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Abstract

In this paper, we examine the dynamic behavior of credit spreads on corporate bond portfolios. We propose an econometric model of credit spreads that incorporates portfolio rebalancing, the near unit root property of spreads, the autocorrelation in spread changes, the ARCH conditional heteroscedasticity, jumps, and lagged market factors. In particular, our model is the first that takes into account explicitly the impact of rebalancing and yields estimates of the absorbing bounds on credit spreads induced by such rebalancing. We apply our model to nine Merrill Lynch daily series of option-adjusted spreads with ratings from AAA to C for the period January, 1997 through August, 2002. We find no evidence of mean reversion in these credit spread series over our sample period. However, we find ample evidence of both the ARCH effect and jumps in the data especially in the investment-grade credit spread indices. Incorporating jumps into the ARCH type conditional variance results in significant improvements in model diagnostic tests. We also find that while log spread variations depend on both the lagged Russell 2000 index return and lagged changes in the slope of the yield curve, the time-varying jump intensity of log credit spreads is correlated with the lagged stock market volatility. Finally, our results indicate the ARCH-jump specification outperforms the ARCH specification in the out-of-sample, one-step-ahead forecast of credit spreads.

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1 Introduction

Accessing and managing credit risk of risky debt instruments has been a major area of interest and concern to academics, practitioners and regulators, especially in the aftermath of a series of recent credit crises such as the Russian default and the Enron and WorldCom collapses. In particular, there has been a fast growing literature on models of credit risk measurement.[1] One measurement issue that is both interesting and challenging is the credit risk of a portfolio of risky bonds. This is especially relevant for banks, pension funds, insurance companies, and bond mutual funds. However, the literature on portfolio credit risk is still new, especially in the area of empirical studies.

In this paper, we examine the time series behavior of credit spreads on corporate bond portfolios and propose an econometric model for these spreads that incorporates portfolio rebalancing, unit root, conditional heteroscedasticity, jumps, and Treasury bond and/or equity market factors. More specifically, we apply this model to option-adjusted spreads (OAS) of Merrill Lynch (ML) corporate bond indices for nine rating/maturity categories over January, 1997 through August, 2002.

There are a few benefits from examining credit spreads of corporate bond indices from a major dealer.[2] First, the ML credit spread indices are representative portfolios with a given rating and maturity range, are updated daily, and are a leading index in credit markets. Yields on several Merrill Lynch corporate bond indices are in fact quoted in the *Wall Street Journal*. A credit spread index is also useful for corporate bond index funds that track a corporate bond index. The accuracy of a tracking model depends crucially on its assumptions on the interest rate and credit spread dynamics of the targeted index (see Jobst and Zenios (2003)).

Second, credit spread indices could serve as the underlying instrument for credit derivatives. There has been a rapid growing market for structured credit products recently, with the total value exceeding $1 trillion in 2001 (Berd and Howard, 2001). Although there are no credit derivatives written directly on credit spread indices in the market at this moment, there are ongoing efforts in the industry to make this happen. For example, Standard & Poor's has announced that "S&P Credit Indices are now available for licensing in association with derivative products, structured notes, reference obligations and other applications" (Standard & Poor's, 1999). To design and price credit derivatives based on

[1] See, for example, Caouette, Altman, and Narayanan (1998), Saunders and Allen (2002), Duffie and Singleton (2003) and references therein.

[2] Three widely followed corporate bond indices covering both the investment-grade and high-yield corporate bond markets are those published by Lehman Brothers, Merrill Lynch, and Salomon Smith Barney (Reilly and Wright, 2001).

a credit spread index, it is important to investigate first the time series properties of the index.

Third, the ML high-yield indices each cover a substantial amount of high-yield issues (see Appendix B). Existing studies on the dynamics of individual bond credit spreads have mostly focused on monthly prices of investment grade bonds because of data constraints (e.g., Duffee, 1999). A study of high-yield indices may help understand the general dynamics of credit spreads in the high-yield bond market.

The dynamics of credit spread indices poses a challenge in the modelling of financial time series. Like equity indices, a corporate bond index is often rebalanced, usually at a monthly frequency, to maintain the qualifying criteria for the index. Although this issue is always ignored in studies of equity indices, we believe that it should not be so in modelling the behavior of a corporate bond index. This is because frequent portfolio rebalancing has two direct impacts on the time series properties of portfolio credit spreads. First, since large movements in credit spreads of an issuer are often accompanied by changes in credit ratings, credit spreads on index rebalancing days are implicitly bounded by some absorbing boundaries for rating based corporate bond indices. Second, the rebalancing leads to changes in the index components and changes in the behavior of the index as well.

The econometric model we propose is flexible enough to capture these special features of corporate bond indices. We assume that after rebalancing, the index credit spread process restarts from a new level and that credit spreads on rebalancing days have a bounded distribution. This distribution is a bounded transformation of the log-normal distribution, and is assumed to be independent of any past history. The absorbing boundaries are essential features of rating based credit spread indices, and should be incorporated into the pricing of any financial products whose payoff is linked to the credit spread of a particular rating class of bonds. To our knowledge, this study is the first to consider the effect of rebalancing on the dynamics of credit spread indices and explicitly estimate the boundaries (induced by rebalancing) of spreads on rebalancing days.

Admittedly, the independence assumption is rather strong, as a considerable number of bonds will likely remain in the index upon rebalancing, but is made for the following reasons. First, we do not have data information about the degree of memory refreshing. Second, the independence assumption facilitates maximum likelihood estimation of the bounds. Third, log-credit spreads behave like unit root processes, but are likely mean reverting in the long run. The independence of the log-spread on rebalancing days allows the log-spread to be a unit root process in between rebalancing days while still being mean-reverting.

We propose a renewal ARX-ARCH-Jump specification to model the behavior of the logarithm of credit spreads between two adjacent rebalancing days. (Non-negativity of credit spreads makes it natural to focus the time series study of credit spreads on the logarithm.) Empirical evidence has documented that credit spreads exhibit volatility clustering and rare extreme movements. For instance, Duffee (1999) notices that failure of incorporating the GARCH effect in the conditional variance of individual bond credit spreads has resulted in model specification errors. Pedrosa and Roll (1998) find strong GARCH effect and non-normality in the distribution of log credit spread changes at the index level. However, a GARCH conditional variance specification for rebalanced credit spread portfolios is not plausible because of the vanishing memory due to rebalancing. For this reason, we adopt an ARCH specification with limited memory. In addition, we incorporate state-dependent jumps in our modelling of the dynamics of credit spreads. The jump probability is allowed to depend on the lagged general market conditions. Similar specifications have been used in studies of the dynamics of interest rates and exchange rates (e.g., Vlaar and Palm, 1993, Bekaert and Gray, 1996, and Das, 2002).

We estimate our model using daily credit spreads of various Merrill Lynch corporate bond indices from December 31, 1996 to August 30, 2002. Our main findings are as follows. First, credit spreads of various indices do not exhibit clear mean-reversion over our sample period. A comprehensive unit root/stationarity analysis is first carried out to determine the integration order of log credit spreads between rebalancing days. The unit root hypothesis is preferred in all the tests we performed. As a result, the empirical behavior of credit spread indices could be characterized as a process where credit spreads behave as unit root process in between rebalancing days, but are regularly pulled back into certain bounds through rebalancing. The unit root property of credit spread indices we have found here in between rebalancing days is consistent with the findings of Pedrosa and Roll (1998) using daily investment-grade option-adjusted spread indices of about one and half years, but contradictory to the clear mean-reversion of individual bond credit spreads reported in Duffee (1999) over a longer time period.

Second, lagged Russell 2000 index returns and changes in the slope of the Treasury yield curve are significant predictors of the credit spread movement. Higher returns on the Russell 2000 index are followed by decreasing credit spreads and the relationship is stronger for lower rated bonds. For investment-grade indices, a steeper yield curve indicates lower spreads on the next day. These results provide evidence that changes in credit spread are driven by changes in market conditions on a daily basis.

Third, we find that jumps play an important role in modelling the dynamics of credit spreads and that the jump intensity depends on the lagged level of CBOE VIX implied volatility index. Jumps affect credit spreads mainly through the conditional volatility of changes in log credit spreads. Our in-sample model diagnostic tests indicate that the model featuring both ARCH effect and jumps describes the data much better than the model without jumps. One-step-ahead forecast comparisons over the most recent three years provide further support on the statistical and economic significance of jumps in the dynamics of credit spreads.

To ensure that the predictable component we have identified in the movement of credit spreads is not due to slow adjustment to market information in the Merrill Lynch trading desk bid prices, we also apply our model specification to daily spreads from the S&P industrial investment-grade and speculative-grade credit indices from January, 1999 through August, 2002. The results are largely consistent with the findings with the Merrill Lynch data.

This article is not the first to examine empirically the daily dynamics of corporate bond credit spread indices, although the model specification introduced here is new. Pedrosa and Roll (1998) study the daily properties of 60 investment-grade option-adjusted spread indices from Bloomberg between October, 1995 and March, 1997. They find that a Gaussian mixture could better describe the empirical distribution of credit spread changes and that the credit spread changes exhibit a GARCH type conditional variance. However, they do not formally model the dynamic behavior of credit spreads and do not consider the index rebalancing effect either.

The remainder of the paper is organized as follows. Section 2 outlines the econometric framework we propose for modelling the dynamics of credit spreads on corporate bond portfolios that are subject to regular portfolio rebalancing. Section 3 describes the Merrill Lynch credit spread data set used in our empirical analysis. Section 4 contains the estimation results and diagnostic and robustness tests. Section 5 discusses the out-of-sample forecast issue and the implications for credit spread risk measurement and management. Section 6 concludes.

2 An Econometric Model of Corporate Bond Credit Spreads

In this section we describe our model of credit spreads on corporate bond portfolios. We will discuss the models for spreads on rebalancing days and on non-rebalancing days separately.

2.1 Model Specifications

Previous studies have documented that the unconditional distribution of credit spread changes exhibits leptokurtosis (Pedrosa and Roll 1998). In various continuous time models such as those used in Duffee (1999), the conditional variance of changes in credit spreads is a function of the level of credit spreads. This method is valid only when the time series under studies is stationary. Duffee (1999) finds that the error terms from these models still exhibit GARCH type effect. Pedrosa and Roll (1998) describe the unconditional distribution of credit spread changes as Gaussian mixtures with fixed probabilities, and the conditional variance is modelled as a GARCH process. However, as mentioned earlier, the GARCH conditional variance specification for rebalanced credit spread portfolios is not plausible because of the vanishing memory. For this reason, we adopt an ARCH specification with limited memory.

Empirical studies have provided ample evidence that the GARCH type specification is generally insufficient to describe the dynamics of financial time series that are featured by occasional large discontinuous movements. Models that incorporate both GARCH feature and jumps have been shown to result in significant model improvements in the studies of exchange rates (Vlaar and Palm, 1993, Bekaert and Gray, 1996) and interest rates (Das, 2002). The credit market is subject to substantial surprises that would induce significant jumps in the credit spread movement. The defaults of the Russian government, Enron and WorldCom are just typical examples of information surprises that would induce jumps in the systematic risk of corporate bond credit spreads. For these reasons, our model incorporates both ARCH and jumps.

Another aspect of our model specification is related to the information content of general market conditions on the systematic risk of corporate bond credit spreads. The BIS (1998) requirements for controlling "spread risk," "downgrade risk" and "default risk" call for credit risk models that fully integrate market risk and credit risk. Jarrow and Turnbull (2000) are among the first to incorporate general market conditions, as reflected in changes in the spot interest rate and equity market indices, into the reduced-form models of corporate bond pricing (e.g. Duffie and Singleton 1999; Jarrow and Turnbull 1995). Huang and Kong (2002) document that changes in the ML index credit spreads are closely correlated with the concurrent changes in interest rates and equity market index. A more interesting issue for credit risk measurement and management purpose is to look at the information content of lagged general market information on the movement of credit spreads. Understanding the predictable component in the daily movement of corporate bond credit spread would help credit risk measurement and management. In our specification, we allow for the movement of credit spreads to depend on lagged market information.

