Imperfect competition and commitment

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Abstract

The degree of competition that a firm faces affects its ability to commit to good behavior. However, the relationship need not be monotonic since competition affects the profits when committed to good behavior (such as efficient high quality) and bad behavior as well as the short-term profits from "cheating". We demonstrate that as a result competition (using two different measures of competition which show qualitatively similar effects) might have non-monotonic effects on a firm's ability to commit. In particular, a firm might choose to operate in a more competitive environment.

1 Introduction

This note, and the examples below essentially make one simple point—that the degree of competition has ambiguous effects on a firm's ability to commit to high quality.

In numerous contexts, such as in professional services industries, it is difficult to write explicit contracts contingent on outcomes. In such cases implicit contracts and reputational considerations can play an important role in ensuring efficient actions. There are numerous exogenous factors which might affect the feasibility of such implicit contracts, or similarly

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of the possibility that a reputational concern can lead to an efficient action.[1] Furthermore, parties may make strategic decisions in order to affect the viability of an implicit contract.[2] In this note, we explore the effect of the degree of competition on a firm's ability to make a credible commitment to exert high effort. In particular, we show that changing the degree of competition that a firm faces has ambiguous and possibly non-monotonic affects on its ability to make such commitments.

We view these results as contributing to a wider literature that seeks to examine the extent to which promoting competition increases welfare. As Nickell (1996) states "Most people believe that competition is a good thing." Despite this common wisdom, empirical and theoretical work has, perhaps surprisingly, been somewhat less sanguine in its assessment.[3] Following Hart (1983), a number of papers have focussed primarily on incentives in a setting where explicit contracts can be written and on effects which are essentially static, that is effects of competition which are manifest in a single period of production.[4] While some of this literature relies on better signals of managerial effort available through comparative performance evaluation in more competitive environments, this affect is absent from the model presented below, in which at the end of a period there is no uncertainty about what happened.[5] We focus instead on dynamic effects; however, we demonstrate that competition can have non-monotonic effects on effort.[6]

A clear and interesting implication of the intuition presented in this note is that there may be occasions where a firm would be better off in a more competitive environment. By

[1] In particular Baker, Gibbons and Murphy (1994) show that as explicit contracting becomes more effective this can render implicit contracts unfeasible.

[2] For example, Baker, Gibbons and Murphy (2002) show that decisions on the allocation of property rights can affect relational contracts and in a related idea, Bernheim and Whinston (1990) show that multi-market contact can make collusion easier to sustain.

[3] See, in particular, Nickell (1996) and Shmidt (1997).

[4] An exception is Meyer and Vickers (1997), discussed in Nickell (1996). Here the role that greater competition plays is to allow for comparative performance evaluation—essentially for better information about the performance of a manager. This can increase reputational incentives but the ratchet effect may imply that rather incentives are dulled.

[5] A number of recent papers consider how industry structure affects the returns to skill and implications for wage inequality (Guadalupe (2003)), contracts (Cuñat and Guadalupe (2003a and b)) and organizational design (Harstad (2003)). We abstract from such considerations below in considering a model in which firms are similar.

[6] Choosing to produce high quality is costly and thus acts as a sort of investment and so our results showing the ambiguous effects of competition on this kind of investment are related to wide literatures on the effects of competition on other kinds of investment. In particular, they are related to a large literature on Schumpeterian innovation and to discussions on the ambiguous effects of market structure on advertising intensity (see, for example, Sutton (1991), Cabral (2000) and Martin (1993)).

encouraging a firm to enter the industry or by making its own product more substitutable with rivals' products, a commitment to produce high quality could become credible and raise profits. [7,8]

2 Preview of results and central intuitions

In general whether reputational considerations will motivate a firm to perform some costly action depends on the trade-off between the short-term gain or saving in not performing the action and the long-term effects of beginning the following period with a relatively low reputation. Supposing that taking the costly action maintains a high reputation, and not undertaking it ensures a low reputation (as is typically assumed to be the case in the literature on relational contracts or equilibria supported by trigger strategies) this trade-off can be summarized by the following inequality which ensures that the firm exerts the costly action:

$$\text{short-term cost of action} \leq \text{discounted value of high reputation - discounted value of low reputation.}$$

One might expect increased competition to reduce profits, both for a high reputation firm and a low reputation firm. In addition inasmuch as increased competition might lead to a smaller market share it might also lead to a lower total cost of producing at high quality—the short term cost. Thus the effect of competition on reputational incentives, as summarized by this inequality above seems ambiguous and will depend on the rate at which the degree of competition affects these profits and costs.

However, it seems clear that if competition is sufficiently severe then the discounted value of having a high reputation might be driven close to zero, which would make taking

[7] Such intuition can easily be borne out through specific examples in the model presented below and such examples are provided in Appendix C.

[8] Other papers have noted that competition can help a firm's ability to commit to behave well, but in these papers, this is essentially a commitment to behave well in the future. For example, Farrell and Gallini (1988) and Shepard (1987) highlight that second-sourcing, or encouraging future competition constrains a monopolist's ability to charge a high price or produce at a low observed quality in the future and so might benefit the monopolist in increasing current sales to customers, who must incur product specific set-up costs. Dudey (1990) and Wernerfelt (1994) argue that firms might locate near each other as a means of committing to price at a reasonably low level and so encourage customers to pay the search costs required to visit the firms. In these models, therefore increasing competition increases a firm's commitment to future good behaviour and this has current benefits. In the model presented in this note, by contrast, the commitment problem is with respect to current unobservable behaviour and so whereas in those papers more competition always helps to overcome the commitment problem, here the effect is non-monotonic.

the costly action unattractive.[9] In this case, a firm's temptation to pursue a fly-by-night or hit-and-run strategy whereby the savings from not undertaking the action outweigh the reputational gains would preclude any expectation that a firm would ever take such action and any reputational incentives. This intuition, that reputational incentives can only be maintained if the firm enjoys some premium above costs has been long recognized and has provoked some discussion on how this observation might be squared with free entry into markets—in essence the resolution was some form of loss-making period of costly signalling in order to establish the reputation initially and which then leads to a period of maintaining reputation and enjoying a price premium (see in particular Klein and Leffler (1981) and Shapiro (1983)).[10]

Given this strong intuition that "too much" competition can damage reputational incentives, it is of some interest to determine whether "some" competition can ever help. Again, there is strong intuition to suggest that it might; in particular a firm in competition faces the prospect of a loss in market share as well as a price drop on losing reputation and so the "punishment" for not exerting effort can be more severe.

Below, we show that both these intuitions have some force in a model where customers and firms make no quality inferences from a firm's quantity decision. In particular, this implies that overall the degree of competition has ambiguous effects on a firm's ability to commit to high quality.[11]

3 Model

In order to consider the effect of the degree of competition a firm faces on its ability to commit to high effort, we introduce a simple model. We suppose that there is only one

[9] In addition a low reputation would be driven out of the market and so the value of having a low reputation would be zero—it cannot fall below this value.

[10] Hörner (2002) considers a model of reputation (that is a model based on beliefs as to the type of a firm) in which price has a signalling role and so even under perfect competition, a price premium above the cost of production is maintained, thereby maintaining the incentives to exert effort and maintain reputation. As discussed at some length below, I suppose that price has no signalling role and so under perfect competition price would be driven down to marginal cost.

