Robust Analysis of Variance: Process Design and Quality Improvement

Avi Giloni

Sy Syms School of Business, Yeshiva University.

Sridhar Seshadri and Jeffrey S. Simonoff

Leonard N. Stern School of Business, New York University

Abstract

We discuss the use of robust analysis of variance (ANOVA) techniques as applied to quality engineering. ANOVA is the cornerstone for uncovering the effects of design factors on performance. Our goal is to utilize methodologies that yield similar results to standard methods when the underlying assumptions are satisfied, but also are relatively unaffected by outliers (observations that are inconsistent with the general pattern in the data). We do this by utilizing statistical software to implement robust ANOVA methods, which are no more difficult to perform than ordinary ANOVA. We study several examples to illustrate how using standard techniques can lead to misleading inferences about the process being examined, which are avoided when using a robust analysis. We further demonstrate that assessments of the importance of factors for quality design can be seriously compromised when utilizing standard methods as opposed to robust methods.

Introduction

In this article we discuss the use of robust analysis of variance (ANOVA) techniques as applied to quality engineering. ANOVA is the cornerstone for uncovering the effects of design factors on performance. In fact, even after decades of use, ANOVA forms the basis for discussion in the most prestigious statistical journals, leading off with the comment that "Analysis of variance (ANOVA) is an extremely important method in exploratory and confirmatory data analysis" (Gelman, 2005, page 1). Indeed, as we shall discuss below, the various proposals for quality engineering all center on ANOVA methodology.

One can hardly mention the field of quality engineering without referring to the seminal work of Taguchi (for discussion see Taguchi and Yokoyama, 1993a). Taguchi's most significant contribution to quality engineering is his development of robust designs. He recognized the importance of the point that products not only be well-built and/or be of inherently high quality, but also that they be able to withstand non-ideal conditions. In order to robustify products, Taguchi suggested the use of experimental methods to select parameters such that the design is insensitive (robust) to variations in the production process, components, and in use. Many examples of this methodology and parameter design can be found in the literature, e.g., Taguchi and Yokoyama (1993b).

Taguchi is credited and lauded for his work on robust design, but some of his suggestions on implementing the methods have received criticism from researchers both within and outside the quality engineering community (see, for example, Box, 1988, and Bisgaard, 1989). On the other hand, even his critics recognize that his techniques can be easily implemented without requiring knowledge of statistical methodologies. Thus, Taguchi-based methods may very well have been adopted more often than general statistical methods by quality engineers. In either case, the methods utilized are based on ANOVA techniques. These techniques can be strongly affected by outliers, particularly since the ratio of observations to estimated coefficients is often small in such designs.

The data analyzed using ANOVA methods are often collected from experiments. There is always the possibility that some observations may contain excessive noise. Thus, even though the primary interest is in understanding how noise affects performance, this must be tempered by the fact that excessive noise during experiments might lead to incorrect inferences. The term "robustness" in the statistics literature is often used to refer to methods designed to be insensitive to distributional assumptions (such as normality) in general, and unusual observations (“outliers”) in particular. The goal is not only to devise methodologies that yield results that are similar to those from standard methods when the underlying assumptions are satisfied, but also are relatively unaffected by rogue observations that are inconsistent with the general pattern in the data. Robust methods have been applied in virtually every area of scientific investigation, including statistical process control; for example, Rocke (1992) developed robust versions of $\bar{X}$ and R control charts, and Stoumbos and Sullivan (2002) described the application of robust methods to multivariate control charts.

In this paper, we discuss how the ideas of robustness can be applied in the context of ANOVA methods in quality engineering. We discuss this with respect to both traditional statistics methods and Taguchi methods. With the existence of statistical software to implement robust ANOVA methods, it is no more difficult to perform these analyses than it is to run an ordinary ANOVA, and we believe that this is an excellent way to verify that the experimental results have not been unduly affected by unusual observations. Confirmation that the standard analysis gives results that are insensitive to unusual observations leads to the avoidance of unnecessary experiments, since there is an additional level of certainty attached to the results; refutation of the standard analysis can lead to reduced costs in that the proper factors for control are more likely to be identified.

