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Offshore & Onshore contracts

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We study the buyer-seller relationship in a supply chain from the perspective of quality. In particular, we examine situations where quality levels of the product can be observed, yet, conformance cannot be enforced unless the buyer engages in costly, legally-binding activity, which we call audit. We assume that the cost of audit is borne by the buyer.

We examine three types of remedies that can be taken in case an audit confirms the existence of quality problems, namely "Full Rebate", "Compensate" and "Repair" contracts. Under the Full Rebate regime, the seller rebates the buyer the purchase price of defective units. Under the Compensate regime (Expectation Damages), the buyer is compensated for his loss of profit due to defects. Finally, under the Repair regime (Specific Performance), the seller is made to repair all the defective units at his own expense. We examine the effects of these contracts on the performance of the supply-chain, and point out the implications for offshore and onshore supply networks, respectively.

Our results indicate conditions where the buyer is not motivated to "squeeze" all the profits out of the supply chain, even in a setting of complete information, and even if the relationship is short-term. We extend the analysis to examine supply-chains which include more than one seller or buyer. Also, as part of the analysis, we study a problem, which we call the "Crime and Punishment" problem that is interesting in its own right.

Key words: quality, audit, supply-chain, newsvendor.

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1. Introduction

Labor arbitrage is often perceived as the principal driving force behind the move to execute processes outside the boundaries of the firm. This is even more pronounced, in cases of offshore outsourcing, where large labor cost differentials are often present. However, a one-dimensional focus on cost ignores many other relevant factors to the outsourcing issue such as quality, service level, delivery schedule, etc. In this paper, we study the the relationship of a firm with other players outside its boundaries in a supply chain from the perspective of quality.

We are interested in examining how factors such as product quality and cost are affected by the contractual features of the supply chain, in particular in the context of an offshore versus onshore decision. We do not assume that an offshore supplier is inherently a lower-quality producer relative to an onshore one. Rather, we examine how the contracting environment itself, in combination with the distance and time delays associated with an offshore supply relation, impacts the performance of the supply chain. The model we present, in the spirit of Plambeck and Taylor (2004) and Aron and Liu (2003), is focused not on the supply contract per se but rather on how the parties respond to deviations from contractual commitments. Thus, our results shed light on the relation between supply chain quality and the effectiveness of the legal system(s) in mediating business disputes among various members of the supply chain.[1]

We study a supply chain in which quality levels are determined by the supplier but are expensive to enforce: The buyer can observe the fraction of defective units supplied by the seller, but is unable to seek remedies unless he is ready to engage in a costly activity, which we call an "audit". Thus, the quality levels selected by the supplier will depend on the likelihood of such an audit and the resulting penalties. One of the interesting features of the analysis is that we specify conditions under which the buyer is not motivated to "squeeze" all the profits out of the supply chain but, rather, is able to pay a higher price in order to induce the seller to provide higher quality. Another interesting result is that the market may split into high-quality and low-quality regions. The incentives of the participants in the low-quality region is particularly surprising. Further, in comparing the attractiveness of an offshore versus onshore option, we show that the buyer must take into account not only the lower purchase cost due to the lower transfer price, but also the corresponding higher quality cost. We observe that the differential in transfer price required to achieve higher profits offshore can be rather high. Our analysis repeatedly uses a sub-problem, which we call the "Crime and Punishment Problem" (CP), that is interesting in its own right and might have applications in many areas including studies of corruption and crime. Finally, we are able to draw interesting conclusions in the setting of multiple buyers or sellers.

The remainder of the paper is structured as follows. We examine the relevant literature in §2. In §3, we introduce the basic model, explaining the structure of the problem and the nature of information asymmetry that we consider. §4 considers properties of the (CP) problem used throughout the paper. In §5, we study single-buyer, single seller games and analyze their implications for outsourcing contracts. We study multi-player games in §6. Finally, we conclude with the insights our analysis has to offer from a managerial perspective.

[1] We are thankful to Erica Plambeck for this observation.

2. Literature Survey

There exists a vast literature on outsourcing and contracts. The relevant literature on the boundaries of the firm can be traced to Coase (1937) who introduced the idea of the cost of transactions. He held that when the cost of exchange in markets is high, it might be cheaper to interact within the firm than through the market. Williamson (1979), in his acclaimed work on TCE (Transaction cost economics) operationalized this concept by identifying the most economical governance structure for several types of idiosyncratic transactions. In particular, he was concerned about situations which give rise to the "hold-up" problem. Grossman and Hart (1986) study firm boundaries in the context of contracts, viewing the costs and benefits of integration in terms of the incentive effects of top management. Hart and Moore (1990) enriched the Grossman and Hart (1986) framework, examining how changes in ownership affect the incentives of employees as well as those of owner managers. Holmstrom and Roberts (1998) hold that investment incentives are not provided by ownership alone, but can be affected by a variety of other factors.

Within the field of Operations Management, the extant literature on contract theory in the context of supply chains is exhaustive. Cachon (2001) is an excellent survey paper on the topic. Below, we briefly discuss the literature that is most relevant to our work.

Anupindi and Bassok (1999) consider a model with two retailers and one manufacturer. They compare two systems: one in which the retailers holds stocks separately and the other in which they cooperate to centralize stocks at a single location. Baiman *et.* al (2000) consider a double moral hazard problem in which the supplier and buyer each take unobserved actions. Lariviere and Porteus (2001) study wholesale price contracts within the context of the Newsvendor problem. Seshadri and Zemel (2003) study supply chains with information asymmetry in which information about sellers is obtained, after the fact, by auditing. Aron and Liu (2003) (see §3 below) study the connection between the structure of the audit cost and the optimal policies of a single buyer and single seller. Kaya and Ozer (2004) study contract manufacturing with unverifiable product quality and hidden information on quality costs. They obtain optimal two-part linear contracts depending on whether the quality is verifiable and the contract manufacturer's quality cost is public. Plambeck and Taylor (2004) study situations in which parties do not always satisfy agreed upon contracts. They compare two different court remedies for breach of contract, namely, "Specific Performance" and "Expectation Damages". Concurrent with our work, Atkins *et.* al (2005) study relational contracts between a retailer and a manufacturer within the framework of a repeated Stackelberg game. They propose that a unilateral deviation by the retailer would result in a breakdown of the relationship for a certain number of periods.

The issue of contract enforcement has also been widely studied by the legal fraternity (Epstein (1989) and Schwartz (1990)). Schwartz (1990) argues that court rulings share the goal of limiting a promisee's recovery to his lost expectation.

3. The Model

Aron and Liu (2003) study a simple supply chain consisting of a single buyer and a single seller. The setting is of complete information. The seller's production cost is linear in the number of good units; defective units are produced free. Demand is known to both the buyer and the seller. The buyer pays the seller a per unit price (on good and defective units alike) which equals the cost of producing a good unit (thus, the only way for the seller to make a profit is to produce some defective units). The buyer's revenue is 0 for a defective unit. The buyer can observe the fraction of defective units supplied by the seller, but is unable to seek remedies unless he is ready to engage in a costly activity, which we call an "audit". If the audit is conducted, the seller is forced to reimburse the buyer for the purchase cost of the defective units.

(AL)'s results are driven by process complexity (and thus audit cost as a function of defect rate) and its effect on the incentives of the supply-chain participants. Our focus, instead, is to examine (i) the effect of information asymmetry with regard to audit cost, and (ii) the inherent nature of the contractual agreements on quality levels. The central question of the paper is whether there exists a cut-off differential in transfer price when offshoring becomes an attractive option and possible drivers for such a result. We also examine the case of several buyers sourcing from a single seller and are able to study the interaction between quality levels and the relative size of the participants. This may be particularly relevant in the retail sector where large buyers can affect the quality of competitors. Conversely, we also study the impact of quality in the case of multiple sellers and a single buyer.

The starting point of our analysis is (AL)'s model with a few small modifications. Specifically, let the demand facing the buyer be D, and the proportion of defective units produced by the seller be p. We assume that the production cost of the buyer is $C(p,D) = (c_0 - \gamma p)D$, with $0 \le \gamma \le c_0$. We refer to c_0 as the cost of production and to γ as the cost of quality. Similarly, the value of units to the buyer is given by $R(p,D) = (v - \omega p)D$, with $\gamma \le \omega \le v$. We refer to v as the value of a perfect unit, and to ω as the value-loss of a defective unit. Finally, denote the per unit price charged by the seller as T. Note that in the case of (AL), $T = c_0$, $\omega = v$ and $\gamma = c_0$. (A list of the symbols used in the paper is provided in Appendix 2.)

We treat the transfer price, T, as a given parameter of the problem, rather than a variable to be determined by the participants of the supply chain. This could be the case, for instance, if T is determined by market forces, outside the influence of the buyer or seller. More generally, this allows us to avoid taking a position as to the relative bargaining power of the two actors and on the question of who sets the price. However, the analysis clearly reveals the preferences of the participants with respect to T, and the "optimum" T from the perspective of each actor. One of the surprising features of our model is that the buyer does not always prefer a lower T, and it is not in his interest to squeeze all the profits out of the supply chain. Rather, in some cases, the buyer is better off paying a premium in order to (indirectly) induce a higher level of quality. Similarly, the seller may sometimes prefer a lower transfer price. These effects hold even in case of complete information.

As stated earlier, we assume that in order to enforce the contract, the buyer must perform a costly activity that we call "audit". We interpret this term in a very general way to correspond to any burdensome activity by the buyer whose purpose is to enforce the contract with respect to quality. This includes activities such as contracting a third party to document a breach of contract or to mediate a dispute, notifying the seller about a problem, returning defective merchandize to the seller, initiating or escalating a conflict or an hostile reaction, initiating a legal action, or even curtailing or terminating a business relationship. The critical feature here is that any of these activities is costly.

In contrast with (AL), we do not assume that the costs of audit vary with p. Rather, in our context, it is likely that they vary with the volume, D. We denote the cost of an audit by $C(D)$, with $c(D) = \frac{C(D)}{D}$. As will be seen shortly, the performance of the supply chain depends critically on the magnitude of $c(D)$. We make no assumptions on the form of $c(D)$, but one may expect that in practice this function is decreasing. In most cases, we suppress the argument D and refer to the (per unit) cost of audit as c.