[11] In recent and related, independent work, Kranton (2003) argues that competitive effects reduce the price premium in a model where consumers can use only current price in addition to past quality to make inferences on the current quality. Thus in Kranton's paper, the first of the two effects highlighted in the paragraphs above is elaborated, that is the role of competition in preventing an equilibrium where high quality is maintained. However, the second effect considered here, the potential for some degree of competition to help in sustaining high quality is precluded by an assumption that a firm that produces low quality in one period can be held down to zero profits in all future periods. Thus the non-monotonic, ambiguous, effect of competition on the ability to commit to high quality—the focus of this note—does not arise.

firm in the industry, Firm A, for which there is a commitment problem. In each period Firm A, chooses whether to produce high quality products at some cost or low quality costlessly. The firm has a commitment problem inasmuch as profits would be higher if it could credibly commit to high quality but in each period customers and rival firms, though they observe the firm's quality in previous periods, cannot observe the firm's quality in the current period until after all purchases have been made. It is assumed that there are n rival firms and that they can only produce high quality.[12]

In each period, all firms set quantities. Given these quantities and consumers' expectations of quality, prices are determined. The realization of the firm's quality in previous periods informs customers' and rivals' expectations of quality. In particular, we suppose that if the firm is ever observed producing low quality then it is supposed that the firm would produce low quality forever—thus we restrict attention to "trigger strategy" equilibria. Moreover, quality realizations are commonly and publicly observed at the end of each period by all customers and all rival firms.

We assume that customers' and rivals' expectations are not affected by any firm's choice of quantity—that is an individual firm's choice of quantity plays no signalling role. Note, in particular, that this assumption precludes the possibility that firms collude in this simple framework.[13] This is an important and perhaps controversial assumption. However, it is certainly plausible to imagine in a richer environment that the current quantity decision of a firm may be hard for customers to observe or in some sense may be less salient to customers and, to me at least, the assumption that it does not affect customers' expectations of quality certainly seems plausible and worth investigating.[14] In effect, we restrict attention to a Markov perfect equilibrium in which the state can take two values corresponding to whether or not Firm A has ever produced low quality.

We work with a linear demand model with quality indices.[15] In this model all consumers (there are assumed to be a measure 1 of consumers) have the same utility function defined

[12] We show in Appendix B that relaxing this assumption and instead supposing that all firms make quality decisions leads to qualitatively similar results.

[13] Kranton (2003) also considers a similar assumption with respect to firms. However, in that model, though customers (as here) can use all past realisations of quality to make inferences about current quality, they can only use the *current* price. Here in contrast they can use neither current nor past prices to make inferences about current quality.

[14] Supposing that customers' compete in bidding for the service that a firm produces would not overcome this problem, since customers would have to have expectations about the behavior of a firm following each realized price. This gives infinitely many possible continuation games with which one could bootstrap particular equilibrium behavior.

[15] See Appendix 2.2 of Sutton (1998) for further details concerning this model.

over the $n+1$ goods in the industry (Firm A's and those of its n rivals) and of a separated outside good available at a fixed price of unity. The following utility function for a consumer in period t ensures that the consumer's demand schedule for each good takes a relatively simple linear form:

$$U = \sum_i (x_{it} - \frac{x_{it}^2}{v_{it}^2}) - 2\sigma \sum_i \sum_{k<i} \frac{x_i}{v_i} \frac{x_k}{v_k} + M, \tag{1}$$

where x_{it} is the quantity of good i consumed, v_{it} is its quality, $\sigma \in [0,1]$ measures the degree of substitutability between different firms' offerings and M denotes the consumption of the outside good which is priced at unity. Writing the consumer's income as Y, it follows that $M = Y - \sum_i p_i x_i$.

In particular, this model implies that the price in period t for firm i's product is given by:

$$p_{it} = 1 - \frac{2x_{it}}{u_{it}^2} - \frac{2\sigma}{u_{it}} \sum_{j \neq i} \frac{x_{jt}}{u_{jt}}. \tag{2}$$

Note that the price will depend not on the actual quality of the good and of rival goods, but rather on their anticipated quality in the period denoted by u_{it}. In particular, all firms but firm A always produce high quality and so $u_{it} = h$ for all $i \neq a$ and for all t.

Firm A can choose either to produce quality l at a cost 0 or quality h at a cost c per unit in each period. As discussed below it is socially more efficient and Firm A would prefer to commit to high quality when $h(1-c) > l$.[16] All firms seek to maximize the discounted value of profits where the discount factor is given by r.

Below, we discuss the relationship between the possibility that an equilibrium in which the strategic firm can commit to exert high effort can be sustained and the different measures of competition, σ and n. First however, it is useful to consider as benchmarks, the static outcomes when quality is observed before purchase rather than after, as is assumed through most of the note.

4 Static benchmarks

Before exploring the model discussed above it is useful to calculate per-period profits in the case where Firm A can commit to high or low quality (or equivalently in the case where

[16] See the last few paragraphs of Section 4.2 and the Appendix for proof of this claim.

customers can observe quality before purchase and rivals can observe Firm A's quantity before deciding their own production).

4.1 Firm A produces high quality

Suppose first that Firm A produces high quality goods, then it produces a_h where a_h maximizes its profit, that is:

$$a_h = \arg\max_x (1 - \frac{2x}{h^2} - \frac{2\sigma}{h}\sum_{j \neq a} \frac{x_j}{h} - c)x. \tag{3}$$

From the first order condition, it follows that

$$a_h = \frac{h^2}{4}(1-c) - \frac{\sigma}{2}\sum_{j \neq i} x_j. \tag{4}$$

Similarly, each of the rival firms i, in equilibrium, chooses x to maximize:

$$(1 - \frac{2x}{h^2} - \frac{2\sigma}{h}\sum_{j \neq i,a} \frac{x_j}{h} - \frac{2\sigma a_h}{h^2} - c)x \tag{5}$$

and so

$$x_i = \frac{h^2}{4}(1-c) - \frac{\sigma}{2}\sum_{j \neq i,a} x_j - \frac{\sigma a_h}{2}. \tag{6}$$

By symmetry $x_j = x_h = a_h$ for all j and so

$$x_h = a_h = \frac{h^2}{4}(1-c) - \frac{\sigma}{2} n x_h. \tag{7}$$

Thus

$$a_h = \frac{h^2(1-c)}{2(2+\sigma n)}. \tag{8}$$

It follows trivially that the profit in this case is given by :

$$\pi_h = \frac{1}{2}\frac{h^2(1-c)^2}{(2+\sigma n)^2}, \tag{9}$$

and the price is given by

$$p_h = \frac{1-c}{2+\sigma n} + c. \tag{10}$$

Note that the quantity, price and profit vary with respect to the parameters of the model in an intuitive way. In particular, profits are increasing in h and decreasing in c, σ and n. In particular, competition (as measured either by n the number of rivals in the industry or σ the degree of substitution between the firm and its rivals) decreases both the price and the quantity and thereby the profits.

4.2 Firm A produces low quality

Suppose that Firm A produces low quality and that the rivals each produce x_{hl}. Then Firm A will choose to produce a_l, where a_l satisfies

$$a_l = \arg\max_x (1 - \frac{2x}{l^2} - \frac{2\sigma}{l} n \frac{x_{hl}}{h})x, \tag{11}$$

and so $a_l = \frac{l^2}{4} - \frac{\sigma l}{2h} n x_{hl}$ so long as this is positive, otherwise $a_l = 0$.

