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Money Transaction, and Portfolio Choice

Pierluigi Balduzzi and Silverio Foresi

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Money, Transactions, and Portfolio Choice

Pierluigi Balduzzi and Silverio Foresi*

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* L. Stern School of Business, New York University.

Abstract

Real money balances are held separately for *consumption* and *portfolio* reasons. When real balances are a state variable in the investor's optimization problem, there is a specific inflation-hedging portfolio. An investor hedges against inflation when the effect of real money holdings on the marginal utility of wealth is negative. We show that an increase in real balances due to inflation has two opposite effects on the marginal utility of wealth. On the one hand, the decrease in real balances reduces consumption, which in turn raises the marginal utility and decreases the marginal cost of consuming: this explains why an investor would normally hedge inflation. On the other hand, the decrease in real balances tends to increase the marginal cost of consuming. When this second effect dominates, we have the somewhat surprising result that the investor reverse-hedges inflation.

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Address: Management Education Center, 44 West 4th Street, Suite 9-190, New York, NY 10012-1126

1 Introduction

The last two decades have seen a tremendous effort of researchers to embed the effects of monetary "complications" in general- and partial-equilibrium models of asset price determination. The focus of this paper is on a popular class of models in which money yields liquidity services. The analysis makes the following contributions. First, it formalizes the fact that the decision of holding money has two facets: it is separately a consumption and a portfolio decision. Second, it shows that if there is a liquidity reason to hold money there is also a separate inflation-hedging portfolio, because real money balances become a state variable for the investor. Third, it shows under which conditions inflation is hedged in a monetary economy.

It is well known that in the standard Arrow-Debreu general equilibrium set-up with a complete menu of frictionless markets there is no role for fiat money. Market frictions are usually advocated to explain why money can be held in spite of being first-order stochastically dominated by nominal bonds as a means of transferring purchasing power from period to period. Monetary theorists have put forward several models which allude to or schematically represent the role of money in facilitating transactions, while retaining the tractability of a framework with perfect markets.

These models go from including money in the utility function or in a transaction-cost function, to imposing a cash-in-advance constraint (CAC). Classical references for the first procedure are Baumol (1952), Tobin (1956), Barro (1976), and Romer (1986), while the second approach is explored in the works of Patinkin (1965), Sidrausky (1967), and it is empirically implemented in Poterba and Rotenberg (1985). In Whalen (1966), a precautionary reason for holding money is coupled with the aforementioned transaction motive. The third approach, the CAC, can be viewed as a special case of either of the first two; this procedure, originally proposed by Clower (1967), has been reintroduced in Lucas (1980, 1982, 1984). Both the first two approaches try to capture the idea that money saves resources in the transaction process and is useful as a non-perishable good: in the first case the resources saved provide direct utility, in the second case they allow for a "loosening" of the budget constraint.

With the possible exception of the CAC, these approaches do not model literally *how* money does facilitate transactions, but are intended to represent parsimoniously the idea that money serves a useful role, and thus it is not a dominated security. Feenstra (1986) provides conditions under which the three approaches are functionally equivalent from the standpoint of the individual optimizing consumer, while Wang and Yip (1992) provide conditions such that the three approaches yield exactly identical equilibrium comparative statics results.

Hence the present paper focuses on the transaction-costs case with little loss of generality.

We show that if trade in a risk-free nominal bond is not precluded, the decision to hold money for liquidity purposes and the decision to hold money for portfolio purposes can be effectively separated: The first decision is completely analogous to that of consuming. The second decision is analogous to that of holding an asset with a one-period sure nominal return.

In a monetary economy one of the mutual funds in the optimal portfolio insures against inflation uncertainty. This portfolio is the one that displays maximum correlation with the rate of return on money. An investor will short this portfolio and hedge against inflation when the effect of money holdings on the marginal utility of wealth is negative.

An increase in real money balances due to lower than expected inflation has two opposite effects on the marginal utility of wealth. On the one hand, a decrease in real money causes a decrease in consumption which raises the marginal utility and decreases the marginal cost of consuming: this explains why an investor would normally hedge inflation. On the other hand, the decrease in real money tends to increase the marginal cost of consuming. When this second effect dominates, we have the somewhat surprising result that the investor reverse-hedges inflation.

