Margin Rules, Informed Trading in Derivatives, and Price Dynamics[1]

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Abstract

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We analyze the impact of option trading and margin rules on the behavior of informed traders and on the microstructure of stock and option markets. In the absence of binding margin requirements, the introduction of an options market causes informed traders to exhibit a relative trading bias towards the stock because of its greater information sensitivity. In turn, this widens the stock's bid-ask spread. But when informed traders are subject to margin requirements, their bias towards the stock is enhanced or mitigated depending on the leverage provided by the option relative to the stock, leading to wider or narrower stock bid-ask spreads. The introduction of option trading, with or without margin requirements, unambiguously improves the informational efficiency of stock prices. Margin rules improve market efficiency when stock and option margins are sufficiently large or small but not when they are of moderate size.

1 Introduction

A number of empirical studies have studied the impact of derivatives trading on the market for the underlying stock. The broad conclusions that have been drawn from these studies are that options listing leads to a reduction in the volatility of stock returns, a reduction in stock bid-ask spreads, and an increase in the informativeness of stock prices.[1]

In contrast to this abundance of empirical research, there are relatively few analytical models that examine the impact of option trading on stock and option prices. Most derivative pricing models assume complete markets where derivatives are redundant securities and hence not traded in equilibrium. But when traders with private information about the underlying stock can choose to trade the stock or the option, then option prices and trades contain valuable information and are no longer redundant. For example, Grossman (1988) argues that even when options can be synthetically replicated by dynamic trading strategies, their absence will prevent the transmittal of information to market participants and lead to real effects such as more volatile stock prices. Similarly, Back (1993) presents a model with asymmetrically informed traders and shows that the introduction of an option causes the volatility of the underlying stock to become stochastic. Easley, O'Hara and Srinivas (1998) develop and test a market microstructure model of informed traders who can trade the stock or the option and present evidence of informed trading in the options market, i.e., certain option trades contain information about future stock price movements. In a departure from these models, Biais and Hillion (1994) examine the impact of option trading on an incomplete market. They show that even though options trading mitigates the market breakdown problem caused by asymmetric information and market incompleteness, its impact on the informational efficiency of the market is ambiguous. Brennan and Cao (1996) use a noisy rational expectations model to demonstrate that the welfare gains that accrue to informed and uninformed traders from multiple rounds of trading in a risky asset can be achieved in a single round of trading by introducing

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[1]For evidence on the reduction in volatility, see Conrad (1989) and Skinner (1989); on the reduction in bid-ask spreads, see Damodaran and Lim (1991) and Fedenia and Grammatikos (1992); and on improved efficiency, see Damodaran and Lim (1991) and Jennings and Starks (1986). See Mayhew (1999) for a more exhaustive list of references.

a quadratic option.

A common deficiency of the above-mentioned studies is that they ignore an important institutional feature of modern markets – the presence of margin requirements when trading stocks and options. When traders are not subject to wealth constraints, they maximize their trading profits and margin requirements do not play a role in their optimal trading strategies. But in more realistic settings where traders do face wealth constraints, the differential margin requirements on these securities can affect their trading strategies and the resulting equilibrium market prices. In this paper, we take a first step in this direction by explicitly characterizing the impact of margins on the strategies of informed traders and on trading prices in the stock and options markets. We start by postulating the existence of informed traders with noisy private signals, exogenous liquidity traders and competitive market makers. We analyze the optimal trading strategies of the informed traders and the equilibrium prices set by the market makers in three different settings:

- Trading is allowed only in the stock market (the *ss* scenario).

- Trading is allowed in the stock and options markets and margin requirements are not binding (the *so* scenario).

- Trading is allowed in the stock and options markets and margin requirements are binding (the *sm* scenario).

The advantage of this setup is that it allows us to examine both the impact of option trading (by comparing the equilibria in the *so* and *ss* worlds) and the impact of margin requirements (by comparing the equilibria in the *sm* and *so* worlds). We show that when option trading is allowed without margin requirements, informed traders face a tradeoff between trading too aggressively in either market and facing larger trading costs (bid-ask spreads) in that market. In equilibrium, they split their trades between the stock and the option although they exhibit a bias towards the stock due to its greater information sensitivity compared with the option (since the option *delta* is less than one). When stock and option margin requirements are added to the picture, the leverage provided by the option may offset the information sensitivity edge of the stock and reduce

or eliminate the informed traders' bias towards stock trading. We show that their optimal trading strategy depends on the relative margin requirements in the two markets and consequently, bid and ask prices in these markets will also be functions of these margin requirements.

We find that the introduction of option trading improves the informational efficiency of stock prices irrespective of whether binding margin requirements are in place or not. Intuitively, even though the addition of option trading enhances the ability of informed traders to disguise and profit from their trades, the informativeness of the trading process is greater because the market can now infer private information from two sources – order flow in the stock *and* option markets. However, a comparison of the *so* and *sm* worlds reveals that the introduction of margin requirements has an ambiguous effect on stock market efficiency and we derive the sufficient conditions for the efficiency of stock prices to be greater in the *sm* world than in the *so* world. These conditions suggest that market efficiency improves with margin requirements if these requirements for the stock and for the option relative to the stock are either large or small. However, market efficiency worsens when these margins take on intermediate values.

On comparing the bid-ask spread for the stock in the *ss* and *so* worlds, we find that the introduction of option trading without margins *increases* the spread. Even though the option market captures trading volume from both informed and liquidity traders, it captures relatively less of the former given their bias towards stock trading. Thus, the relative threat of informed trading in the stock market actually *increases* after the introduction of option trading, causing the market maker to set wider spreads. But this bias does not survive when we introduce margin requirements and we identify the conditions under which stock bid-ask spreads are smaller in the *sm* world than in either the *ss* or *so* worlds. We find that this occurs when stock margins are relatively large and option margins are relatively small.

The impact of margin trading on stock markets is an issue of considerable interest to economists. Garbade (1982) argues that margin trading can create destabilizing *pyramid* effects on stock prices. Chowdhry and Nanda (1998) analytically confirm the validity of this conjecture by presenting a model where margin trading induces market instability. However, the empirical evidence on this

issue is mixed. Consistent with this hypothesis, Hardouvelis (1990) finds that curbing margin trading by increasing margin requirements reduces stock volatility. However, Hsieh and Miller (1990), Seguin and Jarrell (1993), and others find that margin trading has no impact on stock prices or volatility. At the other end of the spectrum, Seguin (1990) presents evidence of margin trading *reducing* stock volatility and improving stock liquidity. These studies examine the effect of margins in a single-asset framework. Our paper adds to this research stream by analyzing the role of margins in a multi-asset (or multiple-market) setting. Specifically, we examine how stock and options margins affect trading strategies and prices in these two markets. This allows us to generate empirical and policy implications on the impact of margin trading on interrelated markets.

The rest of the paper is organized as follows. In Section 2, we describe the basic structure of the model, derive the equilibrium for the *ss* case, and analyze its properties. In Section 3, we introduce option trading into the picture (the *so* case), analyze the resulting equilibrium and compare it to the *ss* case. In Section 4, we introduce binding stock and option margin requirements into the picture (the *sm* case), analyze the resulting equilibrium and compare it to the *ss* and *so* cases. Finally, we present the empirical predictions of our model and conclude in Section 5.

2   The Model With Only Stock Trading

In this section, we develop a trading model in the spirit of Glosten and Milgrom (1985) where agents can trade only in the stock market and are not subject to margin constraints. The sole traded asset in the market is a stock whose future value is uncertain. There are three types of traders in this market: informed traders, liquidity traders and a market maker. All traders are assumed to be risk-neutral and the risk-free interest rate is assumed to be zero. The informed traders receive private signals about the stock's future value and trade based on this information. The liquidity traders are uninformed and have exogenous motives for trade such as portfolio rebalancing. Their presence is necessary to camouflage the informed trades and avoid the *no trade* equilibrium of Milgrom and Stokey (1982). The informed and liquidity traders trade with a competitive market maker who is

assumed to set prices rationally.

The stock's per-share value $\tilde{v}$ depends on the future state of the world θ. There are two possible states of the world in the future, low (L) and high (H), that are equally likely to occur. The stock values are given by v_L and v_H for $\theta = L$ and $\theta = H$, respectively, where $v_L < v_H$. Therefore, the unconditional expected value of the stock is $\bar{v} = (v_L + v_H)/2$. Although the future state of the world is currently unobservable, the informed traders receive identical noisy private signals S about θ, which is either *good news* $(S = G)$ or *bad news* $(S = B)$. The precision of this signal is measured by the probability μ that it is accurate about the state θ, i.e., $Pr(S = G \mid \theta = H) = Pr(S = B \mid \theta = L) = \mu$. Conversely, $1 - \mu$ measures the probability that the signal is inaccurate, i.e., $Pr(S = G \mid \theta = L) = Pr(S = B \mid \theta = H) = 1 - \mu$. In order to ensure that the signal is informative, we assume that $\mu > 0.5$. We also assume that μ is common knowledge.[2]

The sequence of events in the model is as follows. At $t = 0$, the informed traders privately observe a signal S about the future state θ. At $t = 1$, the informed and liquidity traders submit their orders to a market maker who transacts a single, randomly selected order at his quoted bid or ask price. At $t = 2$, the stock price adjusts to reflect the information contained in the actual trade that occured at $t = 1$. Finally, at some distant date $t = 3$, the state of the world θ is realized and publicly observed by all market participants. Note that trade occurs only on date 1 and there is no trade on date 2. This date is introduced only as a modeling device to measure the amount of information revealed by the date-1 trade.

The trading environment in the stock market has the following features. The market maker randomly selects a single order to transact from among the orders submitted to him by the informed and liquidity traders.[3] We denote the fraction of informed and liquidity traders in the market as α and $1 - \alpha$, respectively. The liquidity traders are equally likely to submit buy or sell orders.

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[2]This assumption rules out the possibility of informed traders following a trading strategy where they manipulate the market into thinking they are more or less informed than they really are.

[3]Our single-trade convention is in keeping with the spirit of the Glosten and Milgrom (1985) model where prices are set on an order-by-order basis. Alternatively, we can follow the Admati and Pfleiderer (1989) convention wherein all aggregated sell (buy) orders are transacted at a single bid (ask) price. We can confirm that our results are unchanged under this alternative specification although the market efficiency computations are considerably more messy.