Quality Engineering

Although the development of robust designs is desirable, different ways of attempting to reach this goal have been presented in the literature. Before we discuss how these methods diverge, we first mention their starting point. This starting point is the set of goals (and thus the resulting problems) that Taguchi and collaborators suggested in order to produce robust designs (see for example Taguchi, 1986, and Taguchi and Wu, 1985). These goals/problems are as follows:

1. Even if products conform to specifications, the performance of the product might leave much to be desired, especially when in use by the consumer. Therefore, it is important to minimize the deviation from the target (midpoint) within the specification limits. The resulting problem is that we wish to minimize the deviation of one or more quality characteristics while adjusting its mean or other measure of centrality to a specified target.
2. Since there always will be some variation of the various components within a product, we wish to develop a product with a design that is insensitive to variations in components. The resulting problem is to minimize product sensitivity to component variation.
3. Although a product might have high quality levels, one needs to consider varying environmental conditions. The resulting problem is to minimize product sensitivity to environmental conditions.

The point of departure is how to solve these problems. Both general ANOVA and Taguchi approaches involve techniques based upon experimental designs. The classical statistical approach suggests techniques based upon traditional statistical methods. For a discussion of several such techniques and references to related work, see Box (1988). On the other hand, Taguchi developed a more rigid set of techniques based upon what he refers to as signal-to-noise ratios (S/N ratios). Taguchi suggests using the signal to noise ratio as a metric to obtain a robust design. He suggests several different versions of S/N ratios depending on the various applications. In the case where a particular value is targeted and we wish to minimize the mean square error from the target value (“nominal is better”), it has been shown that the resulting S/N ratio is appropriate when the quality characteristic’s mean is proportional to its standard deviation (Leon, Shoemaker, and Kacker, 1987). Most importantly in our context, the suggested S/N ratios may be very susceptible to outliers, as noted by Box (1988).

Taguchi's approach is similar to Gelman's (2005) proposal to do ANOVA in a hierarchical manner. As Taguchi summarized, the ideal situation is one in which certain factors affect the variability only and others affect the mean response. If the experimental design can uncover that this is the case, then optimizing the S/N ratio is simple. Taguchi thus proposes a two-step strategy:

1. Identify the control factors that have high S/N ratios. Select the combination that gives the highest S/N value.
2. Identify the control factors that have the smallest effect on the S/N ratio and the highest effect on the mean value of the performance variable(s). Use these to adjust the mean response.

The key point for this paper is that many of these tasks are ultimately based on ANOVA methods. Almost invariably, such analyses use standard (least squares) techniques, which are highly sensitive to unusual observations. As such, it is possible that assessments of the importance of factors for quality design can be seriously compromised, leading to waste and inefficiency. In the next section we discuss generally available approaches to robustifying ANOVA (that is, making it more robust). We then look at several examples to illustrate how using standard techniques can lead to misleading inferences about the process being examined, which are avoided when using a robust analysis.

Robust ANOVA Methodolgy

Many techniques for identifying unusual observations and making statistical analyses insensitive to them have been proposed through the years, reflecting the wide range of situations where they apply (see, e.g., Barnett and Lewis, 1994). A prominent approach to making statistical methods more robust is through the use of M-estimation, where estimates are derived through minimization of a robust criterion (Huber, 1964, 1973). We shall illustrate its application in an example utilizing experimental design.

Consider a situation where we are interested in “nominal is better.” The objective is to determinine a robust design of a product whose performance depends on several factors or components. In order to do so, the different factors or components are studied by observing the values of the response variables for various combinations of levels of the factors. Whether one computes an S/N ratio or studies the response values directly, one performs an ANOVA. In an ANOVA, the goal is to explain which factors and their associated levels are most explanatory of the variation in the response variable. This is performed by essentially regressing the response variable against indicator or dummy variables corresponding to the design of the experiment.

In standard ANOVA, the underlying regression estimator is the least squares estimator, where parameters are chosen to minimize the regression sum of squares,

$$\sum_{i=1}^{n} \rho[(y_i - x_i\beta)/\sigma],$$

where x_i is the row vector of predictor values for the i^{th} observation, β is the vector of regression parameters, σ is the standard deviation of the errors, and $\rho(x) = x^2$. Such an estimate can be used to find the correct set of factors along with their appropriate levels (given in the design x) that correspond to a robust design. When there are no outliers in the data, traditional ANOVA methods work quite well. However, when there are outliers, the set of factors and levels that would be suggested as best or appropriate by traditional ANOVA methods may be very incorrect. This is because extreme and/or highly influential response values can easily affect the accuracy of least squares regression (which is the basis for traditional ANOVA). In contrast, in a robust form of ANOVA, the underlying regression estimator is based on a criterion that is more resistant to outliers. There have been many different robust regression estimators proposed in the statistical literature. Although there are substantial differences between different robust estimators in terms of the technical definition of robustness, as well as with the difficulty of their calculation, all such estimators are designed to ensure that unusual observations do not adversely affect the values of the estimated coefficients. M-estimation has been suggested as one approach to constructing a robust regression estimator.