Once an audit is conducted by the buyer, he can "enforce" the contract. We study three types of enforcement agreements: "Full Rebate", "Repair" and "Compensate". The Repair and Compensate contracts are equivalent to the Specific Performance and Expectation Damages contracts studied by Plambeck and Taylor (2004).

1. Full Rebate: The buyer is refunded by the seller for the purchase price of the defective units, TpD. This is equivalent to the buyer paying only for the good units. Note that in this case, $T \geq \gamma$.

2. Repair: The defective units are repaired by the seller at a cost of δpD, with $\delta \geq \gamma$.

3. Compensate: The seller compensates the buyer for the deficit in profit, ωpD, due to defective units.

The physical proximity of the buyer and seller plays an important role when deciding the basic structure of the contract. When the seller is geographically distant from the buyer (offshore), the transportation cost and time delays may preclude the Repair type remedy. In such cases, contracts involving reimbursement, such as 1 or 3 may be more practical. However, the Compensate contract is often impractical to implement since it requires verification of the value-loss parameter, ω. This difficulty is magnified in an offshore environment. Thus, we can expect that the practical remedy in an offshore setting will be of the Full Rebate type. In contrast, in an onshore supply chain, especially when suppliers are located in close proximity to the buyer, the Repair as well as the Full Rebate regime may be used. We include the Compensate contract for the sake of completeness. As it turns out, its behavior is similar to the case of Repair.

Finally, we include some elements of information asymmetry in our model. In particular, we examine the following three sources of randomness:

1. Asymmetric information with respect to audit cost: The buyer knows the audit cost, but the seller does not.

2. Uncertainty about quality yield: There is noise (randomness) in the quality produced by the seller, i.e., the actual quality produced may differ from the quality target selected by the seller.

3. Observation uncertainty: The buyer may be unable to ascertain precisely the percentage of defectives prior to, or even after the audit (due to say, sampling errors).

We assume that the agents are risk-neutral and thus, the third source of uncertainty is trivial in our model. However, the other two require a probabilistic analysis. In order to address the first issue issue, we introduce the following sub-problem which we use throughout the paper. The problem is interesting in itself and has real-life applications, which we discuss.

4. The Crime and Punishment Problem

Consider a risk-neutral decision maker (call him Adam) who wishes to engage in a forbidden activity, which we denote p. Adam prefers p to be as large as possible. However, if he gets too greedy, a “punishment” is invoked. Thus, Adam’s objective is to select p just below the level X which would trigger the punishment. Unfortunately for Adam, he does not know precisely the value of X. Let $F(.)$ and $f(.)$ be the cdf and pdf of Adam’s prior with respect to X. Let $[a,b]$ be the support of X. Assume that $f(x) > 0$ for $x \in [a,b]$.

We model Adam's pay-off using the linear functions:

$$\Pi(p) = \begin{cases} A + \alpha p & p \leq X; \\ B - \beta p & p > X, \end{cases}$$

with $\alpha > 0$, $\beta \geq 0$, $A \geq B$.

Let $\pi(p)$ be the expected payoff:

$$\pi(p) = E[\Pi(p)] = A + \alpha p - (A - B)F(p) - (\alpha + \beta)pF(p). \quad (1)$$

We denote the problem of maximizing $\pi(p)$, $p \in [a, b]$ as the Crime and Punishment problem (CP).

It is instructive to compare the structure of the profit function of the (CP) problem with the classical Newsvendor problem. Fig. 4.1. shows how the seller's profit depends on his ordering quantity in the Newsvendor case. When the ordering quantity is less that the realized demand (which is not known a priori), profit increases linearly with ordering quantity. When a certain cut-off is reached (when the realized demand matches the ordering quantity), the function slopes downward. However, the function is continuous throughout its domain. Fig. 4.2. shows Adam's profit as a function of p in the (CP) problem. Notice that his profit increases linearly with p up to X, and is discontinuous at X. Thus, Adam incurs two penalties when he crosses the punishment trigger-point, X:

1. A fixed penalty, independent of quality choice, $A - B$.
2. A variable penalty, which increases with p.

(Insert Fig. 4.1 and Fig. 4.2)

The (CP) problem occurs commonly in numerous instances involving "forbidden" activities. Examples include low quality being passed by a seller to an unsuspecting buyer; a workman doing sloppy work / being late; a company / individual using aggressive tactics on tax-returns and a street hawker inflating the price of merchandize to a tourist. Another interesting example is the "Entrepreneur-Bureaucrat Problem", in which an entrepreneur has to seek clearance from a bureaucrat for a project. The entrepreneur has private information of the "value" (profit) of the project. The bureaucrat's problem is to decide how much bribe to demand from the entrepreneur. Lambert-Mogiliansky *et. al* (2004) begin their analysis of petty corruption within the framework of this model.

Let

$$\Delta = \frac{A - B}{\alpha + \beta},$$

$$\eta = \min_{[a,b]} (\Delta + p) f(p)$$

and

$$\bar{p} = \begin{cases} F^{-1}(\frac{\alpha}{\alpha+\beta} - \eta) & \eta \leq \frac{\alpha}{\alpha+\beta}; \\ a & \eta > \frac{\alpha}{\alpha+\beta}. \end{cases}$$

Proposition 1. *Let*
P^* be the set of optimal solutions to (CP):

$$\max_{a \leq p \leq b} \pi(p); \tag{2}$$

and P^0 be the set of solutions to the equation

$$F(p) = \frac{\alpha}{\alpha+\beta} - (\Delta + p) f(p). \tag{3}$$

Then,
(a) $P^* \subseteq \{a\} \cup P^0 \subseteq [a, \bar{p}]$,
(b) if $\eta > \frac{\alpha}{\alpha+\beta}$, $P^* = \{a\}$.

Proof. Obviously, any optimal solution to (CP) is either an extreme point of $[a, b]$ or satisfies the first order condition (3). For the second inclusion, note that $\pi'(p) < 0 \; \forall \; p > \bar{p}$. □

Remark 1. Let p_{nv} be the unique solution to

$$F(p) = \frac{\alpha}{\alpha+\beta}$$

in $[a, b]$. Thus, $\bar{p} \leq p_{nv}$.

Lariviere and Porteus (2001) (LP) obtained the equation (3), in the case of $\Delta = 0$, as the solution to the "Selling to the Newsvendor" problem (SN). Note that the problems studied in (CP) and here are not the same: while (SN) involves two decision makers and the source of variability is market demand, which is external to both, (CP) involves one decision maker, and the source of uncertainty is in the trigger level X. Also, the basic structure of (SN) is naturally continuous, while the main building block of (CP) is inherently discontinuous. Nevertheless, the optimization problems are identical.

(LP) provide sufficient conditions that guarantee a unique internal solution, and study its properties under this condition. We will re-examine these conditions later in this section. However, in propositions 2-3 below, we study some monotonicity properties which do not depend on the uniqueness of the solution.

PROPOSITION 2. *(Monotonicity of P^* with penalty and β shifts)*
Consider a (CP) problem P_1, defined by $[a_1,b_1], F_1, \alpha_1, \beta_1$ and $A_1 - B_1$. Let P_2 be derived from P_1 by

$$\beta_2 \geq \beta_1,$$

$$A_2 - B_2 \geq A_1 - B_1,$$

with at least one of the inequalities being strict. Let all other parameters of P_2 be the same as that of P_1. Let p_1^* and p_2^* be any pair of optimal solutions for P_1 and P_2 respectively. Then,
(a) $p_2^* \leq p_1^*$,
(b) Either $p_2^* < p_1^*$ or $p_2^* = p_1^* = a$.

PROOF. Let $\pi'(p)$ be the derivative of the profit function associated with the generic (CP) problem P:

$$\pi'(p) = \alpha - [(A-B) + p(\alpha+\beta)]f(p) - (\alpha+\beta)F(p).$$

Clearly, the function is monotone with $A - B$ and with β, the other parameters being fixed. The result follows from Lemma 1 (see Appendix A). □

In order to examine the effect of shifts in the support of F, we examine the relation between linear transformations and the size of the fixed penalty, $A - B$.

PROPOSITION 3. *(Equivalence of Linear Transformation and penalty shifts (see also (LP)))*
Consider a (CP) problem P_0, defined by $[a_0,b_0], F_0, \alpha_0, \beta_0$ and $A_0 - B_0$.
Let P_1 be derived from P_0 by the linear transformation:
$[a_1,b_1] = [\rho + \lambda a_0, \rho + \lambda b_0]$,
$F_1(\rho + \lambda p) = F_0(p)$,
with all other parameters of P_1 being the same as that of P_0.
Let P_2 be derived from P_0 by the penalty shift:
$A_2 - B_2 = \frac{A_0 - B_0 + (\alpha+\beta)\rho}{\lambda}$,
with all other parameters of P_2 being the same as that of P_0.
Let $P_1^* = \{p_{1i}^*, i = 1,2,...,k\}$ and $P_2^* = \{p_{2i}^*, i = 1,2,...,r\}$ (Wlog, assume that $p_{11}^* \leq p_{12}^* \leq ... \leq p_{1k}^*$ and $p_{21}^* \leq p_{22}^* \leq ... \leq p_{2r}^*$). Then, $k = r$, and $p_{1i}^* = \rho + \lambda p_{2i}^*, i = 1,2,...,k$.

PROOF. Let $\pi_i(p)$ be the objective function for the (CP) problem P_i, $i = 1,2$. Then,

$$\pi_1(p) = A_1 + \alpha_1(\rho+\lambda p) - [(A_1 - B_1) + (\alpha_1+\beta_1)(\rho+\lambda p)]F_1(\rho+\lambda p) \quad (4)$$
$$= (A_1 + \alpha_1\rho) + \lambda[\alpha_2 p - (A_2 - B_2)F_2(p) - (\alpha_2+\beta_2)pF_2(p)] \quad (5)$$

$$= (A_1 - \lambda A_2) + \alpha_1 \rho + \lambda \pi_2(p) \tag{6}$$

where (5) follows from the definition of the parameters and (6) from the expression for $\pi_2(p)$. The result follows from (6). □

Another way of expressing the equivalence of p_1^* and p_2^* is in terms of the penalty probabilities:

$$F(p_{1i}^*) = F(p_{2i}^*), i = 1, 2, 3, ..., k.$$

Combining propositions 2 and 3, we note that a linear transformation of $[a, b]$ would result in a reduction of the probability of punishment iff

$$\rho > \frac{(\lambda - 1)(A - B)}{\alpha + \beta}.$$

The inequality holds, for instance, if $\lambda \leq 1$, $\rho \geq 0$ (with at least one inequality strict). This is satisfied if the intervals $[a_1, b_1]$ and $[a_2, b_2]$ are such that $b_2 \leq a_2 + (b_1 - a_1)$ and $a_2 \geq a_1$.