We assume that all traders submit orders of one share each.[4] Informed traders are conjectured to submit a buy order if they receive good news and a sell order if they receive bad news, i.e., their conjectured trading strategy is given by

$$X^{ss}(S) = \begin{cases} \text{buy stock} & \text{if } S = G \\ \text{sell stock} & \text{if } S = B \end{cases} \tag{1}$$

The market maker transacts a sell order at his quoted bid price B_S^{ss} and a buy order at his quoted ask price A_S^{ss}.[5] He sets his bid and ask prices competitively and rationally, i.e., so as to make zero expected profits on each trade taking into account the information conveyed by the trade. Therefore, he will set $B_S^{ss} = E(\tilde{v} \mid \text{stock sale})$ and $A_S^{ss} = E(\tilde{v} \mid \text{stock buy})$ where he conditions on the information contained in the incoming order.

We define the usual *Bayesian Nash* equilibrium in this market as comprising of the informed trading strategy $X^{ss}(S)$ and the prices $\{B_S^{ss}, A_S^{ss}\}$ that satisfy the following two conditions:

1. Given the market maker's prices $\{B_S^{ss}, A_S^{ss}\}$, the informed traders' strategy $X^{ss}(S)$ maximizes their expected profits.

2. Given the informed trading strategy $X^{ss}(S)$, the market maker sets prices $\{B_S^{ss}, A_S^{ss}\}$ so as to make zero expected profits conditional on the incoming order.

The following proposition characterizes the resulting equilibrium in the *ss* world.

Proposition 1 *In a world where only stock trading is allowed, the equilibrium informed trading strategy $X^{ss}(S)$ is given by equation (1) and the equilibrium bid and ask prices are as follows:*

$$B_S^{ss} = \bar{v} - \frac{\alpha(2\mu - 1)(v_H - v_L)}{2} \tag{2}$$

$$A_S^{ss} = \bar{v} + \frac{\alpha(2\mu - 1)(v_H - v_L)}{2} \tag{3}$$

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[4]The fixed trade size assumption is standard in these microstructure models because the optimal trade size for informed traders who take the bid and ask prices as given is infinite. We can generalize our model to allow the informed traders to choose from among multiple, exogenously specified trade sizes. However, this makes our analysis much more cumbersome (since we now have to compute bid and ask prices for the stock and later, the option, at each trade size) without adding much in the way of new insights.

[5]The subscript S denotes that these are bid and ask prices for the stock and the superscript *ss* indicates that they apply in a world where only stock trading is allowed (the *ss* scenario).

Proof: See the Appendix.

The market maker breaks even on each incoming order by setting a spread between the bid and ask prices. The spread allows him to recoup from the liquidity traders the losses suffered at the hands of the informed traders. The size of the spread is given by:

$$\Delta_S^{ss} = A_S^{ss} - B_S^{ss} = \alpha(2\mu - 1)(v_H - v_L) \tag{4}$$

As expected, the size of the spread is increasing in α and μ. As α increases, informed traders form a greater fraction of the trader population, which increases the threat of informed trading faced by the market maker and he responds by setting a larger spread. Similarly, when μ increases, informed traders pose a greater threat to the market maker because they have more informative signals and this leads to wider spreads.

We can also measure the efficiency of stock prices as the amount of information revealed through trading. Following Kyle (1985), we define market efficiency η as the fraction of the total variability in stock value that is revealed by trading. In other words, it is the ratio of the variances of the post-trade (date-2) stock price $P_{S,t=2}$ and the full information (date-3) stock price $P_{S,t=3}$:

$$\eta = \frac{\text{Var}(P_{S,t=2})}{\text{Var}(P_{S,t=3})} \tag{5}$$

Since the date-3 stock price is v_L or v_H with equal probabilities of 0.5, we can compute $\text{Var}(P_{S,t=3}) = (v_H - v_L)^2/4$. The date-2 stock price is $P_{S,t=2} = B_S^{ss}$ if the date-1 trade is a stock sale and the date-2 price is $P_{S,t=2} = A_S^{ss}$ if the date-1 trade is a stock purchase. Since the probabilities for the state θ, the insider's signal S and liquidity sales/purchases are all symmetric, we can show that $Pr(\text{stock sale}) = Pr(\text{stock buy}) = 0.5$, which implies that $\text{Var}(P_{S,t=2}) = \alpha^2(2\mu - 1)^2(v_H - v_L)^2/4$. Therefore, market efficiency in the *ss* world is:

$$\eta^{ss} = \alpha^2(2\mu - 1)^2 \tag{6}$$

which is increasing in the amount of informed traders (α) and in the quality of their signals (μ).

3   The Impact Of Option Trading

We now expand the model to consider the role of option trading (the *so* case). Suppose the traders in our model have the choice of trading the stock or a put option on the stock with an exercise price of K where $v_L < K < v_H$. The put option provides date-3 payoffs of $K - v_L$ and 0 for the states $\theta = L$ and $\theta = H$, respectively.[6] The sequence of events and the information structure is the same as before except that the single trade transacted on date 1 can be in the stock or options market. Furthermore, informed and liquidity traders now split their trades between the two markets, where the split is exogenously specified for the latter and endogenously derived for the former. As before, there are risk-neutral, competitive market makers in both markets who set prices rationally. These market makers are assumed to observe the order flow in *both* markets when setting prices, which rules out the possibility of arbitrage across markets.

Informed traders, who are a fraction α of the population, choose their strategy by trading off the adverse selection costs of the stock and options market. If they trade too aggressively in one market, the market maker in this market increases their trading cost by widening the spread, which makes it advantageous for them to shift to the other market. Therefore, we conjecture that their equilibrium strategy is a mixed one where they randomize their trading across both markets. We denote their mixing probabilities of trading the stock and the put given a signal $S \in \{B, G\}$ by π_S^{so} and $1 - \pi_S^{so}$, respectively, where $\pi_S^{so} \in [0, 1]$ and pure strategies are feasible. When $S = B$, informed traders are conjectured to either sell the stock or buy the put and their strategy is:

$$X^{so}(B) = \begin{cases} \text{sell stock} & \text{with probability } \pi_B^{so} \\ \text{buy put} & \text{with probability } 1 - \pi_B^{so} \end{cases} \tag{7}$$

When $S = G$, they either buy the stock or sell the put and their conjectured strategy is:

$$X^{so}(G) = \begin{cases} \text{buy stock} & \text{with probability } \pi_G^{so} \\ \text{sell put} & \text{with probability } 1 - \pi_G^{so} \end{cases} \tag{8}$$

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[6]Although we model a put option, we expect our qualitative results to stay unchanged if a call option is modeled instead. Unfortunately, considerations of tractability, in terms of calculating the informed traders' optimal strategy, prevent us from including both call and put option trading simultaneously.

The fraction $1 - \alpha$ of liquidity traders are themselves comprised of a fraction β who trade in the stock market and a fraction $1 - \beta$ who trade in the options market (hedgers, for example). Once again, we assume that the stock (option) liquidity traders are equally likely to buy and sell shares (put options). The stock market maker sets the bid (B_S^{so}) and ask (A_S^{so}) prices for the stock so as to make zero expected profits taking into account the information conveyed by the stock trade, i.e., $B_S^{so} = E(\tilde{v} \mid$ stock sale) and $A_S^{so} = E(\tilde{v} \mid$ stock buy). Similarly, the options market maker sets the bid (B_P^{so}) and ask (A_P^{so}) prices for the put so as to make zero expected profits conditional on the information conveyed by the option trade, i.e., $B_P^{so} = E[(K - \tilde{v})^+ \mid$ put sale] and $A_P^{so} = E[(K - \tilde{v})^+ \mid$ put buy]. The following lemma characterizes these bid and ask prices as functions of informed traders' conjectured strategy $\{\pi_B^{so}, \pi_G^{so}\}$:

Lemma 1 *The zero-profit bid and ask prices set by the stock and options market makers conditional on the informed traders' conjectured trading strategy in equations (7) and (8) are:*

$$B_S^{so} = \bar{v} - \frac{\alpha \pi_B^{so}(2\mu - 1)(v_H - v_L)}{2\beta(1 - \alpha) + 2\alpha \pi_B^{so}} \tag{9}$$

$$A_S^{so} = \bar{v} + \frac{\alpha \pi_G^{so}(2\mu - 1)(v_H - v_L)}{2\beta(1 - \alpha) + 2\alpha \pi_G^{so}} \tag{10}$$

$$B_P^{so} = \frac{(K - v_L)[(1 - \alpha)(1 - \beta) + 2\alpha(1 - \mu)(1 - \pi_G^{so})]}{2(1 - \alpha)(1 - \beta) + 2\alpha(1 - \pi_G^{so})} \tag{11}$$

$$A_P^{so} = \frac{(K - v_L)[(1 - \alpha)(1 - \beta) + 2\alpha\mu(1 - \pi_B^{so})]}{2(1 - \alpha)(1 - \beta) + 2\alpha(1 - \pi_B^{so})} \tag{12}$$

Proof: See the Appendix.

In equilibrium, the informed traders choose a trading strategy that maximizes their profits given on the above prices in the two markets. If they receive the $S = B$ signal, their expected profits from selling the stock is $B_S^{so} - E(\tilde{v} \mid S = B)$ and from buying the put is $E[(K - \tilde{v})^+ \mid S = B] - A_P^{so}$. If they receive the $S = G$ signal, their expected profits from buying the stock is $E(\tilde{v} \mid S = G) - A_S^{so}$ and from selling the put is $B_P^{so} - E[(K - \tilde{v})^+ \mid S = G]$. Since they are conjectured to mix between the stock and the option, they choose π_B^{so} and π_G^{so} in equilibrium so as to equalize their trading profits across these two markets. The following proposition characterizes this equilibrium.

Proposition 2 *In a world where stock and option trading is allowed, the equilibrium informed*

trading strategy $\{X^{so}(B), X^{so}(G)\}$ is given by equations (7) and (8) and the equilibrium prices are given by equations (9)–(12), where $0 < \pi_B^{so} = \pi_G^{so} < 1$ if $\dfrac{v_H - v_L}{K - v_L} < 1 + \dfrac{\alpha}{\beta(1-\alpha)}$ where

$$\pi_B^{so} = \pi_G^{so} = \frac{\beta[(1-\alpha)(1-\beta)(v_H - K) + \alpha(v_H - v_L)]}{\alpha[(1-\beta)(K - v_L) + \beta(v_H - v_L)]} \tag{13}$$

But the equilibrium informed trading strategy is $\pi_B^{so} = \pi_G^{so} = 1$ if $\dfrac{v_H - v_L}{K - v_L} \geq 1 + \dfrac{\alpha}{\beta(1-\alpha)}$.

Proof: See the Appendix.