It is worth noting that the sign of the effect of real money on the marginal utility of wealth is also crucial for the properties of the equilibrium in a representative-agent economy [see the discussion in Orphanides and Solow (1990)].

Section 2 shows the different nature of holding money for precautionary-liquidity reasons and for portfolio purposes, Section 3 describes the investor's optimization problem. In Section 4 we qualify the inflation-hedging behavior. The last section shows how different transaction technologies translate into different hedging decisions.

2 Holding Money: a Consumption and a Portfolio Decision

This section describes a monetary framework in which the choice of holding money for consumption reasons can be effectively separated from the decision of holding money for portfolio reasons. To simplify the exposition we disregard price-index problems arising in multiple-goods settings, since they would give raise to hedging towards specific commodity-price risks [see, for example, Breeden (1979, 1984)].

Assumption 1. There is only one homogeneous consumption good whose price in terms of money is P_t. Unless otherwise stated, all variables are measured in units of the consumption good. The inverse of the rate of price inflation is defined as $r_{m,t} \equiv P_t/P_{t+1}$.

Investment opportunities are as follows:

Assumption 2. At time t the investor allocates her portfolio among four securities: stocks, s_t, which provide real return $r_{s,t+1}$, money, $\hat{m}_t$, which yields real return $r_{m,t+1}$, one-period default-free bonds, b_t, which promise a nominal interest rate R_{t+1} and yield real return $r_{b,t+1} \equiv R_{t+1}\, r_{m,t+1}$, and a risk-free security, f_t, which yields real return $r_{f,t+1}$. (*Returns* stand for *gross* rates of return.) The time subscript t indicates that the variable belongs to the information set at time t, with the exception of R_{t+1} and $r_{f,t+1}$, which are known as of time t. Markets for existing securities are frictionless.

The investor enters period t with wealth w_t and uses it to finance consumption, c_t, and transaction costs, ϕ_t; and to acquire stocks, bonds and money:

$$w_t = c_t + \phi_t + s_t + f_t + b_t + \hat{m}_t. \tag{1}$$

Total expenditures, $c_t + \phi_t$, can be thought of as including noncapital income (negative expenditure), and being net of transfers and taxes.

The value of portfolio wealth at the beginning of period $t+1$ is given by $w_{t+1} = s_t r_{s,t+1} + f_t r_{f,t+1} + b_t r_{b,t+1} + \hat{m}_t r_{m,t+1}$.

Money is a dominated asset, but it is held for its liquidity services. We define money inherited from the previous period as $m_t \equiv \hat{m}_{t-1} r_{m,t}$. Notice the distinction between $\hat{m}_t$ and m_t: $\hat{m}_t$ is the amount of real money balances the investor decides to carry over from period t to period $t+1$; m_t is the amount of money, in real terms, she finds herself with in period t. The latter is a *state* rather than a decision variable, as it is affected by the inflation rate realized between time $t-1$ and t.

Result 1. Under Assumptions 1 and 2, the decision of holding money for liquidity reasons can be effectively separated from the decision of holding money for portfolio purposes. The value of the expected transaction services from real money balances is $\hat{m}_t\ (R_{t+1} - 1)/R_{t+1}$.

Proof: To see this, we only need to show that the investor can think of setting aside a sum for a broadly defined consumption bundle and invest its remaining wealth (investible wealth) among the existing securities. We first rewrite (1) as $w_t - c_t - \phi_t - \hat{m}_t = s_t + f_t + b_t$, and add $\hat{m}_t/R_{t+1}$ to both sides

$$w_t - c_t - \phi_t - \hat{m}_t + \frac{\hat{m}_t}{R_{t+1}} = s_t + f_t + b_t + \frac{\hat{m}_t}{R_{t+1}}.$$

Define *investible* wealth, $\hat{w}_t$, as

$$\hat{w}_t \equiv w_t - c_t - \phi_t - \hat{m}_t + \frac{\hat{m}_t}{R_{t+1}}. \quad (2)$$

