# The Valuation and Hedging of Deferred Commission Asset Backed Securities

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## Abstract

Due to a timing mismatch between fee receipts and commission payments, there is a new and growing market for securities backed by fees from back-end load and level load mutual funds. This paper develops a contingent claims methodology for the valuation of these securities. The resulting security value depends primarily on the current value of fund assets and the fee schedule. The valuation formula also provides an analytical expression for the appropriate strategy for hedging fluctuations in asset value. As a case study, we investigate the hedging performance of an institution that holds a portfolio of these securities.

## 1 Introduction

The mutual fund industry manages over six trillion dollars of assets. These funds are offered by a variety of different sponsors, many of whom either do not market their product directly or are not the sole marketers of their product to potential investors. Instead, these mutual funds rely on brokerage houses to help market and sell their funds. Approximately 14% of these funds have fees which are either back-end loaded ("B shares") or level loaded ("C shares"). The B or C shares are structured in such a way that investors tend to pay the fees to the mutual fund complex at some future date, while the commissions paid to the brokerage house by the mutual fund are paid immediately.

In order to pay the brokerage commissions, mutual fund complexes have generally taken out bank loans, that is, on-balance sheet financing. However, the misalignment of cash flows exposes the mutual fund to risks, such as changes in the fund's net asset value (NAV) and/or fund redemptions, that they may not want to bear. As a result, a number of B- and C-share financing companies have been created as intermediaries between the mutual fund industry and the brokerage houses. In essence, these financing shops front the full expenses of the back-end and level load commissions in return for a portion of the fees from the B and C shares. Many of these companies then securitize these fees, issuing claims on them as a new form of asset backed security, called Deferred Commission Asset Backed Securities (DCABS).

This paper develops a methodology for the valuation and hedging of these DCABS. While the method is specific to the contracts underlying the DCABS, in theory the approach can be applied to the valuation of any institution's holdings of deferred fees, such as a back-end or level load mutual funds.<sup>1</sup> Thus, the applicability of the methodology is widespread. Current valuation methods involve an analysis of expected future cash flows implied by the mutual funds fees, which are then discounted back into present value terms. Because these contracts also involve a number of embedded options, a discounted cash flow (DCF) approach seems antiquated. In this paper, we employ a contingent claims approach to the valuation and hedging of DCABS. Under fairly general assumptions about the distribution of the NAVs of the funds, simple formulas result that make the analysis of DCABS particularly informative.

The key insight underlying our valuation approach is that, in an efficient market, the present value of a claim on the future value of an asset is simply a function of the current value of that asset. The result is that the value of the DCABS can be written in terms

<sup>&</sup>lt;sup>1</sup>More generally, our analysis sheds light on the valuation and risks of mutual fund companies, as well as other money management entities within this important financial services industry.

of three elements: (1) the current value of assets under management, (2) the volatility of this value and the risk-free rate for valuing option-like features, and (3) a set of parameters that describe the fund, including the fee schedules, the expense ratio, the dividend and capital gains distribution rate, the reinvestment rate, and the path of expected future redemptions. For reasonable sets of parameters, the primary factors are the asset value and the fee schedule. We also illustrate the sensitivity of the valuation to the various parameters for a representative fund. Finally, given the relatively simple valuation equation, it is straightforward to derive analytically the appropriate strategy for hedging the risk associated with fluctuations in asset value.

This paper provides several contributions to the current finance literature. First, we describe a potentially important new asset backed security, DCABS, which covers many of the fee structures imposed by the six trillion dollar mutual fund industry. Second, and most important, we provide a new valuation and hedging methodology for DCABS that allows investors, B- and C-share financing companies, and mutual funds themselves an improved way of measuring the returns and risk of their businesses. This methodology is particularly simple and intuitive, and is rooted in modern finance theory à la Black and Scholes (1973). Third, as a case study, we investigate the portfolio of DCABS held by one specific financing company, Constellation Financial Management, which has chosen, in the past, not to securitize the fees, but instead to manage them on their balance sheet. This case study allows us a unique opportunity to evaluate the hedging performance of one institution as it relates to the various risks associated with DCABS.

The paper is organized as follows. Section 2 describes the mutual fund industry as it relates to B and C shares and the market for DCABS. In Section 3, we describe the contract associated with DCABS and the standard approach to valuing these securities in practice. We develop an alternative contingent claims approach to the valuation and hedging of DCABS and illustrate the importance of the various assumptions underlying the model. As a case study, Section 4 describes the portfolio of DCABS of one particular financial institution, and investigates its ability to hedge NAV risk and other risks inherent in DCABS. Section 5 makes some concluding remarks.

## 2 The Mutual Fund Business and DCABS

Over the past decade, the mutual fund industry has experienced exponential growth. Total assets under management have grown from approximately \$0.8 trillion to \$5.5 trillion, which represents an annualised growth rate of 20% (see Mutual Fund Development in 1998, published by the Investment Company Institute). The majority of these assets are in equity and bond funds (about 70%), with money market funds and various hybrid products such as annuities accounting for the remainder. The spectacular growth was driven by both large cash inflows and strong growth in equity markets.

New sales are generated through three channels: (i) direct marketing to institutional investors and individual investors through 401(k) plans, fee-based advisors and wrap accounts (which accounts for 17% of sales), (ii) conventional direct marketing to individual investors (23%), and (iii) indirect marketing through a sales force (the most prevalent method at 60%). This latter category, which accounted for \$897 billion of sales in 1998, is conducted primarily through a network of sales representatives, such as broker-dealers, banks, and insurance agents.

Mutual funds can be divided broadly into three categories according to their fund fees structure: (i) no load funds, (ii) front load funds, and (iii) back or flat load (deferred load) funds. Traditionally the industry has been dominated by front load funds. In recent years, there have been two trends reducing the market share of these funds. First, the fastest growing segment of the industry is no load index funds, characterized by low, flat management fees (e.g., Vanguard's Index Trust 500). Second, there is a shift within the load fund category from front load to deferred load funds. One potential reason for this change is that investors are investing over longer horizons, and for longer holding periods back load funds tend to be cheaper (on an annualized basis). In addition, with back load funds the full 100% of the money invested "works for you" until redemption.

Table 1 provides summary information on mutual funds from the Morningstar database. The total NAV of equity and fixed income mutual funds in December 1998 was \$3.8 trillion. Of this amount, 49% was invested in no load funds, and 51% in load funds. There are almost an equal number of front load and deferred load funds, but since the typical front load fund is larger, about 70% of the funds invested in load funds are invested in front load funds. This difference in fund size is presumably due to the longer history of front load funds, but, by most accounts, current fund flows are dominated by inflows to deferred load funds.