Since $\pi_B^{so} = \pi_G^{so} = \pi^{so}$, informed traders trade with the same intensity whether they get good or bad news. This symmetry follows from two assumptions in our model: the $\theta = L$ and $\theta = H$ states are equally likely and the $S = B$ and $S = G$ signals have the same precision μ. Since the variability of the stock $(v_H - v_L)$ exceeds that of the put option $(K - v_L)$, informed traders prefer to trade the stock rather than the put, because their private information is more valuable when they trade the more volatile security. In other words, informed traders prefer to trade the stock because it is more *information-sensitive* than the option. We can measure this information advantage of the stock over the option by the ratio of their respective volatilities $\dfrac{v_H - v_L}{K - v_L}$, which in our model is just the inverse of the put's *delta* or *hedge ratio*. The above proposition tells us that informed traders will mix between the stock and put if the stock's information advantage is not *too large*. However, if this information edge exceeds a certain threshold, they will switch to a pure strategy of trading only the stock.[7] This threshold value is decreasing in the intensity of stock liquidity trading (β) because when β is high, informed traders find stock trading to be more profitable (due to the availability of more camouflage) and this makes the stock-only pure strategy more likely to occur.

In the mixed strategy equilibrium (which is our focus for the remainder of this section), even though the informed trade in both markets, they still exhibit a relative bias towards the stock. In order to see this, note that when we move from the *ss* to the *so* world by introducing option trading, liquidity traders now split their trades between the stock and the put and their stock trading probability reduces from 1 to β. However, the stock trading probability of informed traders reduces

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[7]Not surprisingly, an equilibrium where the informed traders trade only the option ($\pi_B^{so} = \pi_G^{so} = 0$) does not exist because that requires the option to be more information-sensitive than the stock, i.e., have a delta in excess of one, which is not possible.

from 1 to π^{so}, where we can infer from equation (13) that $\pi^{so} > \beta$. Therefore, the informed traders' stock trading intensity relative to that of the liquidity traders *increases* after the introduction of option trading. This stock trading bias of informed traders is a direct result of stock's greater information sensitivity because if $K = v_H$, the put's delta is one and we can see from equation (13) that $\pi^{so} = \beta$. The comparative statics properties of π^{so} are described in the following corollary.

Corollary 1 *The stock trading intensity of the informed traders π^{so} is increasing in β, decreasing in α and K, and independent of μ.*

Proof: See the Appendix.

These results have an appealing intuition. Informed traders trade more aggressively in the stock market when stock liquidity trading (β) increases because they have more liquidity traders to profit from. But when α increases, the market makers in both markets are more wary of informed trading in their respective order flows and this reduces the informed traders' expected profits in both markets. However, this reduction is greater in the stock market given its greater information sensitivity and so they shift their trading to the relatively more profitable option market, which leads to a decrease in π^{so}. Similarly, an increase in K increases the option's delta and consequently, its information sensitivity. This reduces the stock's information edge and informed traders respond by trading the stock less intensively. Finally, an increase in signal precision μ increases the information advantage of the informed traders. We would expect them to respond by increasing their trading intensity in the more information-sensitive stock market (an increase in π^{so}). However, an increase in μ causes the stock market maker to widen his bid-ask spread more than the option market maker given the former's greater information sensitivity, which induces informed traders to reduce their stock trading intensity (a decrease in π^{so}). In equilibrium, these two effects exactly offset each other leaving π^{so} unchanged.

We now calculate the equilibrium bid-ask spreads in the two markets. On substituting the value for π^{so} from equation (13) into the bid and ask price expressions in equations (9)–(12), we get

$$\Delta_S^{so} = A_S^{so} - B_S^{so} = (2\mu - 1)[\alpha(v_H - v_L) + (1 - \alpha)(1 - \beta)(v_H - K)] \qquad (14)$$

$$\Delta_P^{so} = A_P^{so} - B_P^{so} = (2\mu - 1)[\alpha(K - v_L) - \beta(1 - \alpha)(v_H - K)] \tag{15}$$

Given the informed traders' bias towards trading the stock rather than the option, it is not surprising that the stock market maker sets a wider spread than his option counterpart ($\Delta_S^{so} > \Delta_P^{so}$). Furthermore, a comparison of equations (4) and (14) reveals that $\Delta_S^{so} > \Delta_S^{ss}$, i.e., the introduction of option trading leads to wider bid-ask spreads in the underlying stock. As noted earlier the market maker in the stock faces an increased threat of informed trading (relative to liquidity trading) following the introduction of the option because $\pi^{so} > \beta$ and he responds by setting a wider bid-ask spread. We present the properties of these bid-ask spreads in the following corollary:[8]

Corollary 2 *The stock's bid-ask spread Δ_S^{so} is increasing in α and μ and decreasing in β and K. The option's bid-ask spread Δ_P^{so} is increasing in α, μ, and K and decreasing in β.*

Intuitively, increases in α or μ imply that both market makers face a greater threat from informed traders, either because they are more likely to trade with them or because these traders have better information and so they defend themselves by setting wider bid-ask spreads. An increase in β increases the stock-trading intensity of liquidity traders and reduces the option-trading intensity of informed traders $(1 - \pi^{so})$, which causes both market makers to narrow their spreads. Finally, an increase in K causes informed traders to shift their trading from the stock to the option (because of latter's increased information sensitivity) and the stock (option) market maker responds by setting a smaller (larger) spread.

Once again, we can measure the amount of information revealed through the trading process (market efficiency) using equation (5). The stock price on date 2 after the completion of trading, $P_{S,t=2}$, reflects the information conveyed by the date-1 trade, whether it is in the stock or the option market. We calculate the market efficiency η^{so} in the following proposition and demonstrate that it exceeds the *ss* world's market efficiency.

Proposition 3 *When option trading is introduced, the informational efficiency of stock prices is*

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[8]We omit the proof to conserve space although it follows from a straightforward computation of partial derivatives of Δ_S^{so} and Δ_P^{so} with respect to the relevant variables.

given by:

$$\eta^{so} = \alpha^2(2\mu - 1)^2 \left[\frac{(\pi^{so})^2}{\beta(1 - \alpha) + \alpha\pi^{so}} + \frac{(1 - \pi^{so})^2}{(1 - \alpha)(1 - \beta) + \alpha(1 - \pi^{so})} \right] \tag{16}$$

where π^{so} is as per equation (13). Furthermore, stock prices are more efficient than in a world with only stock trading, i.e., $\eta^{so} > \eta^{ss}$.

Proof: See the Appendix.

The finding that the introduction of option trading improves stock market efficiency is an intuitive one. In the *so* world, informed traders randomize their trading across both the stock and the option. Therefore, the market can now noisily infer their private information from *two* sources – the order flows in the stock and options markets. As a result, more of their private information is revealed in the trading process and market efficiency improves relative to the *ss* world.

4   The Impact of Margin Requirements

We now introduce margin requirements into our model of stock and option trading and consider their impact on trading strategies and market prices. Our analysis so far assumes that informed traders are not subject to wealth constraints and they trade to maximize profits. Margin requirements do not influence their trading strategies in this setting. But in the real world most traders face wealth constraints and we will analyze the behavior of these traders in this section (the *sm* world). Margin rules have a natural role to play in this setting because wealth-constrained informed traders seek to maximize the *expected return*, rather than expected profit, from trading.

The structure of the model is the same as in Section 3 but with informed traders now subject to initial margin requirements. The stock and option margin requirements are modeled as follows:

1. To buy or sell a share of stock, a trader has to invest a fraction m_S of the ask or bid price, respectively.

2. To buy an option, a trader has to invest 100% of the ask price, i.e., options cannot be bought on margin because they are already highly leveraged.

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3. To sell an option, a trader has to invest a fraction m_P of the underlying stock price less the amount (if any) by which the option is out of the money.

These rules broadly conform to regulations in the United States where $m_S = 50\%$ and $m_P = 20\%$ currently. As before, the informed traders' mixed trading strategy is given by equations (7) and (8), where the mixing probabilities are now denoted by $\{\pi_B^{sm}, \pi_G^{sm}\}$. Given this trading strategy, the bid and ask prices in the two markets are once again given by equations (9)–(12) with the appropriate mixing probabilities now being π_S^{sm} rather than π_S^{so}. The informed traders choose $\{\pi_B^{sm}, \pi_G^{sm}\}$ in equilibrium so as to maximize their expected return conditional on the above prices. When they receive the $S = B$ signal, their expected return from selling the stock is $\dfrac{B_S^{sm} - E(\tilde{v} \mid S = B)}{m_S B_S^{sm}}$ and their expected return from buying the put is $\dfrac{E\left[(K - \tilde{v})^+ \mid S = B\right] - A_P^{sm}}{A_P^{sm}}$. But when they receive the $S = G$ signal, their expected return from buying the stock is $\dfrac{E(\tilde{v} \mid S = G) - A_S^{sm}}{m_S A_S^{sm}}$ and their expected return from selling the put is $\dfrac{B_P^{sm} - E[(K - \tilde{v})^+ \mid S = G]}{m_P E(\tilde{v} \mid \text{put sale})}$ assuming that the put is in the money.[9] We now characterize the equilibrium for this model in the following proposition.

