Cross Holding and Imperfect Product Markets[†]

by

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Abstract

We consider a setting in which two firms first choose equity positions in each other's stock (cross holdings) and then compete in an imperfect product market. We demonstrate that cross holdings lead to higher firm profits and higher consumer surplus when the competitors' products are complements. We find that cross holdings lead to lower firm profits and higher consumer surplus when the products are substitutes. This finding is in contrast to the existing literature which establishes that cross holdings leads to higher firm profits and to lower consumer surplus. The contrasting results emerge because we solve for optimal cross holdings, whereas the existing literature considers exogenous cross holdings. In addition, allowing optimal cross holdings improves economic welfare. Furthermore, we demonstrate that cross holdings deter entry when the products are substitutes and facilitate entry when the products are complements.

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1 Introduction

This paper analyzes the role of cross holdings in non-voting stock among firms competing in imperfect product markets, where cross holding is defined as one firm's equity position (long or short) in another firm. Such cross holdings without voting rights arise in several industries. For example, on January 27, 1998, Northwest Airlines paid $500 million in cash and stock for a 14% equity position in Continental Airlines while relinquishing the associated voting right. Similarly, on August 6, 1997, Microsoft announced the purchase of $150 million of non-voting preferred stock of Apple Computer. Microsoft also held a 10% position in RealNetworks since July 1997 but announced selling off these shares because their products are no longer compatible.[1]

In this paper we demonstrate that when the firms' products are complements, optimal cross holdings entail long positions in the rival's equity. This finding is consistent with the cross holdings between Northwest Airlines and Continental Airlines for two reasons. First, these two airlines produce complementary products because they overlap on only eight routes worldwide. Second, although the airline industry does compete on prices, a huge capital investment is required (in airplanes) which implies that Cournot competition is appropriate, see Kreps and Scheinkman [1983]. We also consider the effect of cross holdings on industry structure and demonstrate that prohibiting cross

[1] See Wall Street Journal, November 19, 1998. Microsoft also purchased 5% of the shares in VDO Net Corp., see Wall Street Journal, August 19, 1997. Likewise, in the liquor industry, Guiness (UK) and LVMH Moet Henessy Louis Vuitton (France) held a 24 percent stake in each other's equity in 1988 (see House [1994]). In the telecommunications industry, Telefonica de Espana (TE) holds a 3.5 percent stake in Portugal Telecom who has the option on a 5% stake in a unit of TE (see Nairn [1998] and Wall Street Journal, March 3, 1998).

holdings decreases the likelihood of entry, decreases firm profits, and decreases consumer surplus when the products are complements.[2]

We consider a sequential game where two managers first choose a level of cross holding and then compete in a Cournot duopoly. First, we find that when the products are complements the resulting product market equilibrium exhibits higher quantity, lower price, higher profits, and higher consumer surplus relative to the situation where there are no cross holdings. The increased production arises because cross holdings cause the firm to internalize the positive externality that an increase in its output has on the profitability of the competitor. Second, when the products are substitutes the resulting product market equilibrium exhibits higher quantity, lower price, lower profits, and higher consumer surplus compared to the equilibrium when cross holdings are zero. Further, the interaction between cross holdings and product markets increases economic welfare.

To investigate the robustness of these results to the assumed market structure, we proceed to consider the scenario in which one firm can enter the monopoly market of another firm by incurring a fixed cost. We find that prohibiting cross holdings decreases (increases) the probability of entry when the products are complements (substitutes). Hence, the previous results are robust in the case of complements. However, when the products are substitutes cross holdings deter entry, that is, restrictions on cross holdings increase competition ex ante but decrease competition after market structure has been settled.

[2] The Justice Department has initiated a review of Microsoft's transactions to ensure their compliance with antitrust laws. The results of our model apply to the investment strategy of Microsoft if the products of Apple and Microsoft are complements. This would suggest that the concern of the Justice Department may not be warranted.