As mentioned in the introduction, rebalancing has two direct impacts on the observed credit spreads of a particular rating/maturity index. First, since large movements in credit spreads of an issuer are often accompanied by changes in credit ratings, credit spreads on index rebalancing days are implicitly bounded by some absorbing boundaries for rating based corporate bond indices. Second, rebalancing wipes out part of the memory because of the changes in the index components. The econometric model we propose below is flexible enough to capture these special features of corporate bond indices.

Let S_t be the credit spread of a given credit index/portfolio on day t, and Ω_t denote the information set available at t. Consider first the model specification of credit spreads on rebalancing days. The credit spread S_t when t is a rebalancing day is assumed to be given by:

$$\begin{aligned} S_t &= \alpha + \frac{1}{1/(\beta-\alpha) + \exp(-u_{r,t})}, \qquad (1) \\ u_{r,t} &= \mu_r + \epsilon_{r,t}, \quad \epsilon_{r,t} \sim N\left(0, \sigma_r^2\right) \end{aligned}$$

where α, β, μ_r and σ_r are parameters to be estimated (the subscription r refers to rebalancing days) and $0 \leq \alpha \leq \beta$. It is easy to see that $\alpha < S_t < \beta$ and

$$\lim_{u_{r,t}\uparrow -\infty} S_t = \alpha \qquad (2)$$

$$\lim_{u_{r,t}\uparrow \infty} S_t = \beta \qquad (3)$$

$$\lim_{\alpha\downarrow 0, \beta\uparrow\infty} S_t = \exp(\mu_r + \epsilon_{r,t}). \qquad (4)$$

As said before, under the specification given in Eq. (1), it is implicitly assumed that the distribution of the credit spread on a rebalancing day is independent of the past, so that index rebalancing wipes out all memory, and after each rebalancing, the credit spread starts from a random level within (α, β), albeit following the same process thereafter until the next rebalancing day. Even though this assumption is relatively strong, given the difficulty in quantifying the degree of memory refreshing, our assumption can be regarded as a good one for practical purposes and makes the consistent estimation of α and β easily attainable. Eq. (2) indicates that the distribution of the spread approaches a lognormal distribution if the bounds are relaxed.

Consider next the model specification of credit spreads on non-rebalancing days. Suppose day t is the Jth ($J \geq 1$) day after the last rebalancing day. We will often work with the logarithm of credit spreads on non-rebalancing days because it guarantees the non-negativity of predicted credit spreads from the model. We assume that the log spread, $\ln(S_t)$, conditional on the information set at $t-1$, takes the following ARX-ARCH-Jump specification:

$$\begin{aligned} \ln(S_t)|_{\Omega_{t-1}} &= \mu_0 + \gamma \ln(S_{t-1}) + \sum_{j=1}^{J} \phi_j \left(1 - D_{1,t-j}\right) \ln\left(S_{t-j}/S_{t-j-1}\right) \\ &\quad + \sum_{k=1}^{K} \beta_k x_{k,t-1} + \lambda_t \mu_J + \epsilon_t, \qquad (5) \end{aligned}$$

where

$$\epsilon_t \sim \begin{cases} N\left((1-\lambda_t)\,\mu_J, h_t^2+\sigma_J^2\right) & w.p.\ \lambda_t \\ N\left(-\lambda_t\mu_J, h_t^2\right) & w.p.\ (1-\lambda_t) \end{cases} \tag{6}$$

$$h_t^2 = \varpi_0 + \sum_{p=1}^{P} b_p\,(1-D_{1,t-p})\,\epsilon_{t-p}^2. \tag{7}$$

In the above specification, $D_{1,t}$ is a dummy variable that equals one when day t is a rebalancing day and zero otherwise. This captures the vanishing memory feature from index rebalancing. The exogenous variables $x_k, k = 1, \ldots, K$, represent market factors such as interest rates. h_t^2 is the conditional variance of ϵ_t in the no-jump state and follows an ARCH(P) process where $P \leq J$. For the jump intensity in log credit spreads, we concentrate on the Bernoulli distribution, first introduced in Ball and Torous (1983) and also used in Vlaar and Palm (1993), Bekaert and Gray (1996), and Das (2002). In this structure, the probability that a jump occurs on day t is λ_t and the probability of no jumps on day t is $1-\lambda_t$. Various studies have shown that this structure is tenable for daily frequency data. Conditional on a jump occurrence, we assume the jump size J to be $i.i.d$ and normally distributed with $J \sim N\left(\mu_J, \sigma_J^2\right)$.[3] We also allow the jump probability to depend on lagged exogenous variables such as the volatility of interest rate and the volatility of equity market index. Specifically, the jump probability is assumed to be a logistic function augmented by lagged exogenous variables z_1,...,z_L as follows

$$\lambda_t = \frac{\exp\left(p_0 + p_1 z_{1,t-1} + p_2 z_{2,t-1} + ... + p_L z_{L,t-1}\right)}{1+\exp\left(p_0 + p_1 z_{1,t-1} + p_2 z_{2,t-1} + ... + p_L z_{L,t-1}\right)} \tag{8}$$

where $p_\ell, \ell = 0, \ldots, L$, are parameters to be estimated.

2.2 Estimation Method

Consider first the model of spreads on rebalancing days. We will show that the lower and upper bounds are identified and can be estimated by maximum likelihood. It follows from Eq. (1) that

$$\epsilon_t = \ln\left(S_t - \alpha\right) + \ln\left(\beta - \alpha\right) - \ln\left(\beta - S_t\right) - \mu_r, \tag{9}$$

and thus

$$\frac{d\epsilon_t}{dS_t} = \frac{(\beta-\alpha)}{(\beta - S_t)\,(S_t - \alpha)}. \tag{10}$$

[3] In principle, the mean and variance of the jump size could be allowed to depend on lagged exogenous variables. Our empirical analysis finds that this does not provide any model improvement.

Under the normality assumption of ϵ_t, the probability density function of credit spread on rebalancing day t is given by

$$f\left(S_t\right)_r = \frac{(\beta-\alpha)}{(\beta-S_t)\left(S_t-\alpha\right)} \times \frac{\exp\left[-\frac{1}{2\sigma_r^2} \times \left(\ln\left(\frac{(S_t-\alpha)(\beta-\alpha)}{(\beta-S_t)}\right)-\mu_r\right)^2\right]}{\sqrt{2\pi}\sigma_r}. \tag{11}$$

The estimation of the parameter set $\theta_r = [\alpha, \beta, \mu_r, \sigma_r]$ involves maximizing the log-likelihood function as follows

$$\max_{\theta_r=[\alpha,\beta,\mu_r,\sigma_r]} \quad \mathcal{L} = \sum_{t=1}^{T_r} \ln\left(f\left(S_t|\theta_{\mathbf{r}}\right)_r\right). \tag{12}$$

where T_r is the number of rebalancing day observations in the sample. Of course, the ML estimators of the lower (upper) bound α (β) should be strictly less (greater) than the sample minimum (maximum) of spreads on rebalancing days for a given credit index. In Appendix A we show that the parameters can be identified by the first-order condition $E\left(\partial\mathcal{L}/\partial\theta_{\mathbf{r}}\right)=0$.

Consider next the model of spreads on non-rebalancing days. As can be seen from Eq. (5), our model of credit spreads on non-rebalancing days is specified in terms of the conditional distribution of log credit spreads. Estimation will be done by maximizing the conditional likelihood function.

Let $\theta_{nr} = [\mu_0, \gamma, (\phi_j), (\beta_k), \varpi_0, (b_p), (p_\ell), \mu_J, \sigma_j]$. Let also $I_{1,t}$ be an indicator function that equals one in the event of jump on day t and zero otherwise. It follows from Eq. (5) that the conditional density of credit spread S_t on non-rebalancing days can be written as the following:

$$\begin{aligned} f\left(S_t|\Omega_{t-1},\theta_{\mathbf{nr}}\right)_{nr} &= (1-\lambda)\, f\left(S_t|\Omega_{t-1}, I_{1,t}=0\right) + \lambda f\left(S_t|\Omega_{t-1}, I_{1,t}=1\right) \\ &= (1-\lambda_t)\exp\left(\frac{-\left(\ln\left(S_t\right)-\Psi_{t-1}-\mu_0\right)^2}{2h_t^2}\right)\frac{1}{\sqrt{2\pi h_t^2}S_t} \\ &\quad +\lambda_t \exp\left(\frac{-\left(\ln\left(S_t\right)-\Psi_{t-1}-\mu_0-\mu_J\right)^2}{2\left(h_t^2+\sigma_J^2\right)}\right)\frac{1}{\sqrt{2\pi\left(h_t^2+\sigma_J^2\right)}S_t} \end{aligned} \tag{13}$$

where

$$\Psi_{t-1} \equiv \gamma \ln\left(S_{t-1}\right) + \sum_{j=1}^{J} \phi_j \left(1-D_{1,t-j}\right) \ln\left(S_{t-j}/S_{t-j-1}\right) + \sum_{k=1}^{K} \beta_k x_{k,t-1}$$

As a result, under our model specification the density function of credit spreads on day t can be written as follows:

$$f\left(S_t|\Omega_{t-1}\right) = \begin{cases} f\left(S_t|\Omega_{t-1},\theta_{\mathbf{r}}\right)_r, & if\ D_{1,t}=1 \\ f\left(S_t|\Omega_{t-1},\theta_{\mathbf{nr}}\right)_{nr}, & if\ D_{1,t}=0 \end{cases} \tag{14}$$

Since the parameter sets θ_r and θ_{nr} do not intersect, the estimation of the model can be done separately. Namely, the model in (1) can be estimated using only the rebalancing day sub-sample and the model in (5) estimated using only the non-rebalancing day sub-sample.

3 The Credit Spread Data

The credit spread data used in this study are daily Merrill Lynch option-adjusted spreads of corporate bond indices.[4] Each index is a market value weighted average of individual credit spreads on component bonds within a given maturity, industry, and credit rating category. Daily prices of corporate bonds used to calculate credit spread are based on the bid side of the market at 3:00 PM New York time, and obtained from the Merrill Lynch trading desk. Each index is rebalanced on the last calendar day of each month to maintain its qualifying criteria; see Appendix B for a detailed description of the rebalancing procedure and for the number of issues included in each index on rebalancing days. Issues that no longer meet qualifying criteria for a given index are dropped from the index and new issues that meet the qualifying criteria are included for the following month. The dynamics of credit spreads of a given index thus reflects the spread risk of a corporate bond portfolio that is regularly rebalanced to maintain its characteristics on rating, maturity, and amount outstanding. We believe that these ML data are of high quality since as mentioned earlier, yields on several Merrill Lynch corporate bond indices are quoted in the *Wall Street Journal*.

For investment-grade corporate bonds, we obtain industrial corporate credit spread indices for three maturities, 1-10 years, 10-15 years, and 15+ years and two rating groups, AA-AAA and BBB-A, and as a result, have six indices in total. These indices track the performance of US dollar-denominated investment-grade public debt of industrial sector corporate issuers, issued in the US domestic bond market. For high-yield corporate bonds, Merrill Lynch has credit spread indices for three credit ratings: BB, B and C. The industry composition of these high-yield indices is not available. We have in total 9 series of credit spreads with different rating and maturity categories. The sample period is from December 31, 1996 (the inception date of the Merrill Lynch option-adjusted credit spread indices) to August 30, 2002.

