as a guide to understand aggregate productivity data. It could be parametrized to find out how large the differences in the mix and magnitude of labor-market policies have to be in order to explain the differences in TFP levels among a relevant set of countries. It may also prove to be a useful tool for the econometrician interested in measuring aggregate productivity.

A Appendix

Proof of Proposition 1.

Let $\theta(\phi)$, and $R(\phi)$ denote the solution to (32) and (33) when it exists; and define $\tau(R) = R + \alpha(r+\delta+\lambda)(\tau_h - \tau_f)$. By totally differentiating (32) and (33):

$$\begin{aligned}
\frac{\partial R(\phi)}{\partial \phi} &= \frac{(r+\delta+\lambda)\eta(\theta)}{[\beta\theta q(\theta)/R](\varepsilon/R)^{\alpha}\tau(R) + (r+\delta+\lambda)\eta(\theta)\left[1-\frac{\lambda(\varepsilon/R)^{\alpha}}{r+\delta+\lambda}\right]} \\
\frac{\partial \theta(\phi)}{\partial \phi} &= \frac{-(1-\beta)\theta q(\theta)(1/R)(\varepsilon/R)^{\alpha}\tau(R)}{(r+\delta+\lambda)\eta(\theta)c}\frac{\partial R}{\partial \phi}.
\end{aligned}$$

So $\tau(R) > 0$ is sufficient for $\partial R/\partial\phi > 0$. If $\phi = \phi_\epsilon$, then (32) and (33) have a unique solution, namely $\theta(\phi_\epsilon) = \theta_\epsilon^*$ and $R(\phi_\epsilon) = \varepsilon$. But $\tau(\varepsilon) > 0$ by assumption, so $\partial R(\phi_\epsilon)/\partial\phi > 0$. This and the continuity of $R(\phi)$ implies that $R(\phi) > \varepsilon$ for all $\phi > \phi_\epsilon$. Since $\tau' > 0$, for any $\phi > \phi_\epsilon$, then $\tau(R) > 0$ and therefore $\partial R(\phi)/\partial\phi > 0$ and $\partial\theta(\phi)/\partial\phi < 0$. This establishes parts (b), (c) and (d). In θ-R space, the slopes of the job-destruction and creation conditions are

$$\frac{\beta c}{1-\beta\left[1-\frac{\lambda(\varepsilon/R)^{\alpha}}{r+\delta+\lambda}\right]} > 0, \text{ and } \frac{-c\eta(\theta)(r+\delta+\lambda)R}{(1-\beta)\theta q(\theta)(\varepsilon/R)^{\alpha}\tau(R)} < 0$$

respectively, which establishes (a). Finally, $\phi_\epsilon - \varepsilon > 0$ is equivalent to $\phi_\epsilon - R(\phi_\epsilon) > 0$, which implies (e). ■

Proof of Proposition 2.

Define

$$\Delta = \frac{(\varepsilon/R)^{\alpha}\tau(R)}{(r+\delta+\lambda)R} + \frac{\eta(\theta)}{\beta\theta q(\theta)}\left[1-\frac{\lambda(\varepsilon/R)^{\alpha}}{r+\delta+\lambda}\right].$$

Since $\tau(R) > 0$ by Proposition 1, it follows that $\Delta > 0$ in any equilibrium. By totally differentiating (32) and (33),

$$\begin{aligned}
\frac{\partial R}{\partial \tau_e} &= \frac{-\eta(\theta)}{\beta\theta q(\theta)\Delta} < 0,\ \frac{\partial R}{\partial \tau_f} = -(1/\Delta)\left[(\varepsilon/R)^\alpha + \frac{r\eta(\theta)}{\beta\theta q(\theta)}\right] < 0, \\
\frac{\partial R}{\partial \tau_h} &= (1/\Delta)(\varepsilon/R)^\alpha > 0,\ \frac{\partial R}{\partial \tau_b} = -\frac{\partial R}{\partial \tau_e} > 0,
\end{aligned}$$

and this concludes the proof. ■

Proof of Proposition 3.