In contrast to no load funds, load funds levy a sales charge or commission which is assessed to cover selling costs. Shares are classified by the form of the load: front load ("A shares"), back-end load ("B shares"), or level load ("C shares"). In transactions involving A shares, the traditional arrangement is that the investor pays a sales charge to the broker or financial planner, and this charge is immediately deducted from the investment at the time it is made. These charges range anywhere from 3% to 8.5%. In contrast, B or C shares involve fees at later dates; the broker still receives a sales commission, but it comes from the mutual fund complex rather than immediately from the investor. As such, the mutual fund complex needs to finance these commissions.

The most common of these load funds, the class B shares, move the loads from the time of purchase to the time of redemption, where the load itself gradually diminishes the longer the shares are held. For example, a typical arrangement might be a 4% load if the investor sells the shares during the first year, 3% if sold during the second year or third year, 2% in the fourth year, and 1% in the fifth year. The investor pays no load if the fund is sold after five years. In addition to this protection against *early* redemptions, called Contingent Deferred Sales Charges (CDSC), the fund recoups its costs over the period by adding deferred sales charges to the fund's annual expenses, which makes them higher than they would otherwise be. These charges, called Asset Based Sales Charges (ABSC), fall under the general class of "12(b)-1" fees, which is a method of charging distribution-related expenses (such as marketing costs) directly against fund assets. The term "12(b)-1" refers to the 1980 U.S. Securities & Exchange Commission rule that permits the use of these charges. Standard practice is that after the CDSC period, the fund organizations will automatically convert the class B shares to class A shares (which pay lower annual expenses).<sup>2</sup>

As described above, selling B shares can be *expensive* for mutual fund companies. These companies must pay brokers' commissions upfront, and then wait to collect the money back through the 12b-1 fees (i.e., the ABSCs) and the CDSC revenue streams. The on-balance sheet financing of these commissions via bank loans can be troublesome to the mutual fund complex due to various risk considerations such as the mistiming and risk exposure of the ABSC and CDSC revenues. As a result, a financial services industry has emerged that performs B-share financing. These financing companies perform one primary function – they purchase the future ABSC and CDSC revenue streams from the mutual fund complex and, in the process, assume the risks that go with the revenue stream. The mutual fund complex uses these sales to then pay the brokers' commissions, as well as possibly pocket any remaining proceeds (for their own marketing and sales efforts). The key feature is that the mutual fund complex avoids all the risks associated with back-end loads.

Of course, the financing companies now bear the risks. The cash flow streams incorporate several risks. First, there is a structural risk that the SEC may modify rule 12b-1 either

<sup>&</sup>lt;sup>2</sup>Similar to class B shares, class C shares have no front-end loads; however, their deferred loads are usually a relatively small proportion of the assets (e.g., 1%) and only apply to redemptions made during the first year the investor owns the shares. However, the fund charges a "level load" that is built into a higher overall annual expense charge and continues for as long as the investor owns the shares. For the remainder of the paper, we describe the pricing and hedging of securities backed by B shares since they are the most common of the two loads and C shares can be priced similarly.

by lowering the 12b-1 payment percentage (currently capped at 1% per year), limiting it's practical use, or eliminating it entirely. Clearly, these costs are borne by the holders of the revenue streams. Second, and most important, there is the net asset value (NAV) risk associated with these streams. Since these streams are asset-based fund expenses, decreases in NAV due to either interest rate, currency or stock market movements affect the payment stream. Third, the so-called redemption penalties, i.e., contingent deferred sales charges (CDSCs), are calculated as the lesser of an investor's original deposit or the current net asset value minus all reinvested dividend and capital gain distributions. Thus, if redemptions increase as net asset values decline, the financing company can potentially lose. Finally, CDSCs typically end before B shares are converted to A shares. If investors redeem their holdings after the CDSC period, but before conversion, i.e., before the commissions are fully recovered, then the financing companies bear the brunt of the shortfall.

Within a mutal fund complex or money management division, there may be a number of funds managing a variety of assets. These assets will vary from fixed income to equities to foreign exchange to commodities, and cover domestic or international markets. For class B-share entities, these funds will generally have a particular ABSC schedule and CDSC schedule. As assets come into the mutual fund complex, and this complex must pay corresponding commission fees to brokers, B-share financing companies pay the commissions and receive the ABSC and CDSC schedule. Specifically, the process involves the mutual fund complex selling pools of cash flows from each of their funds. These pools tend to include assets under management which were originated at similar dates. In theory, the B-share financing company will own a number of pools of revenue streams derived from a particular fund. Similar pools are then packaged together and sold off to investors as pass-through securities, i.e., DCABS. Given the value of assets in deferred load funds (see Table 1), the continuing asset flows into these funds, and the benefits in risk-sharing offered by securitization, it is clear that the potential DCABS market is extremely large.

## 3 The Valuation and Hedging of DCABS

### 3.1 The Cash Flows of DCABS

The pools of assets underlying the DCABS will generally face a particular ABSC schedule and CDSC schedule depending on the fund from which they come. While these schedules will differ across mutual funds and mutual fund complexes, a typical schedule is presented in Table 2. In this example, investors pay a fee of 0.75% of their assets under managment each year. If investors redeem any shares, they pay a penalty ranging from 4.0% to 0.0%, depending on the redemption year.

In order to better understand the valuation and hedging of DCABS, let us examine a representative pool of cash flows that originates at time 0 and matures at time T. Any particular cash flow during the T-year period is subscripted by t. Thus, let us define  $ABSC_t$  as the scheduled ABSC at time t (e.g., in year 4,  $ABSC_t = 0.75\%$ ) and  $CDSC_t$  as the scheduled CDSC at time t (e.g., in year 4,  $CDSC_t = 2.0\%$ ). In addition, we will make the following assumptions:

- 1. The net asset value (NAV) of the fund follows a lognormal distribution, with mean  $\mu$  and variance  $\sigma^2$ . (This assumption is necessary for valuing the embedded put option implicit in the CDSC payments, but not the other cash flow components).
- 2. The dividend yield (DIV), capital gains yield (CG), expense ratio (EXP) and dividend/capital gains distribution reinvestment rate (RE) of the fund are all constant. (These parameters generally represent second order effects on the DCABS value, and, for many funds, this assumption is reasonable.) All the inputs are assumed to be annualized and in decimal form, e.g., DIV = 0.02 means an annual dividend yield of 2%.
- 3. The continuously compounded risk-free rate,  $r_f$ , is constant.
- 4. Redemption rates are stochastic, but independent of the NAV of the fund. We define  $RD_t$  as the expected redemption rate as a fraction of outstanding originations at time t.