The original credit spread series we obtained contain data on weekends or holidays. To ensure that the credit spread data we use reflect market information, we restrict our analysis

[4]For a corporate bond with embedded options such as call provisions, the option-adjusted spread calculation begins by using statistical methods to generate a large number of possible interest rate paths that can occur over the term of the bond and measures the resulting impact of the scenarios on the bond's value. By averaging the results of all the scenarios, the implied spread over the Treasury yield curve is determined.

to NYSE trading days. This is done by matching the credit spread data with the $S\&P$ 500 index data over the sample period. When no match is found, we drop the corresponding observation(s) from the credit spread series. This results in 1,425 daily observations for each ML credit spread index. Figures 1-1, 1-2 and 1-3 plot credit spreads of AA-AAA rated indices, BBB-B rated indices and high-yield indices respectively. Figure 2 plots the percentage changes in log credit spreads on non-rebalancing days. One can see from the figure that spread changes exhibit volatility cluster and large spikes, especially during the Asian financial crisis, the Russian and LTCM defaults and the September 11 event. The changes in investment-grade credit spreads are clearly more volatile during the first two years of the sample period.

Before estimating the model, we look at the statistical properties of the credit spread series first. Table 1 shows the basic statistical properties of the 9 credit spread series on non-rebalancing days. Panels A through C contain respectively the summary statistics on credit spread level, basis point changes and log percentage changes in credit spreads. Credit spreads are given in basis points.

The mean and standard deviation of each credit spread series reported here are comparable to those reported in other studies using option-adjusted spreads (Caouette, Altman, and Narayanan, 1998; Kao, 2000). The mean and volatility of credit spreads are generally higher for indices of lower quality and longer maturity. The sample mean of credit spread changes is all insignificantly different from zero. Credit spread changes of all rating/maturity categories show large excess kurtosis.

Because of the index rebalancing, we calculate the first-order serial correlation coefficient $\rho(1)$ of credit spread changes as defined by

$$\rho\left(1\right)=\sum_{t=2}^{T}\left(\Delta s_{t}-\overline{\Delta s}\right)\left(\Delta s_{t-1}-\overline{\Delta s}\right)\left(1-d_{t}\right)/\sum_{t=1}^{T}\left(\Delta s_{t}-\overline{\Delta s}\right)^{2}, \tag{15}$$

where T is the total number of non-rebalancing days, $\overline{\Delta s}$ is the sample mean of credit spread changes, and d_t is a dummy variable that takes on the value one if t is the day right after a rebalancing day, and zeros otherwise. The first order autocorrelation in credit spread changes (both in basis points and in percentage) is significantly negative for investment-grade credit spreads and significantly positive for high-yield credit spreads at 95% significant level. The first-order serial correlation coefficient of squared credit spread changes (both in basis points and in percentage), which is defined in the same way as Eq. (15), is significantly positive at 95% significant level for all investment-grade credit spread series and C rated spread series.

4 Empirical Results

We first present results from testing if mean-reversion exists in the nine ML credit spread series in between rebalancing days. We then report estimation results from our model of credit spreads. Finally, we show results from robustness tests.

4.1 On Mean-Reversion of Credit Spreads

One issue in the estimation of the log credit spread distribution on non-rebalancing days as specified in Eq. (5) is when the log credit spread series is a near unit root process. In this case, the parameter inference associated with γ is non-standard. Below, we detect the integration order of the credit spread in between rebalancing days in our sample.

Reduced-form models of corporate bond pricing such as Duffee (1999) and Duffie and Singleton (1999) typically assume that the expected default loss is governed by a mean-reverting square-root diffusion process. However, empirical evidence on the mean-reverting speed of corporate bond credit spreads, at either individual bond level or index level, is limited. Duffee (1999) reports a median half-life mean reverting speed of less than 3 years estimated with a sample of individual bonds. Using the Augmented Dickey-Fuller (ADF) test, Pedrosa and Roll (1998) could not reject the unit root hypothesis for daily investment-grade option-adjusted credit spread indices of Bloomberg over the period October 5, 1995 to March 26, 1997. Their failure to reject could be due to the relatively short sample period in their study or the fact that the ADF test is known to have a very low power testing against near unit root alternatives.

Below, we perform a comprehensive unit root analysis, using either unit root or stationarity as the null hypothesis, and allowing for structural breaks in the time series. When doing so, we use the whole sample period, including rebalancing day observations. If there is obvious mean-reverting in credit spreads in between re-balancing days, together with the bounded credit spreads distribution on rebalancing days, we would expect more evidence against unit root through these tests.

4.1.1 Standard Unit Root Tests

We begin with two widely used unit root tests in the literature. Let s_t denote the logarithm of the credit spread on day t. We use the log credit spreads in all the tests so that the time series under study is not bounded from below by zero.

The augmented Dickey-Fuller (ADF) test of unit root hypothesis against the stationarity hypothesis is based on the following regression:

$$s_t - s_{t-1} = \alpha + \beta s_{t-1} + \sum_{j=1}^{p} \nu_j \left(s_{t-j} - s_{t-j-1} \right) + \epsilon_t. \tag{16}$$

where ϵ_t is white noise. The null hypothesis of unit root is that $\beta = 0$, while the alternative hypothesis of mean-reverting is $\beta < 0$. Following Said and Dickey (1984) the initial autocorrelation lag p is selected as a function of the sample size: $p = 5N^{1/4}$ where N is the number of observation in the regression. Based on the regression with this p, the optimal p is then selected under the null hypothesis using the Schwartz information criterion (SIC).

Since the assumption made in the ADF test that ϵ_t is white noise may be violated in the credit spread data, we consider another widely used unit root test, the Phillips-Perron (1988) test. Consider

$$s_t = \alpha + \beta s_{t-1} + \epsilon_t, \tag{17}$$

where ϵ_t is a zero-mean stationary process. The null hypothesis of unit root corresponds to $\beta = 1$ and the alternative hypothesis is $\beta < 1$. This test employs a Newey-West type variance estimator of the long-run variance of ϵ_t and is robust to a wide variety of serial correlation and heteroscedasticity.

The estimate of the β coefficient in the ADF test and the Phillips-Perron test are reported respectively in Panels A and B of Table 2. One can see from the table that the unit root hypothesis could not be rejected in any of the 9 credit spread series. The mean-reversion coefficients β in the ADF test are all negative, but insignificantly different from zero. The estimates of β in the Phillips-Perron test are all above 0.99 and the unit root hypothesis is not rejected at the 95% significance level.

4.1.2 Stationarity Tests

It is a well-known empirical fact that the standard unit root tests such as the ADF and Phillips-Perron tests fail to reject the null hypothesis of a unit root in a near unit root economic time series. The null hypothesis is always accepted unless there is strong evidence against it. To avoid this problem, tests have been designed under the null hypothesis that the time series under test is stationary around a long-term mean, against the alternative that the time series has a unit root. We employ two such stationarity tests as a robustness check of the conclusion reached from the ADF and the Phillips-Perron tests.

The first stationarity test we use is developed by Kwiatkowski, Phillips, Schmidt and Shin (1992) (KPSS hereafter). The KPSS test assumes that the time series under test can be decomposed into a random walk and a stationary error term as follows:

$$s_t = r_t + \epsilon_t, \tag{18}$$

$$r_t \;\; = \;\; r_{t-1} + u_t, \tag{19}$$

where the u_t's are $i.i.d\,(0, \sigma_u^2)$. Under the null hypothesis that $\sigma_u^2 = 0$, the process under test is stationary around a long-term mean. A Lagrange multiplier test statistic is designed under the null hypothesis of stationarity and a large value of this statistic leads to the rejection of stationarity hypothesis.

Another stationarity test we use here is proposed by Bierens and Guo (1993). The test is designed with the null hypothesis

$$s_t = \mu + \epsilon_t, \tag{20}$$

against the alternative

$$\Delta s_t = s_t - s_{t-1} = \epsilon_t \tag{21}$$

where ϵ_t is a zero-mean stationary process and μ is the long-term mean. Bierens and Guo (1993) design four types of Cauchy tests against unit root hypothesis, based on an auxiliary linear time trend regression. Large values of these tests would lead to the rejection of the stationarity null hypothesis.

The results of the two stationarity tests are contained in Panel C and Panel D of Table 2. In the KPSS test, the null hypothesis of stationarity is rejected at 95% significance level for all credit spread series. The Bierens and Guo Cauchy tests exhibit similar pictures. The only evidence of stationarity is from the type 3 and type 4 Cauchy tests on the credit spread of the AA-AAA 10-15 years index.

4.1.3 Nonlinear Augmented Dickey-Fuller Test

One possible reason for the non-stationarity shown above could be the presence of structural breaks in the credit spread time series. Perron (1989, 1990) and Perron and Vogelsang (1992) have shown that when a time series has structural breaks in the mean, the unit root hypothesis is often accepted before structure breaks are taken into account, while it is rejected after structural breaks are considered. The fact that our sample includes extraordinary financial and credit events as mentioned earlier makes it very likely to have some structural breaks.

A few unit root tests have been developed for time series with structural breaks. We use the Bierens (1997) nonlinear augmented Dickey-Fuller (NLADF) test here since it allows the trend to be an almost arbitrary deterministic function of time. The test is based on an ADF type auxiliary regression model where the deterministic trend is approximated by a linear function of Chebishev polynomials:

$$\Delta s_t = \beta s_{t-1} + \sum_{j=1}^{p} \nu_j \Delta s_{t-j} + \theta^T P_{t,n}^{(m)} + \epsilon_t, \tag{22}$$

where $P_{t,n}^{(m)} = \left(P_{0,n}^{*}(t), P_{1,n}^{*}(t), ..., P_{m,n}^{*}(t)\right)^{T}$ is a vector of orthogonal Chebishev polynomials. Under the null hypothesis of unit root, $\beta = 0$ and $\theta^T = 0$. The unit root hypothesis is tested based on the t-statistic of β, the test statistic $Am = ((n-p-1)\beta) / \left|1 - \sum_{j=1}^{p} \nu_j\right|$, and the $F-$test of the joint hypothesis that β and the last m components of θ^T are zero. Panel E of Table 2 presents the results of NLADF tests and the associated critical values. The results show that even after any nonlinear trend breaks are taken into consideration, the unit root hypothesis still could not be rejected.

In summary, we can conclude that there is no empirical evidence of mean-reversion in the 9 credit spread series over our sample period in between rebalancing days. As a consequence, the empirical behavior of credit spread indices may be captured by a process in which credit spreads behave as unit root process in between rebalancing days, but are regularly pulled back within certain bounds through rebalancing.

4.2 Estimation Results of the Model of Credit Spreads

The model given in Eq. (5) is rather general. The summary statistics reported in Table 1 suggest that a particular specification of the general model may be adequate for our sample of data. In particular, results of $\rho(1)_{\Delta s}$ and $\rho(1)_{\Delta s^2}$ shown in Table 1 indicate that a specification with AR(1) and ARCH(1) in Eq. (5) may be a good first attempt to capture the autocorrelation in spread changes and conditional variance. As a result, we will estimate an ARX(1)-ARCH(1)-Jump model of log credit spread changes in this subsection. We will perform robustness tests in the next subsection.

4.2.1 Estimated Model

The econometric model introduced in section 2 allows the dynamics of credit spreads to depend on lagged exogenous variables. It has been well recognized that changes in corporate bond credit spreads are closely correlated with the contemporaneous changes in general market and economic conditions, as reflected by changes in interest rate and stock market indices. The exogenous variables we consider include lagged interest rate changes, changes in the slope of the yield curve, Russell 2000 index returns and the CBOE VIX implied volatility. Specifically, we allow the conditional mean of log credit spread changes to depend on lagged interest rate changes Δr, yield curve slope changes $\Delta slope$ and the Russell 2000 index return ret_{rus}. We also allow for the conditional jump probability to depend on lagged level of the CBOE VIX index since we expect to observe more extreme movement in credit spreads in a more volatile equity market.