The quantity produced by a rival firm maximizes

$$(1 - \frac{2x}{h^2} - \frac{2\sigma}{h}((n-1)\frac{x_{hl}}{h} + \frac{a_l}{l}) - c)x. \tag{12}$$

Taking the first order condition and assuming symmetry yields

$$x_{hl} = \frac{h^2}{2(2+\sigma(n-1))}(1-c) - \frac{h\sigma}{l(2+\sigma(n-1)}a_l \tag{13}$$

and substituting back into the earlier expression for a_l yields:

$$a_l = \frac{l}{2}\frac{2l + \sigma l(n-1) - \sigma h(1-c)n}{(2+\sigma n)(2-\sigma)}, \tag{14}$$

so long as

$$2l + \sigma l(n-1) - \sigma h(1-c)n > 0. \tag{15}$$

When Condition (15) fails then $x_l = \pi_l = 0$ and note that condition (15) is more likely to hold the less competitive the environment (the smaller σ and n) since $h(1-c) > l$. When Condition (15) holds then

$$p_l = \frac{2l + l\sigma(n-1) - h\sigma n(1-c)}{l(2+\sigma n)(2-\sigma)} \tag{16}$$

and

$$\pi_l = \frac{1}{2}(\frac{2l + l\sigma(n-1) - h\sigma n(1-c)}{(2+\sigma n)(2-\sigma)})^2. \tag{17}$$

Note, in particular, that in this case the profit for the low quality producer (and both price and quantity) are increasing in l and c and decreasing in h, furthermore it can readily be verified that they are decreasing in n and in σ.[17]

Similarly in the case where condition (15) fails then $\pi_{hl} = \frac{1}{2}(\frac{2h(1-c)-\sigma l}{(2+\sigma n)(2-\sigma)})^2$ as one might expect is increasing in h and decreasing in l, c and n but $\frac{d\pi_{hl}}{d\sigma}$ may be either positive or negative.[18] This result is perhaps not surprising—it is the effect of business stealing from a weaker rival which implies that greater substitution between the two might benefit the stronger competitor.

Note that in the case that condition (15) fails $\pi_h = \frac{1}{2}\frac{h^2(1-c)^2}{(2+\sigma n)^2} > 0 = \pi_l$.

When condition (15) holds then $\pi_h > \pi_l$ if and only if

$$\frac{1}{2}\frac{h^2(1-c)^2}{(2+\sigma n)^2} > \frac{1}{2}(\frac{2l + l\sigma(n-1) - h\sigma n(1-c)}{(2+\sigma n)(2-\sigma)})^2 \tag{18}$$

or equivalently

$$(2 + \sigma(n-1))(h(1-c) - l) > 0 \tag{19}$$

and so in this case $\pi_h > \pi_l$ if and only if $h(1-c) - l > 0$.

In either case if $h(1-c)-l > 0$ then $\pi_h > \pi_l$ and so as earlier claimed if $h(1-c)-l > 0$

[17] For example

$$\frac{d\pi_l}{d\sigma} = -\frac{2l + l\sigma(n-1) - h\sigma n(1-c)}{(2+\sigma n)(2-\sigma)}\frac{4n(h(1-c)-\sigma l) + \sigma^2 n^2(h(1-c)-l) + \sigma^2 nl}{(2+\sigma n)^2(2-\sigma)^2}$$

where $2l + l\sigma(n-1) - h\sigma n(1-c) < 0$ and $h(1-c) > l$ so $(h(1-c) - \sigma l) > 0$ and so $\frac{d\pi_l}{d\sigma} < 0$.

[18] This is also true for $x_{hl} = \frac{1}{2}h\frac{2h(1-c)-\sigma l}{4+2\sigma(n-2)-\sigma^2(n-1)}$ though $p_{hl} = \frac{1}{h}\frac{2h(1-c)-\sigma l}{4+2\sigma(n-2)-\sigma^2(n-1)} + c$ has a different comparative static with respect to c.

Note that

$\frac{d\pi_{hl}}{d\sigma} = (2h(1-c) - l\sigma)\frac{l\sigma^2 n + 4hn(1-c)(1-\sigma) - 4(h(1-c)-l)}{(2+\sigma n)^3(2-\sigma)^3}$

For example when $h(1-c) = 0.9$, $\sigma = 0.1$, $n = 1$ and $l = 0.05$ then $\frac{d\pi_{hl}}{d\sigma} < 0$, but when $h(1-c) = 0.9$, $\sigma = 0.1$, $n = 1$ and $l = 0.1$ then $\frac{d\pi_{hl}}{d\sigma} > 0$ (and in both cases $2l + \sigma l(n-1) - \sigma h(1-c)n > 0$).

then if Firm A could credibly commit to high quality production then it would choose to do so.

5 Commitment

In this section we seek to examine how competition affects the feasibility of an equilibrium in which Firm A commits to high quality. We restrict attention to a Markov perfect equilibrium in which the state is a simple indicator of whether or not Firm A has ever produced low quality and in which quantity has no signalling role. We return to consider this restriction in the concluding section.

First note that in any subgame in which Firm A had previously produced low quality, there is no action which would change the state and so the unique equilibrium strategy in this subgame would be for Firm A to produce low quality in each period and for the static optimal quantities to be produced.

Suppose that Firm A has not produced a low quality good, if in equilibrium in this state it produces high quality, then since the quantities chosen by Firm A and its rivals would not change the state, it is clear that the quantities chosen in each period would be as in the static case in Section 4.1, where Firm A was committed to high effort. In this section, essentially we ask whether this is sustainable.[19] In order to do so first consider the short-term gain from deviating—that is of producing low quality when expected to produce high quality.

The quantity that Firm A produce to maximize profits when producing low quality when rivals and customers anticipate high quality is a_d where a_d satisfies:

$$a_d = \arg\max_x (1 - \frac{2x}{h^2} - \frac{2\sigma}{h^2} n x_h) x. \tag{20}$$

Note that customers expect high quality and as do rival firms so that the price is $(1 - \frac{2a_d}{h^2} - \frac{2\sigma}{h^2} n x_h)$. From the first order condition and substituting for x_h, it can readily be shown that

$$x_d = \frac{h^2}{4} \frac{2 + \sigma c n}{2 + \sigma n}, \tag{21}$$

$$p_d = \frac{1}{2} \frac{2 + \sigma c n}{2 + \sigma n} \tag{22}$$

[19] Note that there always exists an equilibrium in which Firm A always produces low quality.

and profit is given by:

$$\pi_d = \frac{h^2}{8}(\frac{2+\sigma cn}{2+\sigma n})^2. \tag{23}$$

5.1 Condition to ensure no deviation

It follows that a necessary and sufficient condition which ensures that there is a Markov perfect equilibrium in which firm A produces high quality output is given by:

$$\frac{1}{1-r}\pi_h \geq \pi_d + \frac{r}{1-r}\pi_l \tag{24}$$

Equivalently

$$\Delta = \frac{r}{1-r}(\pi_h - \pi_l) - (\pi_d - \pi_h) > 0. \tag{25}$$

We consider the comparative statics of Δ with respect to n and σ, the measures of competition, examining the cases where $\pi_l = 0$ and $\pi_l > 0$ separately. Note that the former case is more likely for high values of σ (when $\sigma > \frac{2l}{h(1-c)n-l(n-1)}$) and for high values of n (when $n > l\frac{2-\sigma}{\sigma(h(1-c)-l)}$). Furthermore as anticipated in the introduction, for large enough n,

$$\Delta = \frac{1}{1-r}\pi_h - \pi_d = \frac{1}{1-r}\frac{1}{2}\frac{h^2(1-c)^2}{(2+\sigma n)^2} - \frac{h^2}{8}(\frac{2+\sigma cn}{2+\sigma n})^2$$

and as $n \to \infty$, $\Delta \to -\frac{h^2c^2}{8} < 0$, that is for large enough n Condition (25) fails.