M-estimation is based on replacing $\rho(.)$ with a function that is less sensitive to unusual observations than is the quadratic (see the discussion of LAD regression below as an example). Carroll (1980) suggested the application of M-estimation to ANOVA models. In this paper we use the implementation of M-estimation that is a part of the `robust` library of the statistical package `S-PLUS` (Insightful Corporation, 2002). This implementation takes $\rho(.)$ to be Tukey's bisquare function,

$$\rho(r;c) = (r/c)^6 - 3(r/c)^4 + 3(r/c)^2, |r| \le c \qquad (1)$$

and $\rho(r;c) = 1$ otherwise, where r is the residual and c is a constant chosen to achieve a specified level of efficiency of the estimate if the errors are normally distributed (in our case we specify 90% efficiency, where the efficiency is defined as the ratio of the large-sample variance of the least squares estimator to that of the robust estimator in the presence of normally distributed errors). The computational algorithm requires an initial estimate, which is taken to be the least absolute deviation (LAD) estimate, which takes $\rho(x) = |x|$ (Portnoy and Koenker, 1997).

LAD regression is itself an example of a robust M-estimator. It has been shown to be quite effective in the presence of fat tailed data (Sharpe, 1971) and its robustness properties have been studied (c.f. Giloni and Padberg, 2004). We note that in order that the resulting M-estimator based on (1) be appropriately robust, it is important that the initial estimate is robust

itself. The reason is that we want the initial estimator to be able to locate or detect the outlying observations by ensuring that they have large absolute residuals. Although the reasons why LAD regression is more robust than least squares are quite technical and complex, one can consider the following intuition. In the case of univariate location, the LAD estimator determines the "center" of the data set by minimizing the sum of the absolute deviations from the estimate of the center, which turns out to be the median. On the other hand, the least squares estimator for the univariate case corresponds to the problem of minimizing the sum of the squared deviations from the estimate of the center, leading to the mean as the estimated center. It is well-known that the median is much more robust to outliers than the mean. Similarly, regression estimators based on minimizing the sum of absolute deviations are more robust than those that are based on squared deviations.
Once LAD provides a provisional estimate that is insensitive to outliers, (1) is used to remove any further influence by bounding the influence of any observation with absolute residual greater than a cutoff (while also increasing the efficiency of the estimator compared to LAD). In fact, the shape of the bisquare function (1) implies that not only is the influence of unusual values bounded, but reduces to zero as the point becomes more extreme, thereby improving performance.

Inference for the resultant models is based on a robust F-statistic,

$$F = \frac{2}{n(p-q)} \sum_{i=1}^{n} \left[\rho\left(\frac{r_{qi}}{\hat{\sigma}_p} \right) - \rho\left(\frac{r_{pi}}{\hat{\sigma}_p} \right) \right],$$

where the subscript p indicates values for the "full" model with p parameters that includes all studied effects, and the subscript q indicates values for the "subset" model with q parameters that omits a specific effect. This is not actually an F-test, since it is referenced to a χ^2 distribution on p-q degrees of freedom. Thus, the importance of an effect is assessed by its effect on the robust criterion ρ, in the same way that in standard ANOVA it is assessed by its effect on the (residual) sum of squares.

We apply this robust ANOVA approach directly in the first two examples. For the third example, we instead suggest a method for robustifying an S/N ratio. When this robust S/N ratio is used as the response variable utilizing standard ANOVA, overall we have a more robust procedure.

Examples

Our first example comes from Adam (1987). The goal of the experiment was to eliminate surface defects on automobile instrument panels. Ten factors were considered potential causes of defects. An L_{12} orthogonal array was chosen to fit the ten variables (plus an error factor). A sample of two parts was generated for each of the 12 runs, and then visually and manually inspected for defects.