This last assumption requires some discussion. Future redemption rates are, of course, unobservable, but in practice, these rates can be estimated from historical data. If redemption rates are either nonstochastic or stochastic but independent of the fund's NAV, then we can simply use the expected redemption rate. However, if redemption rates have a stochastic element which depends on the fund's NAV, then in theory this would induce a nonlinear payoff to DCABS, which would require some model of the joint distribution of NAV and redemptions, i.e., in effect, a two-factor model.

At first glance, there are several reasons why redemptions may depend on the fund's NAV. First, if a fund performs poorly, i.e., low relative NAVs, then one might expect redemptions to increase. Of course, this theory is based on the assumption that fund managers have some ability to beat the market (or the investors' perception that they have this ability or lack thereof). In support of this argument, some recent statistical evidence suggests redemptions increase with declines in NAV (see, for example, Ippolito (1992) and Sirri and Tufano (1998)). However, this evidence is mixed and much weaker for the load funds relevant for our analysis, particularly the back-end load funds with CDSCs. Second, there may be tax planning reasons for redeeming the shares when the NAV of the fund has fallen. In this case, redemptions will behave similarly to the case above. On the other hand, redemptions are likely to depend not on absolute fund performance, but rather on *relative* performance, relative to a benchmark, an index or a designated asset class. If this is the case then, in keeping with our assumption that there is no superior performance ability (alpha) in these funds, redemption should depend on a relative performance measure, which is less correlated with the level of NAV. Hence, we assume that redemptions are stochastic but independent of NAV, and leave to future research the more complex analysis in which redemptions depend on the future path of a fund's NAV.

At any period t during the life of the pool underlying the DCABS, define  $ORIG_t$  as the originated assets remaining at time t,  $AUM_t$  as the dollar value of the assets under management at time t, and  $NAV_t$  as the net asset value (i.e., the fund's "share price") at time t. Note that at origination, the dollar value of assets, the net asset value and origination dollars are all equal, i.e.,  $AUM_0 = NAV_0 = ORIG_0$ . As time passes though, these values begin to diverge. This divergence is important as the cash flows of DCABS depend differentially on all three values. Consider each of these values.

• From time 0 to time t,  $ORIG_t$  equals  $ORIG_0$  adjusted for any redemptions of the original assets. In particular,

$$ORIG_t = ORIG_0 \left[ \prod_{s=0}^{t-1} (1 - RD_s) \right].$$
(1)

• In contrast, the assets under management,  $AUM_t$ , are expected to grow at their gross return rate,  $\mu$ , each period. However, this growth is mitigated by the fund's expense ratio, EXP, and by the fact that not all the dividends or capital gains distributions are reinvested, (1 - RE)(DIV + CG). Furthermore, the amount of these assets decline as redemptions build up. Specifically,

$$AUM_t = AUM_0(1+\mu)^t(1-\eta)^t \left[\prod_{s=0}^{t-1} (1-RD_s)\right]$$
(2)  
where  $\eta \equiv EXP + (1-RE)(DIV + CG).$ 

• The net asset value at time t, NAV<sub>t</sub>, grows like AUM<sub>t</sub>, except reinvested dividends and capital gains are not incorporated into the value. Intuitively, NAV is the share price of the fund's original assets. In particular, its value grows at the gross return less expenses and the total dividend and capital gains distributions:

$$NAV_t = NAV_0(1+\mu)^t (1-\delta)^t \left[\prod_{s=0}^{t-1} (1-RD_s)\right]$$
(3)  
where  $\delta = EXP + DIV + CG.$ 

As mentioned above, the holder of the DCABS receives cash flows from two sources. The first source is from asset based sales charges (ABSCs), i.e., 12b-1 fees, which represent a fixed percentage of assets under management. These revenues are received monthly until either redemption takes place (or eventual conversion to "A" shares, usually in eight years time). With redemptions, the AUM of the fund, and the resulting ABSCs, decline every month thereafter by the percent of the AUM that is redeemed. In particular, at any point in time t, the ABSC cash flows are the product of the ABSC schedule and the assets under management, i.e.,

$$ABSC_tAUM_t.$$
 (4)

The second source of revenues comes from redemption penalties as compensation for the loss of these ABSCs. In particular, if an investor redeems his/her shares within a particular period of time, the holder of the DCABS is entitled to Contingent Deferred Sales Charges (CDSCs). These CDSCs take the form of a fixed percentage of the redeemed NAV; however, this fixed percentage rate falls the longer the shares are held by the investor. Moreover, as mentioned previously, the CDSC is paid as a percentage of the minimum of NAV at purchase or NAV at redemption. Since CDSCs are paid on redemptions on the minimum of origination value and current NAV, the cash flows are

$$CDSC_tRD_t min(ORIG_t, NAV_t).$$

This cash flow can be decomposed into two components

$$CDSC_tRD_t[ORIG_t - max(ORIG_t - NAV_t, 0)].$$
(5)

The first component,  $\text{CDSC}_t \text{RD}_t \text{ORIG}_t$ , represents a fixed cash flow based on the originated assets remaining at time t, the CDSC schedule at time t, and the redemptions at time t. Of course, from above,  $\text{ORIG}_t$  is just equal to the initial dollar value of the assets originated, scaled down by all the redemptions that have taken place prior to that point in time. The second component represents a put on the fund's net asset value,  $\text{NAV}_t$ , with a strike price equal to  $\text{ORIG}_t$ , i.e., the cash flows are identical to the expected payoff on a put

$$CDSC_tRD_tE[max(ORIG_t - NAV_t, 0)].$$

Recall that NAV grows at the gross return less expenses and the total dividend and capital gains distributions, i.e.,  $\delta$ .

#### 3.2 Valuation

The standard approach to valuing DCABS has been a discounted cash flow approach. The DCF approach to valuation calculates expected cash flows along the lines described above, that is, in terms of the three components. The next step is to discount these cash flows back to the current date at the internal rate of return (IRR). The IRR is computed as the discount rate which equates the discounted expected cash flows to the cost at the purchase date. Several problems exist with this methodology. First, the three cash flow components are governed by different risks: (i) the ABSCs by the risk of the underlying assets, (ii) the CDSC component on the original assets by the risk free rate, and (iii) the short position in the put by the rate appropriate for a put option on the underlying assets. Thus, with various discount rates, an IRR is difficult to interpret. Second, the risks of these cash flow components will vary across funds, so that an IRR across funds is also problematic to analyze. Finally, in terms of risk management of the DCABS, it is unclear whether an IRR methodology is consistent with the type of risk measurements that are appropriate.

