NEW YORK UNIVERSITY
STERN SCHOOL OF BUSINESS
FINANCE DEPARTMENT

Working Paper Series, 1994

On the Dynamics and Information Content of Implied Volatility: A Bivariate Time Series Perspective

Bent Jesper Christensen and N.R. Prabhala

FD-94-25

On the Dynamics and Information Content of Implied Volatility:

A Bivariate Time Series Perspective*

Bent Jesper Christensen

Stern School of Business

New York University

New York, NY 10012

N. R. Prabhala

School of Management

Yale University

New Haven, CT 06520

November 1994

Key words: Options; Volatility; Information; Market efficiency

JEL classification: G13; G14

* We thank Interactive Data Corporation for providing the option price data used in the study. Thanks are also due to Stephen Figlewski and Young-Ho Eom for their comments and help, to participants at the 1994 European Finance Association meetings and the discussant Nikunj Kapadia, and to Fallaw Sowell for providing us with software for estimation of the fractional integration models.

Abstract

A new research design is introduced for the empirical analysis of the relationship between implied volatility and *ex-post* realized volatility. The dynamics of volatility are emphasized, and the analysis is cast in terms of non-overlapping data, so that exactly one implied and one realized volatility estimate pertain to each period under consideration. The conclusions from the empirical analysis when using our design are significantly different from those previously reached. Recent literature indicates that implied volatility contains little information about future volatility, beyond that contained in the history of realized volatility. We show that on the contrary, implied volatility efficiently predicts future realized volatility and in particular subsumes the information content of past realized volatility.

1. Introduction

The volatility implied in option prices ('implied volatility') is widely regarded as the option market's forecast of future return volatility, over the life of the relevant option. Viewed thus, implied volatility is a *conditional* volatility forecast, based on information available to option market participants. If option markets are efficient, implied volatility should in fact be an *efficient* forecast of future volatility and should reflect all relevant information available to option markets.[1] A natural question to ask in this context is: how well does implied volatility predict *ex-post* realized volatility? If implied volatility is indeed an efficient predictor of future volatility, it should subsume the information content of all variables in the market's *ex-ante* information set, in explaining *ex-post* realized volatility. In particular, implied volatility ought to subsume the information contained in the entire history of past realized volatility.

Intriguingly, this proposition has found little empirical support in previous research. While implied volatility in isolation does appear to explain future volatility, recent research suggests that its explanatory power is considerably diminished once elements from the history of past volatility are added as explanatory variables [Day and Lewis (1992), Canina and Figlewski (1993), and Lamoureux and Lastrapes (1993)]. By no means has implied volatility been found to subsume the information in past realized volatility; on the contrary, past realized volatility often has as much or greater power than implied volatility in explaining *ex-post* realized volatility. Thus, Day and Lewis (1992) are unable '*... to make strong statements*' about the relative information content of implied volatility. A stronger expression of this view is contained in Canina and Figlewski (1993)—henceforth CF—who conclude that '*... implied volatility has virtually no correlation with future volatility, and it does not incorporate information contained in recent observed volatility.*'

[1] Implied volatility has been interpreted as such in a wide variety of settings [e.g. Day and Lewis (1988), Harvey and Whaley (1992), Latane and Rendleman (1976), Poterba and Summers (1986), and Schmalensee and Trippi (1978)].

The result is striking, in that it is obtained in the context of the most actively traded options—the S&P 100 (OEX) index options. If implied volatility says little about future volatility even in this market, option prices are probably not set according to participants' volatility forecasts, contrary to what conventional option pricing theory would suggest. CF attribute this apparent failure of conventional pricing models to a failure of the underlying frictionless markets assumption. They argue that conventional theory only places very broad bounds on option prices, and that within these bounds, institutional and demand-supply factors determine traded prices. Other researchers [e.g. Lamoureux and Lastrapes (1993)] suggest that such results simply reflect the need for more general option pricing models, such as those accomodating priced stochastic volatility, in order to explain observed option prices.

In this paper, we introduce a new research design for examining this issue, and demonstrate that it leads to significantly different economic conclusions. Broadly speaking, our design differs in three ways from that used in the existing literature. First, we sample implied and *ex-post* volatility at a *lower* frequency than that of previous studies. This serves two purposes. One, it makes the resulting structure consistent with the pricing model used to compute implied volatility, and mitigates any contradiction inherent in viewing implied volatility as the option market's volatility forecast. Two, our design reduces the noise contained in the time series movement of implied volatility and thus improves the power to detect fundamental changes in expected volatility.

The second difference in our analysis is that we account for the *endogeneity* of implied volatility. Previous studies have regarded implied volatility as a variable exogenously determined by option markets. The problem with this approach is that implied volatility is not merely a volatility forecast, but is in fact a full measure of the *price* of an option. Indeed, it is market practice to quote option prices in terms of 'implied volatilities.' Viewed thus as an option's price, implied volatility quite plausibly depends on past levels of both implied and realized volatility. In this case, implied volatility is an endogenous function of the volatility history, not an exogenous variable as previously assumed, and our treatment takes this potential simultaneity into account.

The third difference is that our analysis explicitly accounts for—and sheds light on—the nature of the dynamic properties of volatility. In particular, we account for the concern [e.g. French, Schwert and Stambaugh (1987), Schwert (1990)] that the time series of volatility might be non-stationary.[2]

Briefly, we conduct the analysis in two stages. First, we characterize the individual time series properties of implied volatility[3] of at-the-money OEX call options and of the realized volatility[4] of the underlying index, separately. We document that the two volatility series exhibit similar dynamics, and that they possess some form of long-term memory. Formal statistical tests in the time and frequency domains suggest that both series are non-stationary or at least near non-stationary. Their time series behavior appears to be well-described by the class of persistent 'fractionally integrated' processes. This analysis contributes to and complements (i) the time series analysis of monthly *stock-return* volatility reported in French, Schwert and Stambaugh (1987); (ii) the observation that changes in *implied* volatility are predictable [see Harvey and Whaley (1992)]; and (iii) a developing literature which suggests that stock returns follow fractionally integrated GARCH processes [see Baillie, Bollerslev and Mikkelson (1993)].

In the second stage analysis, we investigate the *relationship* between *ex-ante* implied and *ex-post* realized volatility. We consider two different econometric approaches to the analysis of the joint time series behavior of the two volatility series. Under the first

---

[2] Thus, we mitigate the concern that the explanatory power of implied volatility reported in previous literature may in fact be spurious, an artifact of the non-stationarity of the implied and/or realized volatility series.

[3] In this and other papers in the literature, implied volatility is computed using the Black and Scholes (1973) model.

[4] We use the terms *actual*, *realized*, *ex-post* and *historical* interchangeably in reference to volatility.

approach, we examine the ability of implied volatility to explain future realized volatility using conventional single-equation models, as well as bivariate system specifications that account for the potential simultaneity in the two volatility series. In the second approach, we examine the time series properties of the *residuals* from the regression of realized volatility on implied volatility. Specifically, we test for whether the residuals display any predictability. Under the null that implied volatility aggregates volatility information efficiently, the residuals should exhibit no predictability. This procedure, while similar in spirit to 'cointegration' tests used to assess whether non-stationary (unit root) processes move together over time, is stronger: here, we examine not only whether the regression residuals are stationary (as in cointegration tests), but also whether they are white noise, in order to rule out any predictability in the residual series.

The empirical results are interesting under either approach, and show that there exists an economically and statistically significant relationship between implied and realized volatility, far stronger than previously documented. In fact, implied volatility *subsumes* the information contained in the history of realized volatility in some of the specifications we consider. These results suggest that at least for the S&P 100 index, option markets aggregate volatility information in an efficient manner; the no-arbitrage model of Black and Scholes (1973) does provide a fair first-order characterization of option prices in this market. Our results are quite different from those reported in previous literature. We attribute the difference, in large part, to the differences in experiment design mentioned before, and we discuss the advantages of our approach in understanding the behavior of options markets.

The analysis proceeds as follows. Section 2 describes and motivates the construction of our data set. Section 3 deals with the individual time series properties of the implied and realized volatility series. Besides being of interest in their own right, the time series properties suggest the relative usefulness of the various econometric tests conducted later. Section 4 describes the tests conducted to analyze the relationship between implied and realized volatility, and the results are discussed. Section 5 concludes.

2. Data and Sampling Issues

Our empirical analysis is cast in the context of S&P 100 index ('OEX') options. OEX options began trading on the CBOE in February 1983, initially with quarterly expiration cycles: March, June, September, and December. Exchange traded options with 1-month expiration cycles became available in November 1983, and our two (monthly) data series begin at that time. The data series end in May 1993 and cover a timespan of nine and a half years, or 115 months.

In the remainder of this section, we motivate the sampling scheme employed here, formally describe the data series and then discuss associated measurement error issues.

2.1 Motivating The Sampling Scheme

As in all previous studies in this literature, we use the Black-Scholes (henceforth BS) model to estimate the beginning-of-period implied volatility from call option price data. The fundamental research question is whether implied volatility predicts future volatility, and whether it does so efficiently. To examine this issue empirically, the research design should ideally be such that there is no contradiction inherent in viewing implied volatility as option markets' *ex-ante* volatility forecast over the period in question.

In this context, observe that the BS option pricing model assumes that volatility is either (a) constant; or (b) non-stochastic over an option's life. If this assumption and the standard 'perfect market' assumptions underlying the BS model are correct, implied volatility is indeed a bona-fide estimate of the average return volatility over the relevant option's life. However, if volatility is stochastic, with random shifts over the life of the option, then the pricing formula is different, and the BS implied volatility can no longer be regarded as markets' volatility forecast.[5]

[5] BS implied volatility may be viewed as a measure of an option's *price*—one that controls for the time to expiration of an option, the extent to which it is in or out of the money etc., as discussed in section 3.

Thus, it would appear to be internally inconsistent to use a research design that (i) employs the BS implied volatility as an *ex-ante* volatility forecast; and then (ii) allows volatility to vary stochastically over an option's life. Such inconsistency is characteristic of all previous studies in this literature. On one hand, the studies compute and interpret the BS implied volatility of an option as the conditional expectation of volatility. On the other hand, option prices (i.e. the BS implied volatility) and *ex-post* volatility are sampled several times over an option's life. In effect, this allows for stochastic volatility and potentially leads to the inconsistency noted above.

In this study, we mitigate this inconsistency through the use of non-overlapping data. Our sample is constructed by sampling one call option per month. The implied volatility of the sampled option, computed using the BS formula, serves as our observation of implied volatility for that month. *Ex-post* or realized volatility is computed using daily return data for the month under consideration.

The next option in the time series is sampled only *after* the expiration of the previous option; we do not resample volatility prior to expiration. Over any given option's life, we have exactly one pair of implied and realized volatility estimates. Thereby, we keep the empirical work consistent with the BS assumption that volatility is non-stochastic over an option's life.

We now elaborate on the sampling scheme motivated above and detail the construction of our data series.

2.2 Variable Definitions

By convention, OEX options expire on the third Saturday of every month. We move to the Wednesday that immediately follows, and begin by recording the OEX level on this date—S_t, say. On the same date, we locate the 1-month call option that is closest to the money. We record the price C_t of this call, and its exercise or strike price K_t. This option expires on the third Saturday of the following month $t+1$; the next $(t+1)$ call option is sampled on the Wednesday that immediately follows. An entire sequence of monthly option prices is constructed in this manner. The key feature of this sampling procedure is that there is no overlap in the lifespans of successive options.