Table 1 summarizes the results of the least squares and robust analyses. Effects are given as the estimated expected difference in defects for the second level of the factor versus the first level, with low levels being better. In the standard analysis, factors F (low foam shot weight best), G (diverter use best), and I (use of venting B best) are most important. Although none of the runs are flagged as unusual in the standard analysis (see the left panel of Figure 1, which is a plot of standardized residuals in observation order), in the robust analysis point 17 (panel B of run 5) shows up as clearly outlying (with a response that is too high; see the right panel of Figure 1). The robust analysis identifies the same three factors as important, but the estimated effects are noticeably weaker for all three factors. The reason for this is that run 5 included settings with low foam shot weight, foam diverter presence, and use of venting B, and the unusually high number of defects inflated the magnitude of the effects.

This can be seen in the last two columns of the table, which correspond to standard (least squares) ANOVA with the unusual observation omitted. The estimated effects are the same as those for the robust analysis on all of the data, confirming that the robust approach downweights the effects of the outlier. Interestingly, now factors C (high skin weight best) and E (high foam throughput best) are statistically significant, and have the third- and fourth-highest estimated effects. Thus, the robust analysis allows us to not only identify an unusual outcome, but also recognize that other factors should be examined more closely for potential effects on surface defects.

The second example is from a 2^{6-1} fractional factorial design that was used to investigate the fibril formation (fibrillation) of a polyester tape when it is twisted by two contra-rotating air jets (Goldsmith and Boddy, 1973). The response in this case is the denier (the average weight per unit length of fibril), and smaller values are better. Goldsmith and Boddy noted that apparently the values in rows 4 and 16 were interchanged, so we analyze the corrected data. Table 2 summarizes the standard and robust analyses. Run 12 shows up as outlying in the standard analysis (see the top plot of Figure 2), but if this observation is removed, no further runs show up as unusual (the middle plot of Figure 2). The effects and associated tail probabilities are given in Table 2.

The robust analysis on the full data set flags both runs 12 and 20. Interestingly, Goldsmith and Boddy (1973), using a sophisticated algorithm that sequentially tests whether treating each run as a missing data point changes the implications of the model, identify first run 12 and then run 20 as outlying, which the robust analysis is able to do in one pass. Table 2 also gives the estimated effects and tail probabilities for the robust analysis, and least squares results omitting both outliers. It is apparent that the robust estimates are very similar to the least squares estimates when the outliers are omitted, implying that the tape thickness effect and tape width by type of jet interaction effect are weaker than originally thought, while the type of jet effect and type of jet by air pressure interaction effect are stronger than originally thought.

Our final example is based on the Filippone (1989) and Suh (1990) discussions of the application of the Taguchi method to the design of a passive network filter, as discussed in Chapter 9 of Bagchi (1993). A passive network filter is an electronic circuit that is used to record small displacements, such as that experienced by a strain gauge. The circuit has several components, each of whose values deviate from their nominal value. Only three of the components, labeled R_3, R_2 and C, are control factors, with the remaining six treated as noise factors. The circuit in this example transforms mechanical movement into a deflection of a galvanometer. The deflection can be represented using the formula

$$D = \frac{V_s R_g R_3}{G_{sen}(R_2 + R_g)(R_s + R_3) + R_3 R_s + (R_2 + R_g) R_3 R_s C}.$$

The purpose of this experiment is to calibrate the galvanometer deflection, a “nominal is better” situation in the Taguchi parlance.

The L_9 orthogonal array was used to accommodate three levels of R_3, R_2 and C. The levels are given in Table 3. The remaining factors are assumed to have nominal values as follows: R_s 120 $\pm$ 15% (ohms), V 15 $\pm$ 15% (milliVolts), R_g 98 $\pm$ 15% (ohms) and G_{sen} 657.58 $\pm$ 15% (milliVolts per inch). The data without outliers were created by replicating each of the nine experiments (corresponding to L_9) twenty-seven times. In each replication, the values of the four remaining factors (as well as the control factors) were sampled from a uniform distribution that was centered around the nominal value and had the minimum and maximum values as $\pm$ 15% of the nominal value.

To create the contaminated data, we randomly chose three replications in each experiment corresponding to a value of C = 1,400. This could mimic the situation when some experimental set-up became loose during the particular experiments with this value of C. The deflection produced was scaled up by a factor chosen randomly in the range zero to three using the uniform distribution.