Previous papers consider cross holdings when the products are substitutes.[3] Reynolds and Snapp [1986] and Farrell and Shapiro [1990] compare the product market equilibrium of no cross holding for either firm to the simultaneous deviation of both firms to positive cross holdings. They find that when a firm has a long equity positions in the competitor and the products are substitutes, the result is less quantity produced, higher price, higher profits for both firms, and lower consumer surplus relative to the product market equilibrium obtained when there are no cross holdings. This, however, is not a Nash equilibrium in cross holding choice. For instance, if one firm unilaterally deviates from positive cross holding to zero cross holding it will increase its profits. Farrell and Shapiro [1990] also demonstrate when a firm would voluntarily increase its equity holdings in a rival, assuming that the firm has an initial equity position. If the firm increases this equity position, then own profitability goes down, but the firm makes a profit on the initial exogenous cross holdings since the rivals firm's profitability increases. The firm will want to increase its holdings if the profit on the initial position in the rival's stock out weights the loss on own profits. We demonstrate below, however, that if both firms start without cross holdings, then neither firm will want to increase to long equity positions in the setting considered in Farrell and Shapiro. Thus, their conditions under which a firm will want to increase cross holdings are driven entirely by the assumed initial cross holding position.

The paper proceeds as follows. Section 2 introduces the general model and characterizes the incentives for cross holdings given no initial equity position in the rival. Section 3 presents closed form solutions for firm profitability, consumer surplus, and

[3] See, among others, Flath (1991).

economic welfare when the demand curve is linear. Section 4 endognizes the market structure by considering the effect of restrictions on cross holding on the likelihood of entry. Section 5 summarizes the findings of the paper.

2. The Model

Two all-equity firms, denoted by *i* and *k*, produce differentiated products and compete in a two-stage game without uncertainty. In the first stage the firms simultaneously choose a position (long or short) in the rival's equity correctly anticipating that cross holdings affect the quantity decision that occurs in stage two. In the second stage the firms face Cournot competition, that is, firm managers simultaneously choose quantities. At each stage, firm managers make decisions that maximize the equity value of the firm.[4] We assume that prior to the second stage firm managers observe the cross holdings from the first stage and solve for the subgame perfect Nash equilibrium.

2.1 Operating Profit and Assumptions

In the first stage each firm can choose to take a position in the rival's equity. Let α^i (α^k) denote equity position of firm i (k) in its competitor. Except when explicitly stated, all assumptions are symmetric and we perform analyses only with respect to firm i. These positions are disclosed and the shares are purchased or sold in a competitive, full information capital market, so there is no profit or loss associated with the firm's equity position. We abstract from the control rights of (long) equity positions and consider only

[4] We assume that firms have no debt, and ignore any moral hazard problem that can arise through a shareholder - manager conflict. Thus, maximizing share value is commensurate to maximizing firm value.

silent interests. Without loss of generality, we ignore discounting which ensures that the value of the equity position at the end of the game will equal the purchase price of the position in the first stage. These assumptions allow us to focus on the strategic effect of the equity position on the product market; that is, the only reason for equity positions in our model is their effect on both firms' quantity choice in the product market.

In the second stage the firms play a Cournot game choosing quantities produced, q^i and q^k. At the end of the second stage revenues are determined and profits are realized. Each firm has access to a technology for production with total cost $C^i(q^i)>0$. The production revenue for firm i, denoted $R^i(q^i,q^k)$, is assumed to be twice continuously differentiable. It is assumed the firms' marginal revenues are decreasing. The following assumption on R^i is maintained throughout (the subscripts i and k denote the derivatives with respect to q^i and q^k respectively):

(1) $R^i_i < 0$

We consider two cases of possible interaction between the products. In the first case the products are complements, that is, an increase in q^k causes an increase in total revenue and marginal revenue for firm i. In the second case the products are substitutes, that is, an increase in q^k causes a decrease in total revenue and marginal revenue for firm i. These assumptions are summarized with the following equations:

Case 1 (Complements):

(2a) $R^i_k > 0$

(2b) $R^i_{ik} > 0$

Although we consider only two firms in the model, the results generalize to industries with many firms quite

Case 2 (Substitutes):

(2a') $R^i_k < 0$

(2b') $R^i_{ik} < 0$

The strategy of each firm consists of a position of cross holding for the first stage, and a quantity for the second stage which is a function of the equity positions chosen during the previous stage. Using subgame perfect equilibrium ensures that in the first stage, each firm makes the equity position decision correctly anticipating the rival's equity choice and the corresponding equilibrium outcome of the product market stage. Equilibrium strategies are determined using backward induction; thus, we first establish the firms' optimal quantity decisions given each possible level of cross holdings, and then solve for optimal cross holdings.