Proposition 4 *In a world with stock and option trading and binding margin requirements, the equilibrium informed trading strategy $\{X^{sm}(B), X^{sm}(G)\}$ is given by equations (7) and (8) and the equilibrium prices are given by equations (9)–(12), where the mixing probabilities are*

$$\pi_B^{sm} = \frac{\beta}{2\alpha} \left[\frac{(v_H - v_L)\mathbf{\Psi_1} - m_S \mathbf{\Psi_2}}{m_S(1 - \beta)[\mu v_L + v_H(1 - \mu)] + \mu\beta(v_H - v_L)} \right] \tag{17}$$

$$\pi_G^{sm} = \frac{\beta}{2\alpha} \left[\frac{m_P(v_H - v_L)\mathbf{\Phi_1} - m_S(K - v_L)\mathbf{\Phi_2}}{[\mu v_H + v_L(1 - \mu)][\beta m_P(v_H - v_L) + m_S(1 - \beta)(K - v_L)]} \right] \tag{18}$$

where $\mathbf{\Psi_1} = (1 - \alpha)(1 - \beta) + 2\alpha\mu$; $\mathbf{\Phi_1} = (1 - \alpha)(1 - \beta)(v_L + v_H) + 2\alpha[\mu v_H + v_L(1 - \mu)]$; and $\mathbf{\Psi_2} = \mathbf{\Phi_2} = (1 - \alpha)(1 - \beta)(v_L + v_H)$. The mixed strategy equilibrium exists if and only if the following conditions are satisfied:

$$\frac{\beta(1 - \alpha)}{\beta(1 - \alpha)(v_L + v_H) + 2\alpha[\mu v_L + v_H(1 - \mu)]} < \left(\frac{m_S}{v_H - v_L}\right) < \frac{\mathbf{\Psi_1}}{\mathbf{\Psi_2}} \tag{19}$$

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[9]When the solitary trade in our model is a put sale, the stock market maker rationally sets the stock price to be $E(\tilde{v} \mid \text{put sale})$ and the informed trader's put margin is a fraction m_P of this price. We assume that the put is in the money in this situation, i.e., $K > E(\tilde{v} \mid \text{put sale})$, in order to greatly simplify the algebra. This is a fairly trivial assumption and we can confirm that all qualitative results in this section continue to hold even when the put is out of the money. Furthermore, on substituting from equation (A.8) in the Appendix, we can see that a sufficient condition for the put to be in the money is $K > \mu v_H + v_L(1 - \mu)$, which we will assume is satisfied in this section.

$$\frac{\beta(1-\alpha)(v_L + v_H)}{\beta(1-\alpha)(v_L + v_H) + 2\alpha[\mu v_H + v_L(1-\mu)]} < (\frac{m_S}{m_P})(\frac{K - v_L}{v_H - v_L}) < \frac{\mathbf{\Phi_1}}{\mathbf{\Phi_2}} \qquad (20)$$

When the left (right) hand side inequalities in equations (19) and (20) are reversed, the informed traders trade only in the stock (option) market ($\pi_B^{sm} = \pi_G^{sm} = 1(0)$).

Proof: See the Appendix.

In contrast to the previous section, the trading intensity of informed traders is no longer symmetric in the mixed strategy equilibrium, i.e., $\pi_B^{sm} \neq \pi_G^{sm}$. This is because of the asymmetry in margin requirements between the stock and the option ($m_S \neq m_P$) and also because options can only be sold, not bought, on margin. In order to understand the economic intuition underlying the above proposition, we must understand the tradeoffs faced by the informed agents. *Ceteris paribus*, they prefer to exploit their private information in the stock market rather than in the option market because of the former's greater information sensitivity. However, given their limited wealth, they prefer to trade in the market with the smaller margin requirements in order to increase their leverage and maximize their returns. Further complicating their decision is the fact that if they trade too aggressively in one or the other market to capitalize on its information or margin edge, they will face wider bid-ask spreads in that market. To see these tradeoffs in action, consider their strategy when $S = G$. The ratios m_S/m_P and $(K - v_L)/(v_H - v_L)$ measure the leverage advantage and information sensitivity, respectively, of the option relative to the stock. When either or both of these ratios are large enough so that their product exceeds the threshold $\mathbf{\Phi_1}/\mathbf{\Phi_2}$, then it is more advantageous for them to trade the option and so $\pi_G^{sm} = 0$. But if the product is very small, then the stock is their preferred instrument and they set $\pi_G^{sm} = 1$. For intermediate values, they adopt a mixed strategy of trading the stock (option) with probability π_G^{sm} $(1 - \pi_G^{sm})$ where $\pi_G^{sm} \in \{0, 1\}$. Similarly, when $S = B$, the relative benefit of trading the put for the informed traders is now measured by the ratio $m_S/(v_H - v_L)$ and they trade the put, the stock or mix between the two for high, low and intermediate values, respectively, of this ratio.[10] The threshold values in equations

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[10]Since the put cannot be bought on margin, the relative benefit of trading the put is positively related to the stock's margin requirement m_S (which determines the amount of leverage in the stock) and negatively related to the stock's volatility (which determines its information edge over the option).

(19) and (20) are functions of α and β and they reflect the sensitivities of the stock and options market makers to their respective adverse selection problems. An implication of these arguments is that with margin requirements, informed traders may only trade the option in equilibrium (if it provides substantially more leverage than the stock), which was not possible in Section 3.

In the remainder of this section, we will focus on the mixed strategy equilibrium and compare it to the one that exists when margin rules are absent (Proposition 2). We start by comparing the informed traders' strategies in the two settings. We can show using equations (13), (17) and (18) that the sufficient and necessary conditions for $\pi_B^{sm} > \pi_B^{so}$ and $\pi_G^{sm} > \pi_G^{so}$ are $m_S < \Gamma_1(v_H - v_L)$ and $\frac{m_P}{m_S} > \Gamma_2(K - v_L)$, respectively, where Γ_1 and Γ_2 are functions of the model parameters.[11] In other words, the stock trading intensity of informed traders is greater with margin requirements than without if and only if the stock margin is sufficiently "small" and the option margin is sufficiently "large" relative to the stock margin. Under these conditions, the leverage advantage of the stock relative to the option is large enough to induce informed traders to trade the stock more aggressively. We can also compare the comparative statics properties of π^{sm} as described in the following corollary to those of π^{so} (Corollary 1).

Corollary 3 *The stock trading intensity of informed traders* $\{\pi_B^{sm}, \pi_G^{sm}\}$ *are such that*

1. π_B^{sm} *is increasing in β and μ, decreasing in m_S, increasing (decreasing) in α if $m_S/(v_H - v_L) >$ $(<) (v_L + v_H)^{-1}$, and independent of K and m_P.*

2. π_G^{sm} *is increasing in β and m_P, decreasing in K and m_S, and increasing (decreasing) in α and μ if $m_S(K - v_L)/m_P(v_H - v_L) > (<) 1$.*

Proof: See the Appendix.

As before, informed traders intensify their trading in the stock when stock liquidity trading (β)

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[11]Straightforward, though tedious, algebra gives us the expressions for Γ_1 and Γ_2 as follows:

$$\Gamma_1 = \frac{(1 - \alpha + 2\alpha\mu)(K - v_L) - \beta(1 - \alpha)(2\mu - 1)(v_H - K)}{(1 - \alpha)(v_L + v_H)[\beta(v_H - K) + (K - v_L)] + 2[\mu v_L + v_H(1 - \mu)][\alpha(v_H - v_L) + (1 - \alpha)(1 - \beta)(v_H - K)]}$$

$$\Gamma_2 = \frac{(v_L + v_H) + (2\mu - 1)[\alpha(K - v_L) + (1 - \beta + \alpha\beta)(v_H - K)]}{(v_L + v_H)(K - v_L) + (2\mu - 1)(v_H - v_L)[\alpha(K - v_L) - \beta(1 - \alpha)(v_H - K)]}$$

It is easy to check that Γ_1 and Γ_2 satisfy the feasibility equations (19) and (20).

increases because of the resulting increase in the profitability the stock relative to the option. Another familiar result is the negative relationship between π_G^{sm} and K – an increase in the strike price increases the put's delta and its information sensitivity, which causes informed traders to shift to the option and away from the stock. However, $\frac{\partial \pi_B^{sm}}{\partial K} = 0$ because the informed traders' return from buying the put with bad news is independent of K. An increase in K increases both their profit from buying the put and the put's purchase (ask) price, leaving their return unchanged and thus, they are not more or less eager to trade the option. The relationship between stock trading intensity and α highlights an important distinction between the sm and so worlds. In the so world, π^{so} is negatively related to α but the relationship between π^{sm} and α can be positive or negative depending on parameter values. When α increases, market makers in both markets face an increased threat of informed trading and respond by setting prices accordingly. However, the market maker in the informed traders' preferred security has a relatively stronger response given his greater susceptibility to this threat and therefore, informed traders rationally react by shifting to the less preferred security. The security preference of informed traders depends on two factors - its information sensitivity and its leverage advantage. As we saw earlier, the ratios $(\frac{m_S}{m_P})(\frac{K - v_L}{v_H - v_L})$ and $\frac{m_S}{v_H - v_L}$ measure the net benefit to informed traders of trading the put instead of the stock. The above corollary tells us that if these two ratios are sufficiently "large", $\frac{\partial \pi^{sm}}{\partial \alpha} > 0$ because the market maker in the option (the preferred security) reacts more sharply to the increase in α and so the informed traders increase their stock trading intensity in response. In contrast, the stock is always the preferred instrument for informed traders in the so world (given its greater information sensitivity) and an increase in α leads them to shift away from it and towards the option.

Another difference between the so and sm worlds is in the differing responses of informed traders to a change in μ. When μ increases, informed traders face two conflicting incentives. On the one hand, they want to capitalize on their higher quality signal by trading their preferred security more aggressively. On the other hand, they recognize that the preferred security's market maker responds more sharply to the increased threat of informed trading caused by the higher μ (given his greater sensitivity to this threat) and this will cause them to trade the less-preferred security. In the so

world, these two effects exactly offset each other. In the sm world, the former effect dominates the latter when informed traders receive bad news ($\frac{\partial \pi_B^{sm}}{\partial \mu} > 0$). Informed traders with more precise bad news signals trade the stock more aggressively because it is more information-sensitive than the option and because the option's leverage advantage is minimal (since informed traders with bad news cannot buy the put on margin). But the latter effect dominates the former when informed traders receive good news because $\frac{\partial \pi_G^{sm}}{\partial \mu} > (<) \, 0$ when the option (stock) is their preferred security, i.e., when $(\frac{m_S}{m_P})(\frac{K - v_L}{v_H - v_L})$ is sufficiently large (small).