5.2 Comparative statics with respect to n and σ

Case I: Low quality produces nothing after deviation (condition (15) fails)

In this case

$$\begin{aligned} \Delta = & \frac{1}{1-r}\pi_h - \pi_d \\ = & \frac{1}{1-r}\frac{1}{2}\frac{h^2(1-c)^2}{(2+\sigma n)^2} - \frac{h^2}{8}(\frac{2+\sigma cn}{2+\sigma n})^2 \\ = & \frac{h^2(4(1-c)^2-(1-r)(2+\sigma cn)^2)}{8(2+\sigma n)^2(1-r)} \end{aligned} \tag{26}$$

The effect of a change in the degree of substitution between the offerings of different firms is given by:

$$\frac{d\Delta}{d\sigma} = -2n\frac{h^2(4(1-c)^2 - (1-r)(2+\sigma cn)^2)}{8(2+\sigma n)^3(1-r)} - \frac{h^2 2cn(2+\sigma cn)}{8(2+\sigma n)^2} \tag{27}$$

or, equivalently,

$$\frac{d\Delta}{d\sigma} = -2n\Delta - \frac{h^2 2cn(2+\sigma cn)}{8(2+\sigma n)^2}. \tag{28}$$

It follows that if $\Delta > 0$ then $\frac{d\Delta}{d\sigma} < 0$ but if $\Delta < 0$ then $\frac{d\Delta}{d\sigma}$ may be either positive or negative.

Similar results obtain for the comparative statics in this case with respect to n.[20]

Case B: Low quality produces a positive quantity after deviation (condition (15) holds)

In this case

$$\Delta = \frac{1}{1-r}\pi_h - \frac{r}{1-r}\pi_l - \pi_d, \tag{29}$$

or equivalently,

$$\Delta = \frac{h^2(4(1-c)^2 - (1-r)(2+\sigma cn)^2)}{8(2+\sigma n)^2(1-r)} - \frac{r}{2(1-r)}\left(\frac{2l + l\sigma(n-1) - h\sigma n(1-c)}{(2+\sigma n)(2-\sigma)}\right)^2. \tag{30}$$

The effect of a change in the degree of substitution between the offerings of different firms is given by:

$$\begin{aligned}\frac{d\Delta}{d\sigma} = \; & \tfrac{1}{2}h^2 n\frac{2(c-r)(1-c)+\sigma cn(1-r)(1-c)}{(2+\sigma n)^3(1-r)} \\ & + \frac{r}{(1-r)}n\frac{2l+l\sigma(n-1)-h\sigma n(1-c)}{(2+\sigma n)(2-\sigma)}\frac{4(h(1-c)-l\sigma)-\sigma^2(n-1)l+\sigma^2 nh(1-c)}{(2+\sigma n)^2(2-\sigma)^2}\end{aligned} \tag{31}$$

The second term is always positive, however since the first term may be either positive or negative, the sign of this expression is ambiguous. Similar results apply with respect to

[20]
$$\frac{d\Delta}{dn} = -2\sigma\frac{h^2}{8(2+\sigma n)^3(1-r)}(4(1-c)^2 - (1-r)(2+\sigma cn)^2) - \frac{h^2 2\sigma c(2+\sigma cn)}{8(2+\sigma n)^2}$$

or equivalently

$$\frac{d\Delta}{dn} = -2\sigma\Delta - \frac{h^2 2\sigma c(2+\sigma cn)}{8(2+\sigma n)^2}$$

and so if $\Delta > 0$ then $\frac{d\Delta}{dn} < 0$ but if $\Delta < 0$ then $\frac{d\Delta}{dn}$ may be either positive or negative.

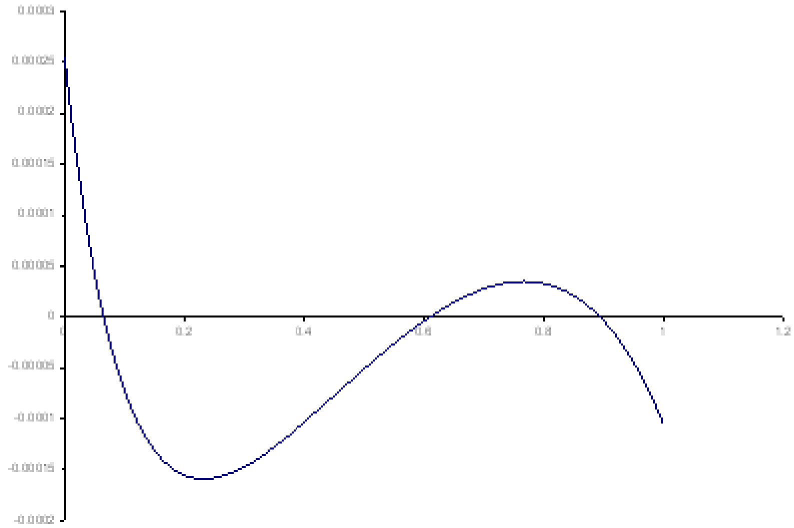

Figure 1: Delta against sigma at h=0.81, l=0.63, c=0.1, n=5 and r=0.485

n.[21]

Thus overall the effect of a change in the degree of competition, measured either by the number of rival of firms in the industry or as the degree of substitution between the products of different of different firms, has ambiguous effects on Δ, or equivalently on the possibility that a Markov Perfect equilibrium in which Firm A would produce high quality can be sustained.

The figures below illustrate that an increase in the degree of competition can have ambiguous effects, holding all other parameters of the model constant. For example, Figure 1 illustrates that when $h = 0.81$, $l = 0.63$, $c = 0.1$, $n = 5$ and $r = 0.485$, then $\Delta > 0$ holds at $\sigma = 0$ but as σ increases, it fails, then holds and then fails again.

Figure 2 illustrates that the effect a change in the number of rivals may have an ambiguous effect on Firm A's ability to commit to producing high quality. The figure, which plots Δ against $\log_2 n$ with $h = 0.8$, $l = 0.77$, $c = 0.02$, $\sigma = 0.5$ and $r = 0.51$, shows that

[21]

$$\frac{d\Delta}{dn} = \frac{1}{2}\sigma h^2 \frac{2(c-r)(1-c) + \sigma cn(1-r)(1-c)}{(2+\sigma n)^3(1-r)} + \frac{r\sigma}{(1-r)} \frac{2l + l\sigma(n-1) - h\sigma n(1-c)}{(2+\sigma n)(2-\sigma)} \frac{2h(1-c) - l\sigma}{(2+\sigma n)^2(2-\sigma)}$$

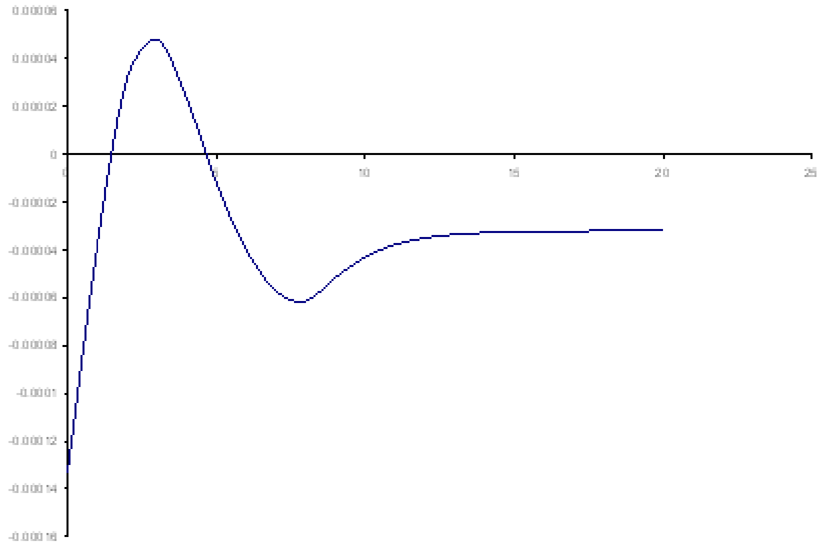

Figure 2: Delta against log(n) at h=0.8, l=0.77, c=0.02, sigma=0.5 and r=0.51

$\Delta > 0$ when n is between 3 and 23, but if there are fewer than 3 or more than 23 rivals then the condition fails.