Changes in credit spreads are generally considered to be negatively correlated with the contemporaneous changes in interest rates and changes in slope of the Treasury yield curve, as has been shown in Duffee (1998). We use the change in the Merrill Lynch Treasury Master Index yield (%) as a proxy for the change in the interest rate. The slope of the Treasury yield curve is approximated by the difference between the ML 15+ years Treasury Index yield (%) and the ML 1-3 years Treasury Index yield (%). Credit spreads also tend to rise when returns on stock market index are low. We choose the Russell 2000 index return ($ret_{rus,t} = \ln(P_{rus,t}/P_{rus,t-1})$) here because it has been shown to be more closely related to credit spread changes than a large-cap index such as the S&P 500 index (Kao, 2000; Huang and Kong, 2002).

Based on the above discussion, we estimate the following ARX(1)-ARCH(1)-Jump model with the sub-sample of credit spreads on non-rebalancing days:

$$\begin{aligned} \ln(S_t) &= \ln(S_{t-1}) + \mu_0 + \phi_1 (1 - D_{1,t-1}) \ln(S_{t-1}/S_{t-2}) \qquad (23) \\ &\quad + \beta_1\, ret_{rus,t-1} + \beta_2\, slope_{t-1} + \beta_3 \Delta r_{t-1} + \lambda_t \mu_J + \epsilon_t, \\ h_t^2 &= \varpi_0 + b_1 (1 - D_{1,t-1})\, \epsilon_{t-1}^2, \qquad (24) \\ \lambda_t &= \exp(p_0 + p_1 * VIX_{t-1}) \,/\, (1 + \exp(p_0 + p_1 * VIX_{t-1})). \qquad (25) \end{aligned}$$

To compare the relative importance of the ARCH specification and jumps in explaining the leptokurtic behavior of spread changes, we estimate both the ARX-ARCH-Jump model and the nested ARX-ARCH model and report results separately. The estimation is done via the (quasi) maximum likelihood method using the GAUSS MAXLIK and CML modules. Both the Broyden, Fletcher, Goldfarb, and Shanno (BFGS) algorithm, and the Berndt, Hall, Hall, and Hausman (BHHH) algorithm are used in the estimation and give the same results.

4.2.2 Results from the Model for Rebalancing Days

Table 3 contains the estimation results of the credit spread distribution on rebalancing days for the nine Merrill Lynch credit spread series. As expected, the estimates of the lower bound α and the upper bound β are close to the sample minimum and maximum of credit spreads, respectively, on rebalancing days. The distance between estimated α and β indicates that even for portfolios that have managed to maintain its rating, minimum amount outstanding and maturity, the upper boundary of the credit spreads could still be three to five times higher than the lower boundary. One can also see from the table that the mean of the innovations, μ_r, increases as the credit rating gets lower, but the estimates of the standard deviation of the innovations, σ_r, do not always increase as credit ratings decrease.

4.2.3 Results from the Model in between Rebalancing Days

Estimation results from the nested ARX(1)-ARCH(1) model for log credit spreads in between rebalancing days are reported in Table 4. Results from the complete ARX(1)-ARCH(1)-Jump model are presented in Table 5.

As shown in the tables, the drift term μ_0 and the mean of the jump size μ_J are mostly insignificant. Even though in the model with jumps, the estimate of μ_J is positive for 8 indices, it is only significant for the BBB-A 15+ years index. This implies that jumps affect credit spreads mainly through the conditional volatility of log credit spreads. There is autocorrelation in the changes of log credit spreads, as suggested by the estimate of the AR(1) coefficient ϕ_1. The positive autocorrelation in log credit spread changes of high-yield indices is clearly a result of the slightly upward trend exhibited by high-yield credit spread indices over the sample period. The negative autocorrelation in log credit spread changes of investment-grade indices suggests the existence of short-run mean-reversion. The estimates of the autocorrelation term are more significant when jumps are considered in the model.

The return on equity market index provides valuable information in forecasting the credit spread of next trading day. The estimated coefficients of lagged Russell 2000 index returns are significantly negative for all indices in the ARX-ARCH model. In the ARX-ARCH-Jump model, the lagged Russell 2000 index returns lose significance for the AA-AAA 1-10 years and 10-15 years indices, but are still significant for all the other seven indices. Not surprisingly, when jumps are included, the estimated coefficients are smaller. The information contained in lagged Russell 2000 index return is also economically significant. Taking the AA-AAA 15+ years index as an example, the estimated coefficient on the lagged Russell 2000 index return in the ARIMAX-ARCH-Jump model is 0.063, and the mean value of the AA-AAA 15+ years index is about 87 basis points over the sample period. Consequently, when evaluated at the mean, a 1% return on the Russell 2000 index predicts a 5 basis points drop in credit spreads on next trading day.

The information contained in the slope of the Treasury yield curve helps predict the movement of credit spreads on investment-grade corporate bonds. The estimated coefficients for high-yield indices are positive and insignificant in all cases. For investment-grade indices, a steepening Treasury yield curve predicts falling credit spreads. The estimated coefficients indicate that, when the difference between the long-term Treasury yield and the short-term Treasury yield increases by 100 basis points, the credit spreads of various investment-grade indices would drop by 1.5% to 3.7%. The insignificance of the yield curve information in the prediction of high-yield credit spreads is consistent with the reported weak contemporaneous relation between changes in high-yield credit spreads and changes in yield curve slope for the period of January 1990 through December 1998 in Kao (2000).

But it is not very clear why credit spreads of high-yield bonds would be less sensitive to the slope of the yield curve than investment-grade bonds.

We do not find any convincing evidence that changes in the interest rate provide any useful information on the next movement of credit spreads. The estimated coefficients on lagged interest rate changes are mostly insignificant.

In summary, the conditional mean of log credit spread changes depends on lagged log credit spread changes, lagged Russell 2000 index returns and lagged changes in the slope of the Treasury yield curve.

We now discuss results on jumps. The estimated mean of the jump size is not significantly different from zero for most indices. Jumps affect credit spreads mainly through the conditional volatility. Consequently, including jumps in the movement of credit spreads results in a sharp decrease in the constant term ϖ_0 and the persistent coefficient b_1 in the ARCH(1) specification. Clearly, part of the time-varying volatility would be better modelled as the result of jumps in the movement of credit spreads than as the result of persistence of the squared innovation in last period. An interesting result is when jumps are modelled in the conditional variance, the ARCH(1) coefficient for high-yield indices becomes significant.

The conditional jump probability is clearly time-varying and depends on the lagged volatility in the equity market. The coefficient on lagged CBOE VIX index in the time-varying jump probability specification is significantly positive for all indices. The sensitivity of conditional jump probability to lagged equity market volatility tends to increase as the credit quality gets lower. This is consistent with the implication of structural models of corporate bond pricing that high-yield bonds behave more like equity. The sample mean of the CBOE VIX index over the sample period is about 26%. When evaluated at the sample mean of the VIX index, the daily jump probability in log credit spreads ranges from 11.2% for the AA-AAA 10-15 years index to 3.1% for the BB index.

Results of the Schwartz and Akaike information criteria shown in the bottom of Table 5 indicate that the ARX-ARCH-Jump model outperforms the ARX-ARCH model in terms of the overall goodness-of-fit. A potential problem might arise when using the likelihood ratio test for the statistical significance of jump behavior in log credit spreads. This is because the parameters associated with jumps cannot be identified under the null hypothesis of no jumps. Hansen (1992) states that unless the likelihood surface is locally quadratic with respect to the nuisance parameters, the LRT statistic is no longer distributed χ^2 under the null hypothesis.[5] A formal test on the null hypothesis would require a series of optimizations over a grid of the nuisance parameters and the computation would be

[5] The LRT statistic ranges from 256 for B rated index to 1011 for BBB-A 1-10 years index.

extremely burdensome. In our case, the fact that the coefficient on lagged equity market volatility is highly significant do provide a certain amount of confidence in the existence of jump behavior in credit spreads. In the next subsection, we present more model diagnostic tests on the ARX-ARCH-Jump model and the nested ARX-ARCH model based on in-sample residuals.

4.2.4 Model Diagnostic Tests

Several specification tests based on in-sample residuals are performed to test the conditional normality of the innovation. We summarize the results in Table 6. Under the ARX-ARCH specification, the standardized residual of the model is standard normally distributed. In the specification involving jumps, the residual of the estimated model is actually a mixture of two normal distributions. To compare the residual distribution from the estimation of the ARX-ARCH-Jump model and the nested ARX-ARCH model, we use the method of Vlaar and Palm (1993). We first calculate the probability of observing a value smaller than the standardized residual. In the jump specifications, this would be a weighted average of the normal cumulative distribution function under the jump state and the no-jump state. Under the null hypothesis of normal mixture, these probabilities should be identically uniformly distributed between 0 and 1. A Pearson chi-squared goodness-of-fit test is then performed on these transformed residual series of each model by classifying the series into g groups based on their magnitudes. Under the null hypothesis, this test statistic is chi-squared distributed with $g-1$ degree of freedom.

Column 4 of Table 6 presents the associated Pearson chi-squared goodness-of-fit test statistic of each model when g equals 20. It is quite clear from the large $\chi^2(19)$ value that the ARX-ARCH normal model is inappropriate for index credit spreads. The smallest value of the $\chi^2(19)$ statistic in the ARX-ARCH model, that of the B credit spread index, is as high as 74.88. The results improve significantly when the specification includes jumps. In the ARX-ARCH-Jump model, the null hypothesis is not rejected at the 5% level for 4 out of the 9 indices, and is not rejected at the 1% level for 8 out of the 9 indices.

We now conduct two diagnostic tests based on the autocorrelation in the standardized residuals of the estimated models. In the specification with jumps, we follow again Vlaar and Palm (1993). Specifically, the standardized residuals are obtained by inverting the standard normal cumulative distribution function based on the probability series in calculating the Pearson chi-squared test. We compute the first-order sample autocorrelation coefficient of the standardized residuals $\rho_\epsilon(1)$, and of the squared standardized residuals $\rho_{\epsilon^2}(1)$ based Eq. (15). The results are presented in columns 5 and 6 of Table 6. It turns out that our specification has removed most of the first-order autocorrelation in spread innovations as reported in Table 1. Although the first-order sample autocorrelation coefficient of the standardized residuals is significant for AA-AAA rated indices, they are typically between 0 and -0.1. The ARCH(1) specification also seems successful in capturing the time dependence of volatility. Except for the AA-AAA 1-10 years index and the BBB-A 10-15 years index, the first-order sample autocorrelation coefficient of the squared standardized residuals is insignificant. In the two cases where they are significant, the estimates are both less than 0.7. Overall, correlation in the residuals and the squared residuals does not pose any challenge to our model specification.

For five indices, the ARX-ARCH-Jump model passes the Pearson $\chi^2\,(19)$ goodness-of-fit test marginally. We explore the possible misspecification by looking at the empirical skewness and kurtosis of the standardized residuals from the model estimation. The first four central moments of the standardized residuals are computed in a joint GMM-system. The normality of the standardized residuals is then tested based on a Wald-test that both the skewness and kurtosis coefficients are jointly equal to zero. The estimated sample skewness, kurtosis and GMM normality test statistic are reported in the last three columns of Table 6. The results in Table 6 tell us that there is no skewness in the standardized residuals from both specifications, and the two specifications differ primarily in modelling the fatness in the tails of the distribution. There is still substantial leptokurtosis in the standardized residuals from the ARX-ARCH specification. The minimum is 7.2 for the C rated index and the maximum is 24.8 for the BBB-A 1-10 years index. The leptokurtosis in the standardized residuals from the ARX-ARCH-Jump specification is also significant for all indices, but the maximum is only 0.84. Again, the transformed residuals from the ARX-ARCH-Jump specification pass the normality test marginally. Since the existing leptokurtosis in the transformed residuals is at such a small magnitude, we are comfortable enough to say that the ARCH-Jump specification well captures the fat tails in the original distribution of credit spread changes.

Overall, there is clear evidence that both jumps and time-varying volatility exist in the daily movement of credit spreads of different credit quality and maturity corporate bond indices. Model diagnostic tests show that the ARX-ARCH-Jump model is strongly preferred over the nested ARX-ARCH and performs rather well for the dynamics of both investment-grade and high-yield indices.