From each observed call price C_t, implied volatility σ_{it} is computed by solving numerically the implicit equation associated with the BS call price formula, i.e.

$$C_t = S_t\, N(d_t) - K_t e^{-\rho_t \tau_t} N(d_t - \sigma_{it}\sqrt{\tau_t})\ , \tag{1}$$

where $d_t = [\log(S_t/K_t)+(\rho_t+\sigma_{it}^2/2)\tau_t]/\sigma_{it}\sqrt{\tau_t}$, with τ_t denoting the time to expiration and ρ_t the interest rate. For the latter, we use the 1-month LIBOR—the inter-bank borrowing rate in the Eurodollar market—as this is probably close to the rate faced by option traders. The typical implied volatility estimate obtained by solving (1) is based on options with about 25 days to expiration.

While implied volatility represents an *ex-ante* volatility forecast, we also compute the *ex-post* return volatility over each option's life. Following Schwert (1990), this *ex-post* measure or 'realized volatility' σ_{ht} is computed as the sample standard deviation of the daily index returns over the remaining life of the option. That is,

$$\sigma_{ht} = \sqrt{\frac{1}{\tau_t}\sum_{k=1}^{\tau_t}(r_{t,k} - \bar{r}_t)^2}\quad , \tag{2}$$

where τ_t is the number of days to expiration, $\bar{r}_t = \tau_t^{-1}\sum_{k=1}^{\tau_t} r_{t,k}$, and $r_{t,k}$ is the index return on day k in month t.

Both volatility measures are expressed in annual terms, to facilitate interpretation. Finally, the empirical analysis uses the logarithms of the volatility series, denoted as $i_t = \log\sigma_{it}$ and $h_t = \log\sigma_{ht}$. All data were obtained from the financial databases of Interactive Data Corporation (IDC).

2.3 Noise in Data

Any estimate of implied volatility is potentially noisy, due to a number of possible measurement errors. We review some of the sources of noise in the measured series, consider how they might affect our results, and discuss how our research design reduces their impact.

The BS formula (1) applies to a European style call option on an underlying asset that is known in advance to pay no dividend prior to expiration of the option. However, OEX options are American-style, and the underlying asset—the S&P 100 index—pays dividends. Nevertheless, there is some justification for our use of the BS formula, on two grounds. First, European and American calls have identical BS values when there are no dividend payments, so in this sense the American feature of the OEX calls causes no problem in itself. With index dividends, we could of course have a positive premium for the American calls over otherwise identical European calls, but since the OEX is an index of 100 stocks and dividend payments are likely to be small and smooth, the deviation from BS pricing for the at-the-money option is likely to be small.

Additional sources of noise in implied volatility include (i) non-synchronicity (within the day) in the measured option and index prices; (ii) bid-ask spreads in option prices; and (iii) the 'wild-card' option embedded in the OEX option [see Harvey and Whaley (1992)]. All these factors potentially reduce the information content of measured implied volatility. Hence, any relationship detected in this study should be interpreted as a conservative lower bound on the true relationship between implied and realized volatility.

Two major features of our experiment design ensure that this downward bias in estimating the relationship is somewhat mitigated. First, our sampling frequency (monthly) is modest, relative to the daily or weekly frequencies used in previous studies. We argue that this improves the signal-to-noise ratio in the system. For instance, day-to-day changes in true expected volatility are small, on average. Thus, a relatively large portion of the day-to-day variation in *observed* volatility would simply be noise—due to sampling error or to the measurement errors discussed above. Indeed, Harvey and Whaley (1992) have shown that such disturbances have a significant impact when data are sampled at a high frequency. By contrast, on a month-to-month basis, fundamental changes in volatility are larger. Hence, at a monthly level, a larger fraction of the variation in observed volatility may be attributed to fundamental changes in volatility. Thus, the signal-to-noise ratio should improve when moving from daily to monthly sampling. Additionally, monthly sampling has some economic appeal, since it corresponds to the approximate frequency at which most macroeconomic news—which are important sources of market volatility—are released.

A second feature of our sampling scheme is that we do not include any options with short terms to expiration. This obviates the need to deal with the options most sensitive to bid-ask spreads and non-synchronicities—those with small prices in dollar terms—and minimizes the value of the 'wild-card' option as a component of overall option value.

It should be emphasized, though, that even if all the errors noted above[6] were first-order, the study of the implied-realized volatility relationship is still meaningful, but now has a different economic interpretation. The BS implied volatility is one measure of an option's *price*, one that controls for option-specific characteristics such as moneyness, time to expiration and so on. To the extent option prices are related in some manner to market participants' volatility expectations, BS implied volatility ought to provide one measure of option markets' volatility forecast. Hence, a study of the implied-realized volatility relationship could simply be viewed as a test of the informativeness of option prices about future return volatility—without necessarily drawing detailed inferences on market efficiency or even on the exact validity of a particular pricing model.

We now examine the dynamics of each volatility series in separation, following which we investigate the relationship between the two series.

3. Time Series Properties of Implied and Realized Volatility

In this section we characterize the time series properties of each of the two volatility series separately. The entire empirical analysis that follows is based on the log volatility series i_t and h_t. Henceforth, we refer to these as 'implied' and 'realized' volatility, respectively, dropping the log prefix.

[6] And others, such as misspecification of the log-normal price process assumed in the BS formula.

3.1 Univariate ARMA Models for Volatility

To gain a first grasp of the fundamental time series properties of volatility, we begin with a conventional Box-Jenkins (1970) analysis of the two series. Figures 1a and 1b display the autocorrelation functions for implied and realized volatility, respectively. For both series, these functions are shaped somewhat similarly to what one might expect for AR(1) series. To consider this further, we fit ARMA(p, q) models of the form

$$\Phi(B)(x_t - \mu) = \Theta(B)\varepsilon_t \ , \tag{3}$$

where x_t represents either of the two series i_t and h_t, μ is the mean parameter, ε_t is white noise, and Φ and Θ are polynomials of order p and q in B, the backshift operator defined by $Bx_t = x_{t-1}$.

Table 1 displays results of fitting AR(1), AR(2) and ARMA(1,1) models to the implied and realized volatility series, in panel A and panel B, respectively. Judging by these results, an ARMA(1,1) process appears to be an adequate descriptor of both volatility series. The Box and Pierce (1970) portmanteau statistics for the fitted ARMA(1,1) models are insignificant, indicating little evidence of any higher order components in either series. Thus, both volatility series appear to exhibit similar dynamics, roughly compatible with low order ARMA specifications. Indeed, the point estimates are similar for both series.

These indications from the analysis in the time domain are of course only tentative, and the picture proves more complicated when we move to the frequency domain. Here, we consider the shape of the spectral densities of the two series, displayed in figures 2a and 2b.[7] For each series, the shape of the spectrum corresponds approximately to what Granger (1966) terms 'the typical spectral shape of an economic variable.' Roughly speaking, the density is high at low frequencies, and tapers down towards the high frequency end. This

[7] The shape of the spectrum $s(\omega) = [\gamma_0 + 2\sum_{k=1}^{\infty} \gamma_k \cos(k\omega)]/(2\pi)$ is estimated by the periodogram $I(\omega_j) = 2[(\sum_t (x_t - \bar{x}) \cos(\omega_j t))^2 + (\sum_t (x_t - \bar{x}) \sin(\omega_j t))^2]/T$, using SAS. Here, γ_k is the autocovariance function and $\omega_j = 2\pi j/T$ is the jth Fourier frequency.

suggests that the volatility series possess significant low frequency (long-term memory) components and might even indicate non-stationarity.[8] In what follows, we implement formal statistical tests designed to detect the nature of this memory property.

3.2 Augmented Dickey-Fuller Unit Root Tests

In this section, we examine whether the volatility series possess unit roots (i.e. whether the AR coefficient ϕ_1 is unity) using procedures suggested by Dickey and Fuller (1979). The existence of a unit root would indicate a strong instance of the type of long-memory property mentioned in the previous subsection, and would imply that the series are non-stationary.

Specifically, the test is for the significance of γ in the augmented Dickey-Fuller (ADF) regression

$$\Delta x_t = \alpha + \gamma x_{t-1} + \sum_{i=1}^{k} \beta_i \Delta x_{t-i} + \xi_t \,, \tag{4}$$

where x_t is either i_t or h_t, and $\Delta = 1 - B$ is the first difference operator. The null hypothesis is that the series x_t is a (non-stationary) unit root process. The specification imposing $\alpha = 0$ allows no drift, whereas a non-zero drift is admitted for general α. In either case, a significantly negative value of γ would reject the null of a unit root and suggest that x_t is stationary. The number of lagged dependent variables k is chosen so as to leave the estimated error terms $\hat{\xi}_t$ serially uncorrelated.[9] Table 2 reports results for both series, with k set to 2.

As the results for both series are similar, we discuss those for implied volatility

---

[8] This is likewise suggested by the large magnitude of the AR coefficient ϕ_1 in table 1, and especially by its dramatic increase after introducing MA terms. French, Schwert and Stambaugh (1987) and Schwert (1987) also argue that the log realized volatility series is non-stationary, for data covering a different period of time.

[9] For both series, the Durbin-Watson statistics are very close to 2.0 whenever k is one or greater. Adding a linear deterministic time trend does not materially change any of our results.

(panel A). When α is restricted to zero (first row), γ (–0.003) is insignificant. Thus, we *cannot reject* the hypothesis that (log) implied volatility is a unit root process with zero drift. On the other hand, when α is unrestricted (second row), conflicting evidence obtains. In this instance, the point estimate of γ (–0.25) is much larger in magnitude and is statistically significant.[10] Based on this, we would *reject* the null hypothesis of non-stationarity in the form of a unit root process with non-zero drift, in favor of a stationary alternative. Thus, there seems to be mixed evidence on whether the two volatility series are stationary or not.[11] Qualitatively similar results obtain for the realized volatility series, as is seen in panel B.

While the stationarity question is of some interest in itself, it also has other important implications. Specifically, it affects (i) the nature of the econometric specifications that would be used to analyze the relationship between implied and realized volatility; and (ii) the distribution of test statistics in both the univariate and bivariate analyses. Hence, we pursue this question in somewhat greater detail.

We first note that the fact that no definite conclusions emerge from the unit root tests might be attributed to their knife-edged nature. The tests admit long-term memory and non-stationarity either at the rather strong level of a unit root process (essentially a random walk), or not at all. Long-term memory and/or non-stationarity may exist in weaker forms. Usually a process is considered to exhibit long-term memory if the spectrum $s(\omega) \to \infty$ as $\omega \to 0$ and the autocorrelation function is not absolutely summable, $\sum_{k=0}^{\infty} |\rho_k| = \infty$. A

[10] If $\alpha = 0$, then the asymptotic distribution of the t-statistic is the same as the distribution of $(\frac{1}{2}(W_1^2 - 1) - W_1 \int W_t \mathrm{d}t)/(\int W_t^2 \mathrm{d}t - (\int W_t \mathrm{d}t)^2)^{\frac{1}{2}}$ where W_t is a standard Wiener process on $[0, 1]$. If $\alpha \neq 0$ the usual t-tables apply (as in the first row of the table).

