UNIVERSITI PUTRA MALAYSIA

ONSET OF CONVECTION IN POROUS MEDIA INDUCED BY TRANSIENT HEAT CONDUCTION

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ONSET OF CONVECTION IN POROUS MEDIA INDUCED BY TRANSIENT HEAT CONDUCTION

By

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		Units in S.I System
$\tilde{a}_c$	Dimensionless wave number	
a	Wave number	(m^{-1})
b	Gap between the vertical planes for Hele-Shaw cell	(m)
d	Beads diameter	(m)
H	Total depth of fluid layer	(m)
g	Acceleration due to gravity	(m^2/s)
h	Heat transfer coefficient	$(W/m^2\ °C)$
h_s	Heat transfer coefficient at surface	$(W/m^2\ °C)$
h'	Enthapi,	(J)
K_e	Permeability	(m^2)
K_l	Permeability	(m/s)
k	Thermal conductivity of fluid	(W/m °C)
k_m	Thermal conductivity of porous media mixture, $k_m = \phi k_f + (1-\phi)k_s$	(W/m.°C)
P	Pressure	(Pa)
q°	Constant heat flux	(W/m^2)
q	Heat flux	(W/m^2)
q_s	Heat flux at surface	(W/m^2)
t_c	Critical time for onset of convection	(s)

T	Temperature	(°C)
T_o	Initial water temperature	(°C)
T_s	Surface temperature at time t	(°C)
ΔT	Temperature difference between top and bottom surface	(°C)
ΔT_s	Temperature difference between initial temperature of the porous media and bottom surface temperature	(°C)
Nu	Nusselt number	
Ra	Rayleigh number	
Y	Gap between the the vertical walls and the thickness of the walls for Hele-Shaw cell	(m)
z	Penetration depth	(m)

Greek Symbols

β	Temperature gradient	(°C/m)
α	Volumetric coefficient of thermal expansion	(K^{-1})
δ	Thickness of effective thermal layer	(m)
κ	Thermal diffusivity	(m^2/s)
κ^*	*Modified thermal diffusivity*, $\kappa^* = km/(\rho C_p)_f$	(m^2/s)
κ_m	Thermal diffusivity of porous mixture, $\kappa_m = k_m/\{\phi(\rho Cp) + (1-\phi)(\rho C_p)_s\}$	(m^2/s)
λ	Wavelength	(m)
ν	Kinematic viscosity	(m^2/s)
μ	Viscosity	(Pa.s)

ρ Density (kg/m^3)

γ Ratio of thermal diffusivity of water to that of other substance.

ϕ Porosity

Γ Aspect ratio

Abbreviation

CFD Computational Fluid Dynamic

CHF constant heat flux boundary condition

FST fixed surface temperature boundary condition

Subscripts

c critical

o initial condition

max maximum

f fluid

s solid

m porous media mixture

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ONSET OF CONVECTION IN POROUS MEDIA INDUCED BY TRANSIENT HEAT CONDUCTION

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In this study a computational fluid dynamics (CFD) package - FLUENT/UNS was adopted to simulate the occurrence of convection in an isotropic porous media. The porous layer was homogenous and bounded by two horizontal rigid surfaces. 2-D simulation for steady state and time-dependent were conducted for the bottom surface with two boundary conditions: i) Fixed Surface Temperature - FST, ii) Constant Heat Flux – CHF. The top surface was maintained at FST boundary condition and the vertical wall was adiabatic.

The 2-D steady state simulations were carried out to investigate the occurrence of convection as predicted by the linear theory. The 2-D time-dependent were conducted to investigate the possibility of adopting Tan and Thorp's transient Rayleigh number theory in deep layer of porous media saturated with water.