The least squares and robust analyses of the level of deflection (not presented here) agree that all three factors are associated with changes in expected deflection level, but the analyses on S/N ratio are noticeably different. This problem allows for a more direct approach to robustifying the standard analysis than is used for the analysis of level (described in the previous section). Instead of estimating the signal-to-noise ratio using the mean and variance of the 27 replicated responses for each experimental setting, as is done in the standard analysis (and which are affected by the outliers in the data), it can be estimated using robust estimates of location and scale. We use the median *M* and the median absolute deviation *MAD* (robust) for this purpose, where

$$MAD = 1.4826 * Median \mid y_i - M \mid,$$

with the constant resulting in consistency for the standard deviation for normally distributed data. As discussed in the previous section, these estimators are much less sensitive to outliers than are the mean and standard deviation, being based on absolute values rather than squares. The standard S/N ratio in this "nominal is better" situation is

$$S/N = 10\log_{10} \bar{X}^2 / s^2 = 20\log_{10} \mid \bar{X} / s \mid,$$

so we define the robust measure of S/N to be

$$S/N_{rob} = 20\log_{10} \mid M / MAD \mid .$$

The results of the analyses are given in Table 4. The standard (least squares) analysis (first column of part (a)) finds marginal evidence for an effect of factor C on S/N, with the highest level (1,400) associated with lower S/N. Unfortunately, this inference is misleading, since it is unduly affected by the outliers. The robust analysis (second column) finds no evidence of any factors being related to the S/N ratio, a finding confirmed by the standard analysis on the uncontaminated data (third column; obviously, in real data situations we would not know which values are actually outliers, but this is possible for these simulated data). The estimated expected S/N values for each of the runs (part (b) of the table) are very similar for the robust analysis and the analysis on uncontaminated data, while it is clear that the standard analysis on the contaminated data gives incorrect answers. The standard analysis would prompt further investigation into factor C when it is not warranted, resulting in wasted effort.

Discussion

In this paper we have discussed how so-called robust analyses, which are insensitive to unusual observations, can be incorporated into the analysis of process design. We do not wish to give the impression that we believe that these robust analyses should take the place of standard ANOVA analyses in this context; rather, we believe that the robust analyses should be undertaken as an adjunct to the standard analyses. If the results are similar, that provides support for the appropriateness of the standard analysis, but if the results are noticeably different, this should prompt closer examination of the data in particular, and the experiment in general, to see if unusual observations have occurred.

We see the potential for further work in this area, particularly in the S/N context. The median and *MAD* are highly resistant to outliers, but are relatively inefficient for normally-distributed data. Using more efficient robust estimates of location (Huber, 1964) and scale (Lax, 1985) in the definition of S/N_{rob} could potentially lead to better performance when outliers are less extreme. An alternative approach to consider is to remove the outliers from the original data, and then calculate S/N in the usual way using the sample mean and variance; when the points are omitted in the proper fashion, this leads to highly robust and efficient estimates of location (Simonoff, 1984) and scale (Simonoff, 1987). Similar robust versions of the suggested Taguchi “larger is better” and “smaller is better” S/N ratios also can be defined.

In this paper we have focused on how response level (through robust ANOVA) and variability (through S/N ratios) can be studied separately, and how the corresponding analyses can be made more robust. A challenging statistical problem is to model level and variability simultaneously, but there has been some recent work on this question (see, e.g., Smyth, 1989, and Rigby and Stasinopoulos, 1996). It would seem that application of these methods to quality engineering problems (particularly “nominal is better” problems) would be a fruitful avenue to explore, and this would also lead to a similar goal of attempting to robustify the methodologies.

References

Adam, C.G. (1987), “Instrument panel process development,” in *Fifth Symposium on Taguchi Methods*, American Supplier Institute, Inc., Dearborn, MI, 93-106. Reprinted in Taguchi, G. and Yokoyama, Y. (1993), *Taguchi Methods: Case Studies from the U.S. and Europe*, Japanese Standards Association, ASI Press, Dearborn, MI, 41-55.

Bagchi, T. P. (1993), *Taguchi Methods Explained*, Prentice-Hall of India, New Delhi, India.

Barnett, V. and Lewis, T. (1994), *Outliers in Statistical Data*, 3rd ed., John Wiley and Sons, New York.

Bisgaard, S. (1989), “Quality engineering and Taguchi methods: a perspective,” *Target*, October 1989, 13-19.

Box, G. (1988), “Signal to noise ratios, performance criteria, and transformations (with discussion),” *Technometrics*, 30, 1-40.