2.2 Product Market Equilibrium

In the product market, firms choose quantities simultaneously taking the equity positions of both firms, α^i and α^k, as given. Each firm chooses a quantity to maximize the value to its shareholders. The shareholders of firm i receive all the operating profits of firm i plus α^i of the operating profits of firm k. Let $\pi^i = R^i(q^i,q^k) - C^i(q^i)$ represent the operating profits of firm i. Firm i's total shareholder value, V^i, is

(3) $V^i = \pi^i + \alpha^i\pi^k$.

Firm i chooses its quantity to maximize equation (3). The first order condition which must be satisfied for firm i is:

(4) $0 = V^i_i = \pi^i_i + \alpha^i\pi^k_i = R^i_i(q^i,q^k) - C^i_i(q^i) + \alpha^i R^k_i(q^i,q^k)$.

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naturally, provided each firm has some degree of market power.

The second order condition that must be satisfied to insure an interior maximum is $V^i_{ii}<0$. In addition we need the following assumption to assure existence and stability of the Nash equilibrium:[5]

(5) $\quad V^k_{kk} V^i_{ii} - V^i_{ik} V^k_{ki} > 0.$

Theorem 1: *For complementary (substitute) products, an increase in α^j causes an increase (decrease) in q^i.*

Theorem 2: *Consider a firm without cross holdings. If the products are complements, an increase in α^j causes an increase in q^k if and only if $R^i_{ki} + \alpha^j R^k_{ki} > 0$. If the products are substitutes, then an increase in α^j causes an increase in q^k if and only if $R^i_{ki} + \alpha^j R^k_{ki} < 0$.*

Proof. See Appendix.

Theorem 1 states how the product market equilibrium is affected by a change in one firm's equity position. The impact of a change in cross holdings on the product market depends on whether the products are substitutes or complements. When firm i increases its cross holding, α^i, the firm increases its emphasis on the profits of the rival when making output market decisions. When the products are complements then firm i will want to increase its quantity, which also has a positive effect on the profits of firm k. Firm k will either increase or decrease quantity depending on the sign of $R^i_{ki} + \alpha^i R^k_{ki}$. Note that if $\alpha^i > 0$ then the term $R^i_{ki} + \alpha^i R^k_{ki}$ will be positive and an increase in α^i will lead to an increase in q^k.

When the products are substitutes an increase in α^i will lead firm i to decrease its quantity, which has a positive effect on the profits of firm k. Again the reaction of firm k to an increase in α^i depends on the sign of $R^i_{ki} + \alpha^i R^k_{ki}$. If $R^i_{ki} + \alpha^i R^k_{ki}$ is negative then

[5] This is a common assumption for these models, see Brander and Lewis [1986]. Intuitively this assumes that when a firm changes its own quantity this has a larger effect on the firm's value than when the competition changes its quantity.

an increase in α^i leads firm k to increase its quantity. Note that this is true when α^i is close to zero. In section 3 we show that when demand is linear and marginal cost is constant then $R^i_{ki} + \alpha^i R^k_{ki} > 0$ if the products are complements and $R^i_{ki} + \alpha^i R^k_{ki} < 0$ if the products are substitutes.

2.3 Equity Market Equilibrium

Prior to the product market stage, both firms are allowed to trade in each other's stock. Each firm chooses an equity position in the rival firm simultaneously. These positions are announced and procured in a competitive capital market. Each firm chooses its equity position, correctly anticipating the choice of the rival firm and with full knowledge of how the two equity positions chosen will affect the product market decision of each firm. At this stage, total shareholder value is the sum of the operating profit of the firm and the net revenue from purchasing the fraction α of the counterpart firm's operating (i.e., the profits less the cost of acquiring the equity position). Put formally,

(6) $V^i = \pi^i + (\alpha^i \pi^k$ - Cost of equity position).