Thus, we can summarize the findings of propositions 2 and 3 by noting that any one of the following changes will decrease the probability of punishment:

1. An increase in the "fixed" penalty $A - B$,
2. An increase in the "variable" penalty β,
3. A reduction in the relative variability $\frac{b-a}{a}$.

The last item is satisfied, for instance, in case of:

a. An increase in the cut-off value, $[a_2, b_2] = x + [a_1, b_1]$, $x > 0$,
b. A reduction in variability, $[a_2, b_2] \subseteq [a_1, b_1]$,

which corresponds to the dominance in the sense of first and second order respectively.

We now briefly discuss conditions for the optimal solution to be unique. Obviously, a sufficient condition for the uniqueness of the optimal solution is that the profit function $\pi(p)$ be concave, i.e., that the derivative function, $\pi'(p)$, be monotone decreasing over $[a, b]$. This translates into the requirement that

$$F(p) + (\Delta + p) f(p)$$

be monotone increasing over $[a, b]$. If $f(p)$ itself can be differentiated, this amounts to the requirement that

$$f'(p) > -\frac{2f(p)}{p + \Delta}.$$

(LP) observed that it is enough to require that $\pi'(p)$ be monotone decreasing wherever it is non-negative. Thus, if $\hat{p}$ is an upper bound on p^*, it is sufficient to require monotonicity over $[a, \hat{p}]$. For the case of $\Delta = 0$, they provide a sufficient condition for this, which they denote the Increasing Generalized Failure Rate (IGFR), namely that

$$\frac{pf(p)}{1 - F(P)}$$

be monotone increasing over $[a, \hat{p}]$. For the case of $\Delta > 0$, the condition easily generalizes into that of requiring monotonicity of:

$$\frac{(p + \Delta)f(p)}{1 - F(p)}.$$

If the monotonicity condition holds, the sufficient condition for the optimal solution to be internal to $[a, b]$ is:

$$\frac{\alpha}{\alpha + \beta} - (\Delta + a)f(a) > 0.$$

We finally study the (CP) problem for the case of the uniform distribution and with $\Delta = 0$. Under these conditions, the optimal solution, p_u^*, is unique and its value can be obtained explicitly. Specifically, let p_{unv} be the solution to the Newsvendor problem with uniform F. Let $r = \frac{\alpha}{\alpha+\beta}$. Thus,

$$p_{unv} = a(1 - r) + br.$$

Proposition 4. $p_u^* = max\{\frac{p_{unv}}{2}, a\}$ *(see (LP)).*

In order to gain some intuition behind the results, we study how p^* varies with the mean, μ, and the spread, θ, of Adam's prior distribution of the trigger level when the other is held constant. With these parameters, p_u^* may be restated as:

$$p_u^* = \begin{cases} \frac{1}{2}\mu + (\frac{r}{2} - \frac{1}{4})\theta & \mu \leq (r + \frac{1}{2})\theta; \\ \mu - \frac{\theta}{2} & \mu \geq (r + \frac{1}{2})\theta. \end{cases}$$

Fig.4.3 shows the dependence of p^* on the mean, μ, when the spread, θ, is held constant. Fig.4.4 shows how p^* varies as a function of the spread, θ, of the distribution when the mean, μ, is held constant. Note that when $r > \frac{1}{2}$, p_u^* increases with θ, for $\theta \geq \frac{\mu}{r+\frac{1}{2}}$. The figure shows the case when $r < \frac{1}{2}$.

(Insert Fig. 4.3 and Fig. 4.4)

Remark 2. A related problem to the (CP) problem can arise when the source of uncertainty is not in the cut-off point x, but there is "noise" in the production process. Consider a seller who chooses a defect rate p, $p \in [a, b]$, where $0 < a < b < 1$ to execute his process. However, the realized process has a defect rate, q, given by: $q = p + \epsilon$. The support of ϵ is such that $q \in [0, 1]$ for every p. Let the cdf and pdf of ϵ be $F(.)$ and $f(.)$ respectively. Since the buyer receives components at a defect rate, q, his auditing decision will depend on whether q is greater or less than the cut-off, x.

This problem can be studied by a generalization of the (CP) problem, called the General Crime and Punsihment Problem (GCP), which differs from the (CP) problem only in the objective function:

$$\Pi(p) = \begin{cases} A + \alpha p - \alpha_1 X & p \leq X; \\ B - \beta p + \beta_1 X & p > X, \end{cases}$$

with $\alpha > 0,\ \beta \geq 0,\ \alpha_1 \geq 0,\ \beta_1 \geq 0,\ A \geq B$. We examine the properties of the (GCP) problem in Sheopuri and Zemel (2005). The reader may note that the Newsvendor, (CP) and "noise" in production problem are special cases of the (GCP) problem.

Though we present the results for the case where the pay-off is linear with the defect rate, the analysis have been extended to non-linear functions as well. However, we feel, the results offer little additional insight and are excluded from the paper.

We now return to the problem outlined in §3. The analysis in §5 and §6 will use the sub-problem that we studied in §4.

5. The Single Buyer, Single Seller Problem

Before we proceed with the analysis of the outsourcing problem outlined in §3, we briefly restate the basic elements of the model. Recall that the seller selects the defect rate p, whereas the buyer decides whether to audit or not. The transfer price, T, is exogenous. The three enforcement agreements that we study are: Full Rebate, Repair and Compensate. The solution approach that we follow is typical to any Stackelberg game. We solve the following two sub-problems in reverse order to answer the questions stated,

1. The Seller's Quality Problem: What quality level should the seller choose?
2. The Buyer's Enforcement problem: Should the buyer audit or not?

In addition, we examine the participation conditions of the buyer and seller.

5.1. The Buyer's Enforcement Problem

The buyer observes the defect rate, p, the transfer price, T and decides whether to audit or not. The buyer's pay-off in the Full Rebate, Repair and Compensate regimes is shown in Table 5.1.

Table 5.1: Pay-offs for the buyer.

	No audit	Audit
Full Rebate	$D[v - \omega p - T]$	$D[v - \omega p - T(1-p) - c(D)]$
Repair	$D[v - \omega p - T]$	$D[v - T - c(D)]$
Compensate	$D[v - \omega p - T]$	$D[v - T - c(D)]$

The buyer audits only if his pay-off from auditing is greater than his pay-off from not auditing. Therefore, in the case of Full Rebate contracts, the buyer audits if $p > \frac{c(D)}{T}$, whereas, for Repair and Compensate contracts, he audits if $p > \frac{c(D)}{\omega}$.

5.2. The Seller's Quality Problem

The seller's problem is to select the level of quality, p, that maximizes his profits, given the transfer price, T, and taking into account the possibility of audit by the buyer. Then, the seller's objective is given by:

Table 5.2: Pay-offs for the seller.

	No audit	Audit
Full Rebate	$D[T-(c_0-\gamma p)]$	$D[T-(c_0-\gamma p)-Tp]$
Repair	$D[T-(c_0-\gamma p)]$	$D[T-(c_0-\gamma p)-\delta p]$
Compensate	$D[T-(c_0-\gamma p)]$	$D[T-(c_0-\gamma p)-\omega p]$

Let the seller's prior distribution of the buyer's audit cost, c, have cdf $F(.)$ and let its support be $[\underline{c},\overline{c}]$. Thus, the seller's problem is equivalent to the (CP) problem with the following parameters:

Table 5.3: Parameters of the Full Rebate, Repair and Compensate contracts.

Contract	$[a,b]$	A_0/B_0	α	β
Full Rebate	$[\frac{\underline{c}}{T},\frac{\overline{c}}{T}]$	$T-c_0$	γ	$T-\gamma$
Repair	$[\frac{\underline{c}}{\omega},\frac{\overline{c}}{\omega}]$	$T-c_0$	γ	$\delta-\gamma$
Compensate	$[\frac{\underline{c}}{\omega},\frac{\overline{c}}{\omega}]$	$T-c_0$	γ	$\omega-\gamma$

Our model is mute on the relevant magnitudes of T, ω and δ. However, one suspects that in practice $T\leq\omega$ and $T\leq\delta$. In this case, propositions 2 and 3 imply that p^* and the probability of audit in the case of the Repair and Compensate regimes will be lower than Full rebate. In particular, if p^* for the later regime is the extreme solution $\frac{c}{T}$, the solution for the former regimes is the extreme point $\frac{c}{\omega}$.

We know from the analysis of the (CP) problem that if the extreme point solution is optimal for a given support $[a,b]$, it will be optimal for any interval with lower relative variability. Define the "low uncertainty" regime as where the extreme point solution is optimal. Similarly, define the "high uncertainty" regime as where the internal solution is optimal. The high uncertainty regime may be solved explicitly only for specific distributions (for example, the uniform distribution) and not in general; whereas the low uncertainty regime is equivalent to the full certainty case, where the seller knows the buyer's audit cost c, and may be solved explicitly. We examine the low uncertainty case first:

Proposition 5. *In the case of the low uncertainty regime:*

Table 5.4

Contract	*Full Rebate*	*Compensate/Repair*
Quality (p)	$\frac{\underline{c}}{T}$	$\frac{\underline{c}}{\omega}$
Audit probability	0	0
Buyer's profit	$[v-\omega\frac{\underline{c}}{T}-T]D$	$[v-\underline{c}-T]D$
Seller's profit	$[T-c_0+\gamma\frac{\underline{c}}{T}]D$	$[T-c_0+\gamma\frac{\underline{c}}{\omega}]D$
Channel's profit	$[v-c_0-(\omega-\gamma)\frac{\underline{c}}{T}]D$	$[v-c_0-(\omega-\gamma)\frac{\underline{c}}{\omega}]D$
Buyer's participation constraint	$T+\omega\frac{\underline{c}}{T}\le v$	$T\le v-\underline{c}$
Seller's participation constraint	$T+\gamma\frac{\underline{c}}{T}\ge c_0$	$T\ge c_0-\gamma\frac{\underline{c}}{\omega}$

We first note that the fraction of defects, p^*, for the Full Rebate regime is given by $\frac{\underline{c}}{T}$ and in the other two cases by $\frac{\underline{c}}{\omega}$. This means that, in each case, p^* increases with the audit cost, $\underline{c}$. Consequently, the channel's profits, as well as the buyer's, decrease with the audit cost while the seller's profits move in the opposite direction. Recall that an offshore supply chain will tend to have a higher value for $\underline{c}$. Also, if $\omega > T$, then the quality is lower (p^* is higher) in the Full Rebate regime as opposed to Compensate or Repair.