Below, we try to rectify this problem by applying a contingent claims approach to the pricing and hedging of DCABS. The fundamental idea is to recognize that DCABS represent a claim on the underlying net asset value of the mutual fund, subject to redemption shocks. In an efficient market, the present value of claims on the future assets of the fund is just the current value of the assets.<sup>3</sup> We can therefore apply modern financial theory to valuing the DCABS by recognizing that these securities are just contingent claims on the underlying assets under management, which have observable market values. This approach relies on the simple insight that the present value of X% of the fund in N years is just X% of the fund today. That point, coupled with a Black-Scholes European option valuation of the embedded put option in the CDSCs, makes the valuation straightforward. Consequently, the three components can be valued today without making assumptions about the expected returns.

<sup>&</sup>lt;sup>3</sup>Professionals might argue that the expense ratios represent, or perhaps even under-represent, investment management skills. That is, the present value of the future assets is greater than the current value due to stock or bond picking abilities, so that the assets are growing at a rate faster than the current market expected returns. We choose instead to believe the volumes of evidence that suggest that mutual funds do not earn excess returns, and that the expense ratios are reflective of transactions costs, rather than some managerial skill (see Carhart (1997), and Elton, Gruber, Das and Hlavka (1993)).

At any time between period 0 and T, the value of the DCABS will depend on a number of variables, including the future ABSC and CDSC schedules, as well as the prior level of redemptions. Consider valuing the DCABS at a particular point in time t by valuing each of the three cash flow components separately. The first cash flow, the ABSC component, is a claim on the future value of the assets under management. Its value is simply the present value of the future value of the assets, i.e., the current value. Thus, using equations (2) and (4), the value can be written as

$$V_{1,t} = \operatorname{AUM}_t \left[ \sum_{\tau=1}^T \operatorname{ABSC}_{t+\tau} (1-\eta)^\tau \left[ \prod_{s=0}^{\tau-1} (1-\operatorname{RD}_s) \right] \right].$$
(6)

The CDSC value reflects two components: (i) the present value of all future redemptions at the CDSC schedule, plus (ii) a downward adjustment for the fact that the CDSC basis may decline if the NAV of the fund declines, either due to expenses or to a fall in the underlying value of the assets. The first CDSC component cash flow is simply the expected redemptions times the original assets under management at the particular CDSC schedule. Thus, the appropriate discount rate is the risk-free rate, giving a value of

$$V_{2,t} = \text{ORIG}_t \left[ \sum_{\tau=1}^T \frac{\text{CDSC}_{t+\tau} \text{RD}_{t+\tau} [\prod_{s=0}^{\tau-1} (1 - \text{RD}_s)]}{(1 + r_f)^{\tau}} \right].$$
(7)

While the investors in DCABS own the fixed CDSC component, recall that they have implicitly written a put option on the fund's NAV, with a strike price equal to the original asset value. In order to value the put option, note that  $\delta$  is defined on the cum-distribution value of the assets. To coincide with option pricing theory we need to translate it into an ex-distribution value as follows

$$\gamma = \frac{\delta}{1-\delta}.$$

We also need the corresponding continuously compounded quantity

$$\gamma^* = \ln(1+\gamma)$$

Thus, NAV can be viewed as the stock price on a dividend paying stock with dividend  $\gamma$ . With the appropriate assumptions about the distribution of NAV, this cash flow can be valued via the usual Black-Scholes analysis. Specifically,

$$V_{3,t} = \left[\sum_{\tau=1}^{T} \text{CDSC}_{t+\tau} \text{RD}_{t+\tau} \prod_{s=0}^{\tau-1} (1 - \text{RD}_s) \left[ \frac{\text{ORIG}_t}{(1 + r_f)^{\tau}} N(-d_2) - \frac{\text{NAV}_t}{(1 + \gamma)^{\tau}} N(-d_1) \right] \right]$$
(8)

where

$$d_{1} = \frac{\ln(\mathrm{NAV}_{t}/\mathrm{ORIG}_{t}) + (r_{f}^{*} - \gamma^{*} + 0.5\sigma^{*2})\tau}{\sigma^{*}\sqrt{\tau}}$$

$$d_{2} = d_{1} - \sigma^{*}\sqrt{\tau}$$

$$\sigma^{*2} = \ln[\sigma^{2} + (1 + \mu)^{2}] - \ln[(1 + \mu)^{2}], \text{ i.e., the variance of log returns}$$

$$r_{f}^{*} \equiv \ln(1 + r_{f})$$

The total value is just the sum of the three components described in equations (6)-(8):

$$V_t = V_{1,t} + V_{2,t} - V_{3,t}. (9)$$

The determinants of equation (9) are the fee schedules (ABSC and CDSC), the value of the underlying assets, redemption rates (RD), and fundamental parameters – DIV, CG, EXP, RE,  $r_f$  and  $\sigma$ .

The effects of most of these factors on the value of DCABS are clear. Higher ABSC or CDSC schedules generate higher cash flows and higher values. Higher dividend and capital gains distributions and higher expenses decrease the value of DCABS. For reinvestment rates of less than 100%, these factors decrease both expected future ABSCs from AUM and expected CDSCs from redemptions. Higher reinvestment rates partially offset the effects of distributions on AUM. A higher risk-free rate reduces the present value of future CDSCs from redemptions and thus lowers the value of DCABS. Increasing volatility increases the value of the put option given to investors and thus lowers the value of the DCABS. The effect of the final factor, redemptions, is less clear and more interesting. An increase in redemptions increases contemporaneous revenues from CDSCs, but decreases future revenues from both ABSCs and CDSCs. The net effect depends on the remaining relative fee schedules, the speed of future redemptions, and AUM relative to ORIG and NAV.

Table 3 illustrates both the direction and magnitudes of these effects via a sensitivity analysis for a valuation on a representative fund at origination. The base case value of \$4.82 is based on \$100 of assets under management, the parameter values given in the table, and the fee schedules in Table 2. Note that more than 75% of the value comes from the ABSC component even though cumulative redemptions reach almost 65% over the life of the security, primarily because these redemptions tend to increase over time as the CDSC schedule falls. The put option given to investors accounts for a decrease in value of only approximately 2.5%.