Recall that the cost of acquiring the equity position is equal to the second stage payoff of the equity position, because there is no uncertainty, no discounting, and investors have rational expectations.[6] Thus, in the first stage, the firm chooses its equity position to maximize operating profits:

(7) $V^i = \pi^i = R^i(q^i,q^k) - C^i(q^i)$

The first order condition for firm i is:

$$dV^i/d\alpha^i = (dR^i/dq^i)(dq^i/d\alpha^i) + (dR^i/dq^k)(dq^k/d\alpha^i) - (dC^i/dq^i)(dq^i/d\alpha^i) = 0.$$

This can be rewritten as:

$$(8) \qquad dV^i/d\alpha^i = [(dR^i/dq^i) - (dC^i/dq^i)]\,(dq^i/d\alpha^i) + (dR^i/dq^k)(dq^k/d\alpha^i) = 0.$$

In general we cannot determine the extent of cross holding (long or short) from the above first order condition. We can, however, show that when the products are related then either firm having zero cross holding is not optimal.

Theorem 3: *Given no cross holdings, each firm manager has an incentive to take a long (short) position in the competitor if the products are complements (substitutes).*

Proof. See Appendix.

This suggests a new rationale for short selling since previous explanations rely on information based trading as in, for example, Diamond and Verrecchia [1987], Committee on Government Operations [1991], and Hansen and Lott [1995].

3. Linear Demand Example

In this section we derive the closed form solution to the game assuming constant marginal cost, C, and the following inverse demand function for firm i:

$$p^i = A - q^i - Bq^k$$

where p^i is the unit price for firm i's product and A and B are constants. A captures the general level of demand in the market, and B characterizes the relation between the firms' products. We assume that $|B|<1$ which means that a change in the firm's own output has more effect on the price the firm receives than a change in the competing firm's output.

[6] See Grossman and Hart [1980].

3.1 Product Market Equilibrium

From the first order condition of equation (4) we derive the following reaction curve for firm i:

$$q^i = \frac{(A - C)}{2} - \frac{(1+\alpha^i)Bq^k}{2}.$$

Solving the two corresponding reaction functions simultaneously yields the optimal quantity as a function of each firm's equity position:

(9) $$q^i = \frac{(A - C)[2 - (1+\alpha^i)B]}{[4 - (1+\alpha^i)(1+\alpha^k)B^2]}.$$

The second order conditions for an optimum holds.

Corollary 1: *If the products are substitutes, then an increase in α^j causes a decrease in q^j and an increase in q^k. If the products are complements, then an increase in α^j causes an increase in q^j and an increase in q^k.*

Corollary 1 illustrates Theorems 1 and 2. In particular, when the products are substitutes (B>0) then firm i will want to decrease its quantity, which has a positive effect on the profits of firm k (see Figure 1). Likewise, when the products are complements (B<0) then firm i will want to increase its quantity, which also has a positive effect on the profits of firm k (see Figure 2). In both cases the added profitability of firm k will induce it to increase its quantity.

3.2 Equity Market Equilibrium

In the first stage, each firm manager chooses the equity position to maximize operating profits, anticipating correctly that in stage 2, the quantity that each firm chooses will satisfy equation (9). Substituting equation (9) and the corresponding optimal quantity for firm k into equation (7), taking the first order condition and solving, yields the following reaction function for equity position:

(10) $$\alpha^i = \frac{(1+\alpha^k)(2-B)B}{(-4+2B+B^2)+\alpha^k(2B+B^2)}.$$

Solving the two corresponding reaction functions simultaneously yields the optimal equity position for each firm.[7]

Theorem 4: *In equilibrium, the optimal amount of cross holding is* $\alpha^i = \frac{-B}{2+B}$ *and each firm produces* $q^i = \frac{(A-C)[2+B]}{4(1+B)}$. *Furthermore,* $\frac{dq^i}{dB} < 0, \frac{dp^i}{dB} < 0,$ *and* $\frac{d\pi^i}{dB} < 0.$

Recalling that |B|<1, it follows immediately from Theorem 4 that if the products are complements (B<0) then the optimal level of cross holdings consists of a long position in the competitor's equity. Likewise, if the products are substitutes (B>0) then the optimal level of cross holdings consists of a short position in the competitor's equity. Our result is in contrast to Flath [1991] who precludes short selling and therefore finds no strategic role of cross holdings under Cournot competition with substitute products. Note that as the degree of substitution increases, the quantity increases, and prices and firm profits decrease.

[7] This results in a second order polynomial with two roots where the second root, $\alpha_i=(2/B)-1$, is a minimum.

Past literature has only examined consumer surplus for exogenous cross holdings. We now examine the welfare properties associated with optimal cross holdings.