Corollary 3 also reports on the sensitivity of the informed traders' strategy to the stock and option margin requirements. Informed traders respond to an increase in the stock margin by trading the option more aggressively ($\frac{\partial \pi^{sm}}{\partial m_S} < 0$) because the relative leverage advantage of the option increases when m_S increases. Similarly, an increase in the option margin reduces the option's leverage edge and leads informed traders to trade the stock more aggressively ($\frac{\partial \pi_G^{sm}}{\partial m_P} > 0$).[12]

We now calculate the equilibrium bid-ask spreads in the sm world and compare them to those in the ss and so worlds. From equations (9)–(12), the stock and option bid-ask spreads are:

$$\Delta_S^{sm} = \frac{\alpha(2\mu - 1)(v_H - v_L)[\beta(1 - \alpha)(\pi_B^{sm} + \pi_G^{sm}) + 2\alpha\pi_B^{sm}\pi_G^{sm}]}{2[\beta(1 - \alpha) + \alpha\pi_B^{sm}][\beta(1 - \alpha) + \alpha\pi_G^{sm}]} \tag{21}$$

$$\Delta_P^{sm} = \frac{\alpha(2\mu - 1)(K - v_L)[(1 - \alpha)(1 - \beta)(2 - \pi_B^{sm} - \pi_G^{sm}) + 2\alpha(1 - \pi_B^{sm})(1 - \pi_G^{sm})]}{2[(1 - \alpha)(1 - \beta) + \alpha(1 - \pi_B^{sm})][(1 - \alpha)(1 - \beta) + \alpha(1 - \pi_G^{sm})]} \tag{22}$$

where π_B^{sm} and π_G^{sm} are given by equations (17) and (18), respectively. From equations (4), (14) and (21), we can show that

$$\Delta_S^{sm} - \Delta_S^{ss} = \frac{\alpha(1 - \alpha)(2\mu - 1)(v_H - v_L)\{\sum\limits_{S=B}^{G} (\pi_S^{sm} - \beta)[\beta(1 - \alpha) + \alpha\pi_{S'}^{sm}]\}}{2[\beta(1 - \alpha) + \alpha\pi_B^{sm}][\beta(1 - \alpha) + \alpha\pi_G^{sm}]}$$

$$\Delta_S^{sm} - \Delta_S^{so} = \frac{\alpha\beta(1 - \alpha)(2\mu - 1)(v_H - v_L)\{\sum\limits_{S=B}^{G} (\pi_S^{sm} - \pi^{so})[\beta(1 - \alpha) + \alpha\pi_{S'}^{sm}]\}}{2[\beta(1 - \alpha) + \alpha\pi_B^{sm}][\beta(1 - \alpha) + \alpha\pi_G^{sm}][\beta(1 - \alpha) + \alpha\pi^{so}]}$$

where $S' = G(B)$ when $S = B(G)$. Therefore, $\Delta_S^{sm} < (>) \, \Delta_S^{ss}$ if $\pi_B^{sm} < (>) \, \beta$ and $\pi_G^{sm} < (>) \, \beta$. Intuitively, the stock market maker narrows (widens) the spread when option trading with margins

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[12]Since informed traders cannot buy the put on margin, their trading strategy with bad news is independent of m_P, i.e., $\frac{\partial \pi_B^{sm}}{\partial m_P} = 0$.

is introduced if he faces an decreased (increased) risk of trading with informed traders relative to liquidity traders. In contrast to the *so* world where this risk is always greater ($\pi^{so} > \beta$) and option trading leads to wider stock spreads, in the *sm* world, the risk can be more or less depending on tradeoff between leverage and information sensitivity in the two markets. We can also infer from the above equations that $\Delta_S^{sm} < (>) \Delta_S^{so}$ if $\pi_B^{sm} < (>) \pi^{so}$ and $\pi_G^{sm} < (>) \pi^{so}$. As expected, stock spreads are narrower (wider) in the *sm* world than in the *so* world if informed traders trade the stock less aggressively with margins. Since we showed earlier that $\pi^{so} > \beta$ and $\Delta_S^{so} > \Delta_S^{ss}$, only one of these two sets of sufficient conditions are binding, i.e., $\Delta_S^{sm} < \Delta_S^{ss} < \Delta_S^{so}$ if $\pi_B^{sm} < \beta$ and $\pi_G^{sm} < \beta$ and $\Delta_S^{sm} > \Delta_S^{so} > \Delta_S^{ss}$ if $\pi_B^{sm} > \pi^{so}$ and $\pi_G^{sm} > \pi^{so}$. On substituting for the equilibrium values of π^{so}, π_B^{sm} and π_G^{sm} and simplifying, we can show that

1. The stock bid-ask spread is lower in the *sm* case than in the *ss* or *so* cases if $\frac{m_P}{m_S} < \frac{K - v_L}{v_H - v_L}$ and $m_S > \dfrac{(1 - \alpha + 2\alpha\mu)(v_H - v_L)}{(v_L + v_H) - \alpha(2\mu - 1)(v_H - v_L)}$.

2. The stock bid-ask spread is greater in the *sm* case than in the *ss* or *so* cases if $\frac{m_P}{m_S} > ,_2 (K - v_L)$ and $m_S < ,_1 (v_H - v_L)$ where $,_1$ and $,_2$ were defined earlier in footnote 11.

These conditions have a straightforward explanation. When stock margins are large and option margins are small relative to stock margins, informed traders trade the option more intensively given its leverage advantage and the resulting reduced threat of informed trading in the stock market lowers the bid-ask spread there compared to the *ss* and *so* cases. Conversely, when stock margins are small and option margins are relatively large, informed traders shift their trading to the stock and the market maker responds by setting large stock spreads. We can derive the properties of the stock and option spreads in the *sm* world by computing the partial derivatives of Δ_S^{sm} and Δ_P^{sm} with respect to the model parameters and the results are described in the following corollary.

Corollary 4 *The stock's bid-ask spread Δ_S^{sm} increases with α, μ and m_P, decreases with K and m_S, and is ambiguous with respect to β. The option's bid-ask spread Δ_P^{sm} increases with α and m_S, decreases with m_P, and is ambiguous with respect to β, μ and K.*

As in the *so* world, an increase in α increases bid-ask spreads in both markets because market makers face a greater adverse selection problem when the fraction of informed traders in the population increases. The comparative statics of Δ_S^{sm} with respect to μ and K are also unchanged from the *so* world and for essentially the same reasons. But whereas μ and K have a positive impact on Δ_P^{so}, their impact on Δ_P^{sm} is ambiguous. Intuitively, an increase in μ has two conflicting effects on the option market maker. On the one hand, it worsens his adverse selection problem since he is trading against better-informed traders and this should cause him to widen the spread by increasing A_P^{sm} and decreasing B_P^{sm}. On the other hand, Corollary 3 tells us that informed traders are less likely to buy the put with bad news and this should cause the market maker to lower A_P^{sm}. As a result of these conflicting effects, $\dfrac{\partial \Delta_P^{sm}}{\partial \mu}$ can be positive or negative.[13] Similarly, $\dfrac{\partial \Delta_P^{so}}{\partial K}$ can be positive or negative because K has an ambiguous effect on B_P^{sm}. An increase in K leaves the put more in-the-money and simultaneously increases the likelihood that informed traders with good news will sell the put ($\dfrac{\partial \pi_G^{sm}}{\partial K} < 0$ from Corollary 3). The former effect tends to increase B_P^{sm} and the latter effect tends to reduce it leaving an ambiguous net effect. In the *so* world, we can show that an increase in K *increases* B_P^{so} because the former effect dominates the latter. However, K has an even larger positive effect on A_P^{so} because both effects work in the same direction now and the net effect of K on the spread is unambiguously positive. While K has a positive effect on A_P^{sm} too, this effect is smaller because it stems solely from the increased in-the-moneyness of the put (since $\dfrac{\partial \pi_B^{sm}}{\partial K} = 0$ from Corollary 3) and it is not enough to overcome the ambiguous effect on B_P^{sm}.

Another contrast between the *so* and *sm* worlds is in the relationship between spreads and the liquidity trading parameter β. This relationship is negative for both markets in the *so* world but can be positive or negative in the *sm world*. The reason for this ambiguity is as follows. *Ceteris paribus*, an increase in stock liquidity trading (β) causes the stock (option) market maker to narrow (widen) his spread. But of course, everything else does not stay constant when β increases. Specifically, informed traders trade the stock (option) more (less) aggressively when β increases (Corollary 3)

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[13]The latter feedback effect of μ on the informed trader's strategy is absent in the *so* world because we know from Corollary 1 that $\dfrac{\partial \pi^{so}}{\partial \mu} = 0$ and that explains why $\dfrac{\partial \Delta_P^{so}}{\partial \mu}$ is unambiguously positive.

and this induces the stock (option) market maker to widen (narrow) the spread, leaving the net impact ambiguous. Finally, the comparative static results on m_S and m_P have a ready intuition. When the stock margin requirement m_S increases, informed traders shift some of their trading to the more-leveraged option, which leads the stock (option) market maker to narrow (widen) his spread. Similarly, an increase in m_P causes informed traders to shift to the stock and this has the opposite effect on stock and option spreads.[14]

Finally, we measure the amount of information revealed by trading, or market efficiency, in the sm case (η^{sm}) using equation (5). The derivation of η^{sm} is analogous to that of η^{so} in equation (16) and we get:

$$\eta^{sm} = \frac{\alpha^2 (2\mu - 1)^2 (\eta_B^{sm} + \eta_G^{sm})}{2} \tag{23}$$

$$\text{where } \eta_S^{sm} = \frac{(\pi_S^{sm})^2}{\beta(1-\alpha) + \alpha\pi_S^{sm}} + \frac{(1 - \pi_S^{sm})^2}{(1-\alpha)(1-\beta) + \alpha(1 - \pi_S^{sm})} \text{ for } S \in \{B, G\} \tag{24}$$

where π_S^{sm} is given by equations (17) and (18). Simple algebraic calculations reveal that $\eta_S^{sm} > 1$ which implies that $\eta^{sm} > \eta^{ss}$ from equations (6) and (23). Therefore, the introduction of option trading improves market efficiency even with binding margin requirements because market participants can infer information not only from stock trades but also from option trades.

But when we compare η^{so} to η^{sm} using equations (16) and (23), we cannot unambiguously conclude that margin rules improve or worsen market efficiency. We can see that $\eta^{sm} > \eta^{so}$ if $\eta_B^{sm} > \eta_B^{so}$ and $\eta_G^{sm} > \eta_G^{so}$ where $\{\eta_S^j; S \in (B, G), j \in (sm, so)\}$ is defined in equation (24). On substituting for the relevant mixing probabilities π_B^j in the above expression for η_B^j, we can show after some tedious algebra that $\eta_B^{sm} > \eta_B^{so}$ if $m_S < \Gamma_1 (v_H - v_L)$ or if $m_S > \Gamma_3 (v_H - v_L)$. Similarly, we can show that $\eta_G^{sm} > \eta_G^{so}$ if $\frac{m_P}{m_S} > \Gamma_2 (K - v_L)$ or if $\frac{m_P}{m_S} < \Gamma_4 (K - v_L)$.[15] Therefore, $\eta^{sm} > \eta^{so}$ if any one of the following four sets of conditions are satisfied:

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[14]The comparative statics on m_S and m_P must be interpreted with caution because they depend on our simplifying assumption of *exogenous* liquidity trading. These results may not obtain in a more general model where the trading behavior of wealth-constrained liquidity traders is also endogenized. In such a model, an increase in m_S (m_P) would induce both informed and liquidity traders to shift to the relatively more leveraged option (stock). This can improve or worsen both market makers' adverse selection problems and cause them to set wider or narrower spreads.