The last two equations imply that investible wealth is allocated to both nominal and real assets: $\hat{w}_t = s_t + f_t + [b_t + (\hat{m}_t/R_{t+1})]$. Define

$$a_{s,t} \equiv s_t/\hat{w}_t, \text{ and } a_{n,t} \equiv [b_t + (\hat{m}_t/R_{t+1})]/\hat{w}_t, \quad (3)$$

as the percentages of stocks and nominal assets in the investible wealth, respectively. Using equations (2) and (3), we can write the law of motion of w_t as

$$w_{t+1} = [a_{s,t}\ r_{s,t+1} + a_{n,t}\ r_{b,t+1} + (1 - a_{s,t} - a_{n,t})\ r_{f,t+1}]\,\hat{w}_t = r_{w,t+1}\ \hat{w}_t. \quad (4)$$

After setting aside $c_t + \phi_t + \hat{m}_t(1 - R_{t+1})/R_{t+1}$, an investor can proceed to chose the portfolio weights $a_{s,t}$ and $a_{n,t}$.□

This result is due to the existence of default-free nominal bonds (Assumption 2) which allow to hedge the inflation risk associated to holding money. Result 1 relies on a simple decomposition. Since we can impute to money a return comparable to that of nominal bonds, the difference between the imputed return and the actual return on money is the cost of holding money rather than bonds: $\hat{m}_t(R_{t+1}-1)/R_{t+1}$. This is also the value of expected transaction services from real money balances. After setting aside $\hat{m}_t(R_{t+1} - 1)/R_{t+1}$ for liquidity purposes—a *consumption* decision—the investor decides the share of nominal securities, a_n—a *portfolio* decision. A similar result was exploited by Fama and Farber (1979), and Kimbrough (1986), and the generality of it was hinted by LeRoy (1984).

3 Investor's Optimization Problem

In this section we describe the investor's optimal consumption and portfolio problem.

Assumption 3. Investors gain utility from consumption. The investor's time horizon is infinite. The period utility function of the investor is given by $U(c_t)$, with continuous derivatives $U_c > 0, U_{cc} < 0$. The objective of the investor is to maximize the expected flow of future utility discounted at the rate $0 < \delta < 1$:

$$\sup E_t \sum_{i=0}^{\infty} \delta^i U(c_{t+i}) \tag{5}$$

subject to the law of motion of wealth (4).

Assumption 4. Real money balances *inherited from the previous period* reduce transaction costs associated with consumption. We assume transaction costs to be described by a convex, twice continuously differentiable function of c_t and m_t, $\phi_t = \phi(c_t, m_t) \geq 0$, where $\phi(0, m_t) = 0$, $\phi_c > 0$, $\phi_m < 0$, $\phi_{cm} < 0$.

While we directly assume a transaction-costs function, other authors, such as Saving (1971) and Kimbrough (1986), study situations where transactions reduce the time available for work and leisure, and money reduces the time the investor must devote to transacting. McCallum (1983) outlines conditions under which the two approaches are equivalent.

As to the shape of the utility and transaction-costs function, we simply imposed conditions that essentially guarantee a unique interior solution to the investor's optimization problem. Feenstra (1986) shows that similar conditions guarantee the equivalence between the transaction-costs and the money-in-the-utility function approach to the introduction of money. The conditions on the transaction-costs function are satisfied, for example, by the Baumol-Tobin model [Feenstra (1986)] $\phi_t = kc_t/m_t$; and by the Cobb-Douglas model $\phi_t = kc_t^{\alpha} m_t^{1-\alpha}$, for $\alpha > 1$ and $k > 0$, which is equivalent to a cash-in-advance constraint when $\alpha \to \infty$ [Marshall (1992)]. The implications of these two models for inflation-hedging decisions will be discussed in the latter Section 5.

Following a "tradition" in finance which dates back to Merton (1971), we proceed to characterize the investor's optimal decision when the sequential problem (5) admits a unique interior solution, and is equivalent to the functional-equation problem

$$J(w_t, m_t) = \sup \ E_t U(c_t) + \delta J(w_{t+1}, m_{t+1}), \tag{6}$$

subject to the law of motion of wealth (4). For simplicity, we look at the case in which the J function is sufficiently smooth.

Assumption 5. We assume that J in (6) gives the value of the supremum in equation (5), $J = \sup E_t \sum_{i=0}^{\infty} \delta^i U(c_{t+i})$, and is a strictly-concave, twice-continuously-differentiable function of the two states w_t and m_t.

Conditions for the equivalence between the sequential and the functional-equation problems are given in Stockey, Lucas, and Prescott (1989), who also provide sufficient conditions for the concavity of the value function. Sufficient conditions for the value function to be twice continuously differentiable when there is a countable number of states are given in Santos (1991).