Theorem 5*: Firm profits are lower when cross holdings are allowed if and only if the products are substitutes. Furthermore, consumer surplus and economic welfare are higher when cross holdings are allowed than when cross holdings are prohibited.*

Proof. See Appendix.

Theorem 5 demonstrates that regulatory restrictions on cross holdings may decrease consumer surplus and economic welfare.

4. Cross Holding and Entry

We now examine how cross holding affects the entry decision. Assume that firm i is the incumbent in the market. Firm k has the ability to enter the market, but must pay a fixed cost E. E is a random variable, drawn from the cumulative distribution function, F(E), which is common knowledge. Firm i does not know the entry costs, E, but firm k will know E prior to making its entry decision. For simplicity, we ignore the possibility of other entrants. If firm k does not enter, then firm i is a monopolist in the product market. If firm k enters then the firms compete through Cournot competition in the product market. Maintaining the assumptions of section 3, both firms are assumed to have constant marginal cost, C, and face the same linear demand curve if they are active in the product market.

Hansen and Lott [1995] also consider how cross holdings affect entry. They demonstrate that a firm considering entering a new market can benefit from cross

holding positions in the incumbent firm because the entrant has private information about its own entry decision. If stock prices are not fully revealing the entrant gets positive trading profits from taking a short (long) position when deciding to enter (not enter). In contrast, our cross holding positions are not information based and decrease firm profits.

4.1 Equilibrium Without Cross Holdings

First we consider the entry decision and resulting product market decisions when cross holding is prohibited. In this situation the game proceeds as follows:

1. Firm k learns E.
2. Firm k decides to enter or stay out.
3. Firm i and firm k (if entered) chose quantity simultaneously.
4. Profits are realized.

First consider the case where firm k does not enter. In this situation, firm i is a monopolist with inverse demand of:

$$p^i = A - q^i$$

and constant marginal cost, C. The well known solution for the product market involves firm i producing $q^i = ½(A-C)$. Profits for firm i are $[½(A-C)]^2$. Next consider the product market result if firm k enters. This situation is identical to the linear demand example above, with $\alpha^i = \alpha^k = 0$. The quantity for firm i can be determined by substituting $\alpha^k = 0$ into equation (3), which yields: $q^{i0} = \left(\frac{A-C}{2+B}\right)$ and firm k produces the same quantity due to the symmetry of this case. The prices for firm i is $p^{i0} = A - (1+B)\left(\frac{A-C}{2+B}\right)$ and firm k

receives the same price. The profits (excluding entry cost for firm k) for each firm are $\frac{(A - C)^2}{[2+B]^2}$.

We can now take a step back and look at the entry decision of firm k. If firm k does not enter it will receive profits of zero. If firm k does enter it will receive total profits of $\frac{(A - C)^2}{[2+B]^2} - E$. Thus, firm k will enter whenever $E < \frac{(A - C)^2}{[2+B]^2}$, and the probability of entry is $F\left(\frac{(A - C)^2}{[2+B]^2}\right)$.

4.2 Equilibrium With Cross Holding

If cross holding is allowed we now assume that prior to the product market decision both firms can choose to take an equity position (long or short) in the rival. The game now proceeds as follows:

1. Firm k learns E
2. Firm k decides to enter or stay out
3. Firm i and k choose cross holding simultaneously
4. Firm i and firm k (if entered) choose quantity simultaneously
5. Profits are realized.

If firm k does not enter the product market results are the same as above. In addition, since we have perfect capital markets there are no incentives for either firm to go long or short in the equity of the competition in stage 3. This means firm i will realize the monopoly profits, and firm k will receive zero profits.

If firm k enters the resulting equity market and product market equilibrium is given by Theorem 4. This yields profits (net of entry costs) of $\frac{(A-C)^2[4-B^2]}{16(1+B)}$ for each firm. Going back to the entry decision, firm k will receive total profits of zero it the firm stays out of the market and total profits of $\frac{(A-C)^2[4-B^2]}{16(1+B)}-E$ if the firm enters the market. This means the firm will enter the market if $E<\frac{(A-C)^2[4-B^2]}{16(1+B)}$ and the probability of entry is $F\left(\frac{(A-C)^2[4-B^2]}{16(1+B)}\right)$.

Theorem 6: *If the products are complements (substitutes) then the ability to cross hold increases (decreases) the probability of entry.*

Proof. See Appendix.