[15]We have previously defined $\mathbf{\Gamma_1}$ and $\mathbf{\Gamma_2}$. The expressions for $\mathbf{\Gamma_3}$ are $\mathbf{\Gamma_4}$ are as follows:

$$\mathbf{\Gamma_3} = \frac{(1 - \alpha + 2\alpha\mu)(v_H - v_L) + \beta(1-\alpha)(2\mu-1)(v_H - K)}{(1-\alpha)(2\mu-1)(v_H - v_L)[(1-\beta)(v_H - K) + (K - v_L)] + 2(K - v_L)[\mu v_L + v_H(1-\mu)]}$$

1. $m_S < \Gamma_1(v_H - v_L)$ and $\frac{m_P}{m_S} > \Gamma_2(K - v_L)$.

2. $m_S < \Gamma_1(v_H - v_L)$ and $\frac{m_P}{m_S} < \Gamma_4(K - v_L)$.

3. $m_S > \Gamma_3(v_H - v_L)$ and $\frac{m_P}{m_S} > \Gamma_2(K - v_L)$.

4. $m_S > \Gamma_3(v_H - v_L)$ and $\frac{m_P}{m_S} < \Gamma_4(K - v_L)$.

On the other hand, the sufficient conditions for $\eta^{so} > \eta^{sm}$ are $\Gamma_1(v_H - v_L) < m_S < \Gamma_3(v_H - v_L)$ and $\Gamma_4(K - v_L) < \frac{m_P}{m_S} < \Gamma_2(K - v_L)$. These conditions together imply that margin rules improve market efficiency if the stock margin and the option margin relative to the stock margin are either large or small. But margin rules worsen market efficiency if the stock and relative option margins are moderate in size. In order to understand these results, we must recognize that both stock and option trades contribute to market efficiency. In fact, the terms $\frac{(\pi_S^j)^2}{\beta(1-\alpha) + \alpha\pi_S^j}$ and $\frac{(1 - \pi_S^j)^2}{(1-\alpha)(1-\beta) + \alpha(1 - \pi_S^j)}$ in equation (24) measure the respective contributions of the stock and option markets to market efficiency and they are increasing functions of the informed traders' stock and option trading intensities π_S^j and $1 - \pi_S^j$, respectively. When informed traders trade the stock (option) more intensively, stock (option) prices become more informative while option (stock) prices become less so and market efficiency improves only if the former effect dominates the latter. When m_S is sufficiently small (large), informed traders with bad news trade the stock (option) so aggressively given its leverage advantage that the additional informativeness of stock (option) trades more than makes up for the reduced informativeness of option (stock) trades and market efficiency improves. Similar arguments apply for informed traders with good news when $\frac{m_P}{m_S}$ is sufficiently large or small. Therefore, extreme values of m_S and $\frac{m_P}{m_S}$ make prices more efficient because the stock or option market witnesses a lot of informed trading, leading to a lot of information revelation. But when the margins are moderate, informed traders are not aggressive in either market and trading is not as informative.

$$\Gamma_4 = \frac{(K - v_L)(v_L + v_H) + (2\mu - 1)(v_H - v_L)[\alpha(K - v_L) - (1 - \alpha)(1 - \beta)(v_H - K)]}{(v_H - v_L)^2[(v_L + v_H) + \alpha(2\mu - 1)(K - v_L) + (\alpha + \beta - \alpha\beta)(2\mu - 1)(v_H - K)]}$$

It is easy to check that Γ_3 and Γ_4 satisfy the feasibility equations (19) and (20). Furthermore, we can also show that $\Gamma_3 > \Gamma_1$ and $\Gamma_4 < \Gamma_2$.

5 Conclusion

In this paper, we modeled the impact of option trading and margin rules on the behavior of traders with private information and on equilibrium prices. In the absence of binding margin constraints, the introduction of option trading leads informed traders to mix between the two markets as they seek to balance the stock's greater information sensitivity against the option's smaller bid-ask spread. In equilibrium, the introduction of the option widens the stock's bid-ask spread because informed traders exhibit a relative bias towards trading the stock. But with binding margin requirements, they may no longer exhibit this bias as they tradeoff the stock's information advantage with the option's relative leverage advantage. Now, option listing shrinks (widens) the stock's bid-ask spread if margin requirements for the stock are large (small) and those for the option are small (large). With or without margin rules, the introduction of option trading improves market efficiency. A policy implication of our model is that regulators can improve market efficiency by setting extreme margin requirements in the stock and options markets rather than moderate ones, i.e., by setting stock margins and relative option margins either large or small.

Some of our model's predictions are consistent with the extant empirical evidence. For example, Jennings and Stark (1986) and Damodaran and Lim (1991) report that stock prices adjust more quickly to information after options are listed on the underlying stocks, which is consistent with our finding that option trading improves market efficiency. The finding by Damodaran and Lim (1991), Fedenia and Grammatikos (1992) and others that option listing leads to a decline in stock bid-ask spreads is consistent with our model because margin requirements on options are much smaller than those on stocks (15% vs 50% currently). Among the new, as yet untested, predictions of our model are that stock (option) bid-ask spreads increase when stock margin requirements decrease (increase) and when option margin requirements increase (decrease). Another new prediction of our model is that the information content of option trades (as measured by their ability to predict future stock price movements) is greater for options with large deltas than for those with small deltas because informed traders prefer to trade the former.

Appendix

Proof of Proposition 1:

We will prove the proposition by first deriving the equilibrium bid and ask prices taking as given the informed traders' strategy in equation (1) and then we will show that this strategy is optimal given the equilibrium prices.

We know that the bid price is given by $B_S^{ss} = E(\tilde{v} \mid \text{stock sale})$ which can be rewritten as:

$$B_S^{ss} = E(\tilde{v} \mid \text{stock sale}) = v_L Pr(\theta = L \mid \text{stock sale}) + v_H Pr(\theta = H \mid \text{stock sale}) \qquad \text{(A.1)}$$

where the conditional probabilities are derived using Bayes' rule as follows.

$$Pr(\theta = L \mid \text{stock sale}) = \frac{Pr(\theta = L)Pr(\text{stock sale} \mid \theta = L)}{\sum_{\theta = L}^{H} Pr(\theta)Pr(\text{stock sale} \mid \theta)} \qquad \text{(A.2)}$$

where $Pr(\theta = L) = Pr(\theta = H) = 0.5$. When $\theta = L(H)$, informed traders sell stock if they receive bad news, which occurs with probability μ $(1 - \mu)$. Since they comprise a fraction α of the trading population, the probability that the market maker transacts a sell order from the informed trader in the two states is $Pr(\text{informed stock sale} \mid \theta = L) = \alpha\mu$ and $Pr(\text{informed stock sale} \mid \theta = H) = \alpha(1-\mu)$. Since liquidity traders comprise a fraction $1-\alpha$ of the population and since they are equally likely to submit a buy or sell order, $Pr(\text{liquidity stock sale} \mid \theta = L) = Pr(\text{liquidity stock sale} \mid \theta = H) = (1 - \alpha)/2$. Therefore, we get $Pr(\text{stock sale} \mid \theta = L) = \alpha\mu + (1 - \alpha)/2$ and $Pr(\text{stock sale} \mid \theta = H) = \alpha(1 - \mu) + (1 - \alpha)/2$. Substituting these probabilities into equation (A.2) gives us:

$$Pr(\theta = L \mid \text{stock sale}) = \frac{(0.5)\left(\alpha\mu + \frac{1 - \alpha}{2}\right)}{(0.5)\left(\alpha\mu + \frac{1 - \alpha}{2}\right) + (0.5)\left(\alpha(1 - \mu) + \frac{1 - \alpha}{2}\right)} = \frac{1 - \alpha + 2\alpha\mu}{2}$$

Noting that $Pr(\theta = H \mid \text{stock sale}) = 1 - Pr(\theta = L \mid \text{stock sale}) = (1 + \alpha - 2\alpha\mu)/2$ and substituting these conditional probabilities into equation (A.1) gives us the bid price as in equation (2).

Similarly, we know that the ask price is given by:

$$A_S^{ss} = E(\tilde{v} \mid \text{stock buy}) = v_L Pr(\theta = L \mid \text{stock buy}) + v_H Pr(\theta = H \mid \text{stock buy}) \qquad \text{(A.3)}$$

Using Bayes' rule once again, we can derive the conditional probabilities in equation (A.3) as $Pr(\theta = L \mid \text{stock buy}) = (1 + \alpha - 2\alpha\mu)/2$ and $Pr(\theta = H \mid \text{stock buy}) = (1 - \alpha + 2\alpha\mu)/2$. On substituting these probabilities into equation (A.3), we get the ask price as in equation (3).

Finally, we need only to demonstrate that the conjectured informed trader strategy in equation (1) is optimal to complete our proof. When $S = B(G)$, the expected profit to an informed trader from her conjectured strategy of selling (buying) a share is $B_S^{ss} - E(\tilde{v} \mid S = B)$ $(E(\tilde{v} \mid S = G) - A_S^{ss})$. We know that $E(\tilde{v} \mid S) = \sum_{\theta=L}^{H} v_\theta Pr(\theta \mid S)$ for $S \in \{B, G\}$. Once again, we can calculate these conditional probabilities using Bayes' rule and we get $E(\tilde{v} \mid S = B) = \mu v_L + v_H(1 - \mu)$ and $E(\tilde{v} \mid S = G) = \mu v_H + v_L(1 - \mu)$. On substituting these values, we get the conjectured trading strategy profit as $B_S^{ss} - E(\tilde{v} \mid S = B) = E(\tilde{v} \mid S = G) - A_S^{ss} = (1 - \alpha)(2\mu - 1)(v_H - v_L)/2 > 0$. This conjectured strategy is optimal if she is not better off from defecting to another strategy. The only two available defection strategies are not trading or trading against her signal. The former gives her zero profits and the latter gives her profits of $E(\tilde{v} \mid S = B) - A_S^{ss} = B_S^{ss} - E(\tilde{v} \mid S = G) = -(1 + \alpha)(2\mu - 1)(v_H - v_L)/2 < 0$ and so informed traders will not defect.

Proof of Lemma 1:

In order to conserve space, we will only derive the bid prices for the stock and put and note that the ask prices are derived in an analogous manner.

