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RISK AND RETURN: AN EQUILIBRIUM APPROACH

Abstract

This paper develops a regime switching, pure exchange economy which duplicates many of the empirical features of the relation between the expectation and volatility of stock returns. The key features of the model are heteroscedasticity in inflation, regimes which mimic the expansionary and contractionary phases of the economy, and transitions between regimes which depend on the level of inflation. These features result in time-varying and asymmetric cross serial correlations between the conditional moments of returns.

1 Introduction

Recent empirical studies (Glosten, Jagannathan and Runkle (1993), Whitelaw (1993)) document a number of stylized facts with regard to the intertemporal relation between risk and return.[1] In particular, they provide support for a negative contemporaneous relation between conditional expected returns and the conditional volatility of returns, and for the potentially important role of nominal interest rates in estimating these conditional moments. In addition, Whitelaw (1993) documents an apparent offset cyclical behavior in the mean and volatility of returns. Volatility appears to lead expected returns with the result that cross serial correlations between the conditional moments exhibit a marked asymmetry and contemporaneous correlations measured over short horizons vary from large positive to large negative values.

It can be argued that these time series properties result from the relation between equity returns, inflation, and output, as these two latter processes vary over the course of the business cycle. Rather than attempting to measure directly the time variations in the underlying processes and modeling the relation between them and returns, the strategy adopted in the literature is to proxy for these complex, and potentially nonlinear, relations using instruments from the financial markets. This approach has the advantage of avoiding the problem of postulating and estimating a model of inflation and output growth, while exploiting the proven predictive power of various financial variables. The disadvantage of the methodology is that it fails to provide a compelling link between returns and the state of the underlying economy, which can be evaluated as either consistent or inconsistent with a particular asset pricing model. The purpose of this paper is to develop an equilibrium model of equity returns and to illustrate the extent to which parameterizations of this model are consistent with the existing empirical evidence.[2] In the process, the links between the state of the economy and the moments of returns are developed and discussed.

The key results are that a relatively simple nominal exchange economy can generate return moment variations and covariations similar to those documented in Whitelaw (1993) and elsewhere. Offset unconditional correlations between the conditional mean and conditional volatility exhibit a

[1]These papers extend earlier work on the subject by Campbell (1987), and French, Schwert and Stambaugh (1987), among many others.

[2]It is already well known that equilibrium models can generate a wide variety of relations between the mean and volatility of returns (see Abel (1988) and Backus and Gregory (1988), among others). The question addressed in this paper is whether the more detailed intertemporal patterns documented recently can be captured by a parsimonious model.

marked asymmetry, and conditional contemporaneous correlations between the moments fluctuate from positive to negative over the course of the cycle. A VAR(1) estimated on the conditional moments from the artificial economy also generates coefficients suggestive of those from the data. The state variables in the model, inflation and aggregate real consumption, are modeled as a VAR. The assumption that drives the results, however, is that the parameters of the VAR can take on two possible sets of values in a regime switching context (Hamilton (1989)). This switching structure permits the necessary asymmetry and heteroscedasticity in the underlying processes, which, in turn, generate asymmetries in the moments of equity returns.

The remainder of the paper is organized as follows. Section 2 develops the asset pricing framework and specializes the specification of the state processes to a switching VAR to facilitate pricing. Section 3 illustrates asset pricing for a simple parameterization and describes the numerical procedure for more complex parameterizations. The behavior of equity returns relative to the state of the economy is explored in Section 4. We attempt to calibrate the model to the empirical evidence in Section 5, and Section 6 concludes.

2 The Asset Pricing Framework

From an asset pricing perspective, consider a nominal version of a pure exchange economy with a single consumption good (Lucas (1978)). Assume further the existence of a representative agent. All assets will be priced according to the first order conditions of this agent, giving the standard pricing equation[3]

$$p_t = \mathrm{E}_t[l_{t,t+1}(p_{t+1} + y_{t+1})], \tag{1}$$

where $l_{t,t+1}$ is the real marginal rate of substitution (RMRS), from time t to $t+1$, defined on the single consumption good, p_t is the real, ex-payoff price of the asset, and y_{t+1} is the asset's real payoff. The price level at any time t, denoted π_t, is simply a unit of account that defines the dollar price of a unit of consumption. This price level can be used to rewrite the pricing equation in nominal terms as

$$P_t = \mathrm{E}_t[l_{t,t+1}(\pi_t/\pi_{t+1})(P_{t+1} + Y_{t+1})], \tag{2}$$

[3]More generally, Harrison and Kreps (1979) show that in the absence of arbitrage there exists a non-negative pricing operator for which this condition holds.

where $P_t = p_t \pi_t$, $Y_t = y_t \pi_t$, and $(\pi_t/\pi_{t+1}) l_{t,t+1}$ is the nominal marginal rate of substitution (NMRS). Denoting the simple nominal return by $R(t,t+1) = (P_{t+1} + Y_{t+1})/P_{t-1}$, the expected return on the asset in excess of the one period nominally riskless rate, $r_f(t,t+1)$, is proportional to the negative of the covariance of this return with the NMRS, i.e.,[4]

$$\mathrm{E}_t[R(t,t+1) - r_f(t,t+1)] = -[1 + r_f(t,t+1)]\mathrm{Cov}_t[l_{t,t+1}(\pi_t/\pi_{t+1}), R(t,t+1)]. \tag{3}$$

This model is not intended to be a perfect representation of the economy. Nevertheless, it is both sufficiently complex to produce insight into the time variation of equity returns and sufficiently simple, under additional assumptions, to preserve tractability.

Assume also the existence of a representative firm, with a capital structure that consists only of debt and equity, whose value is independent of the capital structure and equals the sum of the values of the debt and equity. A Miller-Modigliani world of perfect markets and no bankruptcy costs or taxes is sufficient for these latter conditions to hold. The existence of bankruptcy costs or taxes would complicate the analysis, but it is unlikely to overturn the basic results. Using the identity for the value of the firm (F) as the sum of the values of the debt (D) and equity (S) (i.e, $F_t = D_t + S_t$), and the definition of the leverage ratio ($\mathrm{LR}_t = D_t/S_t$), the return on the firm can be written as

$$R_F(t,t+1) = \frac{1}{1+\mathrm{LR}_t} R_S(t,t+1) + \frac{\mathrm{LR}_t}{1+\mathrm{LR}_t} R_D(t,t+1), \tag{4}$$

where $R_F(t,t+1)$, $R_S(t,t+1)$ and $R_D(t,t+1)$ are the returns on the firm, the equity and the debt respectively, from time t to $t+1$. Solving for the equity return, subtracting the riskless rate and taking expectations conditional on time t information, gives the following relations between the conditional moments:

$$\begin{aligned} \mathrm{E}_t[R_S(t,t+1) - r_f(t,t+1)] &= (1+\mathrm{LR}_t)(\mathrm{E}_t[R_F(t,t+1) - r_f(t,t+1)]) - \\ &\quad \mathrm{LR}_t(\mathrm{E}_t[R_D(t,t+1) - r_f(t,t+1)])\,, \end{aligned} \tag{5}$$

$$\begin{aligned} \mathrm{Var}_t[R_S(t,t+1)] &= (1+\mathrm{LR}_t)^2 \mathrm{Var}_t[R_F(t,t+1)] + \mathrm{LR}_t^2 \mathrm{Var}_t[R_D(t,t+1)] \\ &\quad -2\mathrm{LR}_t(1+\mathrm{LR}_t)\mathrm{Cov}_t[R_D(t,t+1), R_F(t,t+1)]. \end{aligned} \tag{6}$$

[4] This result can also be derived in a similar setting with the growth in the money supply as the second fundamental process, instead of inflation. Imposing a cash-in-advance constraint and appropriately specifying the timing of money endowments and consumption decisions produces a unitary velocity of money and the same pricing relation (Labadie (1989)). Other treatments of money include putting real money balances in the agent's utility function (Danthine and Donaldson (1986)) and specifying transaction costs as decreasing in real money balances (Marshall (1989)).