In order to write the J function as a function of w_t and m_t alone, we need to assume that the investment opportunity set does not vary over time. Allowing for a stochastic investment opportunity set would introduce additional state variables which would appear as arguments of J. This would only complicate the mutual funds result on which the following Result 2 is based, in the obvious way.

The following first-order condition must hold for money to be held optimally:

$$\mathrm{E}_t\left[J_m\left(w_{t+1}, m_{t+1}\right)\ r_{m,t+1} - J_w\left(w_{t+1}, m_{t+1}\right)\ r_{w,t+1}\frac{R_{t+1}-1}{R_{t+1}}\right] = 0,$$

where $r_{w,t+1}$ is defined in (4) above. The price of transaction services per unit of real money balances determines the ratio between the expected utility return on money and the expected utility return on $\hat{w}_t$, the invested wealth:

$$\frac{R_{t+1}-1}{R_{t+1}} = \frac{\mathrm{E}_t\left[J_m\left(w_{t+1}, m_{t+1}\right) r_{mt+1}\right]}{\mathrm{E}_t\left[J_w\left(w_{t+1}, m_{t+1}\right) r_{wt+1}\right]}.$$

This condition makes it clear that without liquidity services money is a dominated asset: a positive nominal rate of interest arises in equilibrium only if the expected utility return of real money balances is also positive.

4 Hedging Inflation Risk

Note that in our framework real money balances inherited from the previous period are an argument of the transaction-costs function. Hence, money is a *state variable* for the investor's optimization problem, and there is specific hedging against the risk that changes in the price level will affect the value of real balances carried over to the next period.

Let $\mathbf{1}$ be a 2-dimensional unit vector, define the vector of portfolio shares $\mathbf{a} \equiv [a_s\ a_n]$, and let $\mathbf{r} \equiv [r_s\ r_b]'$ be the corresponding vector of real returns on stocks and bonds defined in Assumption 2 above. The vector notation allows for an immediate extension to the case of any number of securities. In our notation

$$\mathrm{var}_t\left(\mathbf{r}_{t+1}\right) \equiv \mathrm{E}_t\left[\left(\mathbf{r}_{t+1} - \mathrm{E}_t\left(\mathbf{r}_{t+1}\right)\right)\left(\mathbf{r}_{t+1} - \mathrm{E}_t\left(\mathbf{r}_{t+1}\right)\right)'\right],$$

and

$$\mathrm{cov}_t\left(\mathbf{r}_{t+1}, r_{m,t+1}\right) \equiv \mathrm{E}_t\left[\left(\mathbf{r}_{t+1} - \mathrm{E}_t\left(\mathbf{r}_{t+1}\right)\right)\left(r_{m,t+1} - \mathrm{E}_t\left(r_{m,t+1}\right)\right)\right].$$

Consider first the hedging portfolio $\mathbf{a}_{m,t}$, defined as

$$\mathbf{a}_{m,t} \equiv \frac{\mathrm{var}_t\left(\mathbf{r}_{t+1}\right)^{-1}\mathrm{cov}_t\left(\mathbf{r}_{t+1}, r_{m,t+1}\right)}{\mathbf{1}'\mathrm{var}_t\left(\mathbf{r}_{t+1}\right)^{-1}\mathrm{cov}_t\left(\mathbf{r}_{t+1}, r_{m,t+1}\right)}. \tag{7}$$

Shorting this portfolio provides insurance against inflation uncertainty, as it is the portfolio with maximum positive correlation with r_m. [For the derivation of a maximum-correlation portfolio see, for example, Ingersoll (1987), p. 282.] If an investor is *short* in $\mathbf{a}_{m,t}$, we say she is *hedging* inflation; if she is *long* in $\mathbf{a}_{m,t}$, we say she is *reverse-hedging* inflation.

Result 2. Under Assumptions 1–5 the investor will hedge (reverse-hedge) inflation if

$$J_{wm} < 0 \ (> 0).$$

There is inflation hedging only when money must be held for precautionary-liquidity motives.