The CFD was successful in modeling the onset of convection in saturated porous media. The range of maximum velocity at the onset of convection and the finger shape of the thermal plume were in agreement with the literatures (Horton and Roger 1949, Elder 1968). The maximum Nusselt number based on κ_m for the FST and CHF boundary condition were in the range between 3 – 4, depending on the rate of heat transfer. The steady state and time-dependent simulation results showed no significant difference in the Rayleigh number as predicted by Lapwood (1948) $Ra_c = 39.5$, Ribando and Torrance (1976) $Ra_c = 27.1$ for the FST and CHF boundary condition. The average Rayleigh numbers based on κ_m for the steady state simulation were respectively 32.02 and 32.71 for the FST and CHF boundary conditions. The average transient Rayleigh numbers for the time-dependent simulations were respective 30.90 and 30.04 for FST and CHF boundary conditions respectively.

The deviation of the Rayleigh number may be due to the complexity of the heat transfer in porous media as wide difference of thermal diffusivity of the solid and liquid that are existing in the saturated porous media. Beside this, large temperature difference ΔT_s or heat flux q°, imposed on the porous media to induce the convection was against the assumption of perturbation theory in which allows only a small disturbance or change in density of the fluid and constant fluid properties.

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PERMULAAN PEROLAKAN DALAM MEDIA POROS YANG DIARUH OLEH KONDUKSI HABA

Oleh

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Dalam kajian ini, satu pakaj Pengiraan Dinamik Bendalir (CFD) – FLUENT/UNS telah diguna untuk menyimulasi kejadian perolakan di dalam satu media poros isotropik Media poros ini adalah sekata, dibatasi oleh dinding tegak yang adiabatik dan dua sempadan horizontal yang tegar. Simulasi dalam keadaan mantap dan keadaan fana telah dijalankan untuk sempadan horizontal untuk keadaan i) Permukaan suhu tetap (FST), ii) Fluks haba tetap (CHF). Sempadan datar bahagian atas dikekalkan pada keadaan FST.

Simulasi 2-D dengan keadaan keseimbangan dijalankan untuk mengkaji kejadian perolakan yang diramalkan oleh teori linear. Simulasi 2-D dengan keadaan fana dijalankan untuk mengkaji kesesuaian mengguna teori konduksi yang dikemukakan oleh Tan and Thorpe (1996) dalam media poros yang ditepu dengan air.

CFD adalah berkesan dalam permodelan permulaan perolakan haba dalam media poros yang ditepu dengan air. Julat bagi halaju maximum pada permulaan perolakan haba dan plum haba yang berbentuk jari adalah sependapat dengan kajian yang lain (Horton and Roger 1949, Elder 1968) Nombor Nusselt maximum yang dicapai berdasarkan κ_m untuk sempadan FST and CHF adalah diantara julat 3 – 4, bergantung kepada kadar pemindahan hada. Keputusan simulasi 2-D dalam keadaan mantap dan fana tidak menujukkan perbezaan yang besar untuk nombor Raylegh sebagaimana yang diramal oleh teori iatitu untuk keadaan sempadan FST $Ra_c = 4\pi^2$ (Lapwood, 1948) dan keadaan sempadan CHF $Ra_c = 27.1$ (Ribando dan Torrance,1978). Nombor Rayleigh untuk simulasi keadaan mantap berdasarkan κ_m adalah masing-masing 32.02 dan 32.71 untuk keadaan sempadan FST dan CHF. Manakala, nombor Rayleigh untuk keadaan fana adaalah masing-masing 30.90 dan 30.04 bagi keadaan sempadan FST dan CHF.