To simplify the discussion, assume further that the firm is defined as a series of real output y_t, and that the debt is a single issue of long-term (T-period), riskless, pure discount bonds. The nominal value of the firm and the nominal return on the firm are

$$F_t = \mathrm{E}_t[l_{t,t+1}(\pi_t/\pi_{t+1})(F_{t+1} + y_{t+1}\pi_{t+1})] = \pi_t \sum_{s=1}^{\infty} \mathrm{E}_t[l_{t,t+s}(y_{t+s})] \,, \quad (7)$$

$$1 + R_F(t,t+1) = \frac{F_{t+1} + \pi_{t+1}y_{t+1}}{F_t} = \left(\frac{\pi_{t+1}}{\pi_t}\right) \frac{y_{t+1} + \sum_{s=2}^{\infty} E_{t+1}[l_{t+1,t+s}(y_{t+s})]}{\sum_{s=1}^{\infty} \mathrm{E}_t[l_{t,t+s}(y_{t+s})]} \,. \quad (8)$$

The nominal value of the debt and the nominal return on the debt are

$$D_t = M\mathrm{E}_t[l_{t,t+T}(\pi_t/\pi_{t+T})], \quad (9)$$

$$1 + R_D(t,t+1) = \frac{D_{t+1}}{D_t} = \frac{E_{t+1}[l_{t+1,t+T}(\pi_{t+1}/\pi_{t+T})]}{\mathrm{E}_t[l_{t,t+T}(\pi_t/\pi_{t+T})]}, \quad (10)$$

where M is the face amount of the bond.

Finally, we need to develop the link between real output, real consumption, and the RMRS. For simplicity, assume that the real output of the firm and real consumption of the representative agent are identical.[5] The fact that the stock market represents claims on firms that constitute a large percentage of the productive assets in the economy supports this assumption, although empirically, output and consumption are positively related but not perfectly correlated.

It is immediately clear from (3)-(6) that in this economy the expected return on the stock and the conditional volatility of this return will not necessarily be proportional or even positively related. Thus, parameterizations of this model have the potential to explain the earlier empirical results. The appropriate measure of risk for any asset (for asset pricing purposes) is the covariation of its return with the NMRS. As a result, two major effects drive a wedge between the risk and the volatility of the return on the stock. First, the equity is only one of two claims on the output stream of the firm. Even if the conditional first and second moments of the return on the firm are positively related, equity, as a levered claim, may not possess the same properties. Second, although the real output of the firm and the real consumption of the representative agent are identical, the expected return on the firm depends on the effect of inflation on expectations about future output and the future RMRS, and on the relation between the RMRS and output growth. This latter relation depends on the utility function of the representative agent.

[5]See Cochrane (1991) for a description of a production economy in which this equality holds.

If the representative agent has constant relative risk averse (CRRA) utility over the single consumption good of the form $u(c) = c^{1-\alpha}$ and time preference parameter β, then the RMRS from time t to $t+1$ is $l_{t,t+1} = \beta(\frac{c_{t+1}}{c_t})^{-\alpha}$. The nominal pricing equation then becomes

$$P_t = \beta \mathrm{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\alpha} \left(\frac{\pi_t}{\pi_{t+1}} \right) (P_{t+1} + Y_{t+1}) \right]. \tag{11}$$

This specialization is made keeping in mind the numerical techniques that will be employed later in order to solve for equity prices. CRRA utility permits a relatively simple computation of the moments of equity returns using the discrete state space approximation discussed in Section 3.2. This specialization is necessary to preserve tractability, but the cost is that relative risk aversion cannot change with the level of consumption.

Having already assumed that the real output of the representative firm equals the real consumption of the representative agent, the only step left to complete the specification of the model is to define the laws of motion of inflation and consumption growth. We assume that, at any point in time, the natural logarithms of these state variables follow a vector autoregressive process of order q (VAR(q)) with multivariate normal errors and a constant covariance matrix. However, we also allow for the possibility of a number of different VAR regimes. The state processes follow a specified VAR for a number of periods until a regime switch is triggered. These processes then follow a VAR with different parameters until another switch occurs. In particular a two regime economy is parameterized as

$$\begin{aligned} X_t &= A_0 + S_{t-1}A_0' + \sum_{l=1}^{q}(A_l + S_{t-1}A_l')X_{t-l} + \epsilon_t + S_{t-1}\epsilon_t' \\ & \qquad \epsilon_t \sim \mathrm{MVN}(0, \Sigma) \qquad \epsilon_t' \sim \mathrm{MVN}(0, \Sigma') \qquad \mathrm{Cov}(\epsilon_t, \epsilon_t') = 0 \\ X_t &= \begin{bmatrix} \ln \frac{c_t}{c_{t-1}} \\ \ln \frac{\pi_t}{\pi_{t-1}} \end{bmatrix}, \end{aligned} \tag{12}$$

where S_{t-1} takes on the values 0 or 1 and indexes the regime. Extensions to multiple regimes are straightforward. The evolution of the sequence of random variables $\{S_t\}$ is governed by the transition probabilities

$$\Pr[S_t = 0,\ 1|\Phi_{t-1}] = f(S_{t-1}, \ldots, S_{t-q}, X_{t-1}, \ldots, X_{t-q}), \tag{13}$$

i.e., the probability of being in a given regime next period depends only on a q-history of the regimes and the underlying state variables. This parameterization is a generalization of the switching model in Hamilton (1989), which is also studied in the context of stock returns in Cecchetti, Lam and Mark (1990). In the Hamilton model, the regime affects only the constant term in (12), and the transition between regimes depends only on a history of past regimes. For simple specifications, with each regime having identical VAR coefficients but different covariance matrices, this mechanism permits heteroscedasticity. More complex specifications can capture such features as changes in means and volatilities and their persistence over time.

3 Asset Pricing

Using the pricing equation (11) as applied to the firm and the debt in (7) and (9), and the laws of motion for inflation and consumption (output) growth, it is theoretically possible to calculate the conditional moments of returns on equity. Section 3.1 illustrates these calculations in closed form for a simple specification. For more complex, multi-regime specifications closed form solutions are no longer available, and Section 3.2 discusses a technique that provides approximate numerical solutions.