Proof: Consider the optimal portfolio $\mathbf{a}_t$ which solves the Euler equation

$$\mathrm{E}_t\left[J_w\left(w_{t+1}, m_{t+1}\right)\left(\mathbf{r}_{t+1} - \mathbf{1}r_{f,t+1}\right)\right] = 0.$$

We linearize the first derivative of J around the expected values of its two arguments,

$$\mathrm{E}_t\left(w_{t+1}\right) = [\mathbf{a}_t'\,\mathrm{E}_t\left(\mathbf{r}_{t+1}\right) + \left(1 - \mathbf{a}_t'\mathbf{1}\right)r_{ft+1}]\hat{w}_t,$$

and

$$\mathrm{E}_t(m_{t+1}) = \hat{m}_t\mathrm{E}_t(r_{mt+1}).$$

The optimal portfolio of risky securities $\mathbf{a}_t$ is then (approximately)

$$\begin{aligned}\mathbf{a}_t \approx \ -\ & \mathrm{var}_t\left(\mathbf{r}_{t+1}\right)^{-1}\mathrm{E}_t\left(\mathbf{r}_{t+1} - r_{ft+1}\mathbf{1}\right)\frac{J_w}{J_{ww}\,\hat{w}_t} \\ & -\mathrm{var}_t\left(\mathbf{r}_{t+1}\right)^{-1}\mathrm{cov}_t\left(\mathbf{r}_{t+1}, r_{mt+1}\right)\frac{J_{wm}\hat{m}_t}{J_{ww}\,\hat{w}_t}.\end{aligned} \tag{8}$$

This is a mutual fund result for the optimal portfolio holdings like that of Merton (1971) which would hold exactly in the continuous-time limit of this economy. In the last term of equation (8) we recognize the mutual fund (7): since $\hat{w}_t$ and $\hat{m}_t$ are positive, $\mathbf{a}_{mt}$ is held long (short) if $-J_{wm}/J_{ww} > 0$ (< 0). By the concavity of the J function (Assumption 5) $J_{ww} < 0$, and we have the result.□

Equation (8) provides insights of some generality on the effects of liquidity on optimal portfolio choice. The investor forms her portfolio of risky assets buying shares in two mutual funds. The holding of the first portfolio,

$$\mathbf{a}_{d,t} \equiv \frac{\text{var}_t(\mathbf{r}_{t+1})^{-1} \, \text{E}_t(\mathbf{r}_{t+1} - r_{f,t+1}\mathbf{1})}{\mathbf{1}'\text{var}_t(\mathbf{r}_{t+1})^{-1} \, \text{E}_t(\mathbf{r}_{t+1} - r_{f,t+1}\mathbf{1})},$$

is always positive. This portfolio provides optimal diversification, and its interpretation is identical to that of non-monetary models of optimal portfolio selection. If the investment opportunity set changed with changes in S state variables, we would also have S additional mutual funds [see Merton (1971)].

What sets this monetary framework apart from the standard mutual funds result is the portfolio $\mathbf{a}_{m,t}$ of equation (7), which provides insurance against nominal uncertainty associated with the precautionary/liquidity role of money.

The assumption that only money set aside in advance can be used to reduce transaction costs (Assumption 4) is crucial to have a demand for insurance against inflation. If the liquidity services "produced" by $\hat{m}_t$ could be enjoyed in the current period, the amount of money balances could be decided only on the basis of the current consumption horizon, and the lifetime utility function J would no longer be a function of m_t. Thus the effects of money and inflation on the portfolio allocation problem would disappear.

Finally, equation (8) shows that the weight of the inflation-hedging mutual fund $\mathbf{a}_{m,t}$ in the portfolio depends on the ratio of money to wealth, which is consistent with the intuition that hedging inflation risk is more important when a large portion of the portfolio is held in cash.

We now turn to the issue of what determines the direction of inflation hedging. By the Benveniste and Scheinkman result [see, for example, Stockey, Lucas, and Prescott (1989), p. 84], we have

$$J_w(w_t, m_t) = \frac{U_c(c_t^*)}{1 + \phi_c(c_t^*, m_t)}, \tag{9}$$

where c_t^* denotes optimal consumption. Under Assumption 5, which ensures the smoothness of the value function, the policy function $c_t^*(w_t, m_t)$ is continuously differentiable [see Santos (1991)] and we can write