4.3 Robustness Tests

We have documented the information content of lagged equity market returns, volatility, and yield curve slope. However, it is important to ensure that the predictive power we have identified is not due to the special feature of the Merrill Lynch bid prices. As a robustness test, we fit the model specification used for the Merrill Lynch data to the S&P's credit indices.

Specifically, we use the S&P U.S. industrial investment-grade and speculative-grade credit indices. Both indices contain daily option-adjusted credit spreads (see the S&P publication (1999) for details). The composite market price used to calculate the option-adjusted spread is based on the average bid and ask prices from a number of sources including brokers and dealers. A potential problem of the S&P credit indices is that, unlike other credit spread indices, an issue is substituted by another issue immediately when the original issue no longer meets the selecting criteria on maturity, amount outstanding, rating and liquidity, and when an individual issue "experience a large credit spread fluctuation that is not indicative of the general market trend." The level of the indices is adjusted by a divisor whenever there is a change in the index issue. Consequently, we apply our non-rebalancing day credit spread model to the adjusted spread series of S&P credit indices. Nevertheless, the results from the S&P credit indices would provide additional evidence on the information content of lagged general market conditions.

We estimate the ARX(1)-ARCH(1)-Jump model using the S&P daily credit indices for the time period of December 31, 1998 (the inception date of the indices) to August 30, 2002. The estimated results are reported in Table 7. One can see from the table that similar to the Merrill Lynch indices, the S&P indices also exhibit a relationship between credit spread movements and the lagged equity market return and volatility. In particular, the estimated coefficients on the lagged Russell 2000 index return are significantly negative for both S&P indices. Also, the time-varying jump probability significantly depends on the lagged volatility level in the equity market as measured by the CBOE VIX index.

5 Out-of-Sample Forecast and Implications

In this section we seek to further explore the economic implication of allowing for lagged exogenous variables, conditional heteroscedasticity, jumps in the modelling of credit spreads. We first derive the one-step-ahead prediction formula for the ARX-ARCH-Jump model. We then demonstrate that the model with jumps performs well in forecasting out-of-sample credit spreads. We also discuss the implication of our findings for measuring and pricing of credit risk.

5.1 Out-of-Sample Specification Tests

To avoid over-parameterization of the ARX-ARCH-Jump model, and to establish the economic significance of allowing for jumps in the dynamics of credit spread, we compare the out-of-sample forecast ability of the model with jumps and that of the model without jumps.

Given the model specification in Eqs. (23) and (25), we can form the following one-step-ahead forecast conditional on the information set Ω_{t-1} at time $t-1$:

$$\begin{aligned} E_{t-1}(S_t) &= E_{t-1}\Big[\exp(\ln(S_{t-1}) + \mu_0 + \phi_1(1-D_{1,t-1})\ln(S_{t-1}/S_{t-2}) \\ &\quad + \beta_1 ret_{rus,t-1} + \beta_2 slope_{t-1} + \beta_3 \Delta r_{t-1} + \lambda_t \mu_J + \epsilon_t)\Big] \\ &= S_{t-1}\exp(\phi_1(1-D_{1,t-1})\ln(S_{t-1}/S_{t-2}))\,\exp(\mu_0 + \lambda_t\mu_J) \\ &\quad \times \exp(\beta_1 ret_{rus,t-1} + \beta_2 slope_{t-1} + \beta_3 \Delta r_{t-1})\; E_{t-1}(\exp(\epsilon_t)). \end{aligned} \tag{26}$$

It follows from (6) that

$$\begin{aligned} E_{t-1}(\exp(\epsilon_t)) &= (1-\lambda_t)\exp\left(-\lambda_t\mu_J + 0.5h_t^2\right) \\ &\quad + \lambda_t \exp\left((1-\lambda_t)\mu_J + 0.5\left(h_t^2 + \sigma_J^2\right)\right). \end{aligned} \tag{27}$$

The forecaster based on the model without jumps can be obtained from the above equation by setting λ_t to zero.

In performing the out-of-sample test, we first estimate the ARX-ARCH-Jump model and the nested ARX-ARCH model using the data from January 1997 to December 1999. The estimated parameters are used in the one-step ahead out-of-sample prediction for the non-rebalancing day credit spread in January 2000. The same procedure is repeated each month over the subsequent period. That is, starting with January 2000, on the first non-rebalancing day of each month, the parameters of the model are estimated using all past observation, and the parameters are then used for the credit spread forecast within this month without being updated. In principle, the parameter could be updated each day using past observations. However, this practice will be computationally burdensome. The approach we have adopted is a trade-off between computational convenience and timely updating of new information. Because the estimates of the drift term μ_0, the mean of the jump size μ_J, and the coefficient on lagged interest rate changes are mostly insignificant, we have dropped lagged changes in interest rates, and allowed for both μ_0 and μ_J to be zero in the model we used for forecast. For comparison, we also include the prediction performance of a simple martingale model of credit spreads using just credit spreads of previous day. The difference between forecast and actual credit spreads over the out-of-sample period is summarized in the form of root mean squared error (RMSE) and mean absolute errors (MAE).

The changes in log credit spreads are much more volatile over the first three years than the most recent three years of the sample. This makes it important to allow for a time-varying jump probability. Results in Table 8 show that in the out-of-sample forecast race, the model with time-varying jumps outperform the model without jumps in eight out of the nine different indices. The complicated models outperform the simple martingale model in six (seven) indices in terms of MAE (RMSE).[6] Given the fact that the parameter estimates are only updated monthly for the complicated models, these results are quite encouraging. The out-of-sample forecast results alleviate the fear of over-parameterization and demonstrate the economic significance of allowing for jumps in the dynamics of credit spreads.

5.2 Practical Implications

Our empirical findings from the proposed ARX-ARCH-Jump model of credit spreads have a number of practical implications. First, the econometric model we have proposed for the systematic credit spread risk in corporate bond portfolios directly incorporates the information on the general market condition into the forecasts of the conditional mean and variance of the credit spread. Furthermore, rare extreme movements in credit spreads have been captured by jumps with a time-varying jump probability that depends on equity market volatility. These information could be incorporated into the calculation of the 'value at risk' measure for credit spread risk.

Second, our results shed some light on the effort of reaching superior investment performance through exploring the information content of equity market index returns and volatility on general credit spread movements. Asset price predictability could arise as a result of time-varying risk premium, and not necessarily from information inefficiency. Whatever the reason is, our findings call for further studies on the interaction of equity market risk and corporate bond credit spread risk, and the corresponding strategies that could make use of this information content.

Third, our model could be used for the valuation of credit derivatives written on credit indices. For instance, the model could be potentially used in the valuation of credit spread options.

6 Conclusions

We propose an econometric model to describe the dynamic behavior of credit spreads of corporate bond portfolios. In particular, we develop a method to capture the fact that such portfolios are subjected to rebalancing on a regular basis – an issue that has been ignored

[6] We also compared the forecast errors excluding the month of September 2001 in the sample. The relative performance of the models under consideration is not affected by this although it reduces the RMSE and MAE by 0.1-2 basis points.

in the literature. The proposed model integrates together portfolio rebalancing, changes in general market conditions, conditional heteroscedasticity and jumps. We test the model using daily option-adjusted credit spreads of the Merrill Lynch credit spread indices from December 31, 1996 to August 30, 2002. Empirical results indicate that changes in credit spreads of both investment-grade and high yield bond portfolios exhibit autoregression, conditional heteroscedasticity and jumps. Lagged equity market index returns and changes in the slope of the Treasury yield curve are shown to help predict credit spread changes. The time-varying jump probability is found to be related to the lagged volatility level in the equity market. The statistical and economic significance of jumps and the information content of general market conditions are supported both by in-sample and out-of-sample data.

Given the importance of credit risk management in practice, this study may serve the needs of both investors in corporate bond markets and related regulatory agencies. The estimation method developed here that takes into account the rebalancing of a corporate bond portfolio may be extended to deal with similar issues in equity portfolios.

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A Identification of the Upper and Lower Bounds

Let β and α denote the implicit upper and lower bound, respectively, of credit spreads of a given index on re-balancing days. In our model, the distribution of credit spreads on re-balancing days is

$$S_{r,t} = \alpha + \frac{1}{1/(\beta - \alpha) + \exp(-u_{r,t})}, \tag{28}$$

where $\alpha < S_{r,t} < \beta$, $u_{r,t} = \mu_r + \epsilon_t$ and μ_r is a constant, and ϵ_t is normally distributed with $N\left(0, \sigma_r^2\right)$.

It follows from Eq. (28) that

$$\frac{du_{r,t}}{dS_r} = \frac{(\beta - \alpha)}{(\beta - S_r)(S_r - \alpha)}. \tag{29}$$

Under the assumption that $u_{r,t}$ is normally distributed with $N\left(\mu_r, \sigma_r^2\right)$, the density function of spread on re-balancing days is,

$$h(S_r) = \frac{(\beta - \alpha)}{(\beta - S_r)(S_r - \alpha)} \times \frac{\exp\left[-\frac{1}{2\sigma_r^2} \times \left(\ln\left(\frac{(S_r - \alpha)(\beta - \alpha)}{(\beta - S_r)}\right) - \mu_r\right)^2\right]}{\sqrt{2\pi}\sigma_r}, \tag{30}$$

as follows from the well-known transformation formula for densities. Taking the first order derivative of the log-density function with respect to β given the true parameters α and β, we have:

$$\frac{\partial \ln(h(S_r))}{\partial \beta} = -\frac{(u_r - \mu_r)}{\sigma_r^2} \times \left(\frac{1}{\beta - \alpha} - \frac{1}{\beta - S_r}\right) + \frac{1}{\beta - \alpha} - \frac{1}{\beta - S_r}. \tag{31}$$

From Eq. (28), we have

$$\frac{1}{\beta - S_r} = \frac{\exp(u_r) + (\beta - \alpha)}{(\beta - \alpha)^2}.$$

It follows that

$$\frac{\partial \ln(h(S_r))}{\partial \beta} = \frac{1}{(\beta - \alpha)^2} \times \frac{\left(u_r - \mu_r - \sigma_r^2\right)}{\sigma_r^2} \exp(u_r). \tag{32}$$

Integrating u_r out then yields

$$\begin{aligned} (\beta - \alpha)^2 E\left[\frac{\partial \ln(h(S_r))}{\partial \beta}\right] &= \int_{-\infty}^{\infty} \frac{\left(u_r - \mu_r - \sigma_r^2\right)}{\sigma_r^2} \times \exp(u_r) \frac{\exp\left(-\frac{(u_r - \mu_r)^2}{2\sigma_r^2}\right)}{\sqrt{2\pi}\sigma_r} du_r \\ &= \frac{\exp\left(\frac{2\mu_r\sigma_r^2 + \sigma_r^4}{2\sigma_r^2}\right)}{\sigma_r} \int_{-\infty}^{\infty} y \frac{\exp\left(-\frac{y^2}{2}\right)}{\sqrt{2\pi}} dy = 0. \end{aligned} \tag{33}$$

We can show in a similar fashion that

$$E\left[\frac{\partial \ln(h(S_r))}{\partial \alpha}\right] = 0. \tag{34}$$

B Rebalance of the Merrill Lynch Corporate Bond Credit Spread Indices

In this appendix, we describe certain rules used in rebalance of the Merrill Lynch Corporate Bond Credit Spread Indices. The information is based on a publication from Merrill Lynch (2000). The publication contains information on the Merrill Lynch High Grade U.S. Industrial Corporate Index, the Merrill Lynch U.S. High Yield Master II Index, and a detailed description about the general re-balancing rules used by Merrill Lynch to maintain the qualifying criteria of each index. We believe that the same criteria should hold for the sub-indices we use in this study.

To be included in an index, qualifying bonds must have a fixed coupon schedule and at least one year to maturity. The amount of outstanding required for being on a high-grade index is a minimum of $150 million, while that for being on a high-yield index is a minimum of $100 million.