The intuition of theorem 6 is straight forward. If the products are substitutes then if cross holding is allowed there will be lower profits for each firm in the product market. Because of lower profits in the product market, entry is less attractive relative to the case when cross holding is prohibited. However, if the products are complements then the allowance of cross holding increase firm profits in the product market. This makes entry more likely when products are complements if cross holding is allowed relative to when cross holding is prohibited. This illustrates how cross holdings can affect industry structure.

5. Conclusion

This paper analyzes the incentives and consequences of corporations' trading in the equity of their rivals. We add to the existing literature in several ways. First, we analyze cross holding for companies that produce both complementary and substitute products. Second, we analyze both short and long equity positions. Third, we fully endogenize the cross holding choice and analyze the optimal cross holding conditional on the type of competition. This analysis reveals a new role for short selling, where short selling can be used to commit a firm to an aggressive product market stance. Fourth, we analyze how optimal cross holdings affect consumer surplus and economic welfare. We find that when firms engage in optimal cross holding behavior, both consumer and economic welfare can improve, even though firm profits decrease when the products are substitutes. Finally, we investigate the effect of cross holdings on entry.

APPENDIX

PROOF OF THEOREMS 1 AND 2:

Totally differentiate equation (4) and the corresponding first order condition for firm k with respect to q^i, q^k, and α^i. This gives the following equations:

$$V^i_{ii}dq^i + V^i_{ik}dq^k + V^i_{i\alpha(i)}\, d\alpha^i = 0$$

$$V^k_{ki}\, dq^i + V^k_{kk}\, dq^k + V^k_{k\alpha(i)}\, d\alpha^i = 0.$$

Note that $V^k_{k\alpha(i)} = 0$, so the equations simplify to:

$$V^i_{ii}\, dq^i + V^i_{ik}\, dq^k + V^i_{i\alpha(i)}\, d\alpha^i = 0$$

$$V^k_{ki}\, dq^i + V^k_{kk}\, dq^k = 0.$$

Solving these equations simultaneously gives:

(A1) $dq^i/d\alpha^i = - V^k_{kk}\, V^i_{i\alpha}/H$

(A2) $dq^k/d\alpha^i = V^i_{ki}\, V^i_{i\alpha}/H$

where $H = V^k_{kk}\, V^i_{ii} - V^i_{ik}\, V^k_{ki}$. Note that $H>0$ by assumption and that $V^k_{kk} < 0$ by the second order condition.

Lets now examine the situation when the products are complements. First we determine the sign of (A1). Since $H>0$ by assumption, and $V^k_{kk} < 0$ by the second order condition, the sign of (A1) is the same as the sign of $V^i_{i\alpha}$.

$V^i_{i\alpha} = R^i_k > 0$ by assumption (2a).

Thus, (A1) $= dq^i/d\alpha^i > 0$. Next we determine the sign of (A2). Again, $A>0$ by assumption, and we now know that $V^i_{i\alpha}>0$. Therefore, the sign of (A2) is determined by the sign of V^i_{ki}.

(A3) $V^i_{ki} = R^i_{ki} + \alpha^i R^k_{ki}$

Thus, if $R^i_{ki} + \alpha^i R^k_{ki} > 0$ then $dq^k/d\alpha^i > 0$ and if $R^i_{ki} + \alpha^i R^k_{ki} < 0$ then $dq^k/d\alpha^i < 0$.

Next we consider the situation when the products are substitutes.

First we determine the sign of (A1). $H>0$ and $V^k_{kk} < 0$ as before. However, $V^i_{i\alpha} = R^i_k$ is now negative. This means that the sign of (A1) will be negative. Second, we consider the sign of (A2). We know that $H>0$ and $V^i_{i\alpha} < 0$. Thus, the sign of (A2) is the opposite

sign of $V^i_{ki} = R^i_{ki} + \alpha^i R^k_{ki}$. Therefore, if $V^i_{ki} = R^i_{ki} + \alpha^i R^k_{ki} < 0$ then (A2) is greater than zero, and if $V^i_{ki} = R^i_{ki} + \alpha^i R^k_{ki} > 0$ (A2) is less than zero.