3.1 A Simple Example

Assume that the two state processes ($\{c_t/c_{t-1}\} \equiv \{y_t/y_{t-1}\}$ and $\{\pi_t/\pi_{t-1}\}$) take the following form:

$$\begin{aligned} \ln(c_{t+1}/c_t) &= a_0 + \epsilon_{1,t} \\ \ln(\pi_{t+1}/\pi_t) &= b_0 + \epsilon_{2,t} \\ \epsilon_1 \sim N(0,\sigma_1^2) \qquad & \epsilon_2 \sim N(0,\sigma_2^2) \qquad \mathrm{Cov}(\epsilon_{1,s},\epsilon_{2,t}) = 0 \;\; \forall s,t\,. \end{aligned}$$

In other words, both log consumption (output) growth and log inflation are iid processes, and the two processes are independent. In addition, assume that the debt is nominally riskless. For convenience, we define $Q_t(\tau)$ to be the price at time t of a riskless, τ-period, pure discount bond that pays one unit of the consumption good, i.e.,

$$Q_t(\tau) = \beta^\tau \mathrm{E}_t\left[\left(\frac{c_{t+\tau}}{c_t}\right)^{-\alpha}\left(\frac{\pi_t}{\pi_{t+\tau}}\right)\right] = \beta^\tau \exp[\tau(-\alpha a_0 - b_0 + 1/2\alpha^2\sigma_1^2 + 1/2\sigma_2^2)]. \tag{14}$$

The debt of the firm is riskless, so

$$D_t = MQ_t(\tau) = M\beta^\tau \exp[\tau(-\alpha a_0 - b_0 + 1/2\alpha^2\sigma_1^2 + 1/2\sigma_2^2)], \tag{15}$$

where M is the face amount of the bond. This bond price depends only on the face amount, the maturity, and the parameters of the model, so interest rates are constant and bond returns are non-stochastic.[6] More formally

$$\begin{aligned} \mathrm{E}_t[1+R_D(t,t+1)] &= 1+R_D(t,t+1) = \frac{1}{\beta}\exp\left(\alpha a_0 + b_0 - \frac{\alpha^2\sigma_1^2}{2} - \frac{\sigma_2^2}{2}\right) \\ &= \frac{1}{Q(1)} = 1 + r_f\ , \qquad (16) \\ \mathrm{Var}_t[R_D(t,t+1)] &= 0\ . \qquad (17) \end{aligned}$$

There is no risk to holding long-term debt, so its expected return is equal to the risk-free rate.

From (7), the nominal value of the firm is

$$F_t = \pi_t \sum_{s=1}^{\infty} \beta^s \mathrm{E}_t \left[\left(\frac{c_{t+s}}{c_t}\right)^{-\alpha} (y_{t+s}) \right]. \qquad (18)$$

This economy is stationary in growth rather than stationary in levels. Consequently, the value of the firm is non-stationary, and the variable needed for computing returns is the value–output ratio defined as $F_t/\pi_t y_t$. Recall that y_t is real output, and therefore the denominator is the nominal output of the firm. Note also that this nominal value-output ratio equals the real value-output ratio because F_t/π_t is the real value of the firm. This ratio reduces to[7]

$$\begin{aligned} \frac{F_t}{\pi_t y_t} &= \sum_{s=1}^{\infty} \beta^s \mathrm{E}_t \left[\left(\frac{c_{t+s}}{c_t}\right)^{1-\alpha} \right] \\ &= \frac{\beta \exp((1-\alpha)a_0 + 1/2(1-\alpha)^2\sigma_1^2)}{1-\beta\exp((1-\alpha)a_0 + 1/2(1-\alpha)^2\sigma_1^2)}, \qquad (19) \end{aligned}$$

which is constant over time.

To calculate the return on the firm, note that (8) can be rewritten as

$$1 + R_F(t,t+1) = \frac{\pi_{t+1}}{\pi_t}\frac{y_{t+1}}{y_t}\frac{1+\frac{F_{t+1}}{\pi_{t+1}y_{t+1}}}{\frac{F_t}{\pi_t y_t}}. \qquad (20)$$

The expectation of this return is

$$\begin{aligned} \mathrm{E}_t[1+R_F(t,t+1)] &= \frac{1}{\beta}\exp(\alpha a_0 + b_0 + \alpha\sigma_1^2 - 1/2\alpha^2\sigma_1^2 + 1/2\sigma_2^2) \\ &= \frac{1}{Q(1)}\exp(\alpha\sigma_1^2 + \sigma_2^2). \qquad (21) \end{aligned}$$

[6] Since the prices of riskless bonds are independent of time we will use the notation $Q(\tau)$ instead of $Q_t(\tau)$.

[7] Under the assumption that the value–output ratio is finite, which requires $\exp((1-\alpha)a_0 + 1/2(1-\alpha)^2\sigma_1^2) < 1/\beta$.

The variance of the return on the firm is

$$\mathrm{Var}_t[1 + R_F(t,t+1)] = (\mathrm{E}_t[1 + R_F(t,t+1)])^2(\exp(\sigma_1^2 + \sigma_2^2) - 1). \tag{22}$$

The expected return on the firm exceeds the risk-free rate because the return on the firm is negatively correlated with the NMRS. Although the firm value–output ratio is constant, nominal output and returns are higher when real consumption and inflation are high.

The conditional covariance between the return on the firm and the return on the debt is zero because the return on the debt does not vary at all. Substituting all these results into (5)–(6) yields

$$\mathrm{E}_t[R_S(t,t+1)] - r_f = \frac{1}{Q(1)}(1 + \mathrm{LR}_t)(\exp(\alpha\sigma_1^2 + \sigma_2^2) - 1) \tag{23}$$

$$\mathrm{Var}_t[R_S(t,t+1)] = (1 + \mathrm{LR}_t)^2(\mathrm{E}_t[1 + R_F(t,t+1)])^2(\exp(\sigma_1^2 + \sigma_2^2) - 1). \tag{24}$$

It is tempting to note the positive relation between the expected excess return on the stock and the volatility of this return. Consider, however, that the only stochastic element in both conditional moments is the leverage ratio. They reduce to

$$\mathrm{E}_t[R_S(t,t+1)] - r_f = K_1(1 + \mathrm{LR}_t) \tag{25}$$

$$\mathrm{Var}_t[R_S(t,t+1)] = K_2(1 + \mathrm{LR}_t)^2, \tag{26}$$

where K_1 and K_2 are suitably defined functions of the model's parameters. The expectation and volatility will clearly move together, but only as the maturity or face amount of the debt, or the value of the firm, changes. Interestingly, these effects might induce business cycle fluctuations in the moments of returns. Assuming that over the long run firms adjust debt to maintain a relatively stable leverage ratio, but that adjustments are slow, then persistent consumption (output) growth leads to an increase in firm value, declining leverage, and a lower expectation and volatility of returns. Of course, it makes little sense to talk of expansions or recessions in this iid world, and hence we expand the discussion to more complex models in Section 4. First, however, we turn to the numerical techniques for pricing assets in these complex economies.

3.2 The Discrete State Space Methodology

Consider an n-vector of variables x_t that describes the state of the world at time t and that follows a stationary vector autoregression (VAR) with a single lag:

$$x_{t+1} = A + Bx_t + \epsilon_{t+1}. \tag{27}$$

The assumption of one lag is made for the convenience of exposition only; longer lags can be handled in the same fashion by simply augmenting the vector of state variables. x_t can be approximated by an n-vector of variables $\widehat{x}_t$ each of which takes on only m discrete values. In this new economy there are m^n possible states of the world. The evolution of $\widehat{x}_t$ through time can therefore be described by a $m^n \times m^n$ transition matrix Π whose (i,j) entry is the probability of moving from state i at time t to state j at time $t+1$. The problem, of course, is choosing the discrete values of $\widehat{x}_t$ and the transition probabilities such that $\widehat{x}_t$ best approximates x_t. Tauchen and Hussey (1991) develop such a scheme based on numerical quadrature methods. They choose the discrete points and transition probabilities such that the discretization matches the moments of $\widehat{x}_t$ with those of x_t. They also present an extensive discussion of the theoretical convergence of $\widehat{x}_t$ and functions of $\widehat{x}_t$ to their continuous state space analogs as the number of quadrature points goes to infinity.