The problem is to find a *cdf* H that satisfies $H(R) = 0$ and yields

$$Y = a\left[\sigma_1(\hat{\kappa}K_e)^\rho + \sigma_2 N^\rho\right]^{1/\rho}, \tag{46}$$

where $\rho \in (0,1)$ and a, $\hat{\kappa}$, σ_1 and σ_2 are positive constants. Define $\varsigma(x) = \int_x zh(z)\,dz$ and $s(x) = 1 - H(x)$. Since, in general, $Y = \varsigma(\mu)K_e$ and $N = s(\mu)K_e$, (46) can be rewritten as $\varsigma(x)^\rho = a^\rho\left[\sigma_1\hat{\kappa}^\rho + \sigma_2 s(x)^\rho\right]$. Differentiating both sides of this expression gives $\varsigma(x) = \left(\frac{x}{\sigma_2 a^\rho}\right)^{\frac{1}{1-\rho}} s(x)$. The last two equations yield $s(x) = \hat{\kappa}\left[\frac{1}{\sigma_1}\left(\frac{x}{\sigma_2 a}\right)^{\frac{\rho}{1-\rho}} - \frac{\sigma_2}{\sigma_1}\right]^{-1/\rho}$, which by defining

$$\varepsilon = \sigma_2 a(\sigma_1+\sigma_2)^{\frac{1-\rho}{\rho}} \text{ and } \sigma = \frac{\sigma_1}{\sigma_1+\sigma_2} \tag{47}$$

can be rewritten as $H(x) = 1 - \hat{\kappa}\left[\frac{1}{\sigma}\left(\frac{x}{\varepsilon}\right)^{\frac{\rho}{1-\rho}} - \frac{1-\sigma}{\sigma}\right]^{-1/\rho}$. The requirement that $H(R) = 0$ implies that $\hat{\kappa} = \kappa$ (with κ as defined in Subsection 6.2). After specifying that $H(x) = 0$ for $x < R$, this expression is identical to (45). So by construction, aggregation under (45) yields (46). And after letting $\hat{\kappa} = \kappa$ and making the substitutions in (47), one realizes that (46) is identical to the aggregate in the statement Proposition 3. Finally, verifying that (45) is the truncation of (44) at R concludes the proof. ■

To complete the analysis of Section 5, here I report the effects of all policies on market tightness.

$$\begin{aligned}
\frac{\partial\theta}{\partial\tau_e} &= \frac{-\left(1-\beta\right)\theta q\left(\theta\right)\left(\varepsilon/R\right)^{\alpha}\tau\left(R\right)}{c\eta\left(\theta\right)\left(r+\delta+\lambda\right)R}\frac{\partial R}{\partial\tau_e} > 0,\\
\frac{\partial\theta}{\partial\tau_h} &= \frac{1-\beta}{\beta c}\left[1-\frac{\lambda\left(\varepsilon/R\right)^{\alpha}}{r+\delta+\lambda}\right]\frac{\partial R}{\partial\tau_h} > 0,\\
\frac{\partial\theta}{\partial\tau_b} &= \frac{-\left(1-\beta\right)\theta q\left(\theta\right)\left(\varepsilon/R\right)^{\alpha}\tau\left(R\right)}{c\eta\left(\theta\right)\left(r+\delta+\lambda\right)R}\frac{\partial R}{\partial\tau_b} < 0,\\
\frac{\partial\theta}{\partial\tau_f} &= \frac{1-\beta}{\beta c}\left\{r+\left[1-\frac{\lambda\left(\varepsilon/R\right)^{\alpha}}{r+\delta+\lambda}\right]\frac{\partial R}{\partial\tau_f}\right\}.
\end{aligned}$$

Without additional restrictions the sign of $\partial\theta/\partial\tau_f$ is ambiguous. It is negative in any equilibrium with $\phi > \phi_\epsilon$ if $\delta > r\left(1-\varepsilon\right)/\varepsilon$.

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