Re-balancing takes place on the last calendar day of each month. The adding or dropping decision of any issue will be based on information that is available in the marketplace "up to and including the third business day prior to the last business day of the month." There are 62 re-balancing days in total including the inception date of the indices, December 31, 1996, in our sample.

The table below contains the number of issues that were included in each index on re-balancing days, which are supposed to be the last calendar day of each month since December 31, 1996. If the last calendar day is not a business day of New York Stock Exchange, we use the next available observation.

Number of Issues Included in Each Merrill Lynch Credit Spread Index on Rebalancing Days

This table contains the number of issues that were included in each Merrill Lynch index on rebalancing day. Rebalancing is done on the last calendar day of each month since December 31, 1996 (the inception day of the indexes). When the last calendar day is not a business day of New York Stock Exchange, the first trading day of the next month is used.

Rebalance days	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
19961231	74	7	46	556	83	363	376	467	65
19970131	76	6	46	558	83	364	356	461	66
19970228	75	6	46	570	82	367	358	478	74
19970331	74	6	47	573	81	376	361	482	80
19970430	75	6	47	586	84	381	383	465	79
19970602	75	6	47	598	83	384	378	458	79
19970630	76	6	46	604	84	389	367	454	81
19970731	76	6	45	612	85	397	367	456	76
19970902	80	7	46	621	84	411	367	446	73
19970930	90	7	49	669	91	437	361	451	69
19971031	91	7	51	683	92	447	370	468	69
19971201	89	7	51	691	92	454	372	474	74
19971231	89	7	53	695	92	466	380	476	78
19980202	91	8	55	705	93	488	376	493	84
19980302	91	8	57	707	86	478	408	503	86
19980331	90	8	57	723	88	484	419	515	90
19980430	91	8	59	738	89	488	402	528	90
19980601	88	8	59	755	92	502	384	531	93
19980630	86	7	58	758	91	514	384	523	103
19980731	87	7	58	750	95	517	390	530	107
19980831	86	8	59	766	100	520	392	530	110
19980930	86	8	59	765	98	521	398	541	124
19981102	92	8	60	777	95	526	388	546	137
19981130	89	7	58	786	93	528	390	554	134
19981231	84	8	50	813	94	542	377	537	141
19990201	83	8	51	821	92	542	370	538	141
19990301	81	10	45	831	95	552	380	534	144
19990331	82	9	45	834	93	556	382	532	149
19990430	83	9	47	829	87	552	394	536	155

Rebalance days	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
19990601	82	9	47	841	87	557	386	529	158
19990630	81	9	47	849	87	556	379	529	150
19990802	66	7	44	590	56	423	359	537	139
19990831	70	6	45	611	58	427	361	531	142
19990930	72	6	45	615	57	430	370	552	145
19991101	75	6	45	613	52	426	372	564	147
19991130	73	6	45	630	52	433	373	568	140
19991231	71	6	41	764	61	493	365	563	144
20000131	76	1	36	817	66	530	365	566	136
20000229	78	1	38	858	66	543	382	569	138
20000331	78	1	38	867	66	545	397	600	147
20000501	79	3	43	869	64	539	407	591	148
20000531	77	3	40	870	64	545	402	588	161
20000630	78	3	39	881	64	547	390	600	159
20000731	79	3	39	877	67	546	391	609	165
20000831	78	3	39	890	65	551	396	624	165
20001002	79	3	39	895	67	554	406	619	174
20001031	78	3	39	898	69	552	409	692	207
20001130	74	3	38	896	67	548	396	706	214
20010102	74	4	41	889	65	537	384	703	224
20010131	74	5	35	896	71	534	386	684	253
20010228	77	5	37	923	66	535	398	659	264
20010402	78	4	37	928	67	524	425	651	255
20010430	80	4	37	932	63	521	456	612	276
20010531	81	4	37	942	68	524	453	605	282
20010702	83	4	37	943	65	519	458	578	301
20010731	91	4	37	949	67	522	472	568	298
20010831	89	4	37	963	67	525	495	519	318
20011001	87	4	37	970	64	523	508	525	318
20011031	95	4	44	979	67	511	490	537	313
20011130	89	4	41	1005	68	513	489	523	331
20011231	89	5	44	1005	67	506	495	529	336
20020131	88	5	44	1004	64	508	509	529	328
20020228	82	4	44	1017	62	509	517	564	305
20020331	84	3	42	1028	63	512	523	546	309
20020430	85	2	42	1037	64	509	538	523	315
20020531	86	2	43	1037	62	494	603	520	297
20020630	84	2	44	1041	64	493	663	662	308
20020731	85	2	43	1026	61	480	656	698	303
20020831	85	2	41	1042	63	481	656	702	315
Average	81	6	46	791	77	494	401	547	162

Table 1

Summary Statistics on Merrill Lynch Option-adjusted Credit Spreads

This table reports the summary statistics on daily Merrill Lynch option-adjusted credit spread (OAS) indices on non-rebalancing days. Panels A through C present respectively the summary statistics on credit spread level, changes in credit spreads and changes in log credit spreads. $\rho(1)$ is the first order autocorrelation coefficient. The sample period of daily credit spreads is from 1/02/1997 to 8/30/2002. There are 1358 observations on non-rebalancing days.

Statistics	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
Panel A: Option-adjusted Spreads (Basis Points)									
Mean	62.51	72.03	87.38	136.32	137.45	157.25	319.5	552.33	1317.7
Median	63.668	69.383	90.502	138.756	135.666	153.442	308.45	531.358	1262.645
Std Dev.	20.39	24.49	27.08	54.21	49.96	52.25	125.6	205.77	533.44
Max	103.248	133.32	138.088	239.189	279.223	258.472	725.166	1082.642	2358.557
Min	27.138	16.64	36.17	50.384	52.76	73.994	136.141	280.837	504.05
Panel B: Changes in Credit Spreads ($\Delta S_t = S_t - S_{t-1}$)									
Mean	-0.004	-0.056	0.001	0.127	0.038	0.037	0.34	0.549	1.06
Std Dev.	1.76	3.03	1.83	2.01	2.47	2.03	6.08	8.96	13.79
Skewness	-0.62	-1.79	0.38	1.35	0.68	1.23	7.02	3.36	1.65
kurtosis	20.63	34.27	12.83	15.76	7.85	16.89	116.16	53.7	25.53
Max	10.859	20.23	14.267	18.81	14.77	20.93	106.67	145.81	182.47
Min	-18.7	-41.37	-12.589	-12.43	-12.61	-12.86	-24.8	-48.84	-91.17
$\rho(1)_{\Delta S}$	-0.28	-0.17	-0.17	-0.02	-0.04	-0.01	0.15	0.17	0.17
$\rho(1)_{\Delta S^2}$	0.36	0.05	0.3	0.23	0.31	0.15	0.02	0.01	0.04
Panel C: Changes in Log Credit Spreads ($\Delta s_t = \ln(S_t/S_{t-1}) * 100$)									
Mean	-0.014	-0.063	0.007	0.077	0.024	0.027	0.065	0.082	0.074
Std Dev.	3.74	5.19	2.51	2.03	2.22	1.52	1.54	1.47	1.01
Skewness	-1.78	0.37	-0.31	0.001	0.49	-0.25	3.13	1.81	1.35
kurtosis	47.88	55.6	20.91	27.94	21.31	22.8	41.63	18.91	14.13
Max	29.83	74.91	18.99	15.41	22.88	11.79	22.75	17.47	10.24
Min	-52.43	-60.75	-21.1	-18.36	-16.25	-14.59	-6.24	-5.89	-6.18
$\rho(1)_{\Delta s}$	-0.29	-0.29	-0.22	-0.16	-0.2	-0.14	0.1	0.14	0.2
$\rho(1)_{\Delta s^2}$	0.37	0.4	0.37	0.41	0.42	0.27	0.01	0.01	0.04

Table 2
Unit Root/Stationarity Tests on the Merrill Lynch Option-adjusted Credit Spread Series

This table presents the results of various unit root/stationarity tests on Merrill Lynch option-adjusted credit spreads over the period of 12/31/1996 - 8/30/2002. The estimates and associated t-statistics from the Augmented Dickey-Fuller test and the Phillips-Perron test are reported in Panels A and B. Panel C contains the Lagrangian Multiplier (LM) test statistic from the stationarity test of Kwiatkowski, Phillips, Schmidt and Shin (1992). Panel D reports the four types of Cauchy tests of Bierens and Guo (1993) stationarity test. Panel E presents the results of Bierens (1997) non-linear Augmented Dickey-Fuller tests for unit root.

Test Statistics	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
Panel A: Augmented Dickey-Fuller Test (Critical Value: (5%)=-2.89; (10%)=-2.58)									
β	-0.0042	-0.009	-0.003	-0.001	-0.001	-0.002	-0.005	-0.001	-0.0004
$t-stat$	-1.51	-2.2	-1.53	-0.97	-0.63	-1.18	-0.39	-0.66	-0.33
Panel B: Phillips-Perron Test (Critical Value: (5%)=-14.51; (10%)=-11.65)									
β	0.9925	0.9842	0.996	0.9986	0.9985	0.9985	0.9996	0.9995	0.992
$t-stat$	-5.41	-13.68	-3.6	-1.46	-1.37	-2.33	-1.17	-0.92	-12.04
Panel C: KPSS (1992) Stationarity Test (Critical Value: (5%)=0.463; (10%)=0.347)									
$LM-Stat$	3.23	1.9	2.87	4.12	3.99	3.87	4.14	3.79	4.18
Panel D: Bierens-Guo (1993) Stationarity Tests (Critical Value: (5%)=12.706; (10%)=6.314)									
Type 1	46.33	2.48	374.54	748.52	367.7	607.54	805.6	887	718.59
Type 2	49.61	2.5	541.15	1391.8	1386	1376	1424.7	1424	1425
Type 3	13.62	1.63	29.84	149.36	90.75	122.1	157.6	175.89	205.95
Type 4	30.01	2.82	44.14	189.17	128.28	111.26	118.9	73.88	121.6
Panel E: Bierens (1997) Nonlinear ADF test (Critical Value of t-stat (5%)=-3.97; (10%)=-3.46) (Critical Value of Am (5%)=-27.2; (10%)=-23) (Critical Value of F-test (5%)=4.88; (10%)=5.68)									
β	-0.006	-0.009	-0.003	-0.004	-0.008	-0.003	-0.008	-0.005	-0.007
$t-stat$	-1.32	-1.93	-1.03	-1.35	-2.1	-1.26	-2.36	-2.13	-2.37
Am	-5.1	-9.04	-3.1	-4.84	-9.16	-4.74	-11.34	-9.64	-11.28
F-test	1.25	1.77	1.91	1.69	2.58	1.35	3.06	1.89	3.43

Table 3
Maximum Likelihood Estimates of Credit Spread Distribution on Re-balancing Days

This table presents results of the estimation of Merrill Lynch option-adjusted credit spread indices on index re-balancing days. The distribution of credit spreads S_t for a given index on re-balancing day t takes the following form:

$$S_t = \alpha + \frac{1}{1/(\beta - \alpha) + \exp(-u_{r,t})} \tag{35}$$

where $\alpha < S_t < \beta$, $u_{r,t} = \mu_r + \epsilon_t$, μ_r is a constant, and ϵ_t is normally distributed with $N\left(0, \sigma_r^2\right)$. Maximum likelihood estimates of the parameters for each index and the heterodscadesticity-consistent standard errors are reported below.