[11] If e.g. the AR(1) model is the relevant stationary alternative in the ADF test, then $\alpha = \mu(1 - \phi_1)$ and $\gamma = \phi_1 - 1$, with μ strongly significant in the ARMA models and $\phi_1 < 1$ under stationarity, so $\alpha \neq 0$. Thus, the ADF test imposing $\alpha = 0$ may not admit the relevant alternative and may therefore not be powerful.

unit root is merely one way of modelling this type of behavior. A more flexible vehicle for modelling long-term memory is provided by the class of 'fractionally integrated' processes. This is where our investigation takes us next.

3.3 Fractional Integration Tests

A time series x_t is said to be fractionally integrated [or, to satisfy a 'ARFIMA'—Autoregressive Fractionally Integrated Moving Average—model in the sense of Granger and Joyeux (1980) and Hosking (1981)] if it can be expressed as

$$\Phi(B)(1-B)^d(x_t-\mu)=\Theta(B)\varepsilon_t\ , \tag{5}$$

where as before B is the backshift operator, μ is the mean parameter, Φ and Θ are the AR and MA polynomials, ε_t is white noise, and $0<|d|<1$. The operator $(1-B)^d$ is the infinite order polynomial $\sum_{k=0}^{\infty} b(d,k)(-B)^k$ obtained via binomial expansion, using the binomial coefficients $b(d,k)=\Gamma(d+1)/(\Gamma(d-k+1)\Gamma(k+1))=\prod_{i=1}^{k}(d+1-i)/i$, so that $(1-B)^d=1-dB+\frac{1}{2}d(d-1)B^2-\frac{1}{6}d(d-1)(d-2)B^3+\ldots$. If $d<\frac{1}{2}$ the series $\{x_t\}$ is stationary[12] and for $0<d<1$ it exhibits long memory. We do not provide a full exposition of fractionally integrated series here, but refer the reader to Diebold and Rudebusch (1989), Sowell (1992a, 1992b), Cheung (1993) and Cheung and Lai (1993) for recent discussions of such processes.

Here, we implement two statistical tests to diagnose fractional integration. The first test is based on a procedure developed in Geweke and Porter-Hudak (1983)—henceforth GPH. Under this procedure, the order of fractional integration d is estimated as the slope in the regression

$$\log[I(\omega_j)]=c-d\ \log[4\sin^2(\omega_j/2)]+v_j\ , \tag{8}$$

where $I(\omega_j)$ is the periodogram of the time series x_t at the jth Fourier frequency $\omega_j=2\pi j/T$, $j=1,2,\ldots,q$, and $q<<T$ (here, we chose $q=15$). The test exploits the shape of

[12] Of course, we also need the roots of the equation $\Phi(z)=0$ to be outside the unit circle. This is the usual condition for stationarity of ARMA processes.

the spectral density s at the low frequency end. Formally, $s(\omega) \sim \omega^{-2d}$ as $\omega \to 0$, so that $\log(s)$ is proportional to $d \log(\omega^2)$. Table 3 presents results of the GPH test.

Point estimates of the fractional integration parameter d are 0.49 and 0.47 for the implied and realized volatility series, respectively. These estimates are over two standard errors away from zero and thus provide evidence that the volatility series possess long-term memory.[13]

While these results symptomize the existence of long memory in volatility, they provide no means of assessing the relative importance of long and short run dynamics in describing the evolution of volatility. Is there any evidence of long-run dynamics in volatility, when short-run dynamics of the sort described in section 3.1 are accounted for? Sowell (1992a) shows that this issue can be resolved by simultaneously estimating both the long-memory parameter d and the short-memory ARMA parameters (ϕ, θ). The estimation is accomplished by maximizing the unconditional ARFIMA log likelihood function

$$L(\phi, d, \theta) = -\frac{T}{2}\log(2\pi) - \frac{T}{2}\log(\sigma^2) - \frac{1}{2}\log \mid \Sigma_T \mid -\frac{1}{2\sigma^2}(X - \mu 1)'\Sigma_T^{-1}(X - \mu 1) \,,$$

where $X = (x_1, \ldots, x_T)'$, 1 is a T-vector of ones, and Σ_T is the covariance matrix developed in Sowell (1992b).[14] This is the exact time domain log likelihood for the ARFIMA process, unlike the frequency domain likelihood proposed by Fox and Taqqu (1986), which, though computationally simpler, is only an approximation. Table 4 reports the estimates for the implied volatility series (panel A) and the realized volatility series (panel B) under alternative assumptions about the AR and MA structure. We also report likelihood ratio statistics for three hypotheses of natural interest: (i) $d = 0$; (ii) $d = 1$; and (iii) $(\phi, \theta) = 0$.

[13] Similar conclusions emerge when testing for long-term memory using the modified rescaled range statistic proposed by Lo (1991).

[14] This formulation has two implicit assumptions. One, it assumes that $\{x_t\}$ is normally distributed, which is probably justified for log volatility [see for instance French, Schwert and Stambaugh (1987)]. Second, it assumes that $\{x_t\}$ is stationary, which we ensure by working with the first differences of h_t and i_t in the estimation. This leads to an estimate of $d - 1$ and hence of d.

Three points emerge from the analysis. First, we resoundingly reject the hypotheses $d = 0$ and $d = 1$ for both series, with p-values well below 1%.[15] That d is non-zero suggests that the two series do possess long-term memory; $d \neq 1$ indicates that the long memory in the series is less than that implied by a unit root. Second, the point estimates of d are close to or even larger than $\frac{1}{2}$; the volatility series may well be non-stationary. Finally, observe that we *fail to reject* the hypothesis that the 'short-memory' ARMA parameters (ϕ, θ) jointly vanish; p-values for this hypothesis are of the order 75%.

These results strongly suggest that the dynamics of volatility follow neither low-order ARMA nor unit root processes, but are best characterized by long-memory fractionally integrated processes. In addition, we emphasize the finding that the dynamics of the two volatility series are similar in all aspects of the analysis, suggesting that there is considerable hope of finding a link between the two series in the analysis to follow.

Are the series non-stationary? In fact, while the rejections of ARMA and unit root processes are clear-cut, we cannot resolve the stationarity issue unambiguously based on the available data, as the point estimates of d are too close to $\frac{1}{2}$ to get a significant difference. The evidence does however suggest that the possibility of non-stationarity be taken into account in assessing the relationship between the two series; we do so, in what follows next.

4. The Relationship between Implied and Realized Volatility

This section is organized into four parts, each of which uses a different approach to analyze the implied-realized volatility relationship. We demonstrate that the different approaches lead to similar conclusions, hence underscoring the robustness of our findings.

We begin in section 4.1 with a single equation specification in which *ex-post* realized volatility is modelled as a function of *ex-ante* implied volatility. This is the approach taken

[15] Due to the non-standard setting, the asymptotic χ^2_1 distribution should be modified somewhat, but the statistics (of the order 15 to 85) would clearly be extreme draws.

in all previous work in this area. Section 4.2 adds a second equation, in which implied volatility is modelled as a function of the history of the two volatility series. The resulting structure is estimated as a bivariate simultaneous equation system. Section 4.3 carries out inference within a VAR framework and examines causal relationships between the two volatility series. Finally, section 4.4 examines the properties of the residuals from the regression of realized volatility on implied volatility.

4.1 A Single Equation Analysis

We first analyze the information content of implied volatility within a single equation framework of the type employed in all previous literature. We estimate a specification of the type

$$h_t = \alpha_0 + \alpha_i i_t + e_t \ , \tag{9}$$

where h_t and i_t are defined in section 1. If implied volatility is an efficient predictor of realized volatility, we should find $\alpha_0 = 0$ and $\alpha_i = 1$.

Panel A of table 5 presents estimates from (9). The estimated regression coefficient α_i takes the value 0.65 and is significant. Though less than 1.0, it is much larger than comparable numbers (0.14, 0.22) reported in CF.

Does implied volatility subsume the information contained in past realized volatility? To address this question, we estimate the specification

$$h_t = \alpha_0 + \alpha_i i_t + \alpha_h h_{t-1} + e_t \ . \tag{10}$$

The results are presented in panel B of table 5. Lagged historical volatility h_{t-1} in isolation does explain future volatility h_t (first row in panel B). However, once implied volatility is added as an explanatory variable (second row in panel B), lagged historical volatility is no longer significant. That is, implied volatility *subsumes* the information contained in historical volatility.[16]

[16] Adding more lags and correcting for heteroskedasticity following White (1980) does not alter the

These results are again quite different from comparable results reported by CF. The point estimate of α_i (0.51) is much larger (comparable numbers for this specification in CF are 0.04 and 0.08). CF find that *historical* volatility subsumes the information content of implied volatility, whereas we find that it is *implied* volatility that subsumes the information content of historical volatility.

4.2 A Structural Simultaneous Equation System

In specifications such as (9) and (10), implied volatility is implicitly assumed to be exogenous, an assumption that may be incorrect. One view of implied volatility is that it is a summary measure of an option's *price*, controlling for option specific factors such as term to expiration, the extent to which it is in or out of the money, and so on. In its capacity as an option's price, implied volatility may well be an endogenously determined function of the past option prices (i.e. past implied volatility) as well as past realized volatility.

Motivated by this reasoning, we propose the following simultaneous equation system for the two volatility series:

$$h_t = \alpha_0 + \alpha_i i_t + \alpha_h h_{t-1} + e_{ht} \tag{11}$$

$$i_t = \beta_0 + \beta_i i_{t-1} + \beta_h h_{t-1} + e_{it} \tag{12}$$

Note that equation (11) is in effect a restatement of (10), but that it is now incorporated in a system. Specifications (11)-(12) exploit the structure that implied volatility i_t, which is measured at the *beginning* of month t, cannot be a function of h_t, the 'after-the-fact' volatility for month t. The structural model accounts completely for the potential endogeneity of i_t, including that part (but of course not all) of the contemporaneous shock e_{ht} to realized volatility measurable with respect to the market's information set at the beginning

nature of these results. Further, the Durbin-Watson statistic (2.14) indicates that the standard errors require no adjustment for autocorrelation.

of month t.[17] A bivariate normal distribution is assumed for (e_{ht}, e_{it}), but the correlation coefficient is not restricted to zero, so the system is non-recursive and full simultaneous estimation is called for.

Maximum likelihood estimates for (11)-(12) are presented in table 6. The central question of interest, of course, relates to the sign and significance of α_i in equation (11), which we report in the first row of the table. The point estimate of α_i is 0.90, considerably larger than that obtained in the single equation specification in table 5 (0.51), and also much larger than the comparable CF estimates (0.04 to 0.08). Equally interesting is the fact that α_h, the coefficient for historical volatility, remains economically and statistically insignificant. Once again, implied volatility subsumes the information content of the history of realized volatility.

In fact, observe that our estimate of coefficient α_i for implied volatility (0.90) is not significantly different from 1.0 (the t-statistic for the hypothesis $\alpha_i = 1$ takes the value -0.32, with a p-value of 37% in a one-sided test). Further, we also fail to reject the joint hypothesis that $\alpha_0 = 0, \alpha_i = 1$ (the F-statistic takes the value 1.73, corresponding to a p-value of 18%). Thus, implied volatility passes at least one test for being considered an efficient predictor of realized volatility.

The results also indicate that the single equation analysis is afflicted by classical simultaneous equation bias. Indeed, the cross-equation correlation $\text{corr}(e_{ht}, e_{it})$ is about -0.30 and is significant, underscoring the relevance of adding the simultaneous systems perspective to the single equation viewpoint adopted in previous literature.