In general, the effects of the parameters on the value of the security are significant though not huge. For example, increasing the combined dividend and capital gains yield from 5%

to 10% decreases the total value to \$4.67, or by slightly more than 3%.<sup>4</sup> The majority of this effect comes through the decrease in NAV and the resulting increase in the value of the redemption put option ( $V_3$ ). In contrast, decreasing the reinvestment rate from 90% to 80% only has an effect on the ABSC component ( $V_1$ ) via the effect on AUM. Reducing fund expenses from 2% to 1% increases total value to \$4.96, primarily through the positive effect on future AUM. A large increase in volatility from 15% to 25% significantly increases the value of the redemption put option but has a small overall effect (less than 1.5%) on value. Finally, for these parameters, an increase of 10% in redemptions each year decreases the value of the security to \$4.71. Interestingly, the effects on both the ABSC and CDSC components are very large, but they almost totally offset due to the nature of the fee schedules. Specifically, while early redemptions dramatically reduce ABSCs, they are penalized heavily via high CDSCs. The effects across years are not completely uniform, but standard fee schedules are designed with exactly this issue in mind.

### 3.3 Hedging

B-share financing companies create value through being an intermediary between brokerage houses and mutual funds. In theory, as this market develops, the fees from B shares will fall as all the risks get transferred to the B-share financing companies. Of course, these financing companies need to manage the risks either on their balance sheets or in the interim period as they securitize them and sell them to the marketplace. Either way, for these financing companies to grow and raise capital for their business, it is necessary to hedge their current portfolio of DCABS.

In order to calculate their hedge positions, define

C	vector of futures contract costs
$\Sigma_{AUM,AUM}$	covariance matrix of fund returns
$\Sigma_{Fut,Fut}$	covariance matrix of futures returns
$\Sigma_{AUM,Fut}$	covariance matrix of fund returns with futures returns
SPBR	shadow price of basis risk

Using the valuation formulas in equations (6)-(8) above, it is possible to calculate analytically the change in the value of DCABS for a change in the value of the underlying assets:

$$\frac{\partial V}{\partial AUM} = \frac{\partial V_1}{\partial AUM} + \frac{\partial V_2}{\partial AUM} - \frac{\partial V_3}{\partial AUM}$$

<sup>&</sup>lt;sup>4</sup>Note that the value depends only on the sum of the dividend and capital gains yield. Therefore, the effect is identical regardless of how the increase in yield is divided between the two components.

Note that the second component (the fixed basis CDSC) is independent of the value of the assets; therefore, the derivative is zero. For the ABSC component, we get

$$\frac{\partial V_{1,t}}{\partial \text{AUM}_t} = \left[\sum_{\tau=1}^T \text{ABSC}_{t+\tau} (1-\eta)^{\tau} [\prod_{s=0}^{\tau-1} (1-\text{RD}_s)]\right] > 0$$

For the CDSC put component

$$\frac{\partial V_{3,t}}{\partial AUM_t} = \frac{\partial V_{3,t}}{\partial NAV_t}$$
$$= \left[\sum_{\tau=1}^T CDSC_{t+\tau}RD_{t+\tau}\prod_{s=0}^{\tau-1} (1-RD_s)[(1+\gamma)^{-\tau}(N(d_1)-1)]\right] < 0$$

An increase in the asset value increases the value of the security, both by increasing expected future ABSC revenues and by reducing the value of the put owned by the investors. This latter effect will be small as long as the put is likely to finish out-of-the-money, i.e., when  $N(d_1)$  is close to 1. The precise magnitude depends on the values of the other parameters, but Table 3 provides some suggestive evidence. Specifically, the last column provides the price elasticity (i.e., the percentage change in price over the percentage change in AUM) for each of the scenarios discussed in Section 3.2. The elasticity ranges between 0.82 and 0.94, suggesting that the value of the security moves strongly with the value of the underlying assets but not quite one-for-one. This elasticity is particularly sensitive to two factors – redemptions and yields. High expected redemptions reduce elasticity because they shift value to the fixed CDSC component which does not depend on AUM. In contrast, high yields (dividend or capital gains yields) increase the value of the redemption put option and its probability of finishing in the money, hence increasing elasticity.

Given a set of futures contracts with corresponding costs C, the goal of the hedging exercise is to choose a set of futures positions b to minimize the cost of the residual risk plus the cost of the futures contracts, i.e.,

$$\min_{b} \operatorname{Var}(\epsilon) * \operatorname{SPBR} + C'b$$

where

$$\epsilon = \Delta V - b' \Delta F$$

and  $\Delta V$  and  $\Delta F$  denote the change in value of the securities and the futures contracts, respectively. Define the hedge ratio of each security with respect to the fund assets as

$$h = \frac{\partial V}{\partial AUM} AUM.$$

Then the residual risk can be written

$$Var(\epsilon) = h' \Sigma_{AUM,AUM} h + 2h' \Sigma_{AUM,Fut} b + b' \Sigma_{Fut,Fut} b$$

When costs are ignored, the futures positions which minimize the residual risk are

$$b = -\Sigma_{Fut,Fut}^{-1} \Sigma_{AUM,Fut}' h,$$

which are just the coefficients from an OLS regression of security returns on futures returns. When the costs of the futures contracts are taken into account, the solution is more complex and numerical search procedures may be necessary. However, for the special case in which all the futures positions are negative, the solution is

$$b = -\Sigma_{Fut,Fut}^{-1} \left( \Sigma_{AUM,Fut}' h - \frac{C}{\text{SPBR}} \right).$$

Intuitively, the magnitudes of the futures positions are reduced depending on their costs relative to the cost of not hedging (i.e., the price of basis risk). If either futures costs are high or the price of residual risk is low, then positions are reduced more.

## 4 Constellation Financial Management: A Case Study

In this section we present a case study of Constellation Financial Management, a company whose main business is purchasing deferred commission assets from mutual fund companies.<sup>5</sup> This analysis illustrates some of the relevant practical and methodological considerations in the implementation of the theory developed in this paper. Constellation's strategy is to form a leveraged entity, with DCABS as assets, and bank debt as the largest liability. Given such a strategy, risk management/hedging takes on paramount importance.

### 4.1 The Company

Constellation Financial Management Company, L.L.C. is a New York based company, founded in November 1994. It has provided a total of approximately \$970 million in financing to a total of 31 clients. As of November 1999, the company is servicing \$859 million in distribution fee receivables (based on mark-to-market value) representing over \$23 billion in underlying mutual fund or similar assets.

<sup>&</sup>lt;sup>5</sup>More precisely, Constellation acts as an advisor to FEP Capital and Lightning Finance Limited, the actual owner of the DCABS. We shall refer to these entities collectively as "Constellation" throughout.

The growth in this business is related to the rapid growth in the entire mutual fund industry, as well as to the increased relative popularity of back-load funds among the general class of load funds (see Section 2). Constellation, for example, increased its quarterly origination rate due to ongoing asset flow contracts with existing clients from less than \$5 million in the first quarter of 1996 to nearly \$122 million in the third quarter of 1999, amounting to an annual growth rate of 149%. Figure 1 shows Constellation's quarterly asset origination rate over its history.