Given $\widehat{x}_t$ and the transition matrix Π, the solutions to certain expectation equations become relatively easy to calculate. Assume, for example, that log consumption (output) growth ($\ln[c_{t+1}/c_t]$) and log inflation ($\ln[\pi_{t+1}/\pi_t]$) follow a VAR(1). Assume further that each will be approximated by an m point discretization (yielding m^2 states of the world). Let $\widehat{l}$ and $\widehat{y}$ denote the m^2 by 1 vectors which contain the values that the real marginal rate of substitution and output growth take on in each of the states.[8] Recall that the value of the firm is

$$F_t \;=\; \beta \mathrm{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\alpha} \left(\frac{\pi_t}{\pi_{t+1}} \right) (F_{t+1} + y_{t+1}\pi_t) \right] , \tag{28}$$

or in terms of the value-output ratio

$$\frac{F_t}{\pi_t y_t} = \beta \mathrm{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\alpha} \left(\frac{y_{t+1}}{y_t} \right) \left(\frac{F_{t+1}}{\pi_{t+1} y_{t+1}} + 1 \right) \right] . \tag{29}$$

In the discrete state space world the expectation is simply a summation, and this equation has the solution

$$\widehat{V} = \left[I_{m^2} - (1_{m^2}(\widehat{l}. * \widehat{y})^T). * \Pi \right]^{-1} \left[(1_{m^2}(\widehat{l}. * \widehat{y})^T). * \Pi \right] , \tag{30}$$

where $\widehat{V}$ denotes the vector of value–output ratios (one entry for each state of the world), I_i is an $i \times i$ identity matrix, 1_i is an i-vector of ones, superscript T denotes transpose, and $.*$ is element-by-element matrix multiplication.

[8] The time subscript is dropped because the state vectors are the same in every period.

The solution method for pricing finitely-lived securities such as bonds is somewhat different. As an example, the price of a τ-period, pure discount, riskless bond paying one unit of the consumption good is

$$Q_t(\tau) = \beta^\tau \mathrm{E}_t \left[\left(\frac{c_{t+\tau}}{c_t} \right)^{-\alpha} \left(\frac{\pi_t}{\pi_{t+\tau}} \right) \right] \quad (31)$$

$$= \beta^\tau \mathrm{E}_t \left[\left(\frac{c_{t+1}}{c_t} \right)^{-\alpha} \cdots \left(\frac{c_{t+\tau}}{c_{t+\tau-1}} \right)^{-\alpha} \left(\frac{\pi_t}{\pi_{t+1}} \right) \cdots \left(\frac{\pi_{t+\tau-1}}{\pi_{t+\tau}} \right) \right] , \quad (32)$$

which has the approximate solution

$$\widehat{Q}(\tau) = \Pi \cdot \widehat{N} .* (...(\Pi \cdot \widehat{N} .* (\Pi \cdot \widehat{N}))...) , \quad (33)$$

where $\widehat{Q}(\tau)$ is an m^2 vector of bond prices, one for each state of the world, $\widehat{N}$ is a vector of the NMRS in each state, and the discounting of the marginal rate of substitution vector (which is time independent) occurs τ times.

In addition to permitting solutions for asset prices in complex economies, the discrete state space pricing technique allows for relative simple computations of the conditional mean, variance, and covariance of and between asset returns. The conditional moments are conditional on the specific state of the world as described by the state variables. One final useful property of the discrete state space pricing methodology is that the stationary distribution of the states can be found as the solution to $\Pi P = P$, where P is the vector of stationary probabilities. P can be calculated by making an initial guess and iterating on it using Π.

Throughout, the discussion has focused on an economy described by a single VAR process on the state variables. The extension to multiple regimes of individual VAR processes is relatively straightforward. Within regime transition probabilities, conditional on remaining within the regime, are derived as before. The probabilities of moving between regimes, conditional on the state, must also be defined. The result is an augmented transition probability matrix and an extended matrix of state variables. Computations then proceed as before.

As an illustration, consider again the problem of estimating the value-output ratio, this time in an economy with two regimes. As before, $\widehat{l}$ and $\widehat{y}$ are m^2 by 1 vectors which contain the values of the RMRS and output growth in each of the m^2 states. Let Π_1 denote the transition probabilities between these states, using the VAR parameters of regime 1 and assuming there is only a single regime. In other words, $(\widehat{l}, \widehat{y}, \Pi_1)$ defines a single-regime model, and the rows of Π_1 sum to one.

Similarly, let $(\widehat{l}, \widehat{y}, \Pi_2)$ define a single-regime model under the VAR parameters of regime 2. The state vectors are identical across regimes, but the transition probabilities depend on the parameters of the two VARs. Assume that the probability of moving to regime 2 next period, conditional on being in regime 1, is state independent and equal to p. Similarly, assume the probability of moving from regime 2 to regime 1 is state independent and equal to q. Construct the augmented matrices $\widehat{l}^*$, $\widehat{y}^*$, and Π^* as follows:

$$\Pi^* = \begin{bmatrix} \Pi_1(1-p) & \Pi_1 p \\ \Pi_2 q & \Pi_2(1-q) \end{bmatrix} \quad \widehat{l}^* = \begin{bmatrix} \widehat{l} \\ \widehat{l} \end{bmatrix} \quad \widehat{y}^* = \begin{bmatrix} \widehat{y} \\ \widehat{y} \end{bmatrix}. \tag{34}$$

There are now $2m^2$ states of the world to reflect the fact that for each value of the RMRS and output growth, the necessary information also includes the current regime. Π^* is a valid transition matrix since its rows sum to one. The augmented matrices are used for pricing in (30) and (33).

4 The Behavior of Equity Returns

We next consider the behavior of the expectation and volatility of equity returns in a number of illustrative economies. The focus is on the effects of changes in the conditional moments of inflation and consumption growth on the behavior of and relation between the moments of stock returns. Throughout we maintain the assumptions that the debt is nominally riskless and that the real output of the representative firm and the real consumption of the representative agent are identical.

As the base case scenario consider a VAR(1) on log inflation and log consumption growth defined as follows:

$$\begin{aligned} X_t &= A_0 + A_1 X_{t-1} + \epsilon_t \qquad \epsilon_t \sim \text{MVN}(0, \Sigma) \\ X_t &= \begin{bmatrix} \ln \frac{\pi_t}{\pi_{t-1}} \\ \ln \frac{c_t}{c_{t-1}} \end{bmatrix} \\ A_0 &= \begin{bmatrix} .0021 \\ .0022 \end{bmatrix} \quad A_1 = \begin{bmatrix} .4627 & 0 \\ 0 & -.2019 \end{bmatrix} \quad \Sigma = \begin{bmatrix} .000007 & 0 \\ 0 & .000035 \end{bmatrix}. \end{aligned} \tag{35}$$

These values are estimates from monthly univariate regressions of per capita, constant dollar consumption and the implicit price deflator on a single lag of these variables for the period 3/59-4/89.[9]

[9]These parameter values are not supposed to provide a perfect fit of the data or to replicate the observed features

For ease of exposition, inflation and consumption growth are initially assumed to be independent. The representative agent's coefficient of relative risk aversion is taken to be 2 ($\alpha = 2$), and the time discount factor is .999 ($\beta = .999$). The initial level of output and the price level are normalized to 1, and the debt consists of a single, 12-period, pure discount bond with face value 190. The face value of the bond is chosen to give a leverage ratio of approximately one.