Carroll, R.J. (1980), “Robust methods for factorial experiments with outliers,” *Applied Statistics*, 29, 246-251.

Fillipone, S. F. (1989), “Using Taguchi methods to apply axioms of design,” *Robotics and Computer-Integrated Manufacturing*, 6, 133-142.

Gelman, A. (2005), “Analysis of variance – why it is more important than ever (with discussion),” *Annals of Statistics*, 33, 1–53.

Giloni, A. and Padberg, M. (2004), “The finite sample breakdown point of L1 Regression,” *SIAM Journal of Optimization*, 14, 1028-1042.

Goldsmith, P.L. and Boddy, R. (1973), “Critical analysis of factorial experiments and orthogonal fractions,” 22, 141-160.

Huber, P.J. (1964), “Robust estimation of a location parameter,” *Annals of Mathematical Statistics*, 35, 73-101.

Huber, P.J. (1973), “Robust regression: asymptotics, conjectures and Monte Carlo,” *Annals of Statistics*, 1, 799-821.

Insightful Corporation (2002), *S-PLUS 6 Robust Library User’s Guide*, author, Seattle, WA.

Lax, D.A. (1985), “Robust estimators of scale: finite-sample performance in long-tailed symmetric distributions,” *Journal of the American Statistical Association*, 80, 736-741.

Leon, R.V., Shoemaker, A.C., and Kacker, R.N. (1987), “Performance measures independent of adjustment,” *Technometrics*, 29, 253-285.

Portnoy, S. and Koenker, R. (1997), “The Gaussian hare and the Laplacian tortoise: computability of squared-error versus absolute-error estimators,” *Statistical Science*, 12, 279-300.

Rigby, R.A. and Stasinopoulos, D.M. (1996), “A semi-parametric additive model for variance heterogeneity,” *Statistics and Computing*, 6, 57-65.

Rocke, D.M. (1992), “ $\bar{X}_Q$ and R_Q charts: robust control charts,” *The Statistician*, 41, 97-104.

Sharpe, W.F. (1971), “Mean-absolute deviation characteristic lines for securities and portfolios,” *Management Science*, 18, B1-B13.

Simonoff, J.S. (1984), “A comparison of robust methods and detection of outliers techniques when estimating a location parameter,” *Communication in Statistics – Theory and Methods*, 13, 813-842.

Simonoff, J.S. (1987), “Outlier detection and robust estimation of scale,” *Journal of Statistical Computation and Simulation*, 27, 79-92.

Smyth, G.K. (1989), “Generalized linear models with varying dispersion,” *Journal of the Royal Statistical Society, Ser. B*, 51, 47-60.

Stoumbos, Z.G. and Sullivan, J.H. (2002), “Robustness to non-normality of the multivariate EWMA control chart,” *Journal of Quality Technology*, 34, 260-276.

Suh, N.P. (1990), *The Principles of Design*, Oxford University Press, New York.

Taguchi, G. (1986), *Introduction to Quality Engineering: Designing Quality Into Products and Processes*, Kraus International Publishers, White Plains, NY.

Taguchi, G. and Wu, Y. (1985), *Introduction to Off-Line Quality Control*, Central Japan Quality Control Association, Nagaya, Japan.

Taguchi, G. and Yokoyama, Y. (1993a), *Taguchi Methods: Design of Experiments*, ASI Press, Michigan, USA.

Taguchi, G. and Yokoyama, Y. (1993b), *Taguchi Methods: Case Studies from Japan*, ASI Press, Michigan, USA.

Table 1. Standard (least squares) and robust analyses of automobile instrument panel surface defect data. Columns give the estimated effects for each factor and the *p*-value of the test of significance of the effect.

	Least Squares		*Robust*		*Least squares without outlier*	
Factor	*Effect*	*P*	*Effect*	*p*	*Effect*	*p*
A (foam formulation)	0.02	.998	0.82	.752	0.82	.312
B (venting A)	0.68	.543	-0.12	.180	-0.12	.857
C (skin weight)	-0.83	.460	-1.63	.578	-1.63	.050
D (tooling aid A)	0.03	.976	0.83	.599	0.83	.236
E (foam throughput)	-0.80	.478	-1.60	.294	-1.60	.039
F (foam shot weight)	3.47	.008	2.67	.023	2.67	.009
G (diverter)	-2.20	.067	-1.40	.018	-1.40	.117
H (tooling aid B)	-0.02	.988	-0.82	.837	-0.82	.278
I (venting B)	-2.50	.041	-1.70	.072	-1.70	.056
J (cure time)	1.45	.209	0.65	.581	0.65	.391
Error	0.10	.929	0.90	.298	0.90	.289

Table 2. Standard (least squares) and robust analyses of tape fibrillation data. Columns give the estimated effects for each factor and the *p*-value of the test of significance of the effect.