Parameter	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
α	31.897	27.304	39.832	54.599	53.672	73.905	138.76	288.61	540.99
	(0.218)	(2.371)	(0.525)	(0.31)	(8.09)	(0.88)	(2.99)	(0.76)	(3.82)
β	103.072	139.644	133.4	232.97	281.569	258.21	627.89	1006.14	2346.1
	(0.66)	(5.94)	(0.43)	(0.47)	(8.3)	(1.3)	(7.26)	(4.98)	(9.61)
μ_r	3.82	4.25	4.565	4.65	4.698	4.858	5.337	5.63	7.02
	(0.237)	(0.172)	(0.24)	(0.3)	(0.33)	(0.227)	(0.22)	(0.25)	(0.24)
σ_r	1.898	1.133	1.876	2.286	1.258	1.792	1.613	2.02	1.92
	(0.283)	(0.186)	(0.258)	(0.39)	(0.33)	(0.241)	(0.22)	(0.24)	(0.22)
$ln\,(L)$	-288.47	-309.58	-309.08	-350.58	-357.77	-353.85	-421.76	-437.92	-506.6

Table 4
Maximum Likelihood Estimates of the ARX(1)-ARCH(1) Model of Credit Spreads on Non-rebalancing Days

This table reports the maximum likelihood estimates of the ARX(1)-ARCH(1) model of log credit spreads for the period 01/02/1997 through 08/30/2002. The estimated model is

$$\begin{aligned} \ln(S_t) &= \ln(S_{t-1}) + \mu_0 + \phi_1 (1 - D_{1,t-1}) \ln(S_{t-1}/S_{t-2}) \\ &\quad + \beta_1 ret_{rus,t-1} + \beta_2 slope_{t-1} + \beta_3 \Delta r_{t-1} + \epsilon_t, \end{aligned} \tag{36}$$

where $D_{1,t}$ is a dummy variable that equals one when day t is a rebalancing day and zero otherwise, $ret_{rus,t-1}$ is the lagged Russell 2000 index return, $\Delta slope_{t-1}$ is lagged changes in the slope of yield curve, and Δr_{t-1} is the lagged changes in interest rates. The disturbance ϵ_t has mean zero and conditional variance h_t^2, where h_t^2 is specified as an ARCH(1) process:

$$h_t^2 = \varpi_0 + b_1 (1 - D_{1,t-1}) \epsilon_{t-1}^2. \tag{37}$$

The asymptotic heteroscedasticity-consistent standard errors are reported in parentheses. Bold number indicates significance at the 10% level

Parameter	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
μ_0	0.121	-0.074	0.014	0.082	0.031	0.002	0.066	**0.075**	**0.074**
	(0.133)	(0.143)	(0.061)	(0.049)	(0.057)	(0.034)	(0.083)	(0.039)	(0.037)
β_1	**-0.226**	**-0.287**	**-0.127**	**-0.175**	**-0.137**	**-0.106**	**-0.169**	**-0.165**	**-0.100**
	(0.075)	(0.094)	(0.040)	(0.047)	(0.040)	(0.030)	(0.053)	(0.031)	(0.037)
$\beta_2(*10^2)$	**-4.742**	-4.113	**-3.043**	-2.150	**-4.372**	**-2.035**	2.707	0.413	1.396
	(2.293)	(3.012)	(1.821)	(1.371)	(1.477)	(1.051)	(5.619)	(1.200)	(0.937)
$\beta_3(*10^2)$	-0.805	5.267	-0.228	-0.395	0.964	-0.183	0.095	2.003	0.785
	(3.513)	(3.445)	(1.460)	(1.266)	(1.288)	(1.031)	(1.129)	(1.319)	(0.768)
ϕ_1	-0.261	-0.094	**-0.126**	**-0.151**	-0.096	0.047	0.137	**0.196**	**0.265**
	(0.195)	(0.067)	(0.059)	(0.068)	(0.081)	(0.044)	(0.180)	(0.055)	(0.075)
ϖ_0	**4.226**	**11.104**	**3.351**	**1.660**	**2.290**	**1.227**	**1.936**	**1.980**	**0.786**
	(0.892)	(2.349)	(0.684)	(0.392)	(0.368)	(0.286)	(0.370)	(0.291)	(0.093)
b_1	**0.961**	**0.790**	**0.602**	**0.743**	**0.571**	**0.579**	0.233	0.041	0.236
	(0.288)	(0.355)	(0.195)	(0.246)	(0.175)	(0.194)	(0.450)	(0.038)	(0.223)
$\ln(L)$	-3211.12	-3799.04	-2951.22	-2508.74	-2717.15	-2262.90	-2479.04	-2412.45	-1878.81
BIC	6472.750	7648.577	5952.934	5067.986	5484.810	4576.294	5008.570	4875.404	3808.112
AIC	6436.259	7612.086	5916.443	5031.495	5448.319	4539.803	4972.079	4838.913	3771.621

Table 5
Maximum Likelihood Estimates of the ARX(1)-ARCH(1)-Jump Model of Credit Spreads on Non-rebalancing Days

This table presents the maximum likelihood estimates of the ARX(1)-ARCH(1)-Jump model of log credit spreads for the period 01/02/1997 through 08/30/2002. The estimated model is

$$\begin{aligned} \ln(S_t) &= \ln(S_{t-1}) + \mu_0 + \phi_1 (1 - D_{1,t-1}) \ln(S_{t-1}/S_{t-2}) \\ &+ \beta_1\, ret_{rus,t-1} + \beta_2\, slope_{t-1} + \beta_3 \Delta r_{t-1} + \lambda_t \mu_J + \epsilon_t, \end{aligned} \quad (38)$$

where $D_{1,t}$ is a dummy variable that equals one when day t is a rebalancing day and zero otherwise, $ret_{rus,t-1}$ is the lagged Russell 2000 index return, $\Delta slope_{t-1}$ is lagged changes in the slope of yield curve, and Δr_{t-1} is the lagged changes in interest rates. The disturbance ϵ_t has mean zero and is a mixture of two normal distributions: one is $N\left(-\lambda_t \mu_J, h_t^2\right)$ with probability $(1-\lambda_t)$ in the event of no jumps and the other is $N\left((1-\lambda_t)\mu_J, h_t^2 + \sigma_J^2\right)$ with probability λ_t. h_t^2, the conditional variance of ϵ_t in the no-jump state, is assumed to follow an ARCH(1) process:

$$h_t^2 = \varpi_0 + b_1 (1 - D_{1,t-1})\, \epsilon_{t-1}^2. \quad (39)$$

The jump probability $\lambda_t = \exp(p_0 + p_1 * VIX_{t-1}) / (1 + \exp(p_0 + p_1 * VIX_{t-1}))$. The asymptotic heteroscedasticity-consistent standard errors are in parentheses. Bold number indicates significance at 10% level.

Parameter	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
μ_0	**-0.070**	-0.001	-0.018	-0.016	-0.029	-0.027	0.006	0.039	0.025
	(0.040)	(0.057)	(0.033)	(0.024)	(0.032)	(0.019)	(0.035)	(0.034)	(0.023)
β_1	-0.049	-0.048	**-0.063**	**-0.051**	**-0.074**	**-0.062**	**-0.108**	**-0.133**	**-0.076**
	(0.034)	(0.042)	(0.025)	(0.020)	(0.025)	(0.016)	(0.031)	(0.030)	(0.018)
$\beta_2(*10^2)$	**-3.054**	**-3.68**	**-1.567**	**-1.683**	-1.524	**-1.282**	0.316	0.030	0.572
	(1.277)	(1.735)	(0.978)	(0.662)	(0.988)	(0.553)	(1.026)	(0.824)	(0.614)
$\beta_3(*10^2)$	-1.148	**2.458**	**-1.268**	-0.714	0.624	-0.529	0.072	1.670	**0.989**
	(0.932)	(1.386)	(0.683)	(0.522)	(0.694)	(0.447)	(1.047)	(1.341)	(0.535)
ϕ_1	**-0.243**	**-0.133**	**-0.114**	**-0.133**	-0.045	-0.015	**0.092**	**0.168**	**0.215**
	(0.033)	(0.04)	(0.04)	(0.047)	(0.034)	(0.068)	(0.047)	(0.052)	(0.035)
ϖ_0	**1.311**	**2.258**	**0.771**	**0.411**	**0.798**	**0.280**	**1.122**	**1.305**	**0.478**
	(0.136)	(0.236)	(0.076)	(0.037)	(0.082)	(0.026)	(0.119)	(0.139)	(0.05)
b_1	**0.380**	**0.324**	**0.369**	**0.365**	**0.373**	**0.383**	**0.140**	**0.047**	**0.073**
	(0.063)	(0.06)	(0.082)	(0.07)	(0.05)	(0.079)	(0.049)	(0.02)	(0.028)
p_0	**-4.634**	**-3.604**	**-3.345**	**-5.032**	**-4.132**	**-4.502**	**-5.679**	**-6.124**	**-5.471**
	(0.649)	(0.588)	(0.554)	(0.658)	(0.685)	(0.565)	(1.195)	(1.383)	(1.000)
p_1	**0.077**	**0.059**	**0.049**	**0.090**	**0.065**	**0.087**	**0.086**	**0.103**	**0.112**
	(0.023)	(0.02)	(0.02)	(0.022)	(0.024)	(0.021)	(0.03)	(0.035)	(0.027)
μ_J	1.140	-0.09	0.216	0.557	0.386	**0.769**	1.207	1.070	0.443
	(0.912)	(0.827)	(0.426)	(0.584)	(0.466)	(0.329)	(1.205)	(1.203)	(0.312)
σ_J	**8.708**	**10.146**	**5.283**	**5.027**	**4.667**	**3.346**	**5.015**	**4.152**	**2.189**
	(1.497)	(1.286)	(0.725)	(0.762)	(0.664)	(0.533)	(2.182)	(1.586)	(0.551)
$\ln(L)$	-2803.19	-3329.02	-2564.66	-2003.22	-2428.47	-1844.43	-2265.01	-2279.74	-1736.64
BIC	5685.738	6737.38	5208.669	4085.796	4936.287	3768.203	4609.365	4638.831	3552.626
AIC	5628.395	6680.03	5151.326	4028.453	4878.944	3710.860	4552.022	4581.487	3495.283

Table 6
Model Diagnostic Tests

Table 6 presents various model diagnostic tests for the estimated ARX(1)-ARCH(1)-Jump model and the nested ARX(1)-ARCH(1) model as reported in Table 5. The marginal significance level of the corresponding test statistics are reported in square brakets.