Another feature of interest relates to the estimates from equation (12), reported in the second row of table 6. It is evident from the significance and magnitude of both β_i and β_h that implied volatility i_t depends heavily on lagged implied and realized volatility. This is consistent with our view of implied volatility as an option's price, in which capacity

[17] We can without loss of generality let $e_{ht} = \varepsilon_{ht} + \rho e_{it}$ where $\text{cov}(\varepsilon_{ht}, e_{it}) = 0$. Thus, ρe_{it} is the portion of the realized volatility shock picked up by the market at the time i_t is determined.

it may depend on past values of both realized and implied volatility. The dependence is quite sharply captured here, as indicated by the relatively high R^2 (61%).

4.3 VAR Analysis and Causality Tests

The previous subsection exploited the structure that beginning-of-month t implied volatility i_t cannot be a function of *ex-post* realized volatility for month t (h_t), yet allowed simultaneity through the dependence of h_t on its own 'forecasted' value i_t. Of course, the structural simultaneous system has a reduced-form counterpart, wherein each of h_t and i_t are functions of lagged variables, only. Turning next to this vector-autoregressive (VAR) form of the model, we now obtain a framework appropriate for examining the direction of causality between the two volatility series.

Accordingly, consider the bivariate vector-autoregression (VAR) specification

$$h_t = \alpha_0 + \sum_{k=1}^{L} \alpha_{hk} h_{t-k} + \sum_{k=1}^{L} \alpha_{ik} i_{t-k} + v_{ht} \tag{13}$$

$$i_t = \beta_0 + \sum_{k=1}^{L} \beta_{hk} h_{t-k} + \sum_{k=1}^{L} \beta_{ik} i_{t-k} + v_{it} \tag{14}$$

where L denotes the maximum number of lags and the v's are error terms. We found VAR coefficients α, β associated with lag lengths in excess of $k = 1$ to be economically and statistically insignificant[18] and the residuals $\hat{v}$ associated with one-lag specifications to display virtually no serial correlation. Hence, we used $L = 1$ in implementing the VAR.

Results are reported in table 7-1. Panel A presents estimates of specification (13), where the dependent variable is realized volatility h_t. Panel B presents estimates of specification (14), where the dependent variable is implied volatility i_t. Note that even if the data were generated by the structural model of the previous subsection, the OLS estimates in table 7-1 are not subject to the simultaneous equation bias, since the VAR specification

[18] In absolute terms, two-lag coefficients are a third to a twentieth of the first-order lag coefficients, and associated t-statistics range from 0.25 to 0.65.

with $L = 1$ is exactly the reduced form of the structural model and the regressors are common.

In the VAR system, Granger (1969) causality tests may be used to examine whether i_t causes h_t, and similarly whether h_t causes i_t. For instance, the hypothesis that i_t Granger-causes h_t may be examined by testing whether the coefficients α_{ik} are all zero (for $L = 1$, whether $\alpha_{i1} = 0$). A rejection would indicate that implied volatility i_t Granger-causes realized volatility h_t.

Consider first the results in panel A. The null hypothesis that implied volatility does not Granger-cause future realized volatility is rejected, as evidenced by the significant t-statistic for α_{i1}. This result may appear even more striking when we note that the relevant independent variable in (13) is i_{t-1}, the one month *lagged* implied volatility. In other words, even a stale, one month old implied volatility has non-trivial information content in predicting future realized volatility. The stale implied volatility i_{t-1} and the more recent realized volatility h_{t-1} have roughly the same coefficient in explaining realized volatility h_t.

Further, it is easily seen that the estimates simply represent additional evidence in favor of the structural simultaneous equation model of section 4.2. From table 6, augmenting the 2×2 identity matrix by a (1,2) entry of -0.90 yields the coefficient matrix for contemporaneous variables. Inverting this and multiplying the result onto the coefficients of the lagged variables produces the expected estimates for the reduced form VAR model. In particular, we may anticipate from the structural estimates a coefficient of $0.90 \times 0.44 = 0.40$ for i_{t-1} in table 7-1, panel A. This is within the bounds of estimation error from the actual estimate of 0.33. Similar conclusions hold for other estimates in table 7-1, so the VAR analysis proves to be a useful diagnostic confirming the structural model findings. Essentially, the structural analysis provides an explanation, for instance, of the role of the stale lagged implied volatility in Granger-causing future realized volatility in the VAR model.

Next, consider the panel B results. Based on the significant t-statistic for β_{h1}, we resoundingly reject the hypothesis that realized volatility h_t does not Granger-cause implied volatility i_t. Thus, causality between the two series runs in both directions, consistent with the structural model.[19]

A second notion of causality due to Sims (1972) may also be examined within the VAR framework. Causality in the Sims sense may be detected by regressing implied volatility on *future* realized volatility. Since future realized volatility cannot cause past implied volatility, a significant coefficient in this regression would indicate that implied volatility Sims-causes realized volatility.

The idea becomes more transparent when examining the econometric specifications

$$i_t = \alpha_0 + \alpha_{h+1} h_{t+1} + \alpha_{h-1} h_{t-1} + \alpha_{i-1} i_{t-1} + v_{it} \ , \tag{15}$$

$$h_t = \beta_0 + \beta_{i+1} i_{t+1} + \beta_{h-1} h_{t-1} + \beta_{i-1} i_{t-1} + v_{ht} \ , \tag{16}$$

including both leads and lags on the right hand side. If, for instance, α_{h+1} were statistically significant in (15), the indication would be that implied volatility Sims-causes future volatility. Similarly, a significant value of β_{i+1} would indicate that realized volatility Sims-causes implied volatility. The inclusion of lagged dependent variables was proposed by Geweke, Meese and Dent (1983) to reduce the serial correlation in the error terms v and is related to the conditions for equivalence of Granger and Sims causality [Chamberlain (1982)].

Results of the Sims-causality tests are presented in table 7-2. The relevant t-statistics (for α_{h+1} and β_{i+1}) are significant with p-values of about 5% or less, again indicating causality between the two series in both directions. The evidence that implied volatility Sims-causes realized volatility (panel A) is not strong, but it is significant, and one should bear in mind that a relatively 'stale' implied volatility is being related to a future realized

[19] Thus, we cannot reject the hypothesis that both series are endogeneous. Of course, we do reject strong exogeneity of either series, which requires weak exogeneity as well as Granger non-causality [Engle, Hendry and Richard (1983)].

volatility here: variable i_t is measured about a month *before* the very first observation used in calculating h_{t+1}. Even so, implied volatility has non-trivial information content.

In broad terms, the results of this and the previous two subsections indicate the existence of a significant causal relationship in both directions, between implied and realized volatility. Not only the most recent implied volatility, but also a lagged, one month old implied volatility bears statistically significant information about future return volatility —even when examined in conjunction with more current history of realized volatility.

This summarizes the statistical inference on the implied-realized volatility relationship in the conventional stationary process framework. Residual analysis provides both diagnostic checks on the framework and a vehicle for addressing non-stationarity issues, and this is where we turn next.

4.4 Residual Analysis: Cointegration, Predictability, and Error Correction

In this subsection, we examine the time series properties of the residuals from the regression of realized volatility on implied volatility.

This analysis is motivated by two issues. First, the results of section 3 suggest that the individual volatility series might be non-stationary. Following Engle and Granger (1987)—henceforth EG—a finding that the regression residuals are stationary even when the two volatility series considered individually are not would indicate that the volatility series are cointegrated, i.e. they move together across time. Second, we argue that if implied volatility does efficiently predict future volatility, the time series of regression residuals should be white noise. In other words, the regression residuals should exhibit no short or long-term predictability, under the null of efficiency. Our analysis serves to test this proposition.[20]

---

[20] Stationarity tests for residuals are commonly employed when the individual time series are (non-stationary) unit root processes (see EG) or fractionally integrated processes with $d > \frac{1}{2}$ [Cheung and Lai (1993)] and effectively detect any *long-term* residual dependence. Our economic context requires an

We begin by estimating (9), restated as

$$h_t = \alpha_0 + \alpha_i i_t + e_t \, . \tag{17}$$

We then analyze the time-series properties of the estimated residuals $\{\hat{e}_t\}$ to draw inference on the implied-realized volatility relationship.

4.4.1 Cointegration of Implied and Realized Volatility

We first test whether the implied and realized volatility series are cointegrated. The formal test is based on the ADF regression considered in section 3.2, but now applied to the residual series $\{\hat{e}_t\}$ from (17). That is, we test for the significance of γ in

$$\Delta\hat{e}_t = \alpha + \gamma\hat{e}_{t-1} + \sum_{i=1}^{k} \beta_i \Delta\hat{e}_{t-i} + \xi_t \, . \tag{18}$$

A significantly negative value of γ would imply that we fail to reject the null that implied and realized volatility are cointegrated (see EG). The results are presented in panel A of table 8. We find that $\hat{\gamma}$ is economically and statistically significant.[21] The implication is that if we maintain that h_t and i_t are unit root processes, the two volatility series are cointegrated, and (17) is interpreted as the cointegrating regression.[22]

Cointegration of the two volatility series indicates that they move together across time. Equivalently, the error process $\{e_t\}$ in (17) is stationary and reverts to mean zero, and thereby ensures that the two volatility series do not diverge from each other. However,

---

additional diagnostic for *short-term* residual dependence; there should not be any, if implied volatility aggregates volatility information efficiently. This 'white noise' test clearly has a meaningful economic interpretation whether or not the individual time series are stationary.

[21] The results are invariant to the inclusion of a drift term since the time series average of $\hat{e}_t$ is zero (as (17) includes an intercept).

[22] Following Sargan and Bhargava (1983), these results are also consistent with the Durbin-Watson (1.94) from table 5 (the 5% critical value when h_t, i_t possess unit roots is 0.39).

this does not complete the analysis of the *efficiency* of implied volatility as a predictor of future volatility. If implied volatility does in fact aggregate volatility information efficiently, the error series $\{e_t\}$ should not only be a stationary process (as suggested by the ADF test above)—but indeed just white noise. Such a finding would indicate that the entire ***predictable*** portion of future volatility is efficiently summarized by implied volatility. An empirical investigation of this issue follows next.

4.4.2 Predictability of Regression Residuals and Efficiency

If the regression residuals are in fact white noise, there should be no short or long-memory components in the time series of residuals. In this section, we implement statistical tests to examine this proposition.

We begin by testing for long-memory components in the residuals. Along the lines of Section 3.3, we first use the GPH procedure to estimate the fractional integration parameter d of the series $\{\hat{e}_t\}$. Under the null that long-term memory is absent, d should be zero. The GPH results are presented in panel B of table 8. The point estimate of d is 0.11 and is not significantly different from zero, in line with the hypothesis of no long-term predictability of the residuals.

While the GPH analysis does indicate absence of *long-term* memory, it does not impose enough structure to address the issue of *short-term* memory in the residuals. To test for this possibility, we first fit ARFIMA(p, d, q) models to the residual series. We then test whether the (short memory) ARMA parameters (ϕ, θ) ***and*** the (long memory) fractional integration parameter d all jointly vanish; they should, under the null of efficiency. Results for alternative parametrizations of the underlying AR and MA structures are presented in panel C of table 8.