PROOF OF THEOREM 3:

To show this we examine the situation when a firm has zero cross holding (i.e., $\alpha^i = 0$). If the products are complements we know that $dq^k/d\alpha^i > 0$ from theorem 1 and $R^i_k > 0$ by assumption. In addition, we know that $R^i_i(q^i,q^k) - C^i_i(q^i) = 0$ from equation (4). This means that equation (7) when $\alpha^i = 0$ reduces to:

$$dV^i/d\alpha^i = (dR^i/dq^k)(dq^k/d\alpha^i) > 0.$$

This means the firm can increase its shareholder value by increasing its cross holdings from zero (i.e., by taking a long position in the competitor).

When the products are substitutes and $\alpha^i = 0$, we know that $dq^k/d\alpha^i > 0$ from theorem 1 and $R^i_k < 0$ by assumption. In addition, we know that $R^i_i(q^i,q^k) - C^i_i(q^i) = 0$ from equation (4). This means that equation (7) when $\alpha^i = 0$ reduces to:

$$dV^i/d\alpha^i = (dR^i/dq^k)(dq^k/d\alpha^i) < 0.$$

This means the firm can increase its shareholder value by decreasing its cross holdings from zero (i.e., taking a short position in the competitors stock).

PROOF OF THEOREM 5:

First, we solve the model for the case where no cross holdings are allowed, that is, $\alpha^i = \alpha^k = 0$, indexing the corresponding equilibrium values by superscript 0.

Substituting into equation (3) yields the quantity $q^{i0} = \left(\frac{A-C}{2+B} \right)$ and price $p^{i0} = A - (1+B)\left(\frac{A-C}{2+B} \right)$.

Second, we prove that prices are always lower or, equivalently, quantities are always higher, when cross holdings are allowed.

$A - (1+B)\, q^k = p^i < p^{i0} = A - (1+B)\, q^{k0}$

$\Leftrightarrow \quad \frac{(A - C)[2+B]}{4(1+B)} = q^k > q^{k0} = \frac{(A - C)}{[2+B]}$

$\Leftrightarrow \quad [2+B]^2 > 4(1+B)$

$\Leftrightarrow \quad B^2 > 0$

which is true.

Third, we identify sufficient and necessary conditions for firm profits to be higher when short selling is allowed than when it is prohibited:

$$\frac{(A - C)^2[4 - B^2]}{16(1+B)} = \pi^i > \pi^{i0} = \frac{(A - C)^2}{[2+B]^2}$$

$\Leftrightarrow \quad (4-B^2)[4+4B+B^2] > 16\,(1+B)$

$\Leftrightarrow \quad 16 + 16B + 4B^2 - (4B^2 + 4B^3 + B^4) > 16 + 16\,B$

$\Leftrightarrow \quad B^2\,(4B + B^2) < 0$

$\Leftrightarrow \quad B < 0$

Finally, defining Economic Welfare as

$W^i = S^i + \pi^i = \frac{1}{2}\left(A - p^i\right)\left(q^i - 0\right) + \left(p^i - C\right)q^i$

where S^i is the consumer surplus, we demonstrate that economic welfare is always lower when firms are precluded from cross holdings, that is, $W^i > W^{i0}$.

$$\frac{1}{2}(1+B)\left(q^i\right)^2 + \left(p^i - C\right)q^i = W^i > W^{i0} = \frac{1}{2}(1+B)\left(q^{i0}\right)^2 + \left(p^{i0} - C\right)q^{i0}$$

or

$$\frac{1}{2}(1+B)\left(\frac{(A-C)[2+B]}{4(1+B)}\right)^2 + \frac{(A-C)^2\left[4-B^2\right]}{16(1+B)} > \frac{1}{2}(1+B)\left(\frac{A-C}{2+B}\right)^2 + \left(\frac{A-C}{2+B}\right)^2$$

or

$$[2+B]^4 + 2\left[4-B^2\right][2+B]^2 > 16(1+B)^2 + 32(1+B)$$

or

$$48 + 64B + 24B^2 - B^4 > 48 + 64B + 16B^2$$

or

$$8B^2 > B^4 .$$

PROOF OF THEOREM 6:

The probability of entry is higher when cross holdings are allowed than when cross holdings are prohibited if and only if

$$F\left(\frac{(\mathrm{A}-\mathrm{C})^2[4-\mathrm{B}^2]}{16(1+\mathrm{B})}\right) > F\left(\frac{(\mathrm{A}-\mathrm{C})^2}{[2+\mathrm{B}]^2}\right)$$

or, recalling that |B|<1,

$$[4-\mathrm{B}^2][2+\mathrm{B}]^2 > 16(1+\mathrm{B}) .$$

This reduces to

$- 4B^3 - B^4 > 0$

or $B<0$. Likewise, the probability of entry when cross holding is allowed is less than the probability of entry when cross holding is prohibited when the products are substitutes ($B > 0$).