Next, the buyer's profit increase with the per unit value v, while the seller's decrease with the per unit production cost, c_0. Similarly, the channel's overall profit increase with the margin on a perfect unit, $v-c_0$. Note that if the channel had a centralized authority capable of enforcing the (global) optimal level $p=0$, or equivalently if $\underline{c}=0$, the condition $v-c_0\ge 0$ would be sufficient to enable the chain to operate profitably. However, lacking such authority (or equivalently with $\underline{c}>0$), we note that a necessary condition for the channel to operate is that $v-c_0-(\omega-\gamma)\frac{\underline{c}}{T}\ge 0$ or $v-c_0-(\omega-\gamma)\frac{\underline{c}}{\omega}\ge 0$. This reflects the quality cost of decentralization.

The behavior of the Full Rebate and the Compensate/Repair contracts as a function of the transfer price T are strikingly different. In the latter case, the buyer's profit decreases (linearly) and the seller's increases with T, as expected. The channel's profits are constant, as long as T satisfies the participation constraints of the two parties. This behavior is depicted in Figures 5.1-5.3. The optimal value for T from the perspective of the buyer is $c_0-\gamma\frac{\underline{c}}{\omega}$. This value of price is designed to drain all the profits out of the seller, paying him just enough to (barely) participate. Many suppliers claim that large buyers use exactly this policy in negotiating the transfer price T. Of course, if the seller had the power to set T, his behavior in our model would be symmetric.

Table 5.5: Compensate/Repair contracts at the buyer's optimal transfer price.

Contract	Compensate/Repair
T^*	$c_0-\gamma\frac{\underline{c}}{\omega}$
Quality (p)	$\frac{\underline{c}}{\omega}$
Audit probability	0
Buyer's profit	$[v-c_0-(\omega-\gamma)\frac{\underline{c}}{\omega}]D$
Seller's profit	0
Channel's profit	$[v-c_0-(\omega-\gamma)\frac{\underline{c}}{\omega}]D$

In the case of Full Rebate, the situation is more interesting. First, a necessary condition for the buyer to be profitable for at least *some* values of T is that the value of a unit be above a certain cutoff value $v \geq 2\sqrt{\omega \underline{c}}$. Below this cutoff, the buyer cannot make a profit at any T. We assume below that this condition holds. Next, we consider the seller. There are two cases:

a) $c_0 \geq 2\sqrt{\gamma \underline{c}}$

In this case, the values of T that allow the chain to operate split into two regions which we denote the "quality zone" and "defective zone" respectively (one or both of these zones may be empty). In the quality zone, the transfer price is high and the fraction of defects is low. In the defective zone, the reverse situation applies (an example might be a road-side vendor selling counterfeit watches). Surprisingly, in this zone, the buyer prefers T to be as large as possible. Conversely, the seller prefers T to be low! This is depicted in Fig. 5.4. Note that, in both zones, the channel's profits increase with T.

b) $c_0 < 2\sqrt{\gamma \underline{c}}$

In this case, the seller is profitable at *every* T and the above mentioned two zones merge. Thus, trade is possible in the range where the buyer's profit function is positive (in general). This case is depicted in Figure 5.5. For low values of T, the situation is similar to the defective zone. The buyer prefers high T while the seller prefers low T. Indeed, in the example provided (Fig 5.5), the best T for the seller is the lowest T possible! Note that the buyer's profit is concave with an optimal point at $T^* = \sqrt{\underline{c}\omega}$. The values for the various parameters in this case are given in Table 5.6 below.

Table 5.6: Full Rebate contract at the buyer's optimal transfer price.

Contract	Full Rebate
T^*	$\sqrt{\underline{c}\omega}$
Quality (p)	$\sqrt{\frac{\underline{c}}{\omega}}$
Audit probability	0
Buyer's profit	$[v - 2\sqrt{\underline{c}\omega}]D$
Seller's profit	$[\sqrt{\underline{c}\omega} + \gamma\sqrt{\frac{\underline{c}}{\omega}} - c_0]D$
Channel's profit	$[v - c_0 - \sqrt{\underline{c}\omega} + \gamma\sqrt{\frac{\underline{c}}{\omega}}]D$

(Insert Fig 5.4 and Fig 5.5)

It is interesting to note that while the seller is squeezed out of profits at the buyers optimal transfer price in the Repair and Compensate regime, he makes, in general, positive profits in the case of Full-Rebate contracts. Further, the optimal transfer price for the buyer in the case of Full Rebate contracts (case (b) applies) is independent of the seller's cost of production (see Table 5.6). The result goes against the popular notion that the buyer stands to gain by helping the seller reduce production cost. However, the model shows that the seller gains from reduction in his cost of production. The situation is reverse in the case of Repair and Compensate contracts. Here, the buyer gains by helping the seller cut his cost of production. The seller, on the other hand, is indifferent to any increase or decrease in his cost of production.

We briefly consider the case of the high uncertainty regime in the case of uniformly distributed audit cost. Recall that $r = \frac{\gamma}{T}$, $\frac{\gamma}{\omega}$ and $\frac{\gamma}{\delta}$ respectively for Full-Rebate, Compensate and Repair contracts.

COROLLARY 1. *Let*

$$\underline{c} = 0,$$
$$c' = \frac{r}{2}\overline{c},$$
$$v' = v - \frac{r}{2}c.$$

In the case of the high uncertainty regime with uniformly distributed audit cost:
Table 5.6

Contract	*Full Rebate*	*Compensate*	*Repair*
Quality (p) *Audit probability*	$\frac{c'}{T}$ $\frac{r}{2}$	$\frac{c'}{\omega}$ $\frac{r}{2}$	$\frac{c'}{\omega}$ $\frac{r}{2}$
Buyer's profit *Seller's profit* *Channel's profit*	$[v' - \omega\frac{c'}{T} - T + \frac{r}{2}c']D$ $[T - c_0 - \frac{r}{2}c' + \gamma\frac{c'}{T}]D$ $[v' - c_0 - (\omega - \gamma)\frac{c'}{T}]D$	$[v' - T - (1 - \frac{r}{2})c']D$ $[T - c_0 - \frac{\delta}{\omega}\frac{r}{2}c' + \gamma\frac{c'}{\omega}]D$ $[v' - c_0 - (1 - \frac{r}{2} + \frac{r}{2}\frac{\delta}{\omega} - \frac{\gamma}{\omega})c']D$	$[v' - T - (1 - \frac{r}{2})c']D$ $[T - c_0 - \frac{r}{2}c' + \gamma\frac{c'}{\omega}]D$ $[v' - c_0 - (1 - \frac{\gamma}{\omega})c']D$
Buyer's participation constraint *Seller's participation constraint*	$T + \omega\frac{c'}{T} - \frac{r}{2}c' \leq v'$ $T - \frac{r}{2}c' + \gamma\frac{c'}{T} \geq c_0$	$T + (1 - \frac{r}{2})c' \leq v'$ $T - \frac{\delta}{\omega}\frac{r}{2}c' + \gamma\frac{c'}{\omega} \geq c_0$	$T + (1 - \frac{r}{2})c' \leq v'$ $T - \frac{r}{2}c' + \gamma\frac{c'}{\omega} \geq c_0$

Contrasting the results obtained in Corollary 1 with Proposition 5 provides for an interesting comparison. In both cases, the buyer's (seller's) equilibrium profit is a concave (convex) function of the transfer price, T, in the Full Rebate contracts, whereas it is a linear function of T in the case of Compensate and Repair contracts. Note, however, that the equilibrium quality increases faster with the transfer price, T, in the high uncertainty regime when compared to the low uncertainty regime in the case of the Full Rebate contracts. Further, while the audit probability is zero in the low uncertainty case, it is non-zero in the high uncertainty case for all three contracts.

Offshore or Onshore?

We now contrast the contracts studied in the context of a buyer's decision of whether to offshore or onshore. Fig 5.6 compares the buyer's profit for the Full Rebate and Repair cases, as a function of T (all parameters identical for the two cases). Except for the very high values of T ($T > \omega$), the buyer's profits are higher in the Repair case. As noted earlier, the Repair contract is often unavailable offshore.

It is commonly observed that the drive offshore is fuelled by a lower purchase price, T, available offshore. But is the lower transfer price enough to compensate for the differential in quality? Fig 5.7 demonstrates the magnitude of purchase price differential that is required to make the buyer indifferent between the onshore contract (at higher T) and the offshore contract. This differential can be considerable, especially in the case of low T. In the example depicted (Fig 5.7), for instance, an offshore contract at (approximately) USD 15 is as profitable as an offshore contract at (approximately) USD 25.

(Insert Fig 5.6 and Fig 5.7)

The onshore advantage is even more pronounced in the case that $\underline{c}$ is high in an offshore environment, as is very likely. For high levels of $\underline{c}$, the buyer profits increase as $\underline{c}$ decreases. However, if $\underline{c}$ gets too low, ($\underline{c} << \frac{c_0^2}{4\gamma}$), the quality and defective zones are driven apart and, consequently, the buyer's profits may fall. This phenomenon (the splitting of the quality and defective zones) is mitigated if c_0 is low. Again in an offshore environment, this is typically the case.