DCABS are easily priced and hedged in theory. In practice, however, the mixture of assets is important for risk management purposes, since some DCABS are difficult to hedge even using traded OTC derivatives. Diversification then becomes a key component of actual mark-to-market pricing and risk management. Moreover, diversification is also critical due to the presence of tracking error in mutual fund performance. While for index funds, which are typically no-load, low cost funds, the tracking error is small by design, the tracking error in actively managed funds may be substantial.

As an example, Constellation's receivables portfolio, valued at \$859 million, comprises DCABS from nineteen domestic, three Canadian and four offshore mutual fund families representing shares in over 450 mutual funds. Another, relatively small, portion of the portfolio stems from insurance products such as deferred annuity investment products offered by three insurance companies.

The levered capital structure of FEP Capital, Constellation's asset holding subsidiary, is of interest because of the resulting importance of hedging. The company currently has a \$595 million revolving credit facility arranged by a major commercial bank. The line of credit is used to finance existing assets and purchase new assets. The loan is secured by a first priority lien on these assets. FEP Capital is permitted to borrow up to 95% of the mark-to-market value of its assets (including the hedge position).

This high leverage ratio is related to the historical performance of the hedge (see below). The cost of the credit facility is sensitive to certain coverage tests. Currently, for example, the parameters are such that the company pays 0.375% on undrawn balances and 1.50% over LIBOR on outstanding balances. The company also holds and hedges equity tranches associated with two off-balance-sheet asset backed securitization transactions (see below). The remainder, the equity part of the company's balance sheet, is comprised of initial paid-in capital and retained earnings.

Constellation's final source of funding is securitization. The structure of these transactions is typical to many other securitizations of different asset classes. For example, Constellation's initial securitization transaction, which closed in May 1999, was valued at \$200 million, using a four tranche sequential pay structure. The most senior tranche was rated Aa2 by Moody's and other tranches had ratings between A2 to Ba2, with 89% of the notes carrying investment grade ratings. In September 1999 a second securitization transaction of approximately \$170 million was completed with similar results.

The value of the securitized assets is inherently volatile, since the assets are not hedged. To help provide some protection against this volatility, significant portfolio diversification is built in. The receivables comprise 176 funds from 8 separate families and 237 monthly pools. The funds represent a broad range of investment styles, underlying asset classes, and international exposures. These transactions also validated the mark-to-market valuation approach, in that total cash proceeds approximated the mark-to-market value of the deferred commission assets. The company retained a residual interest (the equity tranche) in the securitization. Proceeds from securitization were used to pay down existing bank loans.

### 4.2 The Reality of Managing the Assets' Risk

When deferred commission assets are purchased, Constellation exposes itself to declines and increases in the mark-to-market value of its receivables. This risk can be divided into three parts: (1) market risk that can be attributed to changes in market-wide factors, (2) specific risk of the underlying mutual funds' performance relative to the relevant factors, and (3) shareholder redemption and reinvestment risk. Due to the amount of leverage, the company attempts to hedge risk. The concern is primarily with market risk, and the attempt is to minimize the volatility of the mark-to-market value of the asset portfolio.

Market risk is reduced in four ways: (1) through asset-type diversification (e.g., non U.S. funds and annuities), (2) through asset-class diversification (e.g., equity, short-term fixed income and long-term fixed income), (3) through the terms of the receivable contracts, and (4) through financial hedging using various derivative contracts. We shall focus on market-based financial transaction that control primarily market risk.

The hedging strategy consists of shorting futures on indices that are determined to be most closely related to the performance of each fund. Because cash flows are affected by fund shareholder behavior, Constellation developed models that are designed to take into consideration predictable redemption rates, reinvestment rates and fee waiver rates over time. The company uses an actuarial analysis of historical observations to generate shareholder behavior predictions. The risk management desk uses futures and options in over 15 indices in the process of hedging with the stated objective of minimizing risk subject to cost constraints. The vast majority of day-to-day transactions are in a handful of listed indices (e.g., S&P 500, S&P MidCap, Russell 2000, Treasury Note and Bond Futures). The company complements these liquid exchange-traded instruments with OTC trades in indices that can better match the asset risks, including MSCI EAFE and Emerging Market indexes and the Lehman High Yield Bond Index.

The procedural aspects of the risk management process are of some interest. Strategies are developed and implemented by Constellation's risk management desk. For each potential strategy, the risk management desk estimates the trading cost and compares it to the cost of an implied equity allocation that would be needed for unhedged risk. If the strategy passes this cost/benefit test, the desk then implements the trades, monitors the positions, calculates daily profits and losses and measures the success of the hedge compared to expectations. The accounting department independently calculates the financial impact of the mark-to-market value of the assets each month and nets it with the hedge results. The net impact is then compared to that reported by the risk management desk in order to provide a system of checks and balances that ensures effective execution of the company's risk/return objectives.

#### 4.3 Hedging Results

The stated goal of the hedging program is to reduce the annualized residual volatility of the value of the DCABS plus the hedge positions to less than 2% of the mark-to-market value of the portfolio. Residual volatility results predominantly from tracking error and from some positions that are left completely unhedged. Tracking error exists due to the presence of fund-specific risk that is not fully diversified, as well as under-/over-hedging of known factors (see below). In addition, some funds generate exposures for which there is no adequate alternative hedge instrument that can be acquired cost-effectively.

Figure 2 shows the hedged and unhedged monthly mark-to-market values of Constellation's portfolio since the beginning of 1996. The most dramatic change in value the company experienced, in both it's hedged and unhedged value, was during the third quarter of 1998. The value of the hedged portfolio declined by 1.77%. The hedge position managed to limit the impact of market movements in the third quarter of 1998 to less than two standard deviations over a one-quarter horizon (which is 1%). Under standard value at risk assumptions this is approximately the type of event that is expected to occur during one quarter in the course of ten years. Specifically, a decline in value of two standard deviations or more is the 2.5 percentile (under normality). Below we show that this impact was primarily concentrated in non-U.S. assets.

It is interesting to look more closely at the effectiveness and economic characteristics of the hedging program. We analyze three monthly return series: asset values, hedge portfolio return, and the difference of these two – the hedging error. Our first goal is to quantify more precisely the quality of the hedging program. In addition, we obtain data on relevant indexes that allow us to better understand *ex post* the sources of the hedging error. The data span the period from January 1996 to April 1999, for a total of 40 monthly returns. All calculations use simple, monthly rates of return.