Before discussing the efficiency test, three observations are in order. First, the point estimates of d for the residual series are considerably smaller than corresponding estimates for the individual series (which are 0.40 or higher; see table 4). Second, estimates of d are approximately equal to those obtained via the GPH procedure. Finally, we reject the hypothesis $d \geq \frac{1}{2}$ for the residual series at a p-value well below 1% (the likelihood ratio statistic ($\sim \chi_1^2$) takes the value 19.20). Maintaining $d \geq \frac{1}{2}$ for the individual volatility series, the result suggests that implied and realized volatility are 'fractionally cointegrated.' This verifies the robustness of the cointegration finding of section 4.4.1.

Turning to the efficiency issue, is there any evidence of predictability in the residual series? More precisely, can we reject the null hypothesis of neither short nor long term predictability, viz. $(\phi, d, \theta) = 0$? Panel C of table 8 displays the likelihood ratio statistics for testing this 'white noise' hypothesis against each of the fitted ARFIMA models. Nowhere can we reject the null hypothesis that all ARFIMA parameters jointly vanish, consistent with the view that implied volatility efficiently aggregates information about future volatility.[23]

4.4.3 A Dynamic Adjustment Mechanism

The evidence from sections 4.4.1–2 indicates that implied and realized volatility move together across time. It is thus plausible that there exists a *dynamic adjustment mechanism* which keeps the two series in tandem. EG show that cointegrated series may be represented in terms of exactly such an adjustment mechanism—an *error correction mechanism* (ECM). It is useful to assess the existence and relevance of such a mechanism in the context of the two volatility series.

Accordingly, we estimate an ECM for the two volatility series, represented as

$$\Delta h_t = b_0 + b_i \Delta i_t + b_{\text{ECM}} \hat{e}_{t-1} + u_t \,. \tag{19}$$

Using lagged residuals $\hat{e}_{t-1}$ from (17), the ECM (19) may be estimated as a simple regression. We expect $b_{\text{ECM}} < 0$ for the past 'errors' $\hat{e}_{t-1}$ to be 'corrected.' Thus, if $\hat{e}_{t-1} > 0$, then h_{t-1} is above its long-run path indicated by $\alpha_0 + \alpha_1 i_{t-1}$ (see (17)), so $b_{\text{ECM}} < 0$ provides the negative pull needed to get the h_t series back to the path.

[23] Identical conclusions emerge from fitting conventional Box-Jenkins models to the residual series.

We report the estimated ECM (19) in panel D of table 8. The point estimate of b_{ECM} is indeed negative, and is both economically and statistically significant. This indicates the existence of a strong dynamic adjustment mechanism that draws the implied and realized volatility series towards each other—precisely what we would expect under the null hypothesis of option market efficiency.

5. Conclusion

The fundamental question addressed in this study is: Does the volatility implied in option prices predict *ex post* realized volatility? Recent literature indicates that implied volatility adds little information beyond that contained in the history of realized volatility, when the two series are sampled on a day-to-day or weekly basis. Such results have been construed as a vote against the joint hypothesis that (i) option markets aggregate volatility information efficiently; and (ii) the Black-Scholes option pricing formula is valid.

This study introduces a new research design to examine the relationship between implied and realized volatility. Our analysis employs a lower (monthly) sampling frequency, and employs non-overlapping data, so that exactly one implied and one realized volatility estimate pertain to every month under consideration. The results are significantly different from those of previous literature, and the difference is robust to variations in the econometric approach: we find that implied volatility does predict future volatility, in isolation as well as in conjunction with the history of past realized volatility. In particular, it *subsumes* the information contained in past realized volatility, in contrast to the findings of previous research.

We attribute the difference in the results to differences in the research design. One obvious difference is the *lower* sampling frequency employed here, and our study highlights an important tradeoff in this context: A low sampling frequency results in fewer data points and thereby a potential loss of estimation efficiency and testing power. On the other hand, with high frequency (e.g. daily) sampling, the individual observations are less informative because of added noise in the system—noise due to bid-ask spreads, non-synchronicities and other frictions, as well as any potential noise from misspecification of the underlying pricing model. In the specific case of options markets, with the accompanying non-linear pricing models, such issues assume first-order importance, and the economic conclusions often depend crucially on the research design adopted.

We favor the particular structure adopted in this paper for three reasons. First, the monthly sampling frequency roughly corresponds to the frequency of most important economic news releases. Second, the non-overlapping sample that results is consistent with the underlying pricing model—the Black-Scholes option pricing model that has been used in every previous study in the literature. Third, the analysis explicitly recognizes the potential endogeneity of implied volatility by incorporating the conventional single equation for the volatility relationship in a complete simultaneous system. We argue that the resulting bivariate time series framework provides the most useful perspective for analyzing the implied-realized volatility relationship.

Our analysis highlights the importance of accounting for these issues and demonstrates that doing so leads to substantial differences in the economic conclusions that emerge. In the particular context here, our results provide much stronger support than previous studies for the null of option market efficiency.

References

Baillie, R. T., T. Bollerslev, and H. Mikkelsen, 1993, Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticty, Working Paper No. 168, Department of Finance, Northwestern University.

Black, F., and M. Scholes, 1973, The Valuation of Options and Corporate Liabilities, *Journal of Political Economy*, 81, 637–654.

Box, G. E. P., and G. M. Jenkins, 1970, *Time Series Analysis, Forecasting and Control*, Holden Day, San Francisco.

Box, G. E. P., and D. A. Pierce, 1970, Distribution of Residual Autocorrelations in Autoregressive-Integrated Moving Average Time Series Models, *Journal of the American Statistical Association*, 65, 1509–1526.

Canina, L., and S. Figlewski, 1993, The Informational Content of Implied Volatility, *Review of Financial Studies*, 6, 659–681.

Chamberlain, G., 1982, The General Equivalence of Granger and Sims Causality, *Econometrica*, 50, 569–581.

Cheung, Y.-W., 1993, Tests for Fractional Integration: A Monte-Carlo Investigation, *Journal of Time Series Analysis*, 14, 33–48.

Cheung, Y.-W., and K. Lai, 1993, A Fractional Cointegration Analysis of Purchasing Power Parity, *Journal of Business and Economic Statistics*, 11(1), 103–113.

Day, T., and C. Lewis, 1988, The Behavior of the Volatility Implicit in Option Prices, *Journal of Financial Economics*, 22, 103–122.

Day, T., and C. Lewis, 1992, Stock Market Volatility and The Information Content of Stock Index Options, *Journal of Econometrics*, 52, 267–287.

Dickey, D. A., and W. Fuller, 1979, Distribution of Estimators for Autoregressive Time Series with Unit Root, *Journal of the American Statistical Association*, 74, 427–431.

Diebold, F. X., and G. Rudebusch., 1989, Long Memory and Persistence in Aggregate Output, *Journal of Monetary Economics*, 24, 189–209.

Engle, R. F., and C. W. J. Granger, 1987, Cointegration and Error-Correction: Representation, Estimation and Testing, *Econometrica*, 55, 251–276.

Engle, R. F., D. F. Hendry, and J.-F. Richard, 1983, Exogeneity, *Econometrica*, 51, 277–304.

Fox, R., and M. Taqqu, 1986, Large-Sample Properties of Parameter Estimates for Strongly Dependent Stationary Gaussian Time Series, *Annals of Statistics*, 14, 517–532.

French, K., G. W. Schwert, and R. Stambaugh, 1987, Expected Stock Returns and Volatility, *Journal of Financial Economics*, 19, 3–30.

Geweke, J., R. Meese, and W. Dent, 1983, Comparing Alternative Tests of Causality in Temporal Systems, *Journal of Econometrics*, 21, 161–194.

Geweke, J., and S. Porter-Hudak, 1983, The Estimation and Application of Long Memory Time Series Models, *Journal of Time Series Analysis*, 4, 221–238.

Granger, C. W. J., 1966, The Typical Spectral Shape of an Economic Variable, *Econometrica*, 34, 150–161.

Granger, C. W. J., 1969, Investigating Causal Relations by Econometric Models and Cross-Spectral Models, *Econometrica*, 37, 424–438.

Granger, C. W. J., and R. Joyeux, 1980, An Introduction to Long Memory Time Series Models and Fractional Differencing, *Journal of Time Series Analysis*, 1, 15–30.

Harvey, C. R., and R. Whaley, 1992, Market Volatility Prediction and the Efficiency of the S&P 100 Index Option Market, *Journal of Financial Economics*, 31, 43–73.

Hosking, J. R. M., 1981, Fractional Differencing, *Biometrika*, 68, 165–176.

Lamoureux, C. G., and W. Lastrapes, 1993, Forecasting Stock Return Variance: Towards Understanding Stochastic Implied Volatility, *Review of Financial Studies*, 6, 293–326.

Latane, H., and R. Rendleman, 1976, Standard Deviation of Stock Price Ratios Implied in Option Prices, *Journal of Finance*, 31, 369–381.

Ljung, G. M., and G. E. P. Box, 1978, On a Measure of Lack of Fit in Time Series Models, *Biometrika*, 65, 297–303.

Lo, A., 1991, Long Term Memory in Stock Prices, *Econometrica*, 59, 1279–1313.

Poterba, J., and L. Summers, 1986, The Persistence of Volatility and Stock-Market Fluctuations, *American Economic Review*, 76, 1142–1151.

Sargan, J. D., and A. Bhargava, 1983, Testing Residuals From Least Squares Regression for Being Generated by the Gaussian Random Walk, *Econometrica*, 51, 153–174.

Schmalensee, R., and R. Trippi, 1978, Common Stock Volatility Expectations Implied by Option Premia, *Journal of Finance*, 33, 129–147.

Schwert, G. W., 1987, Effects of Model Specification on Tests for Unit Roots in Macroeconomic Data, *Journal of Monetary Economics*, 20, 73–103.

Schwert, G. W., 1990, Why Does Stock Market Volatility Change Over Time?, *Journal of Finance*, 37, 1115–1153.

Sims, C. A., 1972, Money, Income, and Causality, *American Economic Review*, 62, 540–552.

Sowell, F., 1992a, Modelling Long-Run Behavior with the Fractional ARIMA Model, *Journal of Monetary Economics*, 29, 277–302.

Sowell, F., 1992b, Maximum Likelihood Estimation of Stationary Univariate Fractionally Integrated Time Series Models, *Journal of Econometrics*, 53, 165–188.

White, H., 1980, A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity, *Econometrica*, 48, 817–838.