	Least squares	*without 1 outlier*	*Robust*		*Least squares*	*without 2 outliers*
Factor	*Effect*	*p*	*Effect*	*p*	*Effect*	*p*
A (tape width)	1.46	.874	1.39	.709	1.30	.271
B (tape thickness)	4.11	<.001	3.56	.006	3.53	<.001
C (type of jet)	-5.88	<.001	-6.79	<.001	-6.87	<.001
D (tape speed)	0.36	.828	0.64	.842	0.77	.522
E (air pressure)	-5.75	.002	-5.98	.007	-5.75	<.001
F (tape tension)	-0.91	.894	-1.04	.773	-1.07	.400
A x C	-2.96	.079	-1.96	.003	-1.81	.105
A x E	0.79	.580	0.14	.051	-0.05	.805
A x F	-0.04	.931	-1.28	.339	-1.19	.447
C x E	3.96	.019	4.88	.011	4.80	.001
C x F	0.79	.643	1.87	.180	1.94	.141
E x F	1.04	.516	0.34	.799	0.20	.871

Table 3. Treatment levels for simulated passive network filter data experiment.

Treatment	*Level*		
	1	2	3
R_3 (ohms)	20	50,000	100,000
R_2 (ohms)	0.01	265	525
C (microfarad, μ)	1,400	815	231

Table 4. Results of S/N analyses for simulated passive network filter data.

(a) ANOVA tables.

Effect	*Standard p*	*Robust p*	*Standard (no outliers) p*
R_3	.438	.348	.700
R_2	.838	.297	.550
C	.079	.207	.492

(b) Estimates of expected S/N ratio for each run.

R_3	R_2	C	*Standard*	*Robust*	*Standard (no outliers)*
20	0.01	1400	7.25	12.87	16.58
50000	265	815	13.61	16.74	16.78
100000	525	231	12.93	18.80	18.56
20	265	231	14.83	17.54	17.01
50000	525	1400	7.37	15.39	17.15
100000	0.01	815	11.59	15.48	17.76
20	525	815	15.31	15.05	17.42
50000	0.01	231	13.75	16.56	17.85
100000	265	1400	4.73	16.80	16.65

Figure 1. Plots of standardized residuals by observation order for automobile instrument panel surface defect data.