As before, the stock bid price B_S^{so} is given by equation (A.1) where the conditional probabilities satisfy Bayes' rule as shown in equation (A.2). But now the possibility of an option trade changes these conditional probabilities. In order to calculate $Pr(\theta = L \mid$ stock sale) as per equation (A.2), note that $Pr(\theta = L) = Pr(\theta = H) = 0.5$. When $\theta = L(H)$, informed traders sell stock as per equations (7) and (8) only if their signal is $S = B$ (probability $= \mu$ $(1 - \mu)$) *and* they choose to trade the stock (probability $= \pi_B^{so}$). Since informed traders comprise a fraction α, we can conclude that $Pr(\text{informed stock sale} \mid \theta = L) = \alpha\mu\pi_B^{so}$ and $Pr(\text{informed stock sale} \mid \theta = H) = \alpha\pi_B^{so}(1 - \mu)$. Irrespective of the state θ, a liquidity trader's stock sale is transacted only if his order reaches the market maker (probability $= 1 - \alpha$), he happens to be a stock liquidity trader (probability $= \beta$), and he wishes to sell, rather than buy, stock (probability $= 0.5$). Therefore, we can infer that $Pr(\text{liquidity stock sale} \mid \theta = L) = Pr(\text{liquidity stock sale} \mid \theta = H) = \beta(1 - \alpha)/2$. These results collectively imply that $Pr(\text{stock sale} \mid \theta = L) = \alpha\mu\pi_B^{so} + \beta(1 - \alpha)/2$ and $Pr(\text{stock sale} \mid \theta = H) = \alpha\pi_B^{so}(1 - \mu) + \beta(1 - \alpha)/2$. On substituting these conditional probabilities into equation (A.2) and

simplifying, we get $Pr(\theta = L \mid \text{stock sale}) = \dfrac{\beta(1-\alpha) + 2\alpha\mu\pi_B^{so}}{2\beta(1-\alpha) + 2\alpha\pi_B^{so}}$ and $Pr(\theta = H \mid \text{stock sale}) = 1 - Pr(\theta = L \mid \text{stock sale}) = \dfrac{\beta(1-\alpha) + 2\alpha\pi_B(1-\mu)}{2\beta(1-\alpha) + 2\alpha\pi_B^{so}}$. On substituting these probabilities into equation (A.1), we can derive B_S^{so} as in equation (9).

The bid price for the put option B_P^{so} is given by:

$$B_P^{so} = E[(K - \tilde{v})^+ \mid \text{put sale}] = (K - v_L).Pr(\theta = L \mid \text{put sale}) + 0.Pr(\theta = H \mid \text{put sale}) \quad \text{(A.4)}$$

since $v_L < K < v_H$. Once again, we use Bayes' rule to derive the conditional probabilities:

$$Pr(\theta = L \mid \text{put sale}) = \frac{Pr(\theta = L)Pr(\text{put sale} \mid \theta = L)}{\sum_{\theta = L}^{H} Pr(\theta)Pr(\text{put sale} \mid \theta)} \quad \text{(A.5)}$$

Using the same arguments as above, we can show that $Pr(\text{informed put sale} \mid \theta = L) = \alpha(1 - \mu)(1 - \pi_G^{so})$; $Pr(\text{informed put sale} \mid \theta = H) = \alpha\mu(1 - \pi_G^{so})$; and $Pr(\text{liquidity put sale} \mid \theta = L) = Pr(\text{liquidity put sale} \mid \theta = H) = (1 - \alpha)(1 - \beta)/2$. This implies that $Pr(\text{put sale} \mid \theta = L) = \alpha(1 - \mu)(1 - \pi_G^{so}) + (1 - \alpha)(1 - \beta)/2$ and $Pr(\text{put sale} \mid \theta = H) = \alpha\mu(1 - \pi_G^{so}) + (1 - \alpha)(1 - \beta)/2$. These probabilities can be substituted into equation (A.5) to derive $Pr(\theta = L \mid \text{put sale})$, which can then be substituted into equation (A.4) to obtain B_P^{so} as given in equation (11).

Proof of Proposition 2:

The informed traders choose $\{\pi_B^{so}, \pi_G^{so}\}$ in equations (7) and (8) so as to be indifferent between the two pure strategies they are mixing between. Therefore, π_B^{so} is chosen in equilibrium so that informed traders expect to make the same profits whether they sell the stock or buy the put, i.e.,

$$B_S^{so} - E(\tilde{v} \mid S = B) = E[(K - \tilde{v})^+ \mid S = B] - A_P^{so} \quad \text{(A.6)}$$

We know from the proof of Proposition 1 that $E(\tilde{v} \mid S = B) = \mu v_L + v_H(1 - \mu)$ and we can show that $E[(K - \tilde{v})^+ \mid S = B] = \mu(K - v_L)$.[16] On substituting these values, the values for B_S^{so} and A_P^{so} from equations (9) and (12), respectively, into equation (A.6) and solving for π_B^{so}, we get the expression as in equation (13). Similarly, we can derive π_G^{so} as in equation (13) by solving

$$E(\tilde{v} \mid S = G) - A_S^{so} = B_P^{so} - E[(K - \tilde{v})^+ \mid S = G] \quad \text{(A.7)}$$

---

[16] We can write $E[(K - \tilde{v})^+ \mid S = B] = (K - v_L).Pr(\theta = L \mid S = B) + 0.Pr(\theta = H \mid S = B) = \mu(K - v_L)$ since $Pr(\theta = L \mid S = B) = \mu$ by Bayes' rule. Similarly, we can derive $E[(K - \tilde{v})^+ \mid S = G] = (1 - \mu)(K - v_L)$.

In order for the mixed strategy equilibrium to exist, the mixing probabilities in equation (13) must be feasible (lie between zero and one). We can see on inspection that they are positive and simple algebraic manipulation tells us that $\pi_B^{so} = \pi_G^{so} < 1$ if $\dfrac{v_H - v_L}{K - v_L} < 1 + \dfrac{\alpha}{\beta(1 - \alpha)}$. If this inequality is reversed, the mixing probabilities are no longer less than one and then informed traders follow the pure strategy of trading only the stock, i.e., $\pi_B^{so} = \pi_G^{so} = 1$.

Proof of Corollary 1:

The results in this corollary can be easily proved by partially differentiating π^{so} in equation (13) with respect to the appropriate variables as we show below:

$$
\begin{aligned}
\frac{\partial \pi^{so}}{\partial \beta} &= \frac{(1 - \alpha)(1 - \beta)^2(v_H - K)(K - v_L) + (v_H - v_L)[\alpha(K - v_L) - \beta^2(1 - \alpha)(v_H - K)]}{\alpha[\beta(v_H - v_L) + (1 - \beta)(K - v_L)]^2} > 0 \\
\frac{\partial \pi^{so}}{\partial \alpha} &= -\frac{\beta(1 - \beta)(v_H - K)}{\alpha^2[\beta(v_H - v_L) + (1 - \beta)(K - v_L)]} < 0 \\
\frac{\partial \pi^{so}}{\partial K} &= -\frac{\beta(1 - \beta)(v_H - v_L)}{\alpha[\beta(v_H - v_L) + (1 - \beta)(K - v_L)]^2} < 0 \\
\frac{\partial \pi^{so}}{\partial \mu} &= 0
\end{aligned}
$$

We can assign a positive sign to $\dfrac{\partial \pi^{so}}{\partial \beta}$ because the condition $\dfrac{v_H - v_L}{K - v_L} < 1 + \dfrac{\alpha}{\beta(1 - \alpha)}$ must be satisfied for the mixed strategy equilibrium to exist.

Proof of Proposition 3:

The date-2 stock price $P_{S, t=2}$ is the bid price B_S^{so} or the ask price A_S^{so}, respectively, if the date-1 trade is a stock sale or purchase. But for an option sale or purchase, $P_{S, t=2} = E(\tilde{v} \mid \text{put sale})$ and $P_{S, t=2} = E(\tilde{v} \mid \text{put buy})$, respectively, where $E(\tilde{v} \mid \text{put sale (buy)}) = v_L Pr(\theta = L \mid \text{put sale (buy)}) + v_H Pr(\theta = H \mid \text{put sale (buy)})$. We calculated these conditional probabilities earlier using Bayes' rule as shown in equation (A.5) and when we substitute for them, we get

$$
E(\tilde{v} \mid \text{put sale}) = \bar{v} + \frac{\alpha(2\mu - 1)(1 - \pi^{so})(v_H - v_L)}{2(1 - \alpha)(1 - \beta) + 2\alpha(1 - \pi^{so})} \tag{A.8}
$$

$$
E(\tilde{v} \mid \text{put buy}) = \bar{v} - \frac{\alpha(2\mu - 1)(1 - \pi^{so})(v_H - v_L)}{2(1 - \alpha)(1 - \beta) + 2\alpha(1 - \pi^{so})} \tag{A.9}
$$

We can now calculate the expected stock price on date 2 as $E(P_{S, t=2}) = B_S^{so}.Pr(\text{stock sale}) + A_S^{so}.Pr(\text{stock buy}) + E(\tilde{v} \mid \text{put sale}).Pr(\text{put sale}) + E(\tilde{v} \mid \text{put buy}).Pr(\text{put buy})$ where the structure of the trading game implies that $Pr(\text{stock sale}) = Pr(\text{stock buy}) = [\beta(1 - \alpha) + \alpha\pi^{so}]/2$ and

$Pr(\text{put sale}) = Pr(\text{put buy}) = [(1-\alpha)(1-\beta) + \alpha(1-\pi^{so})]/2$. Substituting for these probabilities

and for the prices from equations (9), (10), (A.8) and (A.9), we get $E(P_{S,t=2}) = \bar{v}$. We can similarly

calculate the variance of $P_{S,t=2}$ and we get:

$$\text{Var}(P_{S,t=2}) = \frac{\alpha^2(2\mu-1)^2(v_H-v_L)^2}{4}\left[\frac{(\pi^{so})^2}{\beta(1-\alpha)+\alpha\pi^{so}} + \frac{(1-\pi^{so})^2}{(1-\alpha)(1-\beta)+\alpha(1-\pi^{so})}\right] \quad (A.10)$$

On substituting this expression and $\text{Var}(P_{S,t=3}) = (v_H-v_L)^2/4$ into equation (5), we get η^{so} as in

equation (16). Furthermore, $\eta^{so} > \eta^{ss}$ on comparing equations (6) and (16) since simple algebra

reveals that the term inside the square brackets in equation (16) exceeds one.