Spread Index	Model	χ_2 (19)	ρ_ϵ (1)	ρ_{ϵ^2} (1)	*Skewness*	*Kurtosis*	*Normality*
	ARCH	403.1	-0.1	0.071	0.28	15.95	25.66
AA-AAA rated		[0.00]	[0.00]	[0.005]	[0.73]	[0.00]	[0.00]
1-10 Yrs	Jump-ARCH	34.41	-0.096	0.059	-0.037	0.779	4.52
		[0.02]	[0.00]	[0.015]	[0.76]	[0.06]	[0.1]
	ARCH	530.1	-0.072	0.035	0.00	11.29	31.18
AA-AAA rated		[0.00]	[0.004]	[0.102]	[0.99]	[0.00]	[0.00]
10-15 Yrs	Jump-ARCH	60.61	-0.07	0.032	0.06	0.424	6.11
		[0.00]	[0.006]	[0.117]	[0.52]	[0.024]	[0.05]
	ARCH	424.3	-0.038	0.004	0.24	13.32	24.36
AA-AAA rated		[0.00]	[0.08]	[0.44]	0.72	[0.00]	[0.00]
15+ Yrs	Jump-ARCH	31.96	-0.048	0.028	0.059	0.484	6.24
		[0.03]	[0.038]	[0.15]	[0.52]	[0.01]	[0.04]
	ARCH	489.6	-0.05	0.034	0.33	24.8	31.3
BBB-A rated		[0.00]	[0.029]	[0.11]	[0.77]	[0.00]	[0.00]
1-10 Yrs	Jump-ARCH	40.24	-0.022	0.058	-0.024	0.618	7.55
		[0.003]	[0.21]	[0.02]	[0.8]	[0.01]	[0.02]
	ARCH	242.1	-0.04	0.058	0.26	11.99	26.8
BBB-A rated		[0.00]	[0.06]	[0.02]	[0.67]	[0.00]	[0.00]
10-15 Yrs	Jump-ARCH	26.48	-0.016	0.046	0.004	0.65	7.2
		[0.12]	[0.28]	[0.04]	[0.97]	[0.014]	[0.027]
	ARCH	407.5	-0.074	0.024	0.17	20.93	19.16
BBB-A rated		[0.00]	[0.003]	[0.188]	[0.87]	[0.00]	[0.00]
15+ Yrs	Jump-ARCH	31.05	-0.008	0.038	0.045	0.842	8.19
		[0.04]	[0.38]	[0.08]	[0.7]	[0.01]	[0.02]
	ARCH	123.5	-0.058	-0.005	-0.035	21.55	7.63
BB rated		[0.00]	[0.02]	[0.43]	[0.98]	[0.006]	[0.02]
	Jump-ARCH	21.56	-0.013	0.006	-0.047	0.417	4.65
		[0.31]	[0.32]	[0.41]	[0.6]	[0.03]	[0.098]
	ARCH	74.88	-0.042	0.005	-0.302	11.72	6.62
B rated		[0.00]	[0.06]	[0.43]	[0.7]	[0.01]	[0.04]
	Jump-ARCH	13.13	-0.009	0.003	-0.086	0.304	4.16
		[0.83]	[0.37]	[0.45]	[0.26]	[0.05]	[0.12]
	ARCH	90.65	-0.079	-0.004	-0.01	7.2	9.57
C rated		[0.00]	[0.002]	[0.43]	[0.98]	[0.002]	[0.008]
	Jump-ARCH	17.46	-0.034	0.024	0.022	0.314	2.2
		[0.56]	[0.1]	[0.19]	[0.8]	[0.14]	[0.33]

Table 7
Robustness Test Using the S&P Credit Indices

This table presents the maximum likelihood estimates of the ARX(1)-ARCH(1)-Jump model with two daily S&P credit index series over the period 12/31/1998 through 08/30/2002. The estimated model is

$$\begin{aligned}\ln(S_t) &= \ln(S_{t-1}) + \mu_0 + \phi_1 (1 - D_{1,t-1}) \ln(S_{t-1}/S_{t-2}) \\ &\quad + \beta_1\, ret_{rus,t-1} + \beta_2\, slope_{t-1} + \beta_3 \Delta r_{t-1} + \lambda_t \mu_J + \epsilon_t,\end{aligned}$$

where $D_{1,t}$ is a dummy variable that equals one when day t is a rebalancing day and zero otherwise, $ret_{rus,t-1}$ is the lagged Russell 2000 index return, $\Delta slope_{t-1}$ is lagged changes in the slope of yield curve, and Δr_{t-1} is the lagged changes in interest rates. The disturbance ϵ_t has mean zero and is a mixture of two normal distributions: one is $N\left(-\lambda_t \mu_J, h_t^2\right)$ with probability $(1 - \lambda_t)$ in the event of no jumps and the other is $N\left((1 - \lambda_t)\, \mu_J, h_t^2 + \sigma_J^2\right)$ with probability λ_t. h_t^2, the conditional variance of ϵ_t in the no-jump state, is assumed to follow an ARCH(1) process:

$$h_t^2 = \varpi_0 + b_1 (1 - D_{1,t-1})\, \epsilon_{t-1}^2. \tag{40}$$

The jump probability $\lambda = \exp(p_0 + p_1 * VIX_{t-1}) / (1 + \exp(p_0 + p_1 * VIX_{t-1}))$. The asymptotic heteroscedasticity-consistent standard errors are in parentheses. Bold number indicates significance at 10% level.

Parameter	Investment-grade Credit Index	High-yield Credit Index
μ_0	-0.03	0.035
	0.028	0.061
β_1	**-0.102**	**-0.081**
	(0.026)	(0.03)
$\beta_2(*10^2)$	-1.102	0.866
	(0.767)	(0.999)
$\beta_3(*10^2)$	0.742	1.678
	(0.713)	(1.223)
ϕ_1	**0.174**	**0.191**
	(0.058)	(0.0645)
ϖ_0	**0.432**	**0.603**
	(0.094)	(0.101)
b_1	**0.242**	**0.075**
	(0.074)	(0.036)
p_0	**-5.533**	**-4.826**
	(1.581)	(1.785)
p_1	**0.112**	**0.173**
	(0.032)	(0.083)
μ_J	0.362	0.044
	(0.497)	(0.14)
σ_J	**2.314**	**1.446**
	(1.162)	(0.149)
$\ln(L)$	-1201.19	-1479.51
BIC	2477.366	3034.008
AIC	2424.382	2981.024

Table 8
Out-of-Sample Forecast Comparison

This table presents the root mean squared error (RMSE) and the mean absolute error of the actual credit spread and the one-step-ahead predicted credit spread from the ARX-ARCH-Jump model and the nested ARX-ARCH model. Starting from January of 2000, on the first non-rebalancing trading day of each month, the parameters of the model are estimated using all past observations. The parameters are held constant for the one-step-ahead prediction within the month. The initial sample period runs from January, 1997 to December, 1999 and the forecast period is from January, 2000 through August, 2002. The data used are daily observations of Merrill Lynch credit spread indices on non-rebalancing days. MT indicates the simple martingale model of credit spreads. Bold number indicates the smallest value.

Panel A: Root Mean Squared Error

Model	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
Jump-ARCH	**1.447**	**2.605**	**1.909**	2.181	2.708	**2.233**	8.077	**11.081**	**17.494**
ARCH	1.465	2.634	1.919	2.204	2.769	2.24	**8.021**	11.101	17.815
MT	1.497	2.657	1.929	**2.118**	**2.701**	2.255	8.219	11.318	17.686

Panel B: Mean Absolute Error

Model	AA-AAA 1-10 Yrs	AA-AAA 10-15 Yrs	AA-AAA 15+ Yrs	BBB-A 1-10 Yrs	BBB-A 10-15 Yrs	BBB-A 15+ Yrs	BB	B	C
Jump-ARCH	**0.905**	**1.559**	1.17	1.319	1.783	**1.341**	4.543	**7.08**	**11.8**
ARCH	0.949	1.618	1.185	1.389	1.844	1.357	**4.525**	7.092	12.011
MT	0.915	1.561	**1.166**	**1.267**	**1.756**	1.356	4.56	7.268	11.974

Figure 1−1
Merrill Lynch Option−adjusted Credit Spread Indexes (AA−AAA Rated

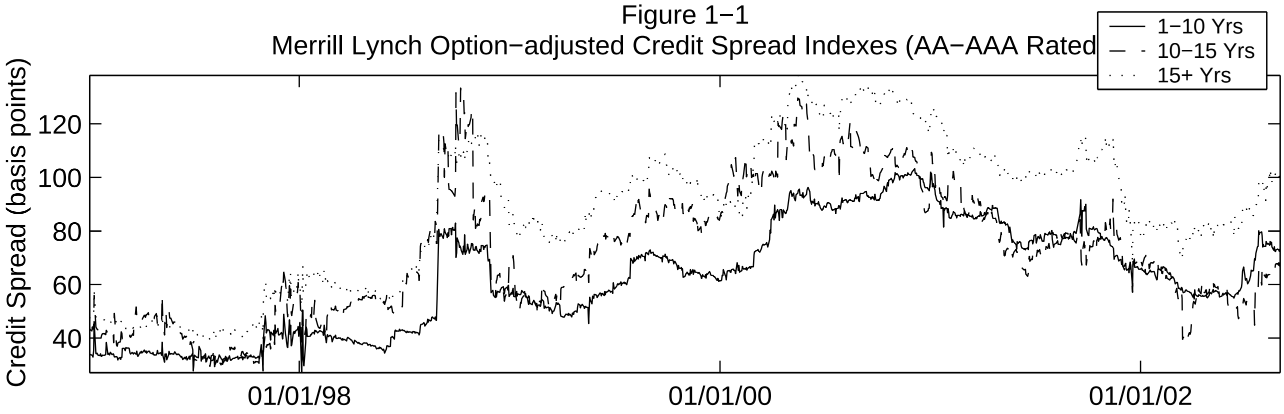

Figure 1−2
Merrill Lynch Option−adjusted Credit Spread Indexes (BBB−A Rated)

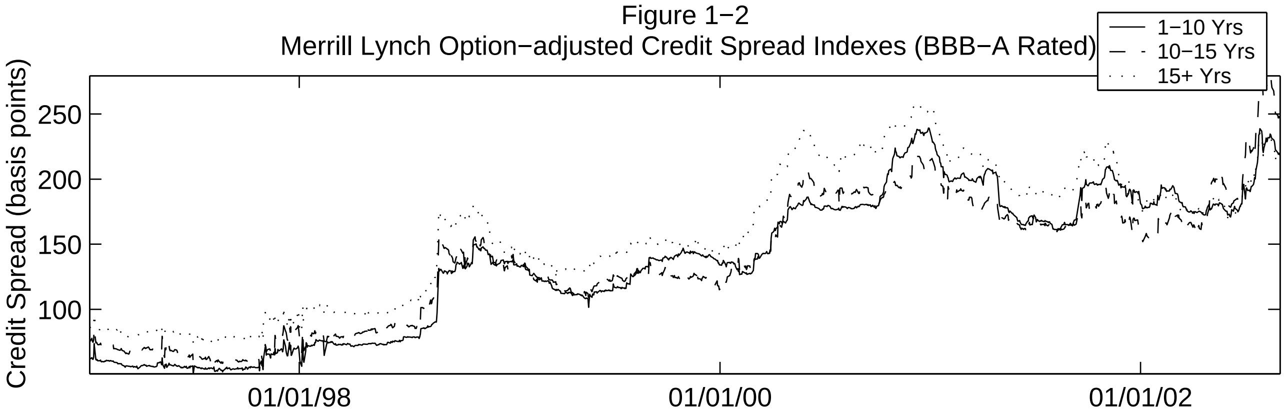

Figure 1−3
Merrill Lynch Option−adjusted Credit Spread Indexes (High−Yield)

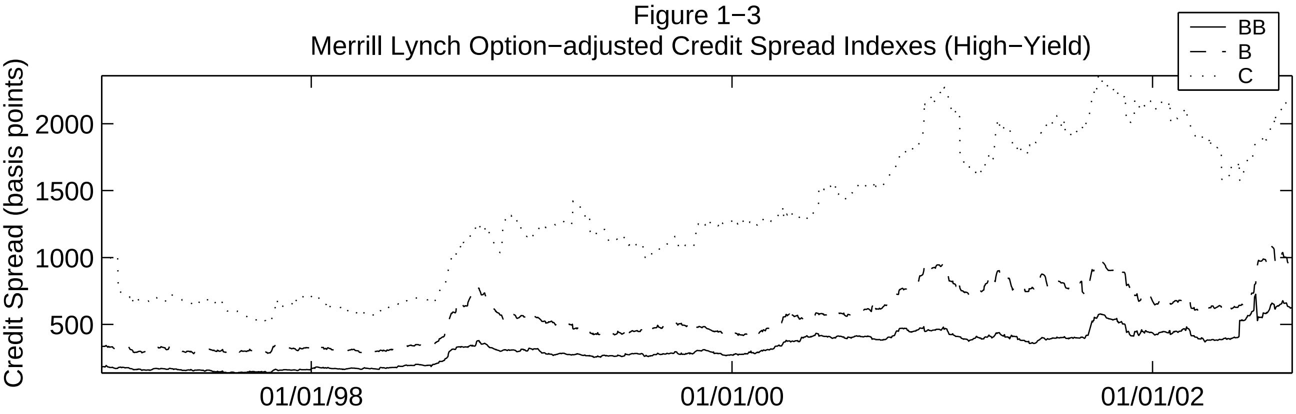

Figure 2: Percentage Changes in Merrill Lynch Option−adjusted Spread Indexes