Table 1[†]

ARMA(p,q) Models For Implied and Realized Volatility

Panel A

Implied Volatility $\{i_t\}$

Fitted Model	μ	ϕ_1	ϕ_2	θ_1	Box-Pierce Statistic Q_{12}	Degrees of Freedom
ARMA(1,0)	-1.92[a]	0.65[a]			9.52	11
	(-31.82)	(9.09)				
ARMA(1,1)	-1.94[a]	0.83[a]		0.34	3.60	10
	(-23.50)	(10.99)		(2.66)		
ARMA(2,0)	-1.93[a]	0.51[a]	0.21[a]		4.46	10
	(-25.70)	(5.61)	(2.25)			
[a] p-value < 0.01; [b] p-value $= 0.05$						

Panel B

Realized Volatility $\{h_t\}$

Fitted Model	μ	ϕ_1	ϕ_2	θ_1	Box-Pierce Statistic Q_{12}	Degrees of Freedom
ARMA(1,0)	-1.99[a]	0.48[a]			17.56[b]	11
	(-35.73)	(5.75)				
ARMA(1,1)	-2.00[a]	0.83[a]		0.49[a]	8.10	10
	(-23.80)	(9.01)		(3.37)		
ARMA(2,0)	-1.99[a]	0.37[a]	0.22[a]		11.09	10
	(-28.78)	(4.03)	(2.37)			
[a] p-value < 0.01; [b] p-value $= 0.05$						

[†] Table 1 reports estimates of ARMA(p,q) models of the form

$$\Phi(B)(x_t - \mu) = \Theta(B)\varepsilon_t$$

fitted to the time series $\{x_t\}$, with $x_t = i_t$ (panel A) or $x_t = h_t$ (panel B), where i_t denotes the natural logarithm of the Black-Scholes implied volatility for at-the-money call options on the S&P 100 index; h_t denotes the natural logarithm of the *ex-post* daily return volatility of the index; ε_t is white noise; $\Phi(B)$ denotes the AR polynomial $1 - \phi_1 B - \phi_2 B^2$; $\Theta(B)$ denotes the MA polynomial $1 + \theta_1 B$; and B denotes the backshift operator. The data consist of $T = 115$ monthly observations on each volatility series, covering the period November 1983 to May 1993. Asymptotic t-values in parentheses.

The Box-Pierce statistics are $Q_{12} = T\sum_{k=1}^{1} 2\hat{\rho}_k^2$ where $\hat{\rho}_k$ are the sample autocorrelations of the estimated residuals $\hat{\varepsilon}_t$. Asymptotically $Q_{12} \sim \chi^2_{df}$ where the degrees of freedom df are given in the table. The related misspecification diagnostic $\tilde{Q}_{12} = T(T+2)\sum_{k=1}^{12} \hat{\rho}_k^2/(T-k)$ proposed by Ljung and Box (1978) produces similar results.

Table 2†

Augmented Dickey-Fuller Tests

Panel A

Implied Volatility i_t

Dependent variable: Δi_t

Independent Variables				Adj.	
Intercept	i_{t-1}	Δi_{t-1}	Δi_{t-2}	R^2	DW
	−0.003	−0.37	−0.16	12%	2.08
	(−0.30)	(−4.02)	(−1.76)		
−0.47	−0.25[a]	−0.22	−0.07	18%	2.04
(−3.00)	(−3.02)	(−2.20)	(−0.78)		
[a] Significant with p-value = 0.05					

Panel B

Realized Volatility h_t

Dependent variable: Δh_t

Independent Variables				Adj.	
Intercept	h_{t-1}	Δh_{t-1}	Δh_{t-2}	R^2	DW
	−0.004	−0.54	−0.28	23%	1.98
	(−0.28)	(−5.84)	(−3.06)		
−0.66	−0.33[a]	−0.33	−0.17	29%	1.95
(−3.20)	(−3.21)	(−2.95)	(−1.73)		
[a] Significant with p-value = 0.05					

† Table 2 reports results of augmented Dickey-Fuller (ADF) tests for the time series $x_t = i_t$ (panel A) and $x_t = h_t$ (panel B), where i_t denotes the natural logarithm of the Black-Scholes implied volatility, for at-the-money call options on the S&P 100 index, and h_t denotes the natural logarithm of the *ex-post* daily return volatility of the index. The data consist of 115 monthly observations on each volatility series, covering the period November 1983 to May 1993.

Numbers in parentheses denote t-statistics for significance tests. The null hypothesis viz. non-stationarity of $\{x_t\}$ is rejected if the coefficient on x_{t-1} is significant. The 5% critical value for the t-statistics in the regressions without intercept is −1.95 [Dickey and Fuller (1979)]. The usual t-tables may be employed for the regressions with intercept if the true value of the intercept is assumed non-zero, i.e. the 5% critical value is −1.66. If the regression includes an intercept but the true intercept is known to be zero, the 5% critical value for t-statistic is −2.89 [Dickey and Fuller (1979)].

Table 3[†]
GPH Test for Fractional Integration

Geweke and Porter-Hudak (1981) Test

Dependent Variable: $\log[I(\omega_j)]$, *with* $I(\omega_j)$ *the periodogram at frequency* ω_j

Independent Variable	*Implied Volatility* (i_t) *Coefficient*	*t-statistic*	*Realized Volatility* (h_t) *Coefficient*	*t-statistic*
Intercept	-5.04^a	-8.79	-4.98^a	-8.69
$-\log[4\sin^2(\omega_j/2)]$	0.49^b	2.21	0.47^b	2.13
^{a} p-value < 0.01; ^{b} p-value $= 0.05$				

[†] Table 3 presents estimates from the regression

$$\log[I(\omega_j)] = c - d\,\log[4\sin^2(\omega_j/2)] + v_j$$

where $I(\omega_j)$ is the periodogram of the time series $\{x_t\}$ ($x_t = i_t, h_t$) at the jth Fourier frequency $\omega_j = 2\pi j/T$ ($j = 1, 2, \ldots, q$), and $q << T$ (here, $q = 15$). The regression coefficient d is an estimate of the order of fractional integration of the series $\{x_t\}$. The data consist of $T = 115$ monthly observations (covering the period November 1983 to May 1993) on each of the series $\{i_t\}$ and $\{h_t\}$, where i_t denotes the natural logarithm of the Black-Scholes implied volatility for at-the-money call options on the S&P 100 index, and h_t denotes the natural logarithm of the *ex-post* daily return volatility of the index. The t-statistics are estimated using that $\mathrm{var}(v_j) \approx \pi^2/6$ in the GPH regression.

Table 4†

ARFIMA(p, d, q) Models for Implied and Realized Volatility

Panel A

Implied Volatility i_t

Model	d	ϕ_1	θ_1	$LRT_{d=0}$	$LRT_{d=1}$	$LRT_{\phi,\theta=0}$
(0, d, 0)	0.55			66.03[a]	23.14[a]	
(1, d, 0)	0.60	0.07		75.14[a]	18.29[a]	0.48
(0, d, 1)	0.59		−0.06	75.40[a]	18.61[a]	0.40
(1, d, 1)	0.60	0.24	0.18	73.39[a]	19.26[a]	0.57

[a] p-value < 0.01

Panel B

Realized Volatility h_t

Model	d	ϕ_1	θ_1	$LRT_{d=0}$	$LRT_{d=1}$	$LRT_{\phi,\theta=0}$
(0, d, 0)	0.40			34.34[a]	37.70[a]	
(1, d, 0)	0.47	0.11		46.84[a]	28.74[a]	1.33
(0, d, 1)	0.48		−0.12	49.04[a]	27.88[a]	1.58
(1, d, 1)	0.48	0.04	0.08	48.63[a]	28.08[a]	1.50

[a] p-value < 0.01

† Table 4 reports estimates of ARFIMA(p, d, q) models of the form

$$\Phi(B)(1-B)^d x_t = \Theta(B)\varepsilon_t$$

fitted to the time series $\{x_t\}$, $x_t = i_t$ (panel A) or $x_t = h_t$ (panel B), where i_t denotes the natural logarithm of the Black-Scholes implied volatility for at-the-money call options on the S&P 100 index; h_t denotes the natural logarithm of the *ex-post* daily return volatility of the index; ε_t is white noise; $\Phi(B)$ denotes the AR polynomial $1-\phi_1 B-\phi_2 B^2$; $\Theta(B)$ denotes the MA polynomial $1+\theta_1 B$; and B denotes the backshift operator. The data consist of 115 monthly observations on each volatility series, covering the period November 1983 to May 1993.

We also report likelihood ratio statistics (columns labelled LRT) for testing three null hypotheses, viz. $d = 0$, $d = 1$ and $(\phi, \theta) = 0$. Under the null, the test statistics are approximately distributed as χ^2_1 for the first two hypotheses and as χ^2_{df} for the last hypothesis, where df denotes the number of restrictions (df = 1, 1, and 2, respectively, in the cases considered).

Table 5[†]

Single Equation Analysis of Implied-Realized Volatility Relationship

Dependent Variable: Realized Volatility h_t

Panel A

Independent Variables Intercept	i_t	h_{t-1}	Adj. R^2	DW
-0.74[a]	0.65[a]		30%	1.91
(-4.10)	(7.01)			
[a] Significant with p-value < 0.01				

Panel B

Independent Variables Intercept	i_t	h_{t-1}	Adj. R^2	DW
-1.03[a]		0.48[a]	22%	2.20
(-6.12)		(5.76)		
-0.67[a]	0.51[a]	0.17	31%	2.14
(-3.65)	(3.84)	(1.52)		
[a] Significant with p-value < 0.01				

[†] Table 5 reports estimates of the single equation specification

$$h_t = \alpha_0 + \alpha_i i_t + \alpha_h h_{t-1} + e_t \ .$$

Here, i_t denotes the natural logarithm of the Black-Scholes implied volatility for at-the-money call options on the S&P 100 index, measured at the beginning of month t; h_t denotes the natural logarithm of the ***ex-post*** daily return volatility of the index, over the life of the option whose log implied volatility is i_t. The data consist of 115 monthly observations on each volatility series, covering the period November 1983 to May 1993. Asymptotic t-values in parentheses.

Table 6†

Simultaneous Equations System for Implied-Realized Volatility Relationship

Dependent Variable	Independent Variables: Intercept	i_t	h_{t-1}	i_{t-1}	Adj. R^2	DW
h_t	−0.40	0.90[a]	−0.07		27%	1.98
	(−1.47)	(2.91)	(−0.32)			
i_t	−0.34[a]		0.44[a]	0.38[a]	61%	2.16
	(−2.77)		(7.31)	(5.23)		
[a] p-value < 0.01; [b] p-value $= 0.05$						

† Table 6 reports full information maximum likelihood (FIML) estimates of the bivariate system

$$h_t = \alpha_0 + \alpha_i i_t + \alpha_h h_{t-1} + e_{ht}$$

$$i_t = \beta_0 + \beta_i i_{t-1} + \beta_h h_{t-1} + e_{it}$$

Here, i_t denotes the natural logarithm of the Black-Scholes implied volatility for at-the-money call options on the S&P 100 index, measured at the beginning of month t; h_t denotes the natural logarithm of the *ex-post* daily return volatility of the index, over the life of the option whose log implied volatility is i_t. The data consist of 115 monthly observations on each volatility series, covering the period November 1983 to May 1993. Asymptotic t-values in parentheses.

Table 7-1[†]

Causality Tests Within VAR Framework

Granger Causality Tests

Panel A

Dependent Variable: Realized Volatility h_t

Independent Variables			Adj.	
Intercept	h_{t-1}	i_{t-1}	R^2	*DW*
-0.70^a	0.33^a	0.33^a	27%	2.02
(−3.56)	(3.38)	(2.91)		
^{a} p-value < 0.01				

Panel B

Dependent Variable: Implied Volatility i_t

Independent Variables			Adj.	
Intercept	h_{t-1}	i_{t-1}	R^2	*DW*
-0.34^a	0.44^a	0.37^a	61%	2.10
(−2.74)	(7.21)	(5.16)		
^{a} p-value < 0.01				

[†] Table 7-1 reports estimates of the VAR specification

$$h_t = \alpha_0 + \alpha_{h1} h_{t-1} + \alpha_{i1} i_{t-1} + v_{ht}$$

$$i_t = \beta_0 + \beta_{h1} h_{t-1} + \beta_{i1} i_{t-1} + v_{it}$$

where i_t denotes the natural logarithm of the Black-Scholes implied volatility for at-the-money call options on the S&P 100 index, measured at the beginning of month t; h_t denotes the natural logarithm of the ***ex-post*** daily return volatility of the index, over the life of the option whose log implied volatility is i_t. The data consist of 115 monthly observations on each volatility series, covering the period November 1983 to May 1993.