In summary, an offshore supply chain is typically characterized by a Full rebate contract, low c_0 and high $\underline{c}$. In contrast, the onshore supply chain can utilize the higher profits observed in the case of Repair contract and also benefits from low $\underline{c}$, while c_0 is higher. In comparing the attractiveness of two such options, the buyer must take into account not only the lower purchase cost due to the lower T, but also the corresponding high quality cost. As we have seen, the differential in transfer price required to achieve higher profits offshore can be rather high.

In all the contracts that we study, we observe that when the unit audit cost function, $c(D)$, is decreasing, there exists economies of scale in auditing. Thus, large buyers can enforce high quality in the supply chain.

6. Multi-player games

We now consider more complicated supply chains involving more than one buyer or one seller. We cover the low uncertainty case; thus the seller chooses the lower support of the distribution of the audit cut-off. For ease of presentation, we use c instead of $\underline{c}$ throughout this section. The insights we derive can be observed in context of simpler supply chains involving one seller and two buyers; and two sellers and one buyer respectively. Consequently, we analyze these cases in detail. However, we indicate in each case how the results would generalize to the one-seller, multi-buyer, and one buyer, multi-seller settings respectively.

We study two regimes, individual enforcement and joint enforcement: In the individual enforcement regime, a player is punished or rewarded contractually for his own actions only. In the joint enforcement regime, a player gains or looses directly based on the actions of other players. This is explained in detail depending on the context in §6.1 and §6.2.

We study in detail the case of Full Rebate contracts in this section. The results for the other two regimes follow directly and are thus excluded.

This section contains 2 parts: In §6.1, we study multi-buyer, single seller games; whereas in §6.2, we analyze multi-seller, single buyer games.

6.1. Multi-Buyer, Single Seller game

Consider two buyers who interact with a single seller. Buyer i orders exogenous quantity, $D_i, i = 1,2$ from the seller. Thus, $D = D_1 + D_2$. Let $c_i(D_i)$ be the per unit audit cost of buyer i. Without loss of generality, assume that $c_1 \leq c_2$. We refer to buyer 1 as the "demanding buyer" and to buyer 2 as the "lax buyer". The seller decides the quality of units to produce. However, he is constrained to produce units at the same quality level for all the buyers. Each buyer decides whether to audit the seller's units or not. In the individual enforcement regime, if buyer i audits and finds the seller's units to be defective, the contract is enforced on the units supplied only to him. In the joint enforcement regime, buyer i may invoke the enforcement clause with respect to the units supplied to all buyers.

Consider first the individual enforcement regime. The seller's profit as a function of his defect rate, p is piece-wise linear:

$$\Pi(p) = \begin{cases} (T - c_0 + \gamma p)D & p \leq \frac{c_1}{T}; \\ ((T - c_0) + p(\gamma - T\frac{D_1}{D}))D & \frac{c_1}{T} < p \leq \frac{c_2}{T}; \\ (T - c_0 - (T - \gamma)p)D & p > \frac{c_2}{T}. \end{cases}$$

When $p < \frac{c_1}{T}$ (region 1), neither buyer audits; when $\frac{c_1}{T} < p \leq \frac{c_2}{T}$ (region 2), buyer 1 audits but buyer 2 does not; while when $p > \frac{c_2}{T}$ (region 3), both buyers audit. Since the seller's profit function is piece-wise linear, an optimal solution for the seller is to choose either $p = \frac{c_1}{T}$ or $p = \frac{c_2}{T}$. An optimal solution is $p = \frac{c_1}{T}$ if

$$[T - c_0 + \gamma\frac{c_1}{T}]D \geq [(T - c_0) + (\gamma - T\frac{D_1}{D})\frac{c_2}{T}]D$$

or

$$\frac{D_1}{D} \geq \frac{\gamma}{T}(\frac{c_2 - c_1}{c_2}). \tag{7}$$

The buyers' participation constraints reduce to

$$V - \omega\frac{c_1}{T} - T \geq 0;$$

while the seller participates if

$$T + \gamma \frac{c_1}{T} - c_0 \geq 0.$$

We now compare the equilibrium quality and profits of the players to the case that each buyer interacts with the seller as in the "single buyer, single seller" setting. If D_1 is large (in the sense that (7) holds), for a given T, the seller's equilibrium defects, p^*, and profits are lower. While the demanding buyer's profits are unchanged, the lax buyer free-rides on the demanding buyer and makes higher profits. As a consequence of higher equilibrium quality, the channel's profits are higher. The results are reverse when D_1 is small.

We now generalize the result to the case of N buyers and 1 seller. Let $T_j = \sum_{i=1}^{j} D_i, j = 1, 2, .., N$. Let $c_i(D_i)$ be the per unit audit cost of buyer i (Wlog, assume that $c_1 < c_2 < ... < c_N$).

In the individual enforcement regime, each buyers' decision is independent of the others, and hence, buyer i audits if $p > \frac{c_i}{T}$. The seller's profit is given by:

$$\begin{aligned} \Pi_S &= \sum_{i:p \leq \frac{c_i}{T}} D_i[T - (c_0 - \gamma p)] + \sum_{i:p > \frac{c_i}{T}} D_i[T(1-p) - (c_0 - \gamma p)] \\ &= D[T - (c_0 - \gamma p)] - \sum_{i:p > \frac{c_i}{T}} D_i T p. \end{aligned}$$

The seller faces the problem:

$$\max_{p \in [0,1]} D[T - (c_0 - \gamma p)] - \sum_{i:p > \frac{c_i}{T}} D_i T p.$$

Proposition 6. *For the N-buyer, single seller game, under the individual enforcement regime,*
(a) $p^* \in \{\frac{c_1}{T}, \frac{c_2}{T}, ..., \frac{c_N}{T}\}$
(b) $\exists\, k \in \{1, 2, ..., N\}$ *s.t. in equilibrium, buyers* $1, 2, ..., k-1 (k > 1)$ *audit,* $k, k+1, ..., N$ *don't audit, seller chooses* $p = \frac{c_k}{T}$*, if*

$$T \geq \gamma D \frac{(c_j - c_k)}{c_j T_{j-1} - c_k T_{k-1}}, \forall j > k$$

and

$$T \leq \gamma D \frac{(c_j - c_k)}{c_j T_{j-1} - c_k T_{k-1}}, \forall j < k.$$

We now consider the joint enforcement regime with two buyers and a single seller. Recall that the buyer with the lower audit cost (c_1) is called the "demanding buyer"; the other being the "lax buyer". Note that when $p \leq \frac{c_1}{T}$ (region 1), there is no audit; when $\frac{c_1}{T} < p \leq \frac{c_2}{T}$ (region 2), the audit is caused by the "demanding buyer"; while in the case when $p > \frac{c_2}{T}$ (region 3), both buyers have an incentive to audit (However, each prefers the other to audit). Thus, the buyer will not select

p in region 2. In region 1, the best p is $\frac{c_1}{T}$: what about region 3? If the buyers can coordinate, at equilibrium one of them will audit. Thus, in this case, one of them will audit and region 3 is not desirable for the seller. If the buyers cannot coordinate, then the seller may be tempted to select p in region 3 in order to provoke an "audit war" between the buyers hoping that none would audit. Such a war can indeed be set up by the seller. However, we show that even though the probability of no audit in region 3 is non-zero, it is not high enough. Thus, the seller's optimal p is conservative, i.e., $\frac{c_1}{T}$ and no audit will result. Note that the "Audit war" is equivalent to the classical game of chicken.

Let x (y) be the probability that the demanding (lax) buyer audits, $1-x$ ($1-y$) be the probability that the demanding (lax) buyer does not audit.

Table 6.1: Pay-offs for the demanding buyer

	Audit (Demanding buyer)	No audit (Demanding buyer)
Audit (Lax buyer)	$D[(v-\omega p-T)+Tp-c_1]$	$D[(v-\omega p-T)+Tp]$
No audit (Lax buyer)	$D[(v-\omega p-T)+Tp-c_1]$	$D[v-\omega p-T]$

In equilibrium, the demanding buyer is indifferent between auditing and not auditing. Thus,

$$y = \begin{cases} 1-\frac{c_1}{Tp} & p > \frac{c_1}{T}; \\ 0 & p \leq \frac{c_1}{T}. \end{cases}$$

Similarly,

$$x = \begin{cases} 1-\frac{c_2}{Tp} & p > \frac{c_2}{T}; \\ 0 & p \leq \frac{c_2}{T}. \end{cases}$$

The seller's profit function is, thus, given by:

$$\Pi(p) = \begin{cases} ((T-c_0)+\gamma p)D & p \leq \frac{c_1}{T}; \\ ((T-c_0+c_1)-(T-\gamma)p)D & \frac{c_1}{T} < p \leq \frac{c_2}{T}; \\ ((T-c_0)-(T-\gamma)p+\frac{c_1 c_2}{Tp})D & p > \frac{c_2}{T}. \end{cases}$$

Note that the profit function is continuous throughout its domain. Further, since $T > \gamma$, the function is decreasing for $p > \frac{c_1}{T}$ and increasing for $p \leq \frac{c_1}{T}$. Thus, the seller selects $p = \frac{c_1}{T}$. The buyers' participation constraints reduce to

$$V - \omega\frac{c_1}{T} - T \geq 0;$$

while the seller participates if

$$T + \gamma\frac{c_1}{T} - c_0 \geq 0.$$

The generalized results for a finite number of buyers ($c_1 \leq c_2 \leq ... \leq c_N$) and a single seller are now given below.

Proposition 7. *For the N-buyer, single seller game under the joint enforcement regime,*
(a) $p^* \in \{\frac{c_1}{T}, \frac{c_2}{T}, ..., \frac{c_N}{T}\}$,
(b) $\exists\ r \in \{1, 2, ..., N\}$ *s.t.*
(1) buyers $r, r+1, ..., N$ *do not audit*
(2) (i) if buyers $1, 2, ..., r-1$ $(r > 1)$ *can coordinate, they select one buyer to audit,*
(ii) if buyers $1, 2, ..., r-1$ $(r > 1)$ *cannot coordinate, there exists a mixed Nash equilibrium in which buyer* i *audits with non-zero probability* q_i,
(3) seller chooses $p = \frac{c_r}{T}$,
(c) if the buyers are identical, i.e., $c_1 = c_2 = ... = c_N$, *then* $\exists$ *a pure strategy Nash equilibrium only.*

The distinction between the equilibrium in the joint enforcement and individual enforcement regimes provides for an interesting comparison. In both regimes, the high audit cost buyers don't audit. However, while in the joint enforcement regime, the low audit cost buyers audit have a randomized equilibrium, each trying to free-ride on the other low audit cost buyers, in the individual enforcement regime, they have a pure-strategy equilibrium.