$$J_{wm}(w_t, m_t) = \frac{U_{cc}(c_t^*)\frac{dc_t^*}{dm_t}[1 + \phi_c(c_t^*, m_t)] - U_c(c_t^*)\left[\phi_{cc}(c_t^*, m_t)\frac{dc_t^*}{dm_t} + \phi_{cm}\right]}{[1 + \phi_c(c_t^*, m_t)]^2},$$

whose sign is undetermined, while J_{ww} is negative by the concavity of the value function (Assumption 5). By Result 2, the investor hedges inflation risk when $J_{wm} < 0$; or when

$$U_{cc}(c_t^*)(dc_t^*/dm_t)[1 + \phi_c(c_t^*, m_t)] - U_c(c_t^*)\left[\phi_{cc}(c_t^*, m_t)(dc_t^*/dm_t) + \phi_{cm}\right] < 0.$$

In realistic situations, dc_t^*/dm_t is positive, and an increase in real money balances has two opposite effects on the marginal utility of wealth. On the one hand, an increase in m_t allows for an increase in consumption, which reduces the marginal utility of consuming and increases the marginal cost of consuming: a *negative* effect on J_w. On the other hand, the increase in m_t tends to reduce the marginal cost of consuming ($\phi_{cm} < 0$): a *positive* effect on J_w. When it is the second effect to dominate, we have the somewhat surprising result that the investor reverse-hedges inflation because an increase in money increases the marginal utility of consumption discounted for marginal transaction costs. When the transaction-costs function is separable, $\phi_{cm} = 0$, the second effect is nil, and the investor unambiguously hedges inflation.

It is well known that the sign of the effect of real money on the marginal utility of wealth is also crucial for the properties of the equilibrium in a representative-agent economy [see, for example, Orphanides and Solow (1990)].

5 Examples

It is useful to consider specific transaction technologies used in the literature to see under which conditions hedging and reverse-hedging arise. Here, we take the investor to be the representative agent of an endowment economy. The period endowment is assumed constant and equal to one. Even if the period endowment is constant inflation is not neutral: inflation (determined in equilibrium) changes the quantity of m_t to be used in transactions and therefore affects consumption and utility. In equilibrium, $1 = c_t^* + \phi(c_t^*, m_t)$, and

$$\frac{dc_t^*}{dm_t} = -\frac{\phi_m(c_t^*, m_t)}{1 + \phi_c(c_t^*, m_t)}.$$

We explore hedging behavior in the following examples within this setting where dc_t^*/dm_t is a continuous function of m_t and it is meaningful to write J_{wm} as assumed in Assumption 5.

Baumol-Tobin transaction technology.

Assume the transactions-cost function is of the Baumol-Tobin type [see Feenstra (1986)]

$$\phi(c_t, m_t) = kc_t/m_t, \; k > 0,$$

which satisfies the conditions of Assumption 4. The direction of the inflation-hedging demand is determined uniquely by the degree of relative risk aversion of the investor. In fact, $dc_t^*/dm_t = -k/m_t^2$, and

$$J_{wm}\left(w_t, m_t\right) = \frac{kU_c\left[1 - \frac{-c_t^* U_{cc}(c_t^*)}{U_c(c_t^*, m_t)}\right]}{m^2[1 + \phi_c(c_t^*, m_t)]^2}$$

is positive or negative depending on the degree of relative risk aversion of the investor being below or above one.

Cobb-Douglas transaction technology.

Assume the transaction-costs function to be of the Cobb-Douglas type:

$$\phi_t = kc_t^\alpha m_t^{1-\alpha}, \quad \alpha > 1, \ k > 0,$$

which also satisfies the conditions in Assumption 4. The effect of m_t on the denominator of J_{wm} depends on the sign of

$$\left(\frac{dc_t^*/c_t^*}{dm_t/m_t} - 1\right) k\alpha(\alpha - 1)(c_t^*)^{\alpha-1} m_t^{-\alpha}.$$

Only when the elasticity of consumption to money is greater or equal to unity, the effect of an increase of m_t on the denominator of equation (9) is non-negative, and the overall sign of J_{wm} is unambiguously negative leading to *inflation hedging.* On the other hand, when the investor exhibits low risk aversion, and the elasticity of consumption to money is small, the sign of J_{wm} is positive and we have *reverse-hedging* of inflation risk. As in Example 1, reverse-hedging occurs when the dominant effect is the reduction in marginal transaction costs as real money balances increase.

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