For ease of exposition, the discrete approximation to the two state variables is limited to two points each. Consequently, we have four possible states of the world which cover all possible combinations of high or low inflation and high or low consumption growth. Because of the coarse nature of the discrete grid, it is unwise to link the results too closely to the underlying continuous state-space VAR. Two discrete points offer only limited control over moments of the state processes, but nevertheless this simple economy does provide insight into the time-variations of equity returns.

In the base case scenario, inflation and consumption (output) growth are independent, hence the firm value-output ratio is independent of inflation. The value of the debt is inversely related to the level of inflation and expected inflation, and therefore the leverage ratio is inversely related to these variables. For these parameters, expected monthly inflation is either .13% or .66%, implying leverage ratios of .941 and .934 for low output growth and .945 and .937 for high output growth (see Table 1).

Table 1: Base Case Scenario: State by State Results

State	Inf	OG	E(Inf)	E(OG)	r_f	LR	$E(R_S - r_f)$	SD(R_S)
1	.126	-.408	.277	.303	.976	.941	.01478	1.3076
2	.658	-.408	.507	.303	1.207	.934	.01476	1.3056
3	.126	.778	.277	.067	.501	.945	.01471	1.3068
4	.658	.778	.507	.067	.935	.937	.01469	1.3048

The state by state values of various variables in the 4 state approximation to the VAR in (35). All values except the leverage ratio are in percent per month. "Inf" and "OG" are inflation and output growth respectively, and $E(\cdot)$ refers to the conditional expectation at time t of the variable from t to $t+1$.

At the same time, high expected inflation raises the risk free rate and the expected excess returns on the debt and the firm via (3). The covariance between the return on the firm and the NMRS is independent of inflation, while the covariance between the return on the debt and

of interest rates and equity returns. Rather, they simply provide a framework within which to qualitatively examine certain interactions.

the NMRS is increasing in inflation but relatively insensitive to it. The covariance is larger for the firm than the debt by a factor of close to 5, but, holding output growth constant, changes in excess returns are smaller than leverage effects. Leverage movements also dominate changes in the variances of returns, as inflation and expected inflation change. Consequently, from (5)-(6), both the expected excess return on the equity and the volatility of this return are negatively related to inflation and expected inflation. See Table 1, state 1 versus state 2 and state 3 versus state 4.

Output growth and expected output growth are inversely related because of the negative coefficient on lagged output growth in the VAR. High output growth and low expected output growth imply low riskless rates and a high value-output ratio for the firm. This latter result holds only for relative risk aversion exceeding 1, because in this case the effect of growth on real discount rates more than offsets the effect on future output. Leverage is higher for high output growth because the debt effect dominates for these parameter values. However, the effect of output growth on expected returns via leverage is dominated by its effect via changes in the riskless rate and the covariance with the NMRS. The riskless rate declines in output growth, and the covariance between the return on the firm and the NMRS increases. The net result is an expected excess equity return that is negatively related to output growth and a leverage ratio that is weakly positively related to the same variable, holding inflation constant. The same relations hold for the volatility of equity returns, but the negative relation is weaker. Compare states 1 and 3 and states 2 and 4 in Table 1.

Two extensions of the base case scenario relax the assumption of zero off-diagonals in the coefficient matrix and the covariance matrix. Consumption and inflation data suggest both that the coefficient on lagged inflation in the consumption growth equation is negative, and that the shocks to these processes are negatively correlated.[10] A negative coefficient of $-.01$ on lagged inflation (all other parameters the same) implies that the firm value-output ratio is positively related to the level of inflation, but the effect of inflation on interest rates is weakened. Although the effect on leverage remains, the link between leverage and the moments of returns is no longer preserved for low output growth. The effect of a negative correlation of -.13 between the shocks to inflation and output growth induces somewhat of the opposite effect. The volatility of interest rates increases while that of the return on the firm declines. The dominance of the inflation induced leverage effect now disappears for high output growth.

[10]See Boudoukh (1993) for an extensive discussion of these empirical issues and the implications of correlations between these processes for the term structure of interest rates.

Of greater interest is the relation between the expectation and volatility of equity returns in this artificial economy. In the stationary discrete state space economy, there are two ways to look at this relation. Given the conditional moments in each state of the world and the long-run probability of each state occurring, it is simple to calculate a long-run, unconditional correlation between the conditional moments. To capture potential changes in the relation over the business cycle, the conditional correlation, conditional on a specific state of the world, is also calculated. At time t, the conditional correlation of the conditional moments from time t to $t+1$ is zero because these moments are, by definition, in the time t information set. Therefore, we consider, at time t, the conditional correlation between the moments from time $t+1$ to $t+2$.[11] There is one conditional correlation for each state of the world, and we can also compute the long-run average of these correlations using the steady state distribution. Note that this average will not equal the long-run correlation above because the unconditional expectation of the conditional correlation will not, in general, equal the unconditional correlation.

For the base case scenario, the conditional correlations are all approximately .57 and the long-run correlation is similar. Nevertheless, as Table 1 indicates, the relation between the moments is not monotonic. A negative correlation between the shocks to the state processes raises all the correlations to above .9. A negative off-diagonal VAR coefficient on lagged inflation also raises the general level of the contemporaneous correlations, but, in addition, it introduces more dispersion. The conditional correlations range from .58 to .83, and the long-run contemporaneous correlation is .74.

All three of the scenarios above have little or no heteroscedasticity in the underlying processes. Our final exercise is to look at two economies in which inflation or output growth are conditionally heteroscedastic. For this analysis, we employ a two-regime structure with eight states of the world. Each four state regime is an approximation to a VAR as before, and the VARs across regimes differ only in their covariance matrices. The probability of switching from one regime to the other is .05, independent of the state of the world.

In the inflation volatility scenario, the first regime is exactly like the base case, while the second regime has inflation variance equal to .00001. Within regimes, the patterns of returns look very similar to the base case, although the increase in volatility affects both the expectation and volatility

[11] Later in the paper we consider correlations other than the contemporaneous correlation, and in the artificial economy they are calculated in a similar manner.

of equity returns. Comparisons across regimes, between low inflation, low output growth states, for example, show the moments of returns moving in opposite directions. As a result, the unconditional correlation is actually negative (-.04), although the conditional correlations are all positive, ranging from .10 to .15. The difference between these results arises because regimes switches are relatively unlikely so conditional correlations put a lot of weight on within regime states. In contrast, the long-run correlation weights both regimes equally.

The output growth volatility scenario is similar except that the second regime has an output growth variance of .00005. This change increases the conditional correlations to almost .9 and the long-run correlation to .97. In this scenario, the conditional moments of equity returns are strongly positively related across regimes.