Figure 1

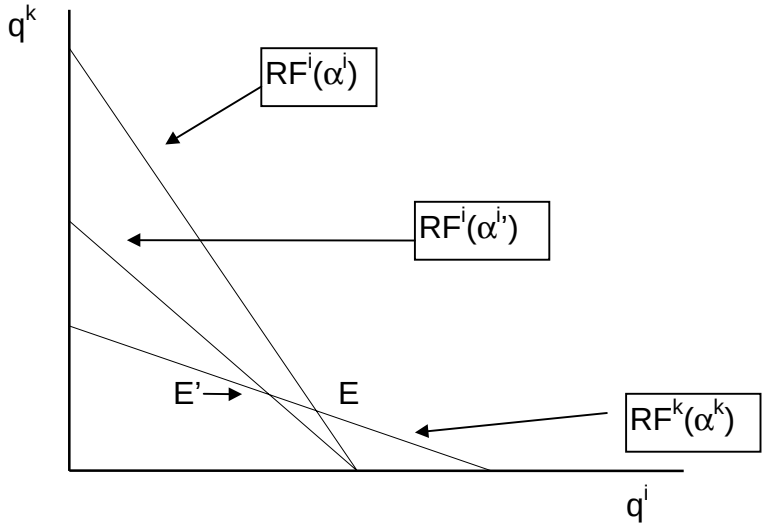

Figure 1 presents the output market reaction functions for each firm when the products are substitutes, B>0. $RF^k(\alpha^k)$ is the reaction function of firm k, for an arbitrary amount of cross holding, α^k. Likewise, $RF^i(\alpha^i)$ is the reaction function for firm i given cross holdings of α^i. This graph shows how the reaction function of firm i changes when it increases cross holdings from α^i to $\alpha^{i'}$. When firm i increases its cross holdings, it increases the weight it puts on the competitor's profits when choosing its own quantity. This causes it to compete less aggressively and moves the output market equilibrium from E to E', which results in lower output from firm i and higher output from firm k.

Figure 2

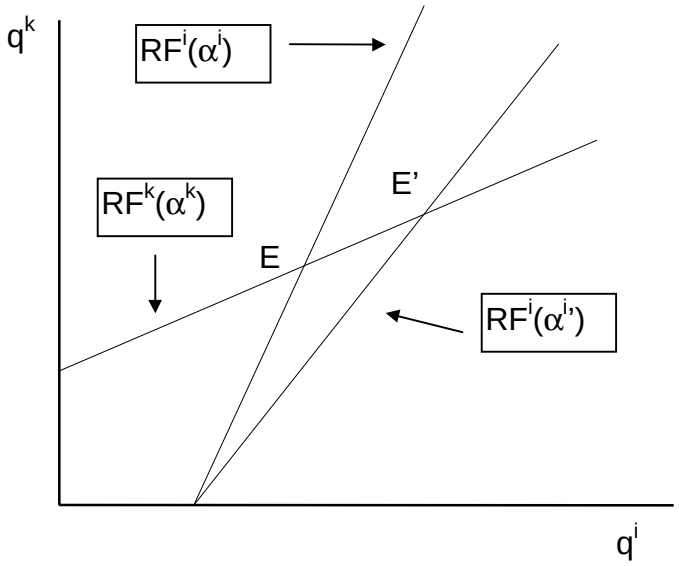

Figure 2 presents the output market reaction functions for each firm when the products are complements, B<0. This graph shows how the reaction function of firm i, $RF^i(\alpha^i)$, changes when it increases cross holdings from α_i to α'_i. When firm i increases its cross holdings it increases the weight it puts on the competitor's profits when choosing its own quantity. When the firms' products are complements, this causes the firm to increase its production, which has a positive effect on the rival firm's profits. The equilibrium moves from E to E' which results in higher output from both firms.

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