A statistically significant value of α_{i1} (β_{h1}) indicates that implied (realized) volatility Granger-causes realized (implied) volatility. Asymptotic t-values in parentheses.

Table 7-2[†]

Causality Tests Within VAR Framework

Sims Causality Tests

Panel A

Dependent Variable: Implied Volatility i_t

Independent Variables				Adj. R^2	DW
Intercept	h_{t+1}	h_{t-1}	i_{t-1}		
-0.21	0.13[a]	0.40[a]	0.34[a]	61%	2.12
(-1.57)	(2.47)	(6.39)	(4.79)		
[a] p-value < 0.01					

Panel B

Dependent Variable: Realized Volatility h_t

Independent Variables				Adj. R^2	DW
Intercept	i_{t+1}	h_{t-1}	i_{t-1}		
-0.21	0.76[a]	0.13	0.05	53%	2.00
(-1.20)	(7.82)	(1.55)	(0.47)		
[a] p-value < 0.01					

[†] Table 7-2 reports estimates of the specifications

$$i_t = \beta_0 + \beta_{h+1} h_{t+1} + \beta_{h-1} h_{t-1} + \beta_{i-1} i_{t-1} + v_{it}$$

$$h_t = \alpha_0 + \alpha_{i+1} i_{t+1} + \alpha_{h-1} h_{t-1} + \alpha_{i-1} i_{t-1} + v_{ht}$$

where i_t denotes the natural logarithm of the Black-Scholes implied volatility for at-the-money call options on the S&P 100 index, measured at the beginning of month t; h_t denotes the natural logarithm of the *ex-post* daily return volatility of the index, over the life of the option whose log implied volatility is i_t. The data consist of 115 monthly observations on each volatility series, covering the period November 1983 to May 1993.

A statistically significant value of β_{h+1} (α_{i+1}) indicates that implied (realized) volatility Sims-causes realized (implied) volatility. Asymptotic t-values in parentheses.

Table 8

Dynamics of the Residual Series

Panel A[†]

Augmented Dickey-Fuller Tests

Dependent Variable: $\Delta\hat{e}_t$

Independent Variables			Adj.	
Intercept	$\hat{e}_{t-1}$	$\Delta\hat{e}_{t-1}$	R^2	DW
0.00	-0.96^a		47%	2.00
(0.02)	(−10.16)			
−0.00	-0.82^a	−0.14	48%	2.03
(−0.03)	(−6.31)	(−1.52)		
^{a} p-value < 0.01				

Panel B[††]

Geweke and Porter-Hudak Test

Dependent Variable: $\log[I(\omega_j)]$, *with* $I(\omega_j)$ *the periodogram of* $\{\hat{e}_t\}$ *at* ω_j

Independent Variable	*Coefficient*	*t-statistic*
Intercept	-4.21^a	−4.89
$-\log[4\sin^2(\omega_j/2)]$	0.11	0.52
^{a} p-value < 0.01		

Panel C[†††]

ARFIMA(p,d,q) **Models for series** $\{\hat{e}_t\}$

Model	d	ϕ_1	θ_1	$LRT_{d,\phi,\theta=0}$	df
(0, d, 0)	0.07			0.86	1
(1, d, 0)	0.16	0.11		1.49	2
(0, d, 1)	0.13		−0.09	1.25	2
(1, d, 1)	0.12	0.25	0.14	1.58	3

Panel D[††††]

Error Correction Mechanism

Dependent Variable: Δh_t

Independent Variables			Adj.	
Intercept	Δi_t	$\hat{e}_{t-1}$	R^2	DW
−0.00	0.39^a	-0.84^a	34%	2.00
(−0.02)	(3.06)	(−7.73)		
^{a} p-value < 0.01				

† Panel A reports results of augmented Dickey-Fuller (ADF) tests conducted on the cointegrating regression residual series $\{\hat{e}_t\}$, obtained by regression of h_t on i_t. Here, i_t denotes the natural logarithm of the Black-Scholes implied volatility for at-the-money call options on the S&P 100 index, measured at the beginning of month t; h_t denotes the natural logarithm of the *ex-post* daily return volatility of the index, over the life of the option whose log implied volatility is i_t. The data consist of 115 monthly observations each volatility series, covering the period November 1983 to May 1993. The null hypothesis of non-stationarity of $\{\hat{e}_t\}$ is rejected (and so cointegration of h_t and i_t supported) if the coefficient on $\hat{e}_{t-1}$ is significant; the 1% and 5% critical values for the relevant t-statistic are −3.98 and −3.42, respectively.

†† Panel B presents estimates of the regression

$$\log[I(\omega_j)] = c - d\ \log[4\ \sin^2(\omega_j/2)] + v_j$$

where $I(\omega_j)$ is the periodogram of the cointegrating regression residual series $\{\hat{e}_t\}$ defined in relation to panel A, evaluated at the jth Fourier frequency $\omega_j = 2\pi j/T$ $(j = 1,\cdots,q)$, $q << T$ (here, $q = 15$ and $T = 115$). The coefficient d is an estimate of the order of fractional integration of $\{\hat{e}_t\}$. The critical values for the t-statistic are close to those from the standard normal table [Cheung and Lai (1993)].

††† Panel C reports estimates of ARFIMA(p,d,q) models of the form

$$\Phi(B)(1-B)^d\hat{e}_t = \Theta(B)\varepsilon_t$$

fitted to the cointegrating regression residual series $\{\hat{e}_t\}$ defined in relation to panel A; $\Phi(B)$ denotes the AR polynomial $1-\phi_1 B$; $\Theta(B)$ denotes the MA polynomial $1+\theta_1 B$; and B denotes the backshift operator. The table also reports likelihood ratio test (*LRT*) statistics for the null hypothesis $\phi,\theta,d = 0$, for each of the fitted ARFIMA models. Under the null, the test statistics are asymptotically distributed as χ^2_{df} where the degrees of freedom *df* are given in the table.

††††In panel D we estimate an error correction mechanism (ECM) of the form

$$\Delta h_t = b_0 + b_i\Delta i_t + b_{\mathrm{ECM}}\hat{e}_{t-1} + u_t\ ,$$

where the cointegrating regression residuals $\hat{e}_t$ are defined in relation to panel A and $\Delta = 1 - B$ is the first-difference operator. Asymptotic t-values in parentheses.

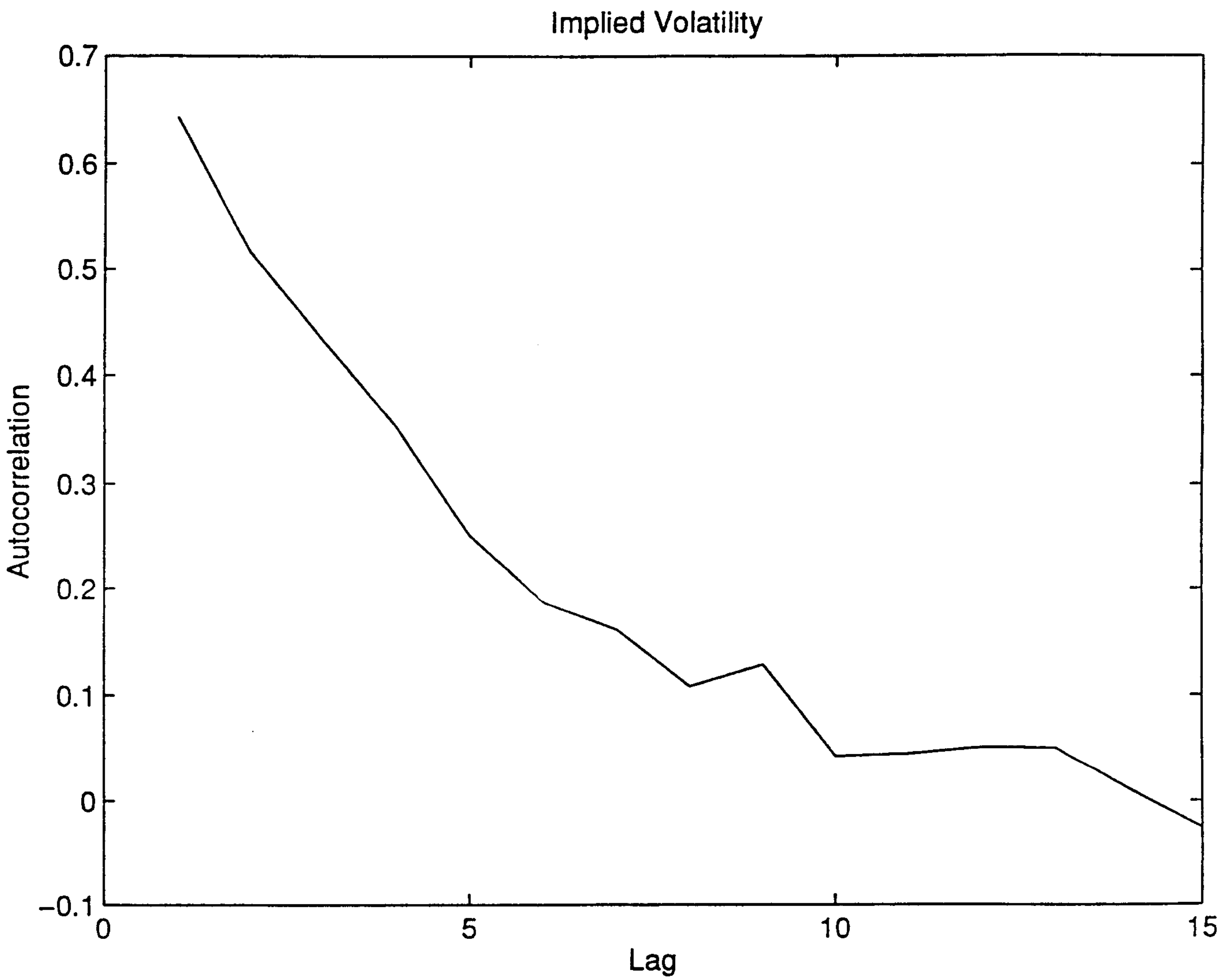

Fig. 1a. Sample autocorrelation function for the time series $\{i_t\}$, where i_t denotes the natural logarithm of the Black-Scholes implied volatility for at-the-money call options on the S&P 100 index. The data consist of 115 monthly observations, covering the period November 1983 to May 1993.