6.2. Multi Seller, Single Buyer game

In this section, we examine a supply chain consisting of a finite number of sellers and a single buyer. The buyer orders quantity, D, from the sellers who are requested to deliver quantities, D_i $(i = 1, 2, ..., M)$, to the buyer. Let γ_i be the cost of quality of seller i. Assume that $\gamma_1 \leq \gamma_2 \leq ... \leq \gamma_M$. We study the individual enforcement and joint enforcement regimes which have similar interpretations as those outlined in §6.1.

In the individual enforcement regime, if the buyer audits seller i at cost $c_i = \frac{C(D_i)}{D_i}$, he may punish that seller for the units supplied to him only.

Proposition 8. *In the the equilibrium of the multi-seller, single buyer game under individual enforcement,*
(a) $p_i = \frac{c_i}{T}$,
(b) the buyer does not audit any seller.

The results of Proposition 8 indicate that under individual enforcement, each seller behaves as in the "single buyer, single seller" setting. Note, however, that if the buyer set his optimal transfer price, externality would be observed in the setting described here.

In the joint enforcement regime, if the buyer audits (the pooled units) at an audit cost $c = \frac{C(D)}{D}$, he may enforce the contract with respect to the units procured from all the sellers.

Proposition 9. *In the the equilibrium of the multi-seller, single buyer game under joint enforcement, if the buyers can coordinate among themselves,*

(a)

$$\frac{\sum_{i=1}^{M} p_i D_i}{D} = \frac{c}{T},$$

(b) the buyer does not audit any seller.

A special case of joint enforcement and coordination among sellers when all sellers provide the same quality level, $p_i = \frac{c}{T}$. Note that the proposition does not cover the case of non-cooperative sellers. In this case, the analysis becomes intractable.

7. Conclusions

We suggest outsourcing contracts within the framework of quality audits. The three contracts that we study are Full rebate, Repair and Compensate. We primarily analyze two kinds of randomness - uncertainty in audit cost and uncertainty in quality yield.

We analyze a sub-problem, called the (CP) problem, which is interesting in its own right and has wide applications, which we discuss. We also identify properties of the (CP) problem and contrast it with the classical Newsvendor problem. Our results throw up some interesting similarities with a different problem studied by Lariviere and Porteus (2001).

In the first step of analyzing offshore and onshore contracts, we study the single buyer, single seller game. The model permits us to capture the essential dynamics of outsourcing relationships. The analysis is then segmented into two cases: the low uncertainty and high uncertainty regime. In the low uncertainty regime, the seller is conservative. In the case of Full Rebate contracts, we find the problem structure (buyer's profit function) to be analogous to that of the "Economic Order Quantity" model. We thus obtain the surprising result that neither does the buyer always prefer a lower transfer price nor is it in his interest to squeeze all the profits out of the supply chain. Similarly, the seller may sometimes prefer a lower transfer price. These effects hold even in case of complete information.

We apply the equilibrium analysis to understand a firms strategic decision of whether to offshore or onshore. We note that an offshore supply chain is typically characterized by a Full rebate contract, low cost of production and high audit cost. In contrast, the onshore supply chain can utilize the higher profits observed in the case of Repair contract, and also benefits from low audit cost, while cost of production is higher. In comparing the attractiveness of an offshore versus onshore option, the buyer must take into account not only the lower purchase cost due to the lower transfer price, but also the corresponding high quality cost. We observe that the differential in transfer price required to achieve higher profits offshore can be rather high. Thus, our results are in line with conventional wisdom that a slight cost differential between onshore and offshore costs of product or service is not sufficient to drive firms to offshore. Note, however, that though this is usually attributed to the "hidden costs of outsourcing", we show that there is another significant cause - the inherent nature of the contracts that govern the relationship.

We also examine the high uncertainty case. Though we provide insights of the structure of the solution in this case, we are unable to provide closed form expressions, except in certain specific cases. We observe here that the seller faces a non-zero audit probability and is not conservative, i.e., it is in his interest to "cheat" more than he would in the low uncertainty regime. We conjecture that a similar model within a multi-period setting may throw up some interesting results with regard to lower or even zero audit probability.

Finally, we extend out analysis to include situations in which there is more than one buyer or one seller. We study two settings: joint enforcement and individual enforcement. In the multi-buyer, single seller joint enforcement case, we demonstrate that an "Audit War" - where multiple buyers have an incentive to audit but none audit, ultimately benefitting the seller - is not an equilibrium. We further show that the "high audit cost" buyers don't audit under both individual and joint enforcement. However, low audit cost buyers audit with probability one under individual enforcement, and probability less than one under joint enforcement. Our results in the multi-seller, single buyer regime indicate that under individual enforcement, each seller behaves as in the "single buyer, single seller" setting, as one may expect.

Appendix A:

LEMMA 1. *Let $f(x)$, $g(x)$ be such that $f'(x) > g'(x)\ \forall\ x \in [a,b]$. Let $X_1 = \{x_1 : x_1 = argmax_{[a,b]} f(x)\}$ and $X_2 = \{x_2 : x_2 = argmax_{[a,b]} g(x)\}$. Let $x_1 \in X_1$ and $x_2 \in X_2$. Then $x_2 \in [a, x_1]$ (see Pp 115, Porteus (2002)).*

PROOF. Define

$$\tilde{g}(x) = g(x) + f(x_1) - g(x_1). \tag{8}$$

Let $\tilde{X}_2 = \{x_2 : x_2 = argmax_{[a,b]} \tilde{g}(x)\}$. Note that $y \in X_2 \iff y \in \tilde{X}_2$.

Assume that $x > x_1$. Then

$$\tilde{g}(x) = \tilde{g}(x_1) + \int_{x_1}^{x} g'(u)du < \tilde{g}(x_1) + \int_{x_1}^{x} f'(u)du = f(x). \tag{9}$$

(9) and $f(x) \leq f(x_1)$ (by definition of x_1) imply that $\tilde{g}(x) < f(x_1)$. However, by (8), $\tilde{g}(x_1) = f(x_1)$. This implies that $x \notin \tilde{X}_2$. Therefore, $x \notin X_2$. □

COROLLARY 2. $x_1 \in [x_2, b]$.

Appendix B: Notation

Symbol	Meaning
p	defect rate
v	revenue from perfect unit for buyer
T	transfer price
ω	value loss of (bad) quality to buyer
c_0	cost of perfect unit for seller
γ	cost of quality for seller
δ	cost of fixing quality
D	demand
c	per unit audit cost
C	audit cost
$F(.), f(.)$	cdf and pdf of audit cost cut-off
P, P^0, P^*	the (CP) problem, solution to FOC, and set of optimal solutions
p^*	element of P^*
π	profit fucntion
X	random variable
α, β	slope of Crime and Punishment lines
A, B	coefficients of Crime and Punishment lines
Δ	$\frac{A-B}{\alpha+\beta}$
η	$\min_{[a,b]}(\Delta+p)f(p)$
$[a,b]$	support of interval of audit cost cut-off
ϵ	noise
λ	slope of linear transformation
ρ	transformation constant
$[\underline{c}, \overline{c}]$	support
θ	spread
μ	mean
r	$\frac{\alpha}{\alpha+\beta}$
p_{nv}	Newsvendor quality
p_{unv}	Newsvendor quality for uniform distribution

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Fig. 4.1: Suppliers profit as a function of his ordering quantity in the Newsvendor problem