Suppose that there is an additional gain to producing high quality (such as pride in producing high quality or reputational spillovers to other products) then there is an equilibrium in which Firm A produces high quality so long as $\Delta > B$. As might be anticipated, given the results on $\frac{d\Delta}{d\sigma}$ in the case where $2l + \sigma l(n-1) - \sigma h(1-c)n < 0$, this might lead to even more peculiar relationships between the possibility of maintaining such an equilibrium and measures of competition. For example Figure 3 illustrates shows that how the condition $\Delta > B$ varies with σ for $h = 0.8$, $l = 0.2$, $c = 0.3$, $n = 3$, $r = 0.405$ and $B = 0.01753$.

6 Caveats

The model in this note is deliberately made simple to make the point that competition has ambiguous effects on reputational incentives starkly. However, there are a number of issues which ought to be considered in refining this intuition in practical application.

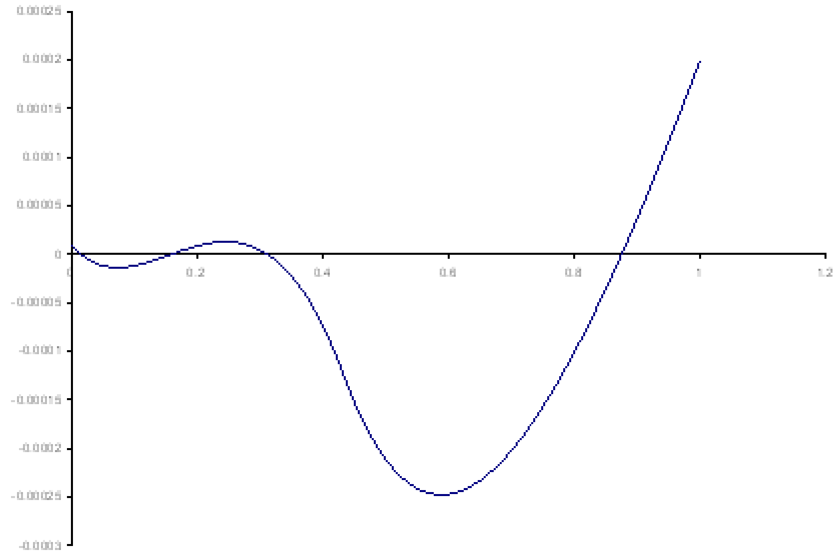

Figure 3: Delta-B against sigma at h=0.8, l=0.2, c=0.3, n=2, r=0.405 and B=0.01753

First, it should be noted that the analysis in this note is partial inasmuch as we take the market structure as exogenous without addressing sunk costs, barriers to entry or other reasons why competition might be limited. In addition, in the model presented above we have focused on a single firm's incentives to produce high quality, taking as given that all other firms in the industry are committed to producing at high quality. We have argued at some length (and notational complexity) that as the degree of competition in the industry changes, the profits to holding a high reputation and the profits to holding a low reputation as well as the cost of maintaining reputation will all change and at different rates. Thus, it seems reasonable to suppose that similar effects might apply if all firms had quality decisions to make in each period, and if the number of firms in the industry was endogenously determined. We prove the first of these conjectures below in Appendix B.[22]

[22] The argument elaborated in the Appendix B is loosely as follows.

Following Abreu (1988), to consider the viability of an equilibrium in which all firms produce high quality, one should consider a most severe feasible punishment continuation equilibrium for a firm that deviates. However, determining which continuation equilibria are feasible is not a trivial exercise (in particular it may not be feasible that all rival firms produce at high quality after one firm has produced at low quality).

Nevertheless, it can be shown in the static version of the model where all firms are committed either to low or high quality, that in an industry with $n+1$ firms, profits for a firm committed either to low or high quality are non-decreasing in the number of firms committed to low quality. It then follows, that Condition (25) defines a harshest (though possibly not feasible) continuation equilibrium and so can be thought of

It should further be noted that collusion does not appear in the model and is left undiscussed. This is perhaps, the most widely discussed topic in formal dynamic models of industry outcomes with a fixed number of firms and so the omission may be a serious one. In part collusion does not arise in the model as a deliberate modelling choice. Clearly the degree of competition in an industry affects firms' ability to collude; however the model presented here is intended to be as simple as possible to illustrate a simple point that competition has ambiguous effects on a firm's incentives to maintain reputation with respect to its customers, which I believe would be robust but perhaps more obscure in a model that allowed for collusion between firms in the industry.

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as a necessary condition for the existence of an equilibrium in which all firms produce high quality in a dynamic version of the model. Since a continuation where all firms produce low quality is always possible, a sufficient condition which ensures the existence of an equilibrium in which all firms produce high quality is given by:

$$\frac{1}{1-r}\pi_h \geq \pi_d + \frac{r}{1-r}\pi_{ll},$$

where $\pi_{ll} = \frac{1}{2}\frac{l^2}{(2+\sigma(n-1))^2}$ is the short-term profit that a firm in the industry earns when all the firms produce low quality and each optimises with respect to the quantity produced.

Examining this condition together with Condition (25) allows us to identify parameters regions in which an equilibrium in which all firms produce high quality can exist and regions where such an equilibrium cannot exist (though there are some regions where the existence of such an equilibrium cannot be determined). Such an analysis is sufficent to show that the existence of such an equilibrium is non-monotonic in the degree of competition. We demonstrate that this is the case for certain parameter values.

A Proofs of results

In this section, we show that if $h(1-c) > l$ then utilitarian social welfare is higher when Firm A is committed to high quality production than when it produces low quality.

When Firm A is committed to high quality then recall $a_h = x_h = \frac{h^2(1-c)}{2(2+\sigma n)}$ from Equation (8), substituting into the utility function (Equation (1)) yields

$$U_h = (n+1)\frac{h^2(1-c)}{2(2+\sigma n)}(1-\frac{1}{h^2}\frac{h^2(1-c)}{2(2+\sigma n)})-\frac{2\sigma}{h^2}\frac{(n+1)n}{2}\frac{h^2(1-c)}{2(2+\sigma n)}\frac{h^2(1-c)}{2(2+\sigma n)}+Y-(n+1)p_h\frac{h^2(1-c)}{2(2+\sigma n)}. \tag{32}$$

Now for Firm A and each of the rival firms

$$\pi_h = (p_h - c)\frac{h^2(1-c)}{2(2+\sigma n)}. \tag{33}$$

So that welfare when firm A is committed to high quality and there are n rivals is given by

$$W_h(n) = (n+1)\frac{h^2(1-c)}{2(2+\sigma n)}(1-\frac{1}{h^2}\frac{h^2(1-c)}{2(2+\sigma n)})-\frac{2\sigma}{h^2}\frac{(n+1)n}{2}\frac{h^2(1-c)}{2(2+\sigma n)}\frac{h^2(1-c)}{2(2+\sigma n)}+Y-(n+1)pc\frac{h^2(1-c)}{2(2+\sigma n)}, \tag{34}$$

or equivalently

$$W_h(n) = \frac{h^2(1-c)^2}{2^2(2+\sigma n)^2}(n+1)(3+\sigma n) + Y. \tag{35}$$

Now suppose that Firm A cannot commit to high quality but instead produces at low quality, there are two cases to consider here.