Sisihan nombor Rayleigh mungkin disebabkan oleh pemindahan haba yang kompleks dalam media poros. Pemindahan haba dalam media poros adalah lebih kompleks daripada cecair disebabkan kewujudan kemeresapan haba yang berbeza bagi matriks pepejal dan cecair. Selain daripada itu, perbezaan suhu dan fluks haba yang tinggi dikenakan ke atas media poros untuk mengaruh perolakan adalah bertentangan dengan "perturbation theory" di mana hanya gangguan atau perubahan ketumpatan yang kecil dibenarkan dalam bendalir dan semua sifat-sifat bendalir yang lain dianggap kekal

CHAPTER I

INTRODUCTION

When a fluid is heated from below or cooled from the top, the density gradient decreases in the direction of the gravity field. If the temperature gradient exceeds a critical value, buoyancy forces will overcome the retarding influence of the viscous force, which will lead to the onset of convection of the fluid. Lord Rayleigh (1916) developed the famous thermal instability criterion for the onset of buoyancy convection based on a linear temperature gradient in a fluid layer. In view that natural convection is induced by time-dependent and non-linear temperature profile, a great number of analytical and experimental works had been done on onset of convection during transient heat conduction in the past (Morton 1957, Lick 1965), Currie 1967). However none of them successfully developed an appropriate transient Rayleigh number for continuous fluid. However recently Tan and Thorpe (1996) had developed a new transient Rayleigh number for the deep fluid for different boundary conditions based on the Biot number.

The knowledge of natural convection in a fluid saturated porous media is of considerable interest because of its importance in geophysical and engineering applications. These included the modeling of geometrical field, high performance insulation for building and cold storage, solar power collection. Beginning with the classic studies of Horton and Roger (1945) and Lapwood (1948), who investigated the critical condition (Rayleigh number) at which the heat transport process changes from purely conduction to convection based on the linear stability analysis, a number of experiments and analytical work were conducted on the of convection in porous media. Among them was Elder (1967), who studied the convective flow for conditions moderated above the critical Rayleigh number. The most recent experimental and numerical studies were concentrated on the structure of the convective flow and the heat transport in porous media by Lister (1990), Straus and Schubert (1988) and Steen and Aidun (1988). Not much of the experimental works had been done for the onset of convection in porous media during the transient heat conduction except Elder (1968), who had performed unsteady-state laboratory experiments on convective flow in Hele-Shaw cell and numerical experiments on the porous media. However he failed to develop a comprehensive concept for the transient Rayleigh number.

In this study, a transient Rayleigh number based on Tan and Thorp (1996) theory for deep fluid was developed for two boundary conditions (FST and CHF) for a semi-infinite porous media saturated with fluid. A computation fluid dynamic (CFD) package FLUENT was used to simulate the onset of convection in a porous media. The objectives of the study were:

i) To simulate the occurrence of convection in porous media as predicted by linear theory for FST and CHF boundary condition

ii) To simulate the time-dependent convection in porous media for FST and CHF boundary condition.

iii) To investigate the possibility of adopting Tan and Thorpe's theory of the transient Rayleigh number in deep layer of porous media saturated with fluid.

CHAPTER II

LITERATURE REVIEW

Steady-State Convection in a Horizontal Layer of Fluid

Rayleigh (1916) did the first analytical work on thermal instability of finite thickness fluid layer. He studied the idealized case of free conducting boundaries with an adverse linear temperature gradient typical of steady-state heat transport. He used a perturbation expansion of the linearized Boussinesq equations, assuming a convection pattern, which varies, sinusoidal in the horizontal direction. In the Boussinesq approximation, the temperature dependence of the fluid properties was neglected, except for thermally induced density difference when they induce buoyant force. He showed that there was a critical Rayleigh number Ra_c, a non-dimensional number now named after him, which divides the conduction and the convection regime. For this boundary condition the critical Rayleigh number was defined as:

$$Ra_c = \frac{g\alpha H^3 \Delta T}{\nu\kappa} = 657$$

Extending to the work of Rayleigh (1916), Jeffreys (1928) predicted that for a layer of fluid confined between two conducting rigid boundary with different constant temperature condition, $Ra_c = 1708$, whereas Low (1929) predicted that if one of the boundary was rigid and the other one is free, $Ra_c = 1108$.

Onset of Convection in a Horizontal Layer of Fluid
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However, in nature, the onset of convection in deep fluid is induced by transient heat conduction with nonlinear temperature profile. This means that the conventional steady-state linear stability analysis is not valid except for very extremely slow cooling or heating rate with finite fluid layer thickness in an artificial laboratory experiment.