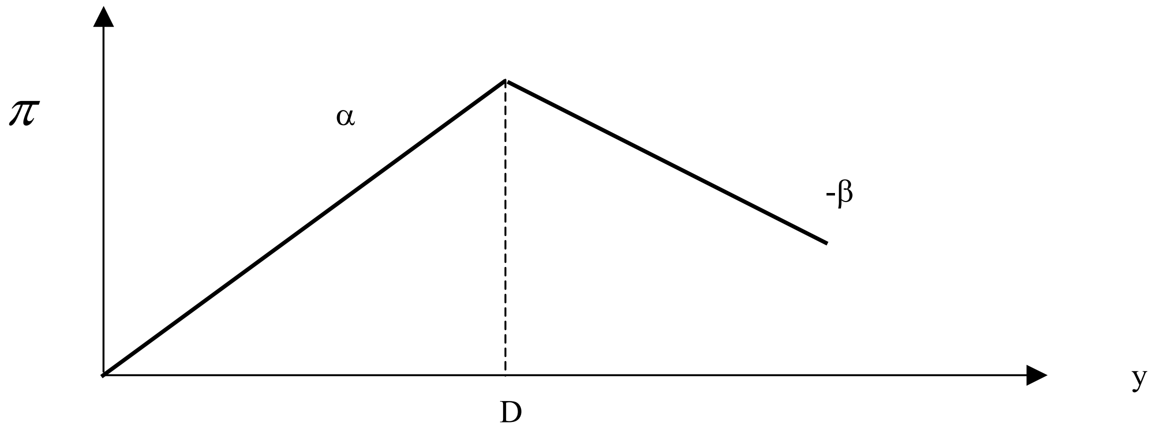


Fig. 4.2: Adam's profit as a function of his defect rate in the (CP) problem

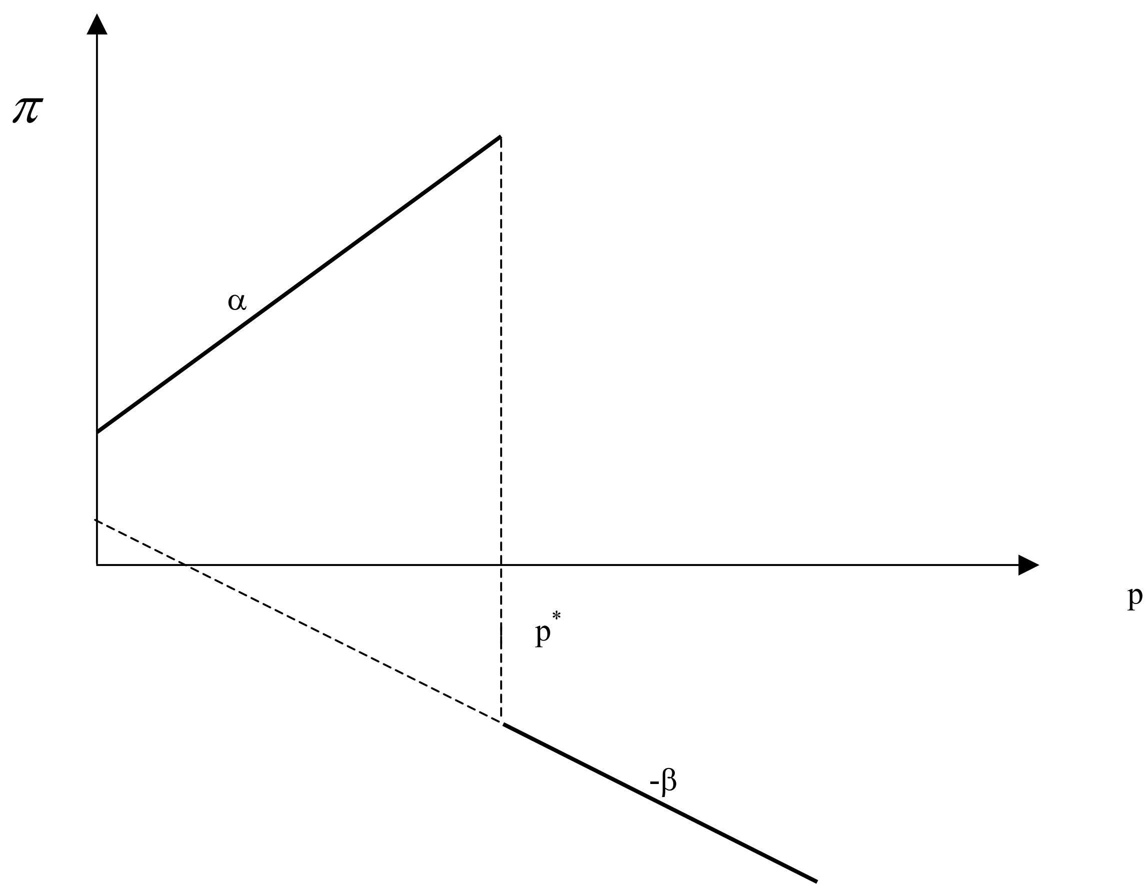


Fig 4.3: Variation of defect rate with the mean, μ of the cut-off distribution in the case of uniform distribution. The spread of the distribution, *θ*, is held constant.

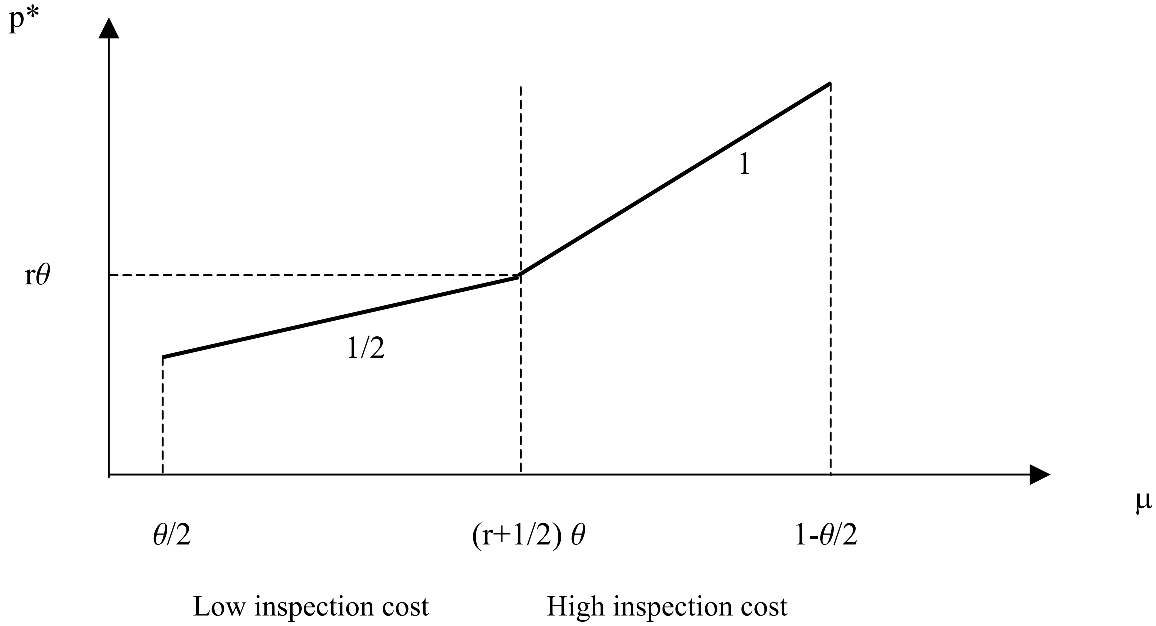


Fig 4.4: Variation of defect rate with the spread of the cut-off distribution in the case of uniform distribution. The mean of the distribution, μ, is held constant.