The divergence of results between the two volatility scenarios is somewhat startling, but it indicates the sensitivity of the results to assumptions about the distributions of the underlying processes. The sensitivity of the model, while disconcerting, has two positive aspects. First, the wide range of results highlights the varied effects of the complex interactions in a dynamic, consumption-based setting. Since the prior empirical results contrast with the implications of the dynamic CAPM (Merton (1980)), it is comforting that the consumption-based model exhibits the potential to accommodate these stylized facts. Second, the sensitivity of the model may permit a better identification of the important factors necessary to generate asymmetry in the conditional moments of equity returns. In particular, these results emphasize the potential importance of changes in inflation volatility in explaining the weak or negative relation between the expectation and volatility of returns. Linking variations in inflation volatility to other variables, in an extension of the independent fluctuations in the example above, provides one avenue for inducing even more complex interactions. This approach is pursued in the following section.

5 Calibration of the Model

The exercise of calibrating the model to the empirical results is a complex task. Restricting the specification to two regimes still leaves 18 VAR parameters, the time discount factor, the level of risk aversion, the parameters governing regime switching, and the parameters determining firm capital structure to be specified. The other dimension of the problem is choosing the features of the data to be matched.

One approach is to estimate the model using the discrete state space methodology and the generalized method of moments (Hansen (1982)). For example, we could attempt to estimate the parameters using various moments of equity returns. The idea is to select the parameters that minimize the distance between the moments in the data and the moments in the model. Although conceptually simple, this procedure is both complex to implement and extremely time consuming to run because of the dimension of the parameter space. Equally important, the goal of this exercise is twofold: (1) to see if asymmetries in the data are consistent with reasonable parameterizations of a rational expectations model, and (2) to gain further economic intuition for the results. For example, we are not interested in the specific implied time series properties of output or in the estimated level of relative risk aversion. Consequently, the strategy pursued is to fix a large number of parameters at intuitively reasonable values and to search over other parameters in order to obtain results that are suggestive of those in the data. A single parameter set that generates interesting results is discussed in the following sections. The focus is on the correlations between the moments of equity returns and the implied parameters of VARs estimated using these moments.

5.1 Setup

Consider an economy with two regimes which, loosely speaking, represent the expansionary and contractionary phases of the business cycle. Regime 1, the expansionary phase, has parameters

$$A_0 = \begin{bmatrix} .0010 \\ .0022 \end{bmatrix} \quad A_1 = \begin{bmatrix} .8 & 0 \\ 0 & .1 \end{bmatrix} \quad \Sigma = \begin{bmatrix} .000010 & 0 \\ 0 & .000010 \end{bmatrix}, \tag{36}$$

for the VAR defined on $X_t = [\ln \frac{c_t}{c_{t-1}} \quad \ln \frac{\pi_t}{\pi_{t-1}}]'$. Periods approximate months, and the long-run mean of the process is $[.0050 \quad .0024]'$. Regime 2, the contractionary phase, has parameters

$$A_0 = \begin{bmatrix} .0011 \\ .0010 \end{bmatrix} \quad A_1 = \begin{bmatrix} .5 & 0 \\ 0 & .1 \end{bmatrix} \quad \Sigma = \begin{bmatrix} .000014 & 0 \\ 0 & .000011 \end{bmatrix}, \tag{37}$$

with long run mean $[.0022 \quad .0011]'$. In addition to having a lower mean, inflation in regime 2 reverts more quickly to its mean, and both processes have a higher conditional variance. The rate of time discount and relative risk aversion of the representative agent are .999 and 2 respectively. The initial price level and level of real consumption are normalized to 1. The firm has one issue of 12-month, pure discount debt outstanding with a nominal face amount of 190. The face amount is set in order to generate a leverage ratio of approximately 1. Finally, within each regime, both processes are approximated by five discrete states giving 25 states for each regime and 50 states in total.

Transitions between the regimes are governed by the level of inflation. In regime 1, the probability of switching to regime 2 is zero if inflation is below its long-run mean. If inflation exceeds its long-run mean, then the probability of a regime switch is

$$P_{1,2} = \frac{.05}{\sigma_{I1}}(\ln \frac{\pi_t}{\pi_{t-1}} - \mu_{I1}) \ , \tag{38}$$

where μ_{I1} and σ_{I1} are the long-run mean and standard deviation of log price level growth in regime 1 respectively. In regime 2 the reverse holds. For inflation above its long-run mean, the probability of a switch is zero. For inflation below its long-run mean, the probability of a switch is

$$P_{2,1} = \frac{.05}{\sigma_{I2}}(\mu_{I2} - \ln \frac{\pi_t}{\pi_{t-1}}) \ , \tag{39}$$

where μ_{I2} and σ_{I2} are defined analogously to μ_{I1} and σ_{I1} above.

This formulation attempts to capture a number of stylized facts from the data. First, inflation and consumption growth both tend to be higher during expansions than contractions. Second, expansions tend to last longer than contractions. This feature of the model relies on the relatively slow mean reversion of inflation in regime 1. Starting at a low level, inflation will increase relatively slowly and therefore trigger a regime switch later on average. Third, inflation is heteroscedastic, and the data suggest that the volatility of inflation is positively related to the level of inflation. This relation is captured partially by the fact that the processes are defined on a log basis so that on a levels basis the conditional volatility increases in the level of the state variable. Heteroscedasticity is also generated by the changing conditional volatility of inflation across regimes. Finally, inflation tends to be high at the end of expansions and low at the end of contractions. This feature is inherent in the regime switching mechanism which relies on the level of inflation relative to its long-run, within-regime mean.

In addition to mimicking certain aspects of the data, the two-regime model is also intended to be suggestive of the potentially important role of nominal interest rates and monetary policy. The empirical results in Whitelaw (1993) are consistent with the explanation that the commercial paper-treasury spread proxies for the stance of monetary policy and that tight monetary policy has an adverse effect on the real economy, perhaps through increases in external borrowing and resulting increases in agency costs. The timing of increases and decreases in the spread suggest further that monetary policy responds to the level of inflation.

5.2 Correlation Results

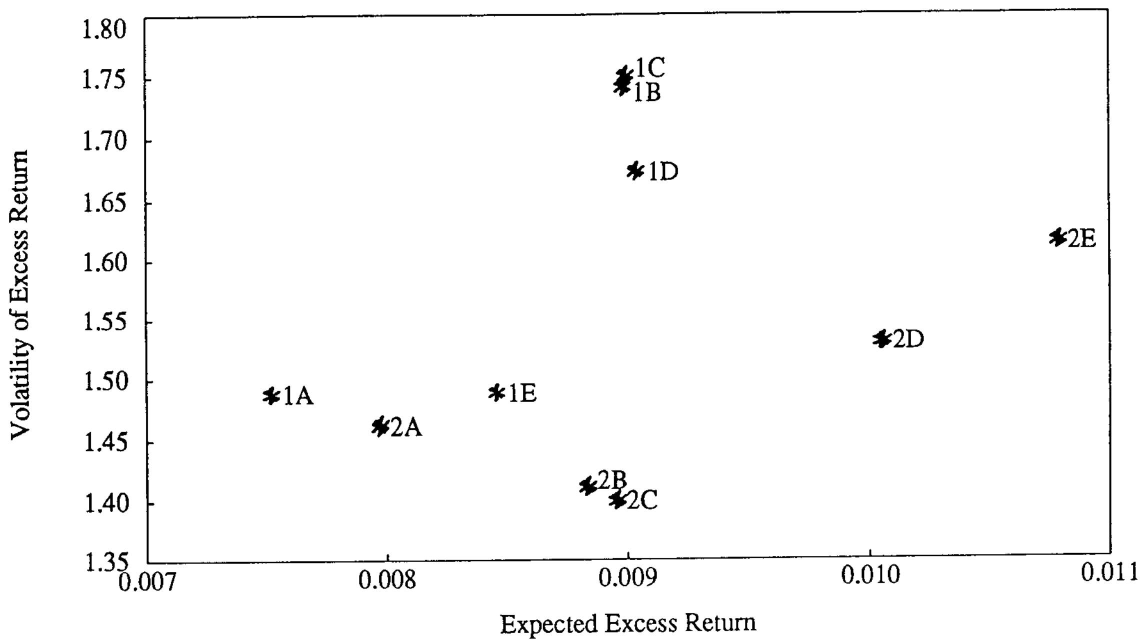
As stated previously, the artificial economy consists of 50 discrete states. In every state all nominal riskless interest rates are positive, and the 1-year (12-period) rate ranges from approximately 6.5% to approximately 12.5%. Given the fixed face amount of nominal debt outstanding, the value of the debt fluctuates similarly. The value of the firm varies from a minimum of about 356 to a maximum of about 368 (for output of 1), but these extremes do not coincide with the extremes in interest rates. As a result, variations in the firm's leverage ratio follow neither variable exactly, and the ratio varies from 0.86 to 0.95 (the firm is 46%-49% debt). All of these values are reasonable from an empirical perspective.