Owing to the above fact, a considerable amount of analytical and experimental investigations had been done in the past to develope the critical transient Rayleigh number. The early studies of the onset of convection during transient heat conduction had been conducted by Morton (1957); Lick (1965); Currie (1967). They had adopted the quasi-steady state model for linear stability analysis, whereas Foster (1965) used the velocity amplification theory in his study to investigate the onset of convection. However none of them were successful in determining the critical Rayleigh number to predict the transient heat convection as pointed out by Tan and Thorpe (1996).

Tan and Thorp (1996) had derived a new transient Rayleigh number, which incorporated the mode and rate of heat transport. The mode of the heat transport is characterized by a thermal boundary condition that determines the Biot number and the corresponding critical Rayleigh number. The new transient Biot number (Tan and Thorpe, 1994 and 1996) is based on an effective thermal depth and depends on the mode and rate of heat transfer.

They adopted Pearson's (1958) definition of Biot number, which is based on the rate of surface heat flux, where

$$Bi = \frac{dq/dT_s}{(dq/dT)_l}$$

When the surface is subjected to a constant heat flux (CHF), i.e $dq/dT_s = 0$, Biot number = 0; Biot number = $\propto$ when the surface temperature is fixed (FST), i.e $dq/dT_s = \propto$. By determining the transient Biot number, the corresponding critical Rayleigh number can be found.

The new transient Rayleigh number is a function of the penetration depth z and the local temperature gradient $\partial T/\partial z$, and is defined as follow:

$$Ra = \frac{g\alpha z^4}{\nu\kappa}\left(\frac{\partial T}{\partial z}\right)_t$$

It is derived from the convection Rayleigh number $Ra = g\alpha z^3 \Delta T/(\kappa\nu)$, by substituting the corresponding $(\partial T/\partial z)_z$ for the Fixed Surface (FST) boundary condition, Constant Heat Flux (CHF) boundary condition and Linear Temperature (LTR) boundary condition as cited by Carslaw and Jaeger (1973). The maximum vertical position z_{max} of the critical transient local Rayleigh number is obtained by differentiating the above equation. The critical Rayleigh number for the FST, CHF and LTR boundary conditions are obtained by substitute the corresponding z_{max} for the FST, CHF and LTR model into $Ra = g\alpha z^3 \Delta T/(\kappa\nu)$. The critical Rayleigh number for the FST, CHF and LTR boundary conditions are obtained as shown in Table (1).

For a fluid layer bounded by a top free surface and bottom solid surface, where the bottom solid surface is at fixed temperature and the top surface of the fluid is cooled by a step change instantaneously to a fixed temperature T_s. The loss of mass by evaporation is negligible, the Biot number is infinity and the corresponding critical Ra_c is 1100 (Low 1929) as shown in Table (2). By substituting the Ra_c = 1100 into the transient Rayleigh number, the critical time t_c, critical wavelength λ_c, critical thermal depth z_{max} for the onset of convection can be obtained. The critical Rayleigh number and Biot number for various boundary conditions are tabulated in Table (2).

Table 1: Equations of maximum transient Ra_c for various boundary conditions. (Tan and Thorpe, 1996)

Fixed surface temperature:	$Ra_{max} = \frac{4.89 g\alpha\sqrt{\kappa t^3}\Delta T_s}{\nu}$
Linear temperature rate:	$Ra_{max} = \frac{1.73 g\alpha B\sqrt{\kappa t^3}}{\nu} = \frac{1.73 g\alpha\sqrt{\kappa t^3}\Delta T_s}{\nu}$
Constant heat flux:	$Ra_{max} = \frac{3.02 g\alpha q^o \kappa t^2}{\nu k} = \frac{2.676 g\alpha\sqrt{\kappa t^3}\Delta T_s}{\nu}$