Proof of Proposition 4:

As before, the equilibrium values of $\{\pi_S^{sm}; S \in (B,G)\}$ are derived such that informed traders are

indifferent between trading the stock or the option. Therefore, π_B^{sm} satisfies:

$$\frac{B_S^{sm} - E(\tilde{v} \mid S = B)}{m_S B_S^{sm}} = \frac{E[(K-\tilde{v})^+ \mid S = B] - A_P^{sm}}{A_P^{sm}} \quad (A.11)$$

where we know from before that $E(\tilde{v} \mid S = B) = \mu v_L + v_H(1-\mu)$ and $E[(K-\tilde{v})^+ \mid S = B] =$

$\mu(K-v_L)$. On substituting these values and the expressions for B_S^{sm} and A_P^{sm} from equations

(9) and (12), respectively, into equation (A.11) and solving for π_B^{sm}, we get its equilibrium value

as shown in equation (17). Similarly, we can derive π_G^{sm} as shown in equation (18) by solving the

following equation:

$$\frac{E(\tilde{v} \mid S = G) - A_S^{sm}}{m_S A_S^{sm}} = \frac{B_P^{sm} - E[(K-\tilde{v})^+ \mid S = G]}{m_P E(\tilde{v} \mid \text{put sale})} \quad (A.12)$$

where $E(\tilde{v} \mid S = G) = \mu v_H + v_L(1-\mu)$, $E[(K-\tilde{v})^+ \mid S = G] = (1-\mu)(K-v_L)$ and A_S^{sm}, B_P^{sm},

and $E(\tilde{v} \mid \text{put sale})$ are as shown in equations (10), (11) and (A.8), respectively.

In order for the mixed strategy equilibrium to exist, the mixing probabilities must lie between

zero and one. We can easily see from equation (17) that the necessary and sufficient conditions for

$0 < \pi_B^{sm} < 1$ are those in equation (19). Similarly, we can see that $0 < \pi_G^{sm} < 1$ if and only if the

conditions in equation (20) are satisfied.

Finally, we can easily see that the informed traders' stock returns are unambiguously greater

than (less than) their option returns when the left hand side (right hand side) inequalities in

equations (19) and (20) are reversed, which implies that they follow the pure strategy of trading only in the stock (option) market.

Proof of Corollary 3:

We prove the corollary by taking partial derivatives of π_B^{sm} and π_G^{sm} with respect to the various parameters in equations (17) and (18), respectively. For π_B^{sm}, we see that $\dfrac{\partial \pi_B^{sm}}{\partial K} = \dfrac{\partial \pi_B^{sm}}{\partial m_P} = 0$ and

$$\frac{\partial \pi_B^{sm}}{\partial \beta} = \frac{2\alpha\mu m_S v_B(v_H - v_L) + (1-\alpha)[m_S(v_L + v_H) - (v_H - v_L)][\mu\beta^2(v_H - v_L) - m_S v_B(1-\beta)^2]}{2\alpha[m_S v_B(1-\beta) + \mu\beta(v_H - v_L)]^2}$$

$$\frac{\partial \pi_B^{sm}}{\partial \alpha} = \frac{\beta(1-\beta)[m_S(v_L + v_H) - (v_H - v_L)]}{2\alpha^2[m_S v_B(1-\beta) + \mu\beta(v_H - v_L)]}$$

$$\frac{\partial \pi_B^{sm}}{\partial \mu} = \frac{\beta(1-\beta)(v_H - v_L)\{2\alpha m_S v_H + (1-\alpha)[\beta - m_S(1-\beta)][m_S(v_L + v_H) - (v_H - v_L)]\}}{2\alpha[m_S v_B(1-\beta) + \mu\beta(v_H - v_L)]^2}$$

$$\frac{\partial \pi_B^{sm}}{\partial m_S} = -\frac{\beta(1-\beta)(v_H - v_L)[\mu\beta(1-\alpha) + \boldsymbol{\Psi_1} v_B]}{2\alpha[m_S v_B(1-\beta) + \mu\beta(v_H - v_L)]^2}$$

where $\boldsymbol{\Psi_1}$ is as defined in Proposition 4 and $v_B = E(\tilde{v} \mid S = B) = \mu v_L + v_H(1-\mu)$. On inspection, we can see that $\dfrac{\partial \pi_B^{sm}}{\partial m_S} < 0$ and $\dfrac{\partial \pi_B^{sm}}{\partial \alpha} > (<) \ 0$ if $\dfrac{m_S}{v_H - v_L} > (<) \ (v_H + v_L)^{-1}$. We can use the inequalities in feasibility equation (19) to show that $\dfrac{\partial \pi_B^{sm}}{\partial \beta}$ and $\dfrac{\partial \pi_B^{sm}}{\partial \mu}$ as shown above are positive.

The partial derivatives of π_G^{sm} in equation (18) with respect to the parameters are as follows:

$$\frac{\partial \pi_G^{sm}}{\partial \beta} = \frac{2\alpha m_S m_P v_G(v_H - v_L)(K - v_L) - (1-\alpha)(v_L + v_H)(1-\boldsymbol{\Phi})[\boldsymbol{\Phi}(1-\beta)^2 - \beta^2]}{2\alpha v_G[\boldsymbol{\Phi}(1-\beta) + \beta]^2}$$

$$\frac{\partial \pi_G^{sm}}{\partial \alpha} = \frac{\beta(1-\beta)(v_L + v_H)(\boldsymbol{\Phi} - 1)}{2\alpha^2 v_G[\boldsymbol{\Phi}(1-\beta) + \beta]}$$

$$\frac{\partial \pi_G^{sm}}{\partial \mu} = \frac{\beta(1-\alpha)(1-\beta)(v_L + v_H)(v_H - v_L)(\boldsymbol{\Phi} - 1)}{2\alpha v_G^2[\boldsymbol{\Phi}(1-\beta) + \beta]}$$

$$\frac{\partial \pi_G^{sm}}{\partial m_S} = -\frac{\beta\boldsymbol{\Phi}(1-\beta)[(1-\alpha)(v_L + v_H) + 2\alpha v_G]}{2\alpha m_S v_G[\boldsymbol{\Phi}(1-\beta) + \beta]^2}$$

$$\frac{\partial \pi_G^{sm}}{\partial m_P} = \frac{\beta\boldsymbol{\Phi}(1-\beta)[(1-\alpha)(v_L + v_H) + 2\alpha v_G]}{2\alpha m_P v_G[\boldsymbol{\Phi}(1-\beta) + \beta]^2}$$

$$\frac{\partial \pi_G^{sm}}{\partial K} = -\frac{\beta\boldsymbol{\Phi}(1-\beta)[(1-\alpha)(v_L + v_H) + 2\alpha v_G]}{2\alpha v_G(K - v_L)[\boldsymbol{\Phi}(1-\beta) + \beta]^2}$$

where $\boldsymbol{\Phi} = \dfrac{m_S(K - v_L)}{m_P(v_H - v_L)}$ and $v_G = E(\tilde{v} \mid S = G) = \mu v_H + v_L(1-\mu)$. On inspection, we can infer that $\dfrac{\partial \pi_G^{sm}}{\partial m_P}$ is positive and $\dfrac{\partial \pi_G^{sm}}{\partial m_S}$ and $\dfrac{\partial \pi_G^{sm}}{\partial K}$ are negative. We can use the inequalities in feasibility condition (20) to show that $\dfrac{\partial \pi_G^{sm}}{\partial \beta} > 0$. Finally, we can easily see that $\dfrac{\partial \pi_G^{sm}}{\partial \alpha}$ and $\dfrac{\partial \pi_G^{sm}}{\partial \mu}$ are positive (negative) if $\boldsymbol{\Phi} > (<) \ 1$.

References

[1] Admati, A.R. and P. Pfleiderer, 1989, Divide and conquer: A theory of intraday and day-of-the-week mean effects, *Review of Financial Studies* 2, 189-223.

[2] Back, K., 1993, Asymmetric information and options, *Review of Financial Studies* 6, 435-472.

[3] Biais, B. and P. Hillion, 1994, Insider and liquidity trading in stock and options markets, *Review of Financial Studies* 7, 743-780.

[4] Brennan, M.J. and H.H. Cao, 1996, Information, trade, and derivative securities, *Review of Financial Studies* 9, 163-208.

[5] Chowdhry, B. and V. Nanda, 1998, Leverage and market stability: The role of margin rules and price limits, *Journal of Business* 71, 179-210.

[6] Conrad, J., 1989, The price effect of option introduction, *Journal of Finance* 44, 487-498.

[7] Damodaran, A. and J. Lim, 1991, The effect of option listing on the underlying stocks' return processes, *Journal of Banking and Finance* 15, 647-664.

[8] Easley, D., M. O'Hara and P.S. Srinivas, 1998, Option volume and stock prices: Evidence on where informed traders trade, *Journal of Finance* 53, 431-465.

[9] Fedenia, M. and T. Grammatikos, 1992, Option trading and the bid-ask spreads of the underlying stocks, *Journal of Business* 65, 335-351.

[10] Garbade, K.D., 1982, Federal Reserve margin requirements: A regulatory initiative to inhibit speculative bubbles, in Paul Wachtel, ed.: *Crises in the Economic and Financial Structure* (Lexington Books, Lexington, Mass.).

[11] Glosten, L.R. and P.R. Milgrom, 1985, Bid, ask and transaction prices in a specialist market with heterogeneously informed traders, *Journal of Financial Economics* 14, 71-100.

[12] Grossman, S.J., 1988, An analysis of the implications for stock and futures price volatility of program trading and dynamic hedging strategies, *Journal of Business* 61, 275-298.

[13] Hardouvelis, G.A., 1990, Margin requirements, volatility, and the transitory component of stock prices, *American Economic Review* 80, 736-762.

[14] Hsieh, D.A. and M.H. Miller, 1990, Margin regulation and stock market volatility, *Journal of Finance* 45, 3-29.

[15] Hull, J.C., 1998, *Introduction to Futures and Options Markets* (Prentice Hall, Englewood Cliffs, New Jersey).

[16] Jennings, R.H. and L.T. Starks, 1986, Earnings Announcements, Stock Price Adjustment, and the Existence of Option Markets, *Journal of Finance* 41, 107-125

[17] Kyle, A.S., 1985, Continuous auctions and insider trading, *Econometrica* 53, 1315-1335.

[18] Mayhew, S., 1999, The impact of derivatives on cash markets: What have we learned?, Working paper, Purdue University.

[19] Milgrom, P.R. and N. Stokey, 1982, Information, trade and common knowledge, *Journal of Economic Theory* 26, 17-27.

[20] Seguin, P.J., 1990, Stock volatility and margin trading, *Journal of Monetary Economics* 26, 101-121.

[21] Seguin, P.J. and G.A. Jarrell, 1993, The irrelevance of margin: Evidence from the Crash of '87, *Journal of Finance* 48, 1457-1473.

[22] Skinner, D.J., 1989, Options markets and stock return volatility, *Journal of Financial Economics* 23, 61-78.