The variables of most interest are the state-by-state values of the conditional expectation and volatility of equity returns. We focus on the 1-month (1-period) values in this analysis. Figure 1 shows a scatter plot of the conditional moments of excess equity returns in which each point represents a mean/volatility pair for a specific state of the world. Points labeled 1x correspond to states in regime 1 (the expansionary phase), and points labeled 2x correspond to states in regime 2 (the contractionary phase).

Note that the points are grouped in clusters of five, hence the appearance of only 10 distinct pairs. These clusters consist of states with identical levels of inflation but different levels of consumption growth. They are labeled xA-xE, where the points are in ascending order, based on the level of inflation. For example, 1A is the group of states in regime 1 with the lowest level of inflation, and 2E is the group of states in regime 2 with the highest level of inflation. The level of consumption growth has relatively little effect on the moments of returns because the process is close to iid given the VAR coefficient of 0.1. Consequently, the variations in returns can be attributed almost solely to fluctuations in the inflation process.

Perhaps the most noticeable feature of Figure 1 is that the points seem to be scattered in a random fashion on the graph. Even though the stationary probability of each point differs, it is clear that there is no strong consistent relation between the expectation and volatility of excess equity returns. This does not imply that there are not times (or states of the world) when a strong

Figure 1: Expected Returns versus Volatility of Returns



A scatter plot of state-by-state expected excess returns versus the volatility of excess returns from the artificial economy.

positive relation between the moments exists. Consider the states of the world in which there is a zero probability of a regime shift in the next period—A, B, and C of regime 1 and C, D, and E of regime 2. The conditional moments are calculated over a single period, and therefore, the moments in those states can be considered within-regime moments. Note that an upward sloping line comes close to connecting the three points within each of these two sets. The same is true of the two points 1D-1E, but 2A-2B exhibit the opposite relation.

These relations are illustrated well by the conditional contemporaneous correlations between the expectation and volatility of returns calculated for each state. Recall from Section 4 that the contemporaneous correlation, conditional on being in state i at time t, is the correlation between the moments defined over the period $t+1$ to $t+2$. Table 2 presents these correlations for the clusters of points in Figure 1. As with the conditional mean and volatility, the conditional correlation varies little over states with the same level of inflation but different levels of consumption growth.

The states in regime 2 are listed in descending order of the level of inflation to emphasize the point that it is not the level of inflation, *per se*, that influences the correlation, but how this variable

Table 2: Conditional Contemporaneous Correlations

State	Correlation	State	Correlation
1A	1.00	2E	.99
1B	.99	2D	.95
1C	.61	2C	.66
1D	.40	2B	-.07
1E	.45	2A	-.21

State-by-state conditional contemporaneous correlations between the expectation and volatility of excess returns from the artificial economy.

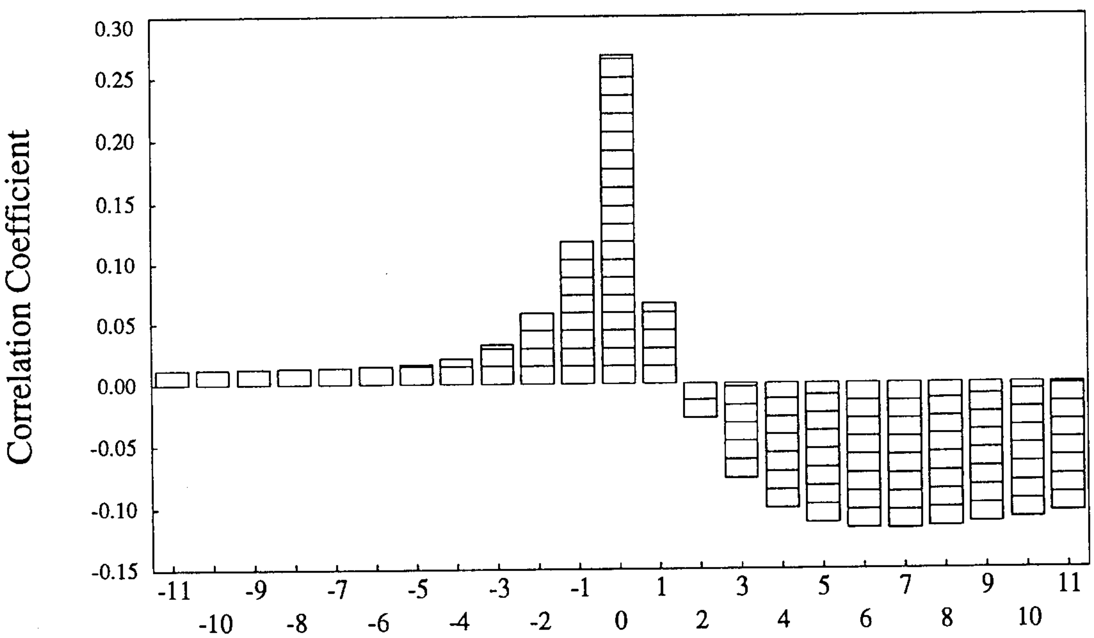
affects the probability of a regime switch. The table also illustrates that the contemporaneous correlation fluctuates over the business cycle. It is low or negative when regime switches are likely, either from expansion to contraction or from contraction to expansion. After a switch, at the beginning of a new regime, the correlation is strongly positive. These fluctuations coincide with the empirical evidence.

A second way of looking at the behavior of returns in the artificial economy is via the unconditional correlations between the lagged and lead values of the conditional moments. Figure 2 shows these correlations for offsets of up to 11 months. The bar above the number 0 on the x-axis represents the contemporaneous correlation, the bar above the number 11 represents the correlation between the expected return at time t and the volatility at time $t + 11$, and the bar above the number -11 represents the correlation between the expected return at time t and the volatility at time $t - 11$. Again the conditional moments are defined over a single period.

The unconditional contemporaneous correlation is positive, and, in fact, the correlation peaks at an offset of 0. This result contrasts with the data in which the correlation peaks at a lag of 18 months. Nevertheless, the marked asymmetry of the correlations in the data is matched by the results from the artificial economy.