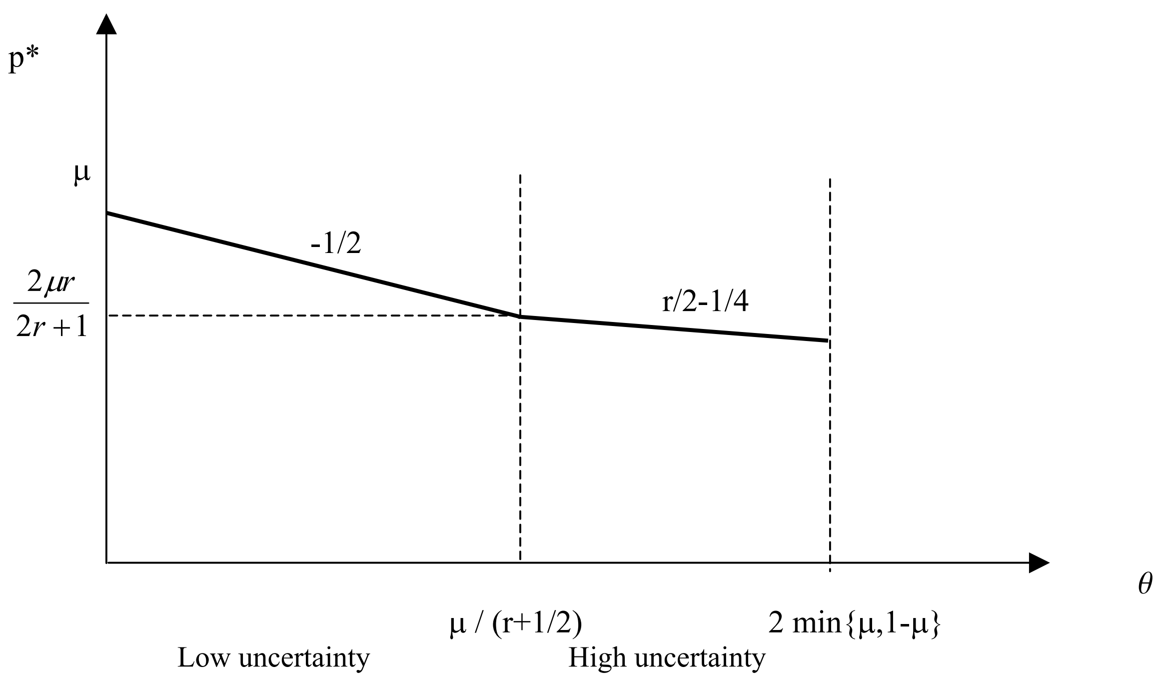


Fig 5.1: Buyers surplus as a function of Transfer price, T

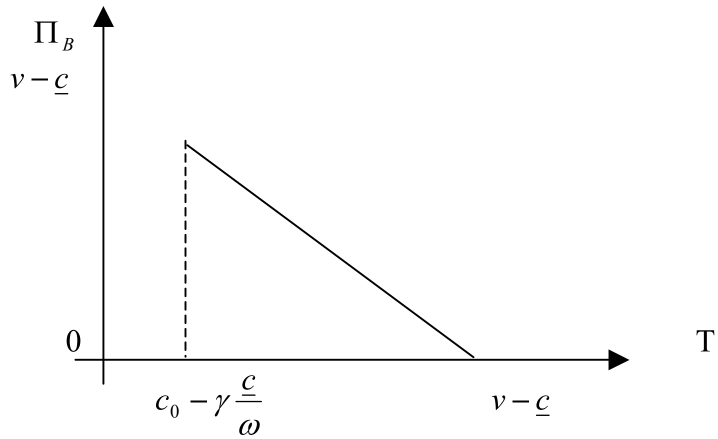


Fig 5.2: Sellers surplus as a function of Transfer price, T

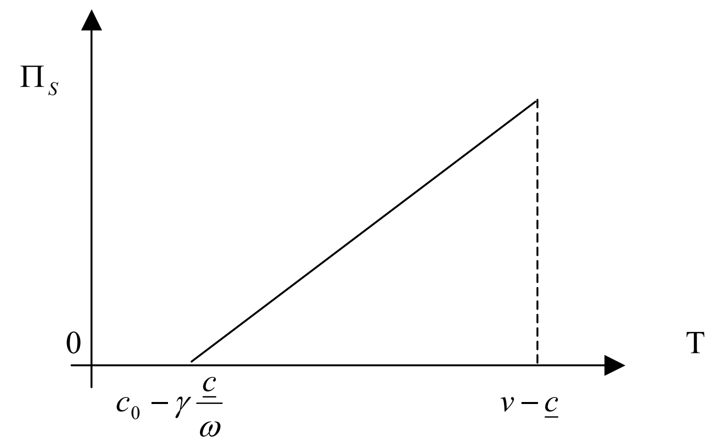
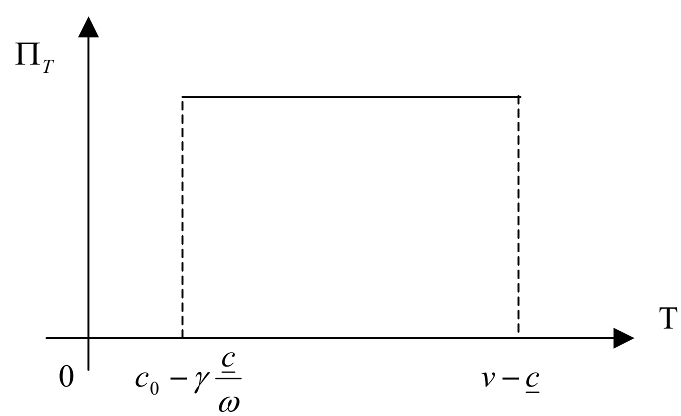
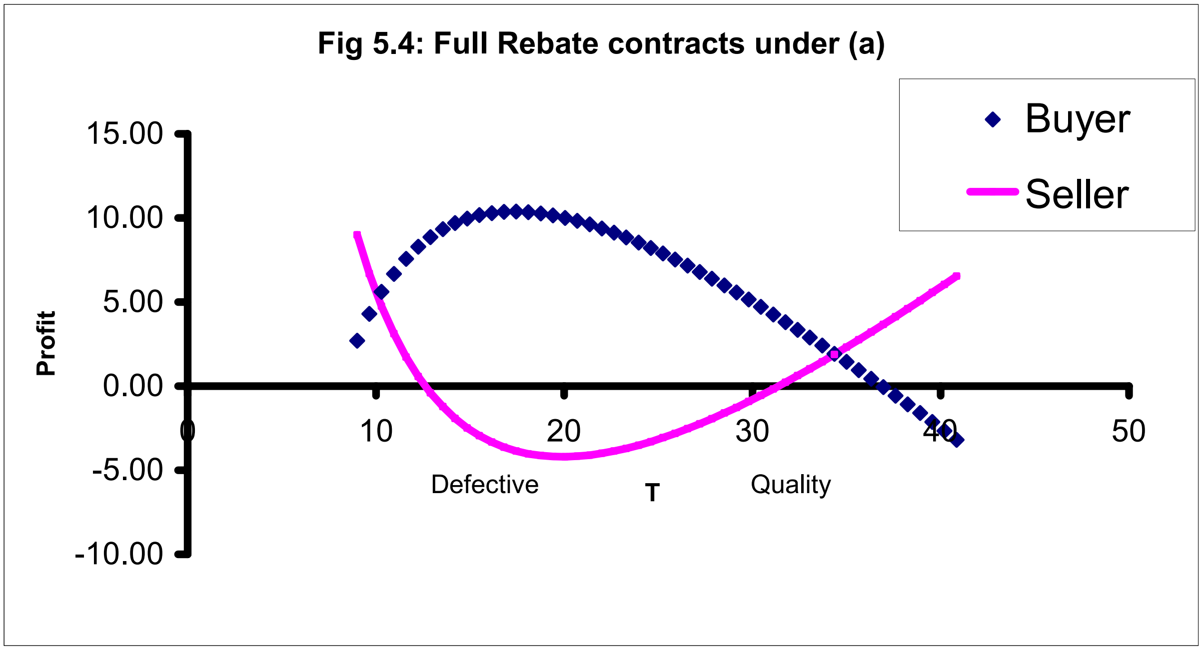
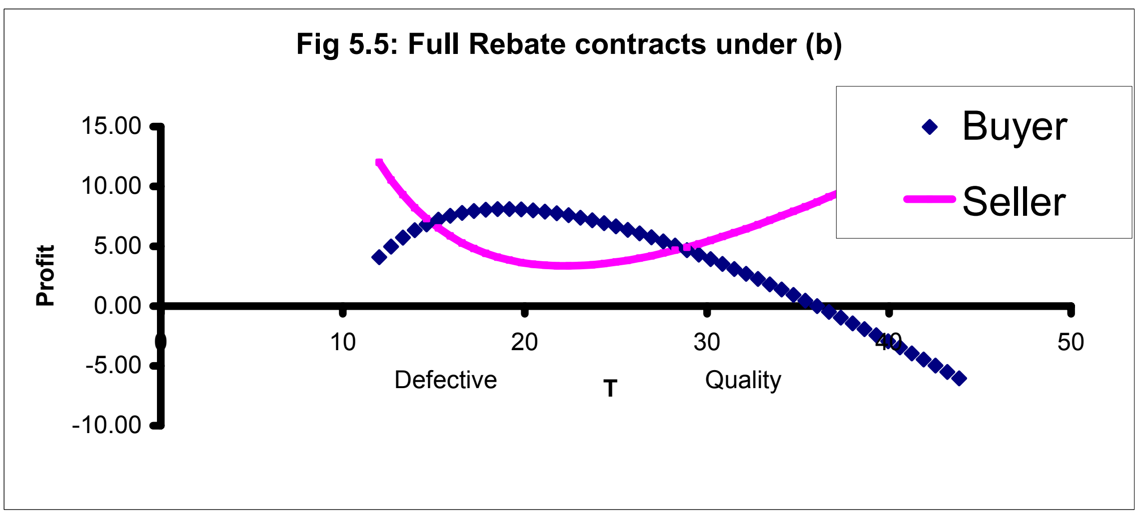


Fig 5.3: Total surplus as a function of Transfer price, T

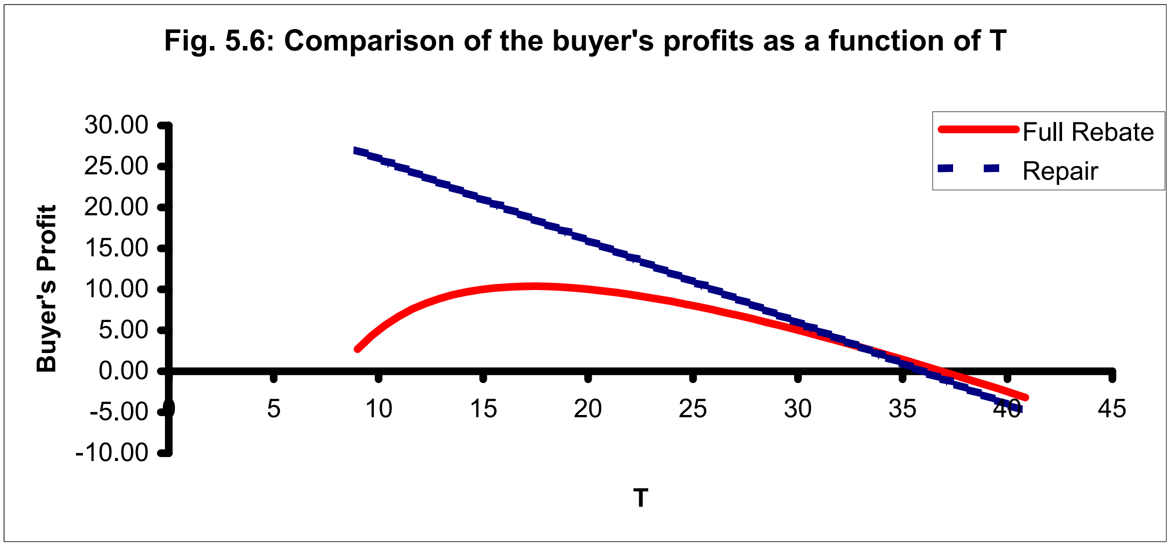




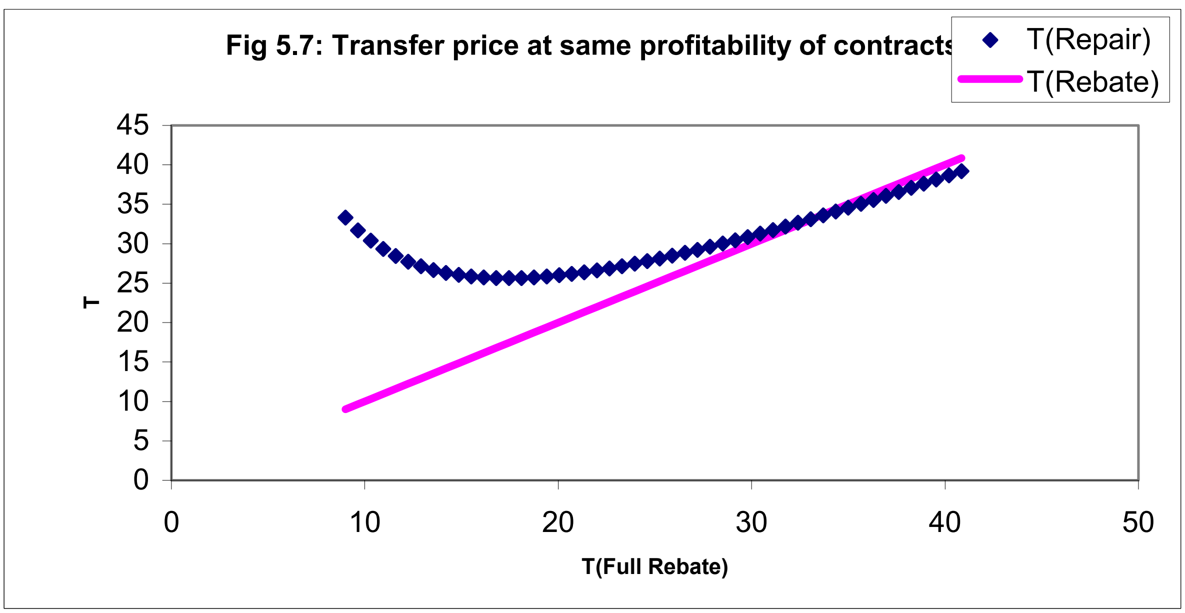
The parameter values for this graph are $v = 45, \varpi = 33.3, \underline{c} = 9, c_0 = \gamma = 44$.



The parameter values for this graph are $v = 46, \varpi = 29.9, \underline{c} = 12, c_0 = \gamma = 41$.



The parameter values for this graph are $v = 45, \varpi = 33.3, \underline{c} = 9$.



The parameter values for this graph are $v = 45, \varpi = 33.3, \underline{c} = 9$.