Table 4 presents summary statistics for the three series. The volatility of the asset and hedge series are 3% and 2.8% per month, respectively, while that of the hedging error series is much lower, 0.5% per month. This is consistent with the stated policy of maintaining a quarterly standard deviation of portfolio return (assets plus hedge) of 1% per quarter. The most striking feature is the high correlation between the asset and hedge series, -0.987. The hedge is nearly perfect. Notice that the hedging error is positively correlated with the asset portfolio. The correlation of 0.512 indicates that the portfolio is slightly under-hedged. *Ex post*, scaling up the hedge portfolio by 7% would have minimized the error. Given the monthly standard deviations and correlations the theoretical minimal error would have been 0.231% per month (instead of 0.512%). It is important to note, again, that these calculations are made *ex post*.

To further examine the quality of the hedge, we obtain data on four potentially relevant factors/indexes. We use monthly returns for the Wilshire 5000 Index, the Morgan Stanley Capital International EAFE Index (which includes all major non-US equity markets including Europe, Australia and the Far East), the MSCI Emerging Market Index (EMER), and Lehman's Government Bond Index (GOV). Table 5 documents the correlations between these "hedging instruments". As could be anticipated, the three equity indexes are highly correlated at the monthly frequency (correlations range between 0.66 and 0.69). The bond index is slightly negatively correlated with these three indexes.

In Table 6 we perform a hedging exercise using univariate regressions of the asset portfolio on the hedging instruments, and, more interestingly, a hedging error analysis using regressions of the hedging error series on the same factors. The Wilshire index has the highest explanatory power for the asset return series, with an  $R^2$  of 0.89. The EAFE and EMER factors have some explanatory power, with  $R^2$ s of 0.55 and 0.56, respectively, while the GOV index has no explanatory power. Given the above results on the tightness of the hedge, it is no surprise that the same regressions for the hedge return series are close to a mirror image of those for the asset series.

Interestingly, the regression analysis of the hedging error series reveals significant explanatory power for all four factors. That is, once most of the risk in the asset portfolio – primarily US market risk – is taken out, the residual depends on all the candidate factors. In terms of  $R^2$  (and correlation, of course) the most relevant factor is the EMER factor. Constellation's portfolio includes a small number of DCABS backed by emerging market funds. Constellation is doing little to hedge this exposure due to its small size and the difficulty and cost of hedging this risk.

In Table 7 we document the results from multiple regressions of various combinations of the hedging instruments. Interestingly, the  $R^2$  from a multiple regression of the asset portfolio on the full set of four hedging instruments is only 0.923. This number may seem surprisingly low, since the hedged portion of the total asset return risk is higher. Specifically, the hedged portion of the total risk can be calculated from the variances of the asset and residual series in Table 4, i.e.,

$$R^2 = 1 - Var(R_{Error})/Var(R_{Asset}) = 1 - 0.517^2/2.992^2 = 0.97.$$

Recall, however, that the actual hedge is implemented and adjusted continuously, which explains the difference. On the other hand, since the hedge is implemented *ex ante*, this is further evidence of the high accuracy of the hedge using the contingent claims approach.

At the same time, as we saw in Table 4, the portfolio is slightly under-hedged, and it is not surprising that the four hedging instruments combined have significant explanatory power for the hedging error. The  $R^2$  for a multiple regression with all four instruments is 0.417. Given the use of the Wilshire index (with an  $R^2$  of 0.227 on its own), each of the additional instruments help explain a significant portion of the hedging error series. For example, a multiple regression of the error series on the Wilshire and the GOV indexes raises the  $R^2$  to 0.347, with a significantly positive beta on the Wilshire index and a significantly negative beta on the GOV index.

Together, these results are remarkable for a couple of reasons. First, the quality of the hedge is surprisingly high. Second, and related, the idiosyncratic risk in the total asset portfolio is suprisingly low. The tracking error that exists in specific funds is diversified away, leaving close to pure market risk, which the contingent claims approach hedges very successfully.

## 5 Concluding Remarks

We develop a framework for the valuation and hedging of DCABS. The formulae we obtain value DCABS using the contingent claims approach and are expressed in terms of the risk-free rate, the current value and volatility of assets under management, and a set of fund-specific characteristics such as the fee schedules, the expense ratio, the dividend and capital gains distribution rate, the reinvestment rate, and the path of expected future redemptions. We show that the most relevant factors are the asset value and the fee schedule. As a case study we investigate the portfolio of fee-backed assets held by Constellation Financial Management, a DCABS financing company. Their success in hedging the market exposure of these assets validates our approach.

While the discussion is specific to DCABS, the framework can be applied more generally. Our analysis sheds light on the valuation and risks of mutual fund companies, as well as other money management entities within the financial services industry. For example, given the consolidation within this industry, many financial institutions now have large holdings of asset management businesses. These businesses are considered "cash cows" to the extent that they generate revenues from fee-based products. However, these revenues are really claims on the underlying assets, i.e., on the stock or bond markets, and thus represent potentially risky claims, irrespective of future redemptions. Therefore, the applicability of this paper's methodology goes well beyond the pricing of the asset-backed security that we discuss.

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Fund Type	Number	Median Value	Total Value	Avg. Size	Percent
	of Funds	(\$MM)	(MM)	(MM)	
ALL SHARES	$10,\!482$	41.1	3,769,537	360	100
No Load	4339	60.6	$1,\!863,\!340$	429	49
Equity	2883	58.3	$1,\!439,\!588$	499	
Fixed Income	1456	65.5	$398,\!609$	274	
Front Load	2940	55.4	$1,\!383,\!123$	470	37
Equity	1660	59.2	$1,\!068,\!542$	644	
Fixed Income	1244	48.9	$298,\!995$	240	
Deferred Load	3268	18.3	$538,\!468$	165	14
Equity	1899	22.5	$389,\!845$	205	
Fixed Income	1333	13.3	$132,\!060$	99	

Table 1: Mutual Fund Summary Information

The table presents a breakdown of mutual fund asset values as of December 1998 by type of asset and fee structure.

YEAR							
1	2	3	4	5	6	7	8
	Contractual ABSC Schedule						
0.75%	0.75%	0.75%	0.75%	0.75%	0.75%	0.75%	0.75%
Contractual CDSC Schedule							
4.0%	3.0%	3.0%	2.0%	1.0%	0.0%	0.0%	0.0%

Table 2: Representative ABSC and CDSC Schedules

The table presents ABSC and CDSC schedules for a representative equity fund.