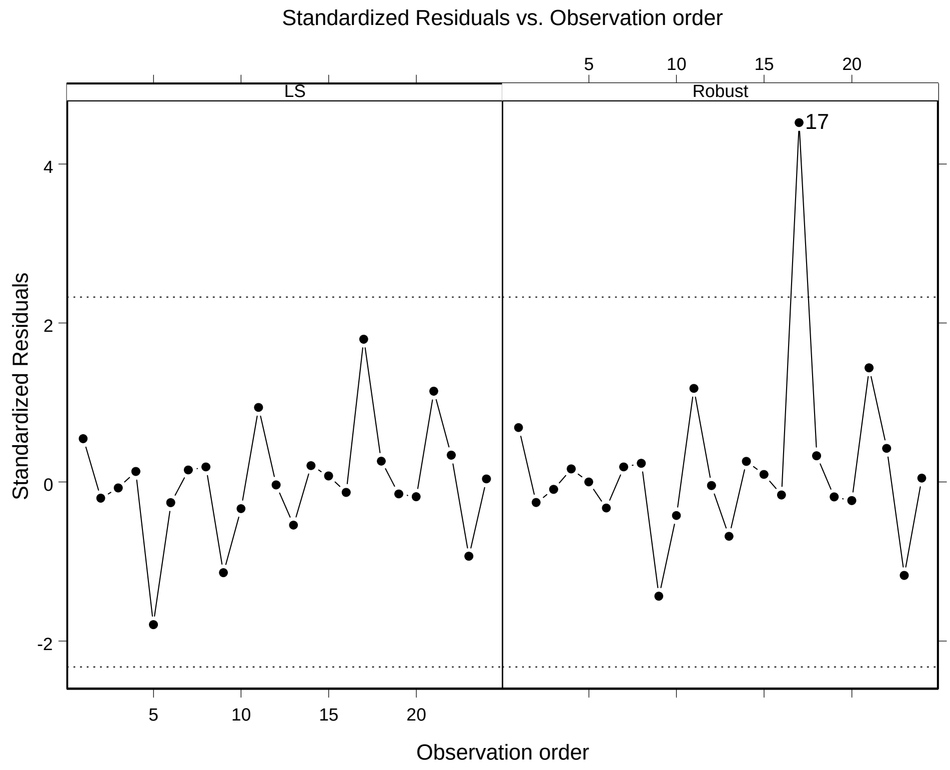
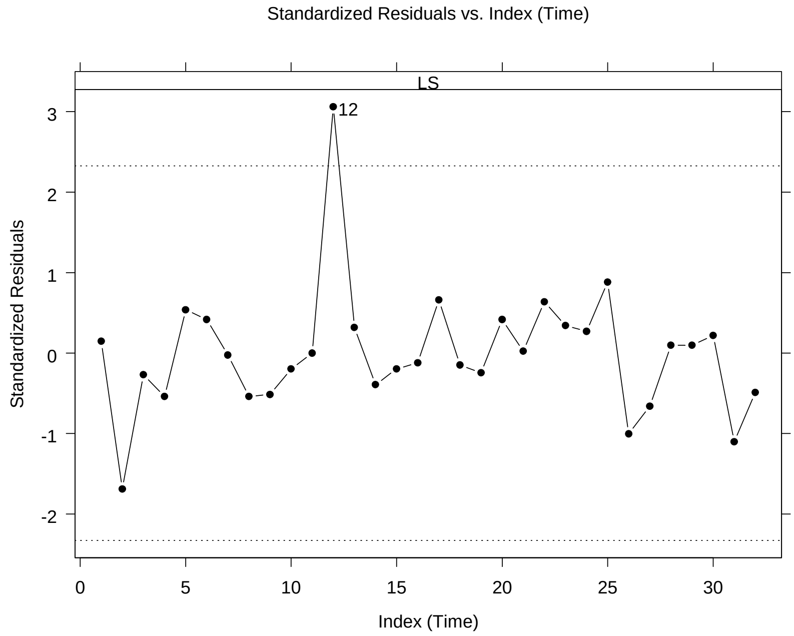
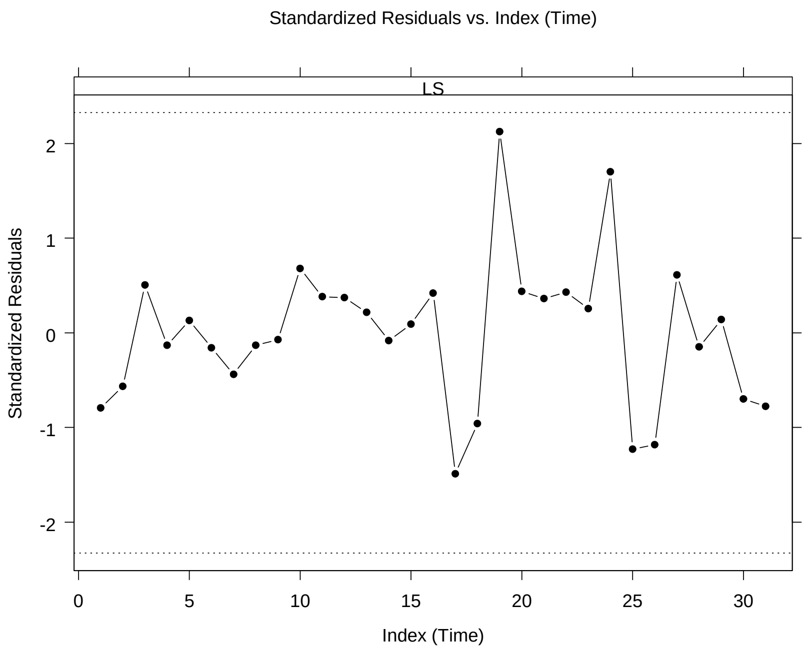


Figure 2. Plots of standardized residuals by observation order for fibrillation data.

Least squares on full data



Least squares omitting run 12



Robust analysis on full data