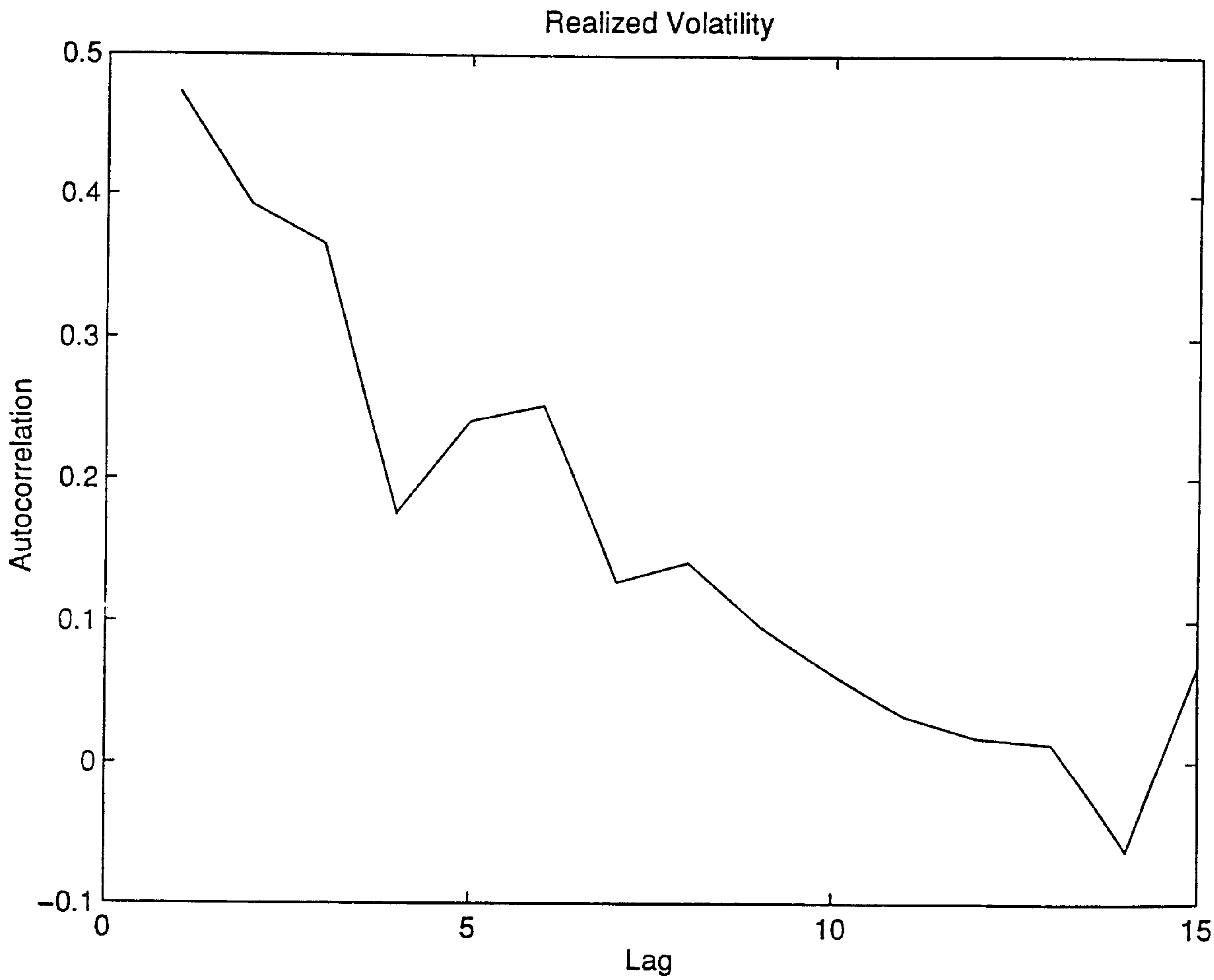

Fig. 1b. Sample autocorrelation function for the time series $\{h_t\}$, where h_t denotes the natural logarithm of the *ex-post* daily return volatility of the S&P 100 index. The data consist of 115 monthly observations, covering the period November 1983 to May 1993.

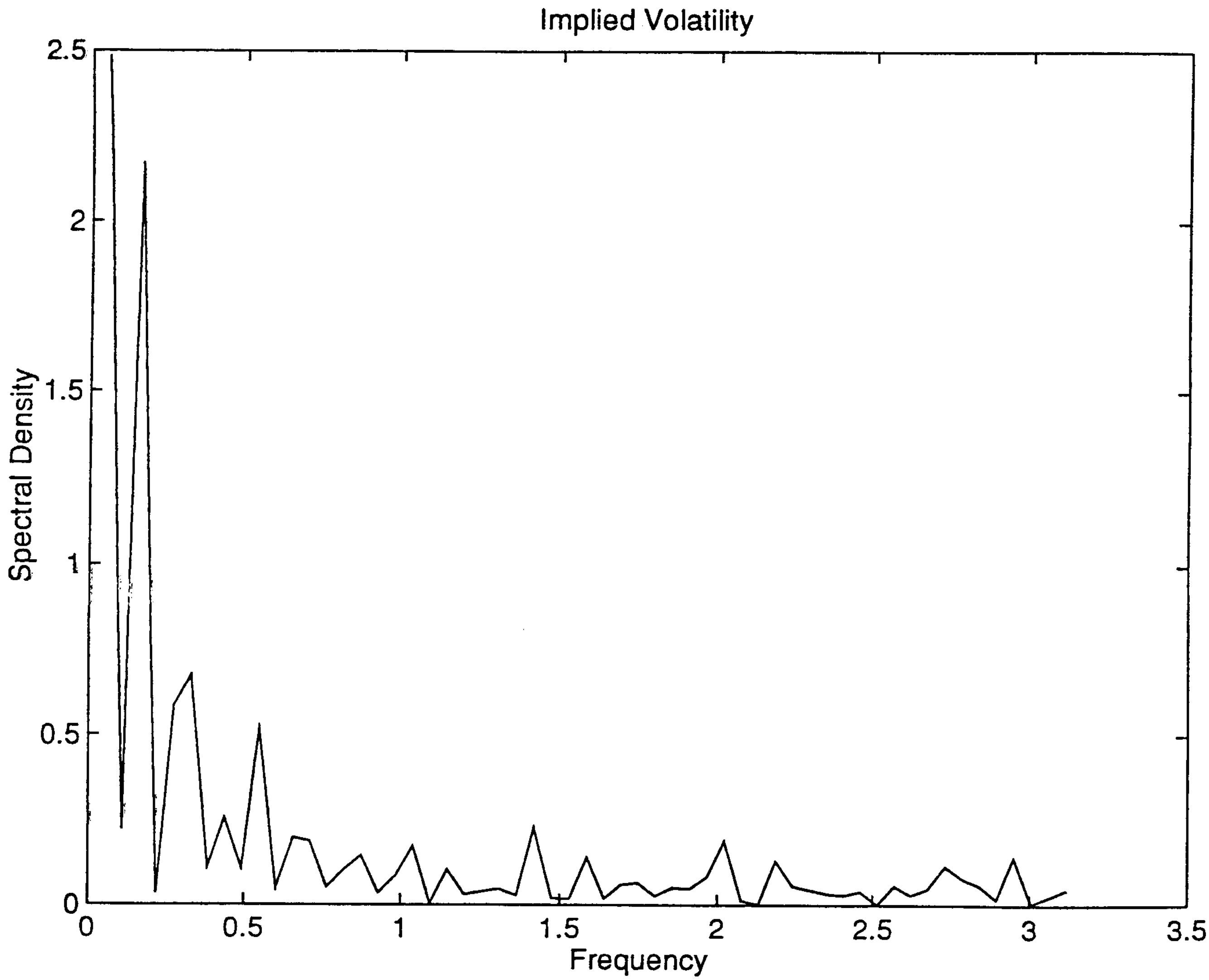

Fig. 2a. Shape of sample spectral density for the time series $\{i_t\}$, where i_t denotes the natural logarithm of the Black-Scholes implied volatility for at-the-money call options on the S&P 100 index. The data consist of 115 monthly observations, covering the period November 1983 to May 1993.

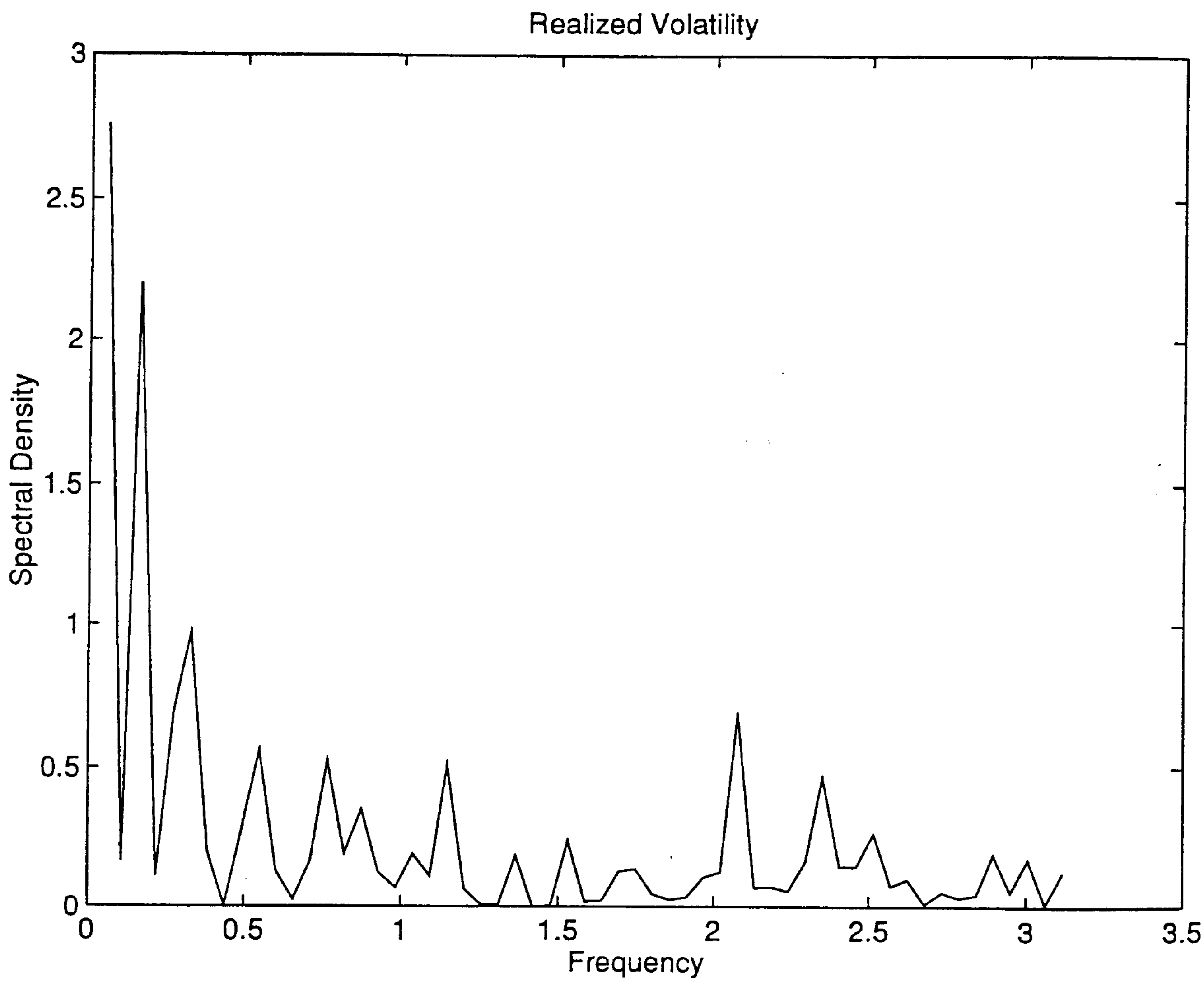

Fig. 2b. Shape of sample spectral density for the time series $\{h_t\}$, where h_t denotes the natural logarithm of the *ex-post* daily return volatility of the S&P 100 index. The data consist of 115 monthly observations, covering the period November 1983 to May 1993.