	V	$V_1$	$V_2$	$V_3$	Elasticity
Base Case	4.822	3.674	1.274	-0.126	0.894
Yield $+5\%$	4.666	3.615	1.274	-0.223	0.943
Reinvestment - $10\%$	4.763	3.615	1.274	-0.126	0.893
Expenses $-1\%$	4.960	3.796	1.274	-0.110	0.886
Volatility $+10\%$	4.757	3.674	1.274	-0.191	0.894
Redemptions $+10\%$	4.707	2.844	2.039	-0.176	0.822

Table 3: Valuation Sensitivity Analysis

The table presents the total value and value of the three components for DCABS (see equations (6)-(9)) for various sets of parameter values. Values are per \$100 of AUM at origination. All values are based on the fee schedules in Table 2. The base case parameters are

$$r_f = 0.06 \quad \sigma = 0.15 \quad \text{DIV} = 0.02 \quad \text{CG} = 0.03 \quad \text{EXP} = 0.02 \quad \text{RE} = 0.90$$
$$\text{RD}_1 = 0.05 \quad \text{RD}_2 = 0.10 \quad \text{RD}_3 = 0.15 \quad \text{RD}_4 = 0.15$$
$$\text{RD}_5 = 0.15 \quad \text{RD}_6 = 0.15 \quad \text{RD}_7 = 0.30 \quad \text{RD}_8 = 0.30$$

For all the other scenarios, all parameters are kept constant with the exception of the change noted in the first column. For example, "Expenses -1%" refers to the scenario in which expenses are reduced from the base case value of 0.02 to 0.01. For the redemption scenario, expected redemptions in all years are increased by 10%.

	Assets	Hedge	Error				
Mean	0.576	-0.630	-0.054				
STD	2.992	2.763	0.517				
Max	4.545	10.732	1.028				
Min	-12.110	-4.205	-1.378				
Correlation							
Assets	1.000	-0.987	0.512				
Hedge	-0.987	1.000	-0.368				
Error	0.512	-0.368	1.000				

Table 4: Assets, Hedge Portfolio and Hedging Error Summary Statistics

The table presents basic summary statistics for the asset portfolio, the hedge portfolio, and the hedging error, i.e., the residual return. Data are for January 1996 to April 1999, for a total of 40 monthly returns. Statistics are for simple, monthly rates of return.

	Wilshire	MSCI EAFE	MSCI EMER	LehGov
Wilshire	1.000	0.673	0.660	-0.016
MSCI EAFE	0.673	1.000	0.689	-0.098
MSCI EMER	0.660	0.689	1.000	-0.192
LehGov	-0.016	-0.098	-0.192	1.000

The table presents the correlation matrix of instruments used for the error analysis. Data are for January 1996 to April 1999, for a total of 40 monthly returns. Statistics are for simple, monthly rates of return.

	Assets			
	Correlation	Beta	SE	$R^2$
Wilshire	0.942	0.498	0.029	0.887
MSCI EAFE	0.745	0.535	0.078	0.554
MSCI EMER	0.749	0.286	0.041	0.561
LehGov	0.043	0.102	0.382	0.002
	Hedge Por	tfolio		
	Correlation	Beta	SE	$R^2$
Wilshire	-0.931	-0.454	0.029	0.867
MSCI EAFE	-0.715	-0.474	0.075	0.511
MSCI EMER	-0.707	-0.249	0.040	0.499
LehGov	-0.113	-0.246	0.351	0.013
	Hedging E	Crror		
	Correlation	Beta	SE	$R^2$
Wilshire	0.477	0.044	0.013	0.227
MSCI EAFE	0.487	0.060	0.018	0.237
MSCI EMER	0.559	0.037	0.009	0.313
LehGov	-0.354	-0.144	0.062	0.126

 Table 6: Univariate Regressions on Indexes

The table presents results from univariate regressions of the asset portfolio, hedge portfolio and residual portfolio on four financial indexes. Data are for January 1996 to April 1999, for a total of 40 monthly returns. Statistics are for simple, monthly rates of return.

	Wilshire	EAFE	EMER	LehGov	$R^2$
Assets	$0.394\ (0.035)$	$0.090\ (0.050)$	$0.066 \ (0.026)$		0.923
Hedge	-0.381(0.040)	-0.073(0.056)	-0.041(0.029)		0.887
Error	$0.013\ (0.018)$	$0.017\ (0.025)$	$0.025\ (0.013)$		0.342
Assets	$0.498\ (0.029)$			$0.138\ (0.128)$	0.891
Hedge	-0.455(0.027)			-0.279(0.122)	0.883
Error	$0.043\ (0.012)$			-0.141(0.054)	0.347
Assets	$0.382\ (0.034)$	$0.091\ (0.047)$	$0.078\ (0.025)$	$0.252\ (0.105)$	0.934
Hedge	-0.364(0.036)	$-0.075\ (0.050)$	-0.059(0.027)	-0.367(0.111)	0.914
Error	$0.018\ (0.017)$	$0.016\ (0.024)$	$0.019\ (0.013)$	-0.115(0.054)	0.417

 Table 7: Multiple Regressions on Indexes

The table presents a multiple regression analysis of the asset portfolio, hedge portfolio and residual portfolio on four financial indexes. Standard errors are in parentheses. Data are for January 1996 to April 1999, for a total of 40 monthly returns. Statistics are for simple, monthly rates of return.

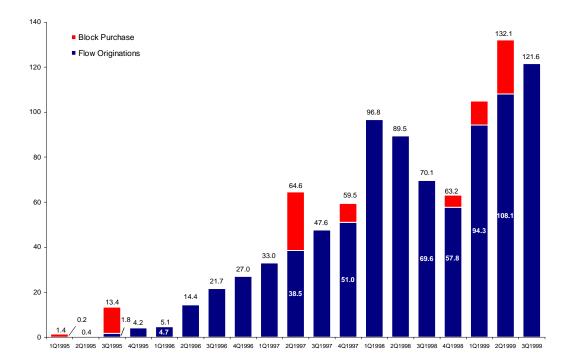
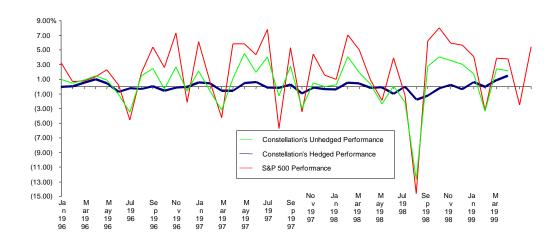


Figure 1: Historical Quarterly Originations Constellation's orgination of fee-backed assets (in \$millions) on a quarterly basis for the period 1995Q1 to 1999Q3.



#### Figure 2: Hedged vs. Unhedged Performance

Unhedged and hedged monthly returns on Constellation's portfolio of fee-backed assets (using marked-to-market valuations) and returns on the S&P500 for the period January 1996 to April 1999.