Table 3 reports results for the estimation of a VAR(1) on the expectation and volatility of returns. As before, the coefficients are calculated based on the partial correlations, and the dependent variables and lagged dependent variables are the actual 1-period conditional moments. The diagonals in the coefficient matrix are both positive as in the actual data. The lagged first moment appears with a large negative coefficient in the second moment equation, while the coefficient on the lagged second moment in the first moment equation is over five orders of magnitude smaller.

Figure 2: Unconditional Correlations Between the Moments of Returns



Unconditional correlations between the conditional expectation and lagged (negative numbers on the x-axis) and led (positive numbers on the x-axis) values of the conditional volatility of returns from the artificial economy.

This difference can be partially attributed to the fact that the volatility is approximately 200 times larger than the expected excess return. The artificial economy exhibits the equity premium puzzle, as one might expect given the inability of the representative agent framework to justify the magnitude of the equity premium in other contexts. Adjusting for this phenomenon, the lagged first moment is still important for predicting the second moment, while the coefficient on lagged volatility in the mean equation is essentially zero. The former result is consistent with the previous empirical results, but the latter result is not.

Table 3: VAR(1) Estimation of the Conditional Expectation and Volatility of Returns

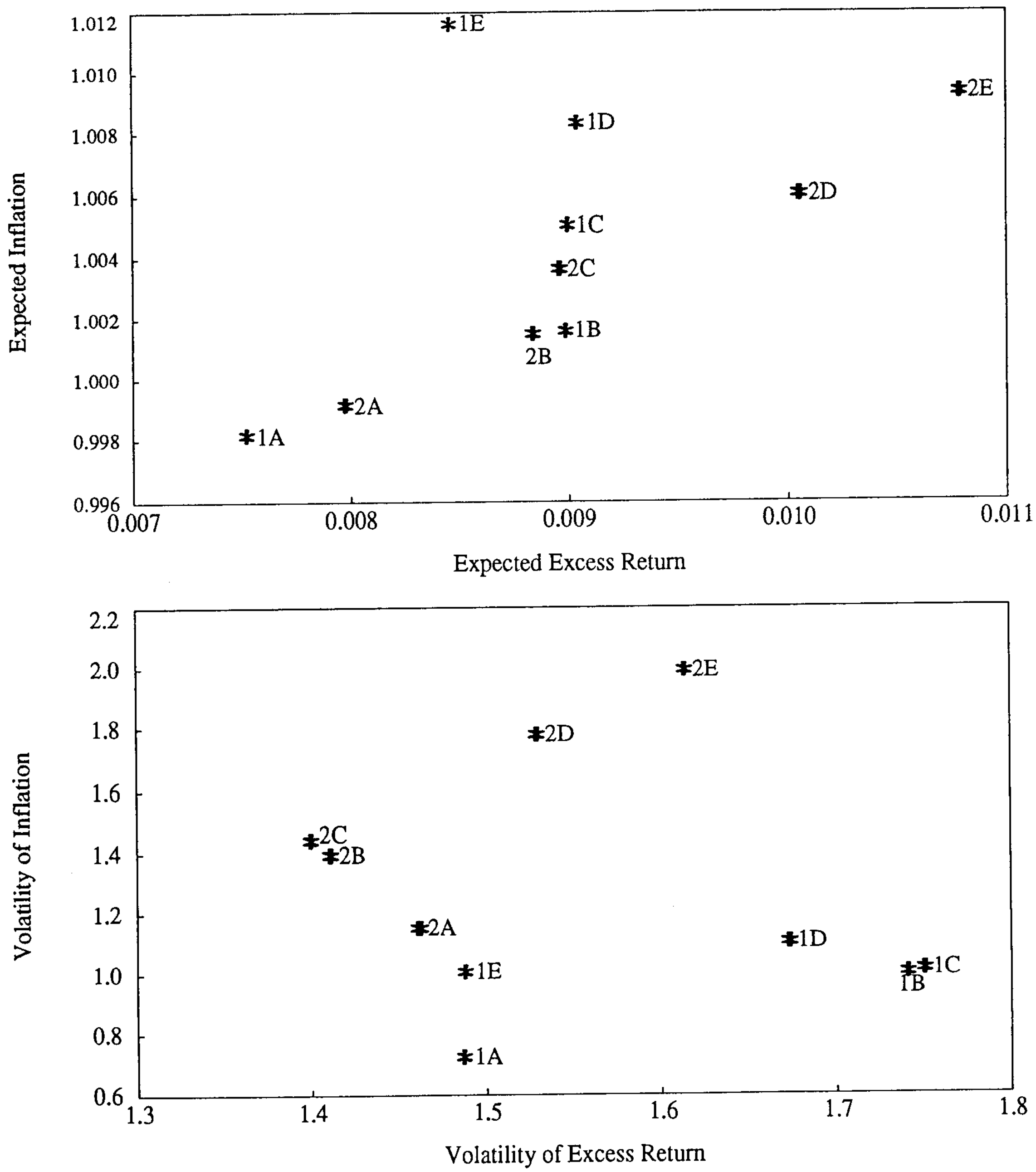
	Explanatory Variables		
	Constant	$m_{t-2,t-1}$	$v_{t-2,t-1}$
$m_{t-1,t}$	.0044	.5097	-.0001
$v_{t-1,t}$	.5651	-42.9119	.8801

A VAR(1) estimation of the expectation and volatility of excess equity returns from the artificial economy.

A final issue that is easily addressed in the artificial economy is the relation between the moments of the underlying processes—inflation and consumption growth—the leverage ratio and the moments of returns. As stated previously, consumption growth plays a limited role for these parameter values, so we will concentrate on inflation. Figure 3 displays scatter plots of the first and second conditional moments of inflation against the corresponding contemporaneous moments of excess equity returns. The same numbering scheme used in Figure 1 is employed in these graphs.

The plots indicate that there is no simple linear or even monotonic relation between the moments of returns and the moments of inflation. Seven or eight of the points (actually 35 to 40 out of the 50 states of the world given the clustering) in the first moment graph lie relatively close to an upward sloping line, but again the possibility of a regime shift pushes points 1D and 1E away from this line. In the second moment graph, there is apparently no evidence of a stable relation. Although not presented here, graphs of the leverage ratio or the moments of consumption growth against the moments of returns exhibit the same, apparently random, scattering of points. The clear conclusion is that even in the two-regime artificial economy the relations between equity returns and the underlying economy are non-linear and complex.

Figure 3: Moments of Inflation versus Moments of Equity Returns



State-by-state scatter plots based on data from the artificial economy. Top: conditional expected inflation versus conditional expected excess equity return. Bottom: conditional volatility of inflation versus conditional volatility of equity return.

6 Conclusion

It appears that an artificial, consumption-based economy can generate many of the features seen in the data. In particular, the model can support conditional correlations that vary from positive to negative depending on the state of the world, unconditional correlations between lagged and led values of the mean and volatility that exhibit a marked asymmetry, and VAR(1) estimates on the conditional moments in which the lagged conditional mean enters with a negative coefficient in the volatility equation. The features of the model which drive these results include the assumed asymmetry of the processes for inflation and consumption growth across the expansionary and contractionary phases of the cycle, the heteroscedasticity of inflation, and the link between the level of inflation and regime switches.

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