Case 1: Condition (15) fails

In this case Firm A produces nothing and so welfare is given by $W_l(n) = W_h(n-1)$.

Now

$$\frac{d}{dn}W_h(n) = \frac{h^2(1-c)^2}{4}\frac{2+\sigma n(1-\sigma)+4(1-\sigma)}{(2+\sigma n)^3} > 0 \tag{36}$$

and so in particular $W_h(n) > W_h(n-1)$

and hence in this case $W_h(n) > W_l$.

Case 2: Condition (15) holds

Then by Equation (14)

$$a_l = \frac{l}{2}\frac{2l + \sigma l(n-1) - \sigma h(1-c)n}{(2+\sigma n)(2-\sigma)} \tag{37}$$

and by Equation (13)

$$x_{hl} = \frac{h}{2}\frac{2h(1-c) - \sigma l}{(2+\sigma n)(2-\sigma)} \tag{38}$$

Industry profits are given by $n(p_{hl} - c)x_{hl} + p_l a_l$ and so in this case

$$\begin{aligned} W_l(n) = \quad & n\tfrac{h}{2}\tfrac{2h(1-c)-\sigma l}{(2+\sigma n)(2-\sigma)}(1 - \tfrac{1}{h^2}\tfrac{h}{2}\tfrac{2h(1-c)-\sigma l}{(2+\sigma n)(2-\sigma)}) \\ & -\tfrac{2\sigma}{h^2}\tfrac{(n-1)n}{2}\tfrac{h}{2}\tfrac{2h(1-c)-\sigma l}{(2+\sigma n)(2-\sigma)}\tfrac{h}{2}\tfrac{2h(1-c)-\sigma l}{(2+\sigma n)(2-\sigma)} - cn\tfrac{h}{2}\tfrac{2h(1-c)-\sigma l}{(2+\sigma n)(2-\sigma)} \\ & +\tfrac{l}{2}\tfrac{2l+\sigma l(n-1)-\sigma h(1-c)n}{(2+\sigma n)(2-\sigma)}(1 - \tfrac{1}{l^2}\tfrac{l}{2}\tfrac{2l+\sigma l(n-1)-\sigma h(1-c)n}{(2+\sigma n)(2-\sigma)}) \\ & -n\tfrac{2\sigma}{hl}\tfrac{h}{2}\tfrac{2h(1-c)-\sigma l}{(2+\sigma n)(2-\sigma)}\tfrac{l}{2}\tfrac{2l+\sigma l(n-1)-\sigma h(1-c)n}{(2+\sigma n)(2-\sigma)} + Y \end{aligned} \tag{39}$$

Equivalently

$$\begin{aligned} W_l(n) = \quad & n\tfrac{1}{4}\tfrac{2h(1-c)-\sigma l}{(2+\sigma n)(2-\sigma)}\tfrac{6h(1-c)+(2h(1-c)-\sigma l)\sigma(n-1)-3\sigma l}{(2+\sigma n)(2-\sigma)} \\ & +\tfrac{1}{4}\tfrac{2l+\sigma l(n-1)-\sigma h(1-c)n}{(2+\sigma n)(2-\sigma)}\tfrac{6l+3\sigma l(n-1)-2\sigma^2 ln+\sigma hn(1-c)}{(2+\sigma n)(2-\sigma)} + Y. \end{aligned} \tag{40}$$

Now consider $(W_h - W_l)$:

$$\begin{aligned} W_h - W_l = \tfrac{h^2(1-c)^2}{2^2(2+\sigma n)^2}(n+1)(3+\sigma n) - n\tfrac{1}{4}\tfrac{2h(1-c)-\sigma l}{(2+\sigma n)(2-\sigma)}\tfrac{6h(1-c)+(2h(1-c)-\sigma l)\sigma(n-1)-3\sigma l}{(2+\sigma n)(2-\sigma)} \\ -\tfrac{1}{4}\tfrac{2l+\sigma l(n-1)-\sigma h(1-c)n}{(2+\sigma n)(2-\sigma)}\tfrac{6l+3\sigma l(n-1)-2\sigma^2 ln+\sigma hn(1-c)}{(2+\sigma n)(2-\sigma)} \end{aligned} \tag{41}$$

rearranging terms, it follows that:

$$\begin{aligned} (W_h - W_l)\tfrac{4(2-\sigma)^2(2+\sigma n)^2}{h(1-c)-l} = \quad & -7\sigma^2 ln - 12\sigma l + 3l\sigma^2 n^2 - l\sigma^3 n^2 + l\sigma^3 n + 12\sigma ln + 3\sigma^2 l + 12l \\ & +ch\sigma^2 n + 3ch\sigma^2 n^2 - \sigma^3 hn^2 c - 3h\sigma^2 n^2 - h\sigma^2 n + \sigma^3 hn^2 - 4\sigma hn \\ & +4\sigma hnc - 12\sigma h + 12\sigma ch + \sigma^3 nh - \sigma^3 nch + 3\sigma^2 h - 3\sigma^2 ch + 12h - 12ch \end{aligned} \tag{42}$$

Equivalently

$$(W_h - W_l)\frac{4(2-\sigma)^2(2+\sigma n)^2}{h(1-c)-l} = \begin{array}{c} -n\sigma h(1-c)(4+\sigma+3\sigma n-\sigma^2(n+1)) \\ +\sigma nl(12-7\sigma-\sigma^2(n-1)+3\sigma n)+3(h(1-c)+l)(4(1-\sigma)+\sigma^2) \end{array} \quad (43)$$

Rearranging terms again

$$(W_h - W_l)\frac{4(2-\sigma)^2(2+\sigma n)^2}{h(1-c)-l} = \begin{array}{c} (3\sigma n-\sigma^2 n)(2l+\sigma(n-1)l-n\sigma h(1-c))-(2-\sigma)l(3\sigma n-\sigma^2 n) \\ -n\sigma h(1-c)(4+\sigma-\sigma^2)+\sigma nl(12-7\sigma+\sigma^2)+3(h(1-c)+l)(4(1-\sigma)+\sigma^2) \end{array} \quad (44)$$

Note that $(3\sigma n - \sigma^2 n) > 0$ and by assumption $(2l + \sigma(n-1)l - n\sigma h(1-c)) > 0$ and so

$$(W_h - W_l)\frac{4(2-\sigma)^2(2+\sigma n)^2}{h(1-c)-l} > \begin{array}{c} -(2-\sigma)l(3\sigma n-\sigma^2 n)-n\sigma h(1-c)(4+\sigma-\sigma^2) \\ +\sigma nl(12-7\sigma+\sigma^2)+3(h(1-c)+l)(4(1-\sigma)+\sigma^2) \end{array}. \quad (45)$$

The right hand side of this inequality it equal to:

$$(2l+\sigma(n-1)l-n\sigma h(1-c))(4+\sigma-\sigma^2)-8l+2l\sigma+ln\sigma(2-\sigma)(1-\sigma)+2l\sigma^2+l\sigma^2(1-\sigma)+3(h(1-c)+l)(4(1-\sigma)+\sigma^2)$$

and since $4 + \sigma - \sigma^2 > 0$, this expression is greater than or equal to

$$-8l + 2l\sigma + ln\sigma(2-\sigma)(1-\sigma) + 2l\sigma^2 + l\sigma^2(1-\sigma) + 3(h(1-c)+l)(4(1-\sigma)+\sigma^2)$$

and since $n > 1$ and we are supposing that since $h(1-c) > l$, this in turn is greater than or equal to

$$-8l+2l\sigma+l\sigma(2-\sigma)(1-\sigma)+2l\sigma^2+l\sigma^2(1-\sigma)+6l(4(1-\sigma)+3\sigma^2) = 4l(4-\sigma)(1-\sigma)+14l\sigma^2 > 0$$

and thus in this case too when $h(1-c) > l > 0$, then $W_h > W_l$.

B All firms make quality decisions

In this section, we modify the model in the main body of the text by assuming that all $n+1$ firms in the industry are symmetric and must make a decision in each period whether to produce high or low quality (that is the n rivals of Firm A are no longer committed to high quality exogenously). The question we address below is whether an equilibrium exists in which all firms in the industry can credibly commit to high quality production.

B.1 Static benchmark: m firms low, $n+1-m$ high

First consider the static benchmark in which product quality is anticipated and can be observed prior to purchase and where of the $n+1$ firms in the industry, m produce low quality and the rest produce high quality.

Suppose that in equilibrium each low quality producer's output is x_{lm} and each high quality producer's output is x_{hm}.

Then for a low quality producer, the x_{lm} is determined by choosing x to maximize:

$$(1-\frac{2x}{l^2}-\frac{2\sigma}{hl}(n+1-m)x_{hm}-\frac{2\sigma}{l^2}(m-1)x_{lm})x$$

Similarly for a high quality producer, the problem is to choose x to maximize

$$(1-\frac{2x}{h^2}-\frac{2\sigma}{h^2}(n-m)x_{hm}-\frac{2\sigma}{hl}mx_{lm}-c)x$$

So the first order conditions yield:

$$x_{hm}=\frac{h^2(1-c)}{2(2+\sigma(n-m))}-\frac{mh\sigma}{l(2+\sigma(n-m))}x_{lm} \tag{46}$$

and

$$1-\frac{4x_{lm}}{l^2}-\frac{2\sigma}{hl}(n-m+1)x_{hm}-\frac{2\sigma}{l^2}(m-1)x_{lm}=0$$

so long as $x_{lm}>0$.

In this latter case

$$x_{lm}=\frac{l}{2}\frac{l(2+\sigma(n-m))-\sigma h(1-c)(n+1-m)}{4+2\sigma(n-1)-\sigma^2 n} \tag{47}$$

Thus if $2l+\sigma l(n-m)-\sigma h(1-c)(n+1-m)>0$ (a condition which is more likely to hold the larger is m)

then

$$x_{lm} = \frac{l}{2}\frac{l(2+\sigma(n-m-1)) - \sigma h(1-c)(n-m)}{4+2\sigma(n-2)-\sigma^2(n-1)} \tag{48}$$

and

$$x_{hm} = \frac{h}{2}\frac{2h(1-c)+h\sigma(m-1)(1-c)-l\sigma m}{(2+\sigma(n+1)-\sigma)(2-\sigma)}. \tag{49}$$

Then

$$\pi_{lm} = \frac{1}{2}(\frac{2l+\sigma l(n-m)-\sigma h(1-c)(n+1-m)}{4+2\sigma(n-1)-\sigma^2 n})^2 \tag{50}$$

which is increasing in m and

$$\pi_{hm} = \frac{1}{2}\frac{(2h(1-c)+h\sigma(m-1)(1-c)-l\sigma m)^2}{(2+\sigma(n+1)-\sigma)^2(2-\sigma)^2} \tag{51}$$

which is increasing in m.

Otherwise, that is if $l(2+\sigma(n-m)) - \sigma h(1-c)(n+1-m) < 0$,

then $\pi_{lm} = 0$ and

$$\pi_{hm} = \frac{1}{2}\frac{h^2(1-c)^2}{(2+\sigma(n-m))^2} \tag{52}$$

which is increasing in m.

Note in particular that consideration of these two cases implies that π_{lm} is increasing in m.[23]

B.2 Necessary and sufficient conditions

That π_{lm} is increasing in m suggests that for a firm producing low quality and anticipated to produce low quality the lowest profits are earned when all other firms produce high quality and so a most severe punishment for defection from a situation when all are producing high

[23]In addition and as one might anticipate π_{hm} is also increasing in m.

π_{hm} is increasing in the region $l(2+\sigma(n-m)) - \sigma h(1-c)(n+1-m) > 0$ and in the region $l(2+\sigma(n-m)) - \sigma h(1-c)(n+1-m) > 0$.

At $l(2+\sigma(n-m)) - \sigma h(1-c)(n+1-m) = 0$ then $l = \frac{\sigma h(1-c)(n+1-m)}{2+\sigma(n-m)}$ and

$\frac{1}{2}\frac{(2h(1-c)+h\sigma(m-1)(1-c)-l\sigma m)^2}{(2+\sigma(n+1)-\sigma)^2(2-\sigma)^2} = \frac{1}{2}\frac{h^2(1-c)^2}{(2+\sigma(n-m))^2}$

so π_{hm} is continuous throughout and so in particular the two regions connect and so π_{hm} is increasing in m.

quality is that following a defection, all the rivals continue producing high quality. Thus a necessary condition for the existence of an equilibrium in which all firms in the industry produce high quality is (25).

However such a continuation may not be feasible, that is it may not be the case that a continuation in which one firm produces low quality and all others produce high can be sustained as an equilibrium. One continuation equilibrium that can always be sustained is that all firms produce low quality for ever. Specifically, we suppose that following a deviation by a firm, in the future all firms produce low quality outputs—it is clear that the most severe punishment that can be sustained in equilibrium is at least as severe as this one. Using the earlier static results it follows that a sufficient condition which ensures that there is an equilibrium in which all firms produce high quality output is given by, it follows that a sufficient condition for an equilibrium in which all firms produce high quality is given by:

$$\frac{1}{1-r}\pi_h \geq \pi_d + \frac{r}{1-r}\pi_{ll}, \tag{53}$$

where $\pi_{ll} = \frac{1}{2}\frac{l^2}{(2+\sigma n)^2}$ is the short-term profit that a firm in the industry earns when all the firms produce low quality and each optimizes with respect to the quantity produced.

Let

$$\Delta_{suff} = \frac{1}{1-r}\pi_h - \pi_d - \frac{r}{1-r}\pi_{ll}. \tag{54}$$

We proceed by considering the comparative statics of Δ_{suff} with respect to n and σ—the measures of competition.

$$\Delta_{suff} = \frac{1}{1-r}\frac{1}{2}\frac{h^2(1-c)^2}{(2+\sigma n)^2} - \frac{h^2}{8}(\frac{2+\sigma cn}{2+\sigma n})^2 - \frac{r}{1-r}\frac{1}{2}\frac{l^2}{(2+\sigma n)^2}. \tag{55}$$

Equivalently

$$\Delta_{suff} = \frac{4h^2(1-c)^2 - (1-r)h^2(2+\sigma cn)^2 - 4rl^2}{8(1-r)(2+\sigma n)^2}. \tag{56}$$

The first derivative with respect to n is given by:

$$\begin{aligned} \frac{d\Delta_{suff}}{dn} &= -\frac{(1-r)h^2 2(2+\sigma cn)\sigma c}{8(1-r)2+\sigma n)^2} - 2\sigma\frac{4h^2(1-c)^2-(1-r)h^2(2+\sigma cn)^2-4rl^2}{8(1-r)(2+\sigma n)^3} \\ &= -\frac{(1-r)h^2 2(2+\sigma cn)\sigma c}{8(1-r)2+\sigma n)^2} - 2\sigma\Delta_{suff} \end{aligned}, \tag{57}$$

or equivalently

$$\frac{d\Delta_{suff}}{dn} = -\frac{(1-r)h^2 2(2+\sigma cn)\sigma c}{8(1-r)2+\sigma n)^2} - 2\sigma\Delta_{suff}. \tag{58}$$

Note in particular that if $\Delta_{suff} > 0$ then $\frac{d\Delta_{suff}}{dn} < 0$ but otherwise it is not clear how to sign this.

The first derivative with respect to σ is:

$$\frac{d\Delta_{suff}}{d\sigma} = -\frac{(1-r)h^2 2(2+\sigma cn)cn}{8(1-r)(2+\sigma n)^2} - 2n\frac{4h^2(1-c)^2 - (1-r)h^2(2+\sigma cn)^2 - 4rl^2}{8(1-r)(2+\sigma n)^3} \tag{59}$$

or equivalently

$$\frac{d\Delta_{suff}}{d\sigma} = -\frac{(1-r)h^2 2(2+\sigma cn)cn}{8(1-r)(2+\sigma n)^2} - 2n\Delta_{suff}. \tag{60}$$

Again if $\Delta_{suff} > 0$ then $\frac{d\Delta_{suff}}{dn} < 0$ but otherwise the sign of $\frac{d\Delta_{suff}}{d\sigma}$ is ambiguous.

Thus increased competition either through an increase in n or an increase in σ cannot make it more likely that $\Delta_{suff} > 0$; however, if there is an additional benefit to non-deviation then the necessary condition $\Delta_{suff} + B > 0$ may have an ambiguous relationship with n and σ.

B.3 Existence of an equilibrium

When the sufficient condition holds, then an equilibrium in which all firms produce high quality can be sustained, however when it fails then the possibility that such an equilibrium can be sustained cannot be ruled out. However, existence of such an equilibrium can be ruled out when the necessity condition fails. Thus by examining the two conditions simultaneously, it is possible to show that the existence of such an equilibrium is not monotonic in the measures of the degree of competition in the industry. Specifically such an equilibrium exists when $\Delta_{suff} + B > 0$ but does not exist when $\Delta + B < 0$; in the intermediate case (that is when $\Delta_{suff} < 0$ and $\Delta > 0$) the conditions are insufficient to prove the existence or non-existence of such an equilibrium).[24]

In the light of earlier results and examples, it is perhaps unsurprising that the effect of an increase in the level of competition (either as an increase in σ or in n) has an ambiguous

[24] It is trivial to show that $\Delta > \Delta_{suff}$ and indeed is a corollary of the result that π_{lm} is increasing in m.

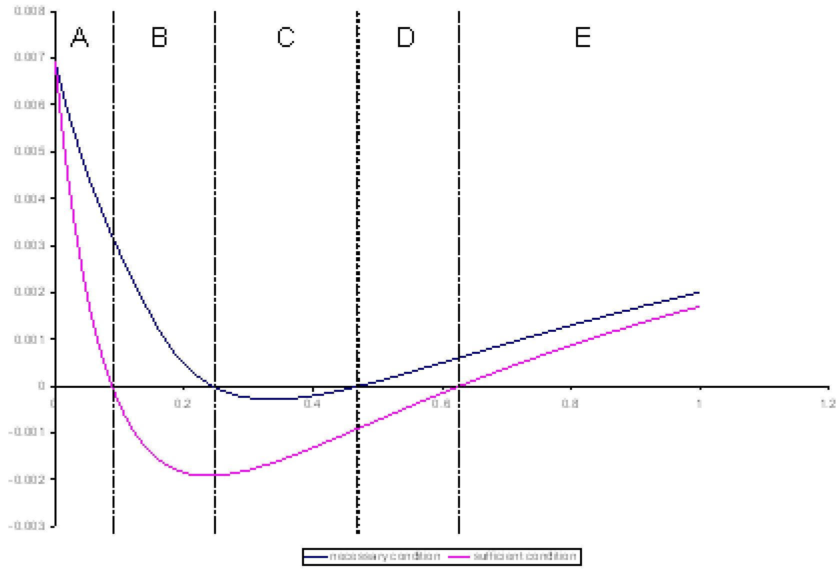

Figure 4: Necessity and sufficiency conditions against sigma at h=0.8, l=0.2, c=0.39, n=8, r=0.6 and B=0.02

effect on the possibility that an equilibrium in which all firms can credibly commit to produce high quality outputs. This is demonstrated by the Figures 4 and 5. In both these figures an equilibrium in which all the firms in the industry can credibly commit to high quality exists in the regions A and E but not in the region C.[25]

C Examples where a firm would prefer more competition

For the case of increasing profits through changing σ, it would be hard to interpret such a change when $n > 1$ (it is not obvious how could an individual firm affect the degree of substitution between two other firms. In the case $n = 1$ where changing the degree of substitution between its own and its rivals products is a more plausible modelling assumption, it is easy to construct examples in which Firm A could benefit from a higher σ. This is the case for example at $h = 0.3$, $l = 0.18$, $c = 0.1$, and σ around 0.54.

Encouraging another firm into the industry may be a plausible modelling assumption (for example in consulting this may be a choice to allow or encourage some partners to spin

[25] In these figures the line "necessary condition" represents $\Delta + B$ and the line "sufficient condition" $\Delta_{suff} + B$.

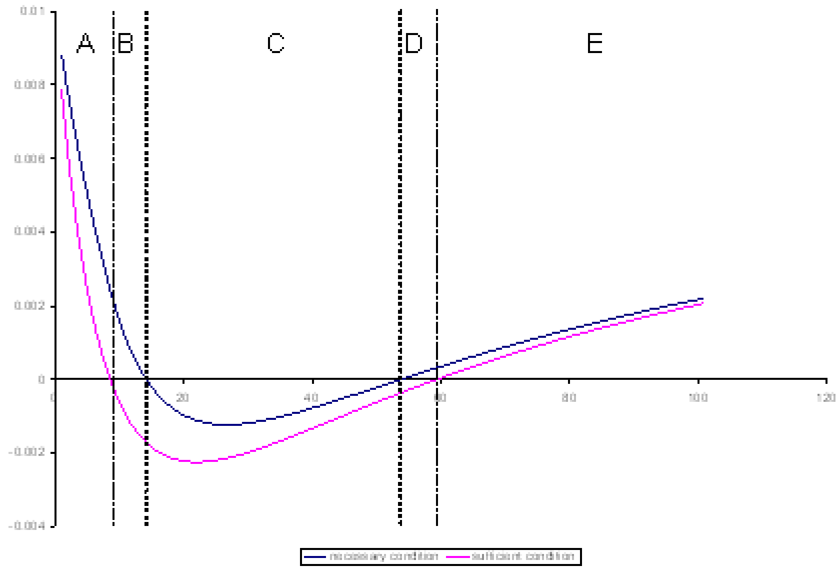

Figure 5: Necessity and sufficiency conditions against n at h=0.9, l=0.2, c=0.3, sigma=0.1, r=0.5 and B=0.017

off or in general it may involve licensing or otherwise diffusing specialist knowledge). If this is possible, again a firm may benefit from doing so. For example, at $h = 0.8$, $l = 0.71$, $c = 0.05$, $\sigma = 0.6$, Firm A can commit to high quality and earn higher per-period profits when $n = 4$ than when $n = 3$ and